Analysis of Aircraft Acceleration Ratios Due to Earth's Curvature and Rotation

Submitted by:
Ajinkya Suranjan Salve
Roll No.: SC24M005
Subject : Atmospheric flight mechanics
Branch: Aerodynamics and flight mechanics

Date:
05 October 2024

1 Introduction

This report analyzes the acceleration ratios (ac/g) at the aircraft's center of gravity (CG) caused by the curvature and rotation of the Earth. The aircraft's speed (Vac) varies between 50 m/s and 1000 m/s, and the altitude ranges from 1 km to 60 km. Earth's gravity is assumed to change with altitude, and Earth's angular velocity is calculated as 2π radians covered in 24 hours. The study also discusses the assumptions of a flat, non-rotating Earth and the limits under which these assumptions hold for the derivation of aircraft equations of motion.

2 Notation

The following symbols are used throughout the analysis:

- g_0: Gravitational acceleration at Earth's surface (9.81 m/s^2)
- g: Gravitational acceleration at height h above Earth's surface
- R_e: Radius of Earth (6,378,000 m)
- h: Altitude (in meters)
- V_r: Resultant velocity (sum of Earth's rotational speed and aircraft velocity Vac)
- Vac: Aircraft velocity
- ac: Centripetal acceleration due to Earth's curvature and rotation
- ac/g: Ratio of centripetal acceleration to gravitational acceleration
- ω: Earth's angular velocity

SectionEarth's Angular Velocity and Gravitational Model

2.1 Earth's Angular Velocity

The Earth completes one full rotation (2 radians) in 24 hours. Therefore, Earth's angular velocity is calculated as:

$$\omega = \frac{2\pi}{86400}\,\text{rad/s}$$

2.2 Gravitational Acceleration

Gravitational acceleration decreases with altitude according to the following equation:

$$g = g_0 \left(\frac{R_e}{R_e + h} \right)^2$$

Where R_e is the radius of the Earth, and h is the altitude. This decrease in gravity with altitude is accounted for in the calculations of centripetal acceleration.

3 Centripetal Acceleration

The centripetal acceleration (ac) due to the Earth's curvature and rotation at a given altitude and velocity is calculated as:

$$ac = \frac{V_r^2}{R_e + h}$$

Where V_r is the resultant velocity, which combines the rotational speed of the Earth and the velocity of the aircraft (Vac):

$$V_r = (R_e \cdot \omega) + Vac$$

4 Ratio of Accelerations

The primary quantity of interest is the ratio of centripetal acceleration to gravitational acceleration (ac/g), which is given by:

$$\frac{ac}{g} = \frac{V_r^2}{(R_e + h) \cdot g}$$

5 Graph and Analysis

The following plot (Figure 1) shows ac/g versus altitude (h) for different aircraft velocities (Vac) ranging from 50 m/s to 1000 m/s.

5.1 Key Observations

- **Minimal Effect of Altitude:** The ratio ac/g remains almost constant with altitude. This suggests that for the range of heights studied (up to 60 km), the effect of altitude on the acceleration ratio is minimal compared to the effect of aircraft velocity.

- **Impact of Velocity:** As the aircraft velocity increases, the ac/g ratio grows significantly. Higher speeds contribute much more to centripetal acceleration compared to altitude variations.

- **Aircraft Speed as a Dominant Factor:** The dominant factor influencing the acceleration ratio is the velocity of the aircraft (Vac). This is evident from the fact that increasing Vac by 100 m/s results in a significant rise in the ac/g ratio.

6 Discussion on Flat and Non-rotating Earth Assumptions

The assumption of a flat and non-rotating Earth simplifies the equations of motion for an aircraft by ignoring the curvature of the Earth and its rotation. This assumption holds well under the following conditions:

- **Low Altitudes:** At low altitudes (below approximately 10 km), the effects of Earth's curvature are minimal, and gravitational variations can be considered negligible. In this region, the flat Earth model holds reasonably well.

- **Low Speeds:** For aircraft flying at low speeds (up to about 300 m/s), the rotational effects of the Earth are small enough to be neglected in the equations of motion.

6.1 Limits of the Flat and Non-rotating Earth Model

As altitude increases beyond 10 km and aircraft speed exceeds 300 m/s, the assumptions of a flat and non-rotating Earth begin to break down:

- **Curvature Effects:** At higher altitudes (above 10 km), the curvature of the Earth becomes increasingly important. Ignoring these effects can lead to significant errors in the calculation of forces and accelerations.

- **Rotational Effects:** For high-speed aircraft (greater than 300 m/s), the Earth's rotation contributes significantly to the resultant velocity, and thus to the centripetal acceleration. The non-rotating Earth assumption is no longer valid in such scenarios.

7 Conclusion

The analysis demonstrates that for aircraft flying at higher speeds and altitudes, the effects of Earth's curvature and rotation must be accounted for in the equations of motion. The flat, non-rotating Earth model holds only for low altitudes and speeds. As altitude and velocity increase, more sophisticated models are required to accurately describe the forces acting on the aircraft.