

ASSIGNMENT-2

A Varun Naik - EE22BTECH11004

Question 12.13.5.10 : A person buys a lottery ticket in 50 lotteries in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize a) atleast once b) exactly once c) atleast twice ?

Solution: Let X be number of winning prizes in 50 lotteries. The trials are Bernoulli trials. X has binomial distribution with $n = 50$ and $p = \frac{1}{100}$

$$q = 1 - p = 1 - \frac{1}{100} \quad (1)$$

$$q = \