Due to symmetry

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

$M = 20\,kg \Rightarrow mg = 196\,N$

$B = \underline{\underline{200}}$

Force equations

$$m\{\dot{u} + qw - rv\} = R(T_{x_1} + T_{x_2}) - (W-B)\sin\theta + D_x$$

$$m\{\dot{v} + ru - pw\} = (W-B)\cos\theta\,\sin\phi + 0 + D_y$$

$$m\{\dot{w} + pv - qu\} = (W-B)\cos\theta\,\cos\phi + R(T_{z_1} + T_{z_2}) + D_z$$

$$D = \frac{1}{2} C_D\, \rho\, A_c\, V^2$$

$\rho = 1000\,kg/m^3$, $A \rightarrow$ frontal area

$Cd \rightarrow$ drag coefficient $= 0.82$

$$D_x = \frac{1}{2} \times 0.82 \times 1000\,\frac{kg}{m^3} \times 0.045\,m^2 \times V\,\frac{m^2}{s^2}$$

$r = 12\,cm$

$\pi r^2 = 0.045$

$(x, y, z, \theta, \phi, \psi) \longrightarrow$ output

$(T_{x_1}, T_{x_2}, T_{z_1}, T_{z_2}) \longrightarrow$ Inputs

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin(\theta) \\ 0 & \cos(\phi) & \cos(\theta).\sin(\phi) \\ 0 & -\sin\theta & \cos(\theta).\cos(\phi) \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

$$L = \dot{p} I_{xx} + rq(I_{zz} - I_{yy}) = 0 \Rightarrow \boxed{\dot{p} = \frac{rq(I_{yy} - I_{zz})}{I_{xx}}}$$

$$M = \dot{q} I_{yy} + rp(I_{xx} - I_{zz}) \Rightarrow$$

$$N = \dot{r} I_{zz} + pq(I_{yy} - I_{xx})$$

Torque equations

$$\dot{q} I_{yy} + rp(I_{xx} - I_{zz}) = (T_{x1} + T_{x2}) z_p + T_{z1} l_{z1} - T_{z2} l_{z2}$$

$$\boxed{\dot{q} = \frac{(T_{x1} + T_{x2}) z_p + (T_{z1} l_{z1} - T_{z2} l_{z2}) + rp(I_{zz} - I_{xx})}{I_{yy}}}$$

$$\dot{r} I_{zz} + pq(I_{yy} - I_{xx}) = (T_{x1} - T_{x2}) y_p$$

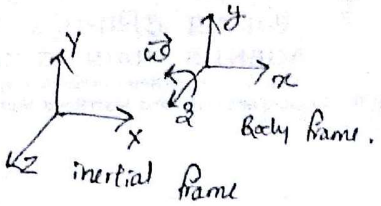
$$\dot{r} = \frac{(T_{x1} - T_{x2}) y_p + pq(I_{xx} - I_{yy})}{I_{zz}}$$

velocities in inertial frame

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = T \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

velocities in body frame

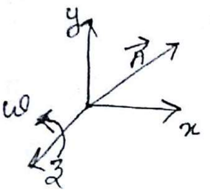
Body frame and inertial frame concepts



Consider Body frame,

Let $(\vec{A})$ in the body frame (constant vector)

Body frame rotating around 'z' axis with an angular velocity of 'ω'.



$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\frac{d\vec{A}}{dt} = \frac{d(A_x\hat{i})}{dt} + \frac{d}{dt}(A_y\hat{j}) + \frac{d}{dt}(A_z\hat{k})$$

$\hat{i}, \hat{j}, \hat{k}$ are in Body frame

so

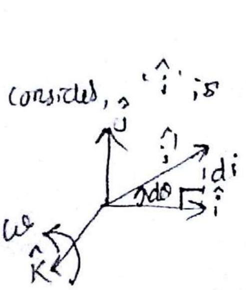
$\frac{d\hat{i}}{dt} \neq 0$

$$= \left(\frac{dA_x}{dt}\hat{i} + \frac{d\hat{i}}{dt}A_x\right) + \left(\frac{dA_y}{dt}\hat{j} + \frac{d\hat{j}}{dt}A_y\right) + \left(\frac{dA_z}{dt}\hat{k} + \frac{d\hat{k}}{dt}A_z\right)$$

$$= \left(\frac{dA_x}{dt}\hat{i} + \frac{dA_y}{dt}\hat{j} + \frac{dA_z}{dt}\hat{k}\right) + \left(A_x\frac{d\hat{i}}{dt} + A_y\frac{d\hat{j}}{dt} + A_z\frac{d\hat{k}}{dt}\right)$$

$$= \left(\frac{d\vec{A}}{dt}\right)_{\text{Body frame}} + \left(A_x\frac{d\hat{i}}{dt} + A_y\frac{d\hat{j}}{dt} + A_z\frac{d\hat{k}}{dt}\right)$$

consider, '$\hat{i}$' is rotated to $\hat{i}$ with '$\vec{\omega}$'



