COMPOSITION OF RELATION

If A, B, C are 3 non-empty sets and R & S the relations from $A \xrightarrow{R} B$ and $B \xrightarrow{S} C$ respectively, then we define ~~the~~ a relation from $A \rightarrow C$, denoted by RoS

$$A \xrightarrow{R} B \qquad\qquad B \xrightarrow{S} C$$

RoS.

$RoS = \{(x,z)$: there exist some $y \in B$ such $(x,y) \in R$ and $(y,z) \in S\}$.

↓

Composition of

Note $(a, b), (b, c)$

if same

then (a, c) you have to write

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Q.1 Composition of Relation

Let $A = \{1, 2, 3\}$, $B = \{p, q, r\}$

$C = \{x, y, z\}$ and

relation $R = \{(1, p), (1, r), (2, q), (3, q)\}$

relation $S = \{(p, y), (q, x), (r, z)\}$

then find RoS

As we know $RoS = \{(x, z) :$

$(x, y) \in R$ & $(y, z) \in S\}$

$(1, p)\ (p, y) \rightarrow$ So $(1, y)$

Same

$(1, p), (q, x)$ leave it

not same

$(1, p)\ (r, z)$ leave it hence

not same

we find only ~~E~~ $(1, y)$

Now we are trying by taking $(1, r)$

$(1, r)\ (p, y)$ no

$(1, r)\ (q, x)$ no

$(1, r)\ (r, z)$ $(1, z)$

same

we find $(1, z)$

Now we are trying (2, 2)

$(2, \underbrace{2) \; (p}, y)$ no

$(2, \underbrace{2) \; (2}_{\text{same}}, x)$ $(2, x)$

$(2, 2) \; (r, z)$ no

So only $(2, x)$ we have

Now last pair is $(3, 2)$

$(3, \underbrace{2) \; (p}, y)$ no

$(3, \underbrace{2) \; (2}_{\text{same}}, x)$ $(3, x)$

$(3, \underbrace{2), (r}_{\text{not}}, z)$ no

So in composition of RoS we have

$RoS = \{(1, y), (1, z), (2, x), (3, x)\}$

Q.2 Find By Matrix method

Let $A = \{3, 4, 5, 6\}$

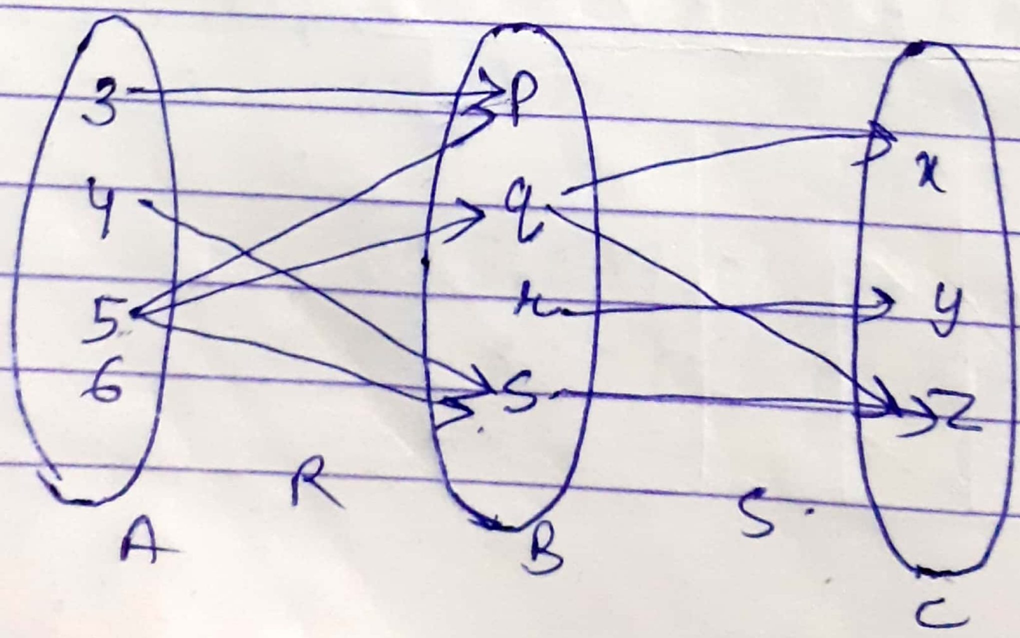
$B = \{p, q, r, s\}$

$C = \{x, y, z\}$.

$R = \{(3, p) (4, s) (5, p) (5, q) (5, s)\}$

$S = \{(q, x), (q, z) (r, y), (s, z)\}$.

$RoS = \{(4, z) (5, x) (5, z)\}$



Matrix Method →

$$M_R = \begin{array}{c} \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccc} p & q & r & s \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{4\times4} \end{array}$$

$$M_S = \begin{array}{c} \\ p \\ q \\ r \\ s \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{4\times3} \end{array}$$

$$M_{R\circ S} = M_R \times M_S$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{array}{c} \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

⇒ Related elements $= \{(4,z)\ (5,x)\ (5,z)\}$

Objective Problem

Q. If R is a relation represented by

$$M_R = \begin{array}{c|ccc} & x & y & z \\ \hline 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}$$

then find (i) M_R^c (Compliment)

(ii) M_R^{-1}

(i) for compliment replace 1 by 0 & 0 by 1

So
$$M_R^c = \begin{array}{c|ccc} & x & y & z \\ \hline 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 \\ 3 & 0 & 1 & 0 \end{array}$$

(ii) To find M_R^{-1} we write transpose of M_R

$$M_R^{-1} = \begin{array}{c|ccc} & x & y & z \\ \hline 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}$$

$\therefore R^{-1} = \{(1,x), (1,z), (2,x), (2,y), (3,z)\}$