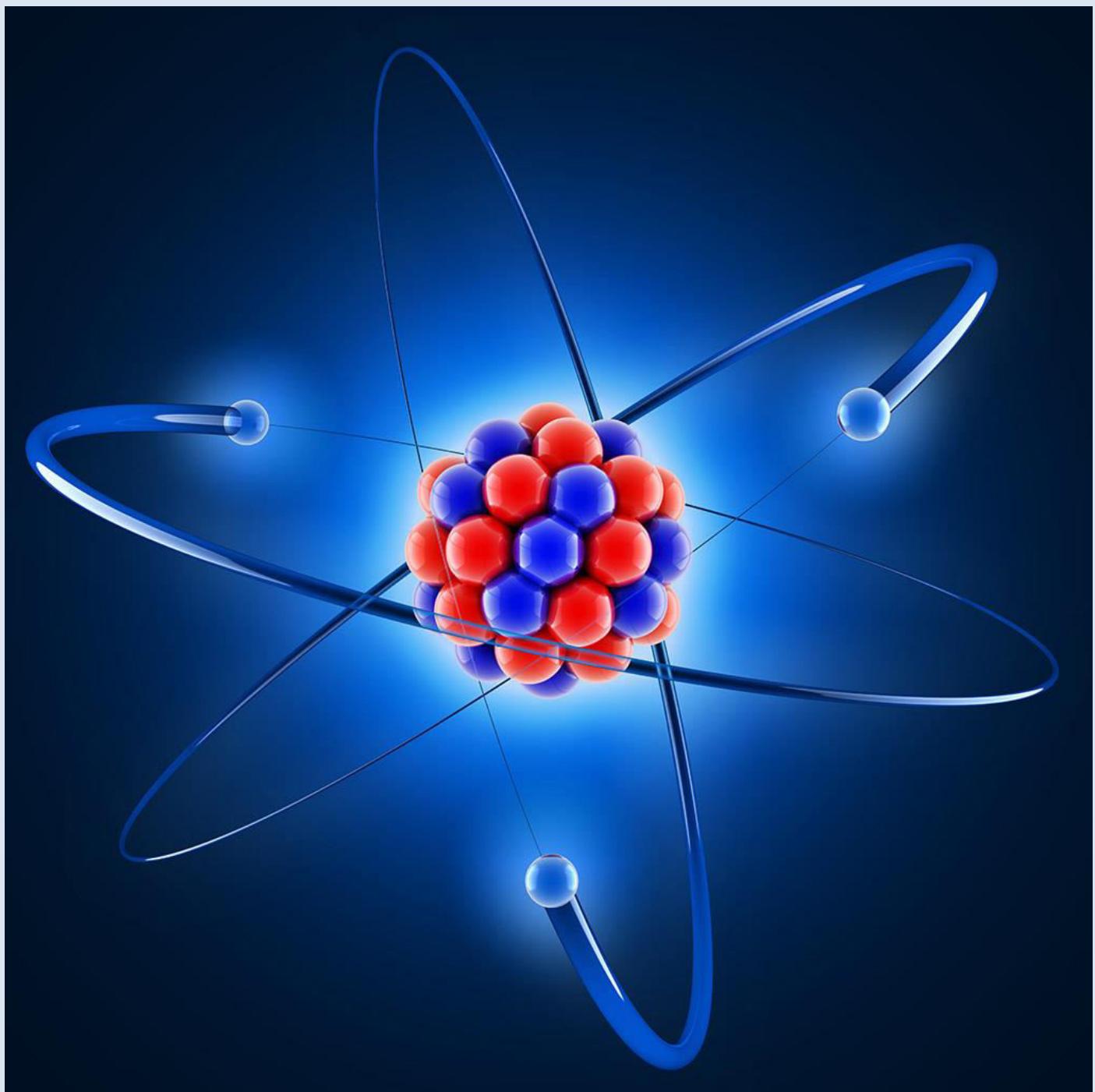


**Chapter–12, 13**

# **Atoms & Nuclei**



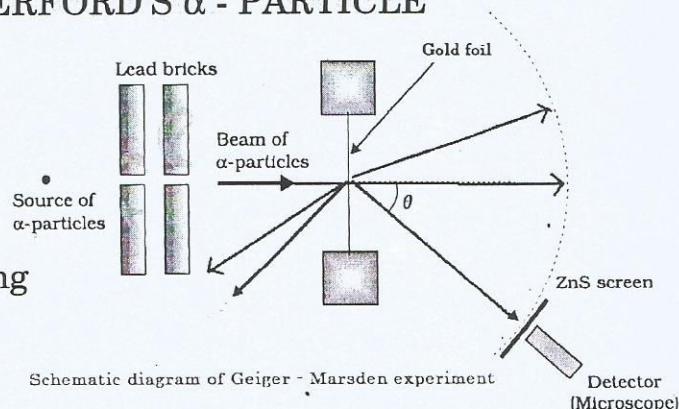
**CBSE CLASS XII NOTES**

**Dr. SIMIL RAHMAN**

## 12. ATOMS

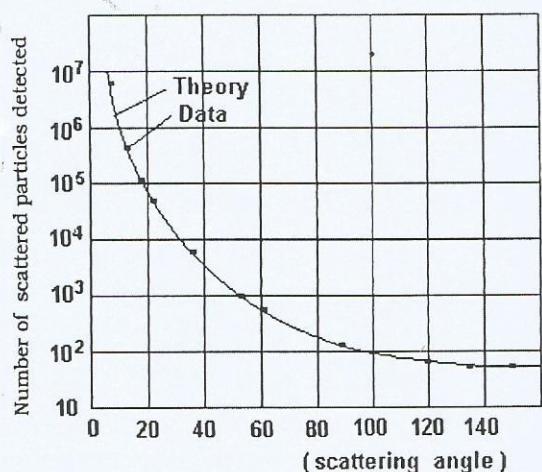
### GEIGER – MARSDEN EXPERIMENT (RUTHERFORD'S $\alpha$ - PARTICLE SCATTERING EXPERIMENT)

