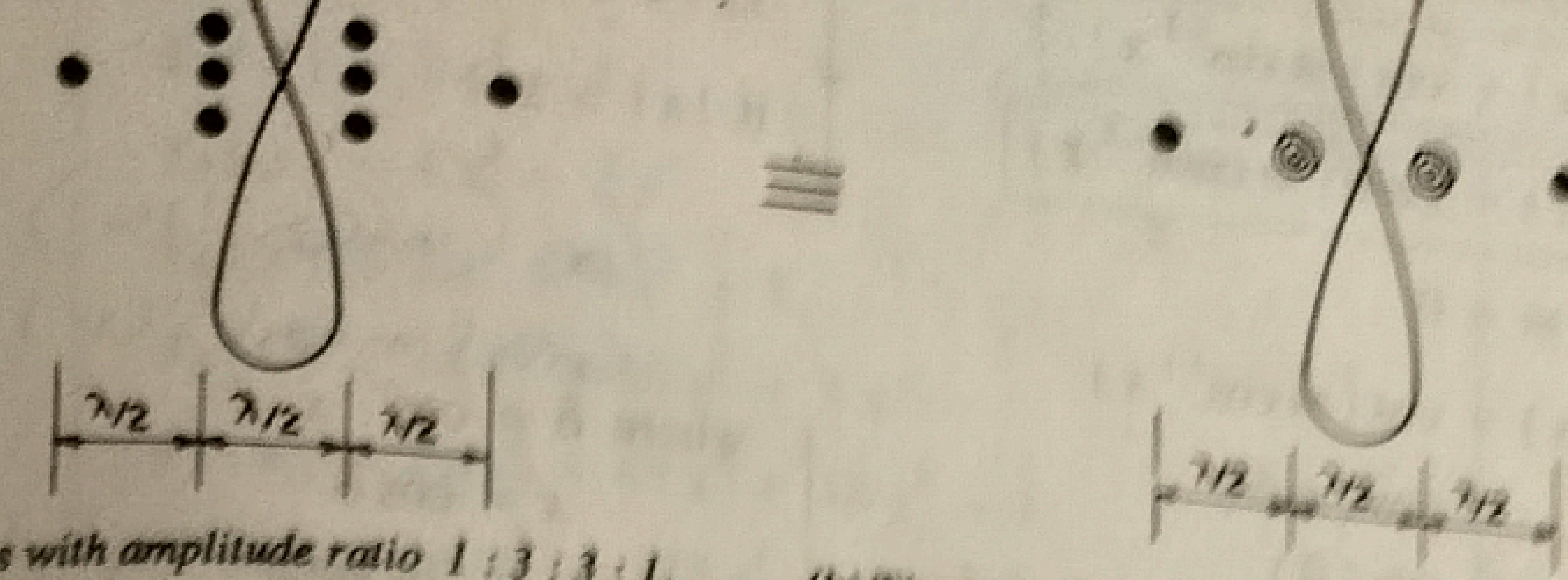


(a) 4 sources with amplitude ratio 1 : 3 : 3 : 1. (b) The same Fig. shown superposition of centre sources.

Fig. 7.29.

now has the three effective sources with amplitude ratio 1 : 2 : 1. Similarly, if this three sources array [Fig. 7.28 (a)] is superimposed with another identical array, then an array of effective four sources with current mplitude ratio 1 : 3 : 3 : 1 is obtained as shown in Fig. 7.29 in which there is no minor lobes. The far field in by principle of multiplicity of Pattern, is given by

$$E_{nor} = \cos^2 (\pi/2 \cos\theta) \quad \text{... (7.111)}$$

In this way, it is possible to have a pattern of any desired directivity without any minor lobes provided the current amplitude sources corresponds to the coefficients of the said binomial series (eqn 7.109) and the far field pattern of n sources is given by

$$E_{nor} = \cos^{(n-1)} (\pi/2 \cos\theta) \quad \text{... (7.112)}$$

Since minor lobes elimination in binomial array takes place at the cost of decrease in directivity in comparison with the same array (of equal amplitude sources) therefore, in practice usually arrays of size uniform are designed as compromise between the uniform and binomial.

7.13.1. *Disadvantages of binomial arrays*

(i) HPBW increases and hence the directivity decreases.

(ii) For design of a large array, larger amplitude ratio of sources is required.

7.14. DOLPH-TCHEBYSCHEFF OR CHEBYSHEV ARRAYS (AMIETE, N/74, M/75, N/76)

In the design of linear inphase antenna arrays of non-uniform amplitudes C.L. Dolph used the Tchebyscheff polynomial and hence the joint name i.e. Dolph-Tchebyscheff arrays. An alternative spelling to this Russian name Tchebyscheff (pronounced as che-bee-shef) is Chebyshev and hence it is also called as Dolph-Chebyshev arrays or simply **Chebyshev arrays.** In the antenna design, it is often desired to achieve the narrowest beam width, besides low side level. However, these characteristics of an antenna system are so related that an attempt in the improvement in the former deteriorates the latter. C.L. Dolph proposed that for a linear inphase broadside arrays, it is possible to minimize the beam width of main lobe (between first nulls) for a specified side-lobe-level and vice-versa i.e. if the beam width between first nulls is specified, then the side-lobe-level is minimized. Thus Dolph-Tchebycheff distribution provides a compromise rather optimum value between the two conflicting properties. *In other words, Dolph-Tchebyscheff arrays produces narrowest beam-width for given side-lobe-level and vice-versa.* Since for a specified side lobe level, narrowest beam width is achieved by this distribution and hence it is considered to be *optimum.*

