Assignment 2: Aircraft Performance Analysis: A Study of Range, Endurance, and Flight Dynamics

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1. Introduction

The goal of this analysis is to study the performance characteristics of an aircraft by evaluating its range, endurance, velocity, and flight path angle across varying altitudes. Using computational techniques, we solve the equations of motion for an aircraft, incorporating the effects of altitude-dependent air density, aerodynamic parameters, and aircraft-specific properties. The study also includes time-domain simulation of aircraft motion to visualize key performance trends.

2. Problem Statement

The problem aims to assess and analyze the following:

1. Maximum range, maximum endurance, velocity, and flight path angle as functions of altitude.

2. Dynamics of velocity, flight path angle, horizontal position, and altitude over time using numerical integration of equations of motion.

The study is based on the aircraft's known physical parameters, aerodynamic properties, and environmental conditions (e.g., air density variation with altitude).

3. Methodology

(Coded in Python Programming Language)

3.1 Equations of Motion

The aircraft's motion is described by the following equations:

$$\frac{dv}{dt} = \frac{-D - W\sin(\gamma)}{m},$$
$$\frac{d\gamma}{dt} = \frac{L - W\cos(\gamma)}{mv},$$
$$\frac{dX_e}{dt} = v\cos(\gamma),$$
$$\frac{dh}{dt} = v\sin(\gamma).$$

These equations are numerically integrated using the `solve_ivp` method to compute the dynamics of velocity, flight path angle, horizontal position, and altitude over time.

3.2 Aircraft Dynamics

The aerodynamic forces, lift (L) and drag (D), are modeled as:

$$L = \frac{1}{2}\rho v^2 S C_L, \quad D = \frac{1}{2}\rho v^2 S \left(C_{D0} + K C_L^2\right),$$

where ρ is air density, S is wing area, C_L is the lift coefficient, C_{D0} is the zero-lift drag coefficient, and $K = \frac{1}{\pi e AR}$ is the induced drag constant.

3.3 Air Density Variation

Air density varies with altitude according to:

$$\rho = \rho_{\text{sl}} \cdot \exp\left(-\frac{h}{\beta}\right),$$

where $\rho_{\text{sl}} = 1.225\,\text{kg/m}^3$ (sea-level density) and $\beta = 9296\,\text{m}$ (scale height).

3.4 Performance Metrics

The following metrics are evaluated:

- **Maximum Range (R_{max}):**

$$R_{\text{max}} = \frac{h}{2\sqrt{C_{D0}K}}.$$

- **Maximum Endurance (E_{max}):**

$$E_{\text{max}} = \sqrt{\frac{\rho_{\text{sl}}}{2(W/S)} \cdot \frac{C_L^{3/2}}{C_D} \cdot 2\beta\left(1 - \exp\left(-\frac{h}{2\beta}\right)\right)}.$$

