

Assignment 1

Problem 1:

Solution:

Perishable product

Purchase cost (to buy a pint) strawberries = c

Selling Price = p

Quantity = q

Profit = $p - c$

Random variable demand = x

Purchase decision (Morning), Distribution CDF 'F'

Strawberries spoil by EOD, loss = $\$p/\text{pint}$.

Optimal quantity, q (Morning Order), $q = F^{-1}((p-c)/p)$

According to Leignitz Integral Rule, $(d/dq) (E[\min\{X-q, 0\}]) = -F(q)$

Approach:

1. Model Free Simulation
2. Model Based Simulation
3. Model Based Optimal
4. Model Free Optimal (Problem 1 is based on this method of approach).

$$\text{Max } q \ E [p^{\min\{X,q\}} - qc]$$

Where, $p^{\min\{X,q\}} = \text{Total Revenue}$

$$\Rightarrow qc = \text{Total Purchase price.}$$

$$\Rightarrow \text{Min } q \ E [-p^{\min\{X,q\}} + qc]$$

To write simulation, For q in $0 \dots 1000$:

$$\Rightarrow \text{Estimate } E[p^{\min\{X,q\}} - qc]$$

Pick q with the largest estimate,

To test, we can assume values for p and q and use Model Free simulation method.

In Model based simulation, Demand distribution is bounded by a curve setting the limit/range of values, whereas in Model free simulation Demand distribution contain only the values without the limit/range.

Applying Formula, $\text{Min } \{x, y\} - a$

$$\begin{aligned} &\Rightarrow \text{Min } \{x - a, y - a\} \\ &\Rightarrow \text{Max } q \text{ } E[p^{\min\{X - q, 0\}} + (p - c) q] \\ &\Rightarrow (d/dq) (E[p^{\min\{X - q, 0\}} + (p - c) q] \text{ where } (p - c) \text{ can be compared to slope} \\ &\quad \text{formula and extracted compared to } (mx + C). \\ &\Rightarrow (d/dq) (E(\min\{X - q, 0\})) \\ &\Rightarrow (d/dq) \int_q^\infty (X - q)w(x)dx \end{aligned}$$

Using Leignitz Integral Rule,

$$\begin{aligned} &\Rightarrow -F(q)p + (p - c) = 0 \\ &\Rightarrow p - c = F(q)p \\ &\Rightarrow ((p - c)/p) = F(q) \end{aligned}$$

$$\Rightarrow F^{-1}((p - c)/p) = q$$

Problem 2:

Solution:

Non-perishable product

Storing product for next day incurs holding cost

To formulate the problem as an MDP,

1. States = Inventory = 0, 1, 2,
2. Actions = Orders
3. Quantity, $q = 1, 2, 3, \dots$ Where q is according to inventory size
4. Rewards = Revenue
5. Transition = Demand
(100 = 100 - X)

$$R(s, a, s')$$

Candy – does not spoil and can be stored.

Variables, h = holding cost, since unsold quantity.

$$\begin{aligned} &\Rightarrow \text{Optimal quantity, } q \text{ to order in case of non-perishable / unsold product varies} \\ &\quad \text{by demand or sale and size of inventory.} \\ &\Rightarrow \text{To represented as an MDP:} \end{aligned}$$

State = X_t on day $t \geq 0$ = size of inventory.

$X = \{0, 1, \dots, M\}$, $M \in \mathbb{N}$, Maximum Inventory size.

⇒ Product unsold during evening and that should be stored in the Inventory can be given as $A = \{0, 1, \dots, M\}$ since there is no probability to store product more than Inventory size.

Since the product is non-perishable and storing is based on size of the inventory, Optimal quantity q to order varies by demand or sale.

A possible assumption containing probability of sales can also be assumed in place of demand resulting in model free approach. The above facts of storing unsold quantity incurring a holding cost does affect the optimality of solution described in the previous question.