CHAPTER 7

Antenna Arrays

7.1. INTRODUCTION (Antenna Arrays or Array of Antenna)

The field radiated by any small linear antenna is un-uniformly distributed in the plane perpendicular to the axis of the antenna. For example, the radiation pattern of an elementary dipole in which maximum radiation takes place at right angle to the axis and the decreases slowly to minimum, as the polar angle decreases towards the axis of dipole. Thus a non-uniform type of radiation pattern may be preferred in many broad-cast services but not at all desirable for point to point communication and preferred-coverage services i.e. services in which it is desired to radiate most of the energy in one particular direction like highly populated area etc. The field strength can be increased in preferred directions by properly exciting Group or array of antennas simultaneously in an arrangement known as array of antennas or simply antenna arrays. Array of antennas in an arrangement, of several individual antennas so spaced and phased that their individual contributions coming in one preferred direction and cancel in all other directions, to get greater directive gain or directivity. **Thus, an antenna array is a system of similar antennas oriented similarly to get greater directivity in a desired direction. It may also be defined as, "A radiating system consisting of several spaced and properly phased radiators".**

Antenna array is one of the common method of combining the radiations from a group or array of milar antennas in which the phenomena of wave-interference is involved. The total field (not power) roduced by an antenna array system at a great distance from it, is the vector sum of the fields produced y the individual antennas of the array system. The relative **phases** of individual field components depend n the relative distance of the individual antennas of the antenna array and, in turn, depend on the irection.

Further, an antenna array is said to be **linear**, if the individual antennas of the array are equally spaced ong a straight line. Individual antennas of an antenna array system is also termed as ELEMENTS. Thus, near antenna array is a system of equally spaced elements. Also, a *uniform linear array is one*, in which the lements are fed with a current of equal magnitude with uniform progressive phase shift along the line. It may, however, be noted that the term "**phase**" **in an antenna arrays and ordinary circuit has same meaning i.e. two currents in two elements are said to be in phase if they reach their maximum values, flowing in the same direction at the same instant.**

Since antennas may be put in various configurations e.g. in straight lines, circles, triangles, rectangle etc. and hence there are large number of possible configurations. However, experience shows that in practice only limited number of antenna arrays are of practical use.

Further, the element in a multi-element array is generally a $\lambda/2$ dipole antenna, the length of which is not necessarily electrical half wavelength. However, variation in electrical length from $\lambda/2$ is within 5% and hence the radiating properties of the element is not affected.

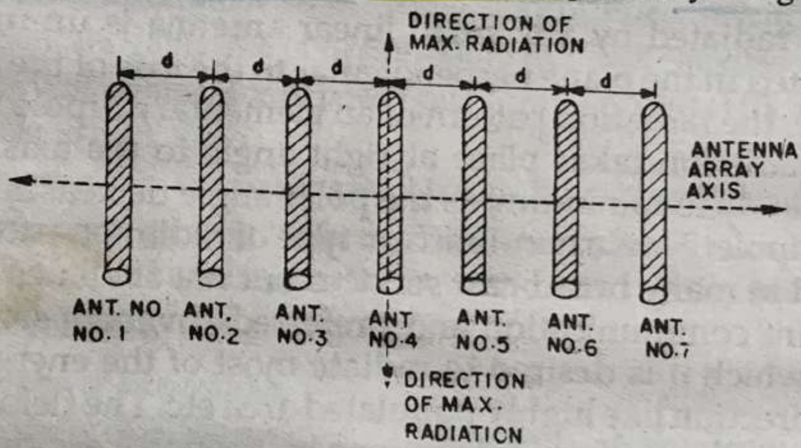
7.2. VARIOUS FORMS OF ANTENNA ARRAYS

Various antenna arrays used in practice are the following :

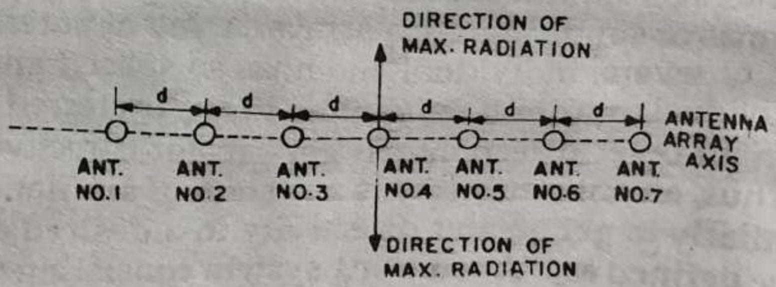
7.2.1. *Broadside Array*

(AMIETE, Nov. 1968, 69, 70, 71, 77, 78, Dec. 1981)

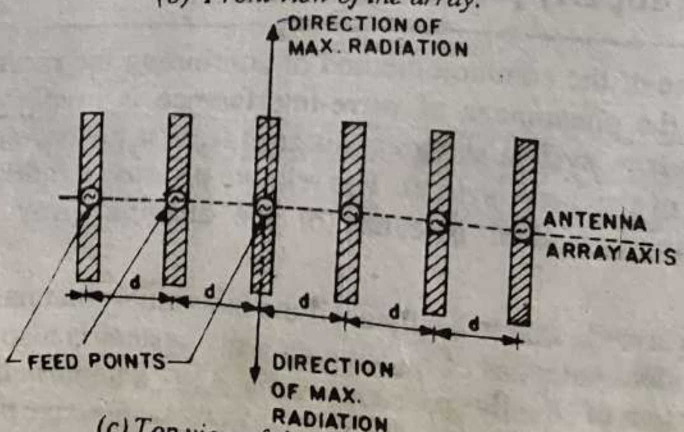
This is one of the important antenna arrays used in practice. Broad-side array is one in which a number of identical parallel antennas are set up along a line drawn perpendicular to their respective axes as shown in Fig 7.1 In the broad-side array, individual antennas (or elements) are equally spaced along a line and each element is fed with current of equal magnitude, all in the same phase. By doing so, this arrangement fires i



(a) Broadside array arrangement.



(b) Front view of the array.



(c) Top view of the Broadside array.

broad-side directions (i.e. perpendicular to the line of array axis) where there are maximum radiations and relatively a little radiations in other directions and hence the radiation pattern broadside array is bidirectional. The broadside array is bidirectional which radiates equally well in either direction of maximum radiations. Therefore, *broadside array may be defined as "An arrangement in which the principal direction radiation is perpendicular to the array axis and also to the plane containing the array element"*.

It may, however, be noted that the bidirectional pattern of a broadside array can be converted into unidirectional by installing an identical array behind this array at distance $\lambda/4$ and exciting it by current leading in phase by 90° or $\pi/2$ radian. Further, broadside arrays may also be arranged in vertical in which case the radiation pattern would be horizontal. In the Fig. 7.1 elements are arranged horizontal and its radiation pattern is vertical normal to the plane of element as in Fig. 7.2 Lastly a **broad-side couplet** is said to form if two isotropic radiators operate in phase thereby they reinforce each other most strongly in the plane right angles to the line joining them.

