(3) The numerical value of directive gain may lie between 0 and ∞ whereas that of directivity be 1 and ∞ and in case of directivity it can not be less than 1.

(4) Directive gain depends entirely on the distribution of radiated power in space whereas the of directivity depends on the solid angle of the far (radiated) field pattern.

(5) Directive gain does not depend on the power input to the antenna and antenna losses and true for the directivity. In other words antenna efficiency is not involved in these two.

(6) The numerical value of directivity of current element and half-wave dipole is respectively 1 1.76 *db*) and 1.64 (or 2.15 *db*).

∴ Directive gain of half-wave dipole over current element = (2.15 – 1.76) *db* = **0.39 *db*.**

6.13. ANTENNA EFFICIENCY (η)

(Kanpur Univ. M.Sc. Phy. (Prev.) 1981, (AMIETE, May 1968, Dec. 1

The efficiency of an antenna is defined as the ratio of power radiated to the total input power supplied to the antenna and is denoted by η or *k*. Thus

$$\text{Antenna efficiency} = \frac{\text{Power Radiated}}{\text{Total Input Power}}$$

or

$$\eta = \frac{W_r}{W_T} = \frac{W_r}{W_r + W_l} = \frac{W_r}{W_T} \times \frac{4\pi\Phi(\theta,\varphi)}{4\pi\Phi(\theta,\varphi)}$$

$$= \frac{4\pi\Phi(\theta,\varphi)}{W_T} \cdot \frac{W_r}{4\pi\Phi(\theta,\varphi)} = G_p \cdot \frac{1}{G_d} = \frac{G_p}{G_d}$$

or

$$\boxed{\eta = \frac{G_p}{G_d} = \frac{W_r}{W_r + W_l}} \quad \text{...(6}$$

where W_r = Power radiated.

W_l = Ohmic losses.

If current flowing in the antenna is I, then $\eta = \dfrac{I^2 R_r}{I^2 (R_r + R_l)}$

or

$$\boxed{\eta\% = \frac{R_r}{R_r + R_l} \times 100} \quad \text{...(6}$$

where R_r = Radiation resistance ; R_l = Ohmic loss resistance of antenna conductor and $R_r + R_l$ = T effective resistance.

It is desirable to have a better radiation characteristics from the antenna and for this loss resista should be as small as possible. The loss resistances may consist of the following, in general.

(i) Ohmic loss in the antenna conductor.

(ii) Dielectric loss.

(iii) I^2R loss in antenna and ground system.

(iv) Loss in earth connections.

(v) Leakage loss in insulation.

Thus antenna efficiency η **represents the fraction of total energy supplied to the antenna wh is converted into electromagnetic waves.**

6.14. EFFECTIVE AREA OR EFFECTIVE APERTURE OR CAPTURE AREA

(AMIETE, Nov. 1977,

A transmitting antenna transmits electromagnetic waves and a receiving antenna receives a frac of the same. The concept of effective area or aperture is best understood by considering an antenna to hav *effective area or aperture over which it extracts electromagnetic energy from the travelling electromagnetic waves.* It may be defined as **the ratio of power received at the antenna load terminal to the Poynting vector (or power density) in Watts/metre² of the incident wave.** Thus

$$\text{Effective area or Effective aperture or Capture area} = \frac{\text{Power received}}{\text{Poynting vector of incident wave}} \quad \text{... 6.57 (a)}$$

or

$$A_e = \frac{W}{P} \equiv A$$

or

$$\boxed{W = PA} \quad \text{... 6.57 ((b)}$$

where W = Power received, in Watts.

P = Poynting vector of incident plane wave, in Watts/m² or power flow per sq. metre for the incident wave.

A = Effective or Capture area or Effective aperture, in m².

Let a receiving antenna be placed in the field of plane polarised travelling waves as illustrated in Fig. 6.7 having an effective area A and the receiving antenna (dipole) is terminated at a load impedance $Z_L = R_L + j X_L$.

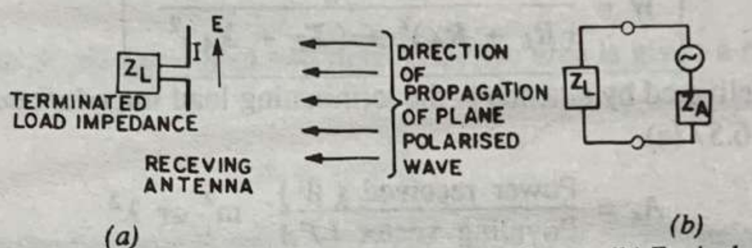


Fig. 6.7 (a). Receiving antenna in the field of plane polarised wave. (b) Equivalent circuit of Fig. (a).

If I be the terminal current, then received power

$$W = I_{rms}^2 R_L \quad \text{... (6.58)}$$

where R_L = Load resistance, in Ω

I_{rms} = Terminal *rms* current

$$\therefore \quad A = \frac{W}{P} = \frac{I_{rms}^2 R_L}{P}$$

Since the antenna extracts energy from incident electromagnetic waves, delivers the same to terminated load impedance Z_L and the power flowing per square metre or Poynting vector is P W/m². This entire system can be replaced by an equivalent circuit [given in Fig. 6.7 (b)] according to Thevenin's theorem. In Fig. 6.7(b)

V = Equivalent Thevenin's voltage.

Z_A = Equivalent Thevenin's impedance.

The voltage V is induced by passing electromagnetic waves which produces current I_{rms} through terminal load impedance Z_L.

$$\therefore \quad I_{rms} = \frac{\text{Equivalent voltage}}{\text{Equivalent Impedance}}$$

$$I_{rms} = \frac{V}{Z_L + Z_A} \text{ Amp.} \quad \text{.... (6.59)}$$

where $Z_A = R_A + j X_A$ = complex antenna impedance.

and $$R_A = R_r + R_l = R_r \text{ if } R_l = 0 \text{ is assumed}$$
$$= \text{Radiation resistance} + \text{loss resistance}$$

Now putting the values of Z_L and Z_A, we have

$$I_{rms} = \frac{V}{(R_L + jX_L) + (R_A + j\,X_A)}$$

or
$$|\,I_{rms}\,| = \frac{|V|}{\sqrt{(R_L + R_A)^2 + (X_L + X_A)^2}} \quad \text{.... 6.6}$$

or
$$|\,I_{rms}\,| = \frac{|V|}{\sqrt{(R_L + R_r + R_l)^2 + (X_L + X_A)^2}} \quad \text{..... 6.6}$$

where
X_L = Load reactance, in Ω
X_A = Antenna reactance, in Ω

Since the Power received by terminal load impedance is given by eqn. 6.58. Therefore, pu eqn. 6.61 (a) in 6.58, we get,

$$W = I_{rms}^2 \cdot R_L$$

$$\boxed{W = \frac{V^2\,R_L}{(R_L + R_A)^2 + (X_L + X_A)^2}} \quad \text{... (6}$$

This is the power delivered by antenna at the terminating load impedance Z_L. Now by definitio effective Aperture A_e, eqn. 6.57 (a)

$$A_e = \frac{\text{Power received } (W)}{\text{Poynting vector } (P)} \text{ m}^2 \text{ or } \lambda^2$$

$$= \frac{V^2\,R_L}{[\,(R_L + R_A) + (X_L + X_A)^2\,]\,P} \text{ m}^2 \text{ or } \lambda^2 \quad \text{From eqn.}$$

Since W is in Watts and P in Watts/metre^2 and hence the unit of A_e is m^2. But if the unit of P is t in terms of wavelength λ (which is convenient) as Watts/(wave length)^2, then A_e is in (wavelength)^2

∴
$$\boxed{A_e = \frac{V^2\,R_L}{[\,(R_L + R_A)^2 + (X_L + X_A)^2\,]\;P}\;\lambda^2} \quad \text{... 6.6}$$

$$A_e = \frac{V^2\,R_L}{[\,(R_L + R_r + R_l)^2 + (X_L + X_A)^2\,]\;P}\;\lambda^2 \quad \text{... 6.6}$$

This is the expression for effective area or aperture or capture area in terms of power densit Poynting vector P), induced voltage V, antenna impedance Z_A and terminating load impedance Z_L. **induced voltage V is maximum when antenna is oriented for max. response and the antenna and incident wave both have the same polarisation.** In eqn. 6.63, $R_A = R_r + R_l$ where R_l accounts for mismatch between Antenna and Z_L and antenna losses.

According to maximum power transfer theorem, maximum power will be transferred from antenn the antenna terminating load if

$$X_L = -\,X_A$$

and
$$R_L = R_A = R_r + R_l \quad \text{... 6.6}$$

or
$$R_L = R_r \quad \text{if } R_L = 0 \quad \text{.... 6.64}$$

Thus maximum power received in Antenna terminating load impedance Z_L can be obtaine conditions of eqn. 6.64 is substituted in eqn. 6.62

$$W_{max} = \frac{V^2 R_L}{4 R_L^2} = \frac{V^2}{4 R_L} = \frac{V^2}{4 R_r}$$

or $$\boxed{W_{max} = \frac{V^2}{4 R_r}} \quad \because R_L = R_r \quad \text{.... (6.65)}$$

This is the maximum power received in antenna terminating load impedance Z_L under the condition of max. power transfer and without antenna loss and corresponding effective aperture is known as maximum effective aperture.

∴ Maximum Effective Aperture

$$(A_e)_{max} = \frac{\text{Max. recd. power}}{\text{Power density of incident wave}} = \frac{V^2}{4 R_r P}$$

$$\boxed{(A_e)_{max} = \frac{V^2}{4 PR_r} \qquad \lambda^2 \text{ or } m^2} \quad \text{.... (6.66)}$$

Sometimes max. effective aperture is simply designated as effective aperture and effective aperture as simply aperture following the 'Institute of Radio Engineers' (IRE) standards. Eqn. 6.66 is the expression for the maximum Effective Aperture in terms of induced voltage, power density of incident wave and radiation resistance of antenna.

Further, the ratio of effective area and max. effective area is given a name known as effectiveness ratio and is denoted by α .

Thus $$\alpha = \frac{A_e}{(A_e)_{max}} \quad \text{.... (6.67)}$$

The values of α lies between 0 and 1, the latter value is of that a perfectly matched antenna of cent per cent efficiency.

In practice, however, the terminated load impedance is generally, not at the antenna terminal as shown in Fig. 6.7 but the load is in the receiver which itself is connected with a transmission line. In such case the load impedance Z_L is the equivalent impedance which appears across the terminals of the antenna. Thus the power delivered to receiver and load impedance Z_L is same if transmission line is lossless and less if the T/L is lossy by an amount equal to loss in the line.

6.14.1. ***Scattering loss Apertures.*** Besides effective aperture, there are other apertures also like scattering aperture (A_s) and loss aperture (A_l) corresponding to considerable losses in radiation or re-radiation resistance (R_r) and antenna loss resistance (R_l) respectively and accordingly they are called as scattering and loss apertures. Mathematically,

$$\text{Scattering Aperture} = A_s = \frac{I_{rms}^{\ 2} R_r}{P}$$

or $$A_s = \frac{V^2 R_r}{[(R_L + R_A)^2 + (X_L + X_A)^2] P} \quad \text{.... (6.68)}$$

and $$\text{Loss Aperature} = A_l = I_{rms}^{\ 2} R_L$$

or $$A_l = \frac{V^2 R_l}{[(R_L + R_A)^2 + (X_L + X_A)^2] P} \quad \text{.... (6.69)}$$

If now the condition of maximum transfer of energy is introduced in eqn. 6.68 from eqn. 6.64, we get

$$(A_s)_{max} = \frac{V^2 R_r}{(R_L + R_r)^2 P} = \frac{V^2 R_r}{4 R_r^2 P} \qquad \because R_A = R_r + R_l = R_r \text{ if } R_l = 0 \text{ but } R_L$$

$$\boxed{(A_s)_{max} = \frac{V^2}{4 R_r P}} \quad \text{....(6}$$

Thus under the condition of maximum transfer of energy maximum scattering aperture and maxi effective aperture are same as seen from eqn. 6.66 and 6.70 i.e.

$$\boxed{(A_s)_{max} = (A_e)_{max}} \quad \text{....(6}$$

Further, the ratio of scattering aperture to effective aperture is given a name known as scattering and is denoted by β. Thus

$$\text{Scattering ratio} = \beta = \frac{A_s}{A_e}$$

The values of β lies between 0 to ∞.

6.14.2. *Collecting Aperture*. Out of power collected by antenna, there are losses as heat in resistance (R_l), radiation resistance (R_r) and antenna loss resistance (R_L) and correspondingly the apertures are Effective. Scattering and Loss. By virtue of conservation of energy these three apertures collectively known as **collecting aperture** and is given mathematically as

$$\text{Collecting Aperture} = A_c = A_e + A_s + A_l$$

or

$$A_c = I_{rms}^2 R_L + I_{rms}^2 R_r + I_{rms}^2 R_l$$

$$= \frac{V^2 R_L}{P[(R_L + R_A)^2 + (X_L + X_A)^2} + \frac{V^2 R_r}{P[(R_L + R_A)^2 + (X_L + X_A)^2]} + \frac{V^2 Rl}{P[(R_L + R_A)^2 + (X_L + X_A)^2}$$

$$\boxed{A_c = \frac{V^2 (R_L + R_r + R_l)}{P[(R_L + R_r + R_l)^2 + (X_L + X_A)^2}} \quad \text{... (6}$$

6.14.3. *Physical Aperture*. Still another type of aperture is known as '**Physical Aperture**' which related to the actual physical size (or cross-section) of the antenna and is denoted by the symbol A_p. Thi why physical aperture is of more meaningful for the antennas of larger physical size or cross-section in t of wavelength like horn, parabolic reflector and mattress multi-element etc. *i.e.* for **some particular type antennas only**, while effective aperture is a unique quantity for *any antenna*. Hence physical aperture may defined as '***the physical cross-section perpendicular to the direction of propagation of incident electromagnetic wave with antenna set for maximum response***'.

