Lecture 19: Multilayer Perceptrons

Artificial Intelligence
CS-GY-6613

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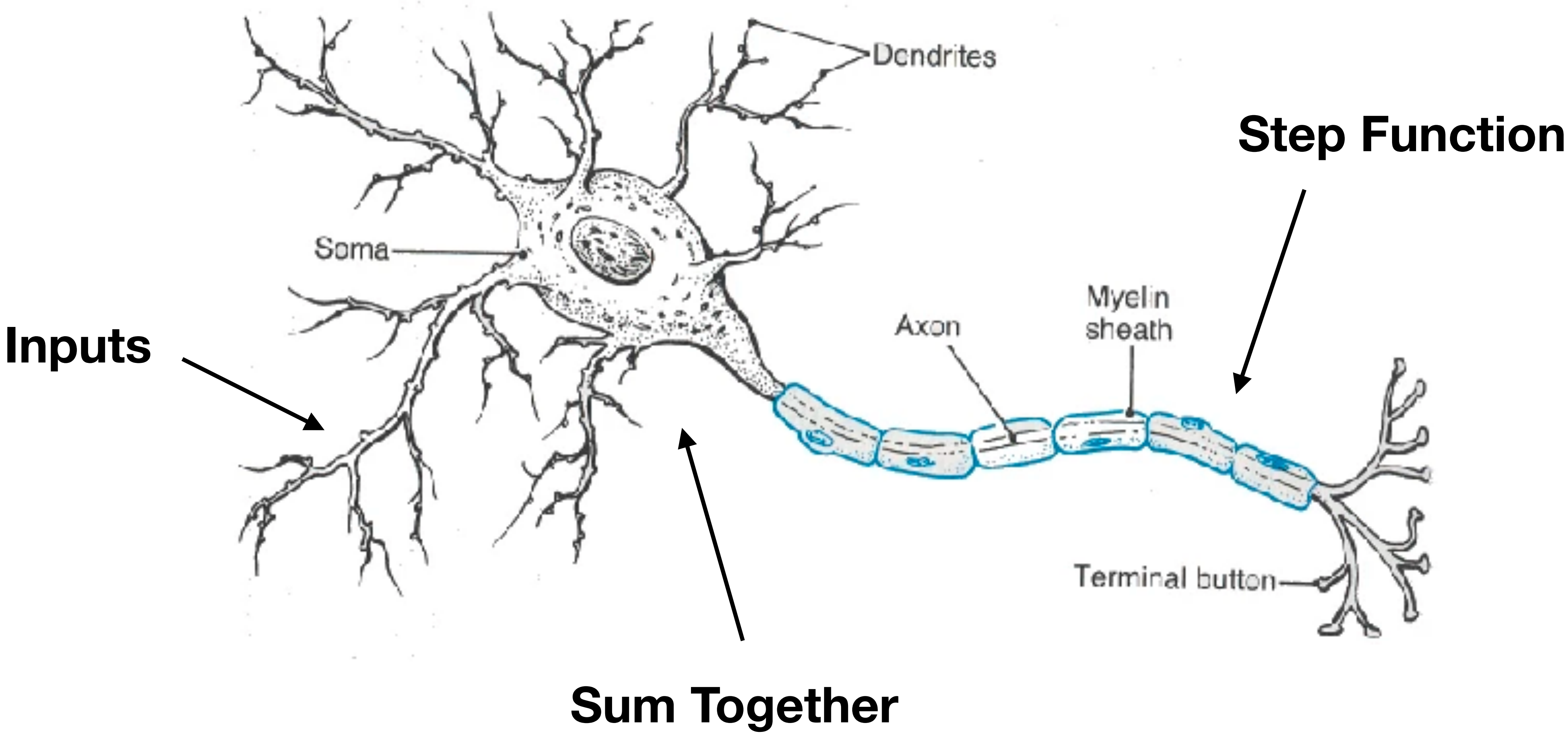
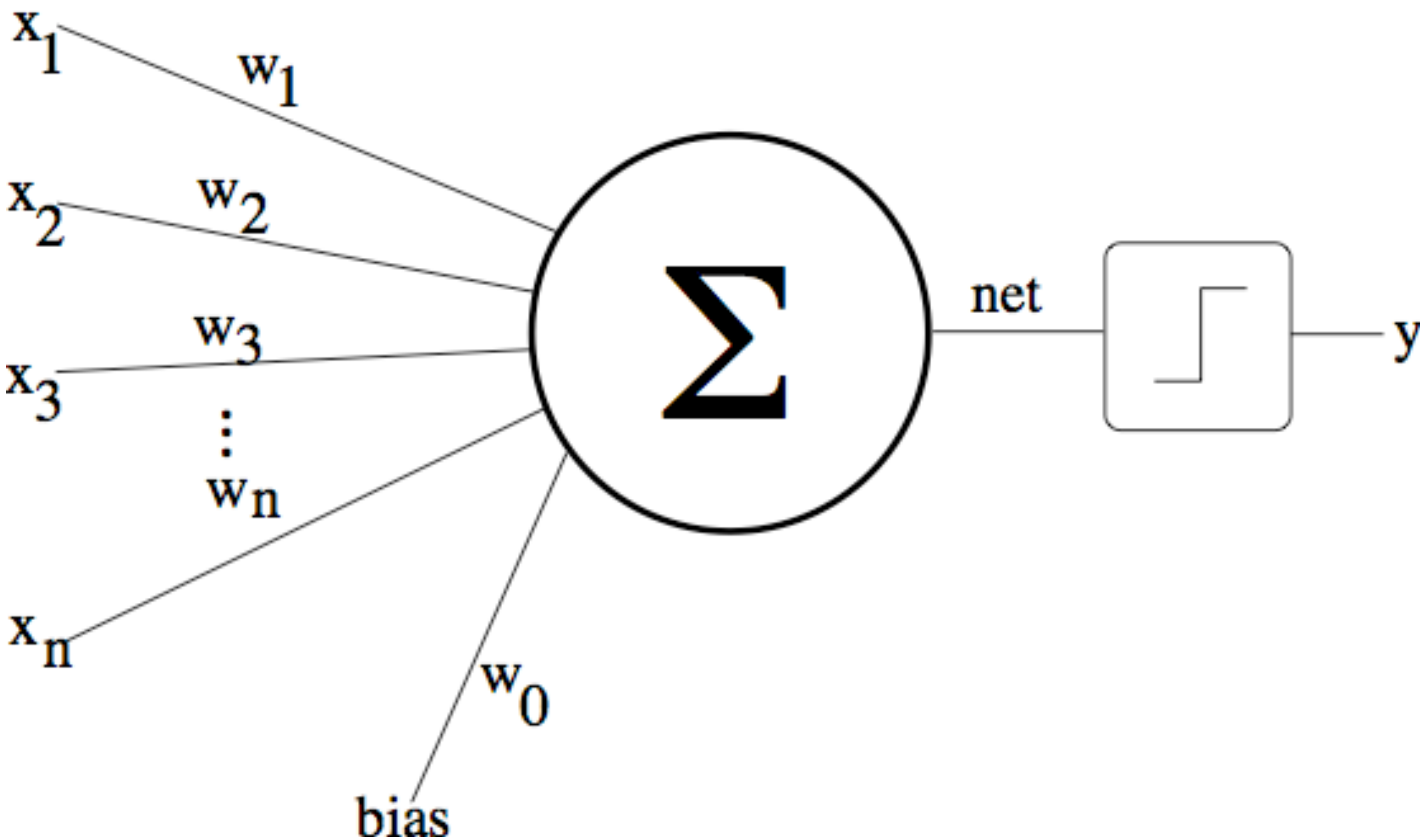
The Neuron



