

# ASEN 2003 LAB 2: BOUNCING BALL EXPERIMENT

*ASEN 2003: Introduction to Dynamics and Systems*  
*University of Colorado at Boulder*

**Assigned** Thursday, Jan. 30  
**Lab Due:** Tuesday, Feb. 11 at 11:59 PM

## I. Objectives

- Gain physical understanding of direct central impact problems and coefficient of restitution
- Derive expressions for coefficient of restitution
- Perform experiments to explore variations of measured results
- Assess experimental results

## II. PROBLEM STATEMENT

When a ball that is not spinning is dropped vertically on a horizontal surface we model the dynamics as a direct central impact of a particle. If the ball is released from rest at an initial height  $h_0$ , the height of subsequent bounces and the total time the ball bounces may be used to estimate the coefficient of restitution of the ball and floor. In this lab assignment, your tasks are to derive the expressions for  $e$  as a function of ball heights, time between bounces, and time to stop bouncing, measure these values several times for different balls, compute and compare your derived estimates of  $e$ , and assess these results based upon experimental error.

## III. THEORY

A ball is released from rest at a height,  $h_0$ , above a horizontal surface. The height of the center of mass for the ball after  $n$  subsequent bounces is termed  $h_n$ . The time required for a single bounce is  $t_n$  and for the ball to stop bouncing is  $t_s$ . (See Figure 1 below)

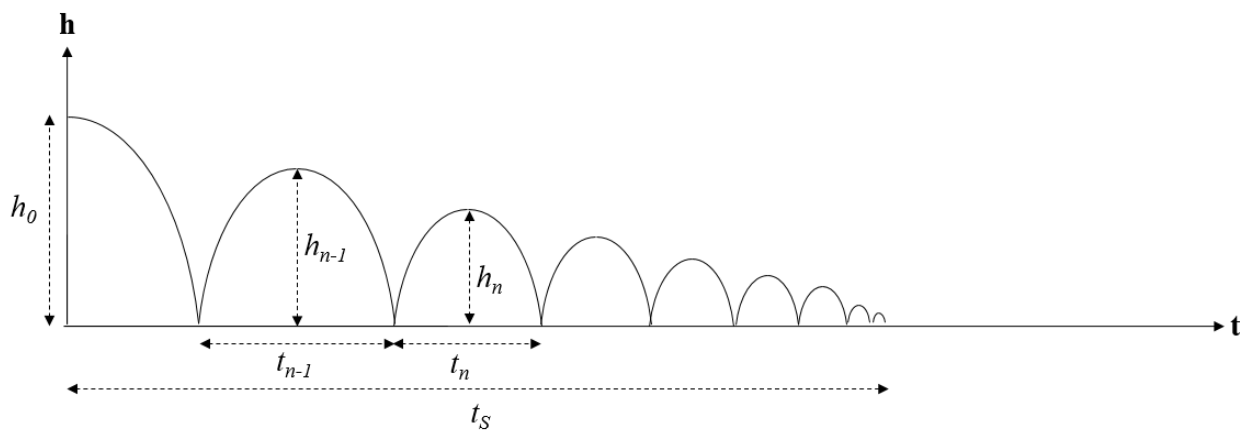


Figure 1. Ball Height Versus Time for a Sequence of Bounces

Starting with the principles of energy and momentum for direct central impacts, the coefficient of restitution can be estimated from three different methods. Derive the expressions given below for the coefficient of restitution for each method. For each method also calculate the sensitivity of the estimate to measurement error. We will be using propagation of errors, if you want to go back and read your notes from ASEN 2012. The sensitivity maps the errors in the measured quantities (i.e. the height or time) to errors in the final estimate of the coefficient of restitution.

Given a ball falling vertically on a horizontal surface, the coefficient of restitution can be derived in various forms. In this lab the three proceeding methods will be used as shown in Equations 2 - 4. Each of these derivations begin with the general equation for the direct central impact coefficient of restitution shown in Equation 1 where  $v_A$  and  $v'_A$  represent the velocity of the Earth before and after the collision.

$$e = \frac{v'_B - v'_A}{v_A - v_B} \quad (1)$$

#### Method 1: Height of Bounce

Estimate of  $e_{height}$  from height of bounces:

$$e = \left( \frac{h_n}{h_{n-1}} \right)^{\frac{1}{2}} = \left( \frac{h_n}{h_0} \right)^{\frac{1}{2n}} \quad (2)$$

#### Method 2: Time of Bounce

Estimate of  $e_{bounces}$  from time of two adjacent bounces:

$$e = \left( \frac{t_n}{t_{n-1}} \right) \quad (3)$$

#### Method 3: Time to Stop

Estimate of  $e_{stop}$  from time to stop bouncing:

$$e = \frac{t_s - \sqrt{\frac{2h_0}{g}}}{t_s + \sqrt{\frac{2h_0}{g}}} \quad (4)$$

## IV. PROCEDURE & RESULTS

Using equipment such as your cell phone, a stop watch, and/or the equipment provided by the PILOT, design experiments to calculate the coefficient of restitution from each of the three methods presented above using one of the supplied ping pong balls or a ball of your choice. Perform at least ten trials of each experiment. Note, the term experiment refers to the application of one of the methods described above to the ball. Thus, you will perform 3 total experiments with at least ten trials per experiment.

