4. **Half-wave Dipole** It is a linear antenna whose length is $\frac{\lambda}{2}$ and the current distribution is assumed to be sinusoidal. It is usually centre-fed.

5. **Quarter-wave Monopole** It is a linear antenna whose length is $\frac{\lambda}{4}$ and the current distribution is assumed to be sinusoidal. It is fed at one end with respect to earth.

3.8 RADIATION MECHANISM

When a transmitting antenna is excited with an alternating voltage, the initial motion is started by the balanced motion of charges in the antenna. Resonant oscillations are produced by the supplied energy. Electric and magnetic fields are generated due to sudden changes in charge. When the charges around the antenna are set in motion first, the other charges are separated from the antenna and they are also set in motion. The disturbance is spread from the antenna into space. The electric and magnetic fields so produced are perpendicular to each other. EM waves have no boundaries. The EM energy decreases as it propagates.

3.9 RADIATION FIELDS OF ALTERNATING CURRENT ELEMENT (OR OSCILLATING ELECTRIC DIPOLE)

The concept of an alternating current element, $I\,dl \cos \omega t$ is of theoretical interest. But the theory developed for this can be extended to practical antennas. The concept of retarded vector magnetic potential **A** is very useful to derive radiation fields of antenna elements including current element.

Derivation of radiation fields consists of the following steps:

1. Write expression for retarded vector magnetic potential.
2. Write expressions for the components of **A** in Cartesian coordinates.
3. Express **A** in components of spherical coordinate system.
4. Obtain the components of **H** from $\mu\mathbf{H} = \nabla \times \mathbf{A}$.
5. Obtain the components of **E** from

$$\dot{\mathbf{E}} = \frac{\partial \mathbf{E}}{dt} = \frac{1}{\epsilon}\nabla \times \mathbf{H} \qquad \text{(as } \mathbf{J} = 0 \text{ for free space)}$$

6. $\mathbf{E} = \frac{1}{\epsilon}\int (\nabla \times \mathbf{H})\, dt.$
7. Identify radiation and near-field terms.

An alternating current element at the origin of a spherical coordinate system is shown in Fig. 3.6.

The retarded vector magnetic potential, $\mathbf{A}(r, t)$ is given by

$$\mathbf{A}(r, t) = \frac{\mu}{4\pi}\int_{\upsilon} \frac{\mathbf{J}(r, t - r/\upsilon_0)}{r} d\upsilon \qquad ...(3.1)$$

As the element is z-directed, **A** is also z-directed. $\frac{r}{\upsilon_0}$ is the delay time.

The **resultant field components** of an alternating current element are

$$H_\phi = \frac{I\,dl \sin\theta}{4\pi}\left[-\frac{\omega \sin \omega t_d}{r \upsilon_0} + \frac{\cos \omega t_d}{r^2}\right]$$

$$E_\theta = \frac{I\,dl \sin\theta}{4\pi \in_0}\left[-\frac{\omega \sin \omega t_d}{r \upsilon_0^2} + \frac{\cos \omega t_d}{r^2 \upsilon_0} + \frac{\sin \omega t_d}{\omega r^3}\right] \qquad ...(3.13)$$

$$E_r = \frac{2I\,dl}{4\pi \in_0}\cos\theta\left[\frac{\cos \omega t_d}{r^2 \upsilon_0} + \frac{\sin \omega t_d}{\omega r^3}\right]$$

$$H_r = 0, \quad H_\theta = 0, \quad E_\phi = 0.$$

3.10 RADIATED POWER AND RADIATION RESISTANCE OF CURRENT ELEMENT

The derivation of expression for radiated power consists of the following steps:

- Obtain field components as in the Section (3.8).
- Obtain expression for radiated power using Poynting vector.
- Obtain average radiated power.
- Obtain total power radiated from $P_T = \oint_S P_{av}\, ds$.
- Identify expression for radiation resistance.

