Q: Prove the impossibility result for leader election for the anonymous ring.

A: The imposiibility result states that there is no anonymous leader election algorithm for ring systems :

The showed result applies for non-uniform algorithms and synchronous rings:
In synchronous systems, messages are delivered in rounds, after which a computation step is performed.
In the case of anonymous ring

1 )all processors always begin in the same state.
2)They execute the same program at the computation step , because they have identical state machine.
3)Same messages are sent by all the processors in every round.

For the purpose of proof, consider a synchronous ring with number of processors n > 1, and assume by contradiction that A is an anonymous
algorithm for electing a leader processor in the ring.

Lemma: For every round "k" of the execution of A on R , the states of all the processors at the end of each round k are the same.

Proof : Applying induction

Base case : K = 0 . All processors will begin in the same start state.

Induction case : If the lemma is true, then it holds for case k-1 as well.
Processors send messages Mr and Ml to the left and right neighbours in round k-1 , in which they are in the same state.

In the next round = (k-1 )+1 = k : all processors will recieve the messages to their left and right respectively.

Therefore, all processors recieve exactly the same messages in round k , because they execute the same program and are in the same state at
end of k.

If this is true then, any processor announces itself as leader at the end of k, then all other processors will also announce themselves
as leaders. A leader election algorithm must have only one leader, but here all processors can claim to be leaders.

This contradicts the assumption that A is a leader election algorithm.