A collimated beam of  $\alpha$  - particles ( helium nucleus) is allowed to fall on a thin gold foil.  $\alpha$  - particles got scattered in all directions and are detected using a rotatable detector consisting a zinc sulphide screen (which scintillates when  $\alpha$  - particles fall on it) and a microscope.



**Observations :** The graph between the number of scattered  $\alpha$  - particles detected vs scattering angle  $\theta$  is shown.

- ✓ Almost all  $\alpha$ -particles went through the gold foil with very slight deflections
- ✓ A small number of  $\alpha$ -particles were scattered by more than  $90^\circ$
- ✓ A very small number of alpha particles rebounded
- ✓ Trajectory (shape of the path) is determined by impact parameter



### Conclusions (Explanations) :

- Most of the atom is empty.
- The deviation of some alpha particles from their original path was due to positive charges within the foil.
- A small number of alpha particles had rebounded or scattered through large angles because they collided with massive and positively charged centre of the atom called Nucleus.

### RUTHERFORD'S NUCLEAR OR PLANETARY MODEL OF ATOM

- ✓ An atom has spherical shape with a positive charge at the center of the atom with almost all the mass of the atom concentrated there – nuclei
- ✓ Electrons – negatively charged- revolves around nucleus in circular orbits.
- ✓ The electrostatic force of attraction between the nucleus and electrons provides the necessary centripetal force for the rotation of electrons.

## DISTANCE OF CLOSEST APPROACH

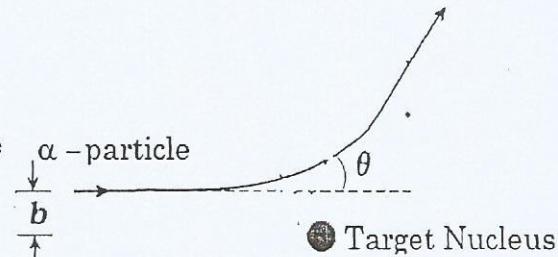
An  $\alpha$  - particle travelling directly towards a nucleus gets decelerated and finally at a certain distance  $r_0$  from the nucleus its velocity becomes zero and gets rebounded.

This distance  $r_0$  is called distance of closest approach. When the  $\alpha$  - particle is far away it has only kinetic energy. At  $r_0$  its entire kinetic energy gets converted into potential energy of  $\alpha$ -particle – nucleus system.

$$\frac{1}{2} m u^2 = \frac{1}{4\pi\epsilon_0} (Z e / r_0) \quad r_0 = \frac{1}{4\pi\epsilon_0} \frac{2 Z e^2}{\frac{1}{2} m u^2}$$

## IMPACT PARAMETER

Impact parameter ( $b$ ) is the perpendicular distance between the initial velocity vector of  $\alpha$  - particle and the centre of the nucleus. The trajectory of  $\alpha$  - particle is determined by impact parameter. If impact parameter is less angle of scattering will be more–more deviation for the  $\alpha$ -particle.



## DRAWBACKS OF RUTHERFORD'S ATOM MODEL

Since *electron* travels in a circular orbit, it is constantly *accelerated*. Accelerated charges radiate electromagnetic waves, which carries away energy. Thus the potential energy of the electron is reduced and it moves closer to the nucleus and finally falls into the nucleus atom is unstable as per Rutherford's model.

According to Rutherford's model, electron can exist anywhere outside the nucleus. So the spectrum of the emitted EM radiation would be continuous. But sharp spectral lines are observed, not a continuum.

## BOHR ATOM MODEL

Bohr's atom model retains all the features of Rutherford's atom model but added some additional postulates in order to overcome the limitations of Rutherford's model.

### **Postulates :**

- (i) Electron moves in circular orbits around the nucleus under the influence of the attractive electric field.

(ii) Not all orbits are allowed. The electron can only have an orbit for which the angular momentum of the electron,  $L$ , is an integer multiple of  $h/2\pi$ . ( $h = 6.63 \times 10^{-34} \text{ J s}$  is the Planck's constant).

$$L = m v r = n h / 2\pi \quad \text{Bohr's quantization condition.}$$

where  $n = 1, 2, 3..$  is the principal quantum number

An electron *does not* emit EM radiation when it is in one of these states (orbits).

Hence these orbits are called stationary states (orbits) or non-radiating orbits

(iii) When electron jumps from higher energy state to lower energy state it emits energy. When electron goes to higher energy state from lower energy state it absorbs energy. The energy of emitted EM radiation

$$hv = E_i - E_f \quad \text{Bohr's frequency condition.}$$

where  $v$  = frequency of the radiation

$E_i$  and  $E_f$  = energy associated with initial and final states respectively

## BOHR'S THEORY APPLIED TO HYDROGEN ATOM

In a hydrogen atom an electron of charge  $-e$  revolves around the nucleus having a charge  $+e$  in a circular orbit. The electrostatic force of attraction between them provides the necessary centripetal force.

