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Mathematical Induction

Principle of Mathematical Induction can be stated as follows

Let $S(n)$ be a statement that involves positive integer $n = 1, 2, 3 --$ then $S(n)$ is true for all positive integer n provided that

1. $S(1)$ is true   Inductive steps
2. $S(k+1)$ is true whenever $S(k)$ is true.

So there are 3 steps of proof using the principle of mathematical induction

Step 1 (inductive base) Verify that $S(1)$ is true.

Step 2 (Inductive hypothesis) Assume that $S(k)$ is true for an arbitrary value of k

Step 3 Inductive Step Verify that $S(k+1)$ is true on basis of the Inductive hypothesis

Q. 1 Show that by mathematical Induction

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

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Solution

Let the Given statement be denoted by $S(n)$

1. Inductive base for $n = 1$

Hence

$$\frac{1}{1\cdot 2} = \frac{1}{1+1} = \frac{1}{2} \quad S(1) \text{ is true.}$$

2. Inductive hypothesis.

Assume that $S(k)$ is true

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{k(k+1)} = \frac{k}{k+1}$$

3. Inductive Step. We wish to show that the statement is true for $n = k+1$

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{(k+1)(k+2)} = \frac{k}{k+2}$$

Using (2) $\underbrace{\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots \frac{1}{k(k+1)}} + \frac{1}{(k+1)(k+2)}$

$$\frac{k}{(k+1)} + \frac{1}{(k+1)(k+2)}$$

$$\frac{k(k+2) + 1}{(k+1)(k+2)} = \frac{k^2 + 2k + 1}{(k+1)(k+2)}$$

$$= \frac{(k+1)^{\cancel{2}}}{(\cancel{k+1})(k+2)} = \frac{k+1}{k+2}$$

Hence $S(k+1)$ is true whenever $S(k)$ is true for all positive integer

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Q.2 Show that

$$1^2+2^2+3^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$$

$n \geq 1$

Solution Let $S(n)$ be the Given Statement

1. Inductive base for $n=1$ we have

$$1^2=\frac{1(1+1)(2+1)}{6}=1 \quad \text{So } S(1) \text{ is true}$$

2. Inductive hypothesis. Assume that $S(k)$ is true.

$$1^2+2^2+3^2+\cdots+k^2=\frac{k(k+1)(2k+1)}{6}$$

3. Inductive step to show that $S(k+1)$ is true

$$1^2+2^2+3^2+\cdots+(k+1)^2=\frac{(k+1)(k+2)(2k+3)}{6}$$

So $1^2+2^2+3^2+\cdots+(k+1)^2=(1^2+2^2+3^2+\cdots k^2)+(k+1)^2$

now using (2)

$$=\frac{k(k+1)(2k+1)}{6}+(k+1)^2$$

$$\frac{k(k+1)(2k+1)+6(k+1)^2}{6}$$

$$\frac{(k+1)[k(2k+1)+6(k+1)]}{6}$$

$$(k+1)[2k^2+k+6k+6]/6$$

$$\frac{(k+1)(2k^2+7k+6)}{6}$$

$$=\frac{(k+1)(k+2)(2k+3)}{6}$$

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hence $S(K+1)$ is true when $S(K)$ is true.

Q3 Show that $7^{2n} + 2^{3n-3} \cdot 3^{n-1}$ is divisible by $25 \ \forall \ n \in N$

Let $S(n) = 7^{2n} + 2^{3n-3} \cdot 3^{n-1}$

We have to show that 25 divides $S(n)$

i.e $S(n) = 25 \times q \quad \forall \ q \in Z$

$S(n) = 49^n + 8^{(n-1)} \cdot 3^{(n-1)}$

$= 49^n + 24^{n-1}$

$S(1) = 49 + 1 = 50 = 25 \times 2$ — (1)

So 25 divides $P(1)$

Now assume 25 divides $S(k)$

then $49^k + 24^{k-1} = 25m$ — (2)

for some $m \in Z$

Now $S(k+1) = 49^{(k+1)} + 24^{k}$

$= 49^k \cdot 49 + 24^k$

using (2) $= 49(25m - 24^{k-1}) + 24^k$

$49 \times 25m - 49 \times 24^{k-1} + 24^k$

$49 \times 25m - 24^{k-1}[49 - 24]$

$= 49 \times 25m - 24^{k-1} \times 25$

$= 25[49m - 24^{k-1}]$ — (3)

$= 25 \cdot q$ where $q = 49m - 24^{k-1}$

hence 25 divides $S(k) \Rightarrow$ 25 divides $S(k+1)$

from 1, 2, & 3 25 divides $S(n) \ \forall \ n \in N$

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