

## mechanics formulation

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### 1 Mechanical equilibrium

First variation with respect to displacement gives:

$$\begin{aligned}\delta_{\mathbf{u}}\Pi(\mathbf{E}) &= \left. \frac{d\Pi(\mathbf{E}(\mathbf{u} + \epsilon\boldsymbol{\omega}^{\mathbf{u}}))}{d\epsilon} \right|_{\epsilon=0} \\ &= \int_{\Omega_0} \left. \frac{df(\mathbf{E}(\mathbf{F}(\mathbf{u} + \epsilon\boldsymbol{\omega}^{\mathbf{u}})))}{d\epsilon} \right|_{\epsilon=0} JdV \\ &= \int_{\Omega_0} \omega_{i,J}^{\mathbf{u}} \frac{\partial f}{\partial F_{iJ}} JdV\end{aligned}\tag{1}$$

Assuming mechanics is quasi-static:

$$\begin{aligned}\delta_{\mathbf{u}}\Pi(\mathbf{E}) &= 0 \\ \Rightarrow \int_{\Omega_0} \omega_{i,J}^{\mathbf{u}} \frac{\partial f}{\partial F_{iJ}} JdV &= 0\end{aligned}\tag{2}$$

### 2 Constitutive equations