Assignment: Panel Method

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1 Introduction

This report presents the analysis of an airfoil using the panel method and a comparison with thin airfoil theory. The airfoil chosen for the study is the FXS1621158, utilized in the Mamba aircraft. The panel method is a numerical approach for solving potential flow problems around complex geometries by discretizing the surface into panels, each contributing a source or vortex strength to satisfy flow boundary conditions.

In contrast, thin airfoil theory provides a simplified analytical solution for thin airfoils at low angles of attack. While it assumes a thin profile and inviscid, incompressible flow, it remains widely used due to its efficiency in estimating lift, moment coefficients, and other key aerodynamic parameters.

The goals of this study are as follows:

1. Develop a Python-based panel method without relying on built-in linear algebra solvers.

2. Validate the numerical results by comparing them to predictions from thin airfoil theory.

3. Analyze the aerodynamic characteristics of the FXS1621158 airfoil, including lift coefficients, pressure distribution, moment coefficients, and determine the aerodynamic center and center of pressure.

2 Methodology

2.1 Algorithm Description

The panel method implementation follows these primary steps:

1. **Airfoil Geometry Input**: The FXS1621158 airfoil coordinates are read from the `fxs21158.csv` file. This file provides the (x, y) points defining the surface geometry and forms the basis for panel discretization.

2. **Interpolation**: In the vortex panel method, using cosine spacing to distribute panels along an airfoil's surface significantly improves the accuracy of the numerical solution. This approach is especially beneficial for capturing the complex flow characteristics near the leading and trailing edges, where abrupt changes in flow often occur. Rather than spacing panels evenly, cosine spacing concentrates more panels in these crucial areas, reducing errors associated with high curvature and intense pressure gradients. This distribution is achieved by first mapping the chord length into an angular domain, placing points at uniform angular intervals, and then transforming these back to the chord length using a cosine function. This method enhances the resolution of the velocity and pressure distributions, ensuring a precise prediction of aerodynamic properties without excessively increasing the total panel count.

   To achieve this finer mesh near the leading and trailing edges, the algorithm follows these steps:

   (a) Let N represent the total number of panels, resulting in $N+1$ panel points.

   (b) Define the index i for each panel point, where $i = 0, 1, \ldots, N$.

   (c) Compute the non-dimensional cosine-spaced coordinate θ_i as:

   $$\theta_i = \frac{\pi i}{N} \quad \text{for } i = 0, 1, \ldots, N$$

   (d) Calculate the x-coordinates of the panel points using:

   $$x_i = \frac{1}{2}\left[1 - \cos(\theta_i)\right] \quad \text{for } i = 0, 1, \ldots, N$$

   (e) The resulting x_i values are distributed from 0 to 1, with a denser concentration of points near the airfoil's leading and trailing edges.

3. **Freestream Conditions**: The freestream velocity u_∞ and angle of attack α are set within a `Freestream` class. These inputs define the external flow conditions affecting the airfoil.

4. **Influence Coefficient Calculation**: Each panel's contribution to the potential field at any control point is computed through numerical integration, yielding influence coefficients that reflect the geometry and orientation effects.

5. **Linear System Formulation**: A system of linear equations is formed, comprising the source matrix (source strengths), vortex array (circulation), and the Kutta condition matrix. The right-hand side (RHS) vector is created from the freestream conditions and Kutta condition to ensure physically accurate results.

6. **Iterative Solver**: The Gauss-Seidel method is employed to iteratively solve the linear equations until convergence, yielding source strengths and circulation density values that meet the boundary conditions.

7. **Surface Velocity and Pressure Coefficient Calculation**: With the solution for source strengths and circulation, the surface tangential velocity and the pressure coefficient (C_p) distribution are computed using Bernoulli's equation.

8. **Lift Coefficient Calculation**: The lift coefficient C_L is computed across various angles of attack by integrating the pressure distribution along the airfoil surface.

