ASSIGNMENT-3

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Question 16.2023: Let X be a random variable with probability density function $f(x) = \begin{cases} \alpha\lambda x^{\alpha-1}e^{-\lambda x^{\alpha}} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$ If the median of X is 1 and the third quantile is 2 then (α, λ) equals

Solution: If median of random variable X is 1, then CDF of X evaluated at 1 is equal to $\frac{1}{2}$
The third quantile is defined as

$$\int_0^{Q3} f(x)dx = \frac{3}{4} \tag{1}$$
$$Given, F(1) = \frac{1}{2} \tag{2}$$
$$F(2) = \frac{3}{4} \tag{3}$$
$$F(x) = \int_{-\infty}^{x} f(x)dx \tag{4}$$
$$x \leq 0, F(x) = 0 \tag{5}$$
$$F(x) = \alpha\lambda \int_0^x x^{\alpha-1}e^{-\lambda x^{\alpha}}dx \tag{6}$$
$$let, x^{\alpha} = y \tag{7}$$

differentiate both sides

$$\alpha x^{\alpha-1}dx = dy \tag{8}$$
$$F(x) = \lambda \int e^{-\lambda y}dy \tag{9}$$
$$= -e^{-\lambda y} \tag{10}$$
$$F(x) = \left(-e^{-\lambda x^{\alpha}}\right)_0^x \tag{11}$$
$$F(x) = 1 - e^{-\lambda x^{\alpha}} \tag{12}$$
$$F(1) = \frac{1}{2} \tag{13}$$
$$\Rightarrow \frac{1}{2} = 1 - e^{-\lambda} \tag{14}$$
$$\lambda = \ln 2 \tag{15}$$
$$F(2) = \frac{3}{4} \tag{16}$$
$$\Rightarrow \frac{3}{4} = 1 - e^{-\lambda 2^{\alpha}} \tag{17}$$
$$\alpha = 1 \tag{18}$$

$\therefore \lambda = \ln 2$ and $\alpha = 1$