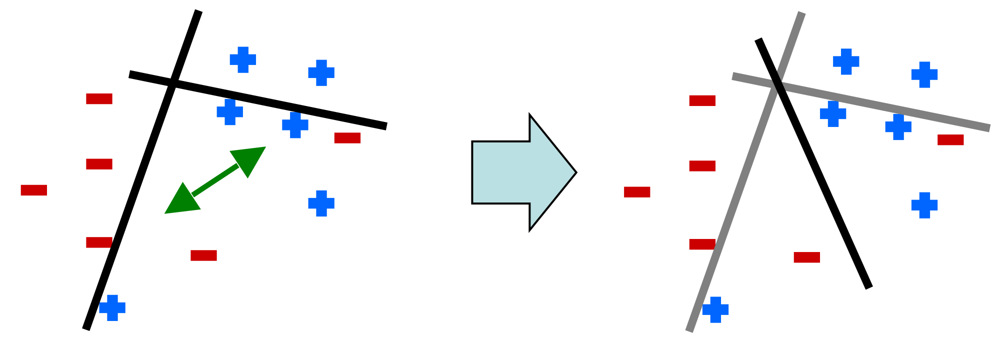
Dendrites
Soma
Axon
Myelin sheath
Terminal button
Step Function
Inputs
Sum Together



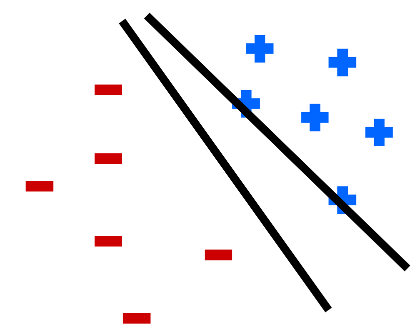
$$f(x) = \begin{cases} 1 & \text{if} \quad \sum_{i=1}^{n} w_i x_i + b > 0 \\ 0 & \text{else} \end{cases}$$

Perceptron problems

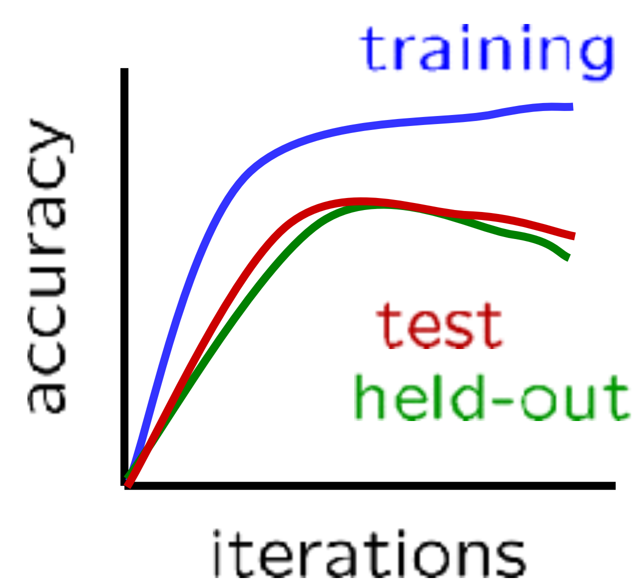
- Noise: if the data isn't separable, weights might thrash



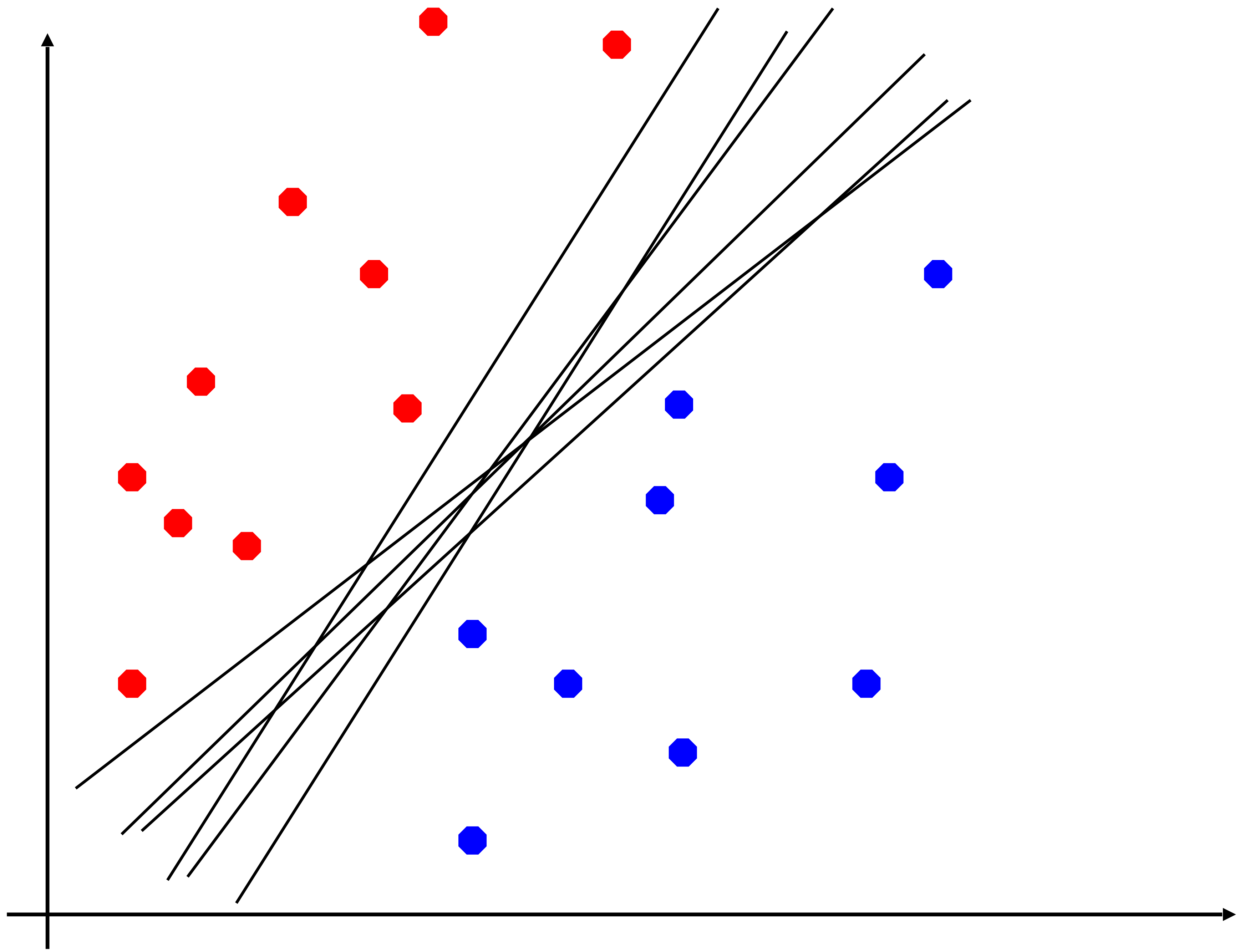
- Mediocre generalization: finds a "barely" separating solution