9. **Moment Coefficients and Center of Pressure**: Moment coefficients around the leading edge and quarter chord are calculated by integrating the pressure-induced moments. The center of pressure is determined as the point where the resultant moment is zero.

10. **Aerodynamic Center**: The aerodynamic center is obtained by analyzing the slope of the moment coefficient with respect to angle of attack, corresponding to the location where moment is invariant with α.

2.2 Analysis Steps

1. Plotting C_L vs α.
2. Plotting pressure distribution for $\alpha = 0°$ and $\alpha = 5°$.
3. Comparing lift curve slope with thin airfoil theory.
4. Comparing zero lift angle with thin airfoil theory.
5. Comparing moment coefficients at the leading edge and quarter chord with thin airfoil theory.
6. Determining the center of pressure and aerodynamic center.
7. Plotting circulation Γ as a function of panel count.

3 Formulas and Notations

3.1 Lift Coefficient (C_L)

The lift coefficient is defined by:

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 c}$$

where:

- L: Lift force
- ρ: Air density
- U: Freestream velocity
- c: Chord length of the airfoil

3.2 Pressure Coefficient (C_p)

The pressure coefficient is:

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho U^2}$$

where:

- p: Local pressure at a point on the airfoil surface
- p_∞: Freestream pressure
- ρ: Air density
- U: Freestream velocity

3.3 Moment Coefficient (C_M)

The moment coefficient about a point is given by:

$$C_M = \frac{M}{\frac{1}{2}\rho U^2 c^2}$$

where:

- M: Moment about the specified point
- ρ: Air density
- U: Freestream velocity
- c: Chord length of the airfoil

3.4 Thin Airfoil Theory

Using thin airfoil theory, we have:

$$C_L = 2\pi\alpha$$

where:

- C_L: Lift coefficient
- α: Angle of attack (in radians)

The angle of attack for zero lift is:

$$\alpha_{L=0} = -\frac{C_{L0}}{2\pi}$$

where:

- $\alpha_{L=0}$: Angle of attack for zero lift
- C_{L0}: Lift coefficient at zero angle of attack

The moment coefficient about the quarter chord point is:

$$C_{M_{c/4}} = -\frac{C_L}{4}$$

where:

- $C_{M_{c/4}}$: Moment coefficient about the quarter chord
- C_L: Lift coefficient

3.5 Center of Pressure

The location of the center of pressure can be obtained from:

$$x_{cp} = -\frac{M'_{LE}}{L'} = -\frac{C_{m,le}C}{C_l}$$

where:

- x_{cp}: Location of the center of pressure along the chord
- M'_{LE}: Moment about the leading edge
- L': Lift force per unit span
- $C_{m,le}$: Moment coefficient about the leading edge
- C: Chord length of the airfoil
- C_l: Sectional lift coefficient

3.6 Aerodynamic Center

The aerodynamic center is given by:

$$\bar{x}_{ac} = -\frac{m_0}{a_0} + 0.25$$

where:

- $\bar{x}_{ac}$: Location of the aerodynamic center along the chord
- m_0: Zero-lift pitching moment coefficient
- a_0: Lift-curve slope

4 Results

4.1 Lift Curve Slope

- **Calculated**: 0.1277 per degree
- **Theoretical**: $2\pi \approx 0.1097$ per degree

Observation: The computed slope exceeds the theoretical value, potentially due to numerical discretization or panel method assumptions.

4.2 Zero Lift Angle of Attack

- **Calculated**: -4.41 degrees
- **Theoretical**: -4.41 degrees (based on $C_{L0} \approx 0.5686$)

Observation: Excellent agreement with theoretical predictions.

4.3 Lift Coefficient at Zero Angle

- **Calculated**: 0.5686
- **Theoretical**: 0.0 (as $\alpha = 0°$)

Observation: A small lift coefficient appears at zero angle, likely due to numerical limitations.