Dolph-Tchebyscheff current amplitude distribution is optimum provided $d \leq \lambda/2$. Dolph's approach indicates that reduction inside lobe cannot be accomplished without sacrifice of antenna performance in some other respect e.g. beamwidth and gain or directivity. It is possible, practically, to design, by this method, a high gain narrow beam antennas for side lobe levels of 20-30 db in VHF and UHF bands specially for radar use. A 20 db level is considered good and 30 db is excellence while it is very difficult to achieve 40 db.

7.14.1. *Tchebyscheff Polynomial.* In defining the Tchebyscheff polynomial the first letter T is used as symbol, as Tchebyscheff being the older spelling. The Tchebyscheff polynomial is defined by eqn.

$$\boxed{T_m(x) = \cos(m \cos^{-1} x)} \qquad |x| < \pm 1 \qquad \text{... 7.113 (a)}$$

and

$$\boxed{T_m(x) = \cosh(m \cosh^{-1} x)} \qquad \text{if } |x| > \pm 1 \qquad \text{... 7.113 (b)}$$

$\cos(m\cos^{-1}x)$ $|x|<1$

$\cosh(m\cosh^{-1}(x))$ $|x|>1$

Let $m = 0$

$$T_0(x) = \cos(m \cos^{-1} x)$$
$$= \cos(m\delta)$$
$$= \cos(0)$$
$$= 1$$

where $\delta = \cos^{-1} x$

$x = \cos\delta$

(A) $x = \cos\psi/2$

$$\therefore \quad \boxed{T_0(x) = 1 \qquad \therefore \ \delta = \psi/2} \qquad \text{... (7.114)}$$

$m=0$

1

$m = 1,$ $T_1(x) = \cos(1.\delta) = \cos\delta = x$

or $\boxed{T_1(x) = x}$ ... (7.115)

x

$m = 2$ $T_2(x) = \cos 2\delta = 2\cos^2\delta - 1$

$2x^2-1$

or $\boxed{T_2(x) = 2x^2 - 1}$ ... (7.116)

$4x^3-3x$

$m = 3$ $T_3(x) = \cos 3\delta = 4\cos^3\delta - 3\cos\delta$

or $\boxed{T_3(x) = 4x^3 - 3x}$ ... (7.117)

$m = 4$ $T_4(x) = \cos 4\delta = 2\cos^2 2\delta - 1$

$$= 2[2\cos^2\delta - 1]^2 - 1 = 2[4\cos^4\delta - 4\cos^2\delta + 1] - 1$$

$$\boxed{T_4(x) = 8x^4 - 8x^2 + 1} \qquad \text{... (7.118)}$$

Further higher terms can be had from the recursion formula

$$\boxed{T_{m+1}(x) = 2x\,T_m(x) - T_{m-1}(x)} \qquad \text{... (7.119)}$$

$\therefore$ For, $T_5(x)$, put $m = 4$

$$T_5(x) = 2x\,T_4(x) - T_3(x)$$
$$= 2x[8x^4 - 8x^2 + 1] - (4x^3 - 3x)$$

$$\boxed{T_5(x) = 16x^5 - 20x^3 + 5x} \qquad \text{... (7.120)}$$

$$T_6(x) = 2x\,T_5(x) - T_4(x) \qquad \text{Put } m = 5$$
$$= 2x[16x^5 - 20x^3 + 5x] - [8x^4 - 8x^2 + 1]$$

$$\boxed{T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1} \qquad \text{... (7.121)}$$

$$T_7(x) = 2x\,T_6(x) - T_5(x)$$
$$= 2x[32x^5 - 48x^4 + 18x^2 - 1] - [16x^5 - 20x^3 + 5x]$$
$$= 64x^6 - 96x^5 + 36x^3 - 2x - 16x^5 + 20x^3 - 5x$$

$$\boxed{T_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x} \qquad \text{... (7.122)}$$

