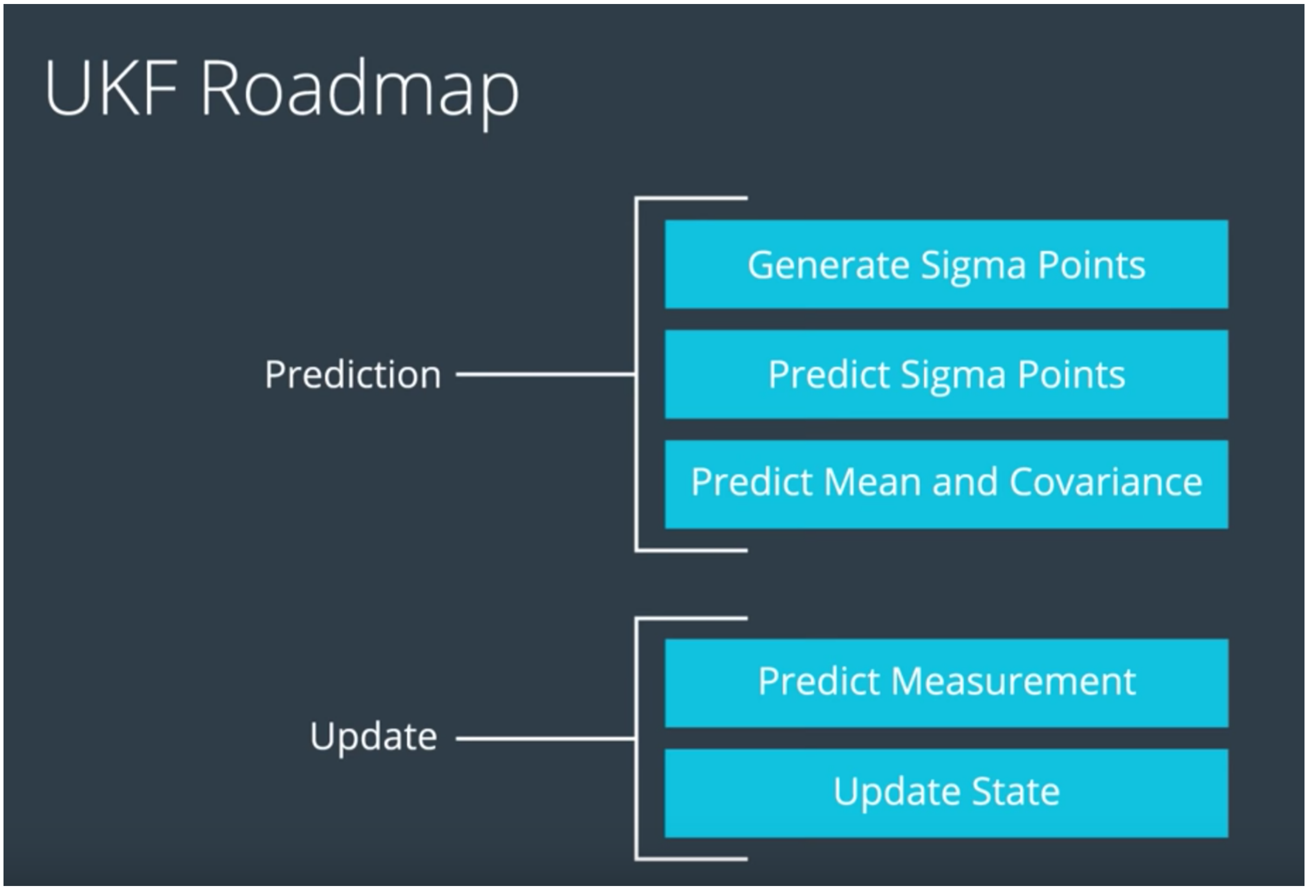
Unscented Kalman Filter Project



Rubric:

https://review.udacity.com/#!/rubrics/783/view

CTRV Model:

Helpful Equations

$$x = \begin{bmatrix} p_x \\ p_y \\ v \\ \psi \\ \dot{\psi} \end{bmatrix}$$

If $\dot{\psi}_k$ is not zero

State = $x_{k+1} = x_k + \begin{bmatrix} \frac{v_k}{\dot{\psi}_k}(sin(\psi_k + \dot{\psi}_k \Delta t) - sin(\psi_k)) \\ \frac{v_k}{\dot{\psi}_k}(-cos(\psi_k + \dot{\psi}_k \Delta t) + cos(\psi_k)) \\ 0 \\ \dot{\psi}_k \Delta t \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}(\Delta t)^2 cos(\psi_k)\nu_a, k \\ \frac{1}{2}(\Delta t)^2 sin(\psi_k)\nu_a, k \\ \Delta t \nu_a, k \\ \frac{1}{2}(\Delta t)^2 \nu \ddot{\psi}, k \\ \Delta t \nu \ddot{\psi}, k \end{bmatrix}$

If $\dot{\psi}_k$ is zero

State = $x_{k+1} = x_k + \begin{bmatrix} v_k cos(\psi_k)\Delta t \\ v_k sin(\psi_k)\Delta t \\ 0 \\ \dot{\psi}_k \Delta t \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}(\Delta t)^2 cos(\psi_k)\nu_a, k \\ \frac{1}{2}(\Delta t)^2 sin(\psi_k)\nu_a, k \\ \Delta t \nu_a, k \\ \frac{1}{2}(\Delta t)^2 \nu \ddot{\psi}, k \\ \Delta t \nu \ddot{\psi}, k \end{bmatrix}$

Generating Sigma Points:

Helpful Equations

$$Xk|k = [Xk|k \qquad X_{k|k} + \sqrt{(\lambda + nx)Pk|k} \qquad X_{k|k} - \sqrt{(\lambda + nx)Pk|k}]$$

remember that $X_{k|k}$ is the first column of the Sigma matrix.

