

HASSE DIAGRAM

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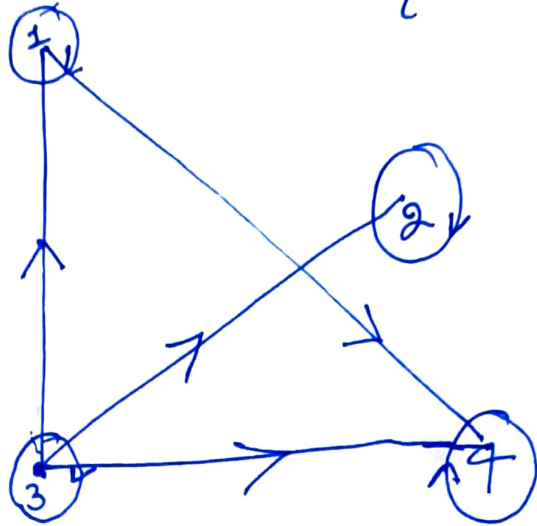
A diagram or graph that is used to represent ~~partial~~ partial order relation (POR) associated with a set is called a Hasse diagram.

Steps to Construct Hasse diagram

- Step-1 Start with directed graph of given relation
- Step 2 Remove the self loop from all vertices.
- Step 3 Remove all the edges whose existence implies transitive Property.
- Step 4 Arrange the Vertices according to their priority.
- Step-5 Remove all the arrows to get Hasse diagram.
- Step-6 If aRb check a precedes b ($a-b$) or b precedes a ($b-a$) then 3 important points
 - (i) if a precedes b , it means a appear before b in Hasse Diagram.
 - (ii) If a succeed b , it means b appears before a
 - (iii) If c precedes a & b and d also precedes a, b it means c & d have the same level.

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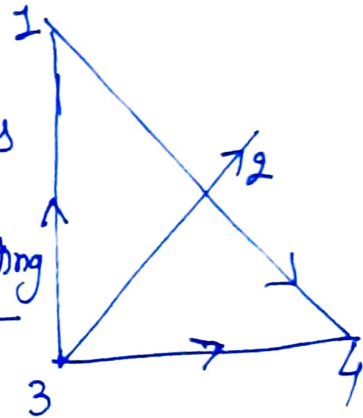
Q.1 When directed graph is given, then draw Hasse diagram for a poset on $A = \{1, 2, 3, 4\}$ and relation R is $R = \{(1,1), (2,2), (3,3), (4,4), (3,1), (1,4), (3,2), (3,4)\}$ (Given)



→ It is given here

Now Step 1 of Hasse diagram

Removing Loops
{ always remove
self loop in starting }

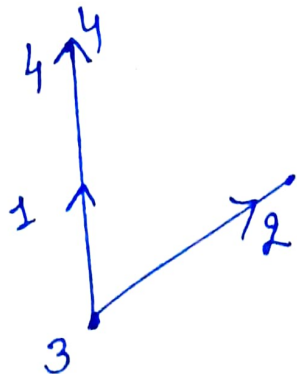


Step 2

Removing transitive property

bcz $(3,1), (1,4) \Rightarrow (3,4)$ will not be there

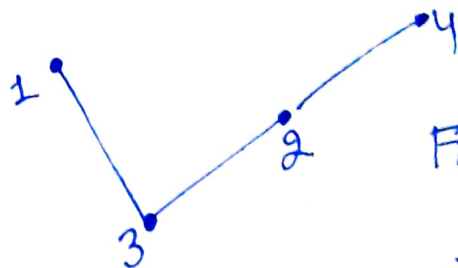
Step 3



Arranging Vertices according to priority

Step 4

Remove all arrows only dot should be there



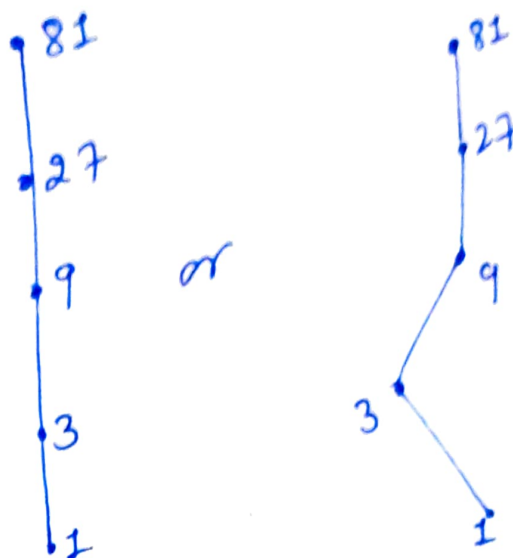
Final Hasse diagram

Case (2) When diagraph is not Given

(3)