Thus summarizing,

$$T_0(x) = 1$$

$$T_1(x) = x \quad \text{... (7.114)}$$

$$T_2(x) = 2x^2 - 1 \quad \text{... (7.115)}$$

$$T_3(x) = 4x^3 - 3x \quad \text{... (7.116)}$$

$$T_4(x) = 8x^4 - 8x^2 + 1 \quad \text{... (7.117)}$$

$$T_5(x) = 16x^5 - 20x^3 + 5x \quad \text{... (7.118)}$$

$$T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1 \quad \text{... (7.119)}$$

$$T_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x \quad \text{... (7.120)}$$

$$T_8(x) = 128x^8 - 256x^6 + 160x^4 - 32x^2 + 1 \quad \text{... (7.121)}$$

$$T_9(x) = 256x^9 - 576x^7 + 432x^5 - 120x^3 + 9x \quad \text{... (7.122)}$$

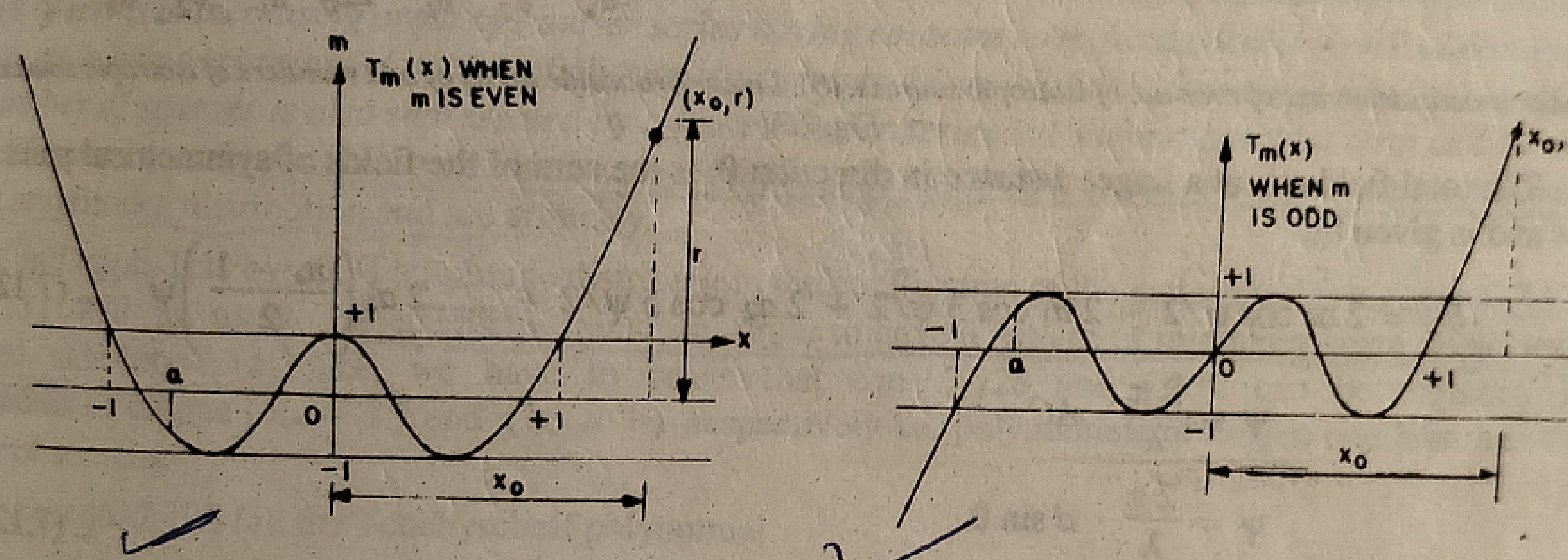
$$\text{... (7.123)}$$

Above eqn. represent Tchebyscheff polynomial and in general can be written as

$$T_m(x) = \cos(m \cos^{-1} x) = \cos(m\,\delta) = \cos(m\,\psi/2)$$

$$\boxed{T_m(x) = \cos m\,\psi/2} \qquad \text{where } \psi/2 = \cos^{-1} x;\ x = \cos\psi/2 \quad \text{... 7.123 (a)}$$

and their general characteristics are shown in Fig 7.30 (a, b). *It is seen from the Tchebyscheff polynomial that the value of m and degree of polynomial is same.*



(a) Tchebyscheff Polynomials when m is even. *(b). Tchebyscheff Polynomial when m is odd.*

Fig. 7.30

From the Fig. 7.30, the following properties of Tchebyscheff polynomial are evident :

(i) All the polynomial in the range $-1 < x < +1$ oscillate between the value -1 and $+1$.

(ii) In the range $|x| < 1$, the mth order polynomial crosses the axis m times.

(iii) In the range $|x| > 1$, the polynomials go on increasing without limit at the rate proportional to x^m.

Thus, as seen, as the x is varied from a point say "a" upto a chosen value x_0 and back to "a", then the function $T_m(x)$ traces out a pattern which consists of –

(i) many small side-lobes,

(ii) one major lobe

(iii) the secondary minor lobes are of same amplitude of unity and are below main lobe by the ratio $1/r$, which can be chosen at will by suitably choosing x_0.

(iv) The side lobes arise in the region $x < 1$.

(v) The main lobe extends far in to the range $x > 1$.

The pattern shown in Fig. 7.29 is called Tchebyscheff or optimum pattern because all the side lobe levels have the same level. Since a optimum pattern is achieved if all the side levels have same amplitude.