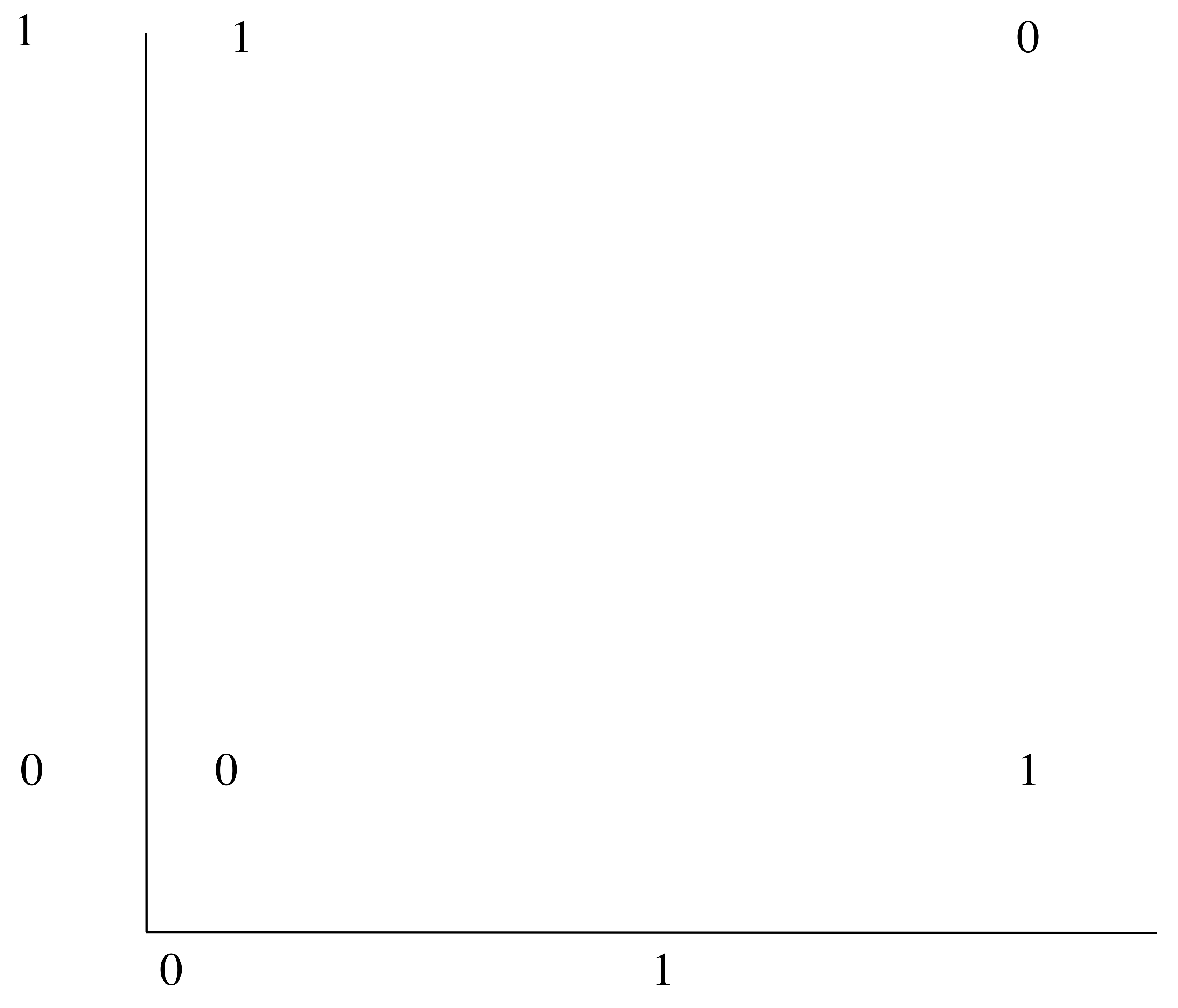
- Overtraining: test / held-out accuracy usually rises, then falls



Which of these separators is optimal?

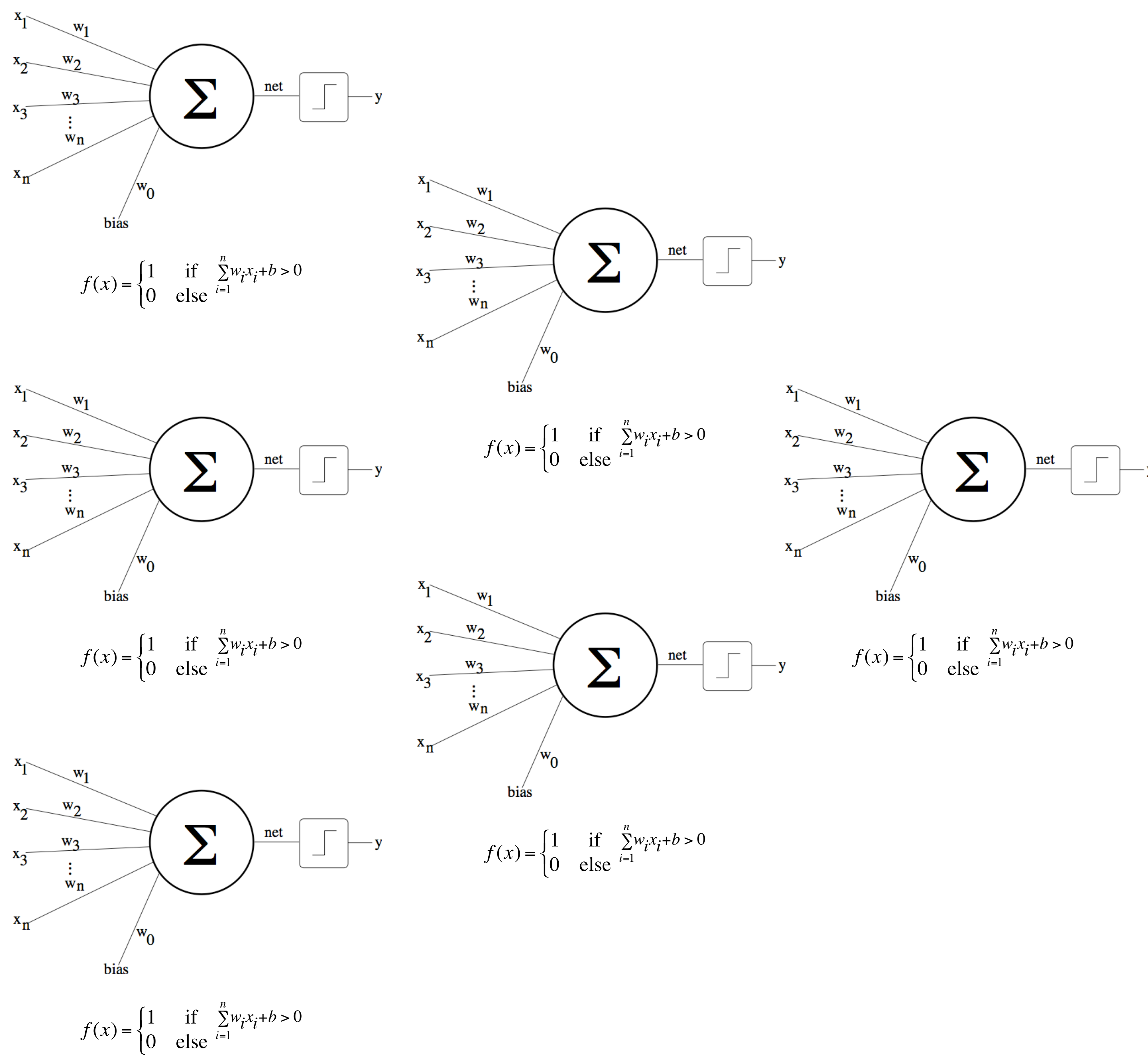


Problem: learn this!



With all of these problems and with so many other learning algorithms, why were people still interested in pursuing this technique?

Could we just train a bunch of perceptrons connected to each other?

