BAN 630: Case Study 2

Scheduling in Emergency Department

Main Chapters:

1. Find the objective of Business Problem
2. Create Integer Linear Programming Optimization Model
3. Set an objective function
4. Set Constraints in the Model, including all Decision Variables are integer
5. Find Optimal Solution Using Solver in Excel
6. Generate Sensitivity Report
7. Create Two-way Solver Table

1. **Formulate an integer programming model that can be used to develop a schedule that will satisfy customer service needs at a minimum total daily cost.**

Solution:

- **Business Objective**:
  Find optimal schedule of attendants at a minimum total daily cost

- **Decision Variables**:

F_{8AM} = Number of Full Time Doctor starting shift at 8 AM
F_{4PM} = Number of Full Time Doctor starting shift at 4 PM
F_{12AM} = Number of Full Time Doctor starting shift at 12 AM
P_{8AM} = Number of Part Time Doctor starting shift at 8 AM
P_{8PM} = Number of Part Time Doctor starting shift at 8 PM

- **Objective Function**:

Minimum Cost to Dublin Hospital from optimal attendance of the Doctors in the hospital:
$1120(F_{8AM}) + 1120(F_{4PM}) + 1120(F_{12AM}) + 504(P_{8AM}) + 504(P_{8PM})$

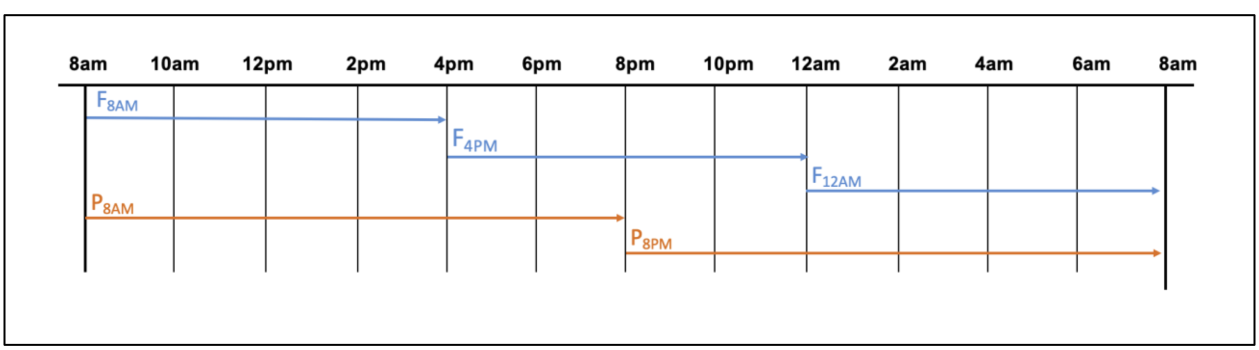
- Note that Full time doctor's hourly cost is $140, Hence $1120 (140*8) is daily cost of Full time Doctor for 8-hour shift. This $1120 will be a coefficient in our objective function for full time Doctors.

- Similarly, Part time doctor's hourly cost is $42, Hence $504 (42*12) is daily cost of Part time Doctors for 12 hours. This $504 will be a coefficient in our objective function for pa t time Doctors.

- **Constraints:**

Constraints	Constraint Equations
Doctors required from 9 Am - 2 Pm	$F_{8AM} + P_{8AM} => 8$
Doctors required from 2 Pm - 6 pm – (A) {2 pm to 4pm}	$F_{8AM} + P_{8AM} => 6$
Doctors required from 2 Pm - 6 pm (B) - {4 pm to 6pm}	$F_{4PM} + P_{8AM} => 6$
Doctors required from 6 Pm - 12Am (A) - {6 pm to 8pm}	$F_{4PM} + P_{8AM} => 10$
Doctors required from 6 Pm - 12Am (B) - {8 pm to 12pm}	$F_{4PM} + P_{8PM} => 10$
Doctors required from 12 Am - 9Am	$F_{12AM} + P_{8PM} => 5$
Non-Negativity Constraints	$F_{8AM}, F_{12AM}, F_{4PM}, P_{8AM}, P_{8PM} => 0$

Note: The constraint for doctors required in 2Pm to 6Pm and, doctors required from 6Pm to 8 have overlapping of shits. Hence then have been broken down w.r.t shift timings. Below is the visual representation for better understanding.