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \quad (1)$$

### Radius of Orbits

$$\text{From (1), } v^2 = \frac{1}{4\pi\epsilon_0} \frac{e^2}{mr} \quad (2)$$

But by Bohr's quantization condition angular momentum  $mvr = nh/2\pi$

$$v = \frac{nh}{2\pi mr} \quad (3)$$

Substituting (3) into (2)

$$\frac{n^2 h^2}{4\pi^2 m^2 r^2} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{mr} \quad \rightarrow$$

$$r = \frac{n^2 h^2 \epsilon_0}{\pi m e^2} \quad (4)$$

Substituting the values of the constants,

$$r = n^2 a_0$$

Where  $a_0 = \frac{h^2 \epsilon_0}{\pi m e^2} = 0.51 \text{ A}^\circ$  (Bohr radius)

Bohr radius is the radius of the first orbit of hydrogen atom. The above equation shows that radius of orbits is quantized i.e. can take only certain specific values.

### Orbital speed of electron

We have,  $m v r = n h / 2 \pi \rightarrow v = n h / 2 \pi m r$

$$v = \frac{n h}{2 \pi m} \cdot \frac{1}{\frac{n^2 h^2 \epsilon_0}{\pi m e^2}} = \frac{n h}{2 \pi m} \frac{\pi m e^2}{n^2 h^2 \epsilon_0} \quad [\text{substituting for } 'r' \text{ from (4)}]$$

Orbital speed  $v = \frac{e^2}{2 n h \epsilon_0}$  (5)

Orbital speed of electron decreases with increase in radius.

### Energy Levels

Total energy of an electron in  $n^{\text{th}}$  orbit = kinetic energy + potential energy

$$\text{Potential energy } U = - \frac{1}{4 \pi \epsilon_0} \frac{e^2}{r} \quad (6)$$

$$\text{Kinetic energy } K = \frac{1}{2} m v^2$$

$$\text{From (1), } m v^2 = \frac{1}{4 \pi \epsilon_0} \frac{e^2}{r} \quad \text{Substituting this, in the equation for kinetic energy,}$$

$$K = \frac{1}{2} m v^2 = \frac{1}{2} \frac{1}{4 \pi \epsilon_0} \frac{e^2}{r} \quad K = \frac{1}{8 \pi \epsilon_0} \frac{e^2}{r} \quad (7)$$

Comparing (6) and (7), it can be seen that  $U = -2K$

$$\text{Total energy } E = K + U = K - 2K = -K$$

$$E_n = - \frac{1}{8 \pi \epsilon_0} \frac{e^2}{r} \rightarrow E_n = - \frac{e^2}{8 \pi \epsilon_0} \frac{\pi e^2 m}{n^2 h^2 \epsilon_0} \quad \text{substituting for } 'r' \text{ using (4)}$$

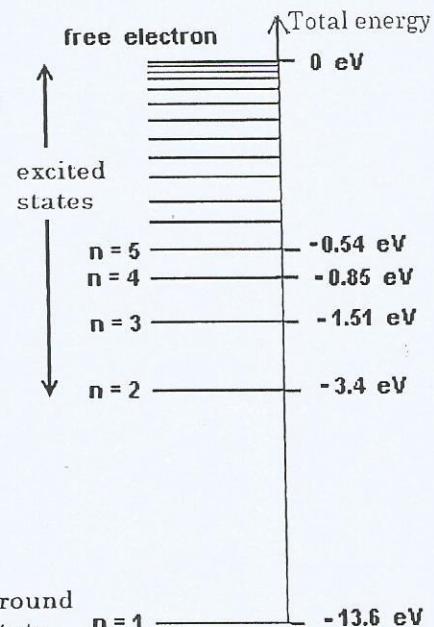
$$E_n = - \frac{m e^4}{8 \epsilon_0^2 n^2 h^2} = - \frac{E_0}{n^2} \quad (8) \quad (\text{Total energy of electron})$$

Where  $E_0 = \frac{m e^4}{8 \epsilon_0^2 h^2} = 13.6 \text{ eV}$  is the energy of the electron in the orbit  $n=1$ .

This is the lowest possible energy of the electron. This energy level is known as **ground state**. The energy levels with  $n=2, 3 \dots$  are known as **excited states** (higher energy states).

Energy increases with increase in 'n' and the energy levels become crowded and almost becomes a continuum at large values of 'n'.

The significance of negative sign in the expression for energy is that the force between electron and nucleus is attractive in nature i.e. electron is bound to the nucleus. Energy must be supplied to free an electron.