In larger cross-section (in wavelength) antennas like horn, parabolic reflector, mattress mu element, the physical aperture is greater than effective aperture which for antenna like short dipole, it is l than the effective aperture. For example, the value of effective aperture is approx. 50% of physical apertu for horn antenna, between 50% to 65% for parabolic reflector and about cent per-cent for large matt multi-element type of antenna. In an ideal size when there is no thermal losses and the field is in phase physical aperture A_p and effective aperture A_e are equal and under this condition maximum directivity achieved *i.e.*

$$A_p = A_e \text{ ..... when no losses} \quad \text{.... (6.}$$

But directivity D and effective aperture, A_e are related by the

$$D = \frac{4\pi}{\lambda^2} \cdot A_e$$

$$\boxed{D_{max} = \frac{4\pi}{\lambda^2} \cdot A_p} \quad \text{From eqn. 6.73} \qquad \text{... (6.74)}$$

Further, the ratio of maximum effective aperture to the physical aperture is given the name as absorption ratio and is denoted by γ. Thus

$$\text{Absorption ratio} = \gamma = \frac{(A_e)_{max}}{A_p} \qquad \text{.... (6.75)}$$

The values of γ lies between 0 and ∞ and being ratio has no unit.

6.15. RELATION BETWEEN MAX. APERTURE AND GAIN OR DIRECTIVITY

(AMIEIE Dec. 1978, Nov. 1977)

The radiation pattern is same for transmitting and receiving antennas, by virtue of reciprocity theorem and hence the idea of directivity, which itself is related with shape of the radiation pattern, is extended for receiving antennas also. It is found, in practice, that the directivity of receiving antennas are directly proportional to the maximum effective apertures.

Let there be two antennas A and B whose directivities and maximum effective apertures are denoted by D_a, D_b and $(A_{ea}')_{max}$ and $(A_{eb}')_{max}$ respectively.

$\therefore$
$$D_a \propto (A_{ea})_{max}$$

and
$$D_b \propto (A_{eb})_{max}$$

$\therefore$
$$\frac{D_a}{D_b} = \frac{(A_{ea})_{max}}{(A_{eb})_{max}} \qquad \text{... (6.76)}$$

But from the eqn. 6.51, the gain and directivity w.r.t. isotropic source or antenna is given by

$$G_0 = kD$$

where
G_0 = Gain of a transmitting or receiving antenna.
k = Efficiency factor.
D = Directivity.

If now the losses of efficiency factor k and mismatch are included, the k can be replaced by effectiveness ratio α, *i.e.*

$$G_0 = \alpha D \qquad \text{.... (6.77)}$$

Let us now assume that G_{oa}, α_a, D_a being the gain, effectiveness ratio and directivity of antenna A and G_{ob}, α_b, D_b the corresponding quantities for the antenna B, then from eqn. 6.77.

$$G_{oa} = \alpha_a D_a \text{ .... for Ant. } A$$

and
$$G_{ab} = \alpha_b D_b \text{ ...... ,, ,, } B$$

$\therefore$
$$\frac{G_{oa}}{G_{ob}} = \frac{\alpha_a D_a}{\alpha_b D_b} = \frac{\alpha_a (A_{ea})_{max}}{\alpha_b (A_{eb})_{max}} \quad \text{from eqn. 6.76} \qquad \text{.. (6.78)}$$

But from definition,

$$\alpha_a = \frac{A_{ea}}{(A_{ea})_{max}} \qquad \text{.... 6.79 (a)}$$

or
$$(A_{ea}) = \alpha_a (A_{ea})_{max} \qquad \text{... 6.79 (b)}$$

and similarly
$$(A_{eb}) = \alpha_b (A_{eb})_{max}$$

Hence from eqn. 6.78

$$\boxed{\frac{G_{oa}}{G_{ob}} = \frac{A_{ea}}{A_{eb}}}$$

when A_{ea} and A_{eb} are the effectiveness apertures of antenna A and B. Let us now assume that antenna an isotropic antenna, then its directivity $D_a = 1$. Putting this condition in eqn. 6.76, we get

$$\frac{D_a}{D_b} = \frac{1}{D_b} = \frac{(A_{ea})_{\max}}{(A_{eb})_{\max}}$$

or

$$\boxed{(A_{ea})_{\max} = \frac{(A_{eb})_{\max}}{D_b}} \quad \text{... 6.}$$

This eqn. suggests that if the maximum effective aperture and directivity of antenna (or any ant B are known, then the ratio of the two will give the maximum effective aperture of *an isotropic* ant Further, if the maximum effective aperture of an isotropic antenna is calculated, then the directivity of antenna can be calculated from the following eqn. *i.e.*

$$\boxed{D_b = \frac{(A_{eb})_{\max}}{(A_{ea})_{\max}}} \quad \text{... 6.80}$$

This states that the directivity of any antenna is nothing but the *ratio of its max. effective apertu the max. effective aperture of an isotropic antenna.*

For example, let us take the case of a short dipole antenna whose max. effective aperture Directivity, if calculated, will be given by $\left(\frac{3}{8\pi}\right)\lambda^2$ and $\frac{3}{2}$, then from eqn. 6.80 (a).

$$(A_{ea})_{\max} = \frac{\frac{3\lambda^2}{8\pi}}{3/2} = \frac{\lambda^2}{4\pi}$$

$$(A_{ea})_{\max} = \frac{\lambda^2}{4\pi} \quad \text{... (6.}$$

Putting this in eqn. 6.80 (b), the desired relation is obtained *i.e.*

$$D_b = \frac{(A_{eb})_{\max}}{\frac{\lambda^2}{4\pi}}$$

or

$$D_b = \frac{4\pi}{\lambda^2}(A_{eb})_{\max}$$

or in general

$$\boxed{D = \frac{4\pi}{\lambda^2}(A_e)_{\max}} \quad \text{.... 6.8}$$

or simply

$$\boxed{D = \frac{4\pi}{\lambda^2}A_{em}} \quad \text{... 6.8}$$

This is the relation between directivity and max. *effective aperture* of an antenna.

6.16. EFFECTIVE LENGTH

(M.Sc. Phy. Lko. Univ. 1979, AMIETE Nov. 1977, Dec. 1981, 88, June 82, Ju

The term '**effective length**' of an antenna represents the **effectiveness of an antenna as radia collector of electromagnetic wave energy**. In other words, effective length indicates how for an ant effective in transmitting or receiving the electromagnetic wave energy.

For a receiving antenna, the effective length may be defined in terms of induced voltage v and incident field. *Effective length is nothing but the ratio of induced voltage at the terminal of the receiving antenna under open circuited condition to the incident Electric field intensity (or strength) E.* Thus.

$$\text{Effective length} = \frac{\text{Open circuited voltage}}{\text{Incident field strength (electric)}}$$

or

$$\boxed{l_e = \frac{V}{E} \text{ metre or wavelength}} \quad \ldots (6.83)$$

l_e is in metre (m) or wavelength (λ) according to whether E is in volt/metre or volt/wavelength. Here V is the induced voltage under open circuited condition at the antenna terminals due to an uniform exciting field E.

Since induced voltage v also depends on the effective aperture and hence effective length and effective aperture of an antenna are related to each other as deduced below. From the eqn. 6.63 (a), the effective aperture is given by

$$A_e = \frac{V^2 R_L}{[(R_A + R_L)^2 + (X_A + X_L)^2]\, P}$$

or

$$V^2 = \frac{A_e \cdot [(R_A + R_L)^2 + (X_A + X_l)^2]\, P}{R_L} \qquad \because P = \frac{E^2}{Z}$$

or

$$V = \frac{\sqrt{A_e [(R_A + R_L)^2 + (X_A + X_L)^2]\, E^2}}{\sqrt{ZR_L}}$$

$\therefore$

$$l_e = \frac{V}{E} = \frac{\sqrt{A_e [(R_A + R_L)^2 + (X_A + X_L)^2]}}{\sqrt{ZR_L}} \quad \ldots 6.84\text{ (a)}$$

under conditions for maximum effective aperture, when

$$X_A = -X_L$$

$$R_A = R_r + R_l = R_L$$

$$R_A = R_r = R_L \text{ if } R_l = 0$$

or

$$l_e = \sqrt{\frac{(A_e)_m (2R_r)^2}{\sqrt{ZR_r}}}$$

$$l_e = 2\sqrt{\frac{(A_e)_m R_r}{\sqrt{Z}}} \text{ m or } \lambda \quad \ldots 6.84\text{ (b)}$$

or

$$\boxed{(A_e)_{max} = \frac{l_e^2 Z}{4 R_r}} \quad \ldots 6.84\text{ (c)}$$

This is the relation between maximum effective aperture and effective length [eqn. 6.84 (b, c)].

Now for the transmitting antenna, the effective length is *that length of an equivalent linear antenna that has the same current I (c) (as at the terminals of the actual antenna) at all the point along its length and that radiates the same field intensity E as the actual antenna.* This is illustrated in Fig. 6.8.

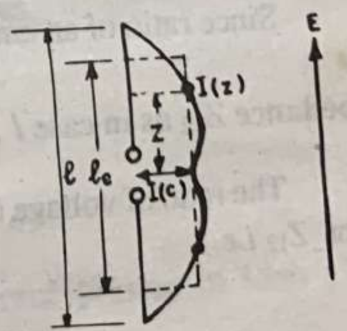


Fig. 6.8. Illustration of effective length for transmitting antenna.

Here
$I(c)$ = Current at the terminals of actual antenna.
$I(z)$ = Current at any point Z of the antenna.
l_e = Effective length.
l = Actual length.

Hence for transmitting antenna,

$$I(c)\, l_{et} = \int_{-l/2}^{+l/2} I(z)\, dz$$

or

$$l_{et} = \frac{1}{I(c)} \int_{-l/2}^{+l/2} I(z)\, dz \quad \text{... 6.8}$$

or

$$\boxed{l_{et} = \frac{2}{I(c)} \int_{0}^{l/2} I(z)\, dz} \quad \text{... 6.8}$$

6.17. RECIPROCITY THEOREM (AMIETE, May 1976, June 1974, 85, 87, 92, Dec. 1979)

The reciprocity theorem is most powerful theorem in circuit and field theories both. The ori theorem is due to Rayleigh Helmholtz which was generalised, to include continuous media, by J.R. Ca That is why it is also known as Rayleigh reciprocity theorem. The reciprocity theorem for antennas is st as follows.

STATEMENT . If an e.m.f. is applied to the terminals of an antenna no. 1 and the curr measured at the terminals of another antenna no. 2, then an equal current both in amplitude and ph will be obtained at the terminals of antenna no. 1 if the same emf is applied to the terminals of ante no. 2.

Or

If a current I_1 at the terminals animals of antenna no. 1 induces an emf E_{21} at the open termi of antenna no. 2 and a current I_2 at the terminals of antenna no. 2 induces an emf E_{12} at the open termi of antenna no. 1, then $E_{12} = E_{21}$ provided $I_1 = I_2$

6.17.1. ***Assumptions.*** It is assumed that

(i) e.m.f.'s are of same frequency.

(ii) Medium between the two antennas are linear, passive and isotropic

(iii) Generator producing emf and the ammeter for measuring the current have zero impedance or if then both the generator and the ammeter impedances are equal.

6.17.2. ***Explanation*** (Fig. 6.9). Let

(1) A transmitter of frequency f and zero impedance be connected to the terminals of antenna no. 2, wh is generating a current I_2 and inducing an emf E_{12} at the open terminals of antenna no. 1 Fig. 6.9

(2) Now the same transmitter is transferred to antenna no. 1 which is generating a current I_1 and induci a voltage E_{21} at the open terminals of antenna no. 2. Fig. 6.9 (d).

Thus according to the statement of reciprocity theorem

$$I_1 = I_2 \text{ provided } E_{12} = E_{21}$$

Since ratio of an emf to current is an impedance, therefore, the ratio $\frac{E_{12}}{I_2}$ is given the name. **Tran Impedance** Z_{12} as in case I , and so also the ratio $\frac{E_{21}}{I_1}$ as **Transfer Impednace** Z_{21} as in II case.

The ratio of voltage (E_1) of one cct to the current I_2 in the second cct is defined as transfer imped Z_T or Z_{12} *i.e.*

$$Z_T = Z_{12} = \frac{E_1}{I_2}$$

Thus from the reciprocity it follows that the two ratios *i.e.* two impedances are equal *i.e.*

$$Z_{12} = Z_{21}$$

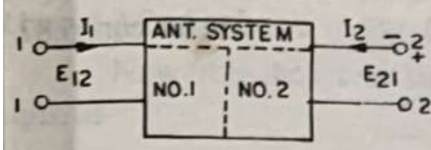


Fig. 6.9. (a)

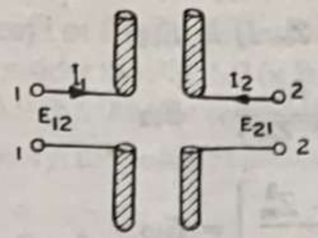
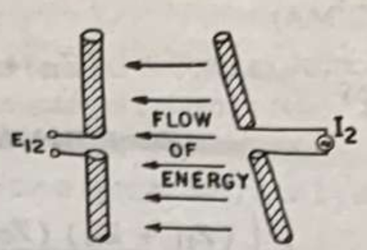


Fig. 6.9. (b)



Ant. No. 1 *Medium* *Ant. No. 2*

Fig. 6.9. (c) current I_2 inducing an emf E_{12} in antenna no. 1.

