10.29. IMPEDANCE MATCHING DEVICES

A transmission line in its role as a connecting links between a transmitter and an antenna or between an antenna and a receiver, affect the efficiency of power transfer. In some cases they may be arranged to apportion a total amount of power among a number of loads. Methods of adjusting the line impedance relations to achieve optimum results in these functions are discussed under the head of matching devices. In communication networks, the element of the network should be designed such that maximum power transfer takes place between the source (or transmitter) to the load (or antenna). This is governed by a well known a.c. circuit theorem known as *'Maximum power transfer theorem'* according to which, the maximum power is absorbed by one network from another network, when the impedance of one is the complex conjugate of the other. This means for maximum transfer of power between source and load, the resistance of the load should be equal to that of the source and the reactance of the load should equal to that of source but of opposite sign i.e. when source is inductive, the load should be capacitive and vice-versa. When this condition is achieved, it is usually referred to as impedance matching and the methods employed to achieve this, are termed as *impedance matching devices*.

10.29.1 ***Principle of Impedance Matching.*** A radio transmitter is in effect a generator having an internal electro-motive force (e.m.f.) which may be complex as

$$\boxed{Z_g = R_g + j X_g} \quad \text{... 10.110 (a)}$$

According to maximum power transfer theorem, maximum power transfer will take place to load, when it is complex congugate i.e.

$$\boxed{Z_g = R_g - j X_g} \quad \text{... 10.110 (b)}$$

The ultimate load for the transmtitter is the antenna but the immediate load may be the input impedance of the transmission line *i.e.*,

$$\boxed{Z_S = R_S + j X_S} \quad \text{... 10.110 (c)}$$

which is connected between the transmitter and the antenna. Hence for maximum transfer of power from the transmitter, it is necessary that

$$\boxed{\begin{aligned} R_g &= R_S \\ X_g &= -X_S \end{aligned}} \quad \begin{aligned} &\text{... 10.111 (a)} \\ &\text{... 10.111 (b)} \end{aligned}$$

and

By Eqns. 10.32 (a) or 10.32 (b), the value of input impedance depends on Z_R, Z_0 and length l. Thus it is possible to choose or adjust the values of these quantities to achieve the desired values of Z_S. However, in practice the desired result is not achieved in this way. For example, because the length l of the line may be determined mainly by the necessary spacing between the transmitter and the antenna and hence will not be independent adjustable. The possible range of Z_0 is also some what practically limited. Lastly, it is usually desired to minimise the *SWR* on the line, it means Z_R must be at least approximately equal to Z_0.

Hence the impedance generally presented at the input terminals of the line will be

$$\boxed{Z_S = Z_0} \quad \text{... (10.112)}$$

Therefore, an impedance transformer is then introduced in between the line input and transmitter output to transform the chracteristic impedance Z_0 into the value needed for maximum transmitter power output or optimum performance where best operation from some stand point other than maximum power is required. Further, the actual load impedance presented by the antenna on the transmission line output terminals may not be equal to the desired value Z_0. So another transformer may be needed between the load end of the line and the antenna terminals and this one must transform the antenna impedance (Z_0 say) into the characteristic impedance Z_0.

Sometimes these transformers of impedances may consist of coupled coil of wire or of coil and condenser circuits specially at low frequencies, However in the higher frequencies region (*i.e.* in MHz region) these impedance transformers usually consists of sections of transmission lines in the various arrangements. Some of the possible arrangements are :

(i) Quarter wave transformer (Impedance inverter),

(ii) Stub matching

This will be discussed now.

(AMIETE, Dec. 1991, 1990)

10.29.2. *Quarter Wave Impedance Inverting Transformer*

Lengths of transmission lines have certain properties which are of special interest for particular values of the length (l) or the load impedance Z_R. From the Eqn. 10.32, it can be seen that a transmission line is, in general, an impedance transformer *i.e.*, if an Z_R is connected at the load end, a different impedance Z_S may be present to a source at the input end

From Eqn. 10.32,

If
$$l = \frac{\lambda}{4};\ \beta = \frac{2\pi}{\lambda}$$

Then
$$\beta l = \text{electric length} = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z_S = Z_0\left[\frac{Z_R\cos\pi/2 + jZ_0\sin\pi/2}{Z_0\cos\pi/2 + jZ_R\sin\pi/2}\right] = Z_0\left[\frac{0 + jZ_0\cdot 1}{0 + jZ_R\cdot 1}\right] \qquad \because \cos\pi/2 = 0,\ \sin\pi/2 = 1$$

$$Z_S = \frac{Z_0\cdot Z_0}{Z_R}$$

or
$$Z_S = \frac{Z_0^2}{Z_R} \qquad \text{... 10.113 (a)}$$

or
$$\boxed{Z_0 = \sqrt{Z_S Z_R}} \qquad \text{... (10.113 b)}$$

or
$$Z_S \propto \frac{1}{Z_R} \qquad \text{... 10.113 (c)}$$

Thus, as clear from Eqn. 10.113 (a), the product of the input impedance (Z_S) and load impedance (Z_R) equals the square of the characteristics impedance (Z_0) of the line. From this Eqn. one infer *that the quarter wavelength line transforms a load impedance Z_R that is smaller than Z_0 into a value Z_S that is larger than Z_0 and vice-versa. Sometimes a quarter wave line is called as impedance inverter for this reason. If Z_R is pure resistive, the input impedance Z_S will also be resistive (i.e. real). This quarter wave transformer is useful when it is desired to transform a resistive impedance into a different resistive impedance value either larger or smaller. The desired transformation is achieved by choosing the appropriate value of Z_0 in accordance* with Eqn. 10.113. The desired value of Z_0 can be obtained by choosing proper value of the ratio of spacing of the line conductors (S) to their diameters. Because the physical dimensions can not practically have an infinite range of values, the transformation ratio of the λ/4 transformer is subject to the practical limitation. However, it is a very useful device because of its simplicity and the ease with which its behaviour is calculated. Length l equal to odd numbers of λ/4 will have the same transformation properties but as the length is made larger the sensitivity to a small change of frequency becomes greater.

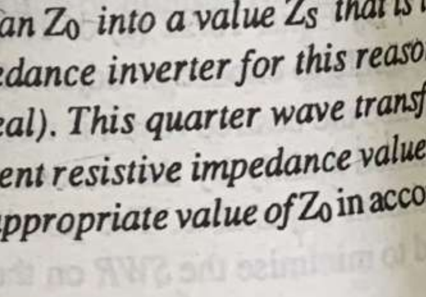

Fig. 10.20. Matching of a λ/2 dipole with transformer.