## SPECTRUM OF HYDROGEN ATOM

At room temperature, hydrogen gas does not emit light. When heated to high temperatures, hydrogen emits radiation. Distinct spectral lines can be observed. From Bohr's frequency condition, when an electron makes a transition from a higher energy level  $E_i$  to a lower energy level  $E_f$ , energy is emitted in the form of radiation. i.e.  $h\nu = E_i - E_f$

$$E_i = -\frac{E_0}{n_i^2} \quad \text{and} \quad E_f = -\frac{E_0}{n_f^2} \quad \text{where} \quad E_0 = \frac{me^4}{8\varepsilon_0^2 h^2}$$

Therefore, energy of emitted radiation

$$h\nu = -\frac{E_0}{n_i^2} - \left(-\frac{E_0}{n_f^2}\right) = -\frac{E_0}{n_i^2} + \frac{E_0}{n_f^2} = \frac{E_0}{n_f^2} - \frac{E_0}{n_i^2} = E_0 \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Substituting the value of  $E_0$      $h\nu = \frac{me^4}{8\varepsilon_0^2 h^2} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$

Frequency of emitted radiation     $\nu = \frac{me^4}{8\varepsilon_0^2 h^3} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$

But, frequency  $\nu = c/\lambda$  ( $\lambda$  =wavelength). Therefore, the above equation becomes

$$\frac{1}{\lambda} = \frac{me^4}{8\varepsilon_0^2 c h^3} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Wavelength of the emitted radiation

$$\frac{1}{\lambda} = R_H \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

where  $R_H = \frac{me^4}{8\varepsilon_0^2 c h^3}$  Rydberg's constant.

$$(R_H = 1.097 \times 10^7 \text{ m}^{-1})$$

## SPECTRAL SERIES OF HYDROGEN ATOM

The spectra of hydrogen atom include the following spectral series. Each series is due to the transition of electron to a particular energy state from higher states.

The wavelengths of the spectral lines can be calculated using the formula

$$\frac{1}{\lambda} = R_H \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

**Lyman Series :** It is in the ultraviolet region and is due to the transition to the first orbit. Therefore,  $n_f = 1$  and  $n_i = 2, 3, 4, \dots$

**Balmer Series :** It is in the visible region and is due to the transition to the second orbit. Therefore,  $n_f = 2$  and  $n_i = 3, 4, 5, \dots$

**Paschen Series :** It is in the infrared region and is due to the transition to the third orbit. Therefore,  $n_f = 3$  and  $n_i = 4, 5, 6, \dots$

**Brackett Series :** It is in the infrared region and is due to the transition to the fourth orbit. Therefore,  $n_f = 4$  and  $n_i = 5, 6, \dots$

**Pfund series :** It is in the infrared region and is due to the transition to the fifth orbit. Therefore,  $n_f = 5$  and  $n_i = 6, 7, 8, \dots$

## de BROGLIE'S EXPLANATION OF BOHR'S QUANTISATION RULE

de Broglie considered electron as a stationary matter wave of wavelength

$$\lambda = h / m v \quad (m - \text{mass of electron}, v - \text{velocity of electron in the orbit}).$$

The stationary states of the electrons in the atom should be such that there is an integral number of a wavelength around the circumference of the orbit. i.e.

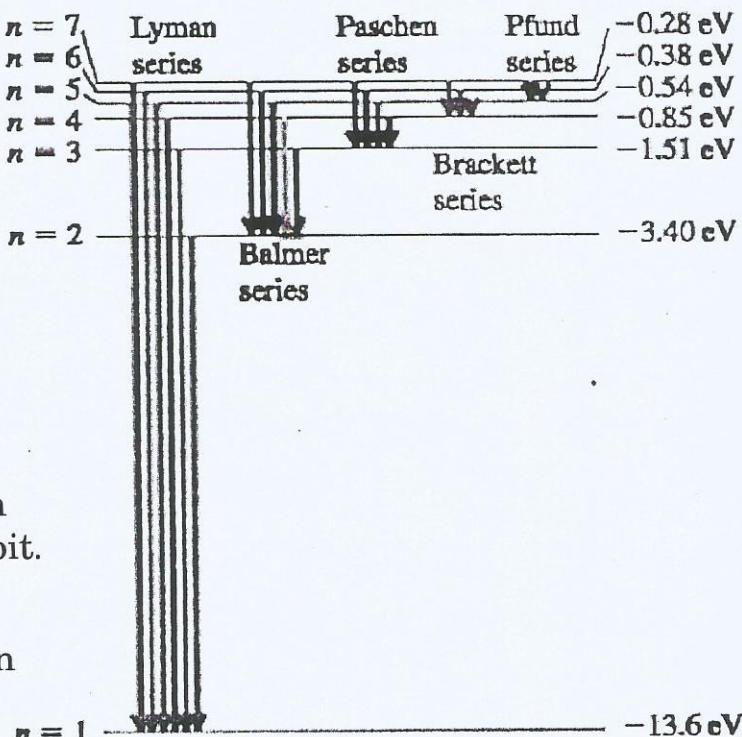
$$2 \pi r = n \lambda \quad (n = 1, 2, 3, \dots)$$

His reasoning was that, unless there are a finite number of complete wavelengths round the orbit, the waves would interfere destructively and atom becomes unstable.

$$2 \pi r = n \lambda = n h / m v \quad (\text{using de Broglie's equation for wavelength})$$

Rearranging,  $m v r = n h / 2 \pi$  which is the Bohr's quantization condition.

\*\*\*\*\*



## **13. NUCLEI**

Nucleus is the positively charged center of an atom. It contains electrically neutral **neutrons** and positively charged **protons**. Protons and neutrons are commonly known as **nucleons**.