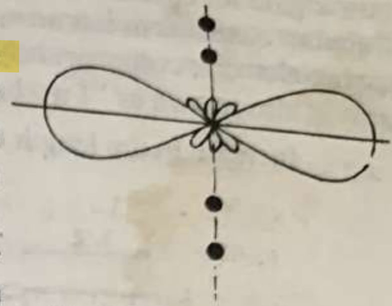


Fig. 7.2. Radiation pattern of Broadside array.

7.2.2. *End Fire Arrays*

(AMIETE. Nov. 1969, 70, 77, 78)

The end-fire array is nothing but broadside array except that individual elements are fed in, out of phase (**usually 180°**). Thus in the end-fire array, a number of identical antennas are spaced equally along a line and individual elements are fed with currents of equal magnitude but their phases varies progressively along the line in such a way as to make the entire arrangement substantially unidirectional. In otherwords, individual elements are excited in such a manner that a progressive phase difference between adjacent elements (in cycles) becomes equal to the spacing (in wavelength) between the elements. Therefore, end fire array may be defined as *"The arrangement in which the principal direction of radiation coincides with the direction of the array axis"*. This is illustrated in Fig. 7.3.

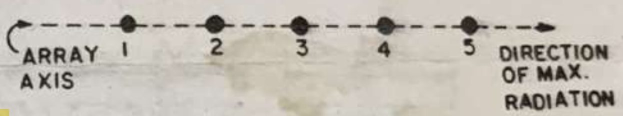


Fig. 7.3. Front view of an end fire array.

It may be noted, however, that an end fire array may be bidirectional also. One such example is a two elements array, fed with equal current, 180° out of phase.

Lastly an end-fire couplet is said to form, if two equal radiators are operated in phase quadrature at a distance of $\lambda/4$ apart.

7.2.3. *Collinear Arrays*

(AMIETE, Nov. 1969)

In collinear array, the antennas are arranged co-axially i.e. antennas are mounted end to end in a single line. In otherwords, one antenna is stacked over another antenna as shown in Fig. 7.4 and 7.5.

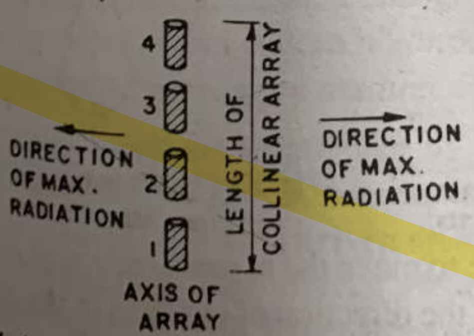


Fig. 7.4. 4 antennas (vertical) arranged collinearly.

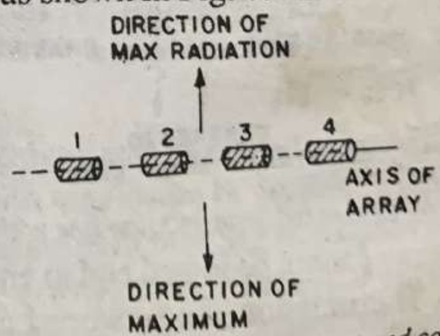


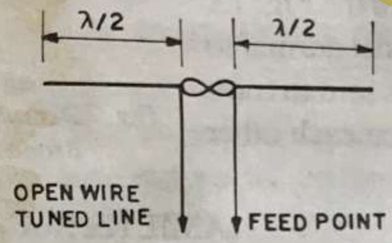
Fig. 7.5. 4 horizontal antennas arranged collinearly.

The individual elements are *fed with equal in phase currents as is the case in the broad side arrays. A collinear array is a broad-side radiators, in which the direction of maximum radiation is perpendicular to the line of antenna.* This arrangement gives radiation pattern which, when viewed through the major axis, closely resembles with the radiation pattern of a broadside array. But the radiation pattern of a collinear array has circular symmetry with its main lobe everywhere perpendicular to the principal axis. That is why, *a collinear array is also sometimes called as broad cast or omnidirectional arrays.*

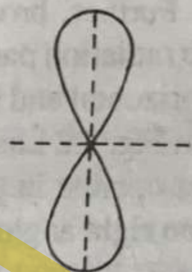
The gain of collinear arrays is maximum when the spacing between elements is of order of

0.3 λ to 0.5 λ but this much space between end to end of elements introduces constructional and feeding problems. Hence, the elements in collinear arrays are operated with their ends very close to each other. The ends of wire antennas of collinear arrays, for example, are joined simply by insulator. The power gain of collinear arrays does not increase in direct proportion of number of collinear elements used. For example power gain for collinear array of 2, 3 and 4 elements are respectively, 1.9 db, 3.2 db and 4.3 db. Hence more than four elements in this array is generally not used as more gain can be obtained by some otherwise device but two elements collinear array is usually used because it helps in multiband operation. Two elements collinear array is also known as **"Two half-waves in phase"** (Fig. 7.6).

Increase in the length of collinear arrays increases the directivity. If 3 or 4 elements collinear arrays



(a)



Radiation Pattern of Two element Collinear Array

(b)

Fig. 7.6. Two elements collinear Arrays.

are to be used, then it becomes essential to connect "phasing stubs" between adjacent elements in order to keep currents in phase in all the elements as shown in Fig. 7.7.

Since in a long wire the direction of current flow reverses in each half wave section and hence elements

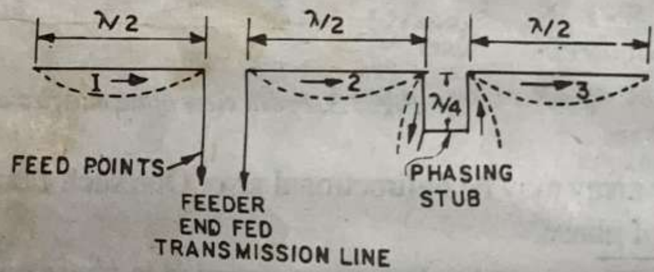


Fig. 7.7 (a). End fed through a transmission line with phasing stub E

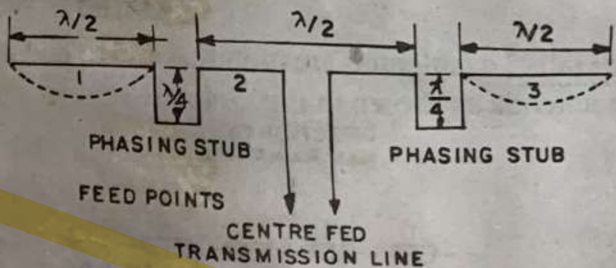


Fig. 7.7 (b). Centre fed, transmission line with phasing stubs (3 elements).