2. Create a spreadsheet model using the formulation from question 1. Solve the model using Excel Solver for the optimal schedule of attendants and interns. Present the optimal schedule and comment on the solution. Will this optimal solution be appropriate for the emergency? department? Explain.

Solution:

Below is the screenshot of output value for Decision Variables and objective function from Solver:

Daily Scheduling

Decision Variables		**Number**	**Cost, $**
Full Time Doctor Starting at 8 AM	F_{8AM} =	0.00	1120.0
Full Time Doctor Starting at 4 PM	F_{4PM} =	0.00	1120.0
Full Time Doctor Starting at 12 AM	F_{12AM} =	0.00	1120.0
Part Time Doctor Starting at 8 AM	P_{8AM} =	10.00	504.0
Part Time Doctor Starting at 8 PM	P_{8PM} =	10.00	504.0

Scheduling based on the model decisions	***8am***	***10am***	***12pm***	***2pm***	***4pm***	***6pm***	***8pm***	***10pm***	***12am***	***2am***	***4am***	***6am***
Full Time Doctors	0	0	0	0	0	0	0	0	0	0	0	0
Part Time Doctos	10	10	10	10	10	10	10	10	10	10	10	10
Total Number of Doctors	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**

Objective

Minimize daily cost, $		10,080.00	

- It can be observed from the above output that minimum cost to Hospital with optimal attendance of doctors will be $10,080. Also, the model has selected only Part time doctors for the optimal solution, 10 Part time Doctors at 8AM, and 10 Part time doctors at 8 PM.

- For emergency, this solution will not be appropriate because a hospital always require few Full time Specialized Doctors in cortical cases. Although mathematically above solution is optimal but in real life, we will always need full time doctors in the hospital

3. **After reviewing the solution in question 2, the medical director has realized that some additional requirements must be incorporated into the schedule. Specifically, she wants to ensure that at least two attendants are working at any 8-hour shift. She also wants to hire no more than 7 interns in each of the 12-hour shift. Provide the mathematical formulation of the additional constraints and incorporate these constraints into the spreadsheet model from question 2. Solve the revised model, present, and explain the new optimal schedule, and compare it with the optimal schedule in question 2.**

Solution:

New Added Constraints:

Required Full time doctors at 8 AM Shift	$F_{8AM} => 2$
Required Full time doctors at 4 PM Shift	$F_{4PM} => 2$
Required Full time doctors at 12 AM Shift	$F_{12AM} => 2$
Required Part time doctors at 8 AM shift	$P_{8AM} <= 7$
Required Part time doctors at 8 pm shift	$P_{8PM} <= 7$

Below is the screenshot of output value for Decision Variables and objective function from Solver:

Daily Scheduling

Decision Variables		***Number***	***Cost, $***
Full Time Doctor Starting at 8 AM	**F_{8AM} =**	**2.00**	**1120.0**
Full Time Doctor Starting at 4 PM	**F_{4PM} =**	**3.00**	**1120.0**
Full Time Doctor Starting at 12 AM	**F_{12AM} =**	**2.00**	**1120.0**
Part Time Doctor Starting at 8 AM	**P_{8AM} =**	**7.00**	**504.0**
Part Time Doctor Starting at 8 PM	**P_{8PM} =**	**7.00**	**504.0**

Scheduling based on the model decisions

	8am	***10am***	***12pm***	***2pm***	***4pm***	***6pm***	***8pm***	***10pm***	***12am***	***2am***	***4am***	***6am***
Full Time Doctors	2	2	2	2	3	3	3	3	2	2	2	2
Part Time Doctos	7	7	7	7	7	7	7	7	7	7	7	7
Total Number of Doctors	**9**	**9**	**9**	**9**	**10**	**10**	**10**	**10**	**9**	**9**	**9**	**9**