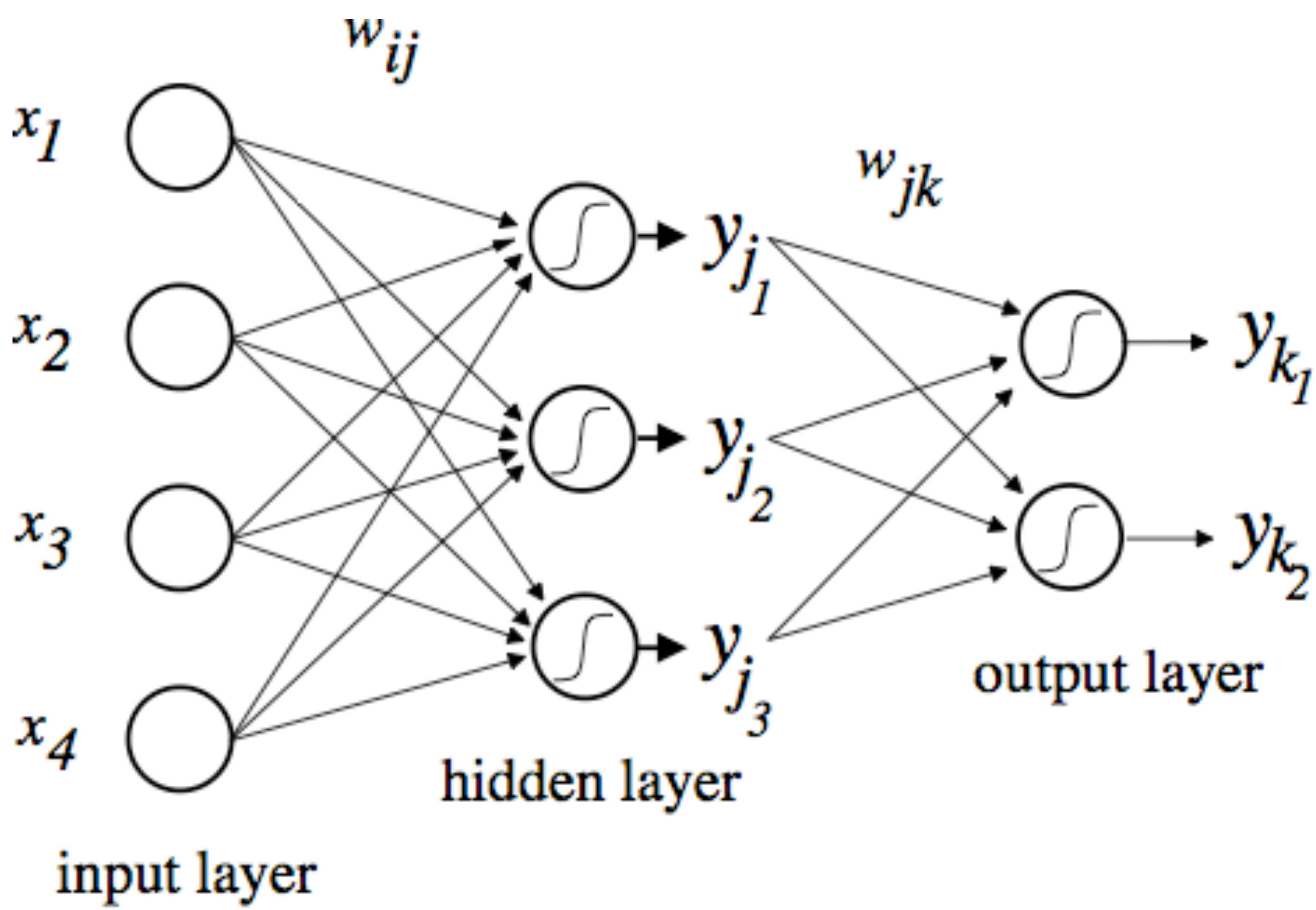


Multilayer Perceptron

"When an axon of cell A is near enough cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A's efficiency, as one of the cells firing B, is increased"

–Donald Hebb

Multi-layer Perceptron (MLP)



Universal Approximator

- George Cybenko in 1989 provided one of the first proofs for the Universal Approximation Theorem for Neural Networks
- States that a feedforward network with at least one hidden layer of a finite size can approximate any continuous function on compact subsets of R^n
- Says nothing about learnability
    - How do you train a hidden layer? What is its loss?

