**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans: False, more employees are between 38 to 44. The standard deviation is 6, it means most of the values are in the range of 32 to 44. Hence number of employees being above 44 is very less.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: False, Most likely false. 36 mployees out of 400 is 9% of total employees. But most of the employees are already between 32 to 44. So to get 36 employees ofr training program may be impossible. New recruits are necessary to have training program under age 30.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: Meaning of 2X1:

µ, in 2X1 = µ + µ = 2µ

σ2 in 2X1 = σ2 + σ2 = 2σ2

on squaring, it will be 4σ2

Hence 2X1 = N(2µ + 4σ2 )

Meaning of (X1 + X2 ) :

µ, in X1 +X2 = µ + µ = 2µ

σ2 in X1 +X2 = σ2 + σ2 = 2σ2

Hence X1 +X2 = N(2µ + 2σ2 ).

Hence, The difference between 2X1 is and (X1 + X2) is 2σ2

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

According to z-score normal distribution, 99% value lie in standard of 2.33 deviation, means +2.33 and -2.33. Hence we can say that values of a and b are:

a = 100 - 2.33\* 20 = 53

b= 100 + 2.33 \* 20 = 147.

Hence Option D is the answer as it is the closest.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Total mean Profits(x) = 5 + 7 = 12 million dollars

Variance = 32  + 42  = 9 + 16 = 25 Million Dollars

Hence, Standard deviation = 5 Million Dollars

To calculate the range of 95% probability

Let ‘p’ be the lowest point of the range and ‘q’ be the highest point of the profit range.

In a standard distribution, the critical values are approximately +or – 1.96 to contain a 95% range of confidence.

P = 12 – 1.96 \* 5 = 2

Q = 12 + 1.96\* 5 = 22