From Eqn. 10.113 (c) input impedance is inversely proportional to the load impedance. Thus if Z_R is high, Z_S will be low and *vice-versa*. Also assuming Z_0 as resistive, if Z_R is capacitive, then Z_S will be inductive and vice-versa. Thus depending upon the Z_R, a $\lambda/4$ transformer line acts as step-up or step down impedance transformer and that is why it is impedance inverter. An excellent example, is matching of a dipole antenna with resistive input impedance R_S is matched with a main transmission line of characteristic impedance Z_0^1 by means of a $\lambda/4$ transformer having characteristic impedance $Z_0 = \sqrt{Z'_0 \cdot R_S}$.

A $\lambda/4$ transformer, however suffer from the disadvantage that it is sensitive to change in frequency, because at a new frequency (*i.e.*, wavelength) the section will no longer be a $\lambda/4$ in length.

10.30. STUB MATCHING (AMIETE, Nov. 1969, May, 1972, 73, 75, 76)

A section of transmission line can be used as matched section by inserting them between load and the source, as seen above. Besides it is also possible to connect sections of open or short circuited line known as *stub* or *tuning stub* in shunt with the main line at a certain point (or points) to effect the matching. The matching with the help of tuning stub or stub is called stub matching and is having the following advantage.

(i) Length (l) and characteristic impedance (Z_0) remains unchanged.

(ii) Mechanically, it is possible to add adjustable susceptance in shunt with the line.

A stub matching is of two types, the single stub matching and double stub matching.

10.30.1. *Single Stub Matching* (AMIETE, Dec. 1981)

A type of transmission line frequently employed for single stub matching is shown in Fig. 10.21. The principal element of this transformer is a short circuited section of line whose open end connected to the main line at a particular distance from the load end, where the input conductance at that point is equal to the characteristic conductance of the line, and the stub length is adjusted to provide a susceptance which is equal in value but opposite in sign, to the input susceptance of the main line at that point, so that the total susceptance at the point of attachment is zero. The combination of stub and line will thus present a conductance which is equal to the characteristic conductance of the line *i.e.*, main length of H.F. transmission line will be matched.

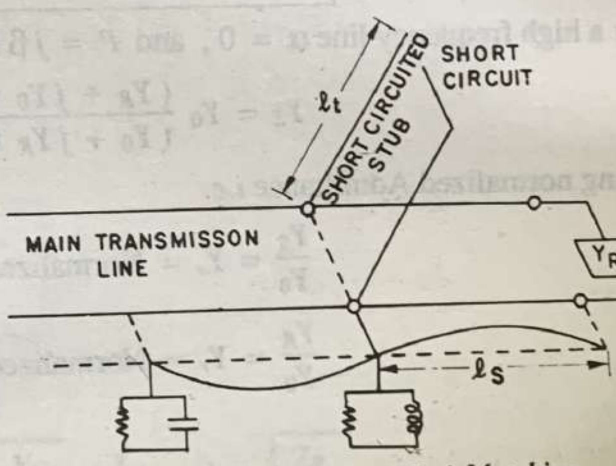

Fig. 10.21. Single Stub Matching.

Let the characteristic admittance of the transmission line be Y_0, which is terminated in a pure conductance Y_R as shown. Since the stub is connected in parallel with the main line so it is better to do the calculation with the admittance rather impedance so that it can directly be added up. When $Y_R \neq Y_0$, as we know standing waves are set up along the line. As we transverse the line from the load towards the generator, the input admittance (looking towards the load) varying from a maximum conductance through a parallel combination of conductance and inductance, a minimum conductance—a parallel comination of conductance and capacitance — and again maximum conductance to minimum conductance and the cycle reparts every $\lambda/2$.

If the line is transversed from the point of maximum (or minimum) conductance to that of minimum (or maximum) conductance, then obviously there will be a point at which the real part of the admittance is equal to the characteristic admittance. If now a suitable susceptance is added (by adding a proper length of short circuited or open circuited line known as stub) in shunt at this point so as to get antiresonance with the susceptance already existing, then upto that point matching has been achieved. Although there exists a mismatch between this point and the load, yet this mismatch over a short length is of insignificance.

Hence it is necessary that the stub should be located as near to the load as possible. Further, the characteristic admittance of the stub so connected in shunt should be same as that of the main line. The mathematical relation for the location of stub (l_s) and the length of short circuited stub (lt) can be started from the input impedance equation of a transmission line e.g.

$$Z_S = Z_0 \frac{Z_R + Z_0 \tanh Pl}{Z_0 + Z_R \tanh Pl}$$

Now converting impedances into admittances by putting

$$Y_0 = \frac{1}{Z_0} \text{ i.e. Characteristic Admittance.}$$

$$Y_R = \frac{1}{Z_R} \text{ i.e. Load Admittance.}$$

and $$Y_S = \frac{1}{Z_S} \text{ i.e. Input Admittance.}$$

we get, $$Y_S = \frac{1}{Z_S} = \frac{(Z_0 + Z_R \tanh Pl)}{(Z_0)(Z_R + Z_0 \tanh Pl)} = Y_0 \frac{\left(\frac{1}{Y_0} + \frac{1}{Y_R} \tanh Pl\right)}{\left(\frac{1}{Y_R} + \frac{1}{Y_0} \tanh Pl\right)}$$

$$\boxed{Y_R = Y_0 \frac{(Y_R + Y_0 \tanh Pl)}{(Y_0 + Y_R \tanh Pl)}} \quad \text{... (10.114)}$$

For a high frequency line $\alpha = 0$, and $P = j\beta$ so that

$$Y_S = Y_0 \frac{(Y_R + jY_0 \tanh \beta l)}{(Y_0 + jY_R \tanh \beta l)}$$

Using normalized Admittance *i.e.*

$$\frac{Y_S}{Y_0} = Y_s = \text{Normalized input admittance.}$$

$$\frac{Y_R}{Y_0} = Y_r = \text{Normalized load admittance.}$$

we obtain,

$$\frac{Y_S}{Y_0} = \frac{\left(\frac{Y_R}{Y_0} + j \tan \beta l\right)}{\left(1 + j \frac{Y_R}{Y_0} \tan \beta l\right)}$$

$$\boxed{Y_s = \frac{(Y_r + j \tan \beta l)}{(1 + jY_r \tan \beta l)}} \quad \text{... (10.1}$$

on rationalization Eqn. 10.115, we get

$$Y_s = \frac{(Y_r + j \tan \beta l)}{(1 + jY_r \tan \beta l)} \times \frac{(1 - jY_r \tan \beta l)}{(1 - jY_r \tan \beta l)}$$

$$Y_s = \frac{Y_r - jY_r^2 \tan \beta l + j \tan \beta l - j^2 Y_r \tan^2 \beta l}{1 + (Y_r \tan \beta l)^2}$$

$$Y_s = \frac{Y_r (1 - j^2 \tan^2 \beta l) + j \tan \beta l (1 - Y_r^2)}{1 + (Y_r \tan \beta l)^2}$$

or
$$\boxed{Y_S = G_S + j\,B_S = \frac{Y_r\,(1 + \tan^2 \beta l) + j \tan \beta l\,(1 - Y_r^2)}{[1 + (Y_r \tan \beta l)^2]}} \qquad \text{... (10.116)}$$

