

PERCENTAGES

Averages
Ratio
TSD
TW
SI-CL
Data Interpretation

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Introduction

- ▶ What is percentage?

In mathematics, a **percentage** is a number or ratio expressed as a fraction of 100..

- ▶ Why percentage?

It helps in comparison by making the same base value for every comparison.

15 out of 50

$$\rightarrow \frac{15}{50} \times 100 = 30\%$$

55 out of 200

$$\frac{55}{200} \times 100 = 27.5\%$$

Shortcuts

Shortcut 1: Splitting of values

a) **20% of 80 = ?**

$$\begin{array}{r} \cancel{100\% \text{ of } 80 = 80} \\ \hline \cancel{10\% \text{ of } 80 = 8} \end{array}$$

$$20\% \text{ of } x = 2 \times 10\% \text{ of } x$$

20% of 80 = 16

$$\begin{array}{r} \downarrow \\ 2 \times 8 \end{array}$$

Q. Try 30% of 60

- A. 15
- B. 18**
- C. 21
- D. None of these

$$\begin{aligned}10\% &\rightarrow 6 \\30\% &\rightarrow 3 \times 10\% \\&= 3 \times 6 = 18\end{aligned}$$

Q) 30% of 62 =

$$10\% \rightarrow 6.2$$

$$\begin{aligned}30\% &\rightarrow 3 \times 6.2 \\&= 3 \times [6 + 0.2] = 18.6\end{aligned}$$

b) 15% of 80 = ?

$$\begin{array}{l} \text{10% of } 80 = \underline{\underline{8}} \\ \text{5% of } 80 = 4 \end{array} \quad \left. \begin{array}{l} \\ \div 2 \end{array} \right\} \div 2$$

15% of 80 = 12



$$8 + 4$$

$$15\% \rightarrow 10\% + 5\%$$

Q. Try 15% of 60

- A. 6
- B. 7
- C. 8
- D. 9

$$\begin{array}{r} 10\% \rightarrow 6 \\ 5\% \rightarrow 3 \\ \hline 15\% \rightarrow 9 \end{array}$$

 =

$$15\% \text{ of } 62 \rightarrow 10\% = 6.2$$

$$5\% = 3.1$$

$$15\% = 9.3$$

$$12 \rightarrow 10 + 2$$

$$12\% \rightarrow 10\% + 2\% \quad \left| \begin{array}{l} 10\% = 0.1 \\ 2\% = 0.02 \end{array} \right.$$

$$\frac{\text{Total } 2\%}{2 \times 1\%}$$

c) **12% of 80 = ?**

$\checkmark 10\% \text{ of } 80 = 8$

$2\% \text{ of } 80 = 1.6$

12% of 80 = 9.6 \checkmark

$1\% \text{ of } 80 = 0.8$

Try 13% of 60

- a. 7.7
- b. 7.8**
- c. 9.1
- d. None

$$10\% + 3 \times 1\%$$

$$\downarrow \qquad \downarrow$$

$$6 \qquad 3 \times 0.6$$

$$= 1.8$$

Q) 11.2% of 60

$$\rightarrow 10\% + 1\% + 0.2\%$$

$$\rightarrow 6 + 0.6 + 0.12$$

$$\rightarrow 6.72$$

$$0.2\% \rightarrow 2 \times 0.1\%$$

$$\rightarrow 2 \times 0.06$$

$$\rightarrow 0.12$$

Q) 11.2% of 62

$$\rightarrow 10\% + 1\% + 0.2\%$$

$$\rightarrow 6.2 + 0.62 + 0.124$$

$$\rightarrow 6.944 //$$

$$45\% \begin{cases} \nearrow 50\% - 5\% \\ \searrow 40\% + 5\% \end{cases}$$

d) 45% of 80 = ?

$$\div 10 \left[\begin{array}{l} 50\% \text{ of } 80 = 40 \\ 5\% \text{ of } 80 = 4 \end{array} \right] \div 10$$

$$45\% \text{ of } 80 = 36 \left[40 - 4 \right]$$

Try 45% of 60

- A. 60
- B. 30
- C. 36
- D. 24 27

$\begin{array}{l} 55\% \text{ of } 80 \\ \downarrow \\ 50\% + 5\% \\ \downarrow \\ 40 + 4 = 44 \end{array}$	$\begin{array}{l} 66\% \text{ of } 80 \\ \downarrow \\ 60\% + 6\% \\ \downarrow \\ 48 + 4.8 \\ = 48 + [4 + 0.8] \\ = 52.8 \end{array}$
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Shortcut 2: Remembering some ratios for %

How do you solve **12.5% of 80 = ?**

10% + 2% + 0.5% ?

— How much time will the following question take?

12.5% of 8.8 = ?