Poynting vector is

$$\mathbf{P} = \mathbf{E} \times \mathbf{H} \text{ watts/m}^2$$

That is, $P_\theta = -E_r H_\phi$ ...(3.14)

From Equations (3.13) and (3.14), we have

$$P_\theta = -\left[\frac{2I\,dl \cos\theta}{4\pi\in_0}\left(\frac{\cos \omega t_d}{r^2 \upsilon_0} + \frac{\sin \omega t_d}{\omega r^3}\right)\right] \times \left[\frac{I\,dl \sin\theta}{4\pi}\left(-\frac{\omega \sin \omega t_d}{r \upsilon_0} + \frac{\cos \omega t_d}{r^2}\right)\right] \qquad ...(3.15)$$

$$= \frac{2I^2\,dl^2 \cos\theta\,\sin\theta}{16\pi^2 \in_0}\left[\frac{\sin^2 \omega t_d}{r^4 \upsilon_0} - \frac{\cos^2 \omega t_d}{r^4 \upsilon_0} - \frac{\sin \omega t_d \cos \omega t_d}{\omega r^5} + \frac{\omega \sin \omega t_d \cos \omega t_d}{r^3 \upsilon_0}\right] \qquad ...(3.16)$$

$$\left.\begin{aligned} 2\sin\theta\cos\theta &= \sin 2\theta \\ \sin\omega t_d \cos\omega t_d &= \frac{\sin 2\omega t_d}{2} \\ \sin^2\omega t_d - \cos^2\omega t_d &= \cos 2\omega t_d \end{aligned}\right\} \qquad \text{...(3.17)}$$

But

Using these identities, Equation (3.16) becomes

$$P_\theta = \frac{I^2\, dl^2 \sin 2\theta}{16\pi^2 \in_0}\left[-\frac{\cos 2\omega t_d}{r^4\, \upsilon_0} - \frac{\sin 2\omega t_d}{2\omega r^5} + \frac{\omega \sin 2\omega t_d}{2r^3\, \upsilon_0^2}\right] \qquad \text{...(3.18)}$$

P_θ in Equation (3.18) represents instantaneous power flow in θ-direction. But the average value of $\cos 2\omega t_d$ or $\sin 2\omega t_d$ over a cycle is zero. Hence, $(P_\theta)_{av} = 0$ at any value of r. This means that power in θ-direction surges back and forth.

Similarly, $P_r = E_\theta\, H_\phi$ ...(3.19)

From Equations (3.13) and (3.18), we get

$$P_r = \frac{I^2\, dl^2 \sin^2\theta}{16\pi^2 \in_0}\left[\frac{\sin\omega t_d \cos\omega t_d}{\omega r^5} + \frac{\cos^2\omega t_d}{r^4\, \upsilon_0} - \frac{\omega \sin\omega t_d \cos\omega t_d}{r^3\, \upsilon_0^2} - \frac{\sin^2\omega t_d}{r^4\, \upsilon_0} - \frac{\omega \sin\omega t_d \cos\omega t_d}{r^3\, \upsilon_0^2} + \frac{\omega^2 \sin^2\omega t_d}{r^2\, \upsilon_0^3}\right] \qquad \text{...(3.20)}$$

Using identities of Equation (3.17), we get

$$P_r = \frac{I^2\, dl^2 \sin^2\theta}{16\pi^2 \in_0}\left[\frac{\sin 2\omega t_d}{2\omega r^5} + \frac{\cos 2\omega t_d}{r^4\, \upsilon_0} - \frac{\omega \sin 2\omega t_d}{r^3\, \upsilon_0^2} + \frac{\omega^2 (1 - \cos 2\omega t_d)}{2r^2\, \upsilon_0^3}\right] \qquad \text{...(3.21)}$$

It is obvious from Equation (3.21), that the average value of P_r is

$$P_{r\,(av)} = \frac{\omega^2 I^2\, dl^2 \sin^2\theta}{32\pi^2 r^2 \in_0 \upsilon_0^3}$$

or

$$P_{r\,(av)} = \frac{\eta_0}{2}\left(\frac{\omega I\, dl \sin\theta}{4\pi r\, \upsilon_0}\right)^2 \text{ watt/m}^2 \qquad \text{...(3.22)}$$

where

$$\eta_0 = \sqrt{\frac{\mu_0}{\in_0}} = 120\pi\Omega$$

The total power radiated ...(3.23)

$$P_T = \oint_{\text{surface}} P_{r\,(av)}\, ds$$

$P_{r\,(av)}$ is independent of ϕ and hence the element of area ds on the spherical shell is given by

$$ds = 2\pi r^2 \sin\theta \; d\theta \qquad ...(3.24)$$

Here, θ varies between 0 and π.