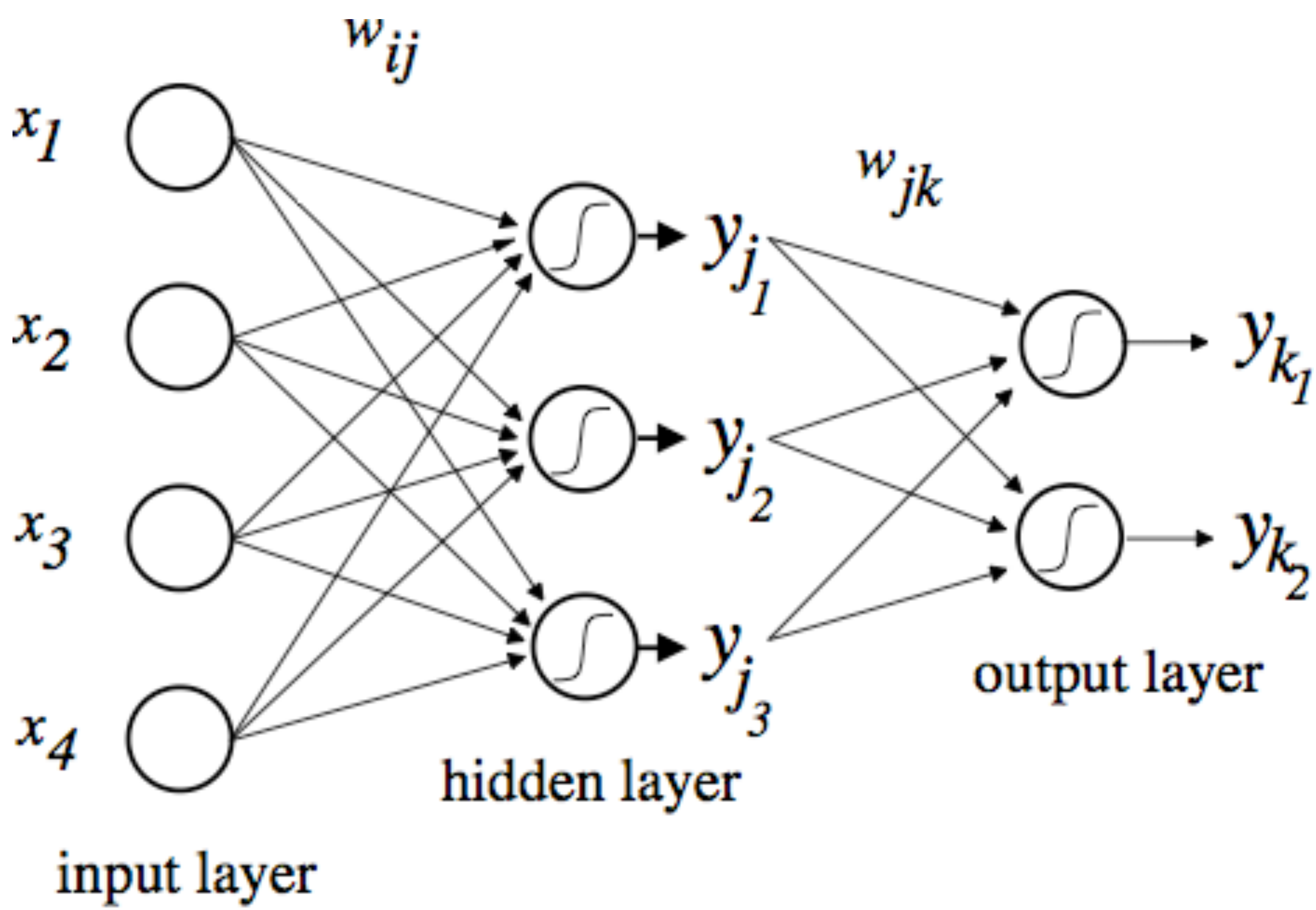
Training a Neural Network

Backpropagation

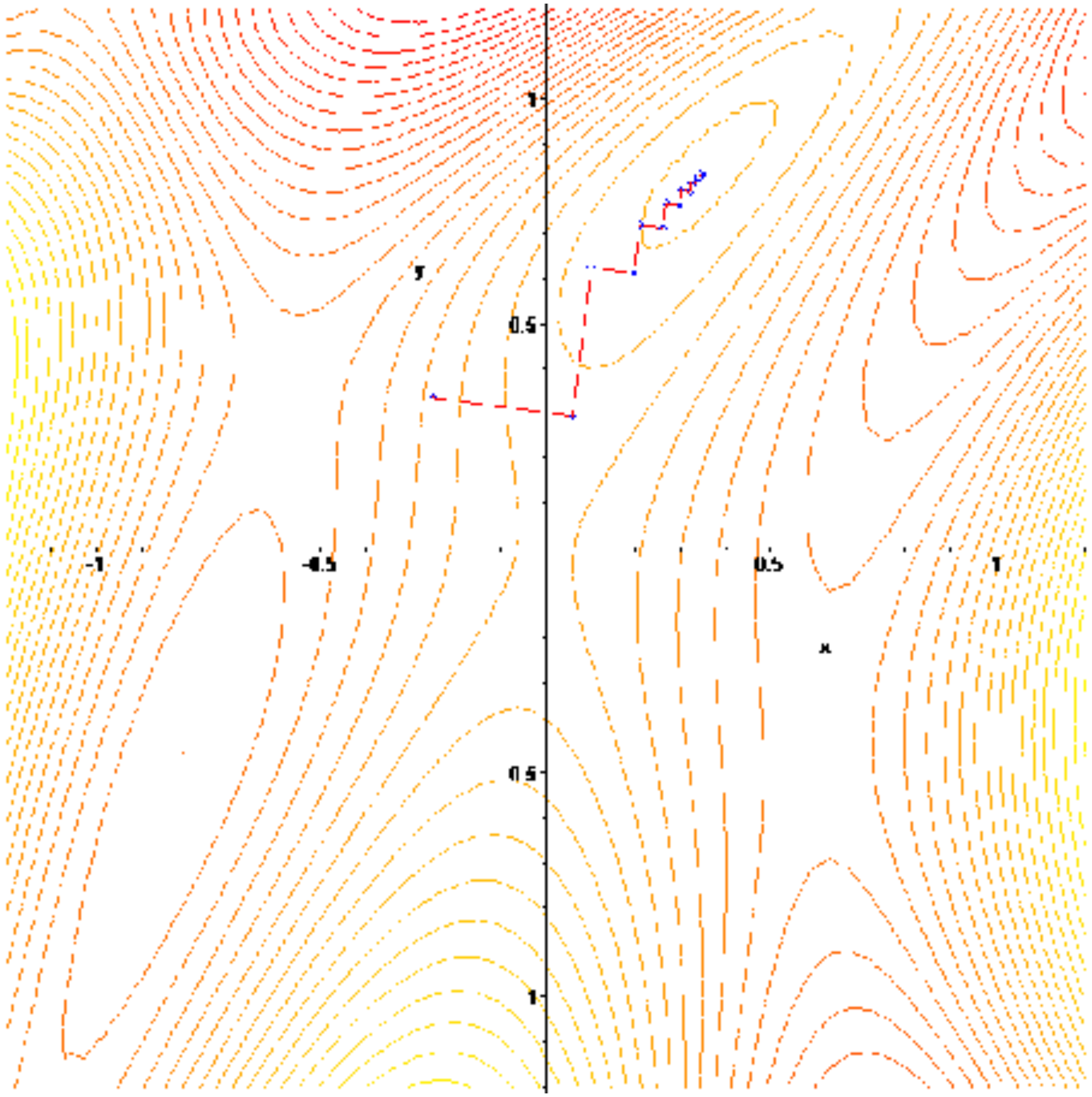
- Forward Pass: present training input pattern to network and activate network to produce output (can also do in batch: present all patterns in succession)

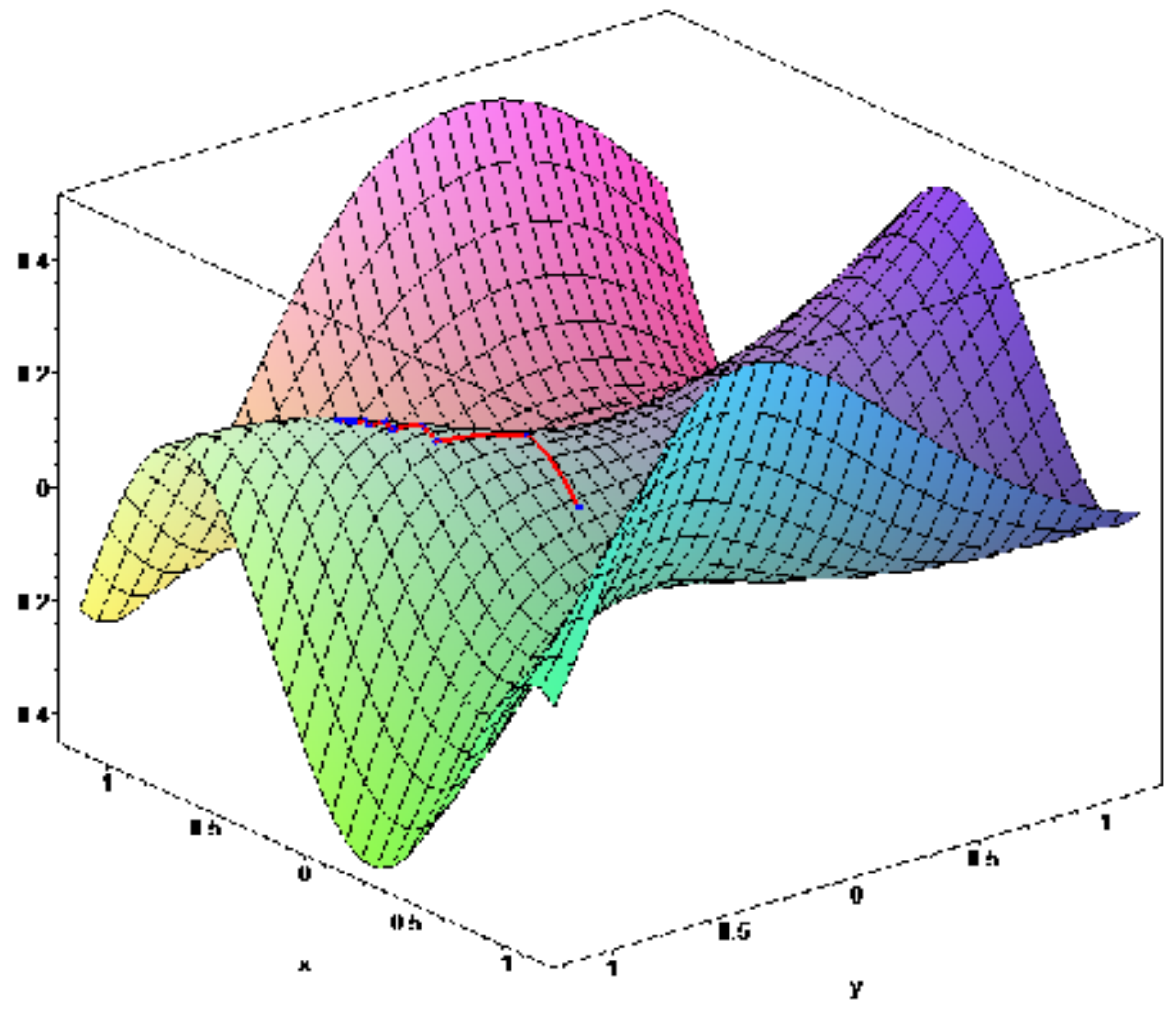
- Backward Pass: calculate error gradient and update weights starting at output layer and then going back

Multi-layer Perceptron (MLP)



Gradient Descent





Gradient Descent

Input : list of n training examples $(x_0, d_0) \ldots (x_n, d_n)$
where $\forall i : d_i \in \{+1, -1\}$

Output : classifying hyperplane w

Algorithm :

Randomly initialize w;

While makes errors on training set **do**

for $(x_i\, d_i)$ **do**

let $y_i = \text{MLP}(w, x_i)$;