### **ATOMIC MASS UNIT**

Atomic mass unit (u) is defined as 1/12<sup>th</sup> of the mass of the carbon (C – 12 ) atom.

$$1\text{u} = \frac{\text{mass of one } {}^{12}\text{C atom}}{12} = \frac{1.992647 \times 10^{-26} \text{ kg}}{12}$$

$$1\text{u} = 1.660539 \times 10^{-27} \text{ kg}$$

$$\text{Mass of proton } m_p = 1.00727 \text{ u}$$

$$\text{Mass of proton } m_n = 1.00866 \text{ u}$$

$$\text{Mass of proton } m_e = 0.00055 \text{ u}$$

Note : 1 u of mass  $\equiv$  931.5 MeV of energy (MeV –million electron volt-unit of energy)

### **NUCLIDE**

A **nuclide** is a specific nucleus characterized by its number of protons Z, its number of neutrons N and its nuclear energy state. Symbolically, it is represented as  ${}^A_Z X$

Where Z – atomic number (number of protons)

A – mass number ( number of protons + number of neutrons ; Z + N )

X – the chemical symbol of the atom

### **TYPES OF NUCLIDES**

Nuclide	Same	Different	Examples
Isotopes	$z$	A and N	Carbon - ${}^6_6 \text{C}^{12}$ , ${}^6_6 \text{C}^{13}$ and ${}^6_6 \text{C}^{14}$ Uranium - ${}_{92}^{233} \text{U}$ , ${}_{92}^{235} \text{U}$ and ${}_{92}^{238} \text{U}$
Isobars	A	$z$ and N	$\text{Pb}^{214}$ and $\text{Bi}^{214}$
Isotones	$N = A - z$	A and Z	${}_{36}^{86} \text{Kr}$ , ${}_{37}^{87} \text{Rb}$
Isomers	$A, Z, N$ ( all )	different radioactive properties	Pair of isomers of ${}_{35}^{80} \text{Br}$

## SIZE AND DENSITY OF THE NUCLEUS

Radius of the nucleus

$$R = R_0 A^{1/3}$$

Where A – mass number of the nucleus  $R_0 = 1.2 \times 10^{-15} \text{ m} = 1.2 \text{ fm}$  (femto metre)

Since nucleus is assumed to be almost spherical in shape,  
volume of the nucleus  $\propto$  (radius)<sup>3</sup> i.e.  $V \propto R^3 \propto A$   
mass of a nucleus  $\propto$  its mass number i.e.  $M \propto A$

Therefore, density (= mass / volume) is independent of mass number 'A' and is almost the same for all nuclei. Nuclear Density  $\approx 2.3 \times 10^{17} \text{ kg m}^{-3}$

## NUCLEAR FORCE

It is the strong attractive force which holds the nucleons together inside a nucleus.

### Characteristics:

- ✓ It is the strongest force.
- ✓ It is independent of charge – the strength of nuclear force is same whether it is between proton-proton or neutron-neutron or proton-neutron.
- ✓ It is a short range force – the range of nuclear force is 2 to 3 fm only.

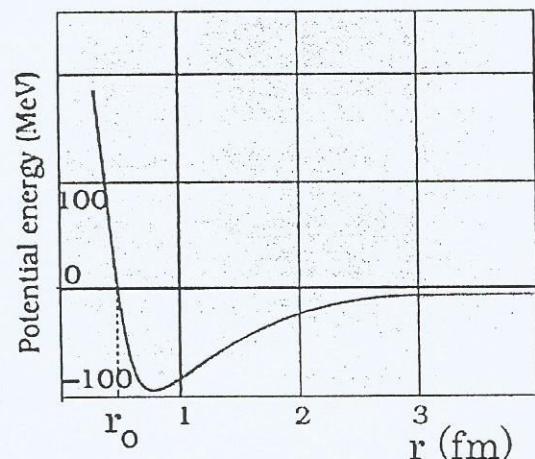
The adjacent graph shows the variation of nuclear potential energy of a pair of nucleons as a function of separation between them.

For separations greater than  $r_0$  potential energy is negative and nuclear force is attractive.

Force of attraction is maximum when separation is around 0.8 fm.

For distances less than  $r_0$  force becomes strongly repulsive as potential energy becomes positive.

- ✓ Nuclear force shows saturation effect. Nucleons interact only with their nearest neighbors. This is the reason why binding energy per nucleon and nuclear density remains constant over a wide range of mass numbers.



## MASS DEFECT

Mass of a stable nucleus is found to be less than the total mass of individual nucleons. The difference between the mass of the nucleus and the total mass of its constituent nucleons is called Mass Defect,  $\Delta M$ .

$$\Delta M = Z m_p + (A - Z) m_n - M$$

$m_p$  – mass of a proton

$m_n$  – mass of a neutron

$Z$  – number of protons

$A$  – mass number     $M$  – mass of the nucleus

## BINDING ENERGY

To hold the nucleons together a large amount of energy is required. The mass defect is converted into the energy which holds the nucleons together. The difference in mass energy between the nucleus and its constituent is called binding energy. *It is also defined as the energy to be supplied in order to break the nucleus into its constituent nucleons.*

Binding energy       $E_b = \Delta M c^2$  (if  $\Delta M$  is in kg)     $c$  – speed of light in vacuum  
                         $E_b = \Delta M \times 931.5 \text{ MeV}$  (if  $\Delta M$  is in amu)

Binding energy is approximately proportional to mass number  $A$  of the nucleus.

## BINDING ENERGY PER NUCLEON

It is the ratio of binding energy of a nucleus to the mass number of the nucleus. It is the average energy per nucleon needed to split a nucleus into its constituent nucleons. It shows how tightly the nucleons are bound together inside a nucleus.

