BAN 630: Case Study 1

Outsourcing Heart Valves

Main Chapters:

1. Find the objective of Business Problem
2. Create Linear Programming Optimization Model
3. Set an objective function
4. Set Constraints in the Model
5. Find Optimal Solution Using Solver in Excel
6. Generate Sensitivity Report
7. Create One way Solver Table

Develop an LP model formulation that addresses the issue of minimizing the monthly cost of purchasing heart valves. Briefly explain the model's decision variables, objective function, and constraints.

Solution:

- **Business Objective**:

Minimize the total monthly cost of the Hayward Care's Purchase of Heart valves.

- **Decision Variables**:

x1 = Number of Heart Valves purchased from Supplier 1
x2 = Number of Heart Valves purchased from Supplier 2
x3 = Number of Heart Valves purchased from Supplier 3

- **Objective Function**:

Minimize Cost of Purchased Heart Valves from all three suppliers:
260(x1) + 180(x2) + 150 (x3)

- Note that 260, 180, and 150 are the cost of each valve from Supplier 1, Supplier 2 and Supplier 3 respectively.
- 260, 180, and 150 are coefficients in the above objective function.

- **Constraints:**

Constraints	Constraint Equations
Total number of Small Heart Valves, >= 400	**0.30(x1) + 0.35(x2) + 0.25(x3) > = 400**
Total number of Medium Heart Valves, >= 300	**0.40(x1) + 0.35(x2) + 0.25(x3) > = 300**
Total number of Large Heart Valves, >= 500	**0.30(x1) + 0.30(x2) + 0.50(x3) > = 500**
Total number of Valves from Supplier 1, >= 300	**x1 >= 300**
Total number of Valves from Supplier 1, 2 and 3 <= 600	**x1, x2, x3 <= 600**
Nonnegativity:	**x1, x2, x3 > 0**

2. Based on the LP model formulation, develop a spreadsheet model, and apply Excel Solver to determine the optimal solution of purchasing the needed heart valves. Present and briefly explain your solution.

<u>Solution</u>:

Below is the screenshot of output value for Decision Variables and objective function from Solver:

Decisions Variables			**Cost per valve in $**	**Proportion of Small Valves**	**Proportion of Medium Valves**	**Proportion of Large Valves**
Number of Heart Valves Units purchased from Supplier 1	**x1=**	**300**	**260.0**	**0.30**	**0.40**	**0.30**
Number of Heart Valves Units purchased from Supplier 2	**x2=**	**525**	**180.0**	**0.35**	**0.35**	**0.30**
Number of Heart Valves Units purchased from Supplier 3	**x3=**	**505**	**150.0**	**0.25**	**0.25**	**0.50**
	Total	**1330**				
Objective Function		**248250.00**				

It can be observed from the above output that optimal solution of purchasing the required number of heart valves, the minimum cost will be $248250. In other words, the minimum cost to purchase the required number of Heart Values from all suppliers to be used by Hayward Care will be $248250.

- Optimal number of heart valves to be purchased from supplier 1 is 300
- Optimal number of heart valves to be purchased from supplier 2 is 525
- Optimal number of heart valves to be purchased from supplier 3 is 505

3A. What should be the cost per valve from Supplier 3 to increase the optimal number of vales purchased from this supplier. Present and briefly explain your results.

<u>Solution:</u>

The cost of Heart Value for a particular supplier is a coefficient in the objective function. Impact of change in coefficient on the value of objective function can be observed in Sensitivity Report from Solver in Excel.

Below is the screenshot of Sensitivity Report generated from Solver with initial cost:

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C4	x1=	300	0	260	1E+30	102.5
C5	x2=	525	0	180	30	90
C6	x3=	505	0	150	150	21.42857143

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B12	Total number of Small Heart Valves, >= 400 LHS	400	450	400	15	31.66666667
B13	Total number of Medium Heart Valves, >= 300 LHS	430	0	300	130	1E+30
B14	Total number of Large Heart Valves, >= 500 LHS	500	75	500	27.14285714	30
B15	Total number of Valves from Supplier 1, >= 300 LHS	300	102.5	300	300	100
B16	Total number of Valves from Supplier 1, <= 600 LHS	300	0	600	1E+30	300
B17	Total number of Valves from Supplier 2, <= 600 LHS	525	0	600	1E+30	75
B18	Total number of Valves from Supplier 3, <= 600 LHS	505	0	600	1E+30	95