The roots of the polynomial corresponds to null and occur when

$$\cos(m\delta) = 0$$

or

$$m\delta = (2k-1)\pi/2$$

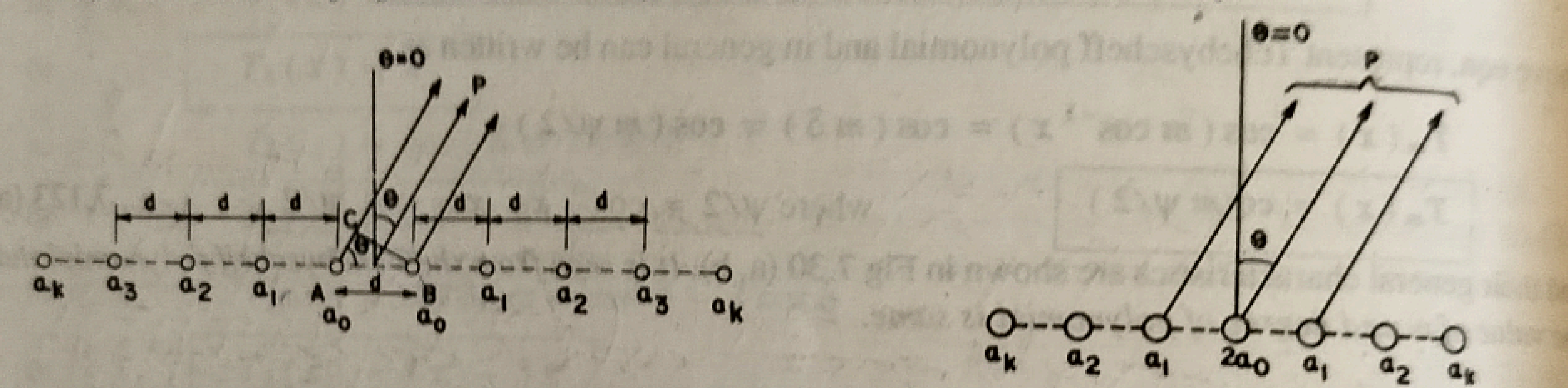
or

$$\delta = \frac{(2k-)\pi}{2m} \quad \text{... (7.124)}$$

where

$$k = 1, 2, 3, \ldots..$$

7.14.2. *Dolph Pattern Method of obtaining Optimum Pattern using Tchebyscheff Polynomial.* Considering now the linear broadside arrays of even and odd number of isotropic sources. Sources are fed in phase having equal distance d and amplitudes are a_0, a_1, a_2 .... a_k Fig 7.31 (a). Unlike earlier, $\theta = 0$, is the direction perpendicular to array with origin at the centre of the array.



(a). Linear broadside arrays of even no. of isotropic sources. (b). Linear broadside array of odd numbers of isotropic sources.
Fig. 7.31

The total field etc. at a larger distance in direction θ is the sum of the fields of symmetrical pairs of sources and is given by

$$E_{t_e} = 2a_0\cos\psi/2 + 2a_1\cos 3\psi/2 + 2a_2\cos 5\psi/2 + \ldots..2a_k\left(\frac{n_e-1}{2}\right)\psi \quad \text{... (7.125)}$$

where

$$\psi = \frac{2\pi}{\lambda}\cdot AC$$

$$\psi = \frac{2\pi}{\lambda}\cdot d\sin\theta \quad \text{... (7.126)}$$

$$n_e = 2, 4, 6, \ldots..2(k+1) \qquad \mid k = 0, 1, 2, 3, \ldots$$

or

$$(n_e - 1) = 2(k+1) - 1 = 2k+1 \quad \text{... 7.127 (a)}$$

or

$$\left(\frac{n_e-1}{2}\right) = \left(\frac{2k+1}{2}\right) \quad \text{... 7.127 (b)}$$

even

Thus

$$\boxed{E_{t_e} = 2\sum_{k=0}^{k=N-1} a_k\cos\left(\frac{2k+1}{2}\right)\psi} \quad \text{... (7.128)}$$

where

$$\frac{n_e}{2} = N$$

Shifting to the linear array of odd number (say n_0) of isotropic sources with uniform spacing (d) Fig. 7.30 (b). The amplitude distribution is symmetrical about centre sources whose amplitudes is assumed to

$2\,a_0$. The amplitudes of other sources are a_1, a_2, a_3 ... etc. The total field E_{t_0} from the odd number of sources at a large distance in direction θ is given is

$$E_{t_0} = 2\,a_0 + 2\,a_1 \cos\psi + 2\,a_2 \cos 2\,\psi + 2\,a_3 \cos 3\,\psi + \ldots\ 2\,a_k \cos\left(\frac{n_0 - 1}{2}\right) \quad \ldots (7.129)$$

where $n_0 = 1, 3, 5 \ldots (2\,k + 1)$

and $k = 0, 1, 2, 3, \ldots$

or $$\frac{(n_0 - 1)}{2} = k \quad \ldots (7.130)$$

odd

Thus $$E_{t_0} = 2 \sum_{k=0}^{k=N} a_k \cos\left(\frac{n_0 - 1}{2}\right)\psi$$

$$\boxed{E_{t_0} = 2 \sum_{k=0}^{k=N} a_k \cos\left(\frac{2\,k\,\psi}{2}\right)} \quad \ldots (7.131)$$

