ISEN 620 - Vaccine Distribution System

Project Phase 1 - IP Model

Group 14

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Executive Summary

The COVID-19 pandemic has changed our lives in ways that we could not have imagined. For the past 18 months, the pandemic has impacted everyone in some or another way. But, the most affected are the vulnerable people including frontline workers, old age people, people having comorbidities and so on. As the Delta Variant of COVID-19 puts more people at risk, there is a newfound desire to administer vaccines to as many people on a priority basis.

In this project, we developed a linear-integer optimization model to automate Vaccine Distribution in the state of Texas. Our model optimizes everything from number of vaccines to order from manufacturers, to allocate these vaccines to warehouses and finally reaches administration sites (say county). We defined a county's performance as equal to the number of priority groups it vaccinated fully. The model accuracy is dependent on each county's ability to vaccinate and so our main target is to improve each county's performance index. Apart from this, we also optimized our model in terms of location of various sites, time aspects and the budget we have.

There can be many conventional algorithms and programming languages to implement this concept. We used the linear-integer model in AMPL modelling software. AMPL is an algebraic modelling language that provides an easy to implement multiply optimization approach. We also make sure to provide a user-friendly interface for entering the input parameters.

In Phase 1 of the report, we covered in detail how we are approaching the problem by defining all the parameters, decision variables, object function and constraints to our linear-integer model and also having the ready basic code structure in AMPL. In the subsequent Phase 2, we will be using all the input parameters in our AMPL code and having a running model which can be used in automating the Vaccine distribution in the state of Texas.

1. Introduction

The Delta variant of COVID-19 is affecting a larger population and they are at the risk of contracting the virus. There is a new breakthrough in the vaccine, and it has to be administered throughout Texas and its citizens. So, in order to effectively administer the vaccine, the Governor of Texas has approached the Texas A&M Industrial Consulting firm to design the efficient supply chain for the distribution and administration of vaccines in the state of Texas. The goal is to vaccinate as many groups as possible of people vulnerable to the virus throughout the state of Texas and maximize the efficiency of vaccination of the least efficient counties in vaccinating the vulnerable population groups in Texas State.

Our approach is to create a tool that focuses on the Integer programming method which suits in making the decisions on vaccinating the vulnerable groups as per the order of vulnerability. Integer programming consists of a set of linear equations and considers variables as integers for making yes or no decisions.

AMPL software for the modelling of the design and CPLEX for the optimizers are considered to solve the problem. AMPL nearly reflects the mathematical syntax form the constraints and the objective for the problem specified and it's easier for the non-programmers to model the problem and input the data to solve the problem. AMPL model is much flexible to solve as we can make the necessary changes as per the needs of the future the data may undergo. AMPL consists of different optimizers such as CPLEX, Gurobi, CBC, FortMp, MINTO which can be used on a case-by-case basis.

The developed model will help the Governor to rectify the shortcomings of vaccinating the vulnerable group of Texas state by improving the efficiency of the least performing counties as the objective by inputting the number of doses the county administered to the vulnerable group in the four weeks of the month in an orderly fashion. Thereby making the State of Texas in achieving the dream of vaccinating its population much earlier than any State in the United States leading to protect the vulnerable group from contracting the virus. So that people can live without the fear and challenges of COVID-19, and we will have to move past this most challenging as the future awaits us to learn from it and move forward.

2. Integer Linear Programming Model

The problem at hand is tackled using the linear integer programming approach to reach an optimal solution for warehouse setup, vaccine distribution and the efficient and cost-effective completion of the vaccination drive at the end of the given time period. Given below are the various aspects of our integer linear programming model developed to achieve the stated purpose.

2.1 Decision Variables

$X1_{ik}$
Number of vaccines bought from manufacturers at the start of a week for i=1,...,N & k=1,...,K

$X2_{ijk}$
Number of vaccines transported from each warehouse to each vaccination center in each week for j=1,...,J & k=1,...,K & i=1,...,N

$X3_{ij}$
Number of vaccines administered at each vaccination center in each week , for i=1,...,N and j=1,...,J

Y_k
= 1 if warehouse k is in use
= 0, otherwise
For k=1,...,K

Z_{mj}
= 1, if Priority group m is fully vaccinated in county j
=0, otherwise
For m=1,...,M and J=1,...,J

