Simulation of discrete-event of an M/M/1 queuing system with bounded, unbounded queue and verification with Analytical results

Evaluation of accuracy of simulated results with theoretical values having defined and undefined queue boundaries

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***Abstract*—Discrete Event Simulation (DES) is one of the types of event simulation methods where operation of the system is represented as a chronological sequence of events. Each event occurs at an instant of time and can trigger new events to be generated. Between consecutive events, no change in the system is assumed to occur. This contrasts with continuous simulation in which the simulation continuously tracks the system dynamics over time. The simulator is a model of physical system that has changes at precise points in simulated time and it utilizes a mathematical/ logical model of a physical system that portrays state changes at precise points in simulated time. The simulator models a First in first out customer queued output port. On analyzing the results of the system, the main ideology that was derived is the experimental results are within 5% of the theoretical results for the bounded, unbounded queue and accuracy can be further improved to the best by using quantile method to obtain confidence intervals.**

I. Introduction

Simulation is defined as creating a virtual environment to study physical problems. It will be performed when an effective option to do real experiment is not available. To analyze bottleneck of current workflow. The method will not be used when system can be read analytically. One of the various methods of simulation is Discrete-Event Simulation which is dynamic, discrete and stochastic. For example, Simulation for queueing in a post office is DES. Mostly, but not limited to, queueing systems: Factory work flow, Freeway traffic simulation, network traffic simulation. Event oriented systems generate next event and put into event queue to sort. It's simulation time advances to next closest event and generally faster than activity oriented systems.

II. M/M/1 Queueing System

The M/M/1 queue is the classic, canonical queueing model. By itself, it usually isn't the right model for most computer systems, but studying it will develop the analysis techniques that can be used for more flexible models. The three-part notation is the preferred way of describing the parameters of an open queueing model. The first letter refers to the distribution of the interarrival times, the second letter to the distribution of the service times and the final value is the number of servers. The letter M refers to Markovian distribution, that is, to the exponential distribution. Therefore, the M/M/1 queue is a model with exponentially distributed interarrival times – which implies that the arrivals are Poisson – exponentially distributed service times with a single server.

III. Theoretical Analysis of DES of An M/M/1 Queueing System

To analyze the M/M/1 queueing system, FIFO tagged customer method has been used. Considering an arbitrarily chosen customer just arriving to the queue. Customer has been tagged and followed through the queue, adding up all the delays that it encounters. The average total time the customer needs to move through the queue, receive its service, and exit is simply the average of all the individual delay sources it encounters on its trip through the system.

A newly arriving tagged customer must wait for three sources of delay:

1) The residual time of the customer in service, if the queue is occupied at the arrival instant.
2) The time for any customers that are waiting in the queue but not being served at the arrival instant.
3) The time for the tagged customer to get its own service.

The arrivals of the M/M/1 queue are Poisson, so the average state of the queue at the instant of an arrival is simply the long-run average state of the queue. Therefore, the probability that the queue is occupied at an arrival instant is simply U, the utilization and the average number of customers waiting but not being served at the arrival instant is $(\bar{Q} - U)\,\hat{s}$. Thus, the mean waiting time can be calculated by the equation, $\bar{w} = \bar{R} - \hat{s}$ where $\bar{R}$ is the average residence time of the system. And for the single server systems with limited waiting room M/M/1/k is a better model that the unbounded queue. The mean waiting time is calculated by the equation, $w = \bar{Q}/\mu(\mu-\bar{Q})$. So, $Ts = 1/\mu$-

IV. Experimental analysis of DES of An M/M/1 queueing system

The experimental evaluation was carried out having unbounded and finite-size queues. It essentially confirms about the state of equilibrium in a stochastic process. At the equilibrium state, also known as steady state, the behavior of the process is independent of the time parameter and the initial value.

The following diagram and table illustrates the patterns that are observed.

