University of Colorado - Boulder

ASEN 2001: Statics, Structures, and Materials

Materials Lab
Due: Wednesday, December 11th, 2019

Statics, Structures, and Materials Lab 3: Composite Wing Beam

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The purpose of this lab is to design a composite wing beam which can withstand a distributed load while also being as lightweight as possible. For this particular lab a composite beam must be designed with extruded foam and balsa wood to attain the lightest and strongest possible beam within the constraints of the lab. The beam is to undergo a loading test using a wiffle tree to simulate the distributed load on a wing. From the experimental data it can be determined what the maximum load is and whether the beam failed in shear or from bending.

I. Introduction

To develop the most efficient wing design, that is, to strike a balance between attaining the strongest beam possible while also having the lightest beam possible within the lab constraints[2], we utilized data[3] from previous load-bearing experiments on other composite wing beams designed to withstand aerodynamic loads on a wing. From this data[3] we can ascertain what the necessary width to be most resistant to the bending stress and shear stress due to a load acting at the center of the beam at each point from the center of the beam to the ends. Using this we designed what we believed to be an optimized composite beam, one in which is only as wide as it needs to be to withstand the maximum stress at each point, which we then were able to test. During the test we used a wiffle tree composed of 8 metal loops, 7 metal bars, and a bucket at the bottom to gradually add mass to until the beam fails. Aerodynamic loading on a wing isn't completely uniform, but the load profile is symmetric and using a wiffle tree this loading can be simulated. We collected data to make sure the wiffle tree setup was fairly precise to minimize error and best simulate wing loading. From the data obtained in the experiment we can then conclude how the beam failed and what the maximum distributed load is that it can bear.

II. Strength Analysis

To perform this experiment we set up a wiffle tree with the distances of the hooks on the components being optimized to simulate the distributed load a wing beam would experience due to lifting forces on it. The beam was cut to a certain shape based on the given test data[3] to maximize the strength at each point along the beam. The testing was completed using a 4-point loading-support configuration shown in figure 1.

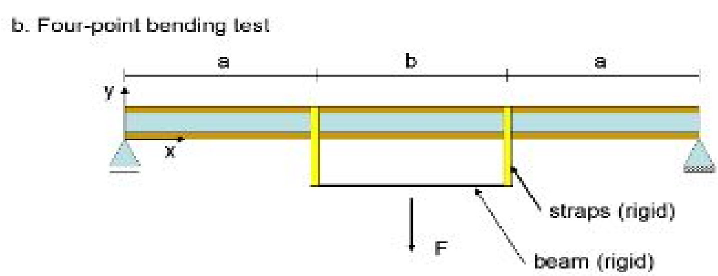


Figure 1: Loading configuration for test cases.

Weights are continuously added to the bottom to gradually increase the force until the beam breaks. At the failure section, the beam can either break under shear force or bending moment. To calculate the failure mode for bending moment and shear force we can use the equations given below.

$$\sigma_{Fail} = -\frac{M_{Fail}c}{(I_b + (E_f/E_b)I_f)}$$

$$\tau_{Fail} = \frac{3V_{Fail}}{2A_f}$$

Using the test data[3] provided, we found our maximum shear force to be $\tau_{Fail} = 7.05 * 10^4$ Pa and our maximum bending moment to be $\sigma_{Fail} = 4.69 * 10^5$ Pa. When performing this experiment, the beam slipped off the table after 7.26 kg (16 lbm) of weights were put in the bucket. This was due to the support being too close to the end of the wing and with the deformation form the added force the wing slipped off the support. We set it back up and after putting 4.54 kg (10 lbm) of weights in the bucket the beam broke very close to the center, 2.86 cm (9/8 inches) from the middle. Calculating for the weight of the wiffle tree setup plus the weights in the bucket the beam withstood a force of 97.2 N. The slippage and the damage which occurred during the fall certainly affected our observations and is a large source of error.