Now Equation (3.24) becomes

$$P_T = \int_0^{\pi} \frac{\eta_0}{2}\left[\frac{\omega I\, dl \sin\theta}{4\pi r v_0}\right]^2 2\pi r^2 \sin\theta \; d\theta$$

$$= \frac{\eta_0\, \omega^2 I^2 dl^2}{16\pi v_0^2}\int_0^{\pi} \sin^3\theta \, d\theta$$

But $\displaystyle\int_0^{\pi} \sin^3\theta \, d\theta = \left(-\cos\theta + \frac{1}{3}\cos^3\theta\right)_0^{\pi} = \frac{4}{3}$

So $$P_T = \frac{\eta_0\, \omega^2 I^2 dl^2}{12\pi v_0^2} \qquad ...(3.25)$$

Here I is the peak value of current.

As $$I = \sqrt{2} I_{\text{eff}}$$

$$I^2 = 2 I_{\text{eff}}^2$$

Thus Equation (3.25) becomes

$$P_T = \frac{\eta_0\, \omega^2 dl^2 I_{\text{eff}}^2}{6\pi v_0^2}$$

or $$\boxed{P_T = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2 I_{\text{eff}}^2 \text{ watts}} \qquad ...(3.26)$$

This is in the form of $P = I^2 R$. Hence the coefficient of I_{eff}^2 has the dimensions of resistance and it is called Radiation Resistance.

Radiation Resistance of Hertzian dipole

$$R_r = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2 \Omega. \qquad ...(3.27)$$

3.11 RADIATION, INDUCTION AND ELECTROSTATIC FIELDS

The field components of current elements from Equation (3.13) are

$$H_\phi = \frac{I\, dl \sin\theta}{4\pi}\left[-\frac{\omega \sin\omega t_d}{r v_0} + \frac{\cos\omega t_d}{r^2}\right]$$

$$E_\theta = \frac{I\,dl \sin\theta}{4\pi\epsilon_0}\left[-\frac{\omega \sin \omega t_d}{r v_0^2} + \frac{\cos \omega t_d}{r^2 v_0} + \frac{\sin \omega t_d}{\omega r^3}\right] \quad ...(3.28)$$

$$E_r = \frac{2I\,dl}{4\pi\epsilon_0}\left[\frac{\cos \omega t_d}{r^2 v_0} + \frac{\sin \omega t_d}{\omega r^3}\right]$$

H_ϕ field consists of $\frac{1}{r}$ and $\frac{1}{r^2}$ terms. $\frac{1}{r^2}$ term dominates over $\frac{1}{r}$ term at points close to the current element. When r is small, $\frac{1}{r^2}$ term is called Induction Field.

On the other hand, $\frac{1}{r}$ term dominates over $\frac{1}{r^2}$ terms when r is large. Here $\frac{1}{r}$ term is called Radiation Field or distant field or far-field.

The expression for E_θ consists of three terms: $\frac{1}{r}, \frac{1}{r^2}, \frac{1}{r^3}$, and the expression for E_r consists of $\frac{1}{r^2}$ and $\frac{1}{r^3}$ terms. These $\frac{1}{r^3}$ term is called Electrostatic Field.

$\frac{1}{r}$ term in E and H fields is called Radiation Field

$\frac{1}{r^2}$ term is called Induction Field

$\frac{1}{r^3}$ term is called Electrostatic Field.