Equating real and imaginary parts we get

$$G_S = \frac{Y_r\,(1 + \tan^2 \beta l)}{1 + (Y_r \tan \beta l)^2} \qquad \text{... 10.117 (a)}$$

and
$$B_S = \frac{\tan \beta l\,(1 - Y_r^2)}{1 + (Y_r \tan \beta l)^2} \qquad \text{... 10.117 (b)}$$

But for no reflection

$$Y_S = G_S + j\,B_S = 1 + j\,0$$

i.e.
$$Y_S = G_S = 1$$

and
$$B_S = 0 \qquad \text{... 10.118 (a)}$$

Thus stub has to be connected where there is no reflection *i.e.* where real part of Eqn. 10.116 [*e.g.* Eqn. 10.117 (a)] is unity. Thus

$$G_S = 1 = \frac{Y_r\,(1 + \tan^2 \beta l)}{1 + (Y_r \tan \beta l_s)^2}$$

or
$$1 + Y_r^2 \tan^2 \beta l_s = Y_r\,(1 + \tan^2 \beta l_s)$$

or
$$Y_r \tan^2 \beta l_s\,(Y_r - 1) = Y_r - 1$$

or
$$\tan^2 \beta l_s = \frac{1}{Y_r} = \frac{1}{\dfrac{Y_R}{Y_0}}$$

or
$$\boxed{\tan \beta l_s = \frac{1}{\sqrt{\dfrac{Y_0}{Y_R}}}} \qquad \text{... 10.119 (a)}$$

or
$$\beta l_s = \tan^{-1} \sqrt{\frac{Y_0}{Y_R}}$$

or
$$l_s = \frac{1}{\beta} \tan^{-1} \sqrt{\frac{Y_0}{Y_R}} = \frac{1}{\beta} \tan^{-1} \sqrt{\frac{Z_R}{Z_0}}$$

or
$$\boxed{l_s = \frac{1}{\beta} \tan^{-1} \sqrt{\frac{Z_R}{Z_0}}} \qquad \because \beta = \frac{2\pi}{\lambda} \qquad \text{... 10.119 (b)}$$

This eqn. gives the location of the stub from the load end as shown in Fig. 10.19.

Now the susceptance at the point of attachment of stub is given by Eqn. 10.117 (b)

$$B_r = \frac{B_S}{Y_0} = \frac{\tan \beta l_s\,(1 - Y_r^2)}{1 + (Y_r \tan \beta l_s)^2}$$

$$\frac{B_S}{Y_0} = \frac{\sqrt{\dfrac{Y_R}{Y_0}}\left(1 - \dfrac{Y_R^2}{Y_0^2}\right)}{1 + \dfrac{Y_R^2}{Y_0^2} \times \dfrac{Y_0}{Y_R}}$$

$$\frac{B_S}{Y_0} = \frac{\sqrt{\left(\frac{Y_0}{Y_R}\right)}\,(Y_0^2 - Y_R^2)}{\left(1 + \frac{Y_R}{Y_0}\right) Y_0^2} = \frac{\sqrt{\left(\frac{Y_0}{Y_R}\right)}\,(Y_0 - Y_R)\,(Y_0 + Y_R)}{\left(\frac{Y_0 + Y_R}{Y_0}\right) Y_0^2}$$

$$\frac{B_S}{Y_0} = \sqrt{\left(\frac{Y_0}{Y_R}\right)}\,\frac{(Y_0 - Y_R)}{Y_0}$$

or

$$\boxed{B_S = \sqrt{\left(\frac{Y_0}{Y_R}\right)}\,(Y_0 - Y_R)} \qquad \text{... (10.120)}$$

Hence the susceptance which should be added at the point of attachment of stub is this B_S. This can be obtained either by an open circuited stub or short circuited stub. The desired length of the stub (l_t) which will provide this susceptance B_S, is readily computed with the help of fundamental Eqn.

$$V_R = V_S \cos \beta l - j Z_0 I_S \sin \beta l$$

Since the short circuited stub has the advantages of

(i) Less power radiation and

(ii) Effective length variation is possible by a shorting bar, therefore, a short circuited stub is invariably used. For a loss-less short circuited stub $V_R = 0$. So the short circuited impedance is

$$\boxed{Z_t = \frac{V_S}{I_S} = j Z_0 \tan \beta l_t}$$

or short circuited admittance is

$$Y_t = \frac{1}{Z_t} = \frac{1}{j Z_0 \tan \beta l_t} \times \frac{j}{j}$$

or

$$\boxed{Y_t = G_t + j B_t = -j Y_0 \cot \beta l_t} \qquad \text{... (10.121)}$$

Susceptance at the point of attachment of short circuited stub is obtained by equating real and imaginary parts from eqn. 10.121.

Thus $\quad G_t = 0$

and $\quad B_t = -Y_0 \cot \beta l_t \qquad \text{... (10.122)}$

where $\quad l_t =$ length of the short circuited stub

Now at the point of attachment, the sum of the line susceptance and the stub susceptance must be zero *i.e.*

$$B_s + B_t = 0$$

or

$$\sqrt{\left(\frac{Y_0}{Y_R}\right)}\,(Y_0 - Y_R) + \{-Y_0 \cot \beta l_t\} = 0$$

or

$$\cot \beta l_t = \left(\frac{Y_0 - Y_R}{Y_0}\right) \sqrt{\frac{Y_0}{Y_R}}$$

$$\cot \beta l_t = (Y_0 - Y_R) \cdot \frac{1}{\sqrt{Y_0 Y_R}}$$

or

$$\tan \beta l_t = \frac{\sqrt{Y_0 Y_R}}{Y_0 - Y_R}$$

$$l_t = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{Y_0\, Y_R}}{Y_0 - Y_R}$$

or

$$\boxed{l_t = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{Z_R\, Z_0}}{Z_R - Z_0}} \quad \text{... (10.124)}$$

This is the equation of length of a short circuited stub required. Stub matching is almost invariably used at a fairly higher frequencies *i.e.* in S.W. range of broadcast band. The basic idea in using stub is to avoid reflections reaching the sending end i.e. source.

10.30.2. *Demerits of Single Stub Matching.* The single stub matching suffers mainly from the two main disadvantages.

(a) The range of terminating impedances which can be transferred is limited. If the terminating impedance (*i.e.* input impedance of antenna, say) changes, then it becomes essential to adjust the position and length of stub as well. It is easier for open wire lines but is inconvenient in case of coaxial lines.

(b) It is useful for a fixed frequency only because as the frequency varies, the position of the stub has to be varied. The change of susceptance, however, does not present any problem because the shorting plug can be moved to the required position. Hence, the single stub matching system is a narrow band system.