$X_{k|k} + \sqrt{(\lambda + nx)Pk|k}$ is the second through $n_k + 1$ column.

$X_{k|k} - \sqrt{(\lambda + nx)Pk|k}$ is the $n_k + 2$ column through $2n_k + 1$ column.

UKF Augmentation:

Augmented State = $x_{a,k} = \begin{bmatrix} p_x \\ p_y \\ v \\ \psi \\ \dot{\psi} \\ \nu_a \\ \nu\ddot{\psi} \end{bmatrix}$

Augmented Covariance Matrix = $Pa, k|k = \begin{bmatrix} Pk|k0 & 0 \\ 0 & Q \end{bmatrix}$

Predict Mean and Covariance:

Weights

$w_i = \frac{\lambda}{\lambda+n_a}, i = 1$

$w_i = \frac{1}{2(\lambda+n_a)}, i = 2...n_\sigma$

Predicted Mean

$x_{k+1|k} = \sum_{i=1}^{n_\sigma} w_i X_{k+1|k,i}$

Predicted Covariance

$P_{k+1|k} = \sum_{i=1}^{n_\sigma} wi(Xk+1|k, i - x_{k+1|k})(Xk+1|k, i - xk+1|k)^T$

Predict Radar Measurements:

State Vector

$$x_{k+1|k} = \begin{bmatrix} p_x \\ p_y \\ v \\ \psi \\ \dot{\psi} \end{bmatrix}$$

Measurement Vector

$$z_{k+1|k} = \begin{bmatrix} \rho \\ \varphi \\ \dot{\rho} \end{bmatrix}$$

Measurement Model

$$z_{k+1|k} = h(xk+1) + wk+1$$

$$\rho = \sqrt{p_x^2 + p_y^2}$$

$$\varphi = arctan(\frac{p_y}{p_x})$$

$$\dot{\rho} = \frac{p_x cos(\psi)v + p_y sin(\psi)v}{\sqrt{p_x^2 + p_y^2}}$$

Predicted Measurement Mean

$$z_{k+1|k} = \sum_{i=1}^{n_\sigma} w_i Z_{k+1|k,i}$$

Predicted Covariance

$$S_{k+1|k} = \sum_{i=1}^{n_\sigma} wi(Zk+1|k,i - z_{k+1|k})(Zk+1|k,i - zk+1|k)^T + R$$

$$R = E(w_k \cdot wk^T) = \begin{bmatrix} \sigma\rho^2 & 0 & 0 \\ 0 & \sigma\varphi^2 & 0 \\ 0 & 0 & \sigma\dot{\rho}^2 \end{bmatrix}$$

UKF Update:

Cross-correlation Matrix

$$T_{k+1|k} = \sum_{i=1}^{n_\sigma} w_i (X_{k+1|k,i} - x_{k+1|k}) \, (Z_{k+1|k,i} - z_{k+1|k})^T$$

Kalman gain K

$$K_{k+1|k} = T_{k+1|k} S_{k+1|k}^{-1}$$

Update State

$$x_{k+1|k+1} = x_{k+1|k} + K_{k+1|k}(z_{k+1} - z_{k+1|k})$$

Covariance Matrix Update

$$P_{k+1|k+1} = P_{k+1|k} - K_{k+1|k} S_{k+1|k} K_{k+1|k}^T$$

Parameter Consistency:

Process model

$$x_{k+1} = f(x_k, \nu_k)$$

Process noise

$$\nu_k = \begin{bmatrix} \nu_{a,k} \\ \nu_{\ddot{\psi},k} \end{bmatrix}$$

Process noise covariance

$$Q = \begin{bmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_{\ddot{\psi}}^2 \end{bmatrix}$$

Measurement model

$$z_{k+1} = h(x_{k+1}) + \omega_{k+1}$$

Radar measurement noise

$$\omega_k = \begin{bmatrix} \omega_{\rho,k} \\ \omega_{\varphi,k} \\ \omega_{\dot{\rho},k} \end{bmatrix}$$

Measurement noise covariance

$$R = \begin{bmatrix} \sigma_\rho^2 & 0 & 0 \\ 0 & \sigma_\varphi^2 & 0 \\ 0 & 0 & \sigma_{\dot{\rho}}^2 \end{bmatrix}$$

Normalized Innovation Squared (NIS)

$$\varepsilon = (z_{k+1} - z_{k+1|k})^T \cdot S_{k+1|k}^{-1} \cdot (z_{k+1} - z_{k+1|k})$$

$$\varepsilon \sim \chi^2$$

df	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$	$\chi^2_{.050}$
1	0.004	0.016	2.706	3.841
2	0.103	0.211	4.605	5.991
3	0.352	0.584	6.251	7.815
4	0.711	1.064	7.779	9.488
5	1.145	1.610	9.236	11.070