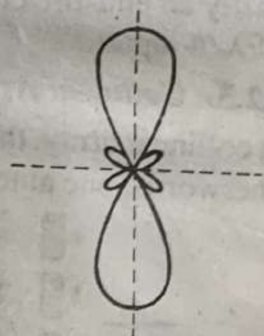


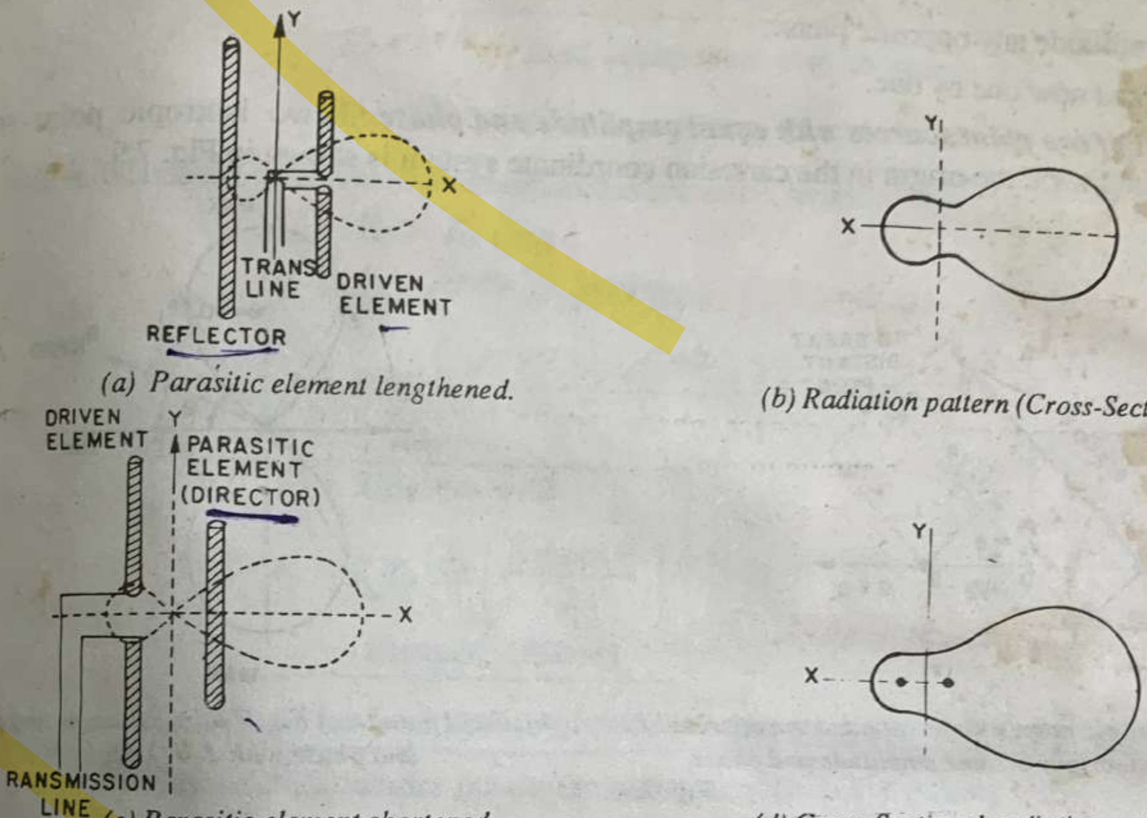
Fig. 7.7 (c). Radiation Pattern of a four elements collinear array.

of collinear array cannot be joined end to end without any device to make the same direction of currents in all the elements. Transmission line between 1 and 2 elements keeps the direction of currents in one direction and phasing stub between 2 and 3 corrects the direction of current in the 3rd element (to the direction of 1 and 2 elements). Because phasing stub is equivalent to a λ/2 (half-wave) section folded on itself to cancel its radiation. Thus *the total length* after the transmission line has become 3 λ/2 in which the centre λ/2 unit acts as "λ/4 *phase reversing stub*". Hence the currents in elements 1, 2, 3 remains in one direction.

In practice, combination arrays, consisting of all the three arrays i.e. broadside, end fire and collinear, are also used to increase further gain and directivity.

7.2.4. *Parasitic Arrays*. In order to ease the problem of fed line, it is, sometime desirable to feed certain antennas of an array parasitically. The element supplied power directly from source (i.e. Transmitter) usually through transmission line is called as **driven element.** But a parasitic element is not fed directly instead parasitic element derives power by radiation from nearby driven element. In other words, parasitic element obtains power solely through electromagnetic coupling with a driven element because of its proximity to that driven element.

The simplest case of a parasitic array is one driven element and one parasitic element and this may be considered as two element array (Fig 7.8). *Multielement arrays having number of parasitic elements are called "Parasitic Arrays" whether driven element is one or more. Hence in parasitic arrays there. is one or more parasitic elements and atleast one driven element to introduce power in the array.*



(a) Parasitic element lengthened. *(b) Radiation pattern (Cross-Section).*

(c) Parasitic element shortened. *(d) Cross-Sectional radiation pattern.*

Fig. 7.8. Two element arrays.

A parasitic array with linear half-wave dipole as elements is normally called as **"Yagi-Uda"** or simply "Yagi" antenna after the name of inventors S. Uda (Japanese) and H. Yagi (English).

The amplitude and phase of the current induced in a parasitic element depends on its tuning and the spacing between parasitic element and driven element to which it is coupled. Variations in the distance between driven element and parasitic element changes the relative phases and this proves to be a very convenient. It helps in making the radiation pattern unidirectional. *A distance of $\lambda/4$ and phase difference of $\pi/2$ radian (or 90°), for example, provides an unidirectional pattern.*

A parasitic element lengthened by 5% w.r.t. driven element acts as reflector and shortened by 5% acts as director as in Fig. 7.8. A reflector makes the radiation maximum in perpendicular direction from parasitic element towards driven element and the director helps in making maximum radiation in perpendicular direction from driven element to the parasitic element (Fig. 7.8). A properly designed parasitic array can provide a larger front to back ratio and assumes special importance when the antenna system is to be rotated in any direction. It is of great practical use specially at higher frequencies between 100-1000 MHz.

7.3. ARRAYS OF POINT SOURCES

In this an antenna is regarded as point source or volumeless radiator. In other words a hypothetical antenna or isotropic or omni-directional or non-directional antenna which occupies zero volume, is considered. For the array theory of point sources, first only two isotropic point sources separated by a distance with different phasing conditions are taken and then the idea is extended for more and finally for n isotropic point sources. Further, the case of non-isotropic but similar to point sources will also be taken which will lead to the principle of multiplication of pattern.

7.3.1. *Arrays of two point sources.* This is the simplest situation in the arrays of isotropic point sources in which it is assumed that the two point sources are separated by a distance (say d) and have the same polarization. Since in array theory of antennas, the superposition or addition of fields from the various sources at a great distance with due regard to phases, is involved and hence the following cases (out of many) will be dealt with Arrays of two isotropic point sources with

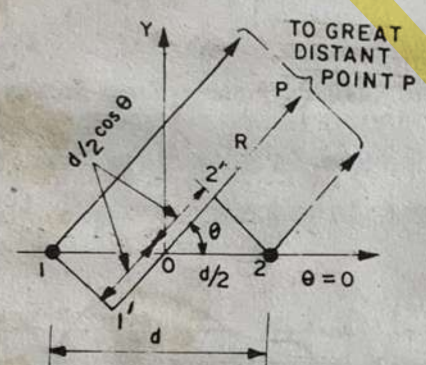
(1) Equal amplitude and phase.