III. Wing Design

To design the wing we wanted to maximize the load capacity that the wing could hold. This in turn is also the lift capacity of the wing. To generate the design, MATLAB[1] was used to handle all of the calculations and to asses the shear and moment stress of the wing. We began by getting an equation for the distributed load $P(x)$ (which is given below) then integrated the equation to get the equation for the shear forces across the wing. Then another integral to generate an equation for the moment. Using these equations and the dimensions of the composite wing an outline was generated of the final design that was the best possible shape for maximum strength and minimum weight.

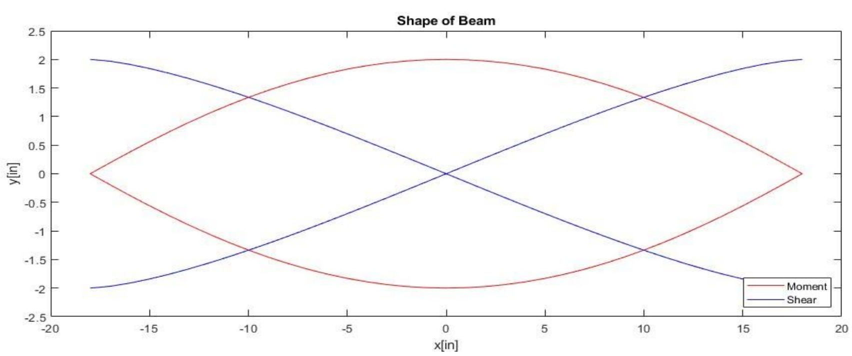
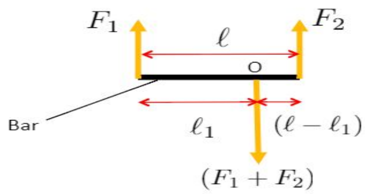


Figure 2. Shear and moment diagram and the necessary beam shape to account for it.

After the optimal shape was traced out and cut the wiffle tree was designed. The goal of the wiffle tree was to subject the wing to near uniform loading. Since the wing is symmetrical only the left side was analyzed and then flipped to get the right side of the wing. Using the equation $p(x) = p_0\sqrt{1-(2x/L)^2}$ we found the centroid of each 4.5'' section of the wing starting with the left edge. Using these centroids and forces we found the location of each metal loop. We then determined the location of the bars using the moment equilibrium equation.



Moment equilibrium of bar:

$$\Sigma M_o = 0 \quad F_1\ell_1 = F_2(\ell - \ell_1) \quad \rightarrow \quad \ell_1 = \frac{F_2\ell}{F_1 + F_2}$$

All of the locations of the loops and bars are given in the table shown here

Placement of the Loops

Location range from left edge (in)	Loop number	Length from left edge of wing (in)
0 to 4.5	L1	2.5514
4.5 to 9	L2	6.8754
9 to 13.5	L3	11.2863
13.5 to 18	L4	15.7863
18 to 22.5	L5	20.2137
22.5 to 27	L6	24.7137
27 to 31.5	L7	29.1246
31.5 to 36	L8	33.4486

6 Inch Bars

Loops used to support	Bar number	Distance from left edge of bar
L1 and L2	B61	3.4018
L3 and L4	B62	3.0484

12 Inch Bars

Loops used to support	Bar number	Distance from left edge of bar
L12 and L34	LB1	6.7566

The 6’’ and 12’’ bars on the right side of the wing are a mirror image of the left so we measured from the right to get the correct placement.

The loading of the wiffle tree was done by attaching a bucket to the bottom of the tree and carefully adding weight until the wing failed due to the force. Smaller weights were placed in the bucket one at a time until the total weight was equal to a larger weight then the smaller weights were replaced with the equivalent larger weight. This was repeated until the failure point of the wing was reached.