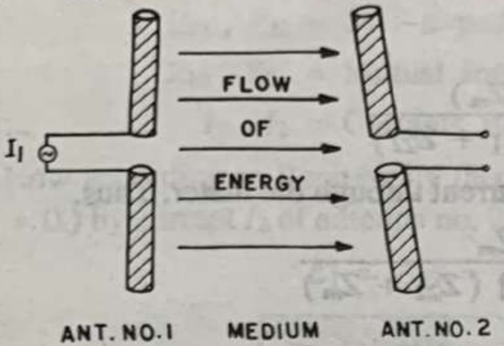


Fig. 6.9. (d) Current I_1 inducing an emf E_{21} in antenna no. 2.

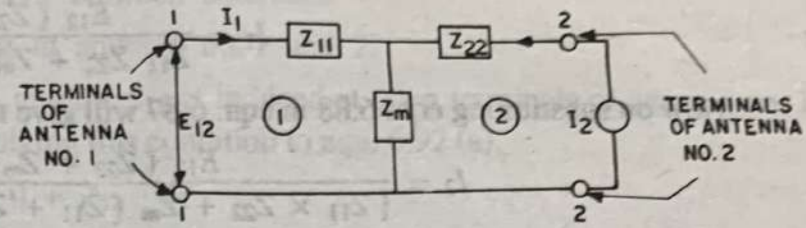


Fig. 6.9 (e). Equivalent—T-network corresponding to 4 terminal network of Fig. 6.9 (c).

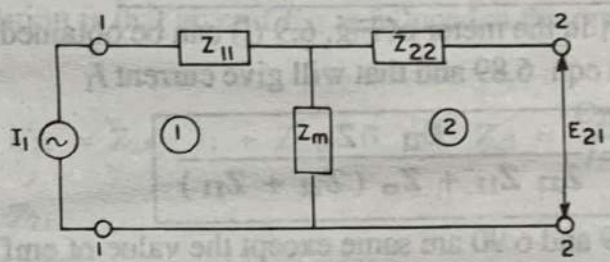


Fig. 6.9 (f). Equivalent—T-network corresponding to 4-terminal network of Fig. 6.9 (d).

This, of course, is nothing but *mutual impedance* (Z_m) between the two antennas. Therefore,

$$\boxed{Z_m = Z_{12} = Z_{21} = \frac{E_{12}}{I_2} = \frac{E_{21}}{I_1}} \quad \text{... 6.86 (a)}$$

or

$$\boxed{\frac{E_{12}}{I_2} = \frac{E_{21}}{I_1}} \quad \text{... 6.86 (b)}$$

6.17.3. *Proof.* To prove the Reciprocity theorem for antennas, the space between the antenna no.1 and antenna no. 2 are replaced by a network of linear, passive and bilateral impedances as shown Fig. 6.9 (e) and 6.9 (f) corresponding to Fig. 6.9 (c) and 6.9 (d) respectively. Because any four terminal network { e.g. Fig. 6.9 (c, d)} can be converted into an equivalent *T*-network {*e.g.* Fig. 6.9 (e.f.)}.

Z_{11}, Z_{22} = Self–impedance antenna no. 1 and 2 respectively.

Z_m = Mutual Impedance between two antennas.

1, 1 = Terminals of antenna no. 1

2, 2 = Terminals of antenna no. 2

Now applying Kirchoff's mesh law to Fig. 6.9 (e).

From loop (2)

$$(Z_{22} + Z_m)\, I_2 - Z_m I_1 = 0 \qquad [\because \text{No voltage source present in loop (2)}]$$

$$\boxed{I_2 = I_1 \frac{Z_m}{(Z_{22} + Z_m)}} \quad \text{...(6.87)}$$

From mesh (1)

or
$$(Z_{11} + Z_m) I_1 - Z_m I_2 = E_{12}$$

or
$$(Z_{11} + Z_m) I_1 - \frac{Z_m^2 I_1}{(Z_{22} + Z_m)} = E_{12}$$

or
$$I_1 \left[\frac{(Z_{11} + Z_m)(Z_{22} + Z_m) - Z_m^2}{(Z_{22} + Z_m)} \right] = E_{12}$$

or
$$I_1 \left[\frac{Z_{11} Z_{22} + Z_{11} Z_m + Z_{22} Z_m + Z_m^2 - Z_m^2}{(Z_{22} + Z_m)} \right] = E_{12}$$

or
$$I_1 = \frac{E_{12} (Z_{22} + Z_m)}{Z_{11} Z_{22} + Z_m (Z_{11} + Z_{22})}$$

Now on substituting eqn. 6.88 in eqn. 6.87 will give the current through the meter. Thus.

$$I_2 = \frac{E_{12} (Z_{22} + Z_m) \cdot Z_m}{[Z_{11} \times Z_{22} + Z_m (Z_{11} + Z_{22})] (Z_{22} + Z_m)}$$

$$\boxed{I_2 = \frac{E_{12} \cdot Z_m}{Z_{11} \times Z_{22} + Z_m (Z_{11} + Z_{22})}} \quad \text{...(6}$$

Similarly, the current I_1 in the meter of Fig. 6.9 (f) can be obtained or by symmetry. Suffix 2 m replaced by 1 and *vice-versa* in eqn. 6.89 and that will give current I_1

$\therefore$
$$\boxed{I_1 = \frac{E_{21} \; Z_m}{Z_{22} \; Z_{11} + Z_m (Z_{22} + Z_{11})}} \quad \text{...(6}$$

Thus, clearly eqns. 6.89 and 6.90 are same except the value of emf (*i.e.* E_{12} and E_{21}). Accordi theorem statement, the theorem is proved if we prove

$$E_{12} = E_{21} \text{ if } I_1 = I_2$$

$\therefore$ Applying the condition

$$I_1 = I_2$$

$$\frac{E_{12} \; Z_m}{[Z_{11} \; Z_{22} + Z_m] (Z_{11} + Z_{12})} = \frac{E_{21} \; Z_m}{[Z_{11} \; Z_{22} + Z_m] (Z_{11} + Z_{22})}$$

or
$$\boxed{E_{12} = E_{21}} \quad \text{..(6}$$

And hence the thorem is proved.

From the reciprocity theorem, it can be proved that the directivity pattern, aperture and termi impedance of an antenna are same whether it is transmitting antenna or receiving antenna. However, **current distribution in receiving antenna is not the same as in transmitting antenna. Further Recipro theorem is equally applicable to two separate the antennas and also for two point of the same anten**

6.17.4. *Limitations :*

(i) Although tthe Rayleigh-Carson theorem is applicable to radio communication but it fails to be te only when the propagation of the radio wave is appreciably effected by the presence of the Ear magnetic field.

(ii) It holds good for all practical radio work but for long distance communication through ionosphe However, still it is expected to apply results averaged over a reasonable interval of time in which c **it can not be expected** to be exactly correct at every given time.

6.17.5. Voltages and Currents Relations

(AMIETE, May 1971)

The circuit theory concept of Rayleigh-Carson theory can be used directly to deduce few terminals properties of antennas. Let us consider the Fig. 6.9 (a, b) in which an antenna system has two "*terminal pairs*" 1-1 and 2-2. As shown in Fig. 6.9 (b), the two terminals pairs are of two dipole antennas.

Now from the circuit theory, the voltages and currents, for linear system, are given by the simultaneous eqns. as

$$E_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{... 6.92 (a)}$$

$$E_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{... 6.92 (b)}$$

where Z_{11}, Z_{22} = Self–impedances of antennas or terminals nos. 11 and 22

Z_{12}, Z_{21} = Mutual Impedances between antennas.

I_1, I_2 = Currents flowing in antenna no. 1 and 2.

Now according to Reciprocity theorem, an emf E_{12} is induced at open terminals of antenna no. 1 (*i.e.* $I_1 = 0$) by current I_2 of antenna no. 2. Applying this condition to eqn. 6.92 (a),

$$E_1 = E_{12} = Z_{11} \cdot 0 + Z_{12} \cdot I_2$$

$$\boxed{Z_{12} = \frac{E_{12}}{I_2}} \quad \text{... 6.93 (a)}$$

Similarly, the second condition is that an emf E_{21} is induced at the open terminals of antenna no. 2 (*i.e.*, $I_2 = 0$) by the current I_1.

$$E_2 \equiv E_{21} = Z_{21} \cdot I_1 + Z_{22} \cdot 0 \quad \text{or} \quad Z_{21} = \frac{E_{21}}{I_1} \quad \text{... 6.93 (b)}$$

But

$$Z_{12} = Z_{21}$$

or

$$\boxed{\frac{E_{12}}{I_2} = \frac{E_{21}}{I_1}} \quad \text{... 6.86 (b)}$$

Thus, currents and voltages at the terminals of two antennas satisfy the reciprocity theorem.

6.18. APPLICATION OF RECIPROCITY THEOREM

The reciprocity theorem may be used to derive the following very important properties of transmitting and receiving antennas.

(1) Equality of directional Patterns. (2) Equality of Directivities.

(3) Equality of Effective Lengths. (4) Equality of Antenna Impedances.

Now these will be taken one by one.

6.18.1. Equality of Directional Patterns

(AMIETE, Dec. 1979)

The directional patterns of transmitting and receiving antennas are identical if all the media are linear, passive, isotropic and the reciprocity theorem holds good.

Proof. Under the mentioned conditions it is to be proved that the transmitting and receiving antennas patterns are identical. For this consider the Fig. 6.10 in which two antennas no. 1 (test antenna) and no. 2 (exploring antenna) are shown. Let antenna no. 1 is transmitting and the antenna no. 2 is receiving.

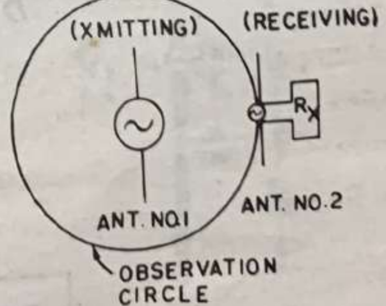


Fig. 6.10. Measurement of Pattern on observation Circle.

The pattern may be either field pattern or power pattern which itself is proportional to square of field pattern.

Considering field pattern, keeping the transmitting antenna no. 1 (*i.e.* test antenna) at the centre of the observation circle, the receiving is linear).

Now, if a voltage (E) is applied at transmitting antenna No. 1 and the resulting current (I) at terminals of receiving antenna No. 2 is measured which will be the indication of *electric field* at the l of antenna no. 2. If the process is reversed *i.e.* the same voltage (E) is applied to antenna no. 2 (which tra and resulting current I is measured at the test antenna no. 1 (which receives). This time the receiving of test antenna no. 2 is obtained while previously to that of antenna no. 1.

Accordingly to reciprocity theorem, for every position of test antenna no. 1 the ratio E/I is the as was in the previous case. **Thus, it is proved that radiation pattern of test antenna no. 1 (transmi observed by moving receiving antenna no. 2 is identical with the radiation pattern obtained wh antenna no. 2 is transmitting and antenna no. 1 is receiving i.e. when the process is reversed.**

A variation of the method may be that instead of reversing the measurement procedure, the re current in exploring or probe antenna no. 2 (receiving) is measured at new location by changing the rad observation circle. It would be found that whatever may be the position of antenna no. 2 the ratio E/I is a same and hence the proof of statement.

Note. In case the radiation is elliptically polarized, then it will contain both the components of e vector (*e.g.* E_θ and E_φ) which are out of phase. In that case, radiation pattern for each polarization (polarization and φ polarization) is plotted separately by keeping the exploring or receiving antenna par that polarization.

6.18.2. *Equality of Directivities.* The directivity D is defined as

$$\text{Directivity} = \frac{\text{Maximum Radiation Intensity}}{\text{Average Radiation Intensity}}$$

or in symbols,

$$D = \frac{\Phi(\theta, \varphi)_{max}}{\Phi_{av}} = \frac{\Phi_m}{\Phi_{av}}$$

But average

$$\text{R.I.} = \frac{\text{Total power radiated, in Watts}}{4\pi \text{ in steradian}}$$

$$\Phi_{av} = \frac{W}{4\pi} \quad \text{W/Sr} \qquad \text{...6}$$

Substituting 6.92 (a) in Eqn. 6.92, we have

$$D = \frac{\Phi(\theta \cdot \varphi)_{max}}{W/4\pi} = \frac{4\pi\,\Phi(\theta, \varphi)_{max}}{W} \qquad \text{...6}$$

But the total power radiated W is given by radiation intensity $\Phi(\theta, \varphi)$ integrated over solid 4 π steradian, *i.e.*

$$W = \iint_{4\pi} \Phi(\theta, \varphi)\, d\Omega \qquad \text{...6}$$

$$\therefore \quad D = \frac{4\pi\,\Phi(\theta, \varphi)_{max}}{\iint_{4\pi} \Phi(\theta, \varphi)\, d\Omega} \equiv \frac{4\pi\,\Phi_m}{\iint_{4\pi} \Phi\, d\Omega}$$

$$= \frac{4\pi}{\iint_{4\pi} \left(\frac{\Phi}{\Phi_m}\right) d\Omega}$$

where $d\Omega$ = Solid angle = $\sin\theta\, d\theta$ ...

$$\boxed{D = \frac{4\pi}{\iint \Phi_n\, d\Omega}}$$

where $$\Phi_n \equiv \Phi_n(\theta, \varphi) = \frac{\Phi(\theta, \varphi)}{\Phi(\theta, \varphi)_{max}} = \frac{\Phi}{\Phi_m}$$

= Normalized power pattern ... (6.96)

Since radiation intensity is a function of φ and θ and can be represented as

$$\Phi = \Phi_m f(\theta, \varphi) \quad \text{... (6.97)}$$

or $$\frac{\Phi}{\Phi_m} = f(\theta, \varphi)$$

$$\frac{\Phi}{\Phi_m} = f_n(\theta, \varphi) = \text{Normalized three dimensional power pattern}$$

Thus eqn. 6.95 can be written as

$$\boxed{D = \frac{4\pi}{\iint f(\theta, \varphi)\, d\Omega}} \quad \text{... (6.98)}$$

Thus, from Eqns. 6.97 and 6.98 it is clear that the directivity depends on the shape of the power pattern only.