(2) Equal amplitude and opposite phase.

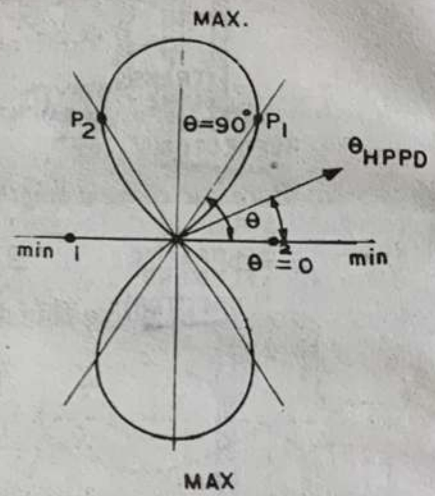
(3) Unequal amplitude any opposite phase.

Let us proceed now one by one.

(1) ***Arrays of two point sources with equal amplitude and phase :*** Two isotropic point sources symmetrically situated w.r.t. the origin in the cartesian coordinate system is shown in Fig. 7.9.



(a) Two isotropic point sources situated symmetrically w.r.t. origin O same amplitude and phase.



(b). Field pattern of Fig. 7.9 (a) i.e. same amplitude and phase with d = λ/2.

Fig. 7.9.

We are to calculate fields at a great distant point, at distance (say R) from the origin O and the origin is taken as reference point for phase calculation. Obviously, waves from source 1 reaches the point P at a latter time than the waves from source 2 because of path difference (1′ 2′) involved between the two waves. Thus the fields due to source 1 lags while that due to source 2 leads. Path difference between the two waves is (1′ 2′) and is given by

$$\text{Path difference} = (1'2') \text{ metres} = \left(\frac{d}{2}\cos\theta + \frac{d}{2}\cos\theta\right) \text{metres}$$

$$= d\cos\theta \text{ metres} \quad \dots \ 7.1\text{(a)}$$

$$= \frac{d}{\lambda}\cos\theta \text{ wavelengths}$$

Then from optics, it is known as ... 7.1 (b)

$$\boxed{\text{Phase angle } (\psi) = 2\pi \text{ (Path difference)}} \quad \dots \ (7.2)$$

$$\therefore \quad \psi = 2\pi\left(\frac{d}{\lambda}\cos\theta\right) \text{radians} = \frac{2\pi}{\lambda} d\cos\theta \text{ radians} \quad \dots \ 7.2\text{(a)}$$

or

$$\boxed{\psi = \beta d \cos\theta \text{ radians}}$$

$$\text{if } \beta = \frac{2\pi}{\lambda} \quad \dots \ 7.2\text{(b)}$$

Also let, E_1 = Far electric field at distance point P, due to source 1

E_2 = Far electric field at distant point P, due to source 2

E = Total electric field at distant point,

and $\psi = \beta\, d \cos\theta$ radians

= Phase angle difference between the fields of the two sources measured at angle θ along radius vector line.

Then, total far field at distant point P, in the direction of θ is given by

$$\boxed{E = E_1 e^{-j\psi/2} + E_2 e^{+j\psi/2}} \quad \text{... (7.3)}$$

where $E_1 e^{-j\psi/2} \equiv$ field component due to source 1 ;

$E_2 e^{+j\psi/2} \equiv$ field component due to source 2;

But, in this case it is assumed that amplitudes are same, hence

$$E_1 = E_2 \equiv E_0 \text{ (say)} \quad \text{... (7.4)}$$

$$\therefore \quad E = E_0 (e^{-j\psi/2} + e^{+j\psi/2})$$

$$= 2\, E_0 \left(\frac{e^{-j\psi/2} + e^{+j\psi/2}}{2} \right) \quad \Big| \because \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \text{ from trigonometry}$$

$$\boxed{\begin{aligned} E &= 2\, E_0 \cos\psi/2 \\ E &= 2\, E_0 \cos\left(\frac{\beta\, d \cos\theta}{2}\right) \\ &\quad \text{(Amp.)} \quad \text{(Phase)} \end{aligned}} \quad \text{7.4 (a)} \quad \text{... 7.4 (b)}$$

This is the equation of far field pattern of two isotropic point sources of same amplitude and phase.

Here the total amplitude is $2\,E_0$ whose maximum value may be 1. By putting $2\,E_0 = 1$ or $E_0 = \frac{1}{2}$, the pattern is said to be normalized. Thus Eqn. 7.4 (b) becomes

$$E = \cos\left(\frac{\beta\, d \cos\theta}{2} \right) = \cos\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \frac{\cos\theta}{2} \right) \quad | \text{ if } d = \lambda/2 \text{ taken}$$

$$\boxed{E = \cos\left(\frac{\pi}{2} \cos\theta \right)} \quad \text{... (7.5)}$$

In order to draw the field pattern, the directions of maxima, minima and half power points must be known, which can be calculated with the help of eqn. 7.5 as follows :

Maxima direction. E is maximum, when $\cos(\pi/2 \cos\theta)$ is maximum, and its maximum value is ± 1.

$\therefore E$ is max. when $\cos(\pi/2 \cos\theta) = \pm 1$

or $\pi/2 \cos\theta_{max} = \pm n\pi$ where $n = 0, 1, 2, \ldots\ldots$

or $\pi/2 \cos\theta_{max} = 0$ if $n = 0$

or $\cos\theta_{max} = 0$ 7.5 (a)

$$\boxed{\theta_{max} = 90^0 \text{ and } 270^0}$$

Minima directions. E is minimum when $\cos(\pi/2 \cos\theta)$ is minimum and its minimum value is 0

$\therefore$ E is minimum. When $\cos(\pi/2 \cos\theta) = 0$

or $(\pi/2 \cos\theta_{min}) = \pm (2n+1)\pi/2$ where $n = 0, 1, 2, \ldots$

or $(\pi/2 \cos\theta_{min}) = \pi/2$

or $\cos\theta_{min} = \pm 1$

$$\boxed{\theta_{min} = 0° \text{ and } 180°} \quad \ldots 7.5\text{ (b)}$$

Half power point direction. At half power points power is $\frac{1}{2}$ or voltage or current is $1/\sqrt{2}$ times the maximum value of voltage or current.

$\therefore$ $\cos(\pi/2 \cos\theta) = \pm 1/\sqrt{2}$

or $\pi/2 \cos\theta_{HPPD} = \pm(2n+1)\pi/4.$ where $n = 0, 1, 2, \ldots$

or $\pi/2 \cos\theta_{HPPD} = \pm \pi/4$

or $\cos\theta_{HPPD} = \pm \frac{1}{2}$

$$\boxed{\theta_{HPPD} = 60°, 120°} \quad \ldots 7.5\text{ (c)}$$

If now the field patterrn bet. E versus θ is drawn for the case $d = \lambda/2$, then the Fig. 7.9 (b) is obtained which is a bidirectional, figure of eight. 360^0 rotation of this figure around x-axis will generate the 3-dimensional space pattern — **a doughnut shape.**

This is the simplest type of "**broad-side array**" and is also known as "**broad-side couplet**" as two isotropic radiators radiates in phase.