IV. Discussion

The analysis conducted resulted in a theoretical failure load of approximately 88 N. During the actual experiment, the wing was able to sustain a load of 97.26 N before failure. The failure which occurred during the testing was neither a shear failure nor a bending failure. While the weights were being applied, the roller on which the wing was clamped to slipped off of the edge of one side of the table. After falling off the table, the wing was still intact; however, the wing was cracked roughly along its midsection in the balsa wood. After assessing the situation, the group continued testing and the wing finally failed through a bending failure with a final load of 44.5 N. It can be confirmed from our initial analysis that the failure must have occurred through bending as the point of failure was approximately 0.028 m from the centerpoint of the wing. At this point, the theoretical shear force should be close to zero, while the bending force should be near its maximum as seen from figures 3 and 4.

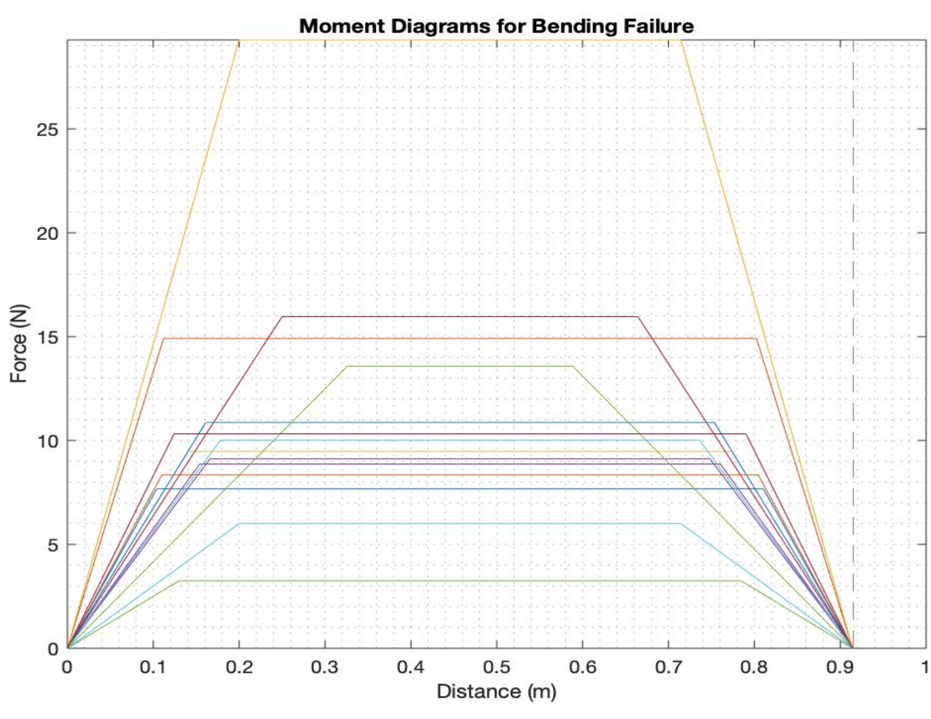


Figure 3. Moment diagrams detailing the force and the distance along the beam.