$d\theta = \frac{d\hat{i}}{1} \Rightarrow d\hat{i} = d\theta \rightarrow$ magnitude of $d\hat{i}$

direction of $d\hat{i} = \hat{\omega} \times \hat{i}$

so, $d\hat{i} = (d\theta) \cdot \hat{\omega} \times \hat{i}$

$\frac{d\hat{i}}{dt} = \left(\frac{d\theta}{dt}\right) \hat{\omega} \times \hat{i}$

$\frac{d\hat{i}}{dt} = (\omega \cdot \hat{\omega}) \times \hat{i}$

$= \vec{\omega} \times \hat{i}$

so,

$$\left(\frac{d\vec{A}}{dt}\right) = \left(\frac{dA}{dt}\right)_B + A_x(\vec{\omega} \times \hat{i}) + A_y(\vec{\omega} \times \hat{j}) + A_z(\vec{\omega} \times \hat{k})$$

$$= \left(\frac{dA}{dt}\right)_B + \omega \times (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})$$

$$= \underbrace{\left(\frac{d\vec{A}}{dt}\right)_B}_{\text{Body frame}} + \underbrace{(\omega \times \vec{A}.)}_{\text{effect of rotation}}$$

'$\vec{A}$' in body frame $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$, derivative of $\vec{A}$ in this rotating plane is not simply derivative of $\left(\frac{dA_x}{dt}\hat{i} + \frac{dA_y}{dt}\hat{j} + \frac{dA_z}{dt}\hat{k}\right) \rightarrow \left(\frac{dA}{dt}\right)_B$, there will be extra terms due to rotation, so, the complete derivative of $\vec{A}$ is

$$\left(\frac{d\vec{A}}{dt}\right) = \left(\frac{d\vec{A}}{dt}\right)_B + (\vec{\omega} \times \vec{A})$$

```
%initializations
clear pos_x pos_y pos_z ori_phi ori_theta ori_si

clc
M=20; %mass of 26 Kg
g=9.8; %9.8 m/s2
W=M*g; %Weight of the body
B=196.2; %Bouyancy force

A=0.045; %frontal area
C=0.82; %Drag coefficient
Ro=997; %density of the water

dt=0.2; %time step

zp=0.1; %z coordinate of the propulsive force
lz1=0.1; %
lz2=0.01;
lx1=0.12;
lx2=0.12;
yp=lx1+lx2;

%moments of inertia
Ixx=10;
Iyy=10;
Izz=10;

%Euler angles
theta=0.01;
phi=0.01;
si=0.01;



%input thrust
Tx1=[0 0 0 0 0 15 15 15 15 15 15 15 15 15 15 15 15 15 15 15 15 15 15 15 15 15 15
15];
Tx2=[0 0 0 0 0 15 15 15 15 15 15 15 15 15 15 15 15 15 15 10 10 10 10 10 10 10 10
10];
Tz1=5;
Tz2=5;

%angular velocities in body frame
p=0;
q=0;
r=0;

%velocities in body frame
u=0;
v=0;
w=0;

%velocities in inertial frame
x_dot=0;
```

```
y_dot=0;
z_dot=0;

%pose in inertial frame
x=0;
y=0;
z=0;



for i=1:28

%drag forces
Dx=0.5*C*Ro*A*(u^2);
Dy=0.5*C*Ro*A*(v^2);
Dz=0.5*C*Ro*A*(w^2);

%derivative of velocities in body frame from equation of motion
u_dot=(((Tx1(i)+Tx2(i))-(W-B)*sin(theta)-Dx)/M)-(q*w-r*v);
v_dot=(((W-B)*cos(theta)*sin(phi)-Dy)/M)-(r*u-p*w);
w_dot=(((W-B)*cos(theta)*cos(phi)+(Tz1+Tz2)-Dz)/M)-(p*v-q*u);

%derivatives of angular velocities in body frame
p_dot=((Iyy-Izz)*r*q)/Ixx;
q_dot=((Tx1(i)-Tx2(i))*zp+(Tz1*lz1-Tz2*lz2)+r*p*(Izz-Ixx))/Iyy;
r_dot=((Tx1(i)-Tx2(i))*yp+(Ixx-Iyy)*p*q)/Izz;

%velocities in body frame
u=u+(u_dot*dt);
v=v+(v_dot*dt);
w=w+(w_dot*dt);
wo=[u;v;w];

%angular velocities
p=p+(p_dot*dt);
q=q+(q_dot*dt);
r=r+(r_dot*dt);
ao=[p;q;r];

%rotation matrix
R=[cos(si)*cos(theta) cos(si)*sin(theta)*sin(phi)-sin(si)*cos(phi)
sin(si)*sin(phi)+cos(si)*cos(phi)*sin(theta);
    sin(si)*cos(theta) cos(si)*cos(phi)+sin(phi)*sin(theta)*sin(si)
sin(theta)*sin(si)*cos(phi)-cos(si)*sin(phi);
    -sin(theta) cos(theta)*sin(phi) cos(theta)*cos(phi)];


%velocites in inertial frame
X_dot=R*wo;
x_dot=X_dot(1);
y_dot=X_dot(2);
z_dot=X_dot(3);

%rotation matrix for angular velocities
```

```
T=[1 0 -sin(theta);
    0 cos(phi) cos(theta)*sin(phi);
    0 -sin(theta) cos(theta)*cos(phi)];

%for derivative of angular velocities in inertial frame
Ao=inv(T)*ao;
phi_dot=Ao(1);
theta_dot=Ao(2);
si_dot=Ao(3);

%euler angles
phi=phi+(phi_dot*dt);
theta=theta+(theta_dot*dt);
si=si+(si_dot*dt);


if(phi>180)
    phi=180;
end
 if (phi<-180)
     phi=-180;
 end
 if(theta>90)
     theta=90;
 end
 if(theta<-90)
     theta=-90;
 end
 if(si>360)
     si=360;
 end
 if(si<0)
     si=0;
 end




%pose in inertial frame
x=x+(x_dot*dt);
y=y+(y_dot*dt);
z=z+(z_dot*dt);

pos_x(i)=x;
pos_y(i)=y;
pos_z(i)=z;

ori_phi(i)=phi;
ori_theta(i)=theta;
ori_si(i)=si;


end
```