Q2 Draw the Hasse Diagram for the Poset $(A, |)$
(here divisibility Given) on Set $A = \{1, 3, 9, 27, 81\}$

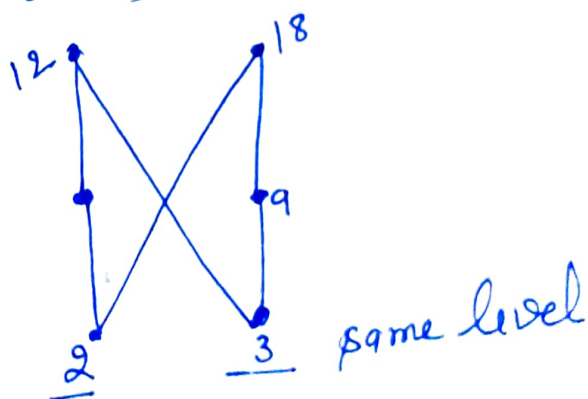
Solution Hasse diagram



Note All other edges will be removed by transitive relations.

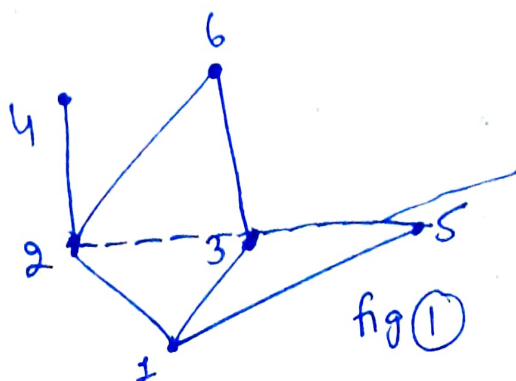
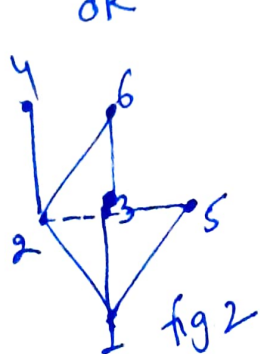
Q3 Draw the Hasse diagram of Set

$A = \{2, 3, 4, 9, 12, 18, 36, 1\}$ Under divisibility



Q4 $A = \{1, 2, 3, 4, 5, 6, \dots\}$

OR



same level

1 should be at the base always

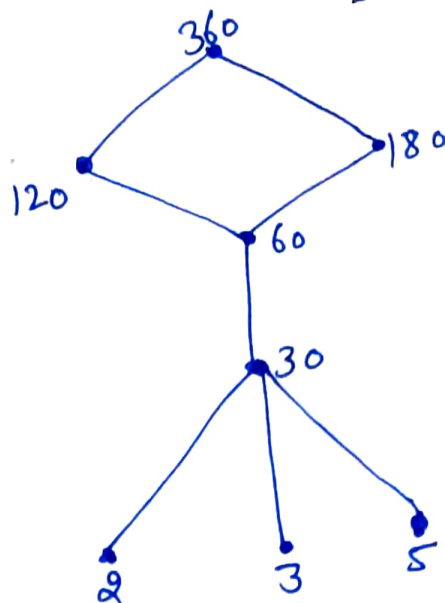
always condition must be satisfied

figure may be different (4)