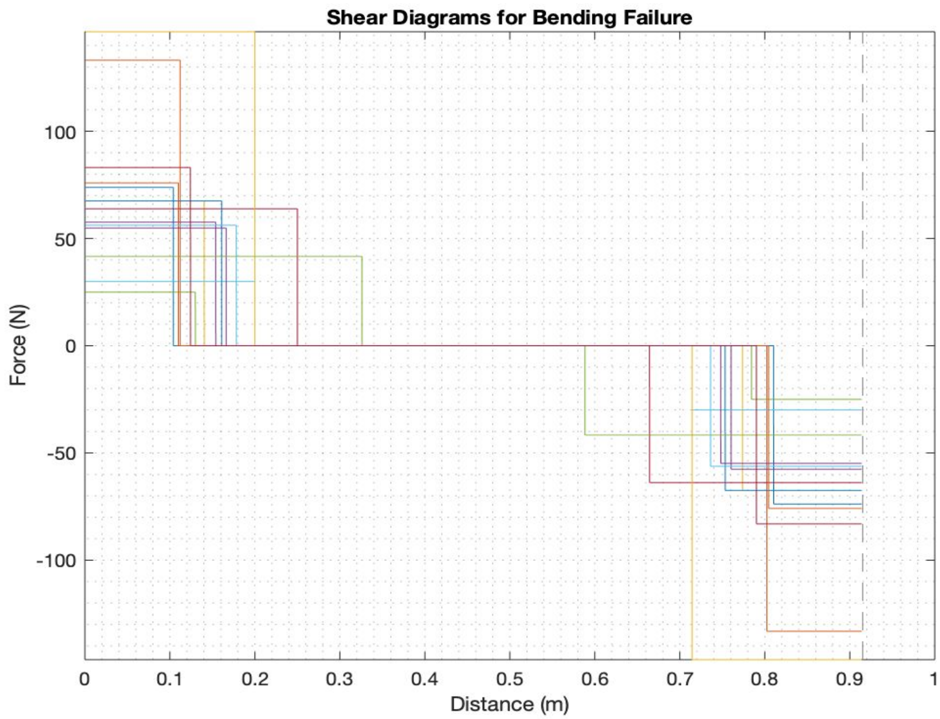


Figure 4. Shear diagrams showing the magnitude of force at a distance along the beam.

In order to create a more realistic model, there would be other types of failure which would need to be accounted for. The first of these would be stress failure. Over a long period of time of the wing flying, stress would cause the material of the wing to wear down and would eventually result in the wing failing in yielding. This could be analyzed and predicted from doing testing on the wing's material and examining a stress vs strain diagram to determine the maximum force the material can take before failing and based on this information either replace the wing after some amount of time, or to choose a different material in order to satisfy the general requirements of the wing. Another type of failure could be due to impact failure. In the event that something struck the wing while it was in use, the wing could suffer damage ranging from anywhere between unharmed to catastrophic. In order for the wing to not fail due to impact, testing would need to be done on the wing using small test chunks of material and using those smaller chunks to test the strength of the wing. If the wing is capable of surviving test chunks large enough to account for the majority of cases in which an impact would occur, this would eliminate a significant amount of risk before flying. Although it would be impossible to predict an impact occurring, it is important to minimize the potential risks in the case it does occur. A final type of mechanical failure is failure due to thermal shock. Thermal shock failure is caused by rapid changes in temperature along certain points. Thermal shock failure can be mitigated by reducing the wing material's thermal expansion coefficient, increasing the material's toughness, or decreasing the material's Young's Modulus.

V. Conclusions

From the data collected we can conclude a couple of things. First, that our experiment is fraught with error due to the beam sliding off the table after initially adding 16 pounds into the attached bucket. Second, we believe our beam failed from bending since the magnitude of shear is nearly zero where the beam broke, about 9/8 in. from the very center. Also, our beam withstood more weight than what was expected. Our calculated weight of failure was approximately 88N, but during our actual tests the beam broke after 97.26N force had been applied. This difference could be accounted for with the 1.5 factor of safety used in designing the wing. In all the beam we designed withstood a greater load than anticipated and only could have withstood a greater bending stress if it was either composed of stronger material or if it were wider in the middle.

VI. References

[1]MATLAB, Software Package, Ver. R2019b, MathWorks, Natick, MA, 2019.

[2]Doostan, A., "ASEN_2001_Lab_3__Fall_2019.pdf", *Statics, Structures, and Materials*.

[3]Doostan, A., "ASEN_2001_Lab_3_Test_Data.xls", *Statics, Structures, and Materials*.