Binding energy per nucleon       $E_{bn} = E_b / A$

## NUCLEAR STABILITY AND BINDING ENERGY PER NUCLEON GRAPH

Stability of a nucleus is determined by the following factors.

- (a) Binding energy per nucleon – higher the binding energy per nucleon higher is the stability.
- (b) Neutron to proton ratio – if the ratio is greater than one, stability is more. The extra neutrons helps to increase the nuclear force and hence to overcome the repulsive force between the protons.

## BINDING ENERGY PER NUCLEON GRAPH

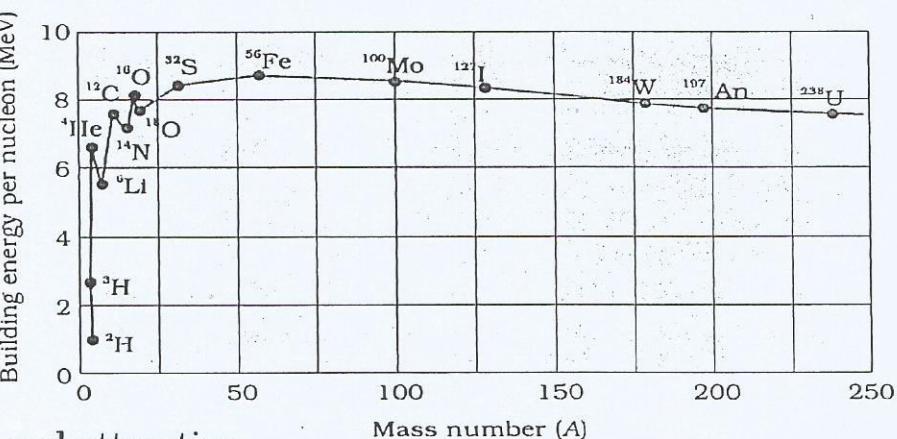
The adjacent graph shows the variation of binding energy per nucleon with mass number.

It can be seen from the graph that :

- ✓ The high binding energy per nucleon values indicates that the nuclear force is very strong and attractive.
- ✓ Binding energy per nucleon remains almost a constant for the middle mass nuclei ( $30 < A < 120$ ) – peaks at Fe-56 (~8.75 MeV). It is due to the fact that the nuclear force is short-ranged and shows saturation effect.

*A nucleon interacts only with its nearest neighbors, the ones which come within the range of the nuclear force (saturation effect). When mass number 'A' increases, the number of nearest neighbors with which a nucleon interact does not change as nuclear force is short-ranged. So the nuclear force experienced by a nucleon almost remains unaffected by the change in mass number. So, the binding energy per nucleon (which is the average energy required to pull out one nucleon against the nuclear force) is independent of mass number and remains almost a constant.*

- ✓ Binding energy per nucleon is lower for lighter ( $A < 20$ ) – exceptions are He-4 , C -12 , O-16 ( $\alpha$ -particle is extremely stable because it is helium nucleus which has high binding energy per nucleon)  
To attain more stability, lighter nuclei combine to form heavier nuclei (fusion) and move towards the middle segment of the graph to attain higher  $E_{bn}$  values.
- ✓ Binding energy per nucleon is lower for heavier nuclei ( $A > 120$ )  
To attain more stability, heavier nuclei split into lighter nuclei (fission) and move towards the middle segment of the graph to attain higher  $E_{bn}$  values.

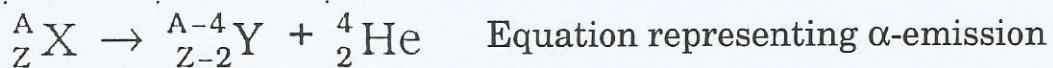


## RADIOACTIVITY

Radioactive decay is a process in which an unstable atomic nucleus loses energy by emitting particles or electromagnetic radiations to become stable. A nucleus emits  $\alpha$ - particles,  $\beta$  – particles or gamma rays to become more stable.

## ALPHA DECAY

An unstable nucleus emits  $\alpha$ -particle (helium nucleus). Mass number of the nuclei decreases by four and atomic number decreases by two.



For  $\alpha$  - decay  $Q = (m_X - m_Y - m_{He}) c^2$  if masses are in kg

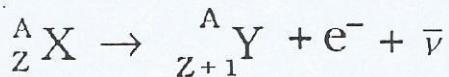
$$Q = (m_X - m_Y - m_{He}) \times 931 \text{ MeV} \quad \text{if masses are in u}$$

$m_X$  – mass of parent nucleus ,  $m_Y$  – mass of daughter nucleus ,  $m_{He}$  – mass of particle

$Q$  is also the net kinetic energy gained during decay or kinetic energy of the products if the initial nucleus was at rest.

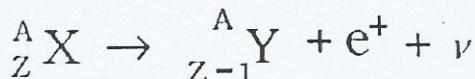
## BETA DECAY

**$\beta^-$  decay** : An unstable nucleus emits electron. Mass number of the nuclei remains unchanged while atomic number increases by one.