- **Velocity (V):**

$$V = \sqrt{\frac{2W}{\rho S C_L}}.$$

- **Flight Path Angle (γ):**

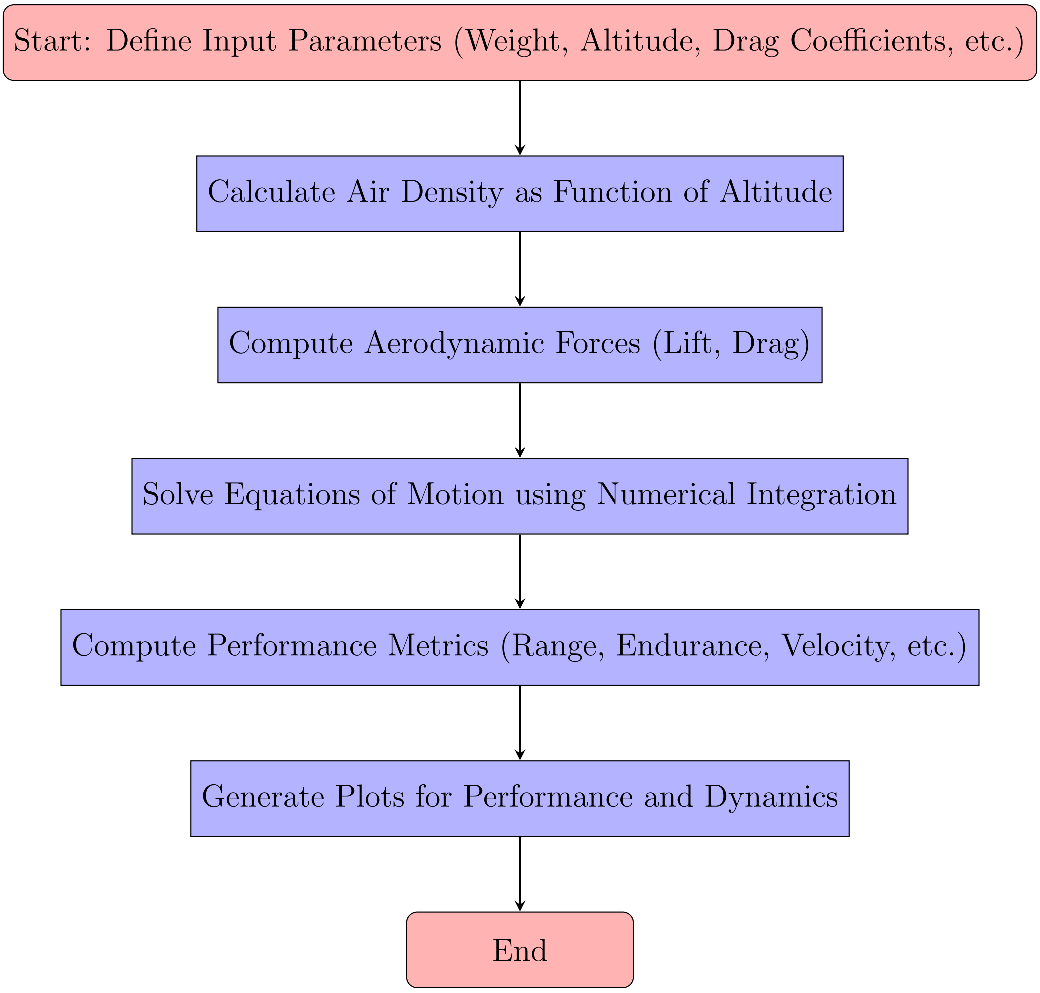
$$\tan(\gamma) = \frac{1}{C_L/C_D}.$$

3.5 Numerical Integration

The equations of motion are solved numerically over a time span of 1000 seconds. The results provide the velocity, flight path angle, horizontal position, and altitude as functions of time.

4. Flowchart of the Python Solution

Below is a flowchart illustrating the workflow used to solve the problem using Python. The steps include defining input parameters, calculating aerodynamic forces, solving equations of motion, and generating performance metrics:



5. Results

5.1 Performance Trends

The analysis reveals several key trends in aircraft performance metrics with respect to altitude:

- **Maximum Range ($R_{\mathbf{max}}$):** 90.7 km for the altitude of 7km.
  The maximum range increases with altitude due to a reduction in drag. At higher altitudes, the thinner air results in less aerodynamic resistance, enabling the aircraft to cover more distance for the same energy expenditure.

- **Maximum Endurance ($E_{\mathbf{max}}$):** 0.68 hours for 10km altitude.
  Maximum endurance decreases with increasing altitude. This is because, while drag decreases, the lift required to sustain the aircraft in thinner air increases energy demands, reducing the duration the aircraft can remain aloft.

- **Velocity (V):** The velocity required for steady flight increases with altitude. To maintain lift in lower-density air, the aircraft must fly faster, as lift is directly proportional to air density and velocity squared.

- **Flight Path Angle (γ):** Flight path angle: -4.35 degrees.
  The flight path angle shows minor variations across altitudes, reflecting adjustments required to balance lift and drag forces under different air density conditions.

5.2 Numerical Integration

The time-domain simulation provides dynamic insights into the aircraft's motion:

- **Velocity vs. Time:** The velocity decreases initially as drag acts on the aircraft but stabilizes over time.

- **Flight Path Angle vs. Time:** The flight path angle evolves steadily, reflecting the aircraft's trajectory under dynamic conditions.

- **Horizontal Position vs. Time:** The horizontal position increases linearly, showing consistent forward motion of the aircraft.

- **Altitude vs. Time:** The altitude changes dynamically, influenced by the velocity and flight path angle, illustrating the climb or descent of the aircraft over time.

6. Discussion

- **Performance Trends:** Maximum range increases with altitude due to reduced drag, while maximum endurance decreases due to reduced lift. Velocity increases with altitude to maintain lift in thinner air, and flight path angle shows minor variations.

- **Time-Domain Results:** Time-series plots reveal consistent trends for velocity, altitude, and position, aligning with theoretical predictions of steady flight.