As an alternative, if reference point in Fig. 7.9 (a) is shifted to say source 1 (instead midway of the array then amplitude of the field pattern remains the same (e.g. $2E_0$) but the phase pattern changes as shown below.

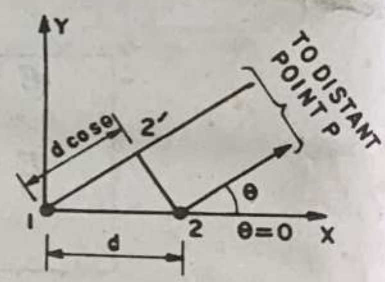


Fig. 7.10. Two point source of equal amplitude and phase, separated by a distance d.

The resultant far field pattern, in this case, is the vector sum of the fields of individual sources at the distant point P,

$\therefore$ $$E = E_1 e^{j0} + E_2 e^{j\psi} = E_1 + E_2 e^{j\psi} \quad \Big| \because e^{j0} = e^0 = 1 \quad \ldots 7.6\text{ (a)}$$

where $\psi = \beta d \cos\theta$

Applying the condition 7.4, we have

$$E = E_0(1 + e^{j\psi}) = E_0 e^{j\psi/2}(e^{-j\psi/2} + e^{j\psi/2}) \quad \Big| \text{ Taking } e^{+j\psi/2} \text{ common}$$

or $$= 2E_0 e^{j\psi/2}\left(\frac{e^{-j\psi/2} + e^{j\psi/2}}{2}\right) = 2E_0 e^{j\psi/2} \cos\psi/2$$

$$\boxed{\begin{array}{l} E = \underset{\text{Amp.}}{(2E_0)} \; \underset{\text{Phase}}{(\cos\psi/2\, e^{j\psi/2})} \\ \text{or} \quad E_{norm} = (\cos\psi/2\; e^{j\psi/2}) \end{array}} \quad \begin{array}{r} \ldots 7.7\text{ (a)} \\ \Big| \because 2E_0 = 1 \quad \ldots 7.7\text{ (b)} \end{array}$$

Thus, comparison of 7.7. (b) and 7.4 (a) indicates that phases are not the same. Eqn. 7.7 (b) may be rewritten as

$$E_{norm} = \cos\psi\left\{\cos\frac{\psi}{2} + j\sin\frac{\psi}{2}\right\}$$

$$= \cos\psi\left\{\angle\frac{\psi}{2}\right\} \quad \ldots 7.8\text{ (a)}$$

Here $e^{j\psi/2}$ or $\angle \psi/2$ represents the variation of phase w.r.t. reference (source 1).

(2) ***Arrays of two point sources with equal amplitude and opposite phase.*** This is exactly similar to above except that point source 1 is out of phase or opposite phase (180°) to source 2 i.e. when there is maximum in source 1 at one particular instant, then there is minimum in source 2 at that instant and vice-versa. Referring to Fig. 7.9 (a), the total far field at distant point P, is given by

$$E = (-E_1 e^{-j\psi/2}) + (+E_2 e^{+j\psi/2})$$

because phase of source 1 and source 2 at distant point P is $-\psi/2$ and $+\psi/2$, since the reference being at between midway of two sources.

But
$$E_1 = E_2 = E_0 \text{ (say)}$$

Then
$$E = E_0 \, 2j \left(\frac{e^{j\psi/2} - e^{-j\psi/2}}{2j} \right)$$

$$\boxed{\mathbf{E = 2jE_0 \sin \psi/2}} \quad \text{... 7.9 (a)}$$

$$\boxed{\mathbf{E = 2jE_0 \sin \left(\frac{\beta d}{2} \cos\theta \right)}} \quad \text{... 7.9 (b)}$$

The Eqn. (7.9) of total far field is similar to that of Eqn. 7.4 but for that it is sine function instead of cosine and the operator j is involved. Presence of operator j simply means that opposite phase brings a phase shift of 90° in the total field. This will be evident by drawing the field pattern. for let $d = \lambda/2$ and and $2E_0 j = 1$

$\therefore$
$$\boxed{E_{norm} = \sin(\pi/2 \cos\theta)} \quad \text{... (7.10)}$$

Maximum directions. Maximum value of sine function is ± 1

$\therefore$ $$\sin(\pi/2 \cos\theta) = \pm 1$$

or $$(\pi/2 \cos\theta_{max}) = \pm(2n+1)\pi/2 \qquad \text{where } n = 0, 1, 2, \ldots\ldots$$

or $$(\cos\theta_{max}) = \pm 1 \qquad \text{if } n = 0$$

$$\boxed{\theta_{max} = 0° \text{ and } 180°} \quad \text{... 7.11 (a)}$$

Minima directions. Minimum value of a sine function is 0

$\therefore$ $$\sin(\pi/2 \cos\theta) = 0$$

or $$\pi/2 \cos\theta_{min} = \pm n\pi \qquad \text{where } n = 0, 1, 2, \ldots\ldots$$

or $$\cos\theta_{min} = 0$$

or $$\boxed{\theta_{min} = 90° \text{ and } -90°} \quad \text{... 7.11 (b)}$$

Half power point directions. For

$\therefore$ $$\sin(\pi/2 \cos\theta) = \pm \frac{1}{\sqrt{2}}$$

or $$\pi/2 \cos\theta_{HPPD} = \pm(2n+1)\pi/4$$

or $$\pi/2 \cos\theta_{HPPD} = \pm \pi/4 \quad \text{if } n = 0$$

or $$\cos\theta_{HPPD} = \pm \frac{1}{2}$$

or $$\boxed{\theta_{HPPD} = 60°, \pm 120°} \quad \text{... 7.11 (a)}$$

From these, it is possible to draw the field pattern which is as shown in Fig. 7.11.

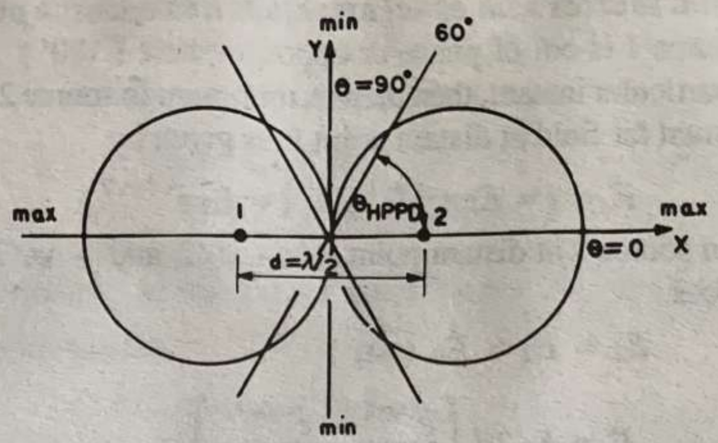


Fig. 7.11. Two point sources with equal amplitude and opposite phase spacing λ/2.