- The sensitivity report states that if the cost of valve from supplier 3 decreases by $21.42 ($22 appx.), then the number of valves from Supplier 3 will change. Hence, we will change the coefficient to $128 and see the result of updated values in the created Linear Programming Model. In similar manner, we can keep decreasing the value of cost (observing rom sensitivity report) until we reach the optimal value of purchasing the valves from supplier 3.

Below is the screenshot of output for Decision Variables and objective function with updated coefficients from Solver:

Decisions Variables			Cost per valve in $	Proportion of Small Valves	Proportion of Medium Valves	Proportion of Large Valves
Number of Heart Valves Units purchased from Supplier 1	x1=	300	260.0	0.30	0.40	0.30
Number of Heart Valves Units purchased from Supplier 2	x2=	457	180.0	0.35	0.35	0.30
Number of Heart Valves Units purchased from Supplier 3	x3=	600	128.0	0.25	0.25	0.50
	Total	1357				
Objective Function		237085.71				

- It can be observed from above table that the value of optimal number of valves to be purchased from supplier 3 is 600, which is also our max allowed limit of purchase.
- We can conclude from above result that the cost per valve from supplier 3 must be **$128** to achieve the optimal purchase of heart values from it.

3B. If Hayward Care would like to decrease the maximum purchase from Supplier 1 to 450 units (currently, it is 600 units), how would this decrease affect/change the optimal solution? Present and briefly explain your results.

<u>Solution:</u>

Updated Constraint: x1 <= 450

Below is the screenshot of output for Decision Variables and objective function with updated constraint from Solver:

Decisions Variables			Cost per valve in $	Proportion of Small Valves	Proportion of Medium Valves	Proportion of Large Valves
Number of Heart Valves Units purchased from Supplier 1	x1=	300	260.0	0.30	0.40	0.30
Number of Heart Valves Units purchased from Supplier 2	x2=	525	180.0	0.35	0.35	0.30
Number of Heart Valves Units purchased from Supplier 3	x3=	505	150.0	0.25	0.25	0.50
	Total	1330				
Objective Function		248250.00				

- It is observed that despite decreasing the maximum allowed limit of purchasing valves from 600 to 450 for Supplier 1, the output of Decision Variable and objective function remain unchanged, i.e., optimal number of valves to be purchased from supplier 3 remain unaffected with the change in constraint (max allowed purchased for supplier 3)
- Optimal number of heart valves to be purchased from supplier 1 is still same 300
- Also, if we try to understand the solution with general sense, it can be observed that cost of each valve is highest for supplier 1, Hence every time our linear programming Model should selected minimum allowed limit of valves as optimal number for supplier 1, Hence change in Maximum limit should not impact the result but minimum limit.

4. **Hayward Care would like to purchase more from Supplier 2, which is the best quality supplier but has the second highest cost per valve. For example, they would like to purchase at least 530 valves from Supplier 2. Present a mathematical formulation of this constraint. Revise the spreadsheet model in question 2 by adding this constraint, identify and present the optimal solution for the revised model. Compare this optimal solution with the optimal solution in question 2 and briefly explain your results.**

Solution:

Added constraint: $x2 >= 530$

Below is the screenshot of output for Decision Variables and objective function with added constraint from Solver:

Decisions Variables			**Cost per valve in $**	**Proportion of Small Valves**	**Proportion of Medium Valves**	**Proportion of Large Valves**
Number of Heart Valves Units purchased from Supplier 1	**x1=**	300	**260.0**	**0.30**	**0.40**	**0.30**
Number of Heart Valves Units purchased from Supplier 2	**x2=**	530	**180.0**	**0.35**	**0.35**	**0.30**
Number of Heart Valves Units purchased from Supplier 3	**x3=**	502	**150.0**	**0.25**	**0.25**	**0.50**
	Total	1332				
Objective Function		248700.00				

- The added constraint has changed the optimal number of valves to be purchased from 525 to 530 for supplier 2.

