

INDIAN INSTITUTE OF INFORMATION TECHNOLOGY, NAGPUR

Course: NUMERICAL METHODS AND PROBABILITY THEORY (NMPT) - MAL 201

Session : July-December 2023

Numerical Methods: Algebraic and Transcendental Equations

1. By using Bisection Method, find an approximate root of the equation $\sin x = 1/x$, that lies between $x = 1$ and $x = 1.5$. Carry out computations upto the 7th stage.
2. Approximate the value of π by solving $\tan \frac{x}{4} - 1 = 0$, using Bisection Method.
3. Find an approximation of the roots of the following equations using the method of False Position correct to three decimal places:
 - (a) $x^3 + x^2 + x + 7 = 0$
 - (b) $x^3 - x - 4 = 0$
4. Find the root of the equation $\cos x = xe^x$ using Regula Falsi method correct to four decimal places.
5. Determine the initial approximations for finding the smallest positive roots of the following equations and use these to find the root correct to three decimal places with Secant method:
 - (a) $x^4 - x - 10 = 0$
 - (b) $x e^x = 1$
6. Find an approximation of the roots of the following equations using the Newton Raphson method correct to three decimal places:
 - (a) $\sin x = 1 - x$
 - (b) $x^4 + x^2 - 80 = 0$
 - (c) $x - \cos x = 0$
 - (d) $x + \ln x = 2$
7. Write the Newton-Raphson iteration formula for finding \sqrt{N} , where N is a real number. Use it to find $\sqrt{5}$ correct to four decimal points.
8. The function $x - 0.2 \sin x - 0.5 = 0$ has exactly one zero in $[0.5, 1]$. Locate the zero correct to six decimal places using Regula Falsi Method, Secant Method and Newton-Raphson method. Compare the solutions.
9. Using the fixed-point iteration method, find the smallest negative root in magnitude of $3x^3 - x + 1 = 0$ correct to four decimal places.
10. Using the method of successive approximations, find the real root of the equation $\cos x = 3x - 1$ correct to three decimal places.

11. Let α and β be two real roots of the equation $x^2 + ax + b = 0$ with condition that $|\alpha| > |\beta|$. Show that the iteration method

$$x_{k+1} = -\frac{ax_k + b}{x_k}$$

is convergent near $x = \alpha$.

12. The equation

$$f(x) = 0.1 - x + \frac{x^2}{(2!)^2} - \frac{x^3}{(3!)^2} + \frac{x^4}{(4!)^2} - \frac{x^5}{(5!)^2} + \dots = 0$$

has one root in the interval $(0, 1)$. Calculate this root correct to 5 decimals.

13. Find the root of the equation $\tan x + \tanh x = 0$ which lies in the interval $(1.6, 3.0)$ correct to three decimal places using Regula-Falsi Method.
14. Find a root of $2x - \log_{10}^x = 7$ correct to three decimal places using the General Iteration Method.