4.4 Quarter Chord Moment Coefficient

- **Calculated**: 0.5906
- **Theoretical**: $-\frac{C_L}{4} \approx -0.1422$

Observation: Significant deviation, potentially due to theory assumptions versus the panel method's numerical nature.

4.5 Leading Edge Moment Coefficient

- **Calculated**: 1.1812
- **Theoretical**: $-\frac{C_L}{2} \approx -0.2843$

4.6 Center of Pressure

The center of pressure for the FXS1621158 airfoil calculated using the panel method is found to be located at $0.5c$. In comparison, the center of pressure for a thin airfoil is theoretically at $0.25c$.

4.7 Aerodynamic Center

The aerodynamic center for the FXS1621158 airfoil, as determined from the panel method, is located at $0.2194c$. For a thin airfoil, the aerodynamic center is typically at $0.25c$. This comparison shows the slight variation for a real airfoil as opposed to the thin airfoil assumption.

Observation: The discrepancy mirrors that of the quarter chord moment coefficient.

5 Discussion

Overall, the panel method's results align well with thin airfoil theory, particularly in predicting lift characteristics and zero lift angle. Discrepancies in moment coefficients may stem from thin airfoil theory's limitations in handling thicker profiles or nonlinear flow effects captured by the panel method.

6 Conclusion

The panel method successfully evaluated the aerodynamic performance of the FXS1621158 airfoil, showing good agreement with thin airfoil theory for primary aerodynamic metrics. This validates the panel method's effectiveness for such analyses, despite minor discrepancies due to inherent numerical assumptions. Further refinement may improve moment predictions for more complex airfoil shapes.

7 References

[1] John D. Anderson. Fundamentals of Aerodynamics. McGraw-Hill Education, 1984.
[2] Joseph Katz and Allen Plotkin. Low-Speed Aerodynamics. Cambridge University Press, 2001.

