Lecture 17: Decision Trees

Artificial Intelligence
CS-GY-6613-I

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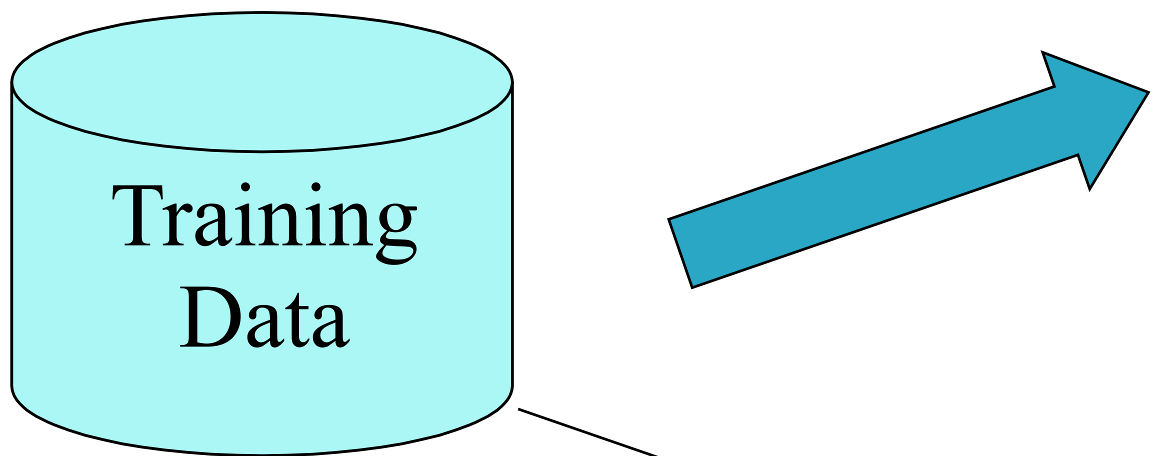
Types of learning

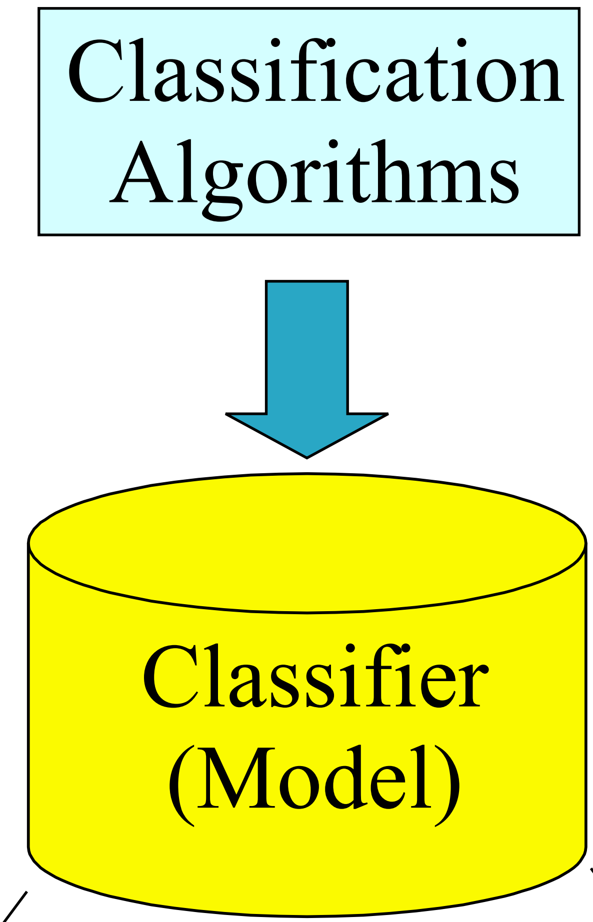
- **Supervised learning**
  Learning to predict or classify labels based on labeled input data

- **Unsupervised learning**
  Finding patterns in unlabeled data

- **Reinforcement learning**
  Learning well-performing behavior from state observations and rewards