If the induction and radiation fields have equal amplitudes, then from Equation (3.28), we have

$$\frac{I\,dl\,\omega \sin\theta}{4\pi r v_0} = \frac{I\,dl \sin\theta}{4\pi r^2}$$

or

$$\frac{\omega}{r v_0} = \frac{1}{r^2} \text{ or } r = \frac{v_0}{\omega} = \frac{\lambda}{2\pi} \approx \frac{\lambda}{6.0} \quad ...(3.29)$$

At a distance of $r = \frac{\lambda}{2\pi}$, induction and radiation fields have equal amplitudes.

3.12 HERTZIAN DIPOLE

Hertzian dipole is an infinitesimal current element Idl which does not exist in real life.

(or)

Hertzian dipole is a short linear antenna which, when radiating, is assumed to carry constant current along its length.

As Hertzian dipole and alternating current elements virtually mean the same, the radiated power and radiation resistance are given by

$$P_T = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2 I_{eff}^2 \text{ watts}$$

$$R_r = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2 \Omega. \qquad \text{...(3.30)}$$

3.13 DIFFERENT CURRENT DISTRIBUTIONS IN LINEAR ANTENNAS

The possible current distributions are:

1. Constant current along its length—valid in Hertzian dipole.
2. Triangular current distribution—valid in short dipole and monopole.

For a triangular current distributions

$$R_r \left(\text{short dipole, } l < \frac{\lambda}{4}\right) \approx 20\pi^2 \left(\frac{l}{\lambda}\right)^2 \Omega \qquad \text{...(3.31)}$$

$$R_r \left(\text{short monopole, } l < \frac{\lambda}{8}\right) \approx 10\pi^2 \left(\frac{l}{\lambda}\right)^2 \Omega$$

3. Sinusoidal current distribution—valid in half-wave dipole.
4. Exact current distribution—This can be determined using the method of moment technique. However, this method is beyond the scope of this book.

3.14 RADIATION FROM HALF-WAVE DIPOLE

Radiated power by half-wave dipole, $P_T = 73.0 I_{eff}^2$

Radiation resistance of half-wave dipole, $R_r = 73\Omega$.

Proof Proof consists of the following steps:

- Write expressions for the assumed current distribution in the element.
- Obtain expression for vector magnetic potential, **A**.
- Obtain **H** from **A**.
- Obtain **E** from $\left(\frac{E}{H}\right) = \eta_0$.
- Obtain average radiated power P_{av}.
- Obtain total power radiated.
- Obtain the value of radiation resistance.

The sinusoidal current distribution is represented by Fig. 3.7.

$$I = I_m \sin\beta(H - Z) \text{ for } z > 0$$

$$= I_m \sin\beta(H + Z) \text{ for } z < 0$$

Here $\quad I_m$ = current maximum.

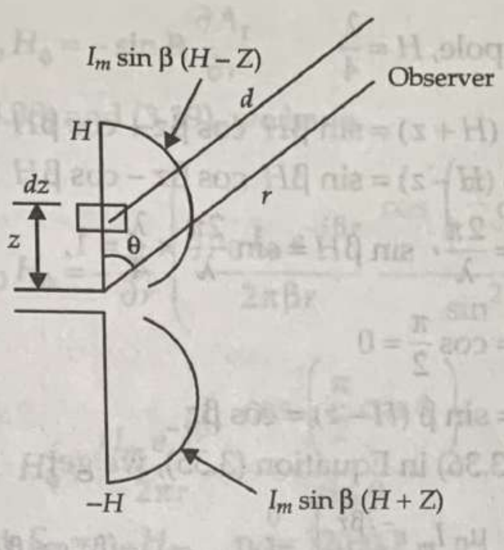

Fig. 3.7 *Dipole*

The vector potential at a point P due to the current element $I\,dz$ is given by,

$$d\mathbf{A} = dA_z\,\mathbf{a}_z = \frac{\mu_0 I e^{-j\beta d}\,dz}{4\pi d}\,\mathbf{a}_z \quad ...(3.32)$$