$loss \leftarrow \text{Mean}((d_i - y_i)^2)$

$w' \leftarrow \text{Backprop}(w, loss)$

$w \leftarrow w - \eta w'$

end

end

x and w are vectors;
i is the instance index

Weight Update Equation

$$\theta_w' = \theta_w - \eta * \frac{\partial TC}{\partial \theta_w}$$

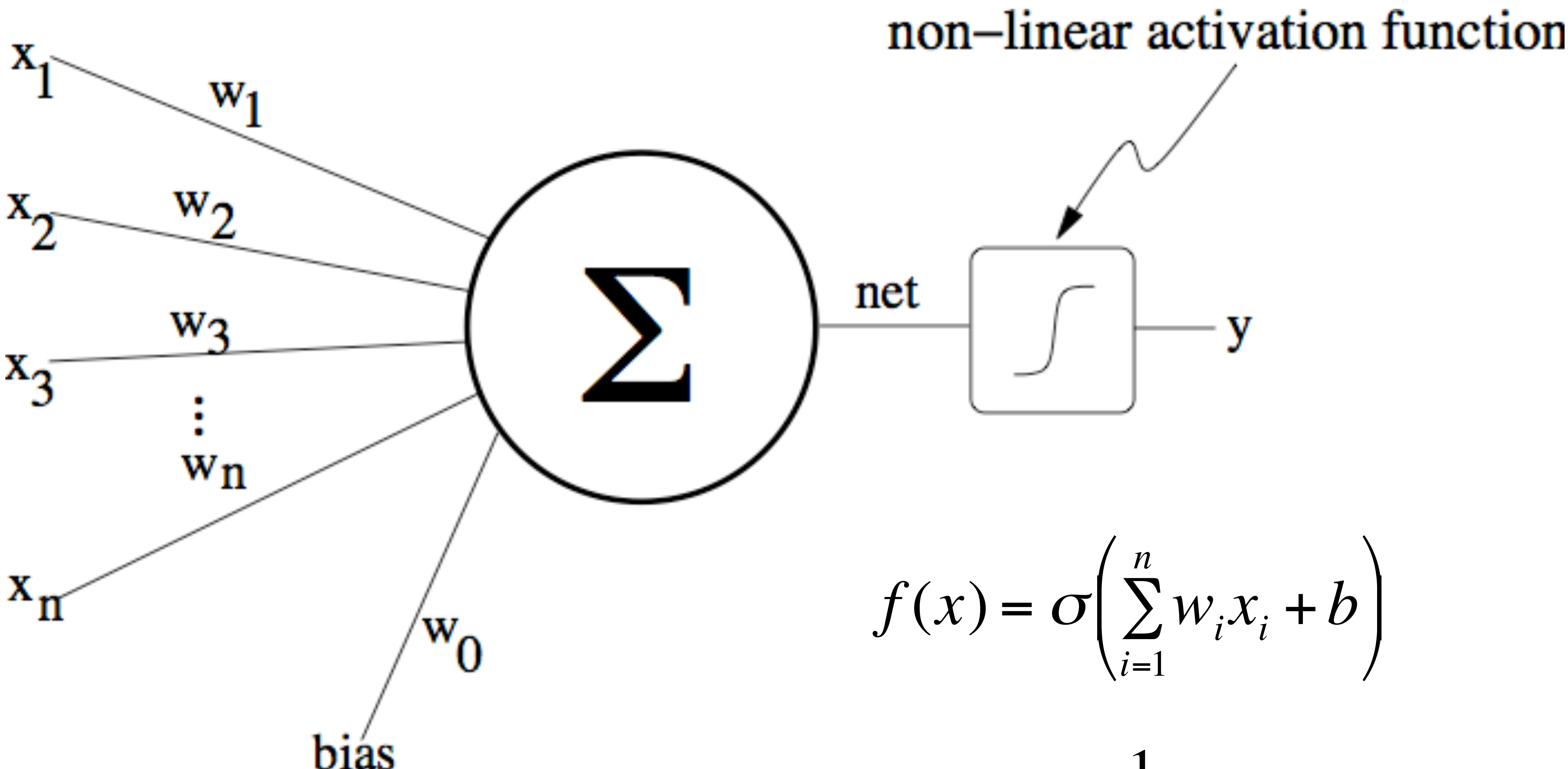
New weight

Learning Rate

```
function BACK-PROP-LEARNING(examples, network) returns a neural network
  inputs: examples, a set of examples, each with input vector x and output vector y
          network, a multilayer network with L layers, weights w_{i,j}, activation function g
  local variables: Δ, a vector of errors, indexed by network node

  repeat
     for each weight w_{i,j} in network do
        w_{i,j} ← a small random number
     for each example (x, y) in examples do
        /* Propagate the inputs forward to compute the outputs */
        for each node i in the input layer do
           a_i ← x_i
        for ℓ = 2 to L do
           for each node j in layer ℓ do
              in_j ← Σ_i w_{i,j} a_i
              a_j ← g(in_j)
        /* Propagate deltas backward from output layer to input layer */
        for each node j in the output layer do
           Δ[j] ← g′(in_j) × (y_j − a_j)
        for ℓ = L − 1 to 1 do
           for each node i in layer ℓ do
              Δ[i] ← g′(in_i) Σ_j w_{i,j} Δ[j]
        /* Update every weight in network using deltas */
        for each weight w_{i,j} in network do
          w_{i,j} ← w_{i,j} + α × a_i × Δ[j]
  until some stopping criterion is satisfied
  return network
```

Activation Functions



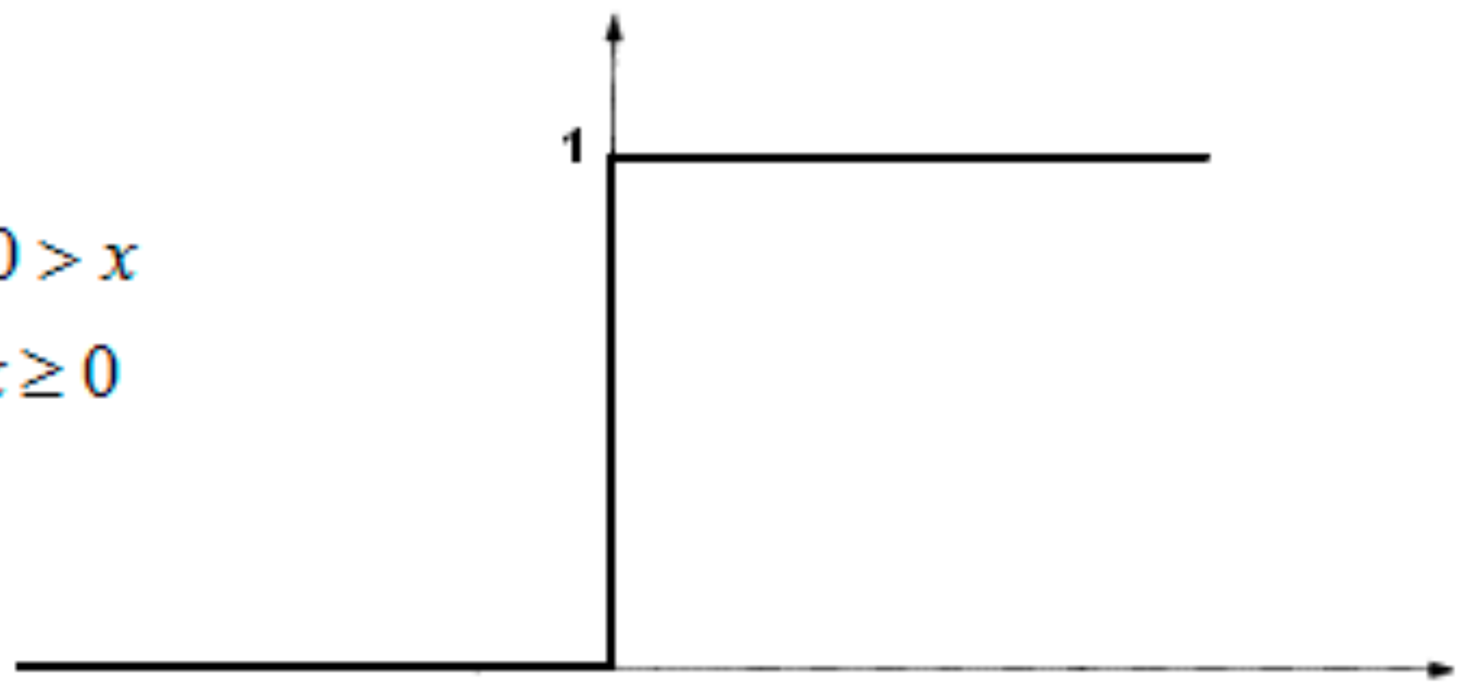
$$f(x) = \sigma\left(\sum_{i=1}^{n} w_i x_i + b\right)$$