where $\left(\frac{n_0 - 1}{2}\right) = N$

Eqn. 7.131 may be treated as finite Fourier series of N terms. Since $k = 0$ gives the constant terms $2\,a_0$ which represents the contribution of centre source. Similarly $k = 1, 2, 3, \ldots$ will give the terms $2\,a_1 \cos\psi$, $2\,a_1 \cos 2\,\psi$ .... which represents the contribution of first pair of sources, second pair of sources respectively and so on, symmetrical about centre source $2\,a_0$. *Therefore, the total field pattern is the summation of series of terms in increasing order of Fourier series having constant term, fundamental term 2nd harmonic term etc. as is being represented in case of alternating currents. Likewise the field pattern [eqn. 7.125], of an even number of sources is also represented by a finite Fourier series but without constant term and having only odd harmonics.* However, in either case, the coefficients of series e.g. a_0, a_1, a_2.... are respectively current amplitudes distribution and are arbitrary.

In Dolph-Tchebyscheff amplitude distribution, the coefficients of the pattern series e.g. eqns. 7.128 and 7.131 will as usual, be uniquely found in order to have a pattern of minimum beam width for a specified side-lobe-level. For, we have to prove that eqn. 7.128 and 7.131 can be regarded as polynomials of degree $(n_e - 1)$ and $(n_0 - 1)$ respectively i.e. polynomials of degree one less than the number of sources.

By eqn. 7.123 (a), the Tchebyscheff polynomial

$$\boxed{T_m(x) = \cos(m\,\delta) = \cos m\,\psi/2} \quad \ldots 7.123\ (a)$$

i.e. $T_m(x) = \cos m\,\psi/2$ is expressed as polynomials of degree m. Further the power series 7.128 and 7.131 are also expressed as polynomials of degrees $2\,k + 1 = (n_e - 1)$ and $2\,k = n_0 - 1$ [eqn. 7.127 (b) and 7.130] respectively. Since eqn. 7.128 and 7.131 are sums of cosine polynomials of the form $\cos m\,\psi/2 = T_m(x)$ i.e. Tchebyscheff polynomial. *This shows that power series (eqn. 7.128 and 7.131) represent the field pattern of a symmetric broadside uniform arrays of n isotropic sources and* are polynomials of degree $(n_e - 1)$ and $(n_0 - 1)$ respectively i.e. number of sources minus one.

Hence if the array polynomials of degree $(n_e - 1)$ or $(n_0 - 1)$ or in general $(n - 1)$ is equated to the Tchebyscheff polynomials of like degree (i.e. $m = n - 1$) and so also the coefficients of array polynomials and Tchebyscheff polynomials, then the amplitude distribution is given by these coefficients of the Tchebyscheff distribution and the field pattern of the array corresponds to the Tchebyscheff polynomial of degree $(m = n - 1)$.

7.14.3. *Steps to be followed while calculating Dolph-Tchebyscheff Amplitude Distribution.* Writing eqn. 7.128 and 7.131 once again, we have

$$E_{t_e} = 2\sum_{k=0}^{k=N} a_k \cos[(2k+1)\psi/2] \qquad \text{number of sources even}$$

and

$$E_{t_o} = 2\sum_{k=0}^{k=N-1} a_k \cos[2k\psi/2] \qquad \text{number of sources odd}$$

The factor 2 in the above array polynomial may be dropped as mostly we are concerned in relative field pattern. Thus

$$E_{t_e} = \sum_{k=0}^{k=N-1} a_k \cos[(2k+1)\psi/2] \qquad \text{...(7.132)}$$

and

$$E_{t_o} = \sum_{k=0}^{k=N-1} a_k \cos[2k\psi/2] \qquad \text{...(7.133)}$$

Assuming n number of sources, the following steps will be followed for calculation of Dolph-Tchebyscheff amplitude distribution.

Step-I. Let the ratio of main lobe to the side lobe level is specified as

$$r = \frac{\textbf{Main lobe maximum}}{\textbf{Side lobe level}} \qquad \text{...(7.134)}$$

The value of r is chosen *i.e.* r is calculated from the eqn.

$$\boxed{\textbf{Side lobe level below main lobe maximum in db} = 20 \log_{10} r} \qquad \text{...(7.135)}$$

Step-II. Now select the Tchebyscheff polynomial $T_m(x)$ of the same degree as the array polynomial (eqns. 7.132 and 7.133). For, if m be the degree of Tchebyscheff polynomial then the degree of array polynomial would be $(n-1)$, where n is the number of sources. Symbolically, therefore,

$$\boxed{T_m(x_0) = T_{n-1}(x_0)} \qquad \text{...(7.136)}$$

After having known the values of $T_m(x_0)$ and r, equate them and solve the equation

$$\boxed{T_m(x_0) = T_{n-1}(x_0) = r, \text{ for } x_0.} \qquad \text{...(7.137)}$$

Sometimes solving of eqn. 7.137 is quite involved, therefore, x_0 is calculated from the following alternate formula

$$\boxed{x_0 = \frac{1}{2}\left[\left\{r + \sqrt{r^2-1}\right\}^{\frac{1}{m}} + \left\{r - \sqrt{r^2-1}\right\}^{\frac{1}{m}}\right]} \qquad \text{...(7.138)}$$

where

$$m = n - 1$$

This formula is specially useful when degree of Tchebyscheff Polynomials is high.