Here d is the distance from the current element to the point P. The total vector potential at P due to all current elements is given by

$$A_z = \frac{\mu_0}{4\pi}\int_{-H}^{H}\frac{I e^{-j\beta d}}{d}\,dz \quad ...(3.33)$$

$$= \frac{\mu_0}{4\pi}\int_{-H}^{0}\frac{I_m \sin\beta(H+z)}{d}\,e^{-j\beta d}\,dz + \frac{\mu}{4\pi}\int_{0}^{H}\frac{I_m \sin\beta(H-z)}{d}\,e^{-j\beta d}\,dz \quad ...(3.34)$$

It is of interest here to consider radiation fields. d in the denominator can be approximated to r. But in the numerator, d is in the phase term and it is given by

$$d = r - z\cos\theta$$

Now Equation (3.34) becomes

$$A_z = \frac{\mu_0}{4\pi}\int_{-H}^{0}\frac{I_m \sin\beta(H+z)}{r}\,e^{-j\beta(r-z\cos\theta)}\,dz + \frac{\mu_0}{4\pi}\int_{0}^{H}\frac{I_m \sin\beta(H-z)}{r}\,e^{-j\beta(r-z\cos\theta)}\,dz$$

$$= \frac{\mu_0 I_m e^{-j\beta r}}{4\pi r}\left[\int_{-H}^{0}\sin\beta(H+z)\,e^{j\beta z\cos\theta}\,dz + \int_{0}^{H}\sin\beta(H-z)\,e^{j\beta z\cos\theta}\,dz\right] \quad ...(3.35)$$

For a half-wave dipole, $H = \frac{\lambda}{4}$

But, $\sin\beta(H+z) = \sin\beta H \cos\beta z + \cos\beta H \sin\beta z$

$\sin\beta(H-z) = \sin\beta H \cos\beta z - \cos\beta H \sin\beta z$

As $\beta = \frac{2\pi}{\lambda}$, $\sin\beta H = \sin\frac{2\pi}{\lambda}\times\frac{\lambda}{4} = 1$,

$\cos\beta H = \cos\frac{\pi}{2} = 0$

So $\sin\beta(H+z) = \sin\beta(H-z) = \cos\beta z$ ...(3.36)

Putting Equation (3.36) in Equation (3.35), we get

$$A_z = \frac{\mu_0 I_m e^{-j\beta r}}{4\pi r}\left[\int_{-H}^{0}\cos\beta z e^{+j\beta z\cos\theta} + \int_{0}^{H}\cos\beta z e^{+j\beta z\cos\theta}\, dz\right] \quad ...(3.37)$$

$$\text{But} \int_{-H}^{0}\cos\beta z e^{+j\beta z\cos\theta}\, dz = \int_{0}^{H}\cos\beta z e^{-j\beta z\cos\theta}\, dz$$

$$A_z = \frac{I_m\mu_0}{4\pi r}e^{-j\beta r}\left[\int_{0}^{\lambda/4}\cos\beta z\,(e^{j\beta z\cos\theta} + e^{-j\beta z\cos\theta})\, dz\right]$$

$$= \frac{I_m\mu_0}{4\pi r}e^{-j\beta r}\left[\int_{0}^{\lambda/4}\cos\{\beta z(1+\cos\theta)\} + \cos\{\beta z(1-\cos\theta)\}\right] dz$$

$$= \frac{I_m\mu_0}{4\pi r}e^{-j\beta r}\left[\frac{\sin\{\beta z(1+\cos\theta)\}}{\beta(1+\cos\theta)} + \frac{\sin\{\beta z(1-\cos\theta)\}}{\beta(1-\cos\theta)}\right]_0^{\lambda/4}$$

$$= \frac{\mu_0 I_m}{4\pi\beta r}e^{-j\beta r}\left[\frac{(1-\cos\theta)\cos\left(\frac{\pi}{2}\cos\theta\right) + (1+\cos\theta)\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right]$$

$$A_z = \frac{\mu_0 I_m}{2\pi\beta r}e^{-j\beta r}\left[\frac{\cos\frac{\pi}{2}\cos\theta}{\sin^2\theta}\right] \quad ...(3.38)$$

