Add Digits

Given a non-negative integer num, repeatedly add all its digits until the result has only one digit.

For example:

Given $\frac{1}{1} = \frac{38}{1}$, the process is like: $\frac{3}{1} + \frac{8}{1} = \frac{11}{1}$, $\frac{1}{1} + \frac{1}{1} = \frac{2}{1}$. Since $\frac{2}{1}$ has only one digit, return it.

Follow up:

Could you do it without any loop/recursion in O(1) runtime?

- 1. A naive implementation of the above process is trivial. Could you come up with other methods?
- 2. What are all the possible results?
- 3. How do they occur, periodically or randomly?
- 4. You may find this Wikipedia article useful.

Credits:

Special thanks to @jianchao.li.fighter for adding this problem and creating all test cases.

Solution 1

The problem, widely known as *digit root* problem, has a congruence formula:

```
https://en.wikipedia.org/wiki/Digital_root#Congruence_formula
```

For base b (decimal case b = 10), the digit root of an integer is:

- dr(n) = 0 if n == 0
- dr(n) = (b-1) if n != 0 and n % (b-1) == 0
- $dr(n) = n \mod (b-1) \text{ if } n \% (b-1) != 0$

or

• dr(n) = 1 + (n - 1) % 9

Note here, when n = 0, since (n - 1) % 9 = -1, the return value is zero (correct).

From the formula, we can find that the result of this problem is immanently periodic, with period (b-1).

Output sequence for decimals (b = 10):

```
~input: 0 1 2 3 4 ...
output: 0 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 1 2 3 ....
```

Henceforth, we can write the following code, whose time and space complexities are both O(1).

```
class Solution {
public:
    int addDigits(int num) {
       return 1 + (num - 1) % 9;
    }
};
```

Thanks for reading. :)

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Solution 2

```
int addDigits(int num) {
   int res = num % 9;
   return (res != 0 || num == 0) ? res : 9;
}
```

The essence of this problem is that $10^n \equiv 1 \pmod{9}$, and thus $an*10^n + ... + a1*10 + a_0 \equiv a_n + ... + a_1 + a_0 \pmod{9}$. This process can be continued until a number less than 9 is gotten, i.e. num % 9. For any digit n, n = n % 9 unless n = 9. The only confusing case is n % 9 = 0, but addDigits(num) = 0 if and only if num = 0, otherwise it should be 9 in fact.

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Solution 3

If an integer is like 100a+10b+c, then (100a+10b+c)%9=(a+99a+b+9b+c)%9=(a+b+c)%9

```
class Solution:
    # @param {integer} num
    # @return {integer}
    def addDigits(self, num):
        if num==0:
            return 0
        return num%9 if num%9!=0 else 9
```

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