It is seen that maxima have shifted 90° along X-axis in comparison to in-phase field pattern. The figure is horizontal figure of 8 and 3-dimensional space pattern is obtained by rotating it along X-axis. Once the arrangement gives maxima along line joining the two sources and hence this is one of the simplest type of **"End fire" 'Array'**.

(3) ***Arrays of two point sources with unequal amplitude and any phase.*** Let us now consider general condition in which the amplitudes of two point sources are not equal and hence any phase difference say α. With reference to Fig. 7.12, let us also assume at source 1 is taken as reference for phase and amplitude

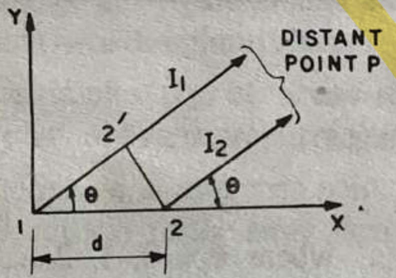


Fig. 7.12 (a). Two point sources with unequal amplitude and any phase difference δ.

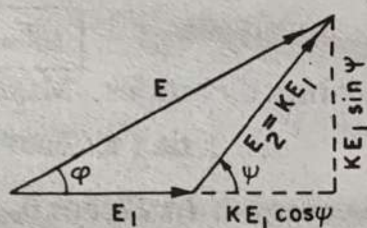


Fig. 7.12 (b). Vector diagram of Fig.. 7.12 (a)

of fields due to source 1 and 2 at a distant point P is E_1 and E_2 in which E_1 is greater than E_2. Then the total-phase difference between the radiations of two sources at point P is given by

$$\boxed{\psi = \frac{2\pi}{\lambda} d \cos\theta + \alpha} \quad \ldots (7.12)$$

where α is the phase angle by which the current (I_2) of source-2 leads the current (I_1) of source 1. If α = 0 or 180° and $E_1 = E_2 = E_0$, then it will correspond to the above two cases resp. just described. The vector diagram is shown in Fig. 7.12 (b). The total fields at P its given by,

$$E = E_1 e^{j0} + E_2 e^{j\psi} = E_1 \left(1 + \frac{E_2}{E_1} e^{j\psi}\right) \quad | \because e^{j0} = e^0 =$$

$$\boxed{E = E_1 (1 + K e^{j\psi})} \quad \ldots (7.13)$$

where $$K = \frac{E_2}{E_1} \quad \ldots (7.14)$$

and since $$E_1 > E_2$$

∴ $$K < 1$$

i.e. $$0 \le K \le 1$$

From Eqn 7.13 magnitude and phase angle (φ) at point P is given by taking its modulus

$$E = \left| E_1 \{ 1 + K(\cos\psi + j\sin\psi) \} \right|$$

∴ $$E = E_1 \sqrt{(1 + K\cos\psi)^2 + (K\sin\psi)^2} \angle\varphi \qquad \text{... 7.15 (a)}$$

or φ = Phase angle at P

where $$\varphi = \tan^{-1} \frac{K\sin\psi}{1 + K\cos\psi} \qquad \text{... 7.15 (b)}$$

It may be noted that if $E_1 = E_2$, $K = 1$ then Eqns. 7.14 and 7.13 become Eqns 7.6 (a).

7.4. NON-ISOTROPIC BUT SIMILAR POINT SOURCES

So far, array of two isotropic point sources were considered but this idea may be extended to the sources which are not-isotropic i.e. non-isotropic provided their field patterns are similar to that of isotropic point source. In other words, fields patterns of non-isotropic must have the same shape and orientation. However, it is not necessary that amplitude of individual non-isotropic source is equal. Such a non-isotropic source is given the name non-isotropic but similar point source. In case, the amplitudes of the individual sources are equal, the sources would be non-isotropic but identical.

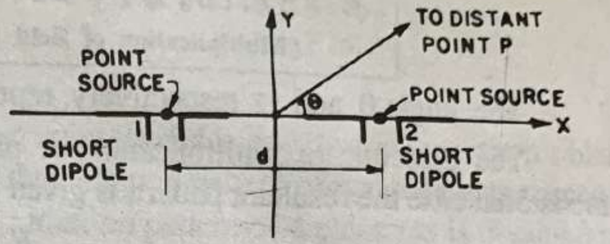


Fig. 7.13. Two non-isotropic (short dipoles and point sources).

Let us now consider two short dipoles which are superimposed over the two isotropic point sources and are separated by a distance.

Let the field pattern of each isotropic point source be given by

$$E_0 = E_1 \sin\theta \qquad \text{... (7.16)}$$

Since it is also possible to obtain such pattern from a short dipole and hence the two types of sources re symmetrically superimposed w.r.t. to origin. From eqn. 7.4 (a), the field pattern of two identical isotropic source is given by

$$E = 2E_0 \cos\psi/2 \qquad \text{... (7.4 a)}$$

where $$\psi = \beta d\cos\theta + \alpha$$

Combining eqn. 7.4 (a) and 7.16, we have

$$E = 2E_1 \sin\theta\cos\psi/2$$

$$E_{norm} = (\sin\theta) \times (\cos\psi/2) \qquad \text{... (7.17 a)}$$

$$E_{norm} = \left(\begin{array}{c} \textbf{Pattern of Individual} \\ \textbf{isotropic source} \end{array} \right) \times \left(\begin{array}{c} \textbf{Pattern of Array of two} \\ \textbf{isotropic point sources} \end{array} \right)$$

This lead to the **principle of multiplication of pattern** as multiplication pattern of individual point source and pattern of array of isotropic point sources gives the field pattern of non-isotropic but similar point sources.

7.5. MULTIPLICATION OF PATTERN (AMIE S/1993, 1994 W/1994, AMIETE, Nov. 1977)

Multiplication of pattern or simply pattern multiplication, in general, can be stated as follows :

"The total field pattern of an array of non-isotropic but similar sources is the multiplication of the individual source patterns and the pattern of an array of isotropic point sources each located at the phase centre of individual source and having the relative amplitude and phase, whereas the total phase pattern is the addition of the phase pattern of the individual sources and that of the array of isotropic point sources."

Here the pattern of the individual sources is assumed to be same whether it is in the array or isolated.

Further the reference point for total phase pattern is the phase centre of the array. The principle of multiplica pattern is applicable for two and three dimensional patterns.

Let E = Total field.

$E_i(\theta, \varphi)$ = Field pattern of individual source.

$E_a(\theta, \varphi)$ = Field pattern of array of isotropic point sources.

$E\,p_i(\theta, \varphi)$ = Phase pattern of individual source.

$E_{pa}(\theta, \varphi)$ = Phase pattern of array of isotropic point source.

Then the total field pattern of an array of non-isotropic but similar source, symbolically, may be writ as

$$\boxed{E = \underbrace{\{E_i(\theta,\varphi) \times E_a(\theta,\varphi)\}}_{\text{(Multiplication of field pattern)}} \times \underbrace{\{E\,p_i(\theta,\varphi) + E\,p_a(\theta,\varphi)\}}_{\text{(Addition of phase pattern)}}} \quad \text{... (7}$$

The angle θ and φ respectively, represent the 'polar' and 'azimuth' angles.