But we have

$$\mu_0 H_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial\theta}A_r\right]$$

$$= \frac{1}{r}\left[\frac{\partial}{\partial r}r(-A_z\sin\theta) - \frac{\partial}{\partial\theta}(A_z\cos\theta)\right]$$

$$\mu_0 H_\phi = -\sin\theta \frac{\partial A_z}{\partial r} \quad \text{...(3.39)}$$

From Equations (3.38) and (3.39), we have

$$\mu_0 H_\phi = -\frac{\partial}{\partial r}\left(\frac{j\mu_0 I_m e^{-j\beta r}}{2\pi\beta r}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right)\sin\theta$$

$$H_\phi = \frac{jI_m e^{-j\beta r}}{2\pi r}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \quad \text{...(3.40)}$$

We also know that $E_\theta = \eta_0 H_\phi$, $\eta_0 = 120\pi\Omega$

$$E_\theta = \frac{j120\pi I_m e^{j\beta r}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

$$= \frac{j60I_m e^{-j\beta r}}{r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right] \quad \text{...(3.41)}$$

The magnitude of E for the radiation field is

$$E_\theta = \frac{60I_m}{r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right] \text{V/m} \quad \text{...(3.42)}$$

E_θ and H_ϕ are in time phase. Hence the maximum value of Poynting vector is

$$P_{\max} = (E_\theta)_{\max}\,(H_\phi)_{\max}$$

$$= \frac{60I_m}{r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right] \times \frac{I_m}{2\pi r}\left(\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right)$$

$$= \frac{30I_m^2}{\pi r^2}\left[\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right] \quad \text{...(3.43)}$$

The average value of Poynting vector is one half of the peak value.

So $$P_{av} = \frac{15I_m^2}{\pi r^2}\left[\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right]$$

or $$P_{av} = \frac{\eta_0 I_m^2}{8\pi^2 r^2}\left[\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right] \quad ...(3.44)$$

Therefore, total power radiated through a spherical surface by half wave dipole is

$$P_T = \oint P_{av}\, ds = \frac{\eta_0 I_m^2}{8\pi r^2}\int_0^{\pi} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\, 2\pi r^2 \sin\theta\, d\theta$$

$$= \frac{\eta_0 I_m^2}{4\pi}\int_0^{\pi}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\, d\theta \quad ...(3.45)$$

But the numerical evaluation of the integral $\int_0^{\pi}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\, d\theta$ by Simpson's or the Trapezoidal rule gives a value of 1.218.

So $$P_T = \frac{\eta_0 I_m^2}{4\pi}$$

$$= \frac{120\pi I_m^2}{4\pi} \times 1.218 = 36.54 I_m^2 \quad ...(3.46)$$

As $I_m = \sqrt{2} I_{eff}$, Equation (3.46) becomes

$$P_T = 36.54 \times 2 \times I_{eff}^2$$

or $$P_T = 73.08\Omega\, I_{eff}^2, \text{ watts} \quad ...(3.47)$$

The coefficient of I_{eff}^2 is the radiation resistance. That is,

$$R_r = 73.08\Omega. \quad ...(3.48)$$

3.15 RADIATION FROM QUARTER-WAVE MONOPOLE

Radiated power of quarter-wave monopole, $P_T = 36.5 I_{eff}^2$ watts Radiation resistance, $R_r = 36.5\Omega$.

Proof Consider Fig. 3.8 in which monopole with current distribution is shown.

Obtain P_{av} exactly as described in half-wave dipole. That is, from Equation (3.44), we have

$$P_{av} = \frac{\eta_0 I_m^2}{8\pi^2 r^2}\left[\frac{\cos^2 \frac{\pi}{2}\cos\theta}{\sin^2\theta}\right] \quad ...(3.49)$$