As seen above, the radiation pattern of an antenna is same whether transmitting or receiving. Therefore, directivities will be same, whether it is calculated from antenna's transmitting pattern or receiving pattern. Hence the term directivity can be applied to both transmitting and receiving antennas, if the value of directivity is same in both the cases.

(AMIETE, May 1977)

6.18.3. *Equality of Effective Lengths*

The maximum effective aperture (or effective aperture simply) of an antenna is given by

$$(A_e)_{max} = \left(\frac{\lambda}{4\pi}\right) D$$

where λ = wavelength

D = Directivity

and hence the term maximum effective aperture can be applied to transmitting antenna and receiving antenna as well.

The value of maximum effective aperture is same for an antenna whether it is transmitting or receiving as illustrated separately below.

For Transmitting Antenna. Effective length for a transmitting antenna is defined as

(i) That length of an equivalent linear antenna that has a current $I(c)$ at all points along its length and that radiates the same electric field strength as the actual antenna in the direction perpendicular to its length, or

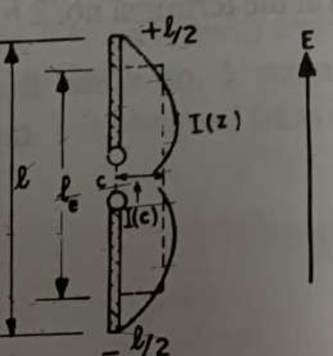
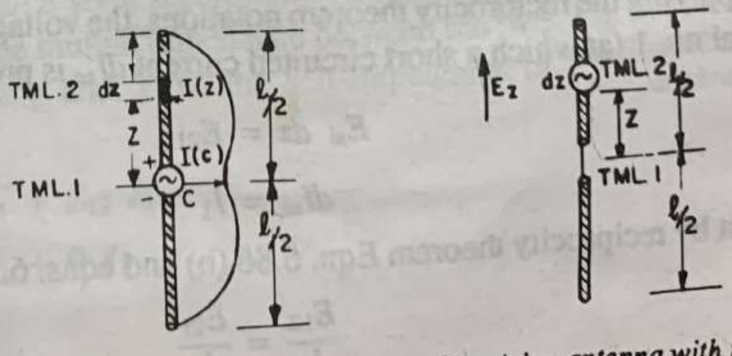


Fig. 6.11.



(a) Transmitting antenna with input at tml no. 2.

(b) Receiving antenna with incident electromagnetic wave of value Ezi at tml. no. 2.

Fig. 6.12.

(ii) The moment of transmitting antenna current distribution divided by the input current, wh moment of the current distribution is defined as the sum of the moments of its current elemen

Symbolically for transmitting antenna,

$$I(c)\ let = \int_{-l/2}^{+l/2} I(z)\, dz$$

or

$$let = \frac{1}{I(c)} \int_{-l/2}^{+l/2} I(z)\, dz \quad ...6$$

where let = Effective length of transmitting antenna.

l = Actual length of antenna.

If an emf E is applied at the centre point C, then currents $I(c)$ at centre point C, and $I(z)$ point (say D in this case) will be produced along the antenna. The value of current at the centre point C i by

$$\boxed{I(c) = \frac{E}{Z_t}} \quad ...$$

where E = Applied voltage at tml. no. 1

Z_t = Antenna terminal impedance.

Applying the reciprocity theorem, if E_{12} voltage is applied at *tml*. no. 1 by shorting the *tml*. no which current I_2 is produced), then

$$E = E_{12}$$

and

$$I(z) = I_2 \quad ...(6$$

must be obtained as to the same antenna terminals as the two voltages E and E_{12} are applied under si condition.

For Receiving Antenna. The effective length of a receiving antenna may be defined as the ra open circuit voltage developed at the terminals of antenna and the received field strength *i.e.*

$$l_{er} = \frac{\text{Open circuit voltage } (V_0)}{\text{Electric field strength } (E)}$$

or

$$\boxed{V_0 = l_{er}\ E}$$

Now considering the same transmitting antenna for receiving case also but with terminal no. 1 is circuited. Let an electromagnetic field of strength E_{zi} is incident on the antenna which causes a vo $E_{zi}\ dz$ to induce in the element dz (*i.e.* tml. no. 2) situated at a distance Z from the centre. This induced vo $E_{zi}\ dz$ can be represented as zero-impedance generator of voltage $E_{zi}\ dz$ in series at point Z, becaus incident or induced voltage $E_{zi}\ dz$ is not dependent on antenna current.

Applying the reciprocity theorem notations, the voltage E_{21} is applied at the terminal no. 2 by sho the terminal no. 1 (at which a short circuited current dI_{sc} is produced), then

$$E_{zi}\ dz = E_{21}$$

and

$$dI_{sc} = I_1 \quad ...(6$$

But by reciprocity theorem Eqn. 6.86 (b) and eqns. 6.99 and 6.101

$$\frac{E_{12}}{I_2} = \frac{E_{21}}{I_1}$$

$$\therefore \quad \frac{E}{I(z)} = \frac{Ezi\ dz}{dI_{sc}}$$

$$\boxed{dI_{sc} = \frac{E_{zi}\ dz\ I(z)}{E}} \qquad \text{... (6.102)}$$

According to superposition theorem, total short circuit current produced at the terminals of receiving ntenna is obtained by integrating eqn. 6.102 along the entire length of antenna.

$$\int dI_{sc} = \int_{-l/2}^{l/2} \frac{E_{zi}\ dz\ I(z)}{E}$$

$$I_{sc} = \frac{1}{E}\int_{-l/2}^{l/2} E_{zi}\ dz\ I(z) \qquad \text{... (6.103)}$$

By Thevenin's theorem, open circuit voltage is given by

$$V_0 = I_{sc}\ Z_t = \left\{\frac{1}{E}\int_{-l/2}^{l/2} E_{zi}\ I(z)\ dz\right\} Z_t$$

$$= \frac{Z_t}{E}\int_{-l/2}^{-l/2} E_{zi}\ I(z)\ dz = \frac{1}{I(c)}\int_{-l/2}^{l/2} E_{zi}\ I(z)\ dz \qquad \text{from eqn. 6.99}$$

For a constant incident field along the entire length of antenna,

$$E_{zi} = E_z \text{ (say)}$$

$$V_0 = \frac{E_z}{I(c)}\int_{-l/2}^{l/2} I(z)\ dz$$

$$\frac{V_0}{E_z} = \frac{\int_{-l/2}^{l/2} I(z)\ dz}{I(c)} \qquad \text{From eqn. 6.85 (a) and 6.83}$$

$$\boxed{l_{er} = l_{et}} \qquad \text{... (6.104)}$$

This proves that maximum effective aperture of an antenna is same whether transmitting or eceiving.

6.18.4. *Equality of Antenna Impedance.* It is to be proved here that impedance of antenna, away from ground and other objects is same whether it is ransmitting or receiving. It may be noted, however, that while transmitting, one point of antenna length is excited whereas during reception the entire length of the antenna is excited. Hence the current distribution during transmission and reception are usually not the same. Further, the manner in which the receiving antenna is excited depends on the direction of the incoming wave. Since the antenna is the same circuit irrespective of mode of excitation and hence the impedance of an antenna remains the same whether transmitting or receiving.

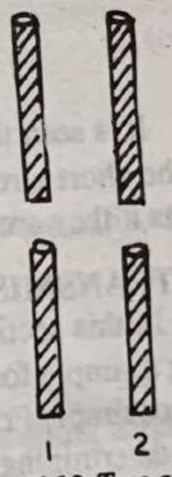


Fig. 6.13. Two antennas no. 1 and 2 with a wide separation.

To prove, let there be two antennas with a wide separation in between (Fig. 6.13). If no. 2 is quite away from no. 1 so that the mutual impedance between the two be neglected while no. 1 antenna is transmitting and, thus, the self-impedance of Antenna no. 1 is obtained from eqn. 6.92.

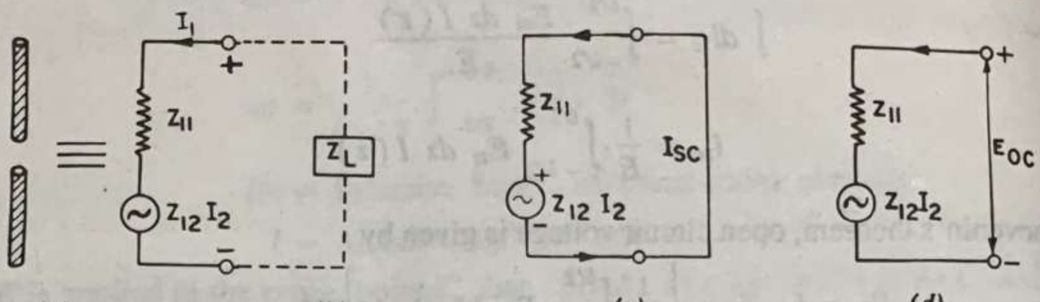
$$E_1 = Z_{11}\ I_1 + Z_{12}\ I_2 \qquad \text{... (6.92}$$

$$= Z_{11}\ I_1 + 0\ I_2 \qquad \because Z_{12} =$$

$$Z_{11} = \frac{E_1}{I_1}$$

$$= \text{Self–impedance of antenna no. 1}$$

But this assumption is not true when antenna no. 1 is receiving because during reception it is the impedance (Z_{12}) between the two antennas which provides coupling. If an equivalent antenna no. 1 is under load (Z_L) with the help of eqn. 6.92, then a little consideration would show that Z_{12} I_2 acts as generator (source) is illustrated in Fig. 6.14.



(a) (b) (c) (d)

Fig. 6.14. (a) Receiving antenna no. 1. (b) Receiving antenna no. 1 under loaded condition equivalent cct thereof. (c) Receiving antenna no. 1 under open circuited condition. (d) Receiving antenna no. under short circuited condition

If now assume that the Antenna no. 2 is quite away from no. 1 so that change in Z_L does not c any change in current I_2 of voltage source Z_{12} I_2, then the voltage source Z_{12} I_2 of Fig. 6.14 (b) acts as an **zero impedance, contant voltage generator.** The equivalent circuit of receiving antenna under open c and short circuited conditions are also shown in Fig. 6.14 (c) and 6.14 (d). In such condition, rece **impedance and transmitting impedance are equal as antenna no. 1 have the terminal behaviou voltage generator with internal impedance Z_{11}.**

$$E = Z_{11} I_1 + Z_{12} I_2$$

∴ $$E_{OC} = Z_{11}\, O + Z_{12} I_2$$ ∴ under open acted condition $I_1 = 0$ ...(6

or $$\boxed{E_{OC} = Z_{12} I_2}$$

$$E = Z_{11} I_1 + Z_{12} I_2$$

$$O = Z_{11} I_{sc} + Z_{12} I_2$$ ∴ under short circuited condition output vo $E = 0$ (and I_1 =

or $$\boxed{I_{sc} = \frac{-Z_{12} I_2}{Z_{11}}}$$ ...(6

It is seen that (Z_{12} I_2) is a voltage source and Z_{11} is the internal impedance and the ratio of the gives the short circuit current. Further, this is not only true for one distant antenna but for any numb antennas if they are away from the antenna whose impedance is being considered.

6.19. TRANSMISSION BETWEEN TWO ANTENNAS

In this section, the utility of aperture concept will be shown in deriving a simple formula popularly known as FRIIS TRANSMISSION formula or simply Friis formula, after the name of H.T. Friis. This formula helps in determining the transmission loss between two antennas in free space. With reference to Fig. 6.15.

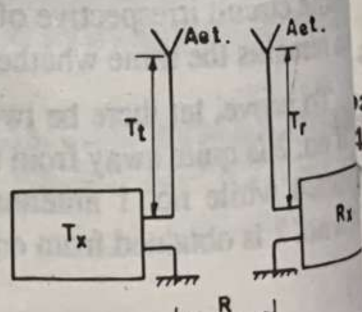


Fig. 6.15. Transmission reception of isotropic transmitting antenna and receiving antenna.

Let R = Distance between transmitting and receiving antennas.

A_{et}, A_{er} = Effective apertures of Transmitting and receiving antennas.

W_t = Total power radiated by isotropic antenna, in Watt.

W_r = Power received at antenna terminal, in Watt.

W_r' = Power received at receiver, in Watt.

λ = wavelength, in metre.

W_t = Transmitted power, in Watts.

W_r = Received power, in Watts.

G_t = Antenna gain.

A_e = Antenna effective aperture · in m^2.