Model construction

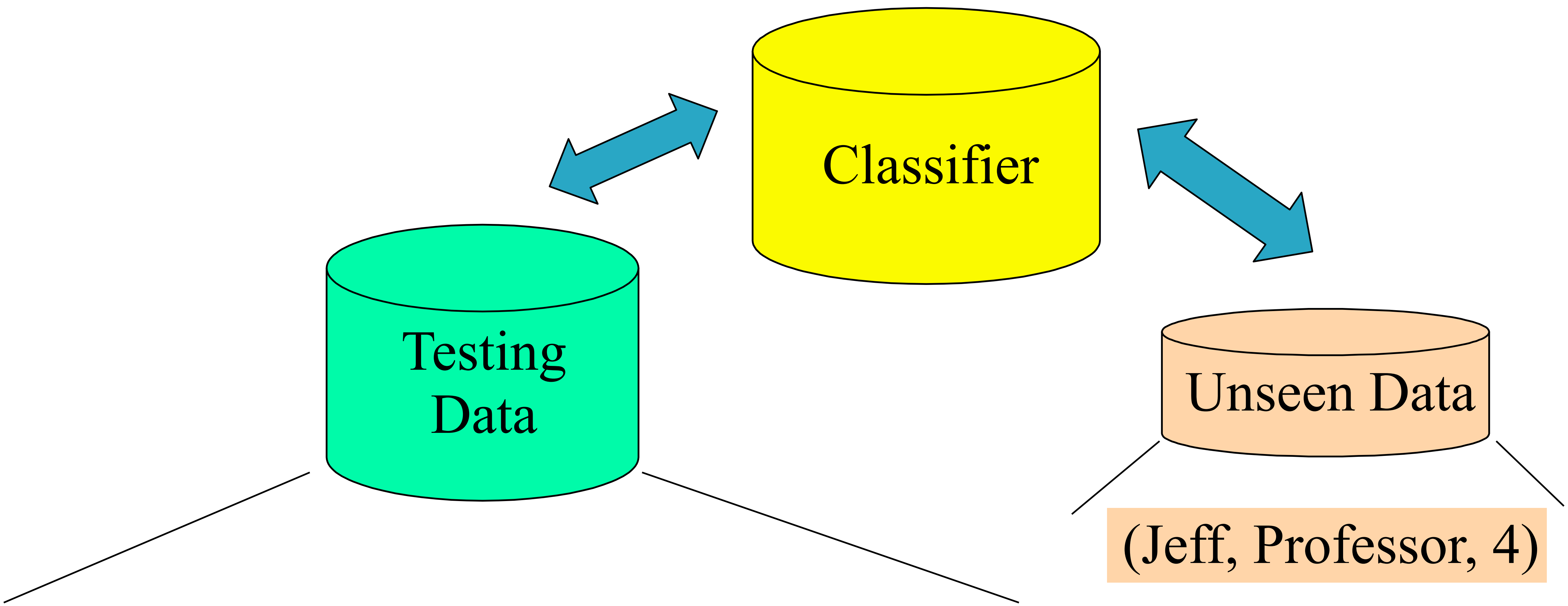




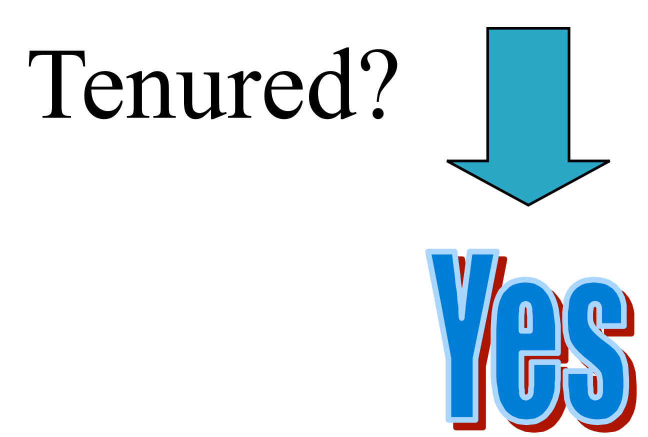
NAME	RANK	YEARS	TENURED
Mike	Assistant Prof	3	no
Mary	Assistant Prof	7	yes
Bill	Professor	2	yes
Jim	Associate Prof	7	yes
Dave	Assistant Prof	6	no
Anne	Associate Prof	3	no