This formula is derived from eqn. 7.137 as follows.

$$T_m(x_0) = r = \cosh(m\cosh^{-1} x_0) \qquad \text{by eqn 7.137 and 7.113} \qquad \text{... 7.113 (a)}$$

Let

$$u = m\cosh^{-1} x_0 \qquad \text{... 7.139 (a)}$$

$$\cosh^{-1} x_0 = \frac{u}{m}$$

or

$$x_0 = \cosh\left(\frac{u}{m}\right) \qquad \text{... (7.140)}$$

or

$$x_0 = \frac{e^{\frac{u}{m}} + e^{-\frac{u}{m}}}{2} \qquad \text{... (7.141)}$$

now from eqn. 7.139 (a)

$$r = \cosh u \quad \text{or} \quad r = \frac{e^{u} + e^{-u}}{2}$$

or

$$2r = e^{u} + e^{-u} \qquad \text{... (7.142)}$$

or

$$2r\,e^{u} = e^{2u} + 1$$

or

$$e^{(u)2} - 2r\,e^{(u)} + 1 = 0$$

or

$$e^{u} = \frac{+\,2r \pm \sqrt{4r^2 - 4 \times 1 \times 1}}{2 \times 1}$$

or

$$e^{u} = r \pm \sqrt{r^2 - 1}$$

taking +ve sign

or

$$e^{u} = r + \sqrt{r^2 - 1} \qquad \text{... (7.143)}$$

From eqn 7.142

$$e^{-u} = 2r - e^{u} = 2r - (r + \sqrt{r^2 - 1})$$

$$e^{-u} = r - \sqrt{r^2 - 1} \qquad \text{.... (7.144)}$$

Putting eqn. 7.143 and 7.144 into eqn. 7.141 we get the alternate formula i.e.

$$\boxed{x_0 = \frac{1}{2}\left[\left\{r + \sqrt{r^2 - 1}\right\}^{\frac{1}{m}} + \left\{r - \sqrt{r^2 - 1}\right\}^{\frac{1}{m}}\right]} \qquad \text{... (7.138)}$$

Step III. Now is the turn for calculation of total field E_t, in polynomial form. As is evident from Fig. 7.29 that is $r > 1$ and so also the value of x_0. But according to eqn. 7.113 (a) x lies in the range $-1 \leq x \leq +1$. This put us in difficulties as x must be in the range $\pm$ 1. This difficulty is overcome by introducing a change of scale as

$$z = \frac{x}{x_0}$$

Thus the condition of eqn. 7.113 (a) is now fulfilled and we get

$$z = \cos \psi/2 = \cos \delta \qquad \text{... (7.145)}$$

Because z restricted to the desired range $-1 \leq z \leq +1$. Now the pattern polynomials eqn. 7.145 and 7.133 are expressed in terms of z by substituting eqn. 7. 145 in them as below.

$$E_{t_e} = \sum_{k=0}^{k=N-1} a_k \cos(2k+1)\,\psi/2 \qquad \text{... (7.132)}$$

$$= a_0 \cos \psi/2 + a_1 \cos 3\,\psi/2 + a_2 \cos 5\,\psi/2 + a_3 \cos 7\,\psi/2 + \ldots\ldots$$

$$E_{t_e} = a_0\,z + a_1\,[\,4z^3 - 3z\,] + a_2\,[\,16^5 - 20z^3 + 5z\,] + a_3\,[\,a_3\,[\,64z^7 - 112z^5 + 56z^3 - 7z\,] + \ldots \qquad \text{... (7.146)}$$

✓ End 6 -

and $$E_{t_e} = \sum_{k=0}^{k=N} a_k \cos(2k\psi/2)$$

$$= a_0 + a_1 \cos 2\psi/2 + a_2 \cos 4\psi/2 + a_3 \cos 6\psi/2 + \ldots\ldots \quad \ldots 7.147\text{ (a)}$$

$$E_{t_e} = a_0 + a_1[2z^2 - 1] + a_2[8z^4 - 8z^2 + 1] + a_3[32z^6 - 48z^4 + 18z^2 - 1] + \ldots \quad \ldots 7.147\text{ (b)}$$

Step IV. Finally, equate array polynomial E_{t_o} or E_{t_e} or E_t in general and Tchebyscheff Polynomial (eqn. 7.136) *i.e.*

$$\boxed{E_t = T_{n-1}(x)} \quad \ldots (7.148)$$

The coefficients a_0, a_1, a_2 ...... etc. are calculated from the eqn. 7.148 which gives Dolph-Tchebyscheff optimum amplitude distribution for the specified side lobe level. For relative current amplitude ratios are taken in the last.

Summarized Steps are as follows :

I. Calculate the value of r from eqn. 7.135

II. Select Tchebyscheff polynomial of the same degree as the array polynomial i.e.

$$\boxed{T_{n-1}(x_0) = r}$$

and solve it for x_0. Alternatively eqn. 7.138 may be used for calculation of x_0.