The principle of multiplication of pattern is true for any number of **similar sources.** For t dimensional case the resultant pattern is given by eqn. 7.4 (a) or 7.17 (a)

$$E = 2E_0 \cos \psi/2 \quad \text{... (7.}$$

$$E = 2E_1 \sin\theta \cos \psi/2 \quad \text{... (7.1}$$

or

$$\boxed{E = E(\theta) \cdot \cos \psi/2} \quad \text{... (7.}$$

Evidently, E_0 is a function of θ say $E(\theta)$. The total field pattern, in this case, is multiplicatio field pattern known as primary and $\cos \psi/2$ the secondary pattern or array factor that the principle is equ applicable to 3 dimensional case also.

The principle of multiplication of patterns provides a speedy method for sketching the patter complicated arrays just by inspection and thus the principle proves to be a useful tool in design of ante arrays. The width of the principle lobe (i.e. width between nulls) and the corresponding width of array pa are same. The secondary lobes are determined from the number of nulls in the resultant pattern. In resul pattern the number of nulls are the sum of nulls of individual pattern and array pattern. This method is e and point by point multiplication of patterns provides the exact pattern of the resultant.

Let us now use the principle to some typical cases.

7.5.1. *Radiation Pattern of 4-isotropic elements fed in phase, spaced $\lambda/2$ apart :*

(AMIETE, May 1

Let the 4--elements of isotropic (or non-directive) radiators are in a linear arrays (Fig. 7.14) in w elements are placed at a distance of $\lambda/2$ and are fed in phase. i.e. $\alpha = 0$. One of the method to get the radia pattern of the array is to add the fields of individual four elements at a distance point P vectorially but ins an alternative method, using the principle of multiplicity of pattern, will be shown to get the same.

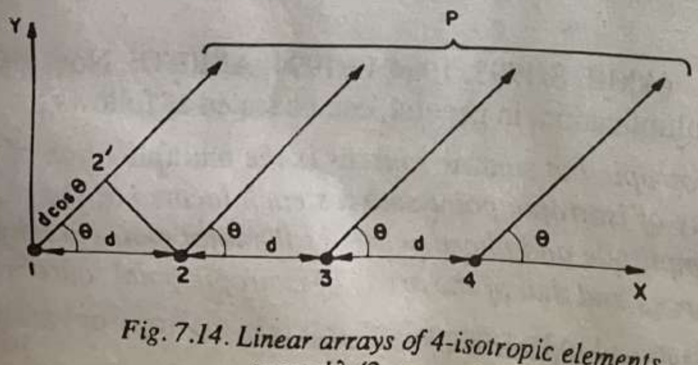


Fig. 7.14. Linear arrays of 4-isotropic elements spaced $\lambda/2$ apart, fed in phase.

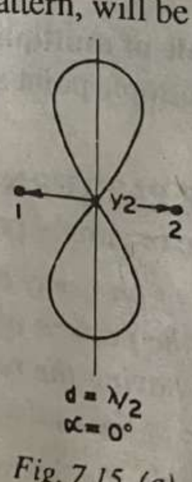


Fig. 7.15. (a)

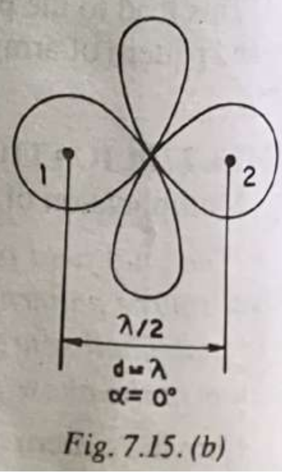
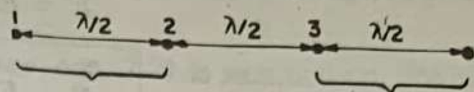
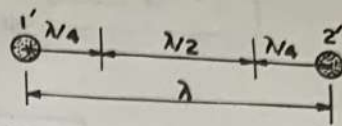


Fig. 7.15. (b)

It is already seen that two isotropic point sources, spaced $\lambda/2$ apart fed in phase provides a bidirectional pattern shown in Fig. 7.9 (b) (figure of eight). Further, the radiation pattern of two isotropic radiation spaced λ apart, fed in phase is known to be as shown in Fig. 7.15.



(a) 4-isotropic elements spaced $\lambda/2$.

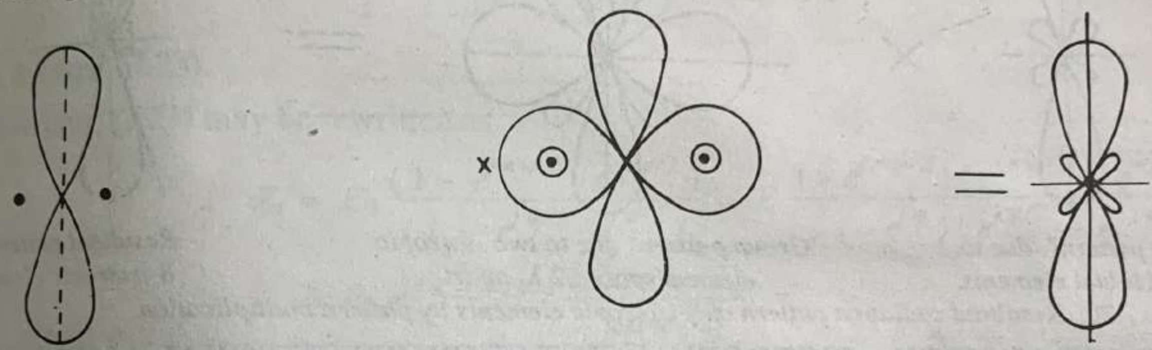


(b) 2-units array, where one unit ($O \equiv 0 \leftarrow \lambda/2 \rightarrow 0$) spaced λ.

Fig. 7.16.

Now elements (1) and (2) are considered as one unit and is considered to be placed between the midway of the elements and so also the elements (3) and (4) as another unit assumed to be placed between the two elements as shown in Fig. 7.16. These two units have the same radiation pattern as in Fig. 7.9 (b) and the radiation pattern of two isotropic antennas of Fig 7.16 (b) spaced λ is shown in Fig. 7.15.

Thus 4 elements spaced $\lambda/2$ have been replaced by two units spaced λ and by doing so, the problem of determining radiation of 4 elements has reduced to find out the radiation pattern of two antennas spaced λ apart. Then according to multiplicity of pattern. The resultant radiation pattern of 4 elements is obtained by multiplying the radiation pattern of individual element Fig 7.9 (b) and array of two units spaced λ (Fig. 7.15) as illustrated in Fig. 7.17.