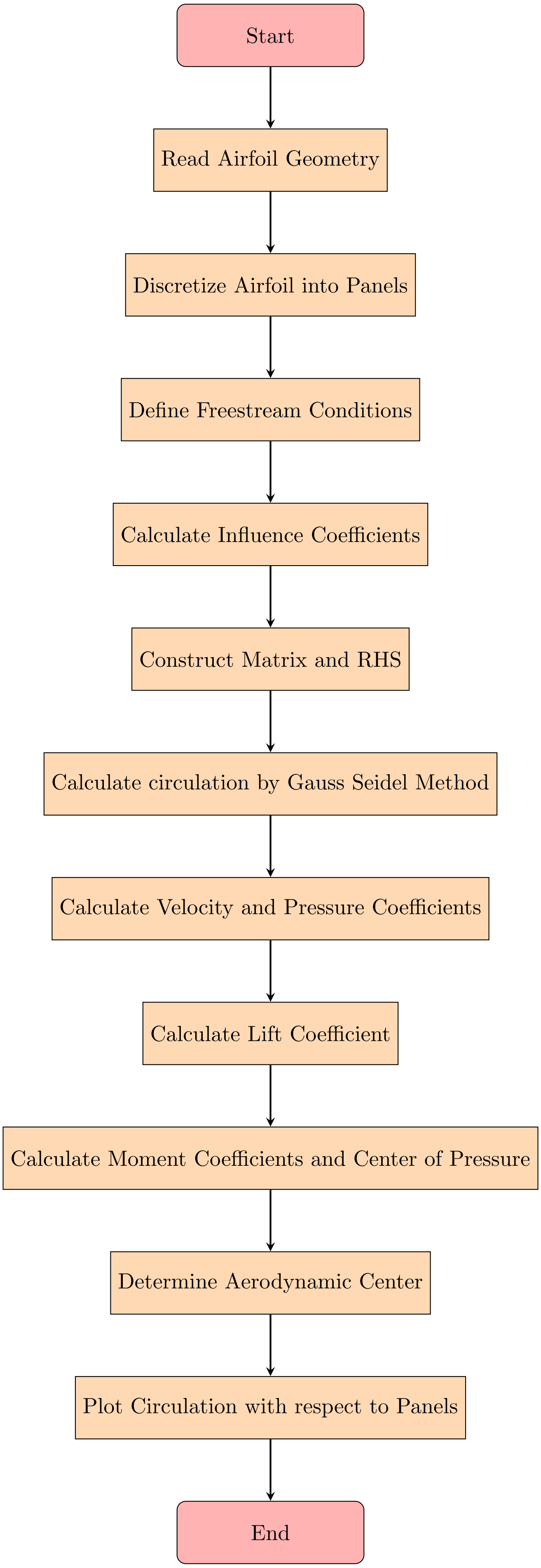

Figure 1: Flow Chart of Panel Method Algorithm

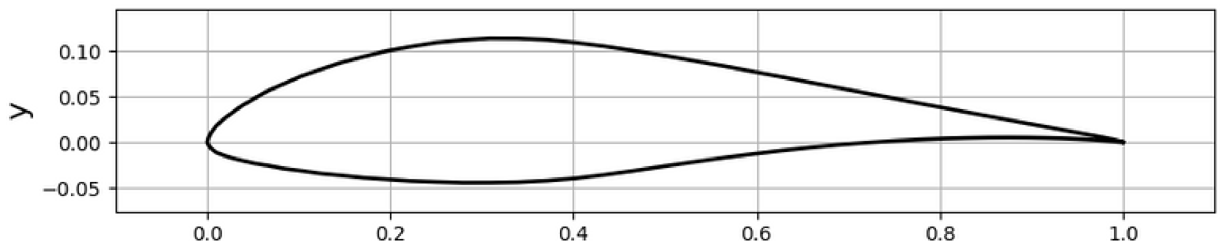

Figure 1: Geometry of Airfoil FXS21158

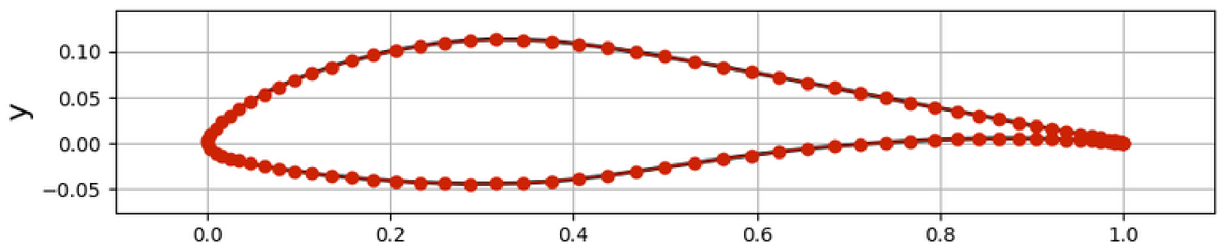

Figure 2: Panels of Airfoil FXS21158