III. Choose array polynomial E_t from eqn. 7.146 or 7.147

IV. Equate Tchebyscheff polynomial. $T_{n-1}(x)$ with Array polynomial E_t (step III) *i.e.*

$$\boxed{T_{n-1}(x) = E_t}$$

and calculate the coefficients and take ratios for relative amplitudes.

7.14.4. *Advantages of Dolph-Tchebyscheff Distribution* (AMIETE, Nov. 1974)

(i) The greatest advantages of Dolph-Tchebyscheff distribution is that it provides a minimum rather optimum beam width for a specified degree side lobe-level reduction.

(ii) It results in side lobes that are all of the same amplitudes unlike uniform distribution in which side lobes near adjacent to the main lobe is largest and others progressively decreases as angle increases from main lobe.

(iii) Tapering is not extreme (as against binomial etc.) i.e. ratio of current between centre element and end element is small which provides ease in feeding design. For example, 8 elements broadside arrays of 26 db down side lobe level reduction provides the amplitude ratio of 1 : 3 : 1 between end and centre element as compared to binomial 1 : 35 where relative amplitudes are 1 : 7 : 21 : 35 : 21 : 7 : 1.

7.14.5. *Beam width between first nulls of chebyshev polynomials patterns.* By definition and properties of Dolph-Tchebyscheff or chebyshev polynomials all the nulls are seen to occur within the range $|x| < 1$. These nulls are nothing but zeros or roots of the chebyshev polynomial within the range $|x| < 1$. Now for an m-element array, the chebyshev polynomial is $T_{m-1}(x)$. Therefore setting $T_{m-1}(x) = 0$, we get the roots of polynomial eqn. 7.113 (a)

$$T_{m-1}(x) = 0$$

or

$$\cos\{(m-1)\cos^{-1}x\} = 0$$

$$\{(m-1)\cos^{-1}x\} = \frac{(2k-1)\pi}{2}$$

where

$$k = 1, 2, 3......$$

or

$$\cos^{-1}x = \left(\frac{2k-1}{2}\right)\pi \cdot \frac{1}{(m-1)}$$

or

$$x = \cos\left[\frac{(2k-1)\pi}{2(m-1)}\right] \qquad \text{... (7.149)}$$

But

$$z = \frac{x}{x_0} = \cos\psi/2 = \cos\delta$$

or

$$x = x_0 \cos\psi/2 \qquad \text{... (7.150)}$$

and

$$\psi = \frac{2\pi}{\lambda} \cdot d \sin\theta$$

$$\psi = \beta\, d \sin\theta \qquad \text{... (7.151)}$$

Hence

$$x = x_0 \cos\psi/2 = \cos\left[\frac{2(k-1)\pi}{2(m-1)}\right].$$

or

$$\cos\psi/2 = \frac{1}{x_0}\cos\left[\frac{(2k-1)\pi}{2(m-1)}\right]$$

or

$$\psi/2 = \cos^{-1}\left[\frac{1}{x_0}\cos\frac{(2k-1)\pi}{2(m-1)}\right]$$

or

$$\frac{\beta a}{2}\sin\theta = \cos^{-1}\left[\frac{1}{x_0}\cos\frac{(2k-1)\pi}{2(m-1)}\right]$$

or

$$\frac{2\pi}{\lambda} \cdot \frac{d\sin\theta}{2} = \cos^{-1}\left[\frac{1}{x_0}\cos\frac{(2k-1)\pi}{2(m-1)}\right]$$

or

$$\sin\theta = \frac{\lambda}{\pi d}\cos^{-1}\left[\frac{1}{x_0}\cos\frac{(2k-1)\pi}{2(m-1)}\right]$$

or

$$\boxed{\theta_{null} = \sin^{-1}\left\{\cos^{-1}\left[\frac{1}{x_0}\cos\frac{(2k-1)\pi}{2(m-1)}\right]\right.} \qquad \text{... (7.152)}$$

The angle θ_{null} for which nulls occur are determined from the eqn. 7.152. The beam width between first null is given by $2\,\theta_{null}$ where $k = 1$ i.e.

$$2\,\theta_{null} = BWFN = 2\sin^{-1}\left\{\frac{\lambda}{\pi d}\cos^{-1}\left[\frac{1}{x_0}\cos\frac{(2\cdot 1-1)\pi}{2(m-1)}\right]\right.$$

$$\boxed{BWFN = 2\sin^{-1}\left\{\frac{\lambda}{\pi d}\cos^{-1}\left[\frac{1}{x_0}\cos\frac{\pi}{2(m-1)}\right]\right.} \qquad \text{... (7.153)}$$