$I1_{ik}$
Inventory in warehouse k at the end of the week i for i=1,...,N

$I2_{ij}$
Inventory in Vaccination Center j at end of the week i for i=1,...,N and j=1,...,J

P
Performance index of the worst-performing vaccination center

2.2 Parameters

Parameter	Description	Index
N	Number of Weeks	
J	Number of Vaccine Centers (Number of counties)	
K	Number Of Candidate Locations for Warehouses (Distribution Centers)	
M	Number of Priority Groups of population	
B	Budget Limit	
Cm_i	No. of Vaccines manufacturer can sell in each week	For i = 1…..,N
Sv	Cost of vaccine per dose	
St_{jk}	Cost per vaccine to transport for each candidate from distribution center to Vaccination center	For k = 1,...,K & j=1,...,J
Cw_k	Storage Capacity for each Distribution Center	For k = 1,...,K
Sc_k	Set-up Cost for each Distribution Center	For k = 1,...,K
Cv_j	Storage Capacity of Vaccination Center	For j = 1,…,.J
Lv_j	No. of Vaccines each Vaccination center can administer	For j = 1,...,J
Sa_j	Storage fee per dose stored at the end of each week for each vaccination center	For j = 1,...,J
Q_{mj}	Population of priority group at each Vaccination center	For m = 1,...,M and j = 1,...,J
Ss_k	Storage fee per dose stored at the end of each week at each distribution center	For k = 1,...,K
G	A very big number	

Table 1. Parameters, description, and indices

Assumptions:

The cost of vaccines per dose(Sv) includes the cost of transportation from manufacturer to each DC

The storage fee per dose stored at the end of each week at each DC is given by the parameter Ss_k

2.3 Objective Function

Maximize the performance of the worst performing county

$$Max\ P$$

The IP model developed for the optimum operation of the vaccination drive is driven by the performance index of each county, while the goal of our drive being to maximize the overall performance of the entire state of Texas and not incentivizing one particular county for spectacular performance. This is being achieved through use of a max-min function, which identifies the worst performing county, and strives to maximize the performance of that county.

2.4 Constraints

2.4.1 Performance of the worst-performing VC

County's performance is measured by their capability to vaccinate as many groups as possible given their priority. The county with the least number of priority groups vaccinated, during a given time period, bounds our performance parameter P.

$P \leq \sum_{m=l}^{M} Z_{mj}$ for all j=1,...,J

2.4.2 Performance index for each population group of each county

For each county j, we have M population groups, prioritized from 1 through M. Only when a priority group with highest priority is fully vaccinated can we vaccinate the next group. We need to check in each county if each of the given 7 population groups are fully vaccinated or not.

2.4.2.1 For Priority Group 1

If the total vaccines administered over the entire time period of vaccination drive in a county exceed the population count of the highest priority vaccination group, the following constraint returns the value of Z_{mj} as 1, Thus, this in turn returns the value of 1 on RHS for constraint 2.4.1 for given county.

$$\sum_{i=1}^{N} X3_{ij} \leq Q_{mj} + G * Z_{mj}$$

for m=1 & j in 1,...,J

2.4.2.2 For Priority Group 2

If the total vaccines administered over the entire time period of vaccination drive in a county exceed the population count of the second highest priority vaccination group, the following constraint returns the value of Z_{mj} as 1 for that population group in the given county, Thus, this in turn returns the value of 2 on RHS for constraint 2.4.1 for given county.

$$\sum_{i=1}^{N} X3_{ij} - Q_{(m-1)j} \leq Q_{mj} + G * Z_{mj}$$

for m=2 & j in 1,...J

2.4.2.3 For Priority Group 3

If the total vaccines administered over the entire time period of vaccination drive in a county exceed the population count of the third highest priority vaccination group, the following constraint returns the value of Z_{mj} as 1 for that population group in the given county, Thus, this in turn returns the value of 3 on RHS for constraint 2.4.1 for given county.

$$\sum_{i=1}^{N} X3_{ij} - Q_{(m-1)j} - Q_{(m-2)j} \leq Q_{mj} + G * Z_{mj}$$

for m=3 & j in 1,...,J

2.4.2.4 For Priority Group 4

If the total vaccines administered over the entire time period of vaccination drive in a county exceed the population count of the fourth highest priority vaccination group, the following constraint returns the value of Z_{mj} as 1 for that population group in the given county, Thus, this in turn returns the value of 4 on RHS for constraint 2.4.1 for given county.