Now this

16.66% of 3.6 = ?

$$25\% \text{ of } 64 = \frac{1}{4} \times 64 \\ = 16.$$

12.5% of 80 = ?

$$\begin{array}{l} \xrightarrow{\div 2} 1 = 100\% \\ \xrightarrow{\div 2} 1/2 = 50\% \\ \xrightarrow{\div 2} 1/4 = 25\% \\ \xrightarrow{\div 2} 1/8 = 12.5\% \end{array}$$

$$\therefore 12.5\% \text{ of } 80 = 1/8 \text{ of } 80 \\ = 10 \quad \checkmark$$

Similarly **12.5% of 8.8** = $\frac{1}{8}$ of 8.8
= **1.1**

16.66% of 3.6 = $\frac{1}{6}$ of 3.6
= **0.6**

Lets learn to convert few more fraction values

1 = **100%**

1/2 = **50%**

1/3 = **33.33%**

1/4 = **25% (half of 1/2)**

1/5 = **20%**

1/6 = **16.66% (half of 1/3)**

1/7 = **14.28%**

1/8 = **12.5% (half of 1/4)**

1/9 = **11.11% (1/3rd of 1/3)**

1/10 = **10%**

1/11 = **9.09%**

$$\begin{array}{l} 100\% \rightarrow 1 \\ \div 3 \left(33.33\% \rightarrow \frac{1}{3} \right) \div 3 \\ \div 3 \left(11.11\% \rightarrow \frac{1}{9} \right) \div 3 \end{array}$$

Note: $\frac{1}{9} \times$ will be in the multiples of 11
 $\frac{1}{11} \times$ will be in the multiples of 9

- ▶ $1/9 = 11.11\%$, $2/9 = 22.22\%$, $3/9 = 33.33\%$
- ▶ $1/11 = 9.09\%$ $2/11 = 18.18\%$, $3/11 = 27.27\%$
- ▶

Shortcut 3:

How do you solve **62% of 150 = ?**

10% to 60%

1% to 2%

Then **60% + 2%** ?

No

a% of b can be written as b% of a- YES or NO???

).

a% of b can be written as b% of a

Proof: $a\%$ of $b = b\%$ of a

$$\frac{a}{100} \times b = b \frac{\cancel{100}}{\cancel{100}} \times a$$

$$\rightarrow ab/100 = ab/\cancel{100}$$

$$\frac{a}{100} \times b$$

$$\begin{aligned}\therefore 62\% \text{ of } 150 &= 150\% \text{ of } 62 \\&= (100\% + 50\%) \text{ of } 62 \\&= 62 + 31 \\&= 93 \quad \checkmark\end{aligned}$$

$$150\% \rightarrow 100\% + 50\%,$$

$\downarrow \quad \downarrow$

62 31

- Why should we interchange 62 and 150?
- Because splitting 150 is easier than splitting 62
- So whenever the right hand side value is easier, interchange the values.

$$\begin{aligned}Q) \quad 8.8\% \text{ of } 12.5 &= 12.5\% \text{ of } 8.8 \\&= \frac{1}{8} \times 8.8 = 1.1\end{aligned}$$

Q. Try 84% of 250 → 250% of 84
↓
 $2 \times 100\% + 50\%$

- a. 168
- b. 200
- c. 210
- d. 250

$$250\% \text{ of } 84 = 25\% \text{ of } 840$$

$$= \frac{1}{4} \times 840$$

$$= 210$$

Q. Try 72% of 90

- A. 63
- B. 64.8
- C. 65.2
- D. None

$$9 \times 10\%$$

$$= 9 \times 7.2$$

$$= 9 \times [7 + 0.2]$$

$$= 9 \times 7 + 9 \times 0.2$$

$$= 63 + 1.8 = 64.8$$

90% of 72

100% - 10%

$$\downarrow \quad \downarrow$$
$$72 - 7.2$$

$$= 72 - [7 + 0.2]$$

$$= (72 - 7) - 0.2$$

$$= 64.8$$

$$\text{Percentage Change} = \frac{\text{Actual change}}{\text{Initial value}} \times 100.$$

► PERCENTAGE INCREASE and PERCENTAGE DECREASE

EXAMPLE- Lets say you have a factory which produced 20 cars in Year 1 and 25 cars in Year 2.

What is the percentage increase from the 1st year to the 2nd ?

What is the increase? 5

From where it is increasing? 20

$$\begin{aligned}\text{Percentage increase} &= 5/20 * 100 \\ &= \frac{1}{4} * 100 \\ &= 25\%\end{aligned}$$

What is the percentage decrease from the 2nd year to the 1st?

What is the decrease? 5
From where it is decreasing? 25

$$\begin{aligned}\text{Percentage decrease} &= 5/25 * 100 \\ &= \frac{1}{5} * 100 \\ &= 20\%\end{aligned}$$

Example: Sachin makes \$5M a week from his job. He earns a raise and now makes \$6M a week. What is the percent increase?

- A. 16.66%
- B. 20%
- C. 25%
- D. 50%

Successive Percentage Change

Example: A car is moving at some constant speed. At first it increases its speed by 25% and then again it increases its speed by 20%. What is the overall percentage increase.