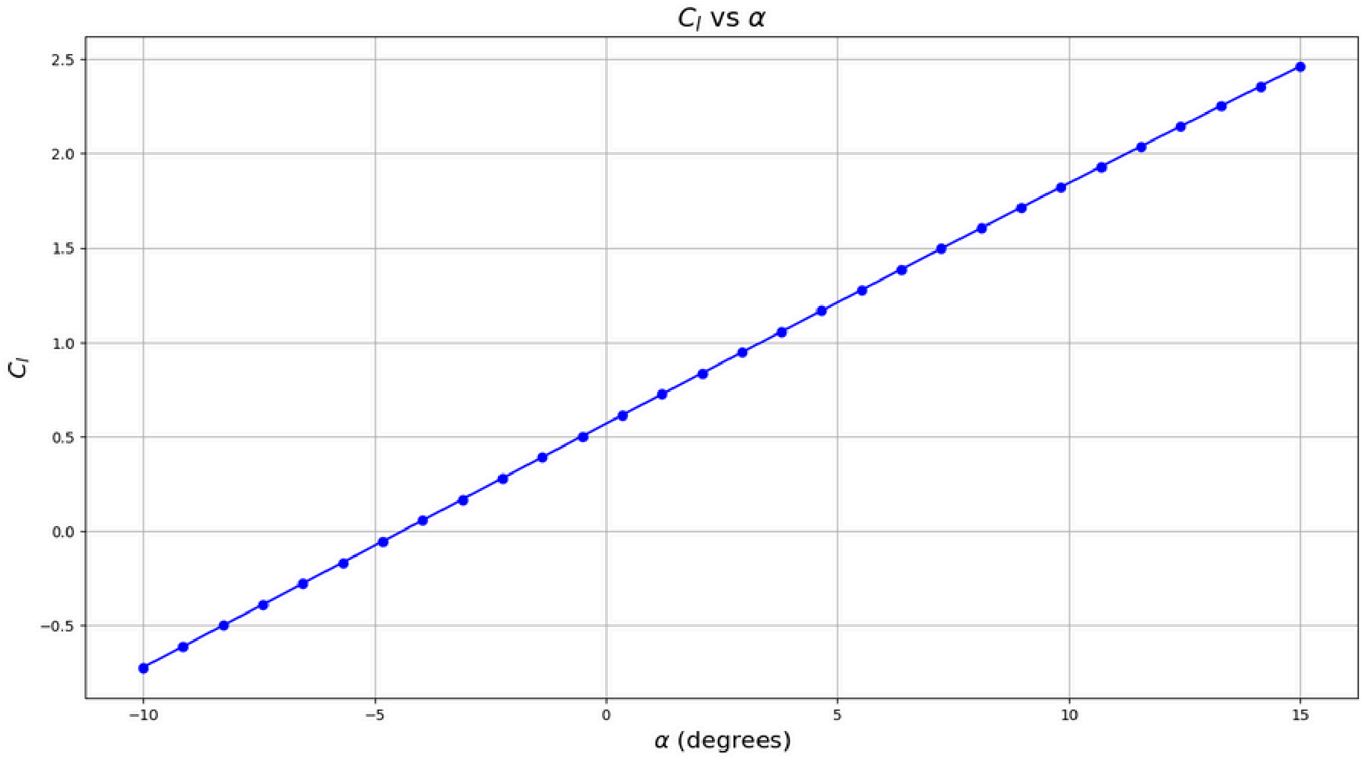

Figure 3: Coefficient of lift v/s Angle of attack

- Slope of Cl vs Alpha curve : 0.12770432641918336 per Degree
- zero lift angle of attack: -4.41 degrees
- Cl at zero angle of attack: 0.5686094465033966 degrees