During  $\beta^-$  decay a neutron changes into an electron, proton and antineutrino. Proton remains in the nucleus (hence Z increases by 1) and the other two particles are emitted.  $n \rightarrow p + e^- + \bar{\nu}$

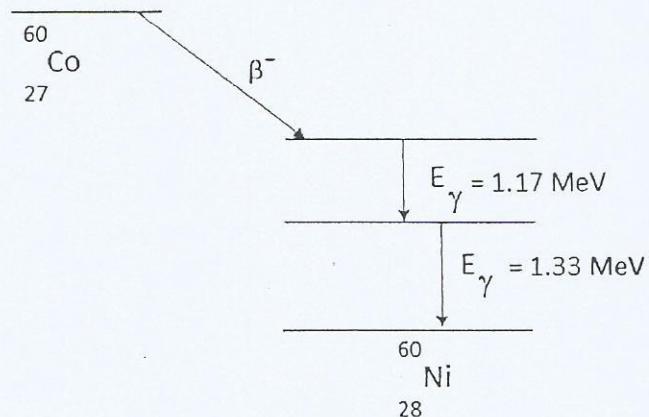
**$\beta^+$  decay** : An unstable nucleus emits positron. Mass number of the nuclei remains unchanged while atomic number decreases by one.



During  $\beta^+$  decay a proton changes into a positron, neutron and neutrino. Neutron remains in the nucleus (hence Z decreases by 1) and the other two particles are emitted.  $p \rightarrow n + e^+ + \nu$

## GAMMA DECAY

In much the same way that electrons in atoms can be in an *excited state*, so can a nucleus. When a nucleus which is in a higher energy state comes to a lower energy state it emits gamma rays.



Energy level diagram showing gamma emission

No change in mass number and atomic number.

Normally occurs after beta or alpha emission.

## Q – VALUE OF A NUCLEAR REACTION

The disintegration energy or the  $Q$ -value of a nuclear reaction is the difference between the initial mass energy of a nucleus and the total mass energy of the decay products.

If  $Q > 0$ , the process is exothermic and occur spontaneously. If  $Q < 0$ , energy has to be supplied to initiate the process.

## RADIOACTIVE DECAY LAW

It states that the number of nuclei undergoing decay per unit time (decay rate) is proportional to the total number of nuclei present in the sample at any instant of time.

$$\text{Rate of decay } -\frac{dN}{dt} \propto N \Rightarrow \frac{dN}{dt} = -\lambda N$$

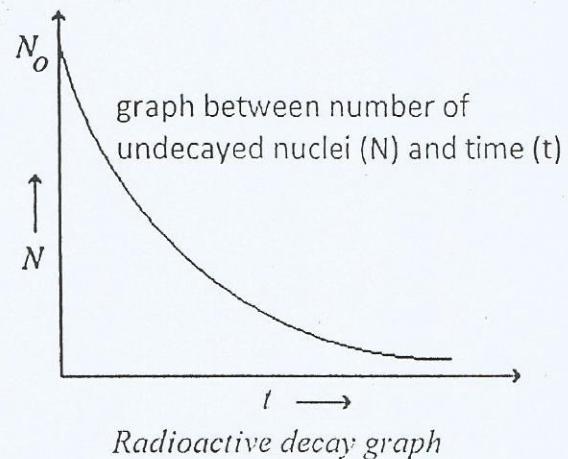
$\lambda$  → decay or disintegration constant – It gives the probability per unit time that any individual nucleus will decay.

$N$  – number of undecayed nuclei at any instant ‘ $t$ ’

$$\text{Rearranging the above equation } \frac{dN}{N} = -\lambda dt$$

$$\text{Integrating } \int_{N_0}^N \frac{dN}{N} = -\lambda \int_{t_0}^t dt \Rightarrow \ln N - \ln N_0 = -\lambda (t - t_0)$$

where  $N_0$  – number nuclei in the beginning, say at time  $t = 0$



$$\ln N - \ln N_0 = -\lambda t$$

$$\ln \frac{N}{N_0} = -\lambda t$$

Taking exponential on both side

$$N(t) = N_0 e^{-\lambda t}$$

Radioactive decay law

**Note :** Decay is faster initially as lots of nuclei will be present in the sample. When the number of nuclei decreases rate of decay also decreases as rate  $- \frac{dN}{dt} \propto N$

## ACTIVITY

Activity of a sample of radioactive material is the total number of nuclei decaying per second.

$$\text{Activity } R = -\frac{dN}{dt}$$

$$R = \frac{d}{dt} (N_0 e^{-\lambda t}) \quad (\text{using } N_0 e^{-\lambda t} = N)$$

$$R = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t} \quad \text{where } R_0 = \lambda N_0$$

$$R = \lambda N_0 e^{-\lambda t} = \lambda N$$

Activity is the total decay rate  $R$  of a sample of radioactive nuclei. Activity vs time graph is also like the above graph. (activity : along y – axis ).

SI unit of activity : Becquerel ( Bq ).

1 Becquerel = 1 decay per second .

Curie is another commonly used unit.

1 Curie =  $3.7 \times 10^{10}$  Bq

## HALF LIFE ( $T_{1/2}$ )

The time it takes for **half the nuclei** of a radioactive substance to decay is called half-life of the radioactive substance.

$$\text{We have, } N = N_0 e^{-\lambda t}$$

According to the definition of half-life, when  $t = T_{1/2}$  then  $N = N_0 / 2$ .