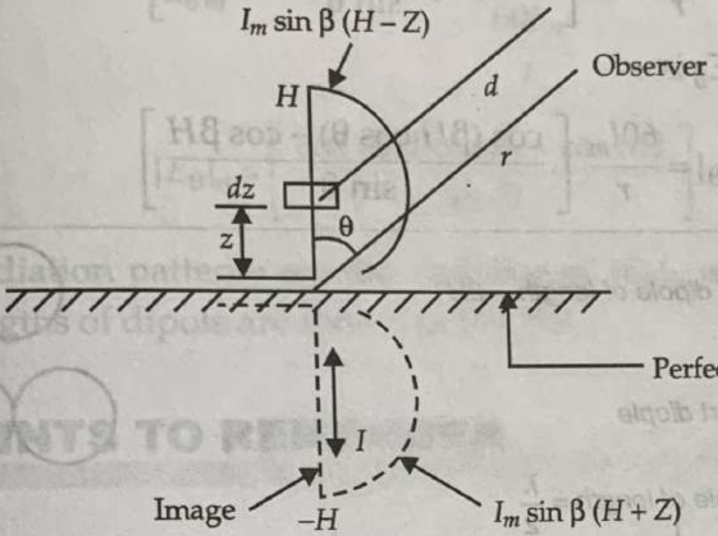

Fig. 3.8 *Monopole with current distribution*

As the monopole is fed with a perfectly conducting plane at one end, it radiates only through a hemi-spherical surface. Therefore, the total radiated power is

$$P_T = \oint P_{av}\, ds$$

$$= \frac{\eta_0 I_m^2}{8\pi r^2}\int_0^{\pi/2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} 2\pi r^2 \sin\theta\, d\theta$$

$$= \frac{\eta_0 I_m^2}{4\pi}\int_0^{\pi/2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} d\theta$$

Numerical evaluation of the integral $\int_0^{\pi/2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta$ by Simpson's or the Trapezoidal rule gives a value of 0.609.

So $$P_T = \frac{\eta_0 I_m^2}{4\pi} \times 0.609$$

$$= 18.27 I_m^2$$

As $$I_m = \sqrt{2} I_{eff} \quad ...(3.50)$$

$$P_T = 36.54 I_{eff}^2 \text{ watts} \quad ...(3.51)$$

The Radiation resistance, $R_r = 36.54\Omega$

3.16 RADIATION CHARACTERISTICS OF DIPOLES

Electric field as a function θ in free space for a dipole of length of $2H$ is given by

$$E_\theta = \frac{j60 I_m e^{-j\beta r}}{r}\left[\frac{\cos(\beta H \cos\theta) - \cos\beta H}{\sin\theta}\right] \qquad \text{...(3.52)}$$

The amplitude of E_θ is

$$|E_\theta| = \frac{60 I_m}{r}\left[\frac{\cos(\beta H \cos\theta) - \cos\beta H}{\sin\theta}\right]$$