After performing the experiments and analyzing the data (see below), identify the method that seems to provide the best results. Your lab report should include characteristics of the ball used in the trials (size, mass, etc).

Finally, conduct three trials with a ping pong ball and record each trial on a cell phone camera or use the provided video footage. After the three trials are conducted, use one of two image tracking software toolkits to determine the coefficient of restitution using all three methods. See Appendix A below for information of the tracking software.

For each experiment write up a clear and concise description of the experimental procedure you developed, including sketches as necessary. The description should be clear enough that other groups could perform your experiments from it. The description can take the form of a numbered or bulleted list where each item clearly describes one step in the process. Describe how you came up with the experiment, justifying your choice of measuring devices and data collection methods. Also, describe important sources of error in your experiments.

## V. ANALYSIS

All computation for this lab is to be done by hand or using MATLAB. For each experiment, calculate the mean, median, and standard deviation, for the data you collected. Describe and compare your results for the different experiments. How consistent are the results of different trials & methods for determining  $e$  for the same ball?

Compute the coefficient of restitution using the three prescribed methods for the three video recorded trials. How do the resulting coefficient of restitution from the recorded trials compare to the original method 1, 2, and 3 trials? Is there more or less error associated with the recorded videos.

Error analysis is a significant component of any experimental study. What are the sources of experimental error in measuring the bounce heights and the stop time? For the experiments you designed, which have the greatest measurement error(s)? Attempt to quantify the measurement error(s) for each experiment. Error in the collected data creates

error in the final estimate of the coefficient of restitution. Calculate the error you expect in the final estimate for the coefficient of restitution. Can you use this error analysis to suggest refinements to your experiment design?

Based on the experimental data and the error analysis, what is your final estimate of the coefficient of restitution? Include bounds on the accuracy of this estimate and justify its calculation.

**Summary of data to be collected:**

- 10 trials for method 1 with ping pong ball
- 10 trials for method 2 with ping pong ball
- 10 trials for method 3 with ping pong ball
- 3 trials for image tracking software

## VI. REPORT CONTENTS

**Title Page** - Lab # and Title, Course Number, Student Names, Date Submitted

### Theory

- Restate Equations 2 -4, with a figure to show what the variables mean.
- State the sensitivity of Equations 2 -4 to measurement errors.
- Utilize the appendices to show the full step by step mathematical derivation but only show and discuss the key parts in this section.

### Procedures

- Provide qualitative descriptions of the experimental procedures
- Provide a step by step description of each experiment
- Explain how you designed each experiment
- List the major sources of error in each experiment

### Results

- List results from the preliminary experiments, collecting data from at least 10 trials per experiment
- Calculate statistics for results
- List results for final experiment with your chosen method and calculate statistics

### Performance Analysis

- Compare and discuss results obtained for the different experiments
- Perform error analysis and compare expected results to data
- Present final estimates of the coefficient of restitution

**Appendix A** - Put your team member participation table in this section.

**Appendix B** - Derivations of the three different methods for finding the coefficient of restitution.

**Appendix C,etc** - Include any MATLAB code you used to compute values or generate plots

### A. Report Grading

10	Theory
20	Procedures
10	Results
25	Analysis
20	Appendices
15	(Style and Clarity - includes title page, table of contents, organization, grammar, spelling)
100	

## Appendix A: Image Tracking Software

As mentioned previously, each group will conduct three trials with a ping pong ball, recording each trial with a camera. After the trials are conducted, the free Tracker software package from Physlets will be used to conduct image tracking on the ping pong ball. To download the Tracker software, make sure your computer is running Java 1.6 or higher and then download the software from the following link: <https://physlets.org/tracker/> **For a complete tutorial of how to use the Tracker software package refer to the Bouncing\_Ball\_Tracker\_Tutorial.PDF file on Canvas.**

Alternatively, you may use the PILOT computer pre-installed Logger Pro software to perform your image analysis. Note: Logger Pro is **ONLY** available on the PILOT computers and is not available for personal computers. **For a complete tutorial of how to use the Logger Pro software package refer to the Bouncing\_Ball\_Logger\_Pro.PDF file on Canvas.**

## Appendix B: Coding Notes

The coding needed for this lab is far less onerous than the roller coaster lab. The main requirements: name and purpose should be provided for each piece of code. Units and variable quantities should be defined at the top of all code (or within the code if that makes more sense).

Once the coefficient of restitution has been computed, the statistical analysis is –for the most part – the same, i.e. mean, standard deviation, standard deviation of the mean.

*No random constants in your code.* Sometimes there are constants that mean something to you, but they mean nothing to the rest of us. If the number of values in your file is 72, then say so. Define it as a variable instead of distributing 72 sporadically in your code.

*Comment your code.* I was given a copy of code written for this lab last year. Literally the only comment in the code is %Generate plots. This is not helpful.