10.30.3. *Length (L_t) and position (L_s) of stub in terms of Reflection coefficient K* It is often convenient to determine L_t and L_s when K is given. To arrive at a relation, let us start with the eqn. 10.93

$$\boxed{Z_s = Z_0 \frac{(1 + K\, e^{-2Pl})}{(1 - K\, e^{-2Pl})}} \quad \text{... (10.61)}$$

For high frequency transmission lines, the attenuation $\alpha = 0$ and hence $P = j\beta$. Further, the reflection coefficient K is a general complex quantity and can be written as

$$K = |K|\, e^{j\varphi}$$

where

$$\varphi = \text{Angle of reflection coefficient } K$$

Therefore eqn. 10.93 can be written

$$Z_S = Z_0 \frac{[1 + |K|\, e^{j\varphi} \times e^{-2(0 + j\beta)\, l}]}{[1 - |K|\, e^{j\varphi} \times e^{-2(0 + j\beta)\, l}]}$$

$$\boxed{Z_S = Z_0 \frac{[1 + |K|\, e^{j(\varphi - 2\beta l)}]}{[1 - |K|\, e^{j(\varphi - 2\beta l)}]}} \quad \text{... (10.125)}$$

As stub is connected in parallel, so it is better to use admittance

$$Y_S = \frac{1}{Z_S} = \text{Input Admittance} = G_S + j\, B_S$$

where

$$G_S = \text{Input conductance}$$

and

$$B_S = \text{Input susceptance}$$

and

$$Y_0 = \frac{1}{Z_0} = G_0 + j\, B_0$$

Thus eqn. 10.125 can be written as

$$Y_S = \frac{1}{Z_S} = \frac{1}{Z_0} \frac{[1 - |K|\, e^{j(\varphi - 2\beta l)}]}{[1 + |K|\, e^{j(\varphi - 2\beta l)}]}$$

$$\boxed{Y_S = G_0 \frac{[1 - |K| \, e^{j(\varphi - 2\beta l)}]}{[1 + |K| \, e^{j(\varphi - 2\beta l)}]}} \quad \ldots (10.1$$

It is assumed here that characteristic impedance is resistive i.e..

$$Z_0 = R_0 = \frac{1}{G_0}$$

where G_0 = Characteristic conductance

Now eqn. 10.126 can be converted to polar coordinate as

$$e^{j\theta} = \cos\theta + j\sin\theta$$

so that
$$Y_S = G_0 \frac{[1 - |K| \{\cos(\varphi - 2\beta l) + j\sin(\varphi - 2\beta l)\}]}{[1 + |K| \{\cos(\varphi - 2\beta l) + j\sin(\varphi - 2\beta l)\}]}$$

$$Y_S = G_0 \frac{[1 - |K| \cos(\varphi - 2\beta l) - |K| j\sin(\varphi - 2\beta l)]}{[1 + |K| \cos(\varphi - 2\beta l) + |K| j\sin(\varphi - 2\beta l)]}$$

Let
$$\varphi' = \varphi - 2\beta l$$

Then
$$Y_S = G_0 \frac{[1 - |K| \cos\varphi' - |K| j\sin\varphi']}{[1 + |K| \cos\varphi' + |K| j\sin\varphi']}$$

Further, let $|K| \cos\varphi' = A$

and $|K| \sin\varphi' = B$

Then
$$Y_S = G_0 \frac{[(1 - A) - jB]}{[(1 + A) + jB]}$$

Now rationalizing, we have

$$Y_S = G_0 \frac{[(1 - A) - jB]}{[(1 + A) + jB]} \times \frac{[(1 + A) - jB]}{[(1 + A) - jB]}$$

$$Y_S = G_0 \frac{[(1 - A)(1 + A) - jB(1 + A) - jB(1 - A) + j^2 B^2]}{[(1 + A)^2 - j^2 B^2]}$$

$$= G_0 \frac{[1^2 - A^2 - jB\{1 + A + 1 - A\} + j^2 B^2]}{[1^2 + A^2 + 2A + B^2]}$$

$$= G_0 \frac{[1 - A^2 - 2jB - B^2]}{[1 + A^2 + B^2 + 2A]}$$

Putting the values, we get

$$Y_S = G_0 \frac{[1 - |K|^2 \cos^2\varphi' - 2|K| j\sin\varphi' - |K|^2 \sin^2\varphi']}{[1 + |K|^2 \cos^2\varphi' + |K|^2 \sin^2\varphi' + 2|K| \cos\varphi']}$$

$$= G_0 \frac{[1 - |K|^2 \{\cos^2\varphi' + \sin^2\varphi'\} - 2|K| j\sin\varphi']}{[1 + |K|^2 \{\cos^2\varphi' + \sin^2\varphi'\} + 2|K| \cos\varphi']}$$

$$Y_S = G_0 \frac{[1 - |K|^2 - 2|K| j\sin\varphi']}{[1 + |K|^2 + 2|K| \cos\varphi']} \quad \ldots (10.12$$

$$\boxed{Y_S = G_S + jB_S = \frac{G_0[\{1 - |K|^2 - 2j|K|\sin\varphi'\}]}{[1 + |K|^2 + 2|K|\cos\varphi]}}$$

Equating real and imaginary parts

$$\boxed{\begin{aligned} G_S &= \frac{G_0\,[\,1 - |K|^2\,]}{1 + 2|K|^2 + |K|\cos\varphi'} \\ B_S &= \frac{G_0\,[-2\,|K|\,\sin\varphi'\,]}{[1 + |K|^2 + 2\,|K|\cos\varphi'\,]} \end{aligned}}$$ ... 10.128 (a)

and ... 10.128 (b)

At the point of attachment of the stub, for no reflection to take place $Z_S = Z_0$ at $l = l_s$

or $$\frac{1}{G_S} = \frac{1}{G_0} \text{ or } \frac{G_S}{G_0} = 1 \text{ and } l = l_s$$

Hence from eqn. 10.128 (a), we get

$$1 = \frac{1 - |K|^2}{1 + |K|^2 + 2\,|K|\cos(\varphi - 2\beta\, l_s)}$$

or $$2\,|K|\cos(\varphi - 2\beta\, l_s) = -\,2\,|K|^2$$

or $$\cos(\varphi - 2\beta\, l_s) = -\,|K|$$

or $$(\varphi - 2\beta\, l_s) = \cos^{-1}(-\,|K|) \quad \text{... (10.129)}$$

But $$\cos^{-1}(-\,|K|) = \cos^{-1}|K| - \pi$$

Hence $$(\varphi - 2\beta\, l_s) = \cos^{-1}|K| - \pi$$

or $$2\beta\, l_s = \varphi + \pi - \cos^{-1}|K|$$

or $$l_s = \frac{\varphi + \pi - \cos^{-1}(|K|)}{2\beta} \qquad \because \beta = \frac{2\pi}{\lambda}$$

or $$\boxed{l_s = \frac{\lambda}{4\pi}\,(\varphi + \pi - \cos^{-1}|K|) \text{ per unit}} \quad \text{... (10.130)}$$

