Rules of Inference :-

1. Law of Detachment or Modus Ponens

$$[(P \to Q) \wedge P] \to Q$$

2. Law of Contraposition or Modus tollens

$$[(P \to Q) \wedge \sim Q] \to \sim P$$

3. Hypothetical Syllogism

$$(P \to Q) \wedge (Q \to R) \to (P \to R)$$

4. Disjunctive Syllogism $(P \vee Q) \wedge \sim P \to Q$

Q 1 If Ram works hard, he will get a Job. Ram works hard
Therefore he will get a job
P: Ram works hard
Q: Ram gets a job

$$(P \to Q), P \vdash Q$$

$$((P \to Q)) \wedge P \to Q$$

P	Q	$P \to Q$	$(P \to Q) \wedge P$	$(P \to Q) \wedge P \to Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

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Q. Test the validity of the argument
If it rains, Ram will be sick
Ram was not sick
It did not rain

Ans.

p : It rains
q : Ram was sick

$P_1 : p \to q$
$P_2 : \sim q$
$Q : \sim p$

$p \to q, \sim q \vdash \sim p$

$(p \to q) \wedge \sim q \to \sim p$ [Law of contra-position]

p	q	$p \to q$	$(p \to q) \wedge \sim q$	$(p \to q) \wedge \sim q \to \sim p$
T	T	T	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	T	T

∴ It is valid

Q. If a man is a bachelor, he is unhappy
If a man is unhappy, he dies young.
Bachelors die young.

Ans.

p : Man is a bachelor
q : He is unhappy
r : He dies young

$P_1 : p \to q$
$P_2 : q \to r$
$P_3 : p \to r$

$p \to q, q \to r \vdash p \to r$

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$(p \to q) \wedge (q \to r) \to (p \to r)$ [Hypothetical Syllogism]

$A \to (p \to r)$

p	q	r	$p \to q$	$q \to r$	$p \to r$	$A \to (p \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	F	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

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(4) Disjunctive Syllogism 4th law

e.g. Either the horse is white or the dog is brown. Horse is not white Therefore the dog is brown.

Disjunctive law

$$(P \vee Q) \wedge \sim P \rightarrow Q$$

P → Horse is whic
Q → Dog is brown
P_1 : $P \vee Q$
P_2 : $\sim P$

∴ : Q

Statement $(P \vee Q), \sim P \vdash Q$

To check Validity

$$(P \vee Q) \wedge \sim P \rightarrow Q$$

We have to Prepare Truth table

Say → A ←

P	Q	PVQ	~P	(PVQ)∧~P	A→Q
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	f	T	F	T

Tautology hence this rule also Verified.