VII. Appendix

Appendix A - MATLAB Code:

- **CODE 1:**

```
%% Import data from spreadsheet
% Script for importing data from the following spreadsheet:
%
%    Workbook: C:\Users\Jeheres something on\Desktop\Lab 3\ASEN 2001 Lab 3
Test Data (1).xls
%    Worksheet: Sheet1
%
% Auto-generated by MATLAB on 04-Nov-2019 09:57:07

%% Setup the Import Options

close all
clear all

opts = spreadsheetImportOptions("NumVariables", 7);

% Specify sheet and range
opts.Sheet = "Sheet1";
opts.DataRange = "A3:G38";
```

```
% Specify column names and types
opts.VariableNames = ["Test", "FN", "am", "wm", "d_fm", "VarName6",
"CommentonFailure"];
opts.SelectedVariableNames = ["Test", "FN", "am", "wm", "d_fm",
"VarName6", "CommentonFailure"];
opts.VariableTypes = ["double", "double", "double", "double", "double",
"double", "string"];
opts = setvaropts(opts, 7, "WhitespaceRule", "preserve");
opts = setvaropts(opts, 7, "EmptyFieldRule", "auto");

% Import the data
Lab3 = readtable("C:\Users\Jeheres something on\Desktop\Lab 3\ASEN 2001
Lab 3 Test Data (1).xls", opts, "UseExcel", false);


%% Clear temporary variables
clear opts

%% Bending and Shear
% Seperate the bending from the shear
 Data = Lab3{(1:24),(1:6)};
for i = 1:length(Data)
    if Data(i,6)==1
        Bending (i,:) = Data(i,2:5);
    else
        Shear(i,:) = Data(i,2:5);
    end
end

%% Create the plots
% Plot the bending and Shear
% get the average failure stress for bending and shear
% Eliminate outlyers in data
Bending(Bending==0)=[];
Bending = reshape(Bending,17,4);
Shear(Shear==0) = [];
Shear=reshape(Shear,7,4);
Bending(7,:)=[];
Bending(10,:)=[];
Shear(6,:)=[];
Shear(4,:)=[];

BendingForce = Bending(:,1);
FDB = Bending(:,4);
ShearForce = Shear(:,1);
```

```
FDS = Shear(:,4);
plot(FDB,BendingForce,'b*')
hold on
plot(FDS,ShearForce,'ro')
xlabel('Failure Distance (m)')
ylabel('Force (N)')
legend('Bending', 'Shear')
hold off


MavgFail = mean(Bending(:,1)); % Average Moment of failure
VavgFail = mean(Shear(:,1)); % Average Shear of failure


%% Failure stresses
% solve for Tau and Sigma
CC = ((1/32)+(3/8))*.0254; % thikness of half of the beam
t = 2*CC; % Thickness of beam
Ef = 2.67 * 10^9;   %Young's Modulus for the foam
Eb = 3.25 * 10^9;   %Young's Modulus for the balsa
Ib = 2* ( (4 * ((.0254/32)^3)/12) + (((t/2)-(.0254/64))^2 * 4 * (.0254
/32))); % moment of inertia for balsa
If = (4 * (3*.0254/4)^3)/12; % moment of inertia of foam
FOS = 1.5;   % Factor of safety