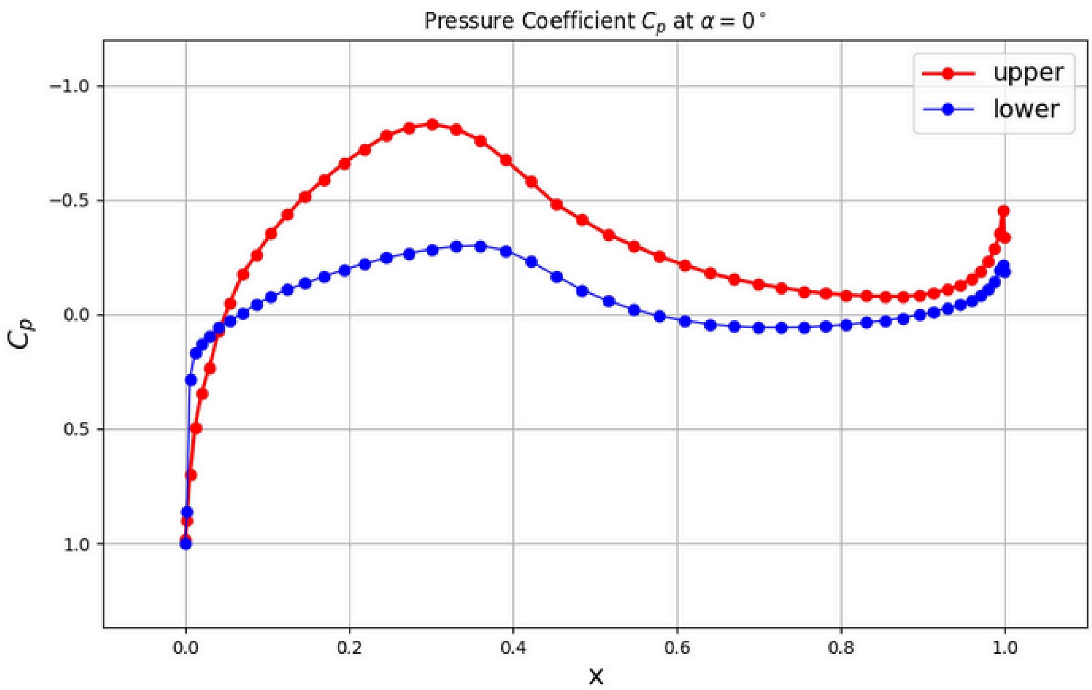

Figure 4: Pressure coefficient v/s chord(X-axis)
at 0 degree Angle of attack

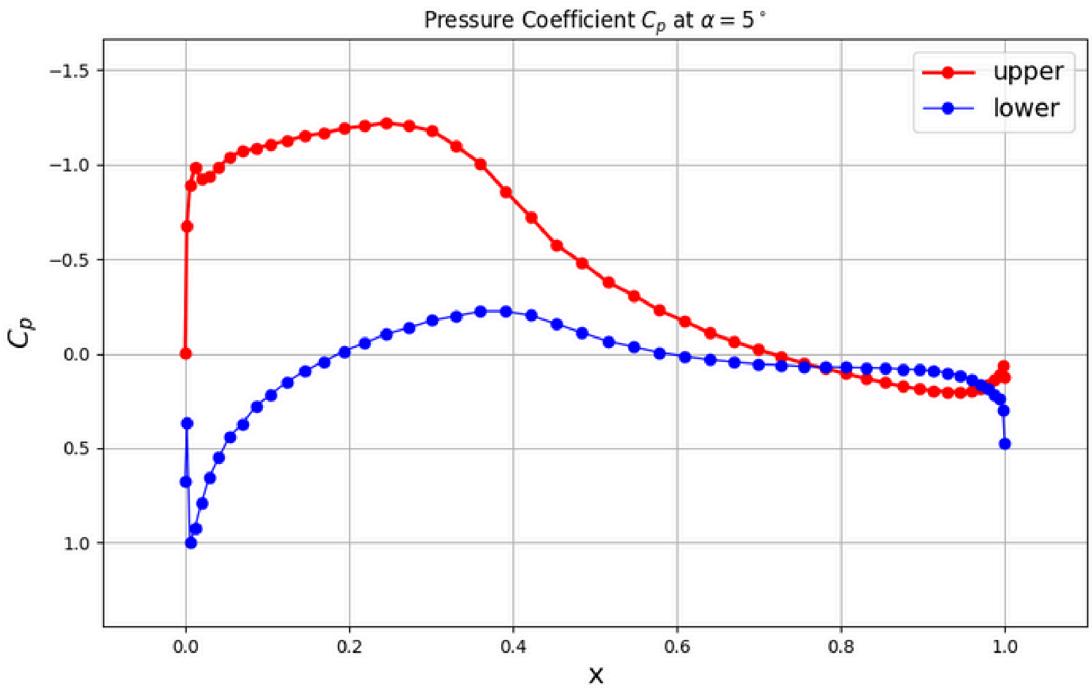

Figure 5: Pressure coefficient v/s chord(X-axis)
at 5 degree Angle of attack