Figure 4.1 Experimental Analysis of DES of an M/M/1 Queueing System

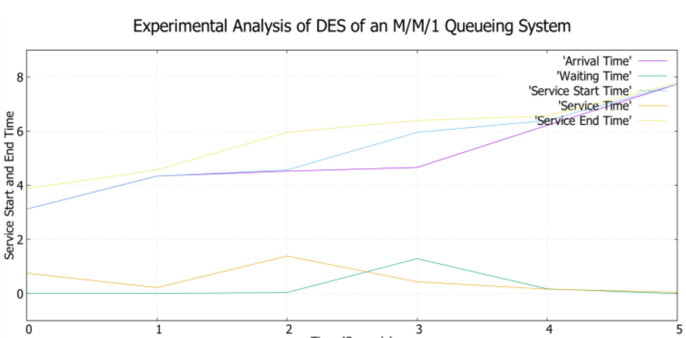

Table 4.1 Analysis of DES of an M/M/1 Queueing System

Arrival Time (S)	Waiting time (S)	Service start time (S)	Service Time (S)	Service end time (S)
3.1262	0	3.1262	0.7576	3.8839
4.3437	0	4.3437	0.2262	4.5700
4.5261	0.0438	4.5700	1.3891	5.9591
4.6643	1.2947	5.9591	0.4393	6.3985
6.2201	0.1783	6.3985	0.1675	6.5661
7.7399	0	7.7399	0.0534	7.7934

Experimental Simulation results are compared with theoretical values calculated using derived formula and they are found to be 0.2528, 0.2438 respectively.

Similarly, the evaluation carried out for unbounded queue was continued with finite queue size and the mean waiting time, 5.5756 obtained through experimental results and 5.5635 obtained theoretically found to be more accurate than the results obtained from the analysis performed where the queue was unbounded.

The following diagram and table illustrates the values observed for evaluation performed with finite queue size,

Figure 4.2 Experimental Analysis of DES of an M/M/1/k Queueing System

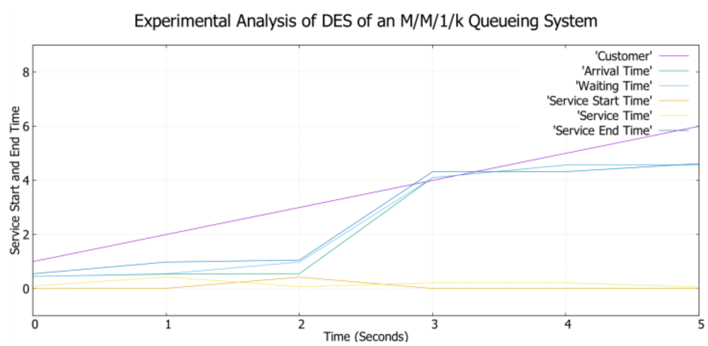

Table 4.2 Analysis of DES of an M/M/1/k Queueing System

Customer	Arrival Time (S)	Waiting time (S)	Service start time (S)	Service Time (S)	Service end time (S)
1	0.4530	0.4530	0	0.0986	0.5517
2	0.5382	0.5517	0.0134	0.4297	0.9814
3	0.5546	0.9814	0.4268	0.0729	1.0543
4	4.1003	4.1003	0	0.2132	4.3136
5	4.5610	4.5610	0	0.2132	4.3136
6	4.5610	4.5610	0	0.0589	4.6199
7	5.3482	5.3482	0	0.4364	5.7846
8	6.9747	7.5882	0.6135	0.4006	7.9885
9	7.3313	7.9888	0.6575	1.4047	9.3936
10	8.7681	9.3936	0.6254	0.4287	9.8223
11	9.9864	9.9864	0	0.3248	10.311
12	11.394	11.394	0	0.1935	11.588

Though the analytical results obtained exceed theoretical values, queue size should be directly proportional to arrival rate to avoid packet loss. And the accuracy can be further improved to the best by using Quantile method to obtain confidence intervals, thereby achieving precision.

References

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[3] Dr. Phillip M. Feldman, *Discrete event simulation* http://phillipmfeldman.org/Python/discrete_event_simulation/