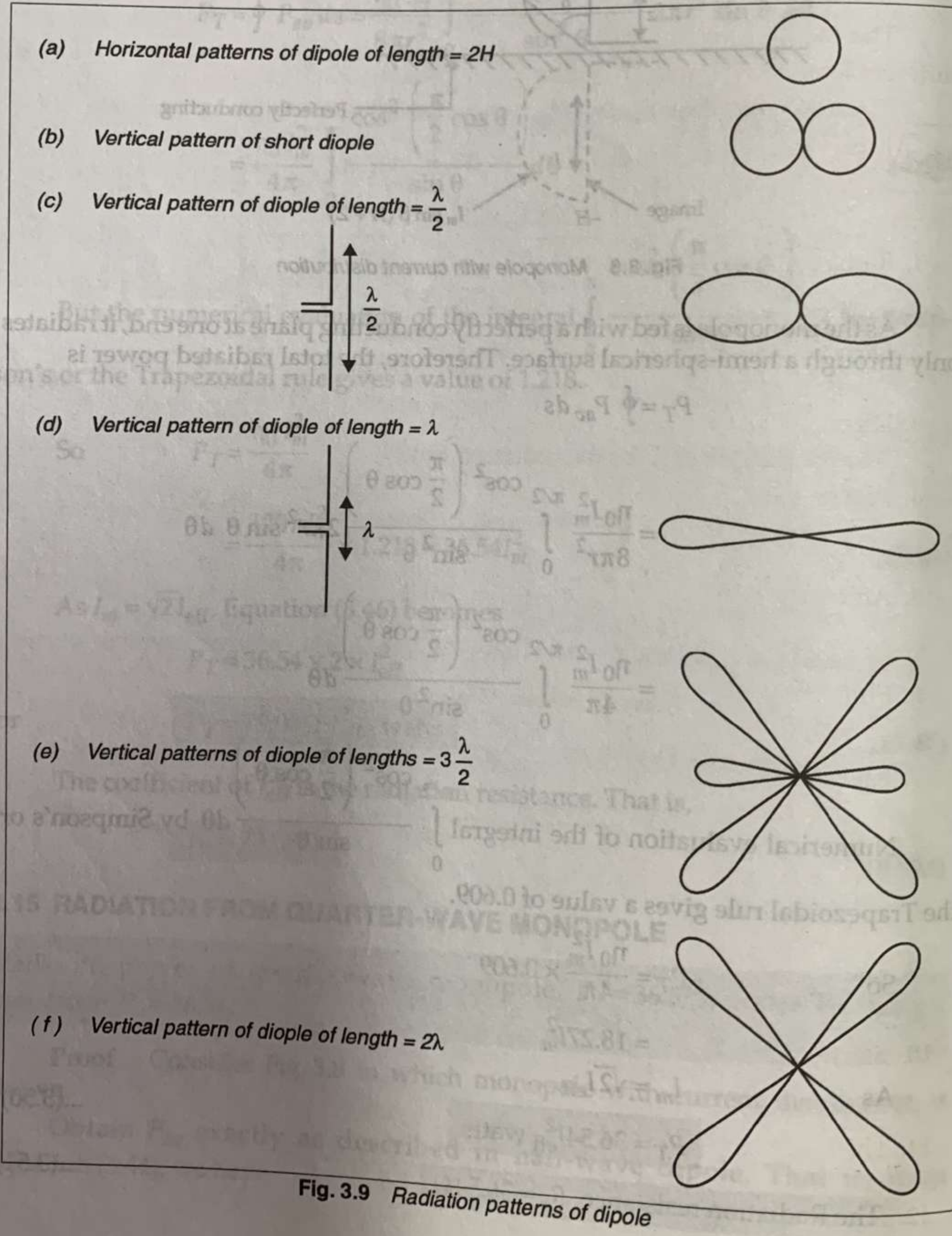

Fig. 3.9 *Radiation patterns of dipole*

The normalised (E_θ) is

$$|E_\theta|_n = \frac{\dfrac{60 I_m}{r}\left[\dfrac{\cos(\beta H \cos\theta) - \cos\beta H}{\sin\theta}\right]}{\dfrac{60 I_m}{r}}$$

So $$|E_\theta|_n = \left[\frac{\cos(\beta H \cos\theta) - \cos\beta H}{\sin\theta}\right] \quad ...(3.53)$$

The radiation patterns are the variation of $|E_\theta|_n$ with θ. These patterns for different lengths of dipole are shown in Fig. 3.9.

POINTS TO REMEMBER

1. Radiation intensity, $RI = \dfrac{r^2 E^2}{\eta_0}$ watts/unit solid angle.

2. Directive gain, $g_d = \dfrac{4\pi \times (RI)}{w_r}$.

3. Directivity, $D = (g_d)_{\max}$.

4. Power gain, $g_p = \dfrac{4\pi \times (RI)}{w_t}$.

5. Antenna efficiency, $\eta = \dfrac{g_p}{g_d}$.

6. Effective area, $A_e = \dfrac{\lambda^2}{4\pi} g_d$ or $A_e = \dfrac{\text{received power}}{\text{power flow of incident waves}}$.

7. Far-field is represented by $\dfrac{1}{r}$ field term.

8. Induction field is represented by $\dfrac{1}{r^2}$ field term.

9. Radiation resistance of Hertzian dipole is $80\pi^2 \left(\dfrac{dl}{\lambda}\right)^2 \Omega$.

10. Electrostatic field is represented by $\dfrac{1}{r^3}$.

11. The far-field and induction field have equal magnitudes at $r = \dfrac{\lambda}{2\pi}$.

12. Radiation resistance of half-wave dipole is 73Ω.