Substituting these values in the above equation and solving

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \Rightarrow 2 = e^{\lambda T_{1/2}} \Rightarrow \ln 2 = \lambda T_{1/2}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

## AVERAGE OR MEAN LIFE ( $\tau$ )

Consider a sample of radioactive nuclei containing  $N_0$  nuclei at time  $t=0$ . The number of nuclei that decay between  $t$  and  $t+dt$  is  $dN = \lambda N dt$ . The life of these nuclei is approximately 't' each. Total life of all these  $dN$  nuclei will be 't dN'.

$$\text{Mean life} = \frac{\text{total life of all the nuclei present in the sample}}{\text{number of nuclei present initially}} = \frac{\int_0^{\infty} t dN}{N_0}$$

$$\text{Mean life } \tau = \frac{\int_0^{\infty} t dN}{N_0} = \frac{\int_0^{\infty} t \lambda N dt}{N_0} = \frac{\int_0^{\infty} t \lambda N_0 e^{-\lambda t} dt}{N_0} = \lambda \int_0^{\infty} t e^{-\lambda t} dt$$

On integration the above equation becomes

$$\text{Mean life } \tau = 1/\lambda = \frac{T_{1/2}}{0.693}$$

It is the time required for a radioactive substance to reduce to 36.7% of the number of nuclei in the beginning.

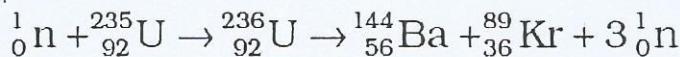
Decay constant  $\lambda$  can also be defined as the reciprocal of mean life.

## NUCLEAR FISSION

Nuclear fission is the splitting of an atomic nucleus into smaller nuclei. It is the working principle behind a nuclear reactor and fission bomb. The final products are radioactive.

Commonly used fissile nuclei : **U-235** and **Pu-239**.

*The total mass of reactants before fission is greater than the total mass after fission. This difference in mass becomes energy.* On an average each U atom releases around 216 MeV of energy

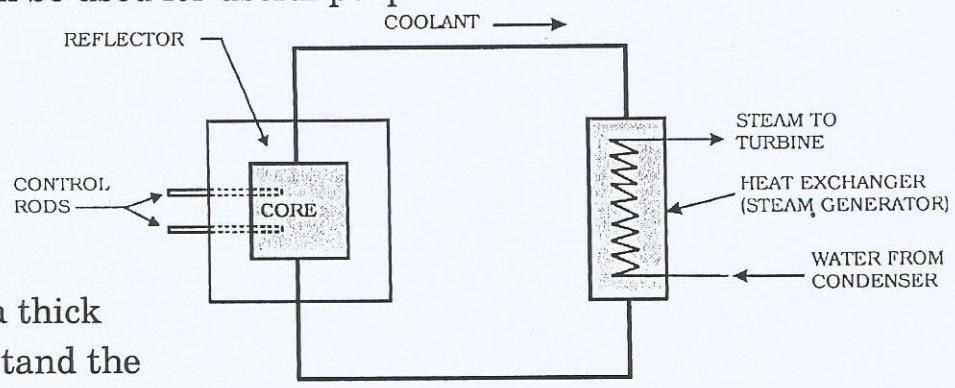


## CHAIN REACTION

The fission of a nucleus of U-235 in a sample containing large number of U-235 nuclei can be initiated by a slow moving neutron. When this nucleus splits further neutrons are produced. These neutrons in turn can cause more nuclei to split. An avalanche effect, called a '**chain reaction**' can then occur releasing large amount of energy in a short period of time. If the chain reaction is controlled so that the power output is constant it can be used for useful purposes.

## NUCLEAR REACTOR

Figure shows a schematic diagram of a nuclear reactor.



Reactor core : It is the site of nuclear fission. It is made of a thick steel vessel designed to withstand the very high pressure and temperature in the core.

Fuel rods : present in the core and contain U-235 or Pu-239.

Control rods : made of boron. When they are placed in-between the fuel rods these absorb neutrons and reduce the rate of fission. Their depth is adjusted to maintain a constant rate of fission.

$$\text{multiplication factor } K = \frac{\text{number of fission produced by a given generation of neutrons}}{\text{number of fission of the preceding generation}}$$

It is the measure of the growth rate of the neutrons in the reactor.

$K = 1$  → critical state – reactor produces energy at constant rate and chain reaction is controlled.

$K < 1$  → sub-critical state – energy production decreases and finally stops

$K > 1$  → super-critical state – energy production increases exponentially and chain reaction becomes uncontrolled.

Moderator : This surrounds the fuel rods and slows neutrons down to make further fission more likely. The moderator can be water / graphite.

Coolant : This transfers the heat energy of the core to the heat exchanger.

Eg : water,  $\text{CO}_2$  gas or liquid Na.

Heat exchanger: Here water is converted into high pressure steam using the heat energy of the coolant and send to the turbines which rotates and produces electricity.

## NUCLEAR FUSION

In fusion two light nuclei combine to become a heavier nuclei releasing energy. More amount of energy is produced than fission and the final products are non-radioactive.

$${}_1^1\text{H} + {}_1^1\text{H} \rightarrow {}_1^2\text{H} + e^+ + \nu + 0.42 \text{ MeV}$$

Limitations : Large amount of energy is required to start a fusion reaction because energy must be supplied to overcome repulsive forces between protons. Extremely high temperatures  $\sim 10^7$  K can provide start-up energy (thermo nuclear fusion). Fusion is the source of energy in stars.

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