Again, from eqn 10.128 (b) the shunt susceptance on a per unit basis is given by

$$\frac{B_S}{G_0} = \frac{[-\,2\,|K|\sin\varphi'\,]}{1 + |K|^2 + 2\,|K|\cos\varphi'\,]}$$

$$= \frac{-\,2\,|K|\sin(\varphi - 2\beta\, l_s)}{1 + |K|^2 + 2\,|K|\cos(\varphi - 2\beta\, l_s)}$$

The input susceptance of the line at the stub location nearest to the load is obtained by putting eqn. 10.129 into eqn. 10.130. Thus

$$\frac{B_S}{G_0} = \frac{-\,2\,|K|\sin(\cos^{-1}|-K|)}{1 + |K|^2 + 2\,|K|\{-\,K|\}} = \frac{-\,2\,|K|\sin(\cos^{-1}|K| - \pi)}{1 + |K|^2 + 2\,|K| \times |-K|}$$

$$\boxed{\frac{B_S}{G_0} = \frac{-\,2\,|K|\sin(\cos^{-1}|K|)}{1 + |K|^2 - 2\,|K|^2}} \quad \text{... (10.131)}$$

In order to find the value of $\sin\{\cos^{-1}(|K|)\}$. Let us put

$$\cos^{-1}(|K|) = \theta$$

or $$|K| = \cos\theta$$

or $$\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - |K|^2}$$

Hence
$$\frac{B_S}{G_0} = \frac{-2\,|K|\sin\theta}{1+|K|^2-2\,|K|^2}$$

$$\frac{B_S}{G_0} = \frac{-2\,|K|\sqrt{1-|K|^2}}{1-|K|^2}$$

or
$$B_S = G_0\,\frac{[-2\,|K|\sqrt{1-|K|^2}\,]}{[1-|K|^2]}$$

$$\boxed{B_S = G_0\,\frac{(-2\,|K|)}{\sqrt{1-|K|^2}}} \qquad \text{...(10.132)}$$

Now, if B_t is the susceptance of the stub and B_t the line susceptance, then in order to cancel the susceptance, both must be zero at the point of attachment. Thus

$$B_s + B_t = 0$$

or
$$B_t = B_s = -\left[G_0\,\frac{\{-2\,|K|\}}{\sqrt{1-|K|^2}}\right]$$

$$B_t = +\,G_0\left\{\frac{2|K|}{\sqrt{2-|K|^2}}\right\}$$

But the susceptance of the short-circuited stub is given by eqn. 10.122 *i.e.*

$$B_t = G_0 \cot\beta\, l_t$$

where l_t = length of short–circuited stub

or
$$G_0 \cot\beta\, l_t = G_0\left\{\frac{2\,|K|}{\sqrt{1-|K|^2}}\right\}$$

$$\frac{1}{\tan\beta\, l_t} = \frac{2\,|K|}{\sqrt{1-|K|^2}} \quad \text{or } \tan\beta\, l_t = \frac{\sqrt{1-|K|^2}}{2\,|K|}$$

or
$$\beta\, l_t = \tan^{-1}\frac{\sqrt{1-|K|^2}}{2\,|K|} \quad \text{or } l_t = \frac{1}{\beta}\tan^{-1}\frac{\sqrt{1-|K|^2}}{2|K|}$$

or
$$\boxed{l_t = \frac{\lambda}{2\pi}\tan^{-1}\frac{\sqrt{1-|K|^2}}{2\,|K|}} \qquad \text{... 10.133}$$

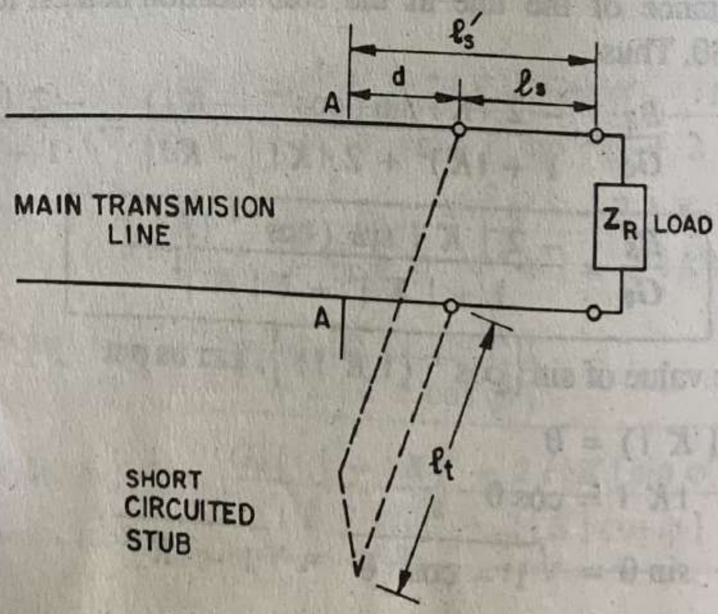

Fig. 10.22. Location of Stub.

From eqn. 10.128 (a), it can be seen that

$$\frac{G_S}{G_0} = \frac{1 - |K|^2}{1 + |K|^2 + 2|K|\cos(\varphi - 2\beta l_s)}$$

This value of $\frac{G_S}{G_0}$ is maximum when denominator is minimum *i.e.* when $\cos(\varphi - 2\beta l_s') = -1$. ... cosine terms should be -1 for maximum value of l_s'. Hence

$$\cos(\varphi - 2\beta l_s') = -1$$

$$\varphi - 2\beta l_s' = \cos^{-1}(-1) \quad \text{or} \quad \varphi - 2\beta l_s' = -\pi$$

$$2\beta l_s' = \varphi + \pi$$

$$l_s' = \frac{\varphi + \pi}{2\beta} = \frac{\varphi + \pi}{2 \times \frac{2\pi}{\lambda}}$$

$$\boxed{l_s' = \frac{\lambda}{4\pi}(\varphi + \pi)} \qquad \text{... 10.133 (b)}$$