For the analysis, first we done the modelling in XY plane (2D analysis). For simplicity as the part of learning ,considered a system with two thrusters in same direction(like a differential drive) in this example.

Matlab code:

```
%initializations
clear pos_x pos_y ori_si vel_u vel_v

%%for the visualization of an object moving in 2D plane
close(figure(1))
ax = axes('XLim',[-50 50],'YLim',[-50 50]);
view(2)
grid on;
[x,y] = cylinder([.2 0.2]);
h(1) = surface(y,x,'FaceColor','yellow');
h(2) = surface(y,x,'FaceColor','blue');
grid on
xlabel('X');
ylabel('Y');
t = hgtransform('Parent',ax);
set(h,'Parent',t)
drawnow

%%

k=1000;
clc
M=20; %mass of 26 Kg
g=9.8; %9.8 m/s2
W=M*g; %Weight of the body
B=196.2; %Bouyancy force

A=0.045; %frontal area
C=0.5; %Drag coefficient
Ro=997; %density of the water

dt=0.1; %time step

lx1=0.12; %length between thruster1 mounted point and centre of mass.
lx2=0.12;
yp=lx1+lx2;

%moments of inertia
Izz=10; %not a calculated value.You need to calculate it :)

%Euler angles
si=0.01;

%input thrust
 Tx2=15;
 Tx1=15;
```

```
%angular velocities in body frame
r=0;

%velocities in body frame
u=0;
v=0;

%velocities in inertial frame
x_dot=0;
y_dot=0;

%pose in inertial frame
x=0;
y=0;

ti=0;



for i=1:k

%drag forces
%Dx=0.5*C*Ro*A*(u^2);
Dy=0.5*C*Ro*A*(v^2);

%derivative of velocities in body frame from equation of motion
u_dot=(r*v);
v_dot=(((Tx1+Tx2)-Dy)/M)-(r*u);

%derivatives of angular velocities in body frame
r_dot=((Tx1-Tx2)*yp)/Izz;

%velocities in body frame
u=u+(u_dot*dt);
v=v+(v_dot*dt);

wo=[u;v];

%angular velocities
r=r+(r_dot*dt);
ao=r;

%rotation matrix
R=[cos(si) -sin(si);
    sin(si) cos(si)];

%velocites in inertial frame
X_dot=R*wo;
x_dot=X_dot(1);
y_dot=X_dot(2);

%for derivative of angular velocities in inertial frame
```

```
si_dot=r;

%euler angles
si=si+(si_dot*dt);

%pose in inertial frame
x=x+(x_dot*dt);
y=y+(y_dot*dt);

pos_x(i)=x;
pos_y(i)=y;

vel_u(i)=u;
vel_v(i)=v;

ori_si(i)=si;

dragF(i)=Dy;

%for plot in 2D plane
translation=makehgtform('translate',[pos_x(i),pos_y(i),0]);
zrotational=makehgtform('zrotate',ori_si(i));
set(t,'matrix',translation*zrotational);
pause(0.1)

end

pos_x=pos_x';
pos_y=pos_y';
ori_si=ori_si';
vel_u=vel_u';
vel_v=vel_v';
dragF=dragF';
time=0:0.1:9.9;
figure(2)
plot(time,vel_v)
grid on
xlabel('Time');
ylabel('Velocity_v');
figure(3)
plot(time,dragF)
xlabel('Time');
ylabel('Force');
```

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