Method 1: Initial speed of the car = x

Speed of the car after 1st increase = $x + 25\% \text{ of } x = 1.25x$

Speed of the car after 2nd increase = $1.25x + 20\% \text{ of } 1.25x = 1.50x$

Initial speed = x

Final speed = $1.50x$

Percentage increase = 50%

Method 2:

Assume the initial speed of the car as 100kmph

Initial speed of the car = 100

Speed of the car after the 1st increase = $100+25=125$

Speed of the car after the 2nd increase = $125+2(12.5)= 150$

Initial speed = 100

Final speed = 150

Percentage increase = 50%

Method 3: Shortcut

If the 1st increase/ decrease is **a%** and the 2nd increase/decrease is **b%**, then the overall increase/decrease % will be

$$\mathbf{a + b + ab/100 \%}$$

In this question **a = 25%** and **b= 20%**

$$\text{Overall increase/decrease} = 25 + 20 + (25)(20)/100$$

$$= 25 + 20 + 5$$

$$= \mathbf{50 \%}$$

Note: If a or b is increase, then include +ve sign

If a or b is decrease, then include -ve sign.

Note : The final answer will be in percentage

Example 5: A city's population was 10,000 at the end of 2008. In 2009, it increased by 25% and in 2010, it decreased by 8%. What was the net percentage change city's population at the end of 2010?

- A. 17%
- B. 15%
- C. 33%
- D. None of these

Solution:

$$25 + (-8) + (25)(-8)/100 \%$$

$$= 25 - 8 - 200/100 \%$$

$$= 25 - 8 - 2 \%$$

$$= 15 \%$$

Question: A fruit seller had some oranges. He sells 70% oranges and still has 420 oranges. How many oranges he had originally?

- A. 1400
- B. 630
- C. 700
- D. 1050

► Solution

From 100% → sold 70%
→ Remaining = 30%.

A/Q, remaining = 420 oranges

$$\Rightarrow \underline{30\%} \rightarrow \underline{420}$$

$$\Rightarrow \underline{10\%} \rightarrow \frac{420}{3} = \underline{\underline{140}}$$

$$\Rightarrow \underline{100\%} \rightarrow \underline{\underline{1400}} //.$$

Question: An agent, gets a commission of 5% on the sales of cloth. If on a certain day, he gets Rs. 12.50 as commission, the cloth sold through him on that day is worth

- A. 125
- B. 250
- C. 500
- D. 1000

Solution

Percent commission = 5 %, Actual commission = Rs 12.5

$$\begin{aligned} \therefore \frac{5\% \text{ of sales}}{10\% \text{ of sales}} &\rightarrow \frac{\text{Rs } 12.5}{\text{Rs } 25} \\ &\times 2 \qquad \qquad \qquad \times 2 \\ 100\% &\rightarrow \text{Rs } 250. \end{aligned}$$

~~✓~~**Question:-** A student has to obtain 33% of the total marks to pass. He got 125 marks and failed by 40 marks. The maximum marks are-

- A. 400
- B. 500
- C. 600
- D. 800

Solution:

Equate percentage value with the marks to get the answer

Percentage Pass mark = 33%

He got 125 marks and need 40 more marks to pass

$$\therefore \text{Pass mark} = 125 + 40 = 165$$

$$\frac{33\%}{\text{Maximum marks}} \rightarrow 165$$

$$\frac{100\%}{\text{Maximum marks}} = 100\%$$

$$33\% \rightarrow 165$$

$$1\% \rightarrow 165/33 = 5$$

$$100\% = 500$$

$$\frac{165}{33} = 5$$

Q. In a test A got 15% of the marks and failed by 7 marks whereas B got 28% and got 32 marks more than the pass mark. What was the pass mark?

- A. 45
- B. 52
- C. 84
- D. 300

Solution: Equate percentage value with the price to get the answer

Percentage of A = 15%

Marks of A = -7 (Deviation from pass mark)

Percentage of B = 28%

Marks of B = + 32

Percentage difference b/w A and B = 13%

Marks difference b/w A and B = 39

$\therefore 13\% = 39 \text{ marks}$

$1\% = 3 \text{ marks}$

Q. A is x% less than B, A is y% less than C. C is k% more than B.
Express k in terms of x and y.

A $\frac{(y-x)100}{100-y}$

B $\frac{(y+x)100}{100-y}$

C $\frac{(y-x)100}{100-x}$

D $\frac{(y-x)100}{100+y}$

Q. In a class 20% of students are below 14 years of age. Out of the remaining students 10% are of the age 14-15 years and ratio of students who are between 15-16 years of age to student above 16 years of age is 3:2. If the number of students who are above 16 years is 72, what is the total number of students in the class?

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A 200

B 250

C 300

D 400

Answer Key

- ▶ Option B



THANK YOU