IF rank = 'professor'
OR years > 6
THEN tenured = 'yes'

Using the model



NAME	RANK	YEARS	TENURED
Tom	Assistant Prof	2	no
Merlisa	Associate Prof	7	no
George	Professor	5	yes
Joseph	Assistant Prof	7	yes



Classification vs prediction

- Classification: binary or nominal labels
  - Examples: pregnant or not, from which country, which type of road sign
- Prediction: continuous labels
  - Examples: future stock price, life expectancy, distance to obstacle

Terminology (supervised learning)

- Each line of data: instance / data point / tuple
- The features of each instance: features / attributes
- That which should be learned: labels / targets
- Each instance has features and a label
- We train on the training set…
- …and test on the testing set

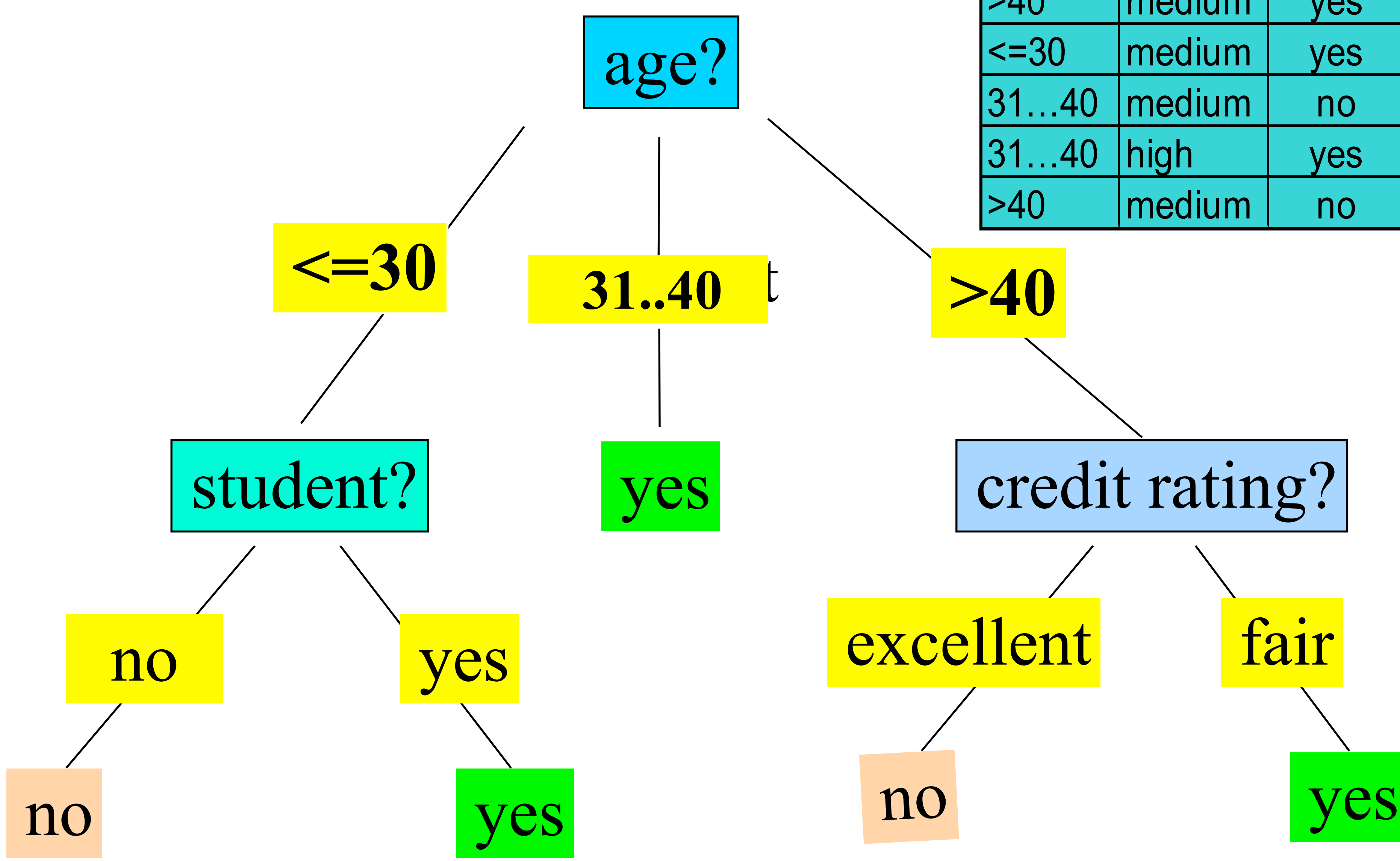
Decision trees

- A popular representation for classifiers
- Human-readable
- Can be learned (induced) efficiently using algorithms based on information theory
  - An eager learning method
- Often yields high-accuracy classifiers

An example

Classify: buys_computer

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no



ID3 Algorithm for Decision Tree Induction

- Tree is constructed in a top-down recursive divide-and-conquer manner
- At start, all the training examples are at the root
- Attributes are categorical (if continuous-valued, they are discretized in advance)
- Examples are partitioned recursively based on selected attributes
- Test attributes are selected on the basis of a heuristic or statistical measure (e.g., information gain)

ID3 Algorithm for Decision Tree Induction

- Stop partitioning when...
  - All samples for a given node belong to the same class, or…
  - There are no remaining attributes for further partitioning – (vote on the leaf) or…
  - There are no samples left

Attribute Selection Measure: Information Gain (ID3/C4.5)

- Select the attribute with the highest information gain
- Let p_i be the probability that an arbitrary tuple in D belongs to class Ci, estimated by $|C_i, D|/|D|$
- Expected information (entropy) needed to classify a tuple in D:

$$Info(D) = -\sum_{i=1}^{m} p_i \log_2(p_i)$$

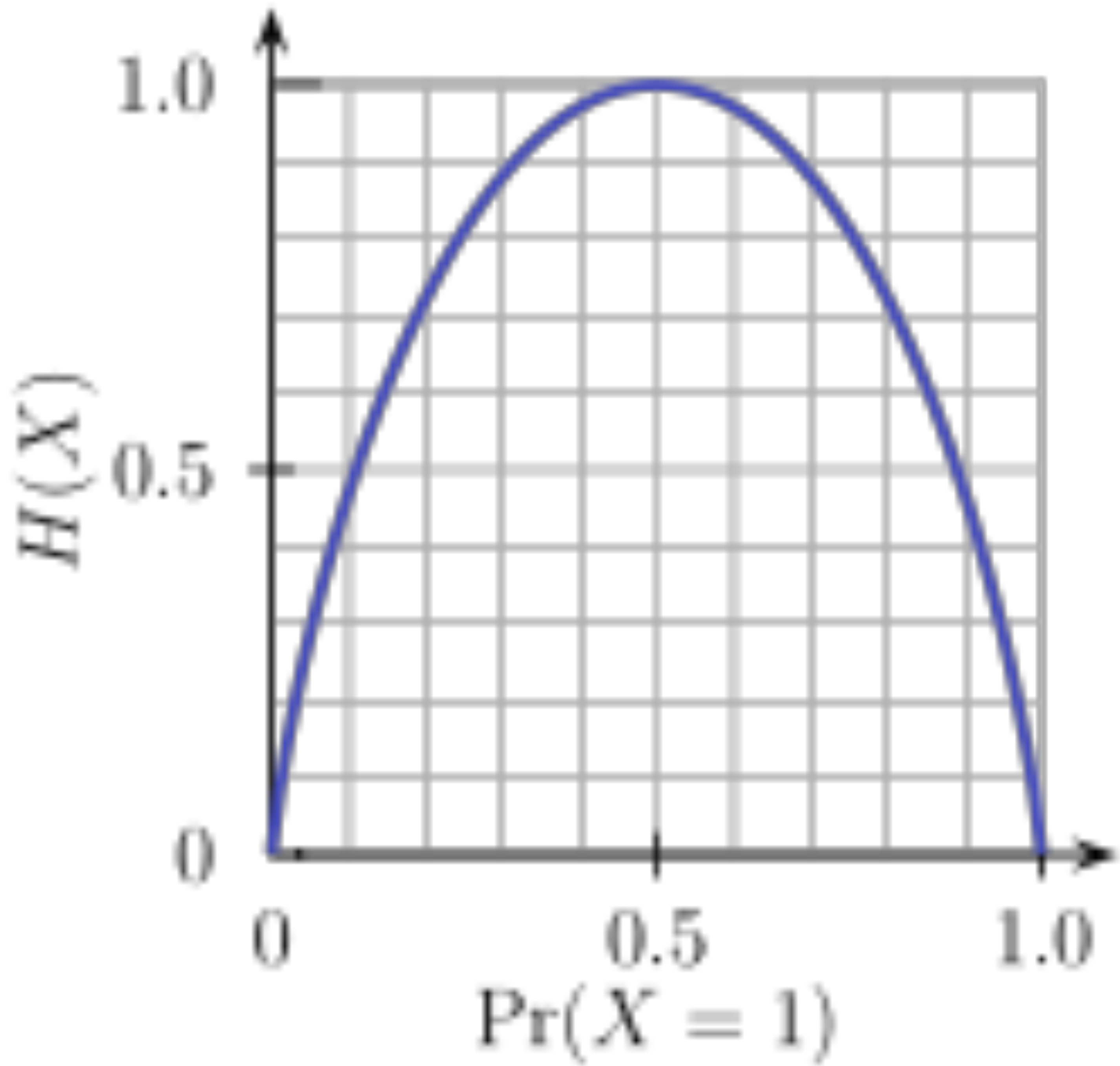
- Information needed (after using A to split D into v partitions) to classify D:

$$Info_A(D) = \sum_{j=1}^{v} \frac{|D_j|}{|D|} \times Info(D_j)$$

- Information gained by branching on attribute A

$$Gain(A) = Info(D) - Info_A(D)$$

Binary entropy function



Attribute Selection: Information Gain

Class P: buys_computer = "yes"

Class N: buys_computer = "no"

$$Info(D) = I(9,5) = -\frac{9}{14}\log_2\left(\frac{9}{14}\right) - \frac{5}{14}\log_2\left(\frac{5}{14}\right) = 0.940$$

age	p_i	n_i	$I(p_i, n_i)$
<=30	2	3	0.971
31…40	4	0	0
>40	3	2	0.971

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

$$Info_{age}(D) = \frac{5}{14}I(2,3) + \frac{4}{14}I(4,0) + \frac{5}{14}I(3,2) = 0.694$$

$\frac{5}{14}I(2,3)$ means "age <=30" has 5 out of 14 samples, with 2 yes'es and 3 no's. Hence

$$Gain(age) = Info(D) - Info_{age}(D) = 0.246$$

Similarly,

$$Gain(income) = 0.029$$

$$Gain(student) = 0.151$$

$$Gain(credit\_rating) = 0.048$$

- Input: A data set, S
  Output: A decision tree

  - If all the instances have the same value for the target attribute then return a decision tree that is simply this value (not really a tree - more of a stump).

- Else

  - Compute Gain values (see above) for all attributes and select an attribute with the highest value and create a node for that attribute.