At a distance l_s' from the load the value of

$$\frac{G_S}{G_0} = \frac{1 - |K|^2}{1 + |K|^2 + 2|K|(-1)}$$

$$= \frac{(1 + |K|)(1 - |K|)}{(1 - |K|)^2} = \frac{1 + |K|}{1 - |K|}$$

$$\frac{G_S}{G_0} = \frac{1 + |K|}{1 - |K|} = VSWR \equiv S$$

$$\boxed{\frac{G_S}{G_0} = S} \qquad \because G_S = \frac{1}{Z_S} = \frac{1}{R_S} \qquad \text{... (10.134)}$$

$$\text{and } G_0 = \frac{1}{Z_0} = \frac{1}{R_0}$$

$$\frac{1}{Z_S} \times Z_0 = S$$

$$\boxed{Z_S = \frac{Z_0}{S}} \qquad \text{... (10.135)}$$

This is the point of maximum $\frac{G_S}{G_0}$ which is recognised as a point of minimum voltage at a distance of l_s' from the load. At a distance l_s from the load, the value of $\frac{G_S}{G_0} = 1$ or $G_S = G_0$. This is the point where stub is attached. The value of l_s has been calculated at eqn. 10.130. Obviously, the value of distance d is given by

$$d = l_s' - l_s = \frac{\varphi + \pi}{2\beta} - \frac{\varphi + \pi - \cos^{-1}(|K|)}{2\beta} \qquad \text{... 10.136 (a)}$$

$$\boxed{\begin{aligned} d &= +\frac{\cos^{-1}(|K|)}{2\beta} \\ d &= \frac{\lambda}{4\pi}\cos^{-1}\left\{\frac{VSWR - 1}{VSWR + 1}\right\} \end{aligned}} \qquad \text{... 10.136 (b)}$$

This shows that stub should be connected at a distance d measured in either direction from the voltage minimum point A. **Normally stub is attached towards load side of that minimum which is nearest to the load voltage standing wave ratio which is existing before the attachment of stub, we may express conveniently the length of the stub in terms of $VSWR$ as follows.** From eqn. 10.133,

$$l_t = \frac{\lambda}{2\pi}\tan^{-1}\frac{\sqrt{1-|K|^2}}{2|K|}$$

But $$|K| = \frac{VSWR-1}{VSWR+1} \equiv \frac{S-1}{S+1}$$

Putting the values, we get

$$l_t = \frac{\lambda}{2\pi}\tan^{-1}\frac{\sqrt{1-\left|\frac{(S-1)}{(S+1)}\right|^2}}{2\sqrt{\frac{S-1}{S+1}}} = \frac{\lambda}{2\pi}\tan^{-1}\frac{\frac{\sqrt{(S+1)^2-(S-1)^2}}{(S+1)}}{2\sqrt{\frac{S-1}{S+1}}}$$

$$= \frac{\lambda}{2\pi}\tan^{-1}\frac{\sqrt{S^2+1+2S-S^2-1+2S}}{2(S-1)} = \frac{\lambda}{2\pi}\tan^{-1}\frac{\sqrt{4S}}{2(S-1)}$$

$$\boxed{l_t = \frac{\lambda}{4\pi}\tan^{-1}\frac{\sqrt{S}}{S-1}\ \text{metres}} \quad \text{...(10.137)}$$

This is the length of the short circuited stub which can be placed d metres towards the load from a point at which a voltage minimum existed before attachment of the stub. The susceptance of the line at d is cancelled and the line appears to be terminated in a pure resistance of value R_0 at that point. This way there will be a smooth line between the generator and the point of attachment of the stub.

Alternatively, the stub may also be attached d metres towards the source from the voltage minimum (AA). The sign of reactance is reversed on this side w.r.t.. the sign for the location nearer the load. In this case, stub length

$$\boxed{l_{t_1} = \frac{\lambda}{2} - l_t} \quad \text{...(10.138)}$$

Normally a short circuited stub is preferred in comparison to an open circuited stub because of greater ease of construction and secondly because of lower loss of energy due to radiation.

10.31. DOUBLING STUB MATCHING (AMIETE, Dec 1981, June 1991)

The two disadvantages of the single stub matching are overcome by using double stub matching as shown in Fig 10.23 in which two short circuited stubs at two fixed point usually $\lambda/4$ apart are utilized. Although positions are fixed but their lengths are independently adjustable. The double stub matching provides wide range of impedance matching.

In a single-stub impedance matching stub is located at a definite point on the line. This requirement frequently calls for placement of the stub at an undesirable place from mechanical point of view. For example, in case of coaxial line it is not possible to locate the location of a voltage minimum without a slotted line section. Thus the placement of single stub at an exact requirement, creates difficulties. In the case of single stub matching two adjustable parameters viz. length and position of the stub.

Alternative way to have the required adjustments is to use the two stubs for matching as

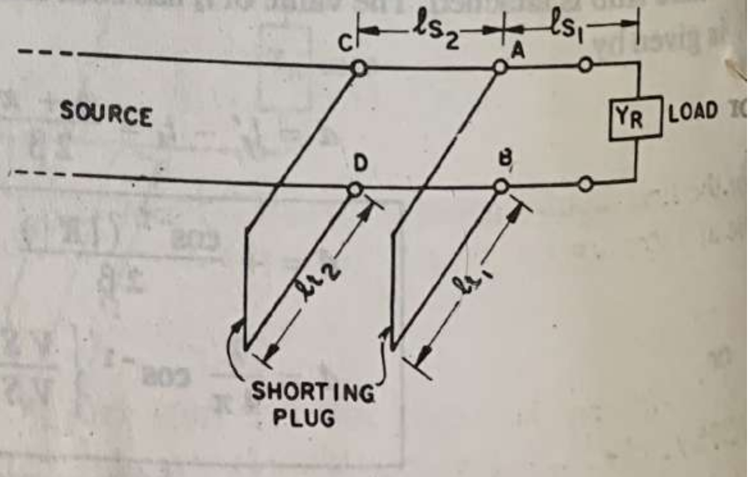

Fig. 10.23. Double stub matching.

shown in Fig. 10.23 in which positions of the two stubs AB and CD are **arbitrary but the two stub lengths are adjustable.** The spacing between two stubs is $0.375\,\lambda$ or $\lambda/4$. Half wave length ($\lambda/2$) spacing is avoided as it places the two stubs in parallel, as result only one effective adjustment is available. The same difficulties are encountered if the two stubs are close together. The separation of $0.375\,\lambda$ or $3/8\,\lambda$ is an optimum. The double stub matcher as shown in Fig. 10.23 is usually employed for coaxial microwave lines. In double stub matcher, two variable are available which provides a very good matching.