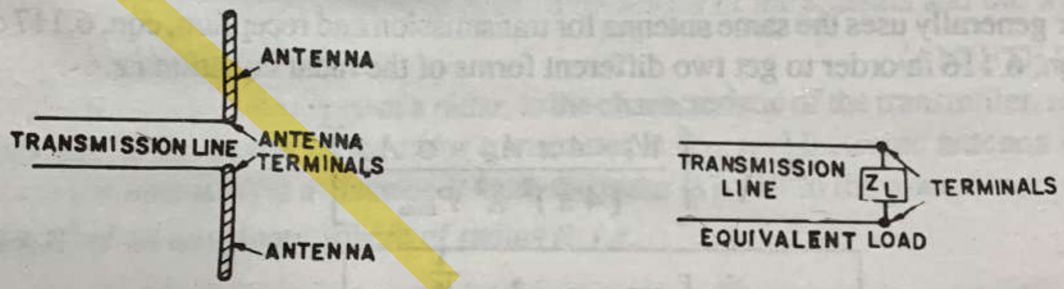
σ = Radar Cross–section, in m^2.

P_{min} = Minimum detectable signal, in Watts.

6.21. SELF IMPEDANCE (AMIETE, May

The important aspect of antenna impedance is the impedance at the point where the transmissio carrying R.F. power from the transmitter is connected. Since at this point input to the antenna is supp therefore, it is called as **Antenna Input Impedance** or since at this impedance the R.F. power fro transmitter is fed so this is also known as *feed point impedance.* Also, at this impedance the transmission operates, so this is known as *Driving point impedance or Terminal impedance. Thus, antenna input or point or driving point or terminal impedance is of considerable importance because it is generally desi supply maximum available power from transmitter to the antenna or to extract maximum amount of rec energy from the antenna.*

The impedance offered by antenna to the transmission line can be represented by a two term network. Next, the entire antenna system can be replaced by an equivalent impedance Z_L as in Fig. 6.16 (



(a) Transmission line with antenna as load. *(b) Transmission line with equivalent load Z_L.*

Fig. 6.16.

Now, if the antenna is *lossless* (*i.e.* without any heat loss) and *isolated* (*i.e.* away from ground other objects), then the *antenna terminal impedance* (Z_L) is same as the self-impedance (Z_{11}) of the ante Mathematically in the said conditions, *self-impedance* of an antenna can be represented by

$$\boxed{Z_{11} = R_{11} + j\,X_{11}} \quad \text{... (6.}$$

where Z_{11} = Self–impedance.

R_{11} = Self–resistance or radiation resistance.

X_{11} = Self–reactance.

For a thin linear half wave centre fed antenna, the self impedance is calculated to be

$$\boxed{Z_{11} = R_{11} + j\,X_{11} = 73 + j\,42.45\ \Omega} \quad \text{.... 6.1}$$

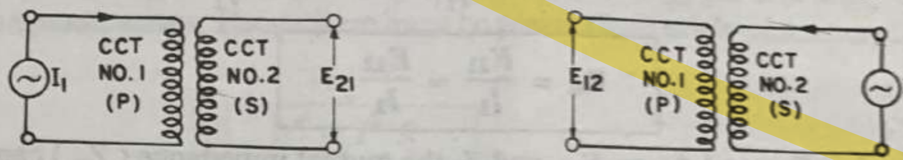
Thus for a lossless and isolated antenna the self-impedance (Z_{11}) and terminal impedance (Z_L equal and as seen in Eqn. 6.119, the self-impedance (Z_{11}) is a complex quantity, the real part (R_{11}) of is given the name **self-resistance or radiation resistance** and the imaginary part (X_{11}) the **self-reacta**

However, if the antenna is in nearby of other objects like active antennas etc., then the ter impedance can still be replaced by a two terminal network but now the terminal impedance gets mo Because besides self-impedance, the mutual impedance too comes in existence due to the currents flow neighbouring active antennas. This is why the *self-impedance of an antenna is defined as its input impe when all other antennas are completely removed i.e. away from it.*

Lastly, it may however be noted that self-impedance of an antenna is always positive and its value is same for both transmission and reception. Further, since the self-impedance Z_{11} contains a reactive term (X_{11}) hence, in order to make the antenna resonant (*i.e.* $X_{11} = 0$) the antenna length can be shortened a bit. By doing this, there is a slight reduction in radiation resistance or self-resistance. For example, for half-wavelength when $X_{11} = 0$, $R_{11} = 70\ \Omega$ instead of $73\ \Omega$.

6.22. MUTUAL IMPEDANCE

In circuit theory, the mutual impedance is defined as the *negative ratio of the voltage induced in one circuit (by a current flowing in the second circuit), to the current in the second circuit, with all the circuits open circuited excepting the second circuit.* For, consider the coupled cct shown in Fig. 6.17.



(a) Closed primary Open secondary. *(b) Closed Secondary Open primary.*

Fig. 6.17.

Thus by definition, mutual impedance is the negative ratio of voltage induced in open secondary (cct no. 2) E_{21} to the current (I_1) flowing in the closed primary, cct no. 1 [Fig. 6.17 (a)] or negative ratio of voltage induced in the open primary (cct no.1). E_{12} to the current (I_2) flowing in the closed secondary (or cct no. 2) as illustrated in Fig. 6.17 (b) *i.e.*

$$Z_{21} = -\frac{E_{21}}{I_1} \quad \text{... 6.120 (a)}$$

and

$$Z_{12} = -\frac{E_{12}}{I_2} \quad \text{... 6.120 (b)}$$

where E_{21} = Open cct voltage across cct no. 2 due to cct I_1 in the closed cct no. 1

and E_{12} = Open cct voltage across cct no. 1 due to closed current I_2 in the closed cct no. 2.

It may, however be noted that **mutual impedance** is not the same as the **transfer impedacne** as used in proof of reciprocity theorem. In fact, transfer impedance is defined as *the ratio of voltage impressed (and not induced) in one cct to the current produced in another cct with all ccts closed.* For, if E is the *voltage impressed* in one cct and current produced in another closed cct is I, then the ratio is transfer impedance (Fig. 6.18).

Then

$$Z_T \text{ or } Z_{T_{12}} = \frac{E_1}{I_2} \quad \text{... 6.121 (a)}$$

or

$$Z_T \text{ or } Z_{T_{21}} = \frac{E_2}{I_1} \quad \text{... 6.121 (b)}$$

It is reciprocity theorem which lead to the definition of transfer impedance and clearly it is not the same as the mutual impedance eqn. 6.121.

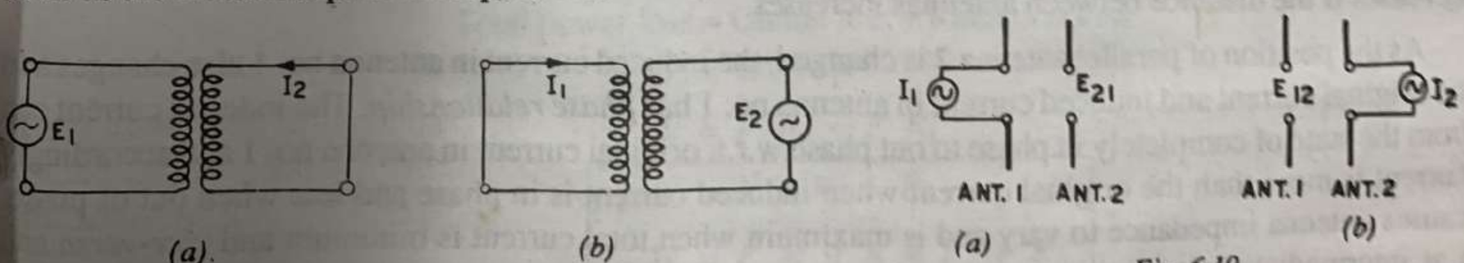


Fig. 6.18. *Fig. 6.19.*

Let us now consider, two coupled antennas, instead of two coupled ccts, as illustrated in Fig. 6.19, where two antennas (no. 1 and no. 2) are separated by a fraction of wavelength and are parallel to each other. Let a current I_1 in the antenna no. 1 induces a voltage E_{21} at the open terminals of antenna no. 2, then the ratio of the two is mutual impedance Z_{21} *i.e.*,

$$Z_{21} = -\frac{E_{21}}{I_1}$$

If the generator is shifted to antenna no. 2 and current I_2 of antenna no. 2 is inducing a voltage E_{12} ... 6.12 the open terminals of antenna no. 1 [Fig. 6.19 (b)], then mutual impedance

$$Z_{12} = -\frac{E_{12}}{I_1} \quad \text{... 6.12}$$

But by reciprocity theorem, the two mutual impedances are equal

$$\therefore \quad Z_{21} = -\frac{E_{21}}{I_1} = Z_{12} = -\frac{E_{12}}{I_2} \equiv Z_m$$

or

$$\boxed{Z_m = \frac{E_{21}}{I_1} = \frac{E_{12}}{I_2}} \quad \text{... 6.12}$$

Thus by knowing E_{21} and I_1 or E_{12} and I_2 the mutual impedance (Z_m) can be calculated and is done analytically.

Thus, like two coupled circuits two or more antennas can also be coupled and when two or m antennas are coupled in a particular arrangement, they are said to form an *antenna array*. In an antenna ar *the input or driving point impedance* not only depends on the self-impedance of an individual antenna but on the mutual impedance between the antennas. In other words, actual impedance of an antenna element is sum of **its self-impedance** — the impedance when all other antennas removed — and its **mutual impeda** with all antennas present in the nearby.

Assuming two antennas (no. 1 and 2) placed side by side in parallel [Fig. 6.19 (a)], when power applied say to no. 1 and current flows in it, a voltage is induced in the nearby antenna no. 2 by electromagnetic field and current starts flowing in this also. The current of no. 2, in turn, will induce a volt in no. 1 causing a current to flow in it. Therefore, the total current in no. 1 is now the vectorial sum of original current and induced current. *The fact that the presence of nearby antenna no. 2 induces a curren antenna no. 1 indicates that presence of antenna no. 2 changes the impedance of antenna no. 1. This effe called mutual coupling and results in mutual impedance*. It is called so because it appears from the *mu effect i.e.* the effect appears in nearby antenna no. 2 due to current flowing in antenna no. 1.

The mutual impedance depends on the

(i) Magnitude of induced current

(ii) Phase relationships between induced and original currents and

(iii) Tuning conditions of second antenna (or nearby antennas).

The value of induced current is largest when the two antennas are parallel and closed to each ot and it decreases as the separation between the two antennas increases. In other words, coupling between t antennas (and so also the mutual impedance) depends on the position of antenna no. 2 (or nearby) antenna and decreases if the distance between antennas increases.

As the position of parallel antenna 2 is changed, the induced current in antenna no. 1 also changes a thus the original current and induced current in antenna no. 1 has *phase relationship*. The induced current c vary from the state of completely in phase to out phase w.r.t. original current in antenna no. 1 and accordin total current is more than the original current when induced current is in phase and less when out of phas This causes antenna impedance to vary and is maximum when total current is minimum and *vice-versa* a hence at intermediate relationships antenna impedance varies from maximum to minimum. *Concludin variation in induced current causes phase of the total current to vary w.r.t. applied voltage. This means t mutual impedance is a complex quantity having resistive and reactive components.*

Hence the presence of antenna no. 2 nearby may cause the impedance of antenna no. 1. to be reacti even if its self-impedance is resistive. *In other words, a antenna tuned at resonance may be detuned by placi* *a second antenna in nearby and the amount of detuning depends on the magnitude and phase of the induced current.*

Further, the *impedance of antenna also depends on the tuning conditions of the nearby antenna.* A detuned nearby antenna may induced additional phase (lead or lag) in the first antenna.

Lastly the *mutual impedance also affects the gain of an antenna as it determines the amount of current which will flow for a given amount of power supplied.*

6.23. RADIATION RESISTANCE

[M.Sc. Phy. (Part-II) 1979, Lko-Univ., AMIETE, Nov. 1969, 70, 71, 74, 77, 92, 93, Dec. 1972, 73, 88]

The antenna is a radiating device, in which the power (*i.e.* energy per unit time) is radiated into space in the form of electromagnetic waves. Hence there must be power dissipation which may be expressed in usual manner as

$$\boxed{W' = I^2 R} \quad \text{... 6.123 (a)}$$

If it is assumed that all this power appears as electromagnetic (or radio) waves, then the power (W') can be divided by square of the current (I^2) *i.e.*

$$\boxed{R_r = \frac{W'}{I^2}} \quad \text{... 6.123 (b)}$$

at the point where it is fed to the antenna and obtain *a fictitous resistance* called as *radiation resistance. It is normally denoted by* R_r or R_a or R_0. The radiation resistance represents a relation between total energy radiated from a transmitting antenna and the current flowing in the antenna (Eqn. 6.123). The radiation resistance (R_r) is thus defined as *that fictitious resistance which, when substituted in series with the antenna, will consume the same power as is actually radiated.*

As a matter of fact, the energy supplied to an antenna is dissipated

(a) In, the form of electromagnetic waves, and

(b) As, ohmic losses in the antenna wire and nearby dielectrics *e.g.* insulators, ground and other surrounding objects.

Though the radiation from an antenna is a desirable and useful phenomena but still as far as antenna is concerned, it represents nothing but a loss just as the energy used in heating the wire of the antenna (*i.e.* ohmic loss) is a loss. Further, the dissipated power in either case is given by $I^2 R$. In case of ohmic losses which appears as heat in the antenna wire, R is the actual ohmic resistance of the antenna wire (say R_l) (hence the power dissipation is $I^2 R_l$) but in case of radiated energy, R is assumed or fictitious resistance, which, if present, would dissipate the power that is actually radiated (*i.e.* $I^2 R_r$) from the antenna. This fictitious or assumed resistance is given the name radiation resistance (R_r). Therefore, the total power loss in the antenna is sum of the two losses *i.e.*

$$\text{Total power loss} = \text{Ohmic loss} + \text{Radiation loss}$$

$$W = W' + W''$$

$$W = I^2 R_r + I^2 R_l$$

$$\boxed{\begin{aligned} W &= I^2 (R_r + R_l) \\ W &= I^2 R \text{ if } R \equiv R_r + R_l \end{aligned}} \quad \text{... (6.124)}$$

However, for $\lambda/2$ antenna the power (W'') lost as heat is very less, only few percent of the total supplied power to the antenna because r.f. resistance of antenna is very less.