$$\sum_{i=1}^{N} X3_{ij} - Q_{(m-1)j} - Q_{(m-2)j} - Q_{(m-3)j} \leq Q_{mj} + G * Z_{mj}$$

for m=4 & j in 1,...,J

2.4.2.5 For Priority Group 5

If the total vaccines administered over the entire time period of vaccination drive in a county exceed the population count of the fifth highest priority vaccination group, the following constraint returns the value of Z_{mj} as 1 for that population group in the given county, Thus, this in turn returns the value of 5 on RHS for constraint 2.4.1 for given county.

$$\sum_{i=1}^{N} \boldsymbol{X3_{ij}} - \boldsymbol{Q_{(m-1)j}} - \boldsymbol{Q_{(m-2)j}} - \boldsymbol{Q_{(m-3)j}} - \boldsymbol{Q_{(m-4)j}} \leq \boldsymbol{Q_{mj}} + \boldsymbol{G} * \boldsymbol{Z_{mj}}$$

for m=5 & j in 1,...,J

2.4.2.6 For Priority Group 6

If the total vaccines administered over the entire time period of vaccination drive in a county exceed the population count of the sixth highest priority vaccination group, the following constraint returns the value of Z_{mj} as 1 for that population group in the given county, Thus, this in turn returns the value of 6 on RHS for constraint 2.4.1 for given county.

$$\sum_{i=1}^{N} \boldsymbol{X3_{ij}} - \boldsymbol{Q_{(m-1)j}} - \boldsymbol{Q_{(m-2)j}} - \boldsymbol{Q_{(m-3)j}} - \boldsymbol{Q_{(m-4)j}} - \boldsymbol{Q_{(m-5)j}} \leq \boldsymbol{Q_{mj}} + \boldsymbol{G} * \boldsymbol{Z_{mj}}$$

for m=6 & j in 1,...,J

2.4.2.7 For Priority Group 7

If the total vaccines administered over the entire time period of vaccination drive in a county exceed the population count of the last priority vaccination group, the following constraint returns the value of Z_{mj} as 1 for that population group in the given county, Thus, this in turn returns the value of 7 on RHS for constraint 2.4.1 for given county.

$$\sum_{i=1}^{N} \boldsymbol{X3_{ij}} - \boldsymbol{Q_{(m-1)j}} - \boldsymbol{Q_{(m-2)j}} - \boldsymbol{Q_{(m-3)j}} - \boldsymbol{Q_{(m-4)j}} - \boldsymbol{Q_{(m-5)j}} - \boldsymbol{Q_{(m-6)j}} \leq \boldsymbol{Q_{mj}} + \boldsymbol{G} * \boldsymbol{Z_{mj}}$$

for m=7 & j in 1,...,J

2.4.3 Budget Limit Constraint:

The administration has a fixed budget to administer vaccines during the time period of the entire vaccination drive. The following constraint considers the storage costs of vaccines in vaccination centers, distribution centers, as well the transportation costs from distribution centers to the counties. Also, how many distribution centers/warehouses should be set up because each warehouse incurred a fixed setup cost is also determined.

$$B \geq \sum_{i=1}^{N}\sum_{k=1}^{K} Sv * X1_{ik} + \sum_{k=1}^{K} Y_k * Sc_k + \sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{K} X2_{ijk} * St_{jk} + \sum_{i=1}^{N}\sum_{j=1}^{J} I2_{ij} * Sa_j + \sum_{i=1}^{N}\sum_{k=1}^{K} I1_{ik} * Ss_k$$

2.4.4 Maximum vaccines transported from potential Distribution Center to Vaccination Centers

In the following constraint, based on the storage capacity of each warehouse and whether the warehouse is set up or not, the maximum vaccines transported from each warehouse to all/ or some of the vaccination centers is determined.

$\boldsymbol{Cw_k * Y_k \geq \sum_{j=1}^{J} X2_{ijk}}$ for all k=1,...,K & i=1,...,N

2.4.5 Limit of vaccination at the Vaccination centers

Each vaccination center has a limit on how many vaccines it can administer on a weekly basis depending upon the administrative constraints of each vaccination center.

