

132 Pattern

Given a sequence of n integers a_1, a_2, \dots, a_n , a 132 pattern is a subsequence a_i, a_j, a_k such that $i < j < k$ and $a_i > a_k > a_j$. Design an algorithm that takes a list of n numbers as input and checks whether there is a 132 pattern in the list.

Note: n will be less than 15,000.

Example 1:

Input: [1, 2, 3, 4]

Output: False

Explanation: There is no 132 pattern in the sequence.

Example 2:

Input: [3, 1, 4, 2]

Output: True

Explanation: There is a 132 pattern in the sequence: [1, 4, 2].

Example 3:

Input: [-1, 3, 2, 0]

Output: True

Explanation: There are three 132 patterns in the sequence: [-1, 3, 2], [-1, 3, 0] and [-1, 2, 0].

Solution 1

We want to search for a subsequence (s_1, s_2, s_3)

INTUITION: The problem would be simpler if we want to find sequence with $s_1 > s_2 > s_3$, we just need to find s_1 , followed by s_2 and s_3 . Now if we want to find a 132 sequence, we need to switch up the order of searching. we want to first find s_2 , followed by s_3 , then s_1 .

IDEA: We can start from either side but I think starting from the end allow us to finish in a single pass. The idea is to start from end and search for a candidate for s_2 and s_3 . A number becomes a candidate for s_3 if there is any number on the left of s_2 that is bigger than it.

DETECTION: Keep track of the largest candidate of s_3 and once we encounter any number smaller than s_3 , we know we found a valid sequence since $s_1 < s_3$ implies $s_1 < s_2$.

IMPLEMENTATION:

1. Have a **stack**, each time we store a new number, we first **pop** out all numbers that are smaller than that number. The numbers that are **popped** out becomes candidate for s_3 .
2. We keep track of the **maximum** of such s_3 (which is always the most recently **popped** number from the **stack**).
3. Once we encounter any number smaller than s_3 , we know we found a valid sequence since $s_1 < s_3$ implies $s_1 < s_2$.

RUNTIME: Each item is **pushed** and **popped** once at most, the time complexity is therefore $O(n)$.

EXAMPLE:

$i = 6$, **nums** = [9, 11, 8, 9, 10, 7, 9], **S1 candidate** = 9, **S3 candidate** = None, **Stack** = Empty

$i = 5$, **nums** = [9, 11, 8, 9, 10, 7, 9], **S1 candidate** = 7, **S3 candidate** = None, **Stack** = [9]

$i = 4$, **nums** = [9, 11, 8, 9, 10, 7, 9], **S1 candidate** = 10, **S3 candidate** = None, **Stack** = [9, 7]

$i = 3$, **nums** = [9, 11, 8, 9, 10, 7, 9], **S1 candidate** = 9, **S3 candidate** = 9, **Stack** = [10]

$i = 2$, **nums** = [9, 11, 8, 9, 10, 7, 9], **S1 candidate** = 8, **S3 candidate** = 9, **Stack** = [10, 9] We have $8 < 9$, sequence found!

EDIT: Thanks @Pumpkin78 and @dalwise for pointing out that the maximum candidate for s_3 is always the recently popped number from the stack, because if we encounter any entry smaller than the current candidate, the function would already have returned.

```
bool find132pattern(vector<int>& nums) {  
    int s3 = INT_MIN;  
    stack<int> st;  
    for( int i = nums.size()-1; i >= 0; i -- ){  
        if( nums[i] < s3 ) return true;  
        else while( !st.empty() && nums[i] > st.top() ){  
            s3 = st.top(); st.pop();  
        }  
        st.push(nums[i]);  
    }  
    return false;  
}
```

written by [jade86](#) original link [here](#)

Solution 2

The idea is that we can use a stack to keep track of previous min-max intervals.

Here is the principle to maintain the stack:

For each number `num` in the array

If stack is empty:

- push a new Pair of `num` into stack

If stack is not empty:

- if `num < stack.peek().min`, push a new Pair of `num` into stack
- if `num >= stack.peek().min`, we first `pop()` out the peek element, denoted as `last`
 - if `num < last.max`, we are done, return `true`;
 - if `num >= last.max`, we merge `num` into `last`, which means `last.max = num`.

Once we update `last`, if stack is empty, we just push back `last`.

However, the crucial part is:

If stack is not empty, the updated `last` might:

- Entirely covered `stack.peek()`, i.e. `last.min < stack.peek().min` (which is always true) && `last.max >= stack.peek().max`, in which case we keep popping out `stack.peek()`.
- Form a 1-3-2 pattern, we are done, return `true`

So at any time in the stack, **non-overlapping Pairs** are formed in descending order by their min value, which means the min value of peek element in the stack is always the min value globally.

```

class Pair{
    int min, max;
    public Pair(int min, int max){
        this.min = min;
        this.max = max;
    }
}

public boolean find132pattern(int[] nums) {
    Stack<Pair> stack = new Stack();
    for(int n: nums){
        if(stack.isEmpty() || n < stack.peek().min ) stack.push(new Pair(n,n))
;
        else if(n > stack.peek().min){
            Pair last = stack.pop();
            if(n < last.max) return true;
            else {
                last.max = n;
                while(!stack.isEmpty() && n >= stack.peek().max) stack.pop();
                // At this time, n < stack.peek().max (if stack not empty)
                if(!stack.isEmpty() && stack.peek().min < n) return true;
                stack.push(last);
            }
        }
    }
    return false;
}

```

written by [leogogogo](#) original link [here](#)

Solution 3

Idea: Find peak and bottom

For every [bottom, peak], find if there is one number $\text{bottom} < \text{number} < \text{peak}$.

```
public class Solution {
    public boolean find132pattern(int[] nums) {
        if(nums.length<3) return false;
        Integer low = null, high = null;
        int start = 0, end = 0;
        while(start<nums.length-1){
            while(start<nums.length-1 && nums[start]>=nums[start+1]) start++;
            // start is lowest now
            int m = start+1;
            while(m<nums.length-1 && nums[m]<=nums[m+1]) m++;
            // m is highest now
            int j = m+1;
            while(j<nums.length){
                if(nums[j]>nums[start] && nums[j]<nums[m]) return true;
                j++;
            }
            start = m+1;
        }
        return false;
    }
}
```

written by [YuTingLiu](#) original link [here](#)

From [LeetCoder](#).