The value of radiation resistance depends on

(i) Configuration of antenna,

(ii) The point where radiation resistance is considered,

(iii) Location of antenna w.r.t. grounds and other objects, and

(iv) Ratio of length of diameter of the conductor used.

(v) Corona discharge — a luminous discharge round the surface of antenna due to ionization of air.

A half-wave dipole antenna has a radiation resistance of 73.2 Ω in free space. The free space is me here that the half-wave dipole antenna is atleast several wavelength away from the earth and other objec practice, however no actual antenna is kept located in free space and hence the radiation resistanc accordingly affected by the presence of nearby objects, the most important being the earth itself.

The presence of ground changes the radiation resistance because the electromagnetic waves radiated from antenna are reflected from ground which (the reflected waves) induced current in the antenna while passing through it. The magnitude and phase of the induced current depends on the position of antenna w.r.t. ground *i.e.* at height above ground. If the height is such the *induced current* is in phase with *antenna current*, then the total current is larger and this results in a series of variation about free space value of radiation resistance. Since the reflected waves are weaker in strength so the fluctuation in radiation resistance decreases as the height (in, λ) is increased. The change of radiation resistance of a λ/2 antenna (horizontal) above perfectly conducting ground is shown in Fig. 6.20. The value of radiation resistance for most of the wire antenna is around 65 Ω and that of rod tubing antennas it is between 55 Ω, to 60 Ω.

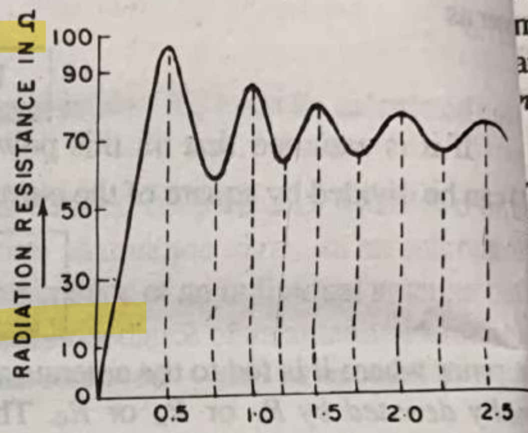


Fig. 6.20. Variation of R_r w.r.t. ground.

The knowledge of radiation resistance of a resonant antenna is important because it act as load for transmitter or for the radio frequency transmission line connecting the transmitter and antenna.

Further, unless otherwise mentioned radiation resistance is meant at the point of current maximum case of ungrounded antenna and to the current at the base of the antenna in case of grounded antennas.

6.24. FRONT TO BACK RATIO

Is defined as the ratio of the power radiated in desired direction to the power radiated in the oppos direction *i.e.*

$$FBR = \frac{\text{Power radiated in desired direction}}{\text{Power radiated in opposite direction}}$$

Obviously, higher the front to Back ratio, the better it is. The FBR changes if frequency of operat of antenna system shifts. Its value tends to decrease if spacing between elements of antennas increases. FBR depends on the tuning conditions or electrical length of the parasitic elements. The higher FBR is achie by diverting the gain of the opposite direction (*i.e.* backward response) to the forward or desired directio adjusting or tuning the length of parasitic elements. Hence the higher value of FBR is achieved at the co sacrifying gain from the opposite direction. *In practice, for receiving purposes adjustments are made to maximum front to back ratio rather than maximum gain.*

6.25. ANTENNA BAND-WIDTH

(AMIETE, Nov. 1971, 72

Like some of the other properties of antenna, there is no unique definition of band-width antenna or antenna system. It is because for the operation of antenna many factors like gain, side-lobe SWR or Front-to-Back-ratio, pattern, impedance and Polarization characteristics etc. are considered and requirements may change when the antenna operates. Therefore, the functional band-width of an ante limited by one (or more) of these factors and accordingly antenna band width may be specified in many dif ways as

(i) Band-width over which the gain is higher than some acceptable value, or
(ii) Band-width over which atleast a given front to back ratio is achieved, or
(iii) Band-width over which the SWR on the transmission line can be maintained below a chosen value.

In other words, it can be said that *antenna band-width is a width (i.e. range) of frequency over which the antenna maintains certain required characteristics like gain, front to back ratio or S.W.R. pattern (shape or direction), polarization and impedance*. In practice, however, these requirements change with the operation of the antenna, therefore, specifications are set to meet the needs of a particular application. For antennas, where increase in side lobe-level decrease in gain and change in impedance value, pattern and polarization characteristics are important, then one of these factors (*e.g.* gain or impedance) determines the low frequency limit and the other factor (*e.g.* change of pattern–shape or direction) the high frequency limit. Hence the band-width of a particular antenna, in general, can then, be defined as *"the band-width within which the antenna maintains a given set of specifications"*.

In general, the band-width of an antenna, as said, mainly depends on its two characteristics *e.g. impedance and pattern*. At low frequency of relatively small dimension ($\lambda/2$ or less) the band-width is usually determined by *impedance variation* because the pattern characteristic is insensitive to frequency i.e. pattern changes less rapidly. Under this condition antenna performance is limited by impedance variation and band-width depends on its reciprocal (*i.e.* "Q" of the antenna). A considerable mathematical analysis will show (Example 6.6) that two frequency limits (*i.e.* ω_2 and ω_1) or band width is given by

$$\Delta\omega \equiv \omega_2 - \omega_1 \equiv \frac{\omega_r}{Q} = \text{Band–width}$$

or
$$\boxed{\Delta\omega = \frac{\omega_r}{Q}} \qquad \because \omega_r = 2\pi f_r \qquad \text{... 6.125 (a)}$$

or
$$\boxed{\Delta f = \frac{f_r}{Q}} \qquad \Delta\omega = 2\pi\Delta f \qquad \text{... 6.125 (b)}$$

or
$$\Delta f \;\alpha\; \frac{1}{Q} \qquad \text{.... 6.125 (c)}$$

where f_r = Centre or resonant or design frequency.

$$\boxed{Q = 2\pi \frac{(\text{Total energy stored by antenna})}{(\text{Energy dissipated or radiated per cycle})}} \qquad \text{... 6.125 (d)}$$

Thus, the lower the "Q" of the antenna the higher the band-width and vice-versa.

For antennas of larger dimensions in wavelength (like thick cylindrical antennas or biconical antennas or antennas arrays except super-directive arrays *i.e.* arrays designed to give supergain) impedance characteristic may be satisfactory over a wide band and it is *pattern characteristic that determines the limits of frequencies. In this case the design statements are formulated in terms of beam-width and side-lobe-level requirements.*

For the antennas of about one wavelength dimension (*i.e.* neither larger nor small) band-width is limited by either on impedance or on pattern characteristic depending on the particular application.

Now the recent researches have led to the development of *Frequency-Independent antennas like log-periodic antennas etc. which have unlimited band-width where lower and upper frequencies limits are specified independently. In such cases, the band width is represented by a ratio of highest to lowest operating frequency.* For example, band width of broad band antennas, like those of log Periodic, 20 : 1 is attained with ease and 100 : 1 with careful design. Band width generally of low and moderate values are expressed *in terms of Percentage of centre frequency*

$$\text{B.W.\%} = \frac{\text{Operating range}}{\text{Centre frequency}} \times 100$$

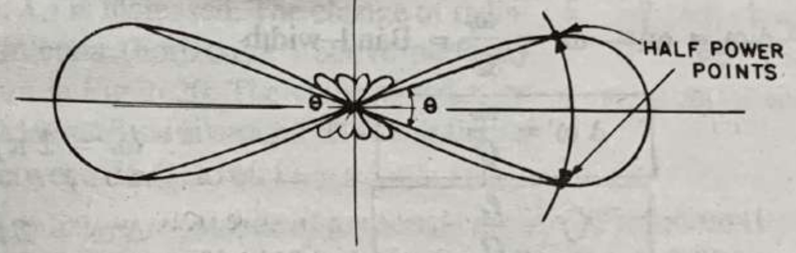
For example suppose an antenna operates between minimum frequency of 98 MHz to maxi frequency of 102 MHz, then its band width is 4 MHz. In terms of percentage

$$\text{B.W.} = \frac{4 \text{ MHz}}{100 \text{ MHz}} \times 100 = 4\%$$

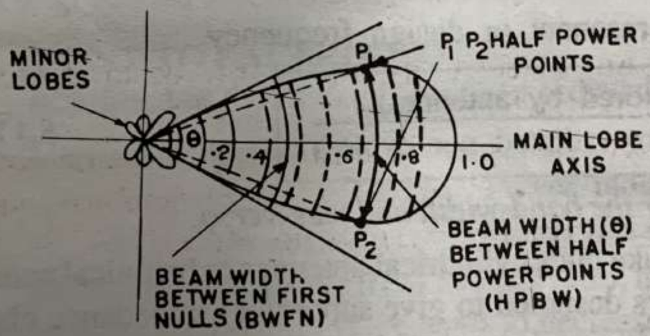
6.26. ANTENNA BEAM-WIDTH

(AMIETE, Nov. 1965, 69, 70, May 1970)

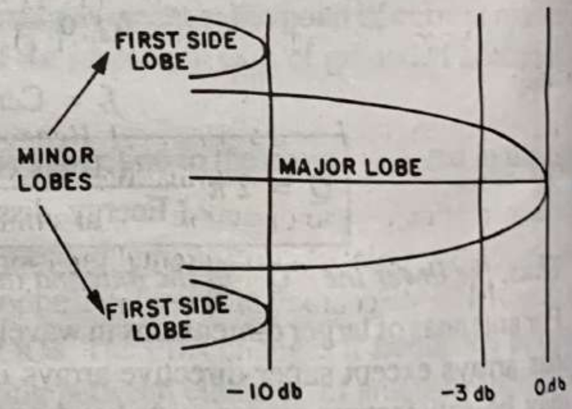
Antenna beam-width is a measure of directivity of an antenna. *Antenna beam-width is an ang width in degrees, measured on the radiation pattern (major lobe) between points where the radiated po has fallen to half its maximum value*. This is called as *"beam width" between half power point or half p beam width (HPBW) because the power at half power points is just half*. Half power beam width is also kn as **"3-db beam width"** because at half power points, the power is 3-db down of the maximum power of the major lobe. Further, at these half power points' the field intensity (*i.e.* voltage) equals $1/\sqrt{2}$ or (times its maximum value or 3-db down from maximum value. Beam width of the major lobe of particular (usually half power points) is one of the way to describe conveniently the radiation pattern of an antenna function of angular width, as the radiation pattern itself is a function of direction. Consider the radiation pat shown in Fig. 6. 21 (a) and let P_1 and P_2 be the half power points, then the angle $P_1 \; OP_2$ is **'beam width the antenna.** Therefore, antenna beam-width can be defined as **"The angular width (in degrees) of major lobe between the two directions at which the radiated or received power is one half the maxim power".**



(a). Beam-width angle P_1OP_2 .



(b) Beam-width on polar co-ordinate on linear scale.



(c) Beam width on rectangular co-ordinates on logarithmic scale (decibel).

Fig. 6.21.

In Fig. 6.21 (a, b), for example θ° represents the beam-width of the lobe. With a idea to highligh structure of minor lobes, the radiation pattern is shown in Fig. 6.21 (c) plotted on decibel scale (logari where 0 - db corresponds to main lobe maximum 3-db to half power points. Sometimes radiation patte also described in terms of angular width between first nulls or first side lobes, known as *beam-width betw first nulls and is abbreviated as (BWFN) or beam width - 10 db down* from the pattern maximum.

The directivity (D) is related with beam solid angle (Ω_A) or Beam area (B) by eqn. 6.48 as

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{B} \qquad ...($$

Since the radiation pattern or lobe is a 3-dimensional and hence the major lobe area approxim may be given by the product of beam-widths in horizontal and vertical planes or E plane and H planes

$B \simeq (HPBW)$ in horizontal plane $\times$ (HPBW) in vertical plane.... square radians

or $\simeq (HPBW)$ in E plane $\times$ $(HPBW)$ in H plane

$$B \simeq \theta_E \times \theta_H \text{ ...... Square radians if } \theta_E \text{ and } \theta_H \text{ in radians.} \quad \text{... (6.126)}$$

$\therefore$

$$D = \frac{4\pi}{\theta_E \; \theta_H} \text{ where } \theta_E \text{ and } \theta_H \text{ in radians} \quad \text{... 6.127 (a)}$$

$\because$ 1 radian = 57.3°

or

$$D = \frac{4\pi \times (57.3)^2}{\theta_E^\circ \; \theta_H^\circ} \text{ square degrees} \quad \text{where } \theta_E \text{ and } \theta_H \text{ are in degrees}$$

or

$$\boxed{D = \frac{41{,}257}{\theta_E^\circ \times \theta_H^\circ}} \quad \text{... (6.127 (b))}$$

This formula is very approximate and applies to antennas having almost equal and narrow (*i.e.* about 20 degrees) beam-width and also there is no more minor lobes. The errors increases if the beam widths of E and H planes increases.

$$\log 4 = 0.60206$$
$$\log \pi = 0.49715$$
$$\underline{2 \log 57.3 = 3.51630}$$
$$= 4.61551$$
$$\therefore \quad D = A' \log 4.61551$$
$$= 0.41257 \times 10^5 = 41257$$

The factors affecting the beam width of an antenna are

(i) The shape of radiation pattern,

(ii) The wavelength, and

(iii) Dimensions (*e.g.* radius of aperture etc. specially in case of horn-antenna) etc.