$$\sigma(net) = \frac{1}{1 + e^{-net}}$$

Activation Functions

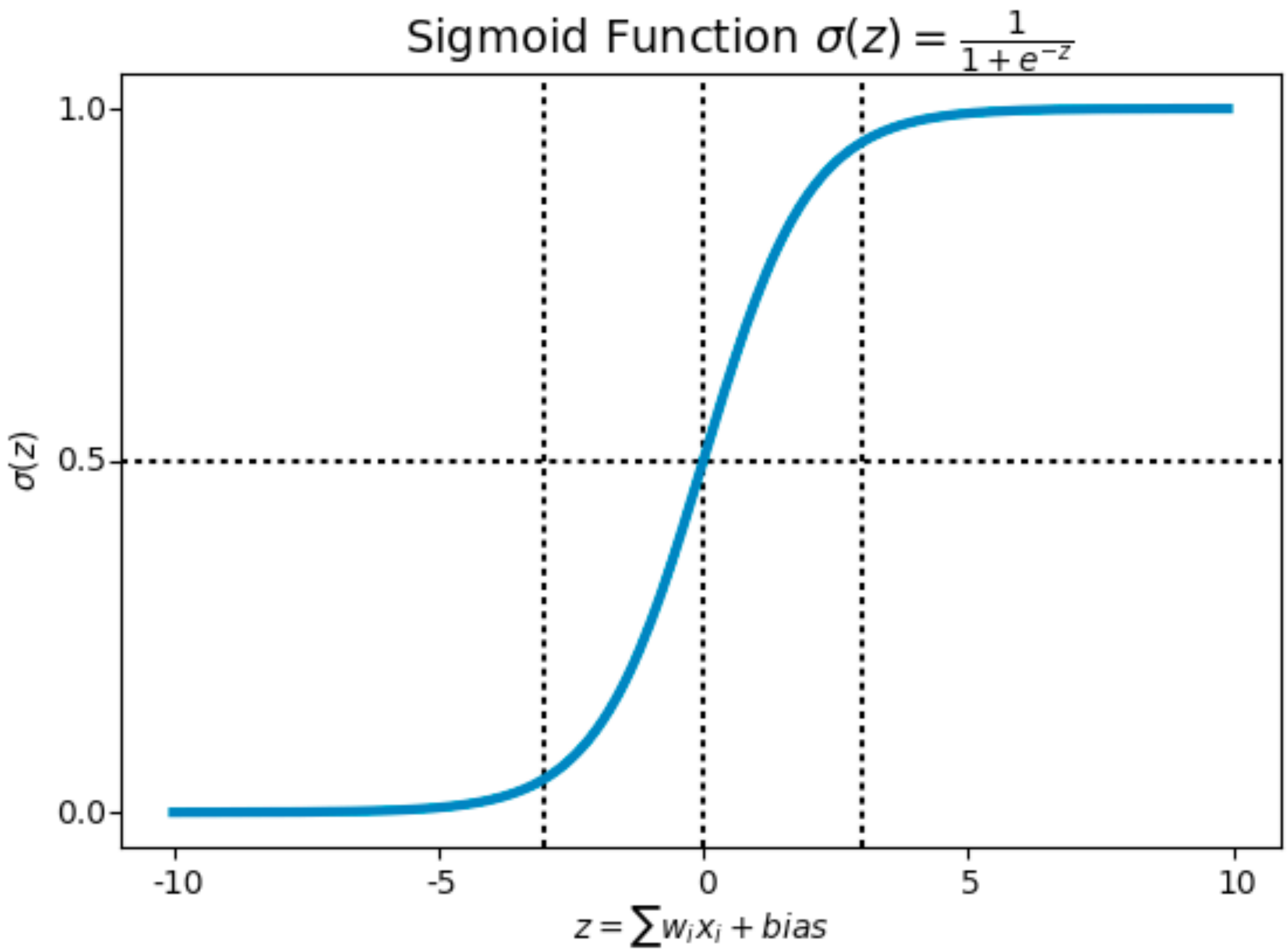
Unit step (threshold)

$$f(x) = \begin{cases} 0 \text{ if } 0 > x \\ 1 \text{ if } x \geq 0 \end{cases}$$



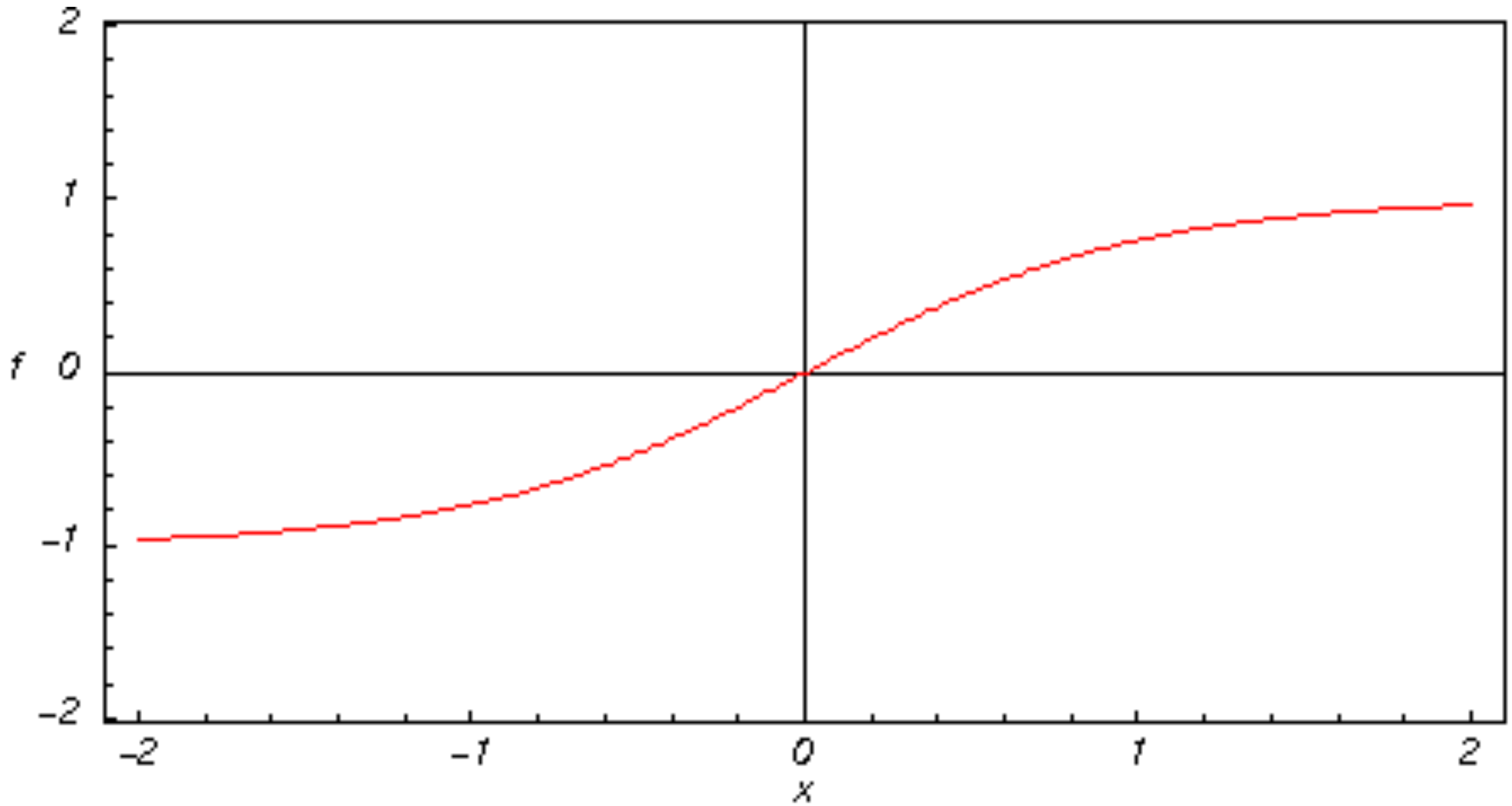
Derivate: N/A

Activation Functions



Derivative: σ'(z) = σ(z)(1-σ(z))