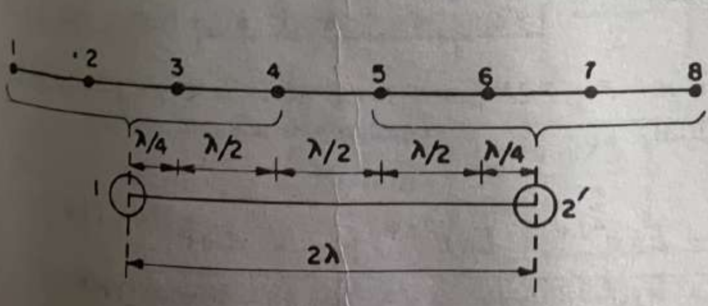


Individual (i.e. unit pattern) due to two individual element) *i.e. Group pattern due to array of two (isotropic).* *Resultant pattern of 4 isotropic elements.*

Fig. 7.17. Resultant radiation patern of 4-isotropic elements by Pattern multiplication.

In place of isotropic (non-directional) if the array is replaced by an non-isotropic (i.e. directional) antennas, then the radiation pattern Fig. 7.15 must be accordingly modified.

7.5.2. *Radiation pattern of 8-isotropic elements fed in phase, spaced $\lambda/2$ apart*. As above the principle can be applied to broad-side linear array of 8-isotropic elements also as shown in Fig. 7.18. In this case 4-isotropic elements are assumed to be one unit and then to find the radiation pattern of two such units spaced a distance 2λ apart. The radiation pattern of isotropic element is just seen in Fig. 7.17 and radiation pattern of two isotropic antennas spaced 2λ apart fed in phase can be calculated from eqn. 7.20 (to be discussed later).



(a) Linear Arrays of 8-isotropic elements spaced $\lambda/2$.
(b) Equivalent two units array spaced 2λ,
(where one unit $O \equiv \leftarrow \lambda/2 \rightarrow \lambda \rightarrow \leftarrow \lambda/2 \rightarrow$)

Fig. 7.18.

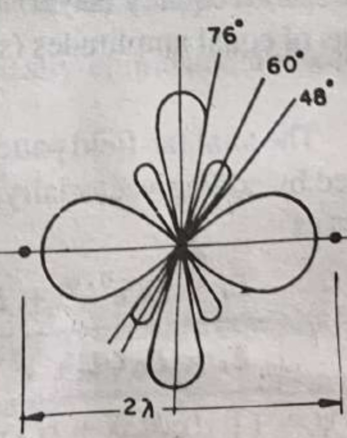


Fig. 7.19. Radiations pattern of isotropic radiators spaced 2λ.

$$E = E_0 (1 + e^{j\psi} + e^{2j\psi} + \ldots\ldots)$$

$$\boxed{E = E_0 \sum_{n=1}^{N} e^{j(n-1)\psi}}$$

where

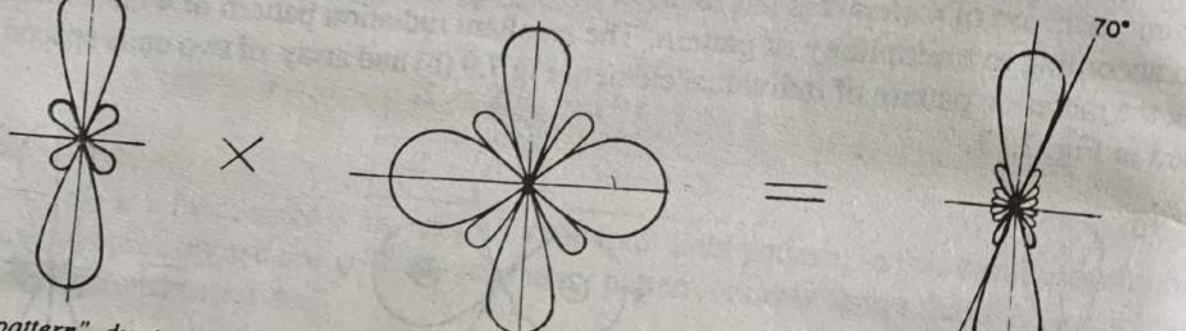
$$\psi = \beta d \cos\theta + \alpha \qquad \because \alpha = 0$$

$$= \beta d \cos\theta \qquad \because d = 2\lambda$$

$$\boxed{\psi = 4\pi\cos\theta} \qquad \ldots (7$$

which is as shown in Fig. 7.19.

Thus the radiation pattern of 8 isotropic elements is obtained by multiplying the unit pattern individual elements as already obtained in Fig. 7.17 and Group pattern of two isotropic radiators spaced is as shown in Fig. 7.19 and hence the resultant (Fig. 7.20).



"Unit pattern" due to 4-individual elements.

"Group pattern" due to two isotropic element spaced 2 λ apart.

Resultant pattern of 8-isotropic elements.

Fig. 7.20. Resultant radiation pattern of 8-isotropic elements by pattern multiplication.

7.6. LINEAR ARRAY WITH n ISOTROPIC POINT SOURCES OF EQUAL AMPLITUDE AND SPACING

(AMIETE June 74, 79, 90, 91, Dec. 73, 81, 82,

As said, an array is said to be linear, if the individual elements of the array are spaced equally a line and uniform, if the same are fed with currents of equal amplitude and having an uniform progres phase shift along the line. Often higher frequencies, for point to point communication, a single narrow beam of the radiation pattern is required which is usually obtained by multiunit linear arrays.

We shall now calculate the pattern of a linear array of n isotropic point sources in which point sources are spaced equally (say d) and are fed within phase currents of equal amplitudes (say E_0) as shown in Fig. 7.21.

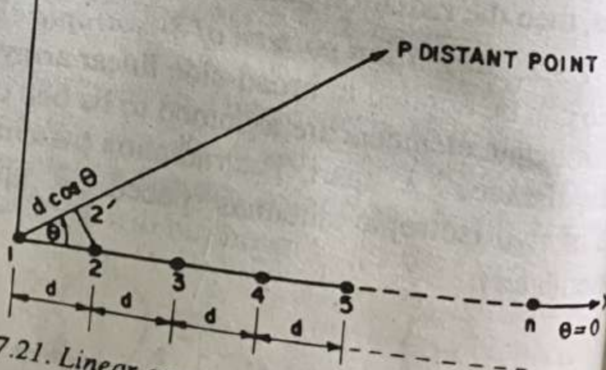


Fig. 7.21. Linear array with n isotropic point sources w equal amplitude and spacing

The total far-field pattern at a distant point P is obtained by adding vectorially the fields of individual sources as

$$E_t = E_0 e^{0j\psi} + E_0 e^{2j\psi} + E_0 e^{3j\psi} + E_0 e^{4j\psi} + E_0 e^{5j\psi} + \ldots\ldots\ E_0 e^{j(n-1)\psi}$$

$$\boxed{E_t = E_0 (1 + e^{j\psi} + e^{2j\psi} + e^{3j\psi} + e^{4j\psi} + e^{5j\psi} + \ldots\ldots\ldots\ e^{(n-1)\psi}} \quad e^{jn\psi} \qquad \ldots($$

where $\psi = (\beta d \cos\theta + \alpha)$ radian.

= Total phase difference of the fields at point P from adjacent sou

and α = Phase difference in adjacent point sources