Objective

Minimize daily cost, $		**14,896.00**

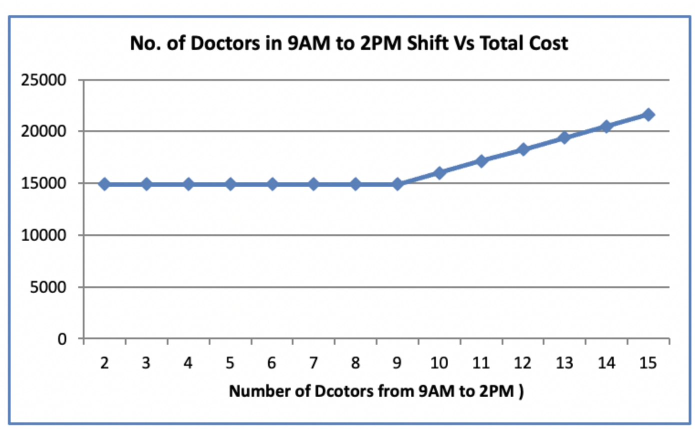
- It can be observed from the above output that minimum cost to Hospital with optimal attendance of doctors have increased to $14,896. Also, the model has reduced number of Part time doctors for the optimal solution and considered the constraints of least full-time doctors from each shift.
- Comparing it with Answer 2 Solution, we can see that cost has increased from $10,080 to $14,896, and Full-time doctors have been included in the Attendance as well after adding new constraints.

4. For the optimal model in question 3, use Solver Table to analyze the effect of changing the minimum number of attendants (doctors) from 9 am - 2 pm (currently equal to 8) from 2 to 15 (with an increment of 1) on the minimum total cost, and number of attendants and interns in respective time shifts. Present and briefly explain the results of your sensitivity analysis.

<u>Solution:</u>

Below is the screenshot of optimal total cost w.r.t change in doctors required in 9 AM to 2 PM Shift:

Doctors in 9AM t0 2 PM Shift	Minimum Total Cost	F8am	F4pm	F12am	P8am	P8pm
2	14896	2.00	3.00	2.00	7.00	7.00
3	14896	2.00	3.00	2.00	7.00	7.00
4	14896	2.00	3.00	2.00	7.00	7.00
5	14896	2.00	3.00	2.00	7.00	7.00
6	14896	2.00	3.00	2.00	7.00	7.00
7	14896	2.00	3.00	2.00	7.00	7.00
8	14896	2.00	3.00	2.00	7.00	7.00
9	14896	2.00	3.00	2.00	7.00	7.00
10	16016	3.00	3.00	2.00	7.00	7.00
11	17136	4.00	3.00	2.00	7.00	7.00
12	18256	5.00	3.00	2.00	7.00	7.00
13	19376	6.00	3.00	2.00	7.00	7.00
14	20496	7.00	3.00	2.00	7.00	7.00
15	21616	8.00	3.00	2.00	7.00	7.00



- It is observed that if we do sensitivity analysis for the number of doctors in 9 AM to 2 PM shift from 2 to 15. The value of objective function remains same for 2 to 9 Doctors in the shift, but later we see increase in the value of objective function with each addition of doctors because our optimal value of a Decision Variable changes after 9 Doctors.

- We can also observe that change in above constraints impacts only the number of Full time Doctors working in the 8 AM shift. Because morning Full time Doctors works till 4 PM. So, more morning Doctors can cover this requirement of doctors greater than 9 in the 9 AM to 2 PM Shift. One question can be raised is that Why this requirement is not being filled by part time doctors since their cost is less, the answer is because model has already satisfied maximum number of part time doctors constraint in each shift which is 7.