Since the beam has different widths in different planes through the beam axis, the axis is the direction of maximum radiation *i.e.* a line is drawn from the antenna to the nose of the radiation pattern. Hence the beam-width of the antenna is given in two planes perpendicular to each other as per the customary when the beam is linearly polarized, the E plane and H plane is used for the purpose as above, because they are perpendicular. Accordingly these beam width is also, sometimes, called as **E - plane beam-width and H plane beam-width.**

6.26.1. ***Practical Importance of Beam-Width.*** An antenna having narrow beam if used for reception, it may lead to the determination of the direction from which the signal is reaching. In this way it provides information on the direction of transmitter. For, the antenna beam must be adjustable or steerable *i.e.* capable of being pointed in different directions. Obviously, for *a direction finding applications*, a narrow beam is desirable and accuracy of direction finding is inversely proportional to beam-width

$$\left(\because D \; \alpha \; \frac{1}{B} \right)$$

Hence association of beam-width with DF accuracy is one of the important aspect of the beam-widt

Sometimes, a receiver is unable to discriminate between desired and undesired signals and in th case pointing a narrow receiving antenna beam in the direction of desired signal is helpful. This result in high gain of the antenna at the same time reduced gain for the unwanted signals. This may provide necessa discrimination. However, if there is wide separation between wanted and unwanted signals, then a relative wide beam-width will suffice.

Since gain or directivity and beam-width of an antenna are related by eqn. 6.127

$$D = \frac{4\pi}{\Omega} = \frac{4\pi}{B} = \frac{41{,}257}{\theta_E^\circ \; \theta_H^\circ}$$

$$D \; \alpha \; \frac{1}{\text{Beam-width}}$$

or

Hence narrower the beam-width, the higher the gain or directivity. *So a narrow beam-width pro a desirable property i.e. gain.*

Still there are situations in which wider beam-width is needed instead of narrow-beam width e broadcasting station where all round *i.e.* 360° coverage is required. In this case, any narrowing of the bea get gain must be done in vertical plane. Since the narrowing of beam in vertical plane is not possible at broad and low frequency band due to the large size of antennas but in television and *FM* broadcasting, (in VH UHF bands) narrowing of beam in vertical plane, while maintaining 360° coverage, is generally used.

6.27. ANTENNA BEAM EFFICIENCY

Antenna beam efficiency (*BE*) is the parameter that is frequently used to judge the quali transmitting and receiving antennas. For an antenna with its major lobe coincident with *z*-axis ($\theta = 0$) 6.4 (a), the beam efficiency is defined as,

$$BE = \frac{\text{Power transmitted (or received) within cone angle } \theta_1}{\text{Power transmitted (or received) by the antenna}} \quad \text{... 6.128}$$

where θ_1 = half angle of the cone within which the percentage of the total power is to be found.

Eqn. 6.128 (a) can be written as

$$BE = \frac{\int_0^{2\pi}\int_0^{\theta_1} \Phi(\theta, \varphi) \sin\theta \, d\theta \, d\varphi}{\int_0^{2\pi}\int_0^{\pi} \Phi(\theta, \varphi) \sin\theta \, d\theta \, d\varphi} \quad \text{... 6.128}$$

If θ_1 is chosen as angle where the first nulls or minimum occurs, the beam efficiency will indicate amount of power in the major lobe compared to the total power. A very high beam efficiency (between nulls or the minimum) normally in the high 90's is necessary for antenna used in radiometry, astronomy, ra and other application where received signal through the minor lobes must be minimised.

In terms of beam area (Ω_A), the beam efficiency is defined as *the ratio of the main beam a (Ω_M) to the total beam area (Ω_A) i.e.*

$$BE \text{ or } (\varepsilon_M) = \frac{\Omega_M}{\Omega_A} = \frac{\text{Main beam area}}{\text{Total beam area}}$$

where the total beam area Ω_A consists of the main beam area (or solid angle) Ω_M and the minor-lobe area solid angle) Ω_m *i.e.*

$$\boxed{\Omega_A = \Omega_M + \Omega_m} \quad \text{... 6.129}$$

$$\text{Total beam area} = \text{Main beam area} + \text{Minor lobe area.} \quad \text{... 6.129}$$

Divide throughout eqn. 6.129 (a) by Ω_A, we have

$$1 = \frac{\Omega_M}{\Omega_A} + \frac{\Omega_m}{\Omega_A}$$

$$\boxed{1 = \varepsilon_M + \varepsilon_m} \quad \text{... (6.1}$$

where $$\varepsilon_m = \frac{\Omega_m}{\Omega_A} = \text{Stray Factor} = \frac{\text{Minor–lobe area}}{\text{Total beam area}} \quad \text{... 6.131}$$

& $$\varepsilon_M = \frac{\Omega_M}{\Omega_A} = \text{Beam Efficiency} \quad \text{... 6.13}$$

6.28. ANTENNA BEAM AREA OR BEAM SOLID ANGLE Ω (Roorkee Univ. 1981

An area *ds* of the surface of a sphere as seen from the centre of the sphere subtends a solid Ω (Fig. 6.22). The total solid angle subtended by the sphere is 4 π steradians (or square rad bbreviated as S_r.

From Fig. 6.22 incremental area ds of the urface of sphere is given by

$$ds = (r \sin\theta \, d\varphi)(r \, d\theta)$$

$$= r^2 \sin\theta \, d\theta \, d\varphi$$

$$\boxed{ds = r^2 \cdot d\Omega \ \text{m}^2} \quad \text{... (6.132)}$$

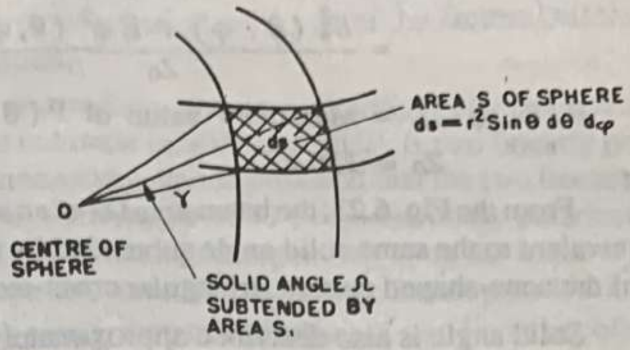


Fig. 6.22. Solid angle Ω subtended by area S.

ere $d\Omega$ = Solid angle subtended by area ds

$$d\Omega = \frac{ds}{r^2} \quad S_r$$

$$d\Omega = \frac{4\pi r^2}{r^2} \quad S_r \qquad \because ds = 4\pi r^2 = \text{Area of sphere}$$

$$\boxed{d\Omega = 4\pi} \quad \text{... (6.133)}$$

$$1 \text{ Steradian} = \frac{d\Omega}{4\pi}$$

$$1 \text{ Steradian} = \frac{\text{Solid angle of sphere}}{4\pi}$$

$$= 1 \,(\text{radian})^2 = \left(\frac{180^\circ}{\pi}\right)^2 \quad (\text{degrees})^2$$

$$= \frac{180 \times 180}{3.1416 \times 3.1416} (\text{deg.})^2 = \frac{32{,}400}{3.1416 \times 3.1416} (\text{deg.})^2$$

$$= \frac{10313.216}{3.1416} (\text{deg.})^2$$

$$\boxed{1 \text{ Sr.} = 3282.7909 \,(\text{deg.})^2} \quad \text{...(6.134)}$$

nce π radian = 180°

$$1 \text{ radian} = \left(\frac{180^\circ}{\pi}\right)$$

erefore, 4π Steradians $= 3282.7909 \times 4\pi \,(\text{deg.})^2$

$$= 13131.163 \times 3.1416 \,(\text{deg.})^2 = 41252.861 \,(\text{deg.})^2$$

$$\boxed{4\pi S_r \cong 41253 \,(\text{deg.})^2 = \text{Solid angle in a sphere}} \quad \text{... (6.135)}$$

The beam area (or beam solid angle) Ω_A for an antenna is therefore, given by the integral of the rmalised power pattern over a sphere ($4\pi S_r$) or

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} P_n(\theta, \varphi) \, d\Omega \, S_r$$

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} P_n(\theta, \varphi) \cdot \sin\theta \, d\theta \, d\varphi \quad S_r$$

ere $P_n(\theta, \varphi)$ = Normalised power pattern.

$$\boxed{P_n(\theta, \varphi) = \frac{P(\theta, \varphi)}{P(\theta, \varphi)_{max}}} \quad \text{... (6.135)}$$

$P(\theta, \varphi)$ = Poynting Vector

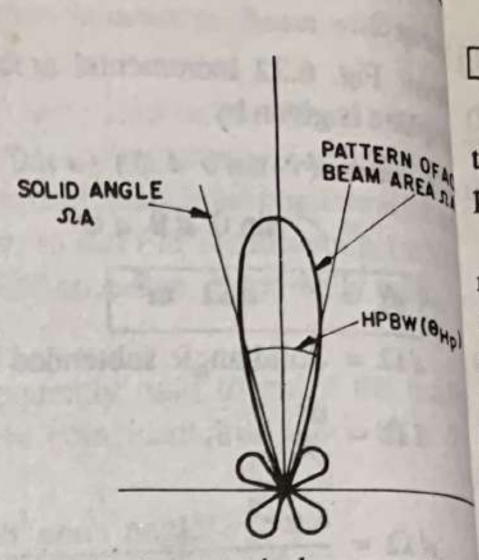
$$= \frac{E_\theta^2(\theta, \varphi) + E\varphi^2(\theta, \varphi)}{Z_0} \text{ W/m}^2$$

$P(\theta, \varphi)_{max}$ = Maximum value of $P(\theta, \varphi)$

and $Z_0 = 120\,\pi$.

From the Fig. 6.23, the beam area Ω_A of an actual pattern is equivalent to the same solid angle subtended by the spherical cap of the cone-shaped pattern (triangular cross-section).

Solid angle is also described approximately in terms of the angles subtended by the Half-power-points of the main lobe in principal planes.



$$\boxed{\Omega_A = \theta_{HP}\ \varphi_{HP}\ (S_r)} \quad \text{... (6.136)}$$

Fig. 6.23. A symmetrical power pattern of with equivalent solid angle Ω_A

where θ_{HP} = *HPBW* in *E–plane* or θ plane.

and φ_{HP} = *HPBW* in *H–plane* or φ plane.

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6.29. POLARIZATION

According to customary and convention the polarization is described in terms of Electric V in

Hence *Polarization or plane of polarization of a radio wave can be defined by the direction in w* eve *electric vector **E** is aligned during the passage of atleast one full cycle.*

Since Electric vector ***E*** and magnetic vector ***H*** are mutually perpendicular and this electron wave propagate in the perpendicular direction as shown in Fig. 6.24 in which mutually perpendicular di of electric vector, magnetic vector and propagation are illustrated.

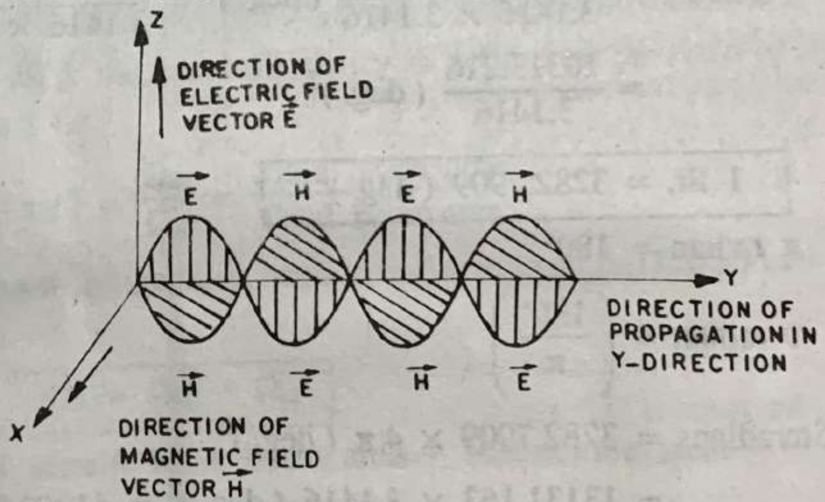

Fig. 6.24. Electromagnetic waves in free space.

The magnetic fields surround the wire and perpendicular to it, it means the electric field is pa the wire. This configuration is, applied even after the radiation of waves from the wire and hence the of polarization. Polarization refers to the physical orientation of the radiated electromagnetic waves *A electromagnetic wave is said to linearly polarized if they all have the same alignment in space.* I figure the wave is *linearly polarized or vertically polarized* as all the electric field vectors are vertical in space above earth, electric field vector ***E*** is vertical or lies in the vertical plane, the wave is sai *vertically polarized.* Similarly, if ***E*** is in horizontal plane, the wave is said to be *horizontally po* Polarization is a characteristics of most of the antenna that they radiate linearly polarized (i.e. verti horizontally) waves. The direction of antenna and polarization is alike *i.e. if an antenna is vertica radiate vertically polarized waves and a horizontal antenna, horizontally polarized waves.* This is w often said that *antennas are vertically polarized antennas or horizontally polarized antenna acco whether they are producing vertically polarized or horizontally polarized wave respectively.*

The initial polarization of electromagnetic waves is determined by the orientation of antenna itself in the space. Hence in the design of an antenna, the type of polarization is one of the factor. Different types of polarizations are useful in different types of application.