$\boldsymbol{Lv_j \geq X3_{ij}}$ for all i in 1,....,N & j=1,.....,J

2.4.6 Maximum Selling Capacity of Manufacturer each week

The manufacturer has a fixed supply chain that can produce vaccines within a certain limit every week. Thus, the maximum vaccines bought by the warehouses depends on this limit of vaccines the manufacturer can sell for each week.

$Cm_i \geq \sum_{k=1}^{K} X1_{ik}$ for all i in 1,...,N

2.4.7 Vaccines transported to Distribution Center from Manufacturer each week

Once the manufacturer produces the vaccines, they are sold and transported to distribution centers. But the quantity of vaccines a distribution center can procure every week depends on its maximum storage capacity and previous leftover inventory.

$\boldsymbol{X1_{ik} \leq Cw_k * Y_k - I1_{ik}}$ for all i in 1,....,N & k in 1,...,K

2.4.8 Vaccines transported from Distribution Center to Vaccination Center each week

Once a distribution center receives vaccines from the manufacturer, it is transported to different vaccination centers every week. However, the number of vaccines procured by the vaccination center are determined by the following constraint considering the storage capacity of the corresponding vaccination center and the inventory leftover from the previous week.

$\boldsymbol{\sum_{k=1}^{K} X2_{ijk} \leq Cv_j - I2_{ij}}$ for all i in 1,...,N & j in 1,...,J

2.4.9 Inventory at Warehouse

The difference between the number of vaccines each warehouse procures from the manufacturer and the quantity transported to different vaccination centers is considered in the below constraint for week 2 and onwards.

$\boldsymbol{I1_{ik} = I1_{(i-1)k} + X1_{ik} - \sum_{j=1}^{J} X2_{ijk}}$ for all i in 2,...,N & k in 1,...,K

2.4.10 Inventory at Vaccination Center

The quantity of vaccines a vaccination center receives from the warehouse versus how many it can be administered during the previous week gives us the inventory leftover at the start of a given week for week 2 and onwards.

$I2_{ij} = I2_{(i-1)j} + \sum_{k=1}^{K} X2_{ijk} - X3_{ij}$ for all i in 2,...,N & j in 1,...,J

2.4.11 Inventory at Warehouse at End of Week 1

The inventory leftover at the start of each week is given by the following constraint, which is nothing but the difference between the vaccines the given warehouse procured from the manufacturer and the vaccines transported from the given warehouse to all the vaccination centers for week 1 only.

$I1_{ik} = X1_{ik} - \sum_{j=1}^{J} X2_{ijk}$ for i=1 & k in 1,...,K

2.4.12 Inventory at Vaccination Center at end of Week 1

The inventory leftover from the previous week at the start of each week is given by the following constraint, given by the difference between total vaccines incoming from all the warehouses in the given vaccination center and the vaccines administered for week 1 only.

$I2_{ij} = \sum_{k=1}^{K} X2_{ijk} - X3_{ij}$ for i=1 & j in 1,...,J

2.4.13 Non-negativity constraints

None of the decision variables can be negative. Also, apart from the binary decision variables Z_{mj} & Y_k, all other decision variables are integer variables, as the number of vaccines cannot be fractional values, but only whole numbers.

$X1_{ik} \geq 0, integer$ for i=1,...,N & k=1,...,K

$X2_{ijk} \geq 0, integer$ for i=1,...,N, j=1,...,J & k=1,...,K

$X3_{ij} \geq 0, integer$ for i=1,...,N & j=1,...,J

$I1_{ik} \geq 0, integer$ for i=1,...,N & k=1,...,K

$I2_{ij} \geq 0, integer$ for i=1,...,N & j=1,...,J

$P \geq 0, integer$

Y_k *is binary* for k=1,...,K

Z_{mj} *is binary* for m=1,...,M & j=1,...,J

3. *References*

a. Rastegar, M., Tavana, M., Meraj, A., & Mina, H. (2021). An inventory-location optimization model for equitable influenza vaccine distribution in developing countries during the COVID-19 pandemic. Vaccine, 39(3), 495–504. https://doi.org/10.1016/j.vaccine.2020.12.022

b. Winston, W. L., & Goldberg, J. B. (2004). Operations research: applications and algorithms (Vol. 3). Belmont: Thomson Brooks/Cole.

c. ISEN 620 –Material posted for Fall 2021 coursework on CANVAS (Lectures, videos, handouts)