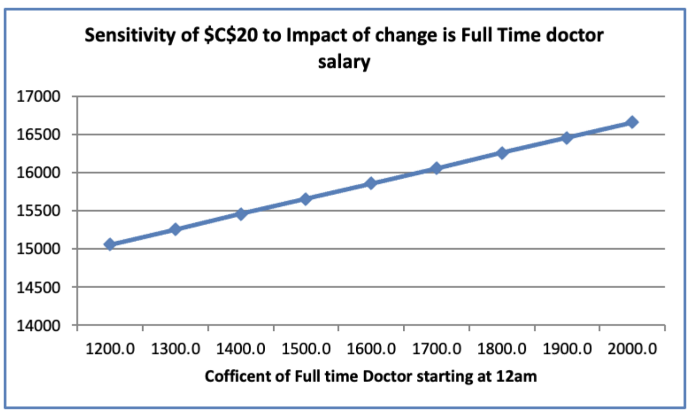
5. **For the optimal model in question 3, provide a two-way sensitivity analysis using Solver Table for parameters of your choice. Present and briefly explain the results of your sensitivity analysis**

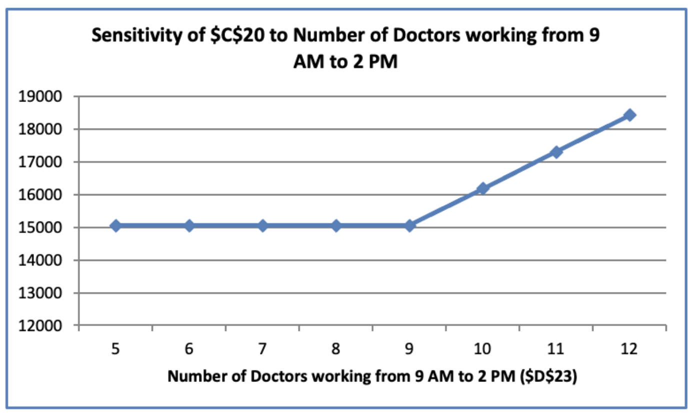
<u>Solution:</u>

I have changed two parameters for my Two-way sensitivity Table:

1. For Full time Doctors working in night Shift (12 AM), the cost should be higher. So, I am checking the sensitivity if cost of 8 hour (coefficient) for F_{12AM} ranges $1200 to $2000 with increment of 100 i.e., Per hour cost from $150 to $250
2. Also, Doctors requirement in the 9AM to 2PM ranges between 5 to 12 with increment of 1.

Below is the screenshot of result from Two-way Sensitivity from Solver Table:





<u>Note: C20 here represents the Objective function Value</u>

- It can be observed from the left chart the minimum cost for doctors increases as we increase the cost for Full time Doctors working in the night. Note that this cost increases as we have constraint of selecting at least two full time doctors in the night 12 am shift.

- From the table we also see that the change in cost of Full-time doctors in the night does not change the value of any of our decision Variables, this change in the optimal minimum cost is coming purely from the change in coefficient for F_{12AM} i.e. change in the cost

C3	1200.0	1300.0	1400.0	1500.0	1600.0	1700.0	1800.0	1900.0	2000.0
5	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
6	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
7	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
8	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
9	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
10	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
11	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
12	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00

C4	1200.0	1300.0	1400.0	1500.0	1600.0	1700.0	1800.0	1900.0	2000.0
5	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
6	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
7	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
8	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
9	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
10	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
11	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
12	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00

C5	1200.0	1300.0	1400.0	1500.0	1600.0	1700.0	1800.0	1900.0	2000.0
5	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
6	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
7	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
8	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
9	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
10	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
11	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
12	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00

C6	1200.0	1300.0	1400.0	1500.0	1600.0	1700.0	1800.0	1900.0	2000.0
5	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
6	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
7	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
8	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
9	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
10	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
11	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
12	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00

C7	1200.0	1300.0	1400.0	1500.0	1600.0	1700.0	1800.0	1900.0	2000.0
5	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
6	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
7	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
8	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
9	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
10	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
11	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00
12	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00	7.00

- The other parameter shows same result as in Answer 4. As can be seen in above highlighted portion the number of full-time doctors in the morning 8 AM shift increases once the requirement of doctors goes past 9. It does not change any other Decision Variables to meet the constraints.