Q.5

$$A = \{2, 3, 5, 30, 60, 120, 180, 360, '\}'$$

Hasse diagram



Q.6

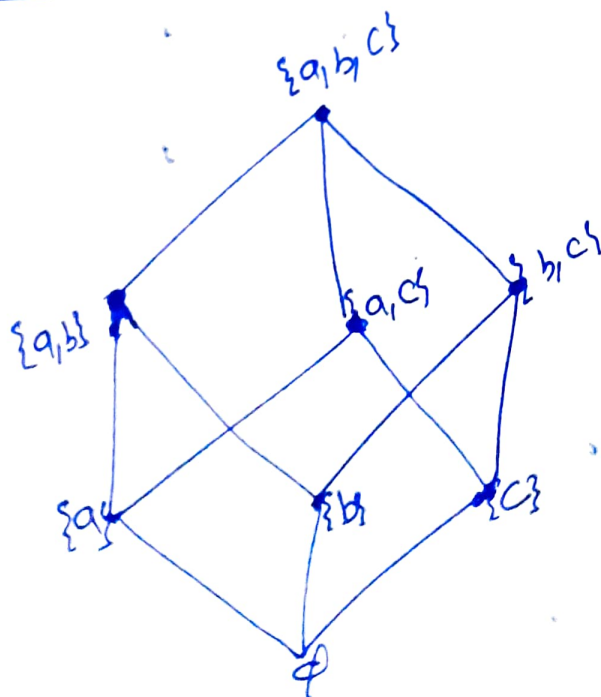
Draw Hasse diagram for poset $\{A, B\}; A \subseteq B\}$ on power set $P(S)$ on set S i.e. $(P(S), \subseteq)$ where $S = \{a, b, c\}$

Note here condition is subset

First power set will be

$$P(S) = \{\{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}, \emptyset\}$$

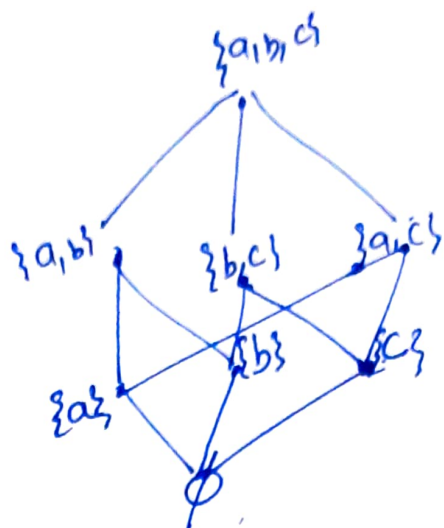
Hasse diagram



Note \emptyset must be at base
a, b, c position may be change

but remember $a \subseteq b$
& $a \subseteq c$ so we join a to (a, b) & (a, c)
Similarly others
 (a, b) (b, c) so
due to transitivity
Can't join a to c
Similarly others in every pair

Other figure of Hasse diagram

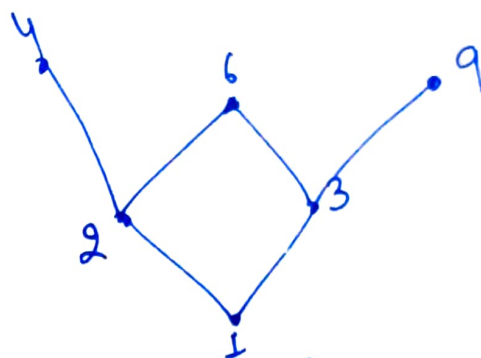


Both are Correct
So don't be Confused
here Condition is
Subset \subseteq

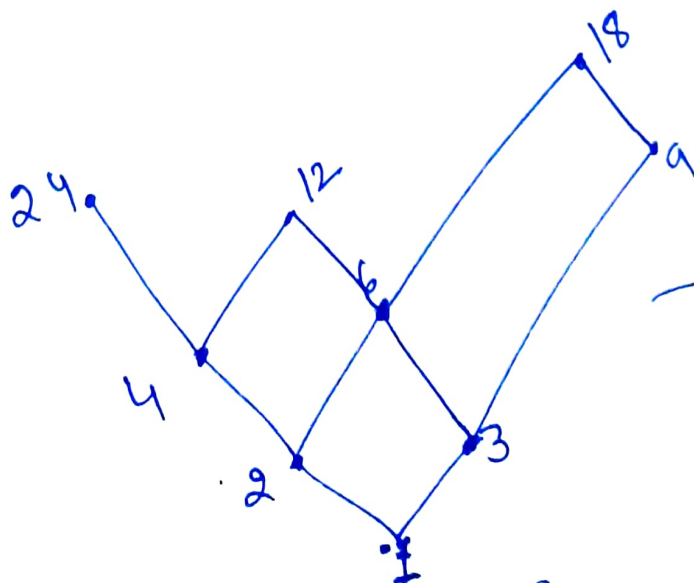
Q.7 $A = \{1, 2, 3, 4, 6, 9, '\}'$

Hasse diagram

~~In last step arrow~~



Q.8 $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$
be ordered by divisibility



Transitivity must
not be there for
any pair is Very
Important

e.g $1R2, 2R6$ so $1R6$ will
never be there