7.14.6. *Half power beam width (HPBW) and minor lobes maxima of chebyshev polynomials patterns.* The HPBW may be determined using the definition and properties of Dolph-Tchebyscheff polynomials in connection with array pattern. It is known that main lobe is totally associated with the range $|x| \geq 1$. Therefore, in order to find the HPBW, equate that part of the chebyshev polynomial governing the range $|x| \geq 1$ to $r/2$ *i.e.*

$$\cosh\{(m-1)\cosh^{-1}x\} = \frac{r}{\sqrt{2}}$$

or $$(m-1)\cosh^{-1}x = \cosh^{-1}\left(\frac{r}{\sqrt{2}}\right)$$

or $$\cosh^{-1}x = \frac{\cosh^{-1}\left(\frac{r}{\sqrt{2}}\right)}{(m-1)}$$

or $$x = \cosh\left[\frac{\cosh^{-1}\left(\frac{r}{\sqrt{2}}\right)}{(m-1)}\right] \qquad \text{... (7.154)}$$

But $$z = \frac{x}{x_0} = \cos\psi/2$$

or $$x = x_0\cos\psi/2$$

or $$x_0\cos\psi/2 = \cosh\left[\frac{\cosh^{-1}\left(\frac{r}{\sqrt{2}}\right)}{(m-1)}\right]$$

or $$\cos\psi/2 = \frac{1}{x_0}\cosh\left[\frac{\cosh^{-1}\left(\frac{\gamma}{\sqrt{2}}\right)}{(m-1)}\right]$$

or $$\psi/2 = \cos^{-1}\left[\frac{1}{x_0}\left\{\cosh\frac{\cosh^{-1}\left(\frac{r}{\sqrt{2}}\right)}{(m-1)}\right\}\right]$$

or $$\psi = 2\cos^{-1}\left[\frac{1}{x_0}\cosh\left\{\frac{\cosh^{-1}\left(\frac{r}{\sqrt{2}}\right)}{(m-1)}\right\}\right] \qquad \text{... 7.154 (a)}$$

But $$\psi = \beta d\sin\theta$$

and Let θ_{HP} = Semi-half power beam width angle, then

$$\sin\theta_{HP} = \frac{\psi}{\beta d}$$

or $$\theta_{HP} = \sin^{-1}\left(\frac{\psi}{\beta d}\right)$$

$$\boxed{\theta_{HP} = \sin^{-1}\left\{\frac{2}{\beta d}\cos^{-1}\left[\frac{1}{x_0}\frac{\cosh\left(\cosh^{-1}\frac{r}{\sqrt{2}}\right)}{(m-1)}\right]\right\}}$$

Hence HPBW is given by $2\,\theta_{HP}$

i.e.

$$\boxed{HPBW = 2\,\theta_{HP} = 2\sin^{-1}\left\{\frac{\lambda}{\pi d}\cos^{-1}\left[\frac{1}{x_0}\frac{\cosh\left(\cosh^{-1}\frac{r}{\sqrt{2}}\right)}{(m-1)}\right]\right\}} \qquad \text{... (7.155)}$$

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The minor lobes maxima is determined by differentiating eqn. 7.113 (a) and setting it to zero i.e.

$$\frac{d}{dx}\left\{T_{(m-1)}(x)\right\} = 0$$

$$\frac{d}{dx}\left\{\cos\left[(m-1)\cos^{-1}x\right]\right\} = 0 \qquad \text{... 7.156 (a)}$$

where

$$\cos^{-1}x = \delta$$

or

$$x = \cos\delta$$

Now from eqn. 7.156 (a)

$$\sin\left[(m-1)\cos^{-1}x\right] = 0$$

or

$$\sin\left[(m-1)\cos^{-1}x\right] = \sin k\pi$$

or

$$(m-1)\cos^{-1}x = k\pi$$

$$\cos^{-1} = \frac{k\pi}{(m-1)}$$

or

$$x = \cos\left(\frac{k\pi}{m-1}\right)$$

But

$$z = \frac{x}{x_0} = \cos\psi/2$$

or

$$x_0\cos\psi/2 = x = \frac{\cos k\pi}{(m-1)}$$

or

$$\cos\psi/2 = \frac{1}{x_0}\frac{\cos k\pi}{(m-1)}$$

or

$$\psi/2 = \cos^{-1}\left[\frac{1}{x_0}\frac{\cos k\pi}{(m-1)}\right]$$

or

$$\psi = 2\cos^{-1}\left[\frac{1}{x_0}\cos\left(\frac{k\pi}{(m-1)}\right)\right] \qquad \text{... 7.156 (b)}$$

But

$$\beta d\sin\theta = \psi$$

or

$$\sin\theta_{NM} = \frac{\psi}{\beta d}$$

or

$$\sin\theta_{NM} = \frac{2\cos^{-1}\left[\frac{1}{x_0}\cos\left(\frac{k\pi}{m-1}\right)\right]}{\beta d}$$

or

$$\boxed{\theta_{NM} = \sin^{-1}\left[\frac{\lambda}{\pi d}\cos^{-1}\left\{\frac{1}{x_0}\cos\left(\frac{k\pi}{m-1}\right)\right\}\right]} \qquad \text{... (7.157)}$$

where θ_{NM} = Angles at which null. maxima occur.

7.15. CONTINUOUS ARRAYS

Continuous arrays or continuous aperture distribution is the array of infinite number of point sources separated by infinitesimally small distances between them. Such arrays are normally known as continuous arrays because individual point sources or element are so close together that they form an essentially