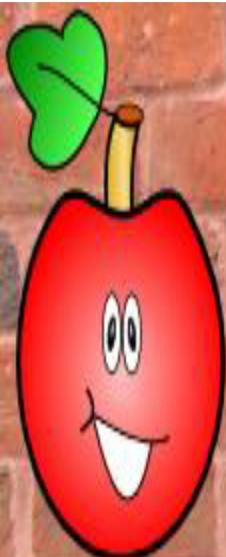


# AVERAGES



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~~General equation~~

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# 1) INTRODUCTION



Average of n values is equal to the sum of n values divided by the total number of values (n)

Averages = Sum of Observations /Number of Observation

$$\text{Avg} = \text{Sum}/n$$

$$\begin{aligned} & \text{Average of } x, y, z \\ &= \frac{x+y+z}{3} . \end{aligned}$$

# 1.1 FORMULA

Numbers <i>natural</i>	Sum	Average (Sum/n)
First n numbers	$n(n+1)/2$	$(n+1)/2$
First n odd numbers	$n^2$	$n$
First n even numbers	$n(n+1)$	$(n+1)$
First n square numbers	$n(n+1)(2n+1)/6$	$(n+1)(2n+1)/6$
Consecutive numbers	$n(\text{first term} + \text{last term})/2$	$(\text{first term} + \text{last term})/2$



↓  
Arithmetic Progression

It's a sequence of numbers where the difference of any 2 numbers remains the same OR the next term is obtained by adding a constant number to the previous term

eg → 1, 2, 3, 4, 5, ... → AP  
 5, 10, 15, 20, 25, ... → AP

Q) What is the average of first 100 multiples of 8?

$$\rightarrow 8 \times 1, 8 \times 2, 8 \times 3, \dots, 8 \times 100$$

$$\text{Av.} = \frac{8 + 800}{2} = \frac{808}{2} = 404$$

$$\text{Av.} = \frac{\text{Sum}}{n}$$

Q) What is the average of first 150 natural numbers?

$$\Rightarrow \frac{150 + 1}{2} = 75.5$$

**Note:** Average of consecutive numbers can also be written as  
 $(2^{\text{nd}} \text{ term} + 2^{\text{nd}} \text{ last term})/2$  or  $(3^{\text{rd}} \text{ term} + 3^{\text{rd}} \text{ last term})/2$  and so on.

For odd number of consecutive values, the middle term will be the average.

**Example 1.** Find the average of following number

1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21

- A) 9
- B) 11
- C) 12
- D) 13

$$Av = \frac{1+21}{2}$$





**Example 2.** The average of 20 numbers is zero. Of them, at the most, how many may be greater than zero?

- A) 0
- B) 1
- C) 10
- D) 19

Let the 20 nos be  $x_1, x_2, x_3, x_4, \dots, x_{20}$

$$\frac{x_1 + x_2 + x_3 + \dots + x_{20}}{20} = 0 \Rightarrow x_1 + x_2 + x_3 + \dots + x_{20} = 0$$

$x_1$  to  $x_{19} \rightarrow \text{Sum} = a$

$x_{20} \rightarrow \text{Value} = -a$



**Example 3.** The average of 7 consecutive numbers is 20.  
The largest of these numbers is :

- A) 27
- B) 26
- C) 24
- D) 23



## 2) TYPE 1 PROBLEMS COMBINED AVERAGE

Class A

0 ← Avg.

Class B

100 ← Avg.

What is the combined average? ✓

50 marks?? ✓

NO

→ will be 50 only  
when both the  
classes have equal no.  
of Student

We cannot determine the average without knowing the number of students in each class

The combined average depends on the number of students and the average in each class

Example: There are 36 students in class A whose average is 30kg and 24 students in class B whose average is 40kg. What will be the average if the classes are combined?