SigmaFail = MavgFail*CC/(Ib+(Ef/Eb)*If) % Average of Sigma fail
SigmaAllow = SigmaFail/FOS

TauFail = (3*VavgFail)/(2*(4*.0254)*(.75*.0254)) % Tau Fail
TauAllow = TauFail/FOS

%% Plot the optimal beam shape
% use symbolic variables to create the best shape

% Parameters
L = 36 * 0.0254;

% Symbolic variables
syms x p0

% Force per unit length
qx = 4 * 0.0254 * p0 * sqrt(1-(2*x/L)^2);

% Reactions
R = -int(qx,x,-L/2,L/2)/2;
```

```
% Shear force
Vx = R + int(qx,x,-L/2,x);

% Moment
Mx = int(Vx,x,-L/2,x);

% Bending moment at x=0
M0 = subs(subs(Mx,x,0),p0,1);

%width function
t = ((1/16) + (3/4))*.0254; %thickness of the beam
tb = .0254/32;  %Thickness of the balsa
tf = 3*.0254/4; %Thickness of the foam
Ef = 2.67 * 10^9;   %Young's Modulus for the foam
Eb = 3.25 * 10^9;   %Young's Modulus for the balsa
wx = (Mx*t/2)/(SigmaAllow * ( (2*tb*((1/12)+((t-tb)/2)^2)) +
(Ef/Eb)*((tf^3)/12) ));

% Evaluate at a grid
xgrid = -L/2:1*0.0254:L/2;
p0_computed = 4.3258*10^4;

Mgrid = (2*.0254/415.3)* subs(subs(Mx,p0,p0_computed),x,xgrid);
Vgrid = (2*.0254/1578)* subs(subs(Vx,p0,p0_computed),x,xgrid);

% Plot moment diagram
plot(xgrid/.0254,Mgrid/.0254,'R',xgrid/.0254,-Vgrid/.0254,'B')
hold on
xline(-18);
xline(-13.5);
xline(-9);
xline(-4.5);
xline(0);
plot(xgrid/.0254,-Mgrid/.0254,'R')
plot(xgrid/.0254,Vgrid/.0254,'B')

xlabel('x[in]');
ylabel('y[in]');
title('Shape of Beam') % Title of figure
legend('Moment','Shear','Location','SouthEast'); % Create legend in lower
left corner

%% Wiffle Tree design
% This section does the calculations to design the wiffle tree
% weights of the wiffle tree components in grams
```

```
Loops = 8 * 63;
AL6Bar = 4 * 184;
AL12Bar = 2 * 292;
AL18Bar = 403;
WiffleWeight = (Loops + AL6Bar + AL12Bar + AL18Bar) * 0.00980665 % The
force in N of the wiffle tree

a = 18;
l = 36;
F0 = p0_computed*sqrt(1-(2*a/l)^2);
b=13.5;
F1 = p0_computed*sqrt(1-(2*b/l)^2);
c = 9;
F2 = p0_computed*sqrt(1-(2*c/l)^2);
d = 4.5;
F3 = p0_computed*sqrt(1-(2*d/l)^2);
e = 0;
F4 = p0_computed*sqrt(1-(2*e/l)^2);

% loop placement in inches measured from left support
L1 = (F2*4.5)/(F1 + F2)
L2 = (F3*4.5)/(F3 + F2);
L2 = L2 + 4.5
L3 = (F4*4.5)/(F4 + F3);
L3 = L3 + 9
L4 = (F4*4.5)/(F4 + F3);
L4 = L4 + 13.5
% placement of loops on the right side of the beam measured from right
% support
L5 = 36 - L4
L6 = 36 - L3
L7 = 36 - L2
L8 = 36 - L1

%6 inch bars
B61 = (F2*6)/(F1 + F2)
B62 = (F4*6)/(F4 + F3)


% 12 inch bar
F12 = F1 + F2;
F34 = F3 + F4;
LB1 = (F34*12)/(F12 + F34)  % from left side
```