Besides linear polarization, antenna may also radiate *circularly or elliptically polarized waves*. In recent years circular polarization has become quite common in VHF and UHF. If two linearly polarized are simultaneously produce in the same direction from the same antenna provided that the two linear polarization are mutually perpendicular to each other with a phase difference of 90°, then circularly polarized waves are produced. Circular polarization may be right handed or left handed depending upon the sense of rotation *i.e.* phase difference is positive or negative. Circular polarization results only when the amplitudes of two linearly polarized waves are equal. If the amplitudes are not equal, then combination of two linearly polarized waves will produce Elliptically polarized wave.

Further, the undesired radiation from an antenna is called as *Cross Polarization*. The cross polarization, for linearly polarized antennas, is perpendicular to the intended radiation. Antennas are also called as *vertically or horizontally polarized antennas and not vertical or horizontal*. VLF, LF, MF and some of the HF antennas are made vertically polarized due to the closeness of earth. *But there are some advantages also in using horizontally polarized antennas, as man-made noises have usually vertical polarization.*

Thus, as seen, polarization from an antenna may be linearly, Elliptically or circularly but polarization in different portion of the total antenna pattern may be different *e.g.* polarization of major and minor lobes even be different.

Ordinarily simplest antennas transmit or receive linearly polarized waves. However, at VLF it is practically not possible to transmit horizontally polarized waves successfu''y, as it will be cancelled by the radiation from the image antennas in the earth. On the other hand, vertically polarized wave propagate successfully at these frequencies and hence at these frequencies (below 1000 kHz) vertically polarized waves are the only practicable mode.

At television broadcasting frequencies (between 54 to 890 MHz) horizontal polarization has been adopted as standard. In all the microwave frequencies (above 1000 MHz) there is a little basis for a choice of horizontal and vertical polarization. Final decision, however, is taken according to the type of application. But in any case the polarization should be same in the transmitting and receiving antennas.

Lastly, the circular polarization is increasingly becoming popular in recent years as it is advantageous in VHF, UHF and microwaves applications *e.g.* in radar for minimising the "clutter" echoes received from raindrops, transmission and reception between artificial satellite and earth etc. In fact, when VHF and microwaves signals are transmitted through ionosphere, then rotation of polarization vectors take place, the amount usually unpredictable. Hence if linearly polarized waves are transmitted, then it is better to have circularly polarized receiving antenna or *vice versa*. It would be still better if both ends have circularly polarized antenna for maximum efficiency.

6.30. ANTENNA TEMPERATURES (T_A)

Every object with a physical temperature above absolute zero (0° K = – 273° C) radiates energy. Associated with the radiation resistance is also an Antenna tempeature (T_A). For a lossless antenna, antenna temperature has nothing to do with the physical temperature of the antenna proper but is related to the temperature of distant regions of space (and nearer surroundings) coupled to the antenna via radiation resistance. Actually, the *antenna temperature is not so much an inherent property of the antenna as it is a parameter that depends on the temperature of the regions the antenna is "looking at". In this sense, a receiving antenna may be regarded as a remote sensing, temperature-measuring device. Both the antenna temperature (T_A) and radiation resistance (R_r) are single valued scalar quantities.*

The Noise power per unit band-width available at the terminals of a resistor of resistance R and temperature T is given by (Fig. 6.25).

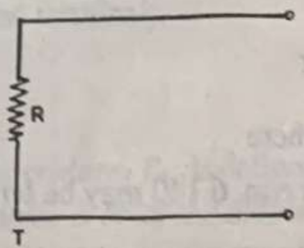

Fig. 6.25. Resistor (R) at temperature T.

$$\boxed{p = kT \text{ Watt/ Hertz}} \quad ...(6.1$$

where p = Power per unit bandwidth in Watt/Hertz.

k = Boltzman's Constant = 1.38×10^{-23} J/k.

T = Absolute temperature of resistor in °K.

If the resistor R is replaced by a lossless antenna of radiation resistance R in an **anechoic chamb** as shown in Fig. 6.26, at temperature T, the noise power per unit bandwidth, available at the terminals unchanged. That is to say that if the resistance R is replaced by a lossless resonant antenna of Radiati resistance R, the impedance presented at the terminals is unchanged. However, the noise power will not same unless the antenna is receiving from a region at the temperature T.

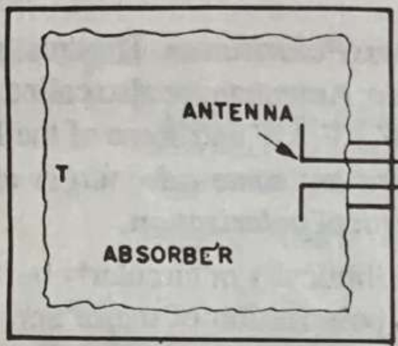

Fig. 6.26. Antenna at ANECHOIC CHAMBER at temperature T.

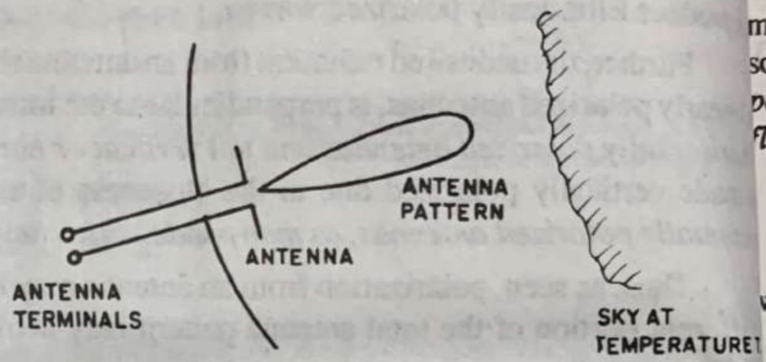

Fig. 6.27. Antenna and sky temperature.

Now if the antenna is removed from the ANECHOIC (NO ECHO) CHAMBER and pointed at a sk of temperature T, the noise power at the terminals is the same as for the two previous cases. It is assumed he that entire antenna pattern "Sees" the sky temperature T.

If the power per unit bandwidth p is independent of frequency, the total power (P) is obtained b multiplying by the bandwidth (B) *i.e.*

$$\boxed{P = kTB \text{ Watts}} \quad ...(6.13$$

where P = Total power, in Watt.

B = Bandwidth, in Hertz.

Let the antenna shown in Fig. 6.27 has an effective aperture A_e and that its beam is directed at a sourc of radiation which *produces a power density per unit bandwidth or flux density (S) at the antenna*. The pow received from the source is given by

$$\boxed{P = S A_e B \text{ Watts}} \quad ...(6.139$$

where S = Power desity per unit bandwidth in w/m² Hz

A_e = effective–aperture, in m²

B = Bandwidth, in Hz

Equating eqn. 6.138 and 6.139 the power density per unit bandwidth or flux density from the sourc at the antenna is

$$P = SA_e B = kTB$$

or

$$\boxed{S = \frac{kT_A}{A_e} \text{ W/m}^2 \text{ H}_z} \quad ...(6.140)$$

where T_A = Antenna Temperature due to the source, in °K

or eqn. 6.140 may be written as

$$\boxed{T_A = \frac{SA_e}{k} \text{ °K}} \quad ...6.141 (a)$$

In receiving with a radio telescope it is convenient to express the received power as power flux density. Therefore dividing the received power per unit bandwidth by the effective aperture A_e of the antenna gives the flux density S or

$$S = \frac{p}{A_e} = \frac{k\,T_A}{A_e}$$

or

$$\boxed{T_A = \frac{S \cdot A_e}{k}} \qquad \text{... 6.141 (b)}$$

The units of flux density is same as for the Poynting vector per unit bandwidth and hence flux density may be regarded as a measure of the Poynting vector per unit bandwidth received from the distant celestial source of radio emission. Most of the celestial sources radiate unpolarised wave. *Any antenna irrespective of polarisation characteristics can receive only half the incident power of an unpolarised wave, so the actual flux density should be twice of that given by eqn. 6.140 i.e.*

$$\boxed{S = \frac{2\,k\,T_A}{A_e}} \qquad \text{.. (6.142)}$$

where $k = 1.38 \times 10^{-23}$ J/k

T_A = Antenna noise temperature, in °K

A_e = Effective aperture of antenna, in m^2.

If the celestial source extent Ω_S is small compared to the antenna-beam solid angle Ω_A and the antenna beam is aligned with source, the observed flux density is given by eqn. 6.140.

If the angular size of the source is small compared to Ω_A and its magnitude is known, then it is possible to determine the source temperature T_S very simply from eqn.

$$\boxed{T_A = \frac{\Omega_S}{\Omega_A}\,T_S}$$

or

$$T_S = \frac{\Omega_A}{\Omega_S}\,T_A \qquad \text{... (6.143)}$$

where Ω_A = Antenna beam solid angle, in S_r ; Ω_S = Source solid angle in S_r ; T_A = Antenna noise temperature and T_S = Source temperature in °K.

In case the receiver itself has a certain noise temperature T_r due to thermal noise in the receiver components the *system noise power at the receiver terminals* is given by

$$\boxed{P_S = k\,(T_A + T_r)\,B_N} \qquad \text{.... 6.144 (a)}$$

where

$$\boxed{T_{sys} = T_A + T_r} \qquad \text{... 6.144 (b)}$$

P_S = System noise power, at receiver terminals.

T_A = Antenna noise temperature, at receiver terminals.

T_{rs} = Receiver noise temperature, at receiver terminals.

$T_{sys} = (T_A + T_{rs})$ = Effective system noise temperature, at receiver terminals.

B_N = Bandwidth.

6.31. EQUIVALENT NOISE TEMPERATURE OF ANTENNA

The noise introduced by a network may also be expressed as *effective noise temperature*, T_e, is define *as that fictional temperature at the input of the network which would account for the noise* ΔN *at the outpu*... *Sometimes another parameter noise figure* ... ΔN *is the additional noise introduced by the network itself.*

used. The *noise figure* F, is related with effective *noise temperature* T_e as

$$F = 1 + \frac{T_e}{T_0} \text{ or } (F - 1) = \frac{T_e}{T_0}$$

$$\boxed{T_e = (F - 1)\, T_0} \qquad \text{... (6.14}$$

where T_e = Noise Temperature effective, in °K;

T_0 = 290° K (273 + 17) °K.

F = Noise figure, (dimensionless).

The noise figure F in decibel is given by

$$\boxed{(F)_{db} = 10 \log_{10} F} \qquad \text{... (6.14}$$

Example 6.1. *Calculate the dirrectivity of an isotropic antenna.*

Solution. Directivity D is given by $D = \dfrac{4\pi}{\Omega_A}$

For isotropic antenna $\Omega_A = 4\pi$ $\qquad \therefore D = \dfrac{4\pi}{4\pi} = 1$ and $D = 1$.

Thus directivity of an isotropic antenna is unity which is the smallest.

Example 6.2. *Calculate the maximum effective aperture of a short dipole.* (AMIETE Dec. 19

Solution. It is assumed that

(i) Short dipole is coincident with x-axis as shown.

(ii) Plane polarized wave is travelling along y-axis as shown and inducing current along the x-axis antenna which is constant throughout the length of the dipole and in same phase.

(iii) Length of the short dipole is small in comparison to wave-length *i.e.* $dl << \lambda$

(iv) Antenna losses are zero *i.e.*

$$R_L = R_r + R_l \quad \text{or } R_L = R_r \qquad \because R_l =$$

The maximum effective aperture is given by eqn. 6.66.

$$(A_e)_{\max} = \frac{V^2}{4\pi P R_r} \qquad \text{... (6.6}$$

where V = Induced voltage ; P = Poynting vector and R_r = Radiation resistance

Thus $(A_e)_{\max}$ can be calculated if above factors are known.

(1) The induced voltage V in the short dipole dl is maximum when it is parallel to the incident electric field $\boldsymbol{E}$. Thus effective value of V is given by the product of effective length of short dipole (dl) and Electric field intensity E, *i.e.*

$$V = E \cdot dl \qquad \text{... (6.147)}$$

(2) The Poynting vector or Power density of incident wave at the short dipole is given by

$$P = \frac{E^2}{\eta} \text{ W/m}^2 \qquad \text{... (6.148)}$$

where E = Electric field intensity ; η = Intrinsic impedance of free space and P = Poynting vector.

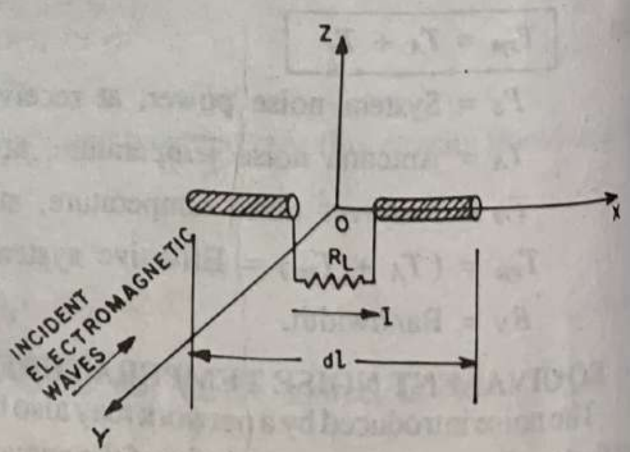

Fig. 6.28. Short dipole with uniform current I along its length and terminated by a load resistance R_L.