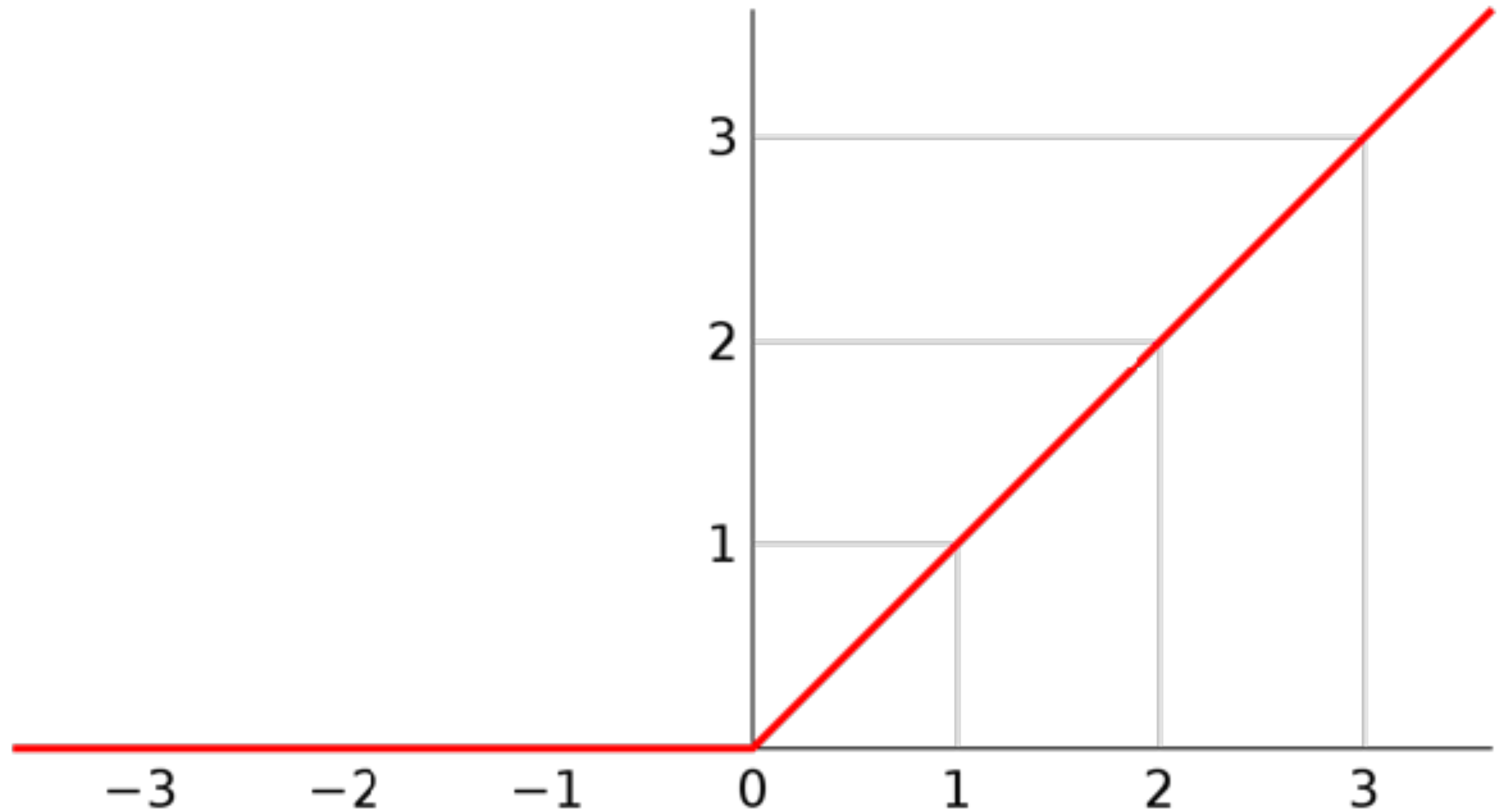
Activation Functions



Hyperbolic Tangent

Derivative: $\tanh'(z) = 1-\tanh(z)^2$

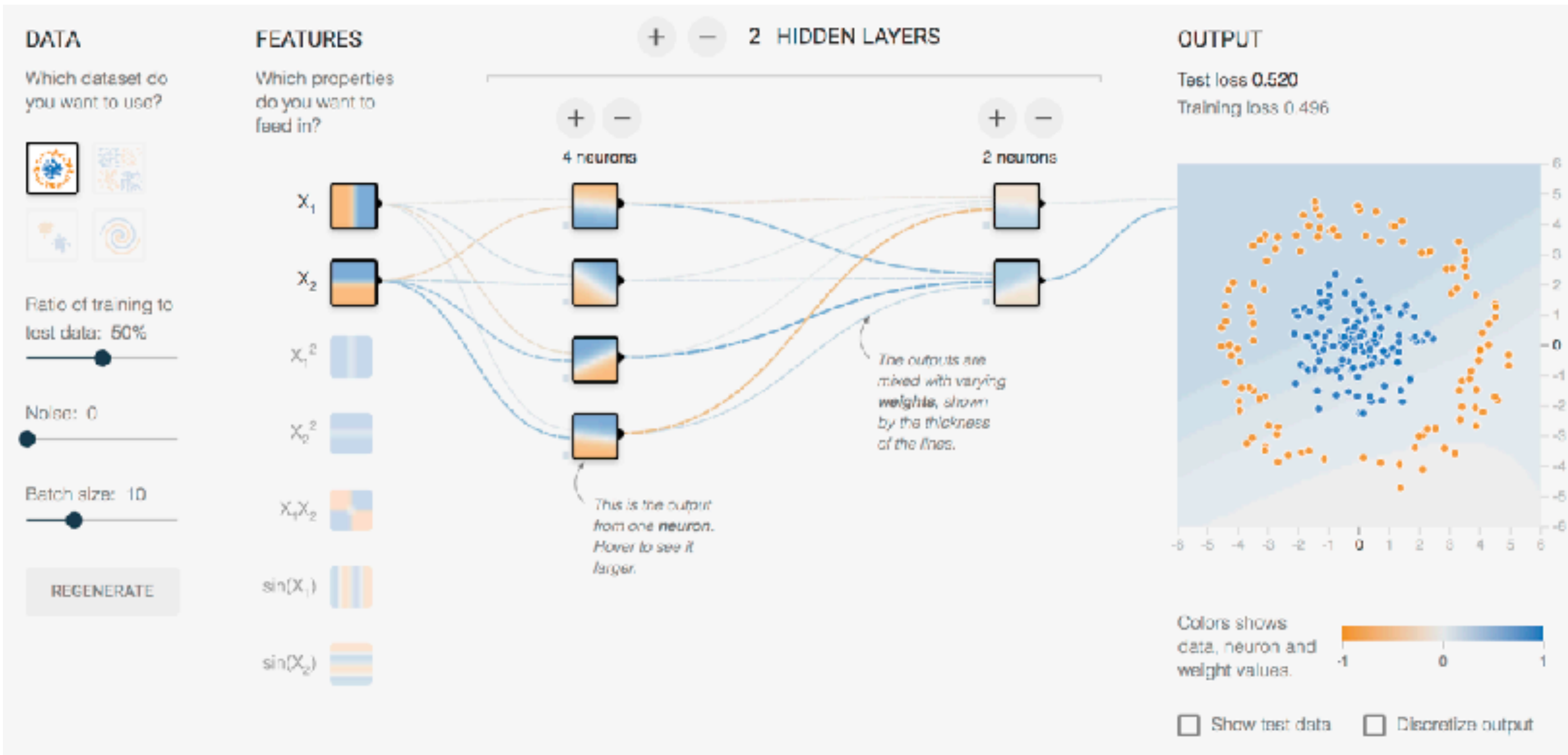
Activation Functions



Rectified Linear Units
(ReLu)

Derivative: 1, if x > 0; otherwise 0

Elements of a Neural Network



http://playground.tensorflow.org/

Perceptron problems

- Noise: if the data isn't separable, weights might thrash
- Multiple layers allow it to work on non-linearly separable data
- Mediocre generalization: finds a "barely" separating solution
- More data and a continuous loss function greatly improve generalization
- Overtraining: test / held-out accuracy usually rises, then falls
- Large data, many layers, and batch training help