- **CODE 2:**

```
      %% ASEN 2001 - Lab 3 Group 6
% Ajay Dhindsa
%
%
%
%% Housekeeping
clear;
clc;
close all
E_F = 0.035483*10^9 ; % Pa Foam
E_W = 3.2953*10^9 ; % Pa Wood
%% Read-in Data
original_data = readtable('ASEN_2001_Lab_3_Test_Data.xls');
LabGroup = table2array(original_data(1:21,1));
NumData = table2array(original_data(1:21,2:5));

original_data(7,:) = [];  % Group 07 - Bad Data - No Breaking
original_data(10,:) = []; % Group 11 - Bad Data - No Comment
original_data(22:33,:) = []; % Getting rid of unncessary comments at
bottom


BendFailure =
contains(original_data.CommentOnFailure,{'Bending','bending'});
ShearFailure =
contains(original_data.CommentOnFailure,{'Shear','shear','perpendicular'})
;

BendData = NumData(BendFailure,:);
ShearData = NumData(ShearFailure,:);
%% Moment and Shear Calculations
BarLength = 36/39.37; % in to m
FoamLength = (3/4)/39.37;
MaxBendLength = BarLength - 2 * BendData(:,2); % meter
MaxShearLength = BarLength - 2 * ShearData(:,2); % meter
WoodLength = (1/32)/39.37;
CSWidth = 4/39.37;
CSBendDataWidth = BendData(:,3);
CSShearDataWidth = ShearData(:,3);


%[BendFail_ShearDiagram, BendFail_MaxShear, BendFail_MomentDiagram] =
diagram(BendData);
%[ShearFail_ShearDiagram, ShearFail_MaxShear, ShearFail_MomentDiagram] =
diagram(ShearData);
BendFail_ShearDiagram = {};

for i = 1:length(MaxBendLength)
    syms x
```

```
    BendFail_ShearDiagram{i} = piecewise(0 < x < BendData(i,2),
BendData(i,1)/2, BendData(i,2) < x < BendData(i,2) + MaxBendLength(i), 0 ,
MaxBendLength(i) + BendData(i,2) < x < BarLength, -BendData(i,1)/2);
    BendFail_MaxShearFail(i) =
double(max(abs(subs(cell2sym(BendFail_ShearDiagram(i)),linspace(0,BarLengt
h,1000)))));
    BendFail_MomentDiagram(i) = piecewise(0 < x < BendData(i,2),
BendData(i,1)/2 * x, BendData(i,2) < x < MaxBendLength(i) + BendData(i,2),
BendData(i,2) * BendData(i,1)/2, MaxBendLength(i) + BendData(i,2) < x <
BarLength, (-BendData(i,1)/2 * (x - BarLength)));
end

ShearFail_ShearDiagram = {};

for i = 1:length(MaxShearLength)
    syms x
    ShearFail_ShearDiagram{i} = piecewise(0 < x < ShearData(i,2),
ShearData(i,1)/2, ShearData(i,2) < x < ShearData(i,2) + MaxShearLength(i),
0 , MaxShearLength(i) + ShearData(i,2) < x < BarLength,
-ShearData(i,1)/2);
    ShearFail_MaxShearFail(i) =
double(max(abs(subs(cell2sym(ShearFail_ShearDiagram(i)),linspace(0,BarLeng
th,1000)))));
    ShearFail_MomentDiagram(i) = piecewise(0 < x < ShearData(i,2),
ShearData(i,1)/2 * x, ShearData(i,2) < x < MaxShearLength(i) +
ShearData(i,2), ShearData(i,2) * ShearData(i,1)/2, MaxShearLength(i) +
ShearData(i,2) < x < BarLength, (-ShearData(i,1)/2 * (x - BarLength)));
end
%% Plots
figure(1);
fplot(BendFail_MomentDiagram, [0, 1])
title('Moment Diagrams for Bending Failure')
xlabel('Distance (m)')
ylabel('Force (N)')
grid minor

figure(2)
fplot(BendFail_ShearDiagram, [0, 1])
title('Shear Diagrams for Bending Failure')
xlabel('Distance (m)')
ylabel('Force (N)')
grid minor

figure(3)
fplot(ShearFail_MomentDiagram, [0, 1])
title('Moment Diagrams for Shear Failure')
xlabel('Distance (m)')
ylabel('Force (N)')
grid minor
```

```
figure(4)
fplot(ShearFail_ShearDiagram, [0, 1])
title('Shear Diagrams for Shear Failure')
xlabel('Distance (m)')
ylabel('Force (N)')
grid minor
```