Comparison with Linear Model of Answer 2:

	Before changing S2 Minimum limit	After changing S2 Minimum limit
Valves from Supplier 1	**300**	**300**
Valves from Supplier 2	**525**	**530**
Valves from Supplier 3	**505**	**502**
Total Valves	**1330**	**1332**
Total Cost	**248250.00**	**248700.00**

From above comparison, we can observe the following:

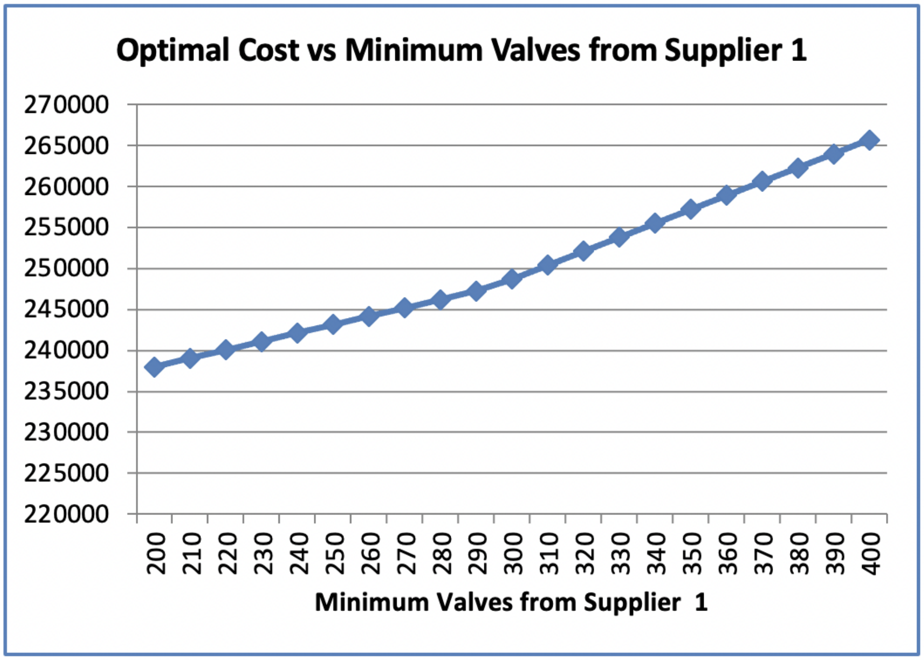
1. Optimal number of valves to be purchased from supplier 2 has increased from 525 to 530.
2. Also, Optimal number of valves to be purchased from supplier 3 has been affected to achieve the business objective under given constraints. The optimal purchase of valves from supplier 3 has been decreased from 505 to 502.
3. Overall number of valves to be purchased from all suppliers have also been decreased by 2.
4. The minimum cost to purchase the Heart valves from all suppliers in given constraints has increased from $248250 to $248700

5. **For the revised model in question 4, apply Solver Table to investigate the effect on the optimal total cost of changing the minimum number of valves purchased from Supplier 1 from 200 to 400 units (currently it is 300 units) with an increment of 10 units. Present and briefly explain your results.**

Solution:

Below is the screenshot of optimal total cost w.r.t change in minimum no. of valves from Supplier 1

Min. Valve from Supplier 1	Optimal Cost
200	238000.00
210	239025.00
220	240050.00
230	241075.00
240	242100.00
250	243125.00
260	244150.00
270	245175.00
280	246200.00
290	247225.00
300	248700.00
310	250400.00
320	252100.00
330	253800.00
340	255500.00
350	257200.00
360	258900.00
370	260600.00
380	262300.00
390	264000.00
400	265700.00



- We can see the optimal cost changes for each instance of change in minimum valves constrain from supplier 1.
- At 200, the optimal cost is $238000, whereas for at 400, the optimal cost is $265700
- If we observe our Model and constraints, we know that among 3 suppliers, Supplier 1 has highest cost, hence the model should always select minimum allowed number of valves from supplier 1 to achieve the objective. Hence optimal price is increased with each increase of instance in the min. valves from supplier 3 because the optimal value (new minimum value of valve) is changing.