If the first stub whose length is l_{t_1} is located at AB at distance of l_{s_1} from the load, the normalized input admittance is then given by eqn. 10.115.

$$(Y_S)_{AB} = \frac{Y_r + j\tan\beta l_{s_1}}{1 + jY_r\tan\beta l_{s_1}} \qquad \text{... (10.115)}$$

Rationalizing, we get

$$(Y_S)_{AB} = \frac{Y_r + j\tan\beta l_{s_1}}{1 + jY_r\tan\beta l_{s_1}} \times \frac{1 - jY_r\tan\beta l_{s_1}}{1 - jY_r\tan\beta l_{s_1}}$$

$$= \frac{Y_r + j\tan\beta l_{s_1} - jY_r^2\tan^2\beta l_{s_1} + Y_r\tan^2\beta l_{s_1}}{1 - (jY_r\tan\beta l_{s_1})^2}$$

$$= \frac{Y_r\,(1 + \tan^2\beta l_{s_1}) + j\tan\beta l_{s_1}\,(1 - Y_r^2)}{1 + Y_r^2\tan^2\beta l_{s_1}}$$

$$= \frac{Y_r\sec^2\beta l_{s_1}}{1 + Y_r^2\tan^2\beta l_{s_1}} + j\,\frac{(1 - Y_r^2)\tan\beta l_{s_1}}{1 + Y_r^2\tan^2\beta l_{s_1}}$$

$$(Y_S)_{AB} = (G_S)_{AB} + j\,(B_S)_{AB} \qquad \text{... (10.139)}$$

where

$$(G_S)_{AB} = \frac{Y_r\sec^2\beta l_{s_1}}{1 + Y_r^2\tan^2\beta l_{s_1}} \qquad \text{... 10.140 (a)}$$

&

$$(\beta_S)_{AB} = \frac{(1 - Y_r^2)\tan^2\beta l_{s_1}}{1 + Y_r^2\tan^2\beta l_{s_1}} \qquad \text{... 10.140 (b)}$$

and

$$Y_s = \frac{Y_S}{Y_0} = \text{Normalised input admittance}$$

when a stub having a susceptance B_1 is added to this point AB, then new admittance will be say

$$\boxed{(Y_S)_{CD} = (G_S)_{AB = CD} + j\,(B_S)_{CD}} \qquad \text{... (10.141)}$$

Because only the susceptance value is changed by the addition of the stub, while the conductance par remains unchanged *i.e.* $(G_S)_{AB} = (G_S)_{CD}$. Then $(Y_S)_{CD}$ should be such that admittance Y at CD is equa to $1 + j\,\beta_2$. The stub length at CD is adjusted such that the new value of $(Y_S)_{CD}$ is unity to effect the matching For smooth operation of the line, the input admittance of the line looking towards the load at CD should b ($Y_s = 1$)

or

$$Y_s = \frac{Y_S}{G_0} = 1 \text{ or } Y_s = G_0 \qquad \text{... (10.14}$$

or the line should appear terminated in its characteristic impedance at that point. Hence the point CD shou be at a location on the line having a per unit admittance of

$$Y_s = \frac{Y_S}{G_0} = 1 \pm jB_2 \qquad \text{... (10.14}$$

It may be noted that two stubs are usually at fixed points normally separated by a distance $0.375\,\lambda$. *The stub AB nearest to the load is adjusted to make the real part of the admittance at the points C* *equal to the characteristic conductance of the line, in absence of the second stub (AB). This stub (AB) is then adjusted to produce zero susceptance at the points CD.*

Double stub matcher is usually connected between the load and the main transmission line to ensure the shortest possible length of mismatched line. The mismatch occurs between *CD* and the load end. In order that this mismatch portion on the line is minimum, the total distance $l_{s_1} + l_{s_2}$ should be as small as possible. This is the reason the stub *AB* is kept nearest to the load The two stubs whose lengths are individually controllable are shunted across the line near the load as shown in Fig. 10.24. This arrangement has the advantages that trial and error adjustment of the impedance matching system can be made without the necessity of providing a connection that can slide along the main transmission line which makes it suitable for coaxial transmission line. However, there is a disadvantage of the system that the range of the load impedances that can be matched to the transmission line is limited.

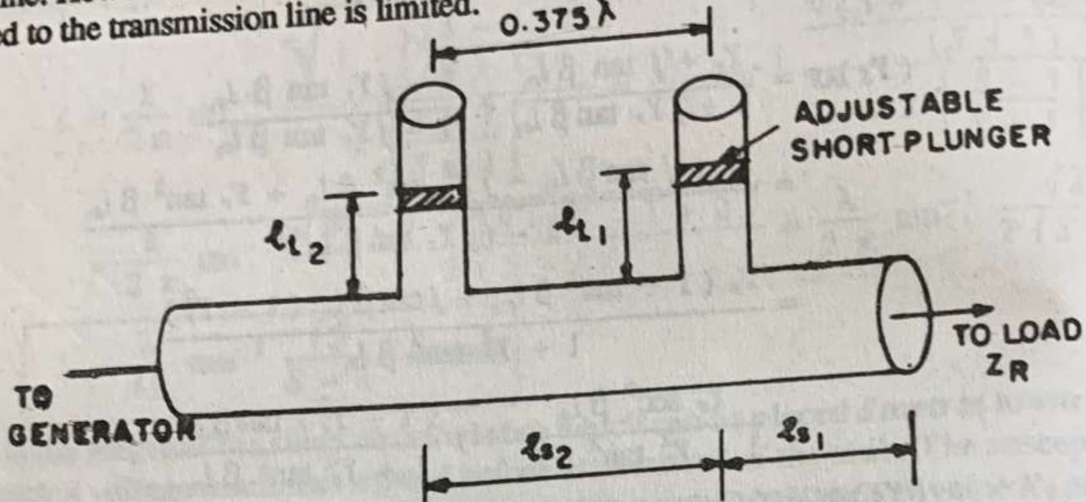

Fig. 10.24. Double Stub impedance matching system.

By double stub matching, it is possible to reduce VSWR below 1.2. If almost perfect matching under all conceivable conditions of VSWR and load impedance Z_R is required, a triple stubs, adjustable in length and placed at 0.125 λ apart may be used. The *above discussions are* basis for the location and adjustment of the two stubs. It may be noted that it is not possible to assign any arbitrary value to l_{s_1} and l_{s_2} to calculate double stub problems analytically. However, the value of l_{s_1} and l_{s_2} may be determined with the help of smith chart. The length of stubs are in the range of

$$l_{t_1} = 0.348\ \lambda \text{ and } l_{t_2} = 0.11\ \lambda \qquad \text{... 10.144 (b)}$$

10.32. TYPES OF HIGH-FREQUENCY TRANSMISSION LINES (FEEDERS)

The are many types of H.F. transmission lines which are in general use, of these, the most important types are :

(i) Coaxial cable (ii) Open wire

(iii) Twin lead or ribbon lead.