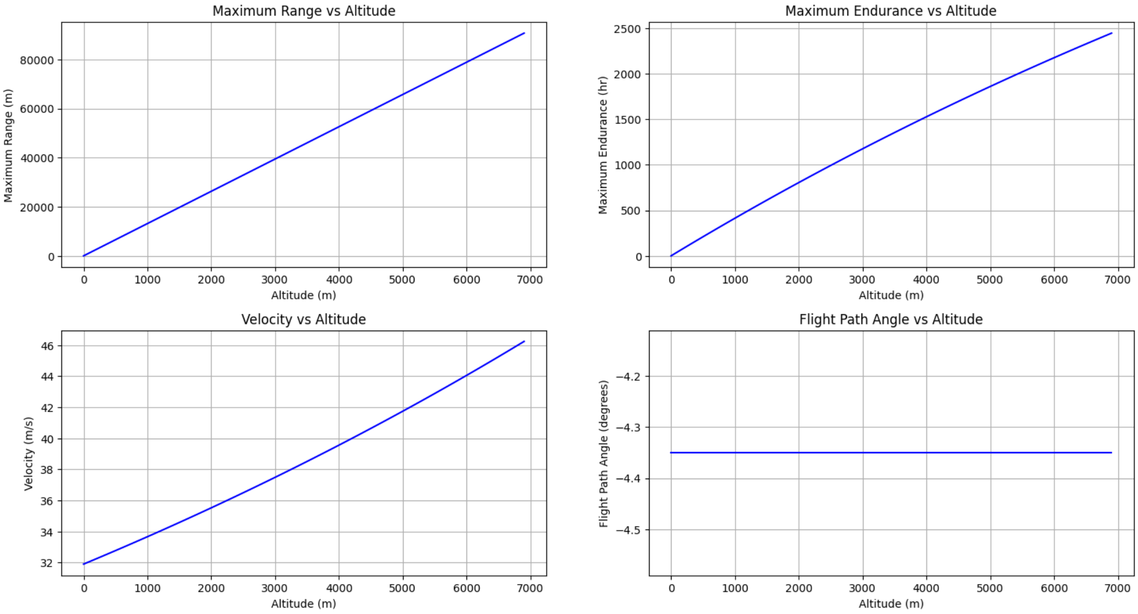


Figure 1: Time-Domain Simulation Results

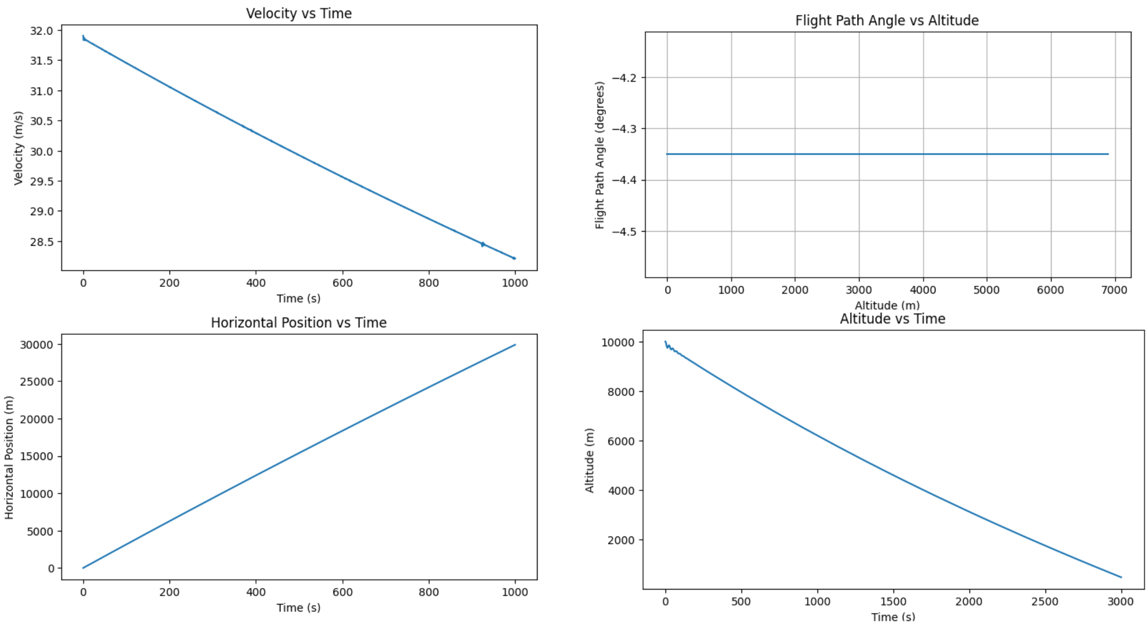


Figure 2: Performance Metrics vs. Altitude

- **Limitations:** The study assumes constant aerodynamic properties and neglects factors like engine thrust, fuel consumption, and real-world atmospheric effects.

7. Conclusion

This analysis demonstrates the effects of altitude on aircraft performance, providing insights into optimal cruising conditions for maximizing range and endurance. Numerical integration validates the equations of motion and serves as a basis for more advanced studies.

8. References

- John D. Anderson, *Introduction to Flight*, 8th edition, McGraw-Hill Education / Asia, 2016.
- Mohammad H. Sadraey, *Aircraft Performance: An Engineering Approach*, CRC Press, 2017.