  - Make a branch from this node for every value of the attribute

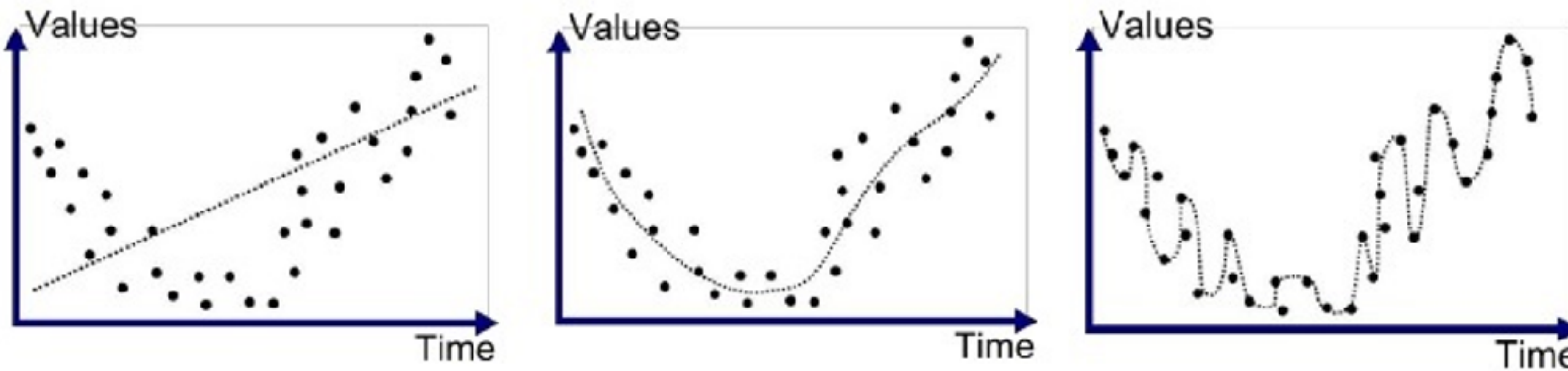
  - Assign all possible values of the attribute to branches.

  - Follow each branch by partitioning the dataset to be only instances whereby the value of the branch is present and then go back to 1.

function DECISION-TREE-LEARNING($examples$, $attributes$, $parent\_examples$) **returns** tree

    if $examples$ is empty **then return** PLURALITY-VALUE($parent\_examples$)
    else if all $examples$ have the same classification **then return** the classification
    else if $attributes$ is empty **then return** PLURALITY-VALUE($examples$)
    else
        $A \leftarrow \mathrm{argmax}_{a \in attributes}$ IMPORTANCE(a, $examples$)
        $tree \leftarrow$ a new decision tree with root test A
        for each value v_k of A **do**
            $exs \leftarrow \{e : e \in examples$ **and** $e.A = v_k\}$
            $subtree \leftarrow$ DECISION-TREE-LEARNING(exs, $attributes - A$, $examples$)
            add a branch to $tree$ with label $(A = v_k)$ and subtree $subtree$
        return $tree$

Overfitting (in supervised learning)



Underfitted | Good Fit/Robust | Overfitted

- Fitting the data too well, including the noise
- Reduces accuracy on unseen data

Overfitting and pruning

- Overfitting: An induced tree may overfit the training data (specific instance of a concept that applies to all supervised learning)
  - Too many branches, some may reflect anomalies due to noise or outliers
  - Poor accuracy for unseen samples
- Three approaches to avoid overfitting
  - Prepruning: Halt tree construction early– do not split a node if this would result in the goodness measure (e.g. accuracy on a validation set) falling below a threshold
  - Postpruning: Remove branches from a “fully grown” tree—get a sequence of progressively pruned trees
    - Use a set of data different from the training data to decide which is the “best pruned tree”
  - Limit depth or size

What to do with numerical attributes?

- Create a set of bins, where each bin becomes one attribute value (e.g. income <30k, 30-40k, >40k)
  - Equal depth: All the bins have the same number of instances in them
  - Equal width: All the bins have the same range
  - More sophisticated ways, including looking at where there are big discontinuities in the range of the input data etc

Can we do better?

- It's hard to train good classifiers
- Can we replace quality with quantity?
  - Train many classifiers and combine them somehow
  - Called *ensemble learning*