## 2.I GENERAL EQUATION → Weighted Average

$$\text{Avg}_{36} = 30 \quad \text{Avg}_{24} = 40$$

$$\text{Total weight of class A} = 30 \times 36 \rightarrow \text{Total sum of class A} = (\text{Av.}) \times (\text{No. of observ'})$$

$$\text{Total weight of class B} = 40 \times 24$$

✓ Overall average =  $\frac{(\text{Total weight of class A}) + (\text{Total weight of class B})}{\text{Number of students in class A} + \text{Number of students in class B}}$

$$= \frac{(30 \times 36) + (40 \times 24)}{36 + 24} \rightarrow \frac{30 \times 36 + 40 \times 24}{36 + 24} = 34$$

★ D.v. Average = 
$$\frac{S_A + S_B}{n_A + n_B} = \frac{n_A \times A_A + n_B \times A_B}{n_A + n_B}$$

Total sum of Class A =  $63 \times 32$ , for class B =  $21 \times 44$

**Example 5.** In class A there are 63 students whose average is 32, and in class B there are 21 students whose average is 44, then find the overall average?

- A) 33  
B) 35  
C) 36  
D) 38

$$\frac{63}{21} = 3$$

Method 1 :- Overall Average =  $\frac{63 \times 32 + 21 \times 44}{63 + 21}$

Method 2 :- Find the ratio of the no. of Students

$$63 : 21 \rightarrow 3 : 1$$



$$= \frac{2016 + 924}{84}$$

$$= \frac{2940}{84} = 35$$

Now, you can assume that there were 3 students in class A for every 1 student in class B

$$\text{Overall Avg} = \frac{3 \times 32 + 1 \times 44}{3+1} = \frac{96 + 44}{4} = \frac{140}{4} = 35$$

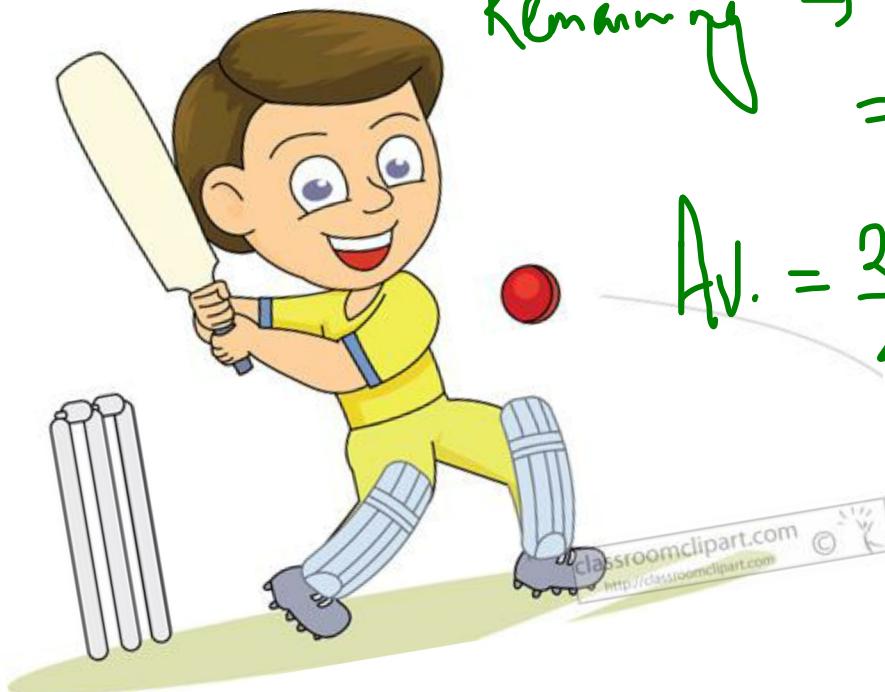
**Example 6.** In a first 10 overs cricket game run rate was 3.2. what should be run rate in the remaining 40 overs to reach the target 282 runs?

