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Total No. of Questions : 5] [Total No. of Printed Pages : 5

EW-67

B.Tech. I[st] Semester (CSE, IT & Electronics)

Examination, 2021-22

Engeening Mathematics-I

Paper - BE - 101

Time : 3 Hours] **[Maximum Marks : 60**

Note : - Attempt all the questions. All question carry equal marks.
Each questions has an internal choice.

1. (a) Prove that $\int_0^{\pi/4} \log_e(1+\tan x)\,dx = \pi/8 \log_e 2$

(b) Show that $\int_0^\infty \frac{x^c}{c^x} dx = \frac{\lfloor c+1}{(\log c)^{c+1}}$

OR

(a) Evaluate

$$\lim_{n\to\infty}\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\left(1+\frac{n}{n}\right)\right\}^{\frac{1}{n}}$$

(b) Find the volume of the spindle shaped solid generated by revolving the astroid.

$x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis.

2. (a) Evaluate $\lim_{x\to\pi/2} \frac{1+\cos 2x}{(\pi-2x)^2}$

(b) Verify Rolle's theorem for the function $f(x) = x^3 - 6x^2 + 11x - 6$ in the interval $[1, 3]$

OR

(a) Expand the function $f(x) = \sin^{-1}\frac{2x}{1+x^2}$ in ascending powers of x by using maclaurins theorem upto term containing x^5.

(b) Discuss the maxmimum or minimum values of $x = x^3 + y^3 - 3axy$.

3. (a) Test the convergence of the seris :

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots\ldots$$

(b) Discuss the converserice of the series :

$$\sum \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n$$

OR

(a) Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$.

(b) Find the half- range cosine series for the function $f(x) = (x - 1)^2$ in the interval $0 < x < 1$.

4. (a) If F is a field them show that $w = \{(a_1, a_2, 0); a_1, a_2, \in F\}$ is a subspace of $V_3(F)$.

(b) Find a basis fot the subspace spanned by the vectors $(1, 2, 0), (-1, 0, 1), (0, 2, 1)$ in $V_3(R)$.

OR

(a) Show that the system of three vecors (1, 3, 2), (1, - 7 -8),

(2, 1, -1) of $V_3(R)$. is linearly dependent.

(b) If $T : V_3(R) \to V_3(R)$ is linear transformation defined by T (a, b, c) = {3a, a-b, 2a + b + c} then show that $(T^2 - I)(T - 3I) = \hat{0}$.

5. (a) Find the rank of the materix.

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

(b) Find the eigen values and eigen vactors of the maxtrix.

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

OR

(a) Using cayley - Hamiltaon theorem, find the in verse of the matrix.

$$A = \begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$$

(b) Investigate for consitency of the following equations and if possible find the solutions :

$4x - 2y + 6z = 8$

$x + y - 3z = -1$

$15x - 3y + 9z = 21$

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