10.31.1. *Coaxial cable* (AMIETE, Nov. 1960)

If one wire of open wire line is placed inside a larger hollow concentric conductor the arrangement is called as coaxial cable. It is called so as both the conductors (outer and inner) have the same axis (Fig. 10.25). In practice, the central conductor is held in position accurately by insulating material which may be a solid core, disc or breads strung along the axis of the conductor. Equal and opposite current flows in inner and outer

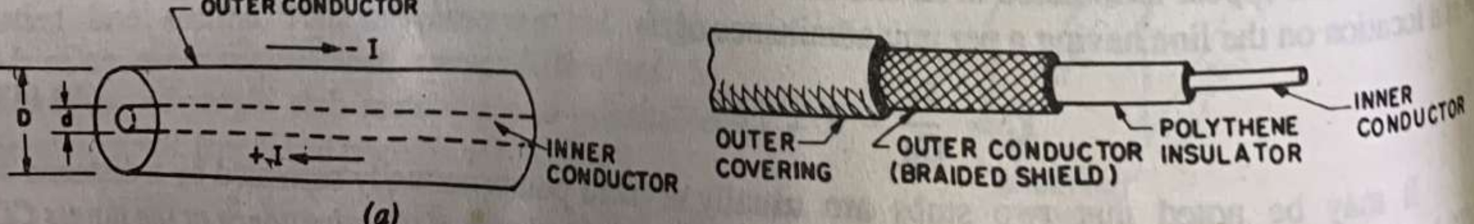

Fig. 10.25. Coaxial Cables.

10.33. BALANCE TO UNBALANCE TRANSFORMATION (AMIETE, May 1978, Dec. 1978)

Sometimes grounded antennas or monopole antennas or Marconi antennas are fed through a two-wire transmission line and an ungrounded antenna system that is symmetrical w.r.t. ground is fed through a coaxial transmission line. *An open wire transmission line, being symmetrical w.r.t. ground is a balanced transmission line while coaxial cable is an unbalanced transmission line.* In either case, whether power is to be delivered to a grounded antenna through a two-wire transmission line or conversely a coaxial cable is used to deliver power to an ungrounded antenna, *it becomes necessary to convert between a balanced system which is symmetrical w.r.t. ground and an unbalanced system in which one side is grounded (i.e. not symmetrical w.r.t. ground). It is because a two-wire transmission line (a balanced transmission line) feeds grounded antenna (an unbalanced antenna). Likewise an unbalanced coaxial cable feeds an ungrounded antenna which is balanced antenna.*

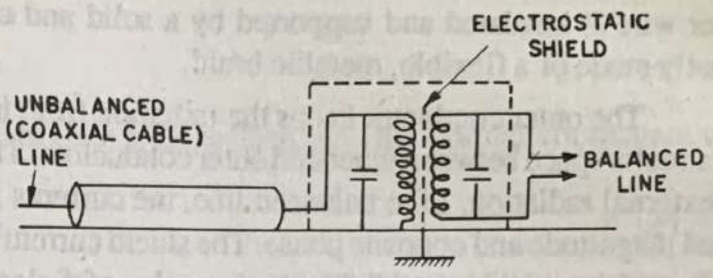

Fig. 10.27. Balanced to unbalanced transformation with the help of a tuned transformer.

A method used to effect such a transformation is to employ a transformer shown in Fig. 10.27. Here the winding associated with the balanced system is symmetrically arranged w.r.t. a grounded electrostatic shield so that stray capacitances enevitably present will not introduce unbalance.

10.34. BALUNS (AMIETE, May 1976, 1991)

Stands for **Balancing unit.** At frequencies high enough for resonant lines to be practical, arrangements of the types shown in Fig. 10.28 are employed. They are known as balun. A balun is an impedance transformer designed to couple a balanced transmission line and unbalanced transmission circuit (or antenna). The impedance transformation is accomplished generally with convertional techniques.

However, the conversion between a balanced system and an unbalanced system requires special techniques. The operation of the balun can be understood by considering that the voltage is applied to the balun from balanced side. Since this voltage is applied across the conductors A—B of the coaxial cable, therefore, it is transferred to the coaxial system without change.

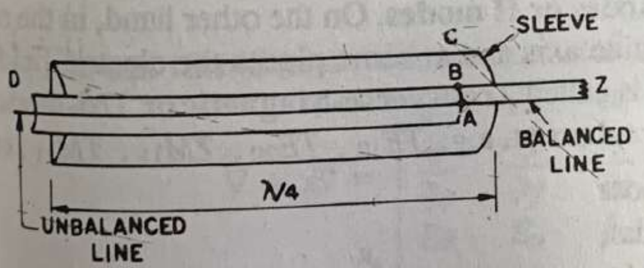

Fig. 10.28. Illustration of arrangements for transforming a balanced system (i.e. symmetrical w.r.t. ground) to an unbalanced system (in which one side is grounded) i.e. balun.

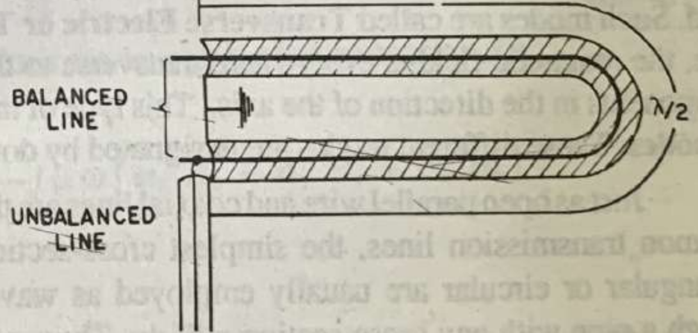

Fig. 10.29. Half-wave-line balun.

At the sametime, the fact that conductor B is an extension of the outer conductos D of the coaxial cable does not introduce an unbalance in the balanced system at conductor B because when the sleeve is exactly at $\lambda/4$ long the impedance across B—C approaches infinity. Thus sleeve C acts as an extension of conductor D and remains at ground potential, while point B is free to assume any potential the balance system desires it to have. Of various other basic types of baluns possible another type is shown Fig. 10.29.

10.35. BALANCE AND UNBALANCE LINES

A balance line consists of two parallel conductors which are separated a fixed distance apart by some low loss material. When the R.F. Currents in each wire of the line are of equal magnitude and opposite phase the field around the line is small and a very minimum of power escapes from the line in the form of radiation. On the other hand, the electrical field surrounding a line having unequal or improperly phased currents will be large. The field of the line may interact with that of the antenna, which will distort the pattern of the antenna.

In any event, the radiated field of the line may be considered to be valuable power loss before it reaches the antenna. Since there is a minimum of solid dielectric between the wires, the dielectric losses of a good **two-wire balanced transmission line** are low in comparison to a coaxial line having greater amount of dielectric material.

An unbalanced line is a Coaxial Cable which consists of a wire inside a tabular outer conductor. The inner wire is insulated and supported by a solid and continuous dielectric material. The outer conductor is usually made of a flexible, metallic braid.

The outer conductor keeps the radiation from inner conductor minimum. All the fields exist between the annular space between inner and outer conductors. This makes coaxial cable a perfectly shielded line having no external radiation. Like balanced line, the currents in the inner and outer conductor and the shield are of equal magnitude and opposite phase. The shield current flows only on the inner surface of the shield. The outer surface of the shield is '*cold*'. The higher value of dielectric loss of the coaxial unbalanced line is compensated by the fact that coaxial cable has no radiation loss, as does the balanced line.