- A) 6.25
- B) 6.5
- C) 6.75
- D) 7

1<sup>st</sup> 10 overs → 32 runs.

Remaining →  $282 - 32 = 250$  runs

$$\text{Av.} = \frac{250}{40} = 6.25$$



### 3) TYPE 2 PROBLEMS

Change in average → Difficult

- A Example: Average of 5 students marks is 30. If one student having 90 mark is added to the team then what will be the new average ?

#### 3.I GENERAL EQUATION

✓ Avg<sub>5</sub> = 30

No of students = 5 ✓

$$\text{Sum} = (\text{Average}) \times (n)$$

Sum<sub>5</sub> = 30 × 5

= 150 ✓

Sum<sub>6</sub> = 150 + 90

= 240 ✓

Avg<sub>6</sub> = 240 / 6

= 40 ✓



## **3.II EQUAL DISTRIBUTION METHOD:**

→ All the problems in this concept are solved by assuming all the values as average itself.

**Step1:** Assume all the values to be 30.

30 30 30 30 30

If the new mark is also 30 then the average will remain the same.

30 30 30 30 30 30

**Step2:** Finding the extra values-

But the actual new mark is 90, which means extra 60 is added to the values.



Extra Value  
= New no - Old average

### Step3: Distributing the extra values equally-

The extra 60 should be divided equally among 6 values as 10 each.

---

$$\begin{array}{ccccccc} 30 & 30 & 30 & 30 & 30 & 30 \\ +10 & +10 & +10 & +10 & +10 & +10 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 40 & 40 & 40 & 40 & 40 & 40 \end{array}$$

Change in average =  $\frac{\text{Extra Value}}{\text{fixed no. of people}}$

$$= \frac{60}{6} = 10$$

∴ the new average is 40

Example 7. Average of 4 students marks is 50 and one student having marks has 200, is added to the team , what is new average ?

- A) 70
- B) 80
- C) 90
- D) None

Step 2:- Extra Value =  $200 - 50 = 150$

Step 3:- Change in average =  $\frac{\text{extra}}{n} = \frac{150}{5} = 30$

New Avg =  $50 + 30 = 80$



**Example 8.** Average of 6 students marks is 60 , what is the new average if a student of marks 110 is taken out ?

- A)50
- B)55
- C)60
- D)None

Step 2:- Extra Value =  $110 - 60 = 50$  marks  
and

Step 3:- Change in average =  $\frac{\text{Extra}}{n} = \frac{50}{5} = 10$

$$\begin{aligned}\text{New Avg} &= 60 - 10 \\ &= 50\end{aligned}$$

**\*If the number of values is unknown:**

$$\text{Avg}_n = 30$$

$$\text{Avg}_{n+1} = 40$$

New number = 90

Old values      **30    30    30.....**  
                     **+10    +10    +10.....**

New values      **40    40    40.....**



Total extra value added is 60 (30 → 90)

This 60 is divided as 10 each which means there should be  $60/10 = 6$  values

∴ New values,  $n+1 = 6$

Old values,  $n = 5$

**Example 9.** A batsman having average 40 makes 90 runs in his last inning thereby his average increases by 2. Find the number of matches he has played.

- A)10
- B)50
- C)25
- D)24



**Example 10.** The average marks of 12 students increases by 3 if a new student having mark 79 is included. The average mark of the students is?

- A) 37
- B) 40
- C) 43
- D) 82

**Example 11.** The average weight of 8 men having average weight 40 kg is increased by 2 kg when a new man is included. The weight of the new man is

- A) 56
- B) 58
- C) 96
- D) 98

Step 1 :- Assume all values to be 40 kg each.

Step 2 :- Extra Value = ?

$$\text{Change in average} = \frac{\text{Extra Value}}{n}$$

$$\Rightarrow 2 \times 9 = \text{Extra Value} = 18 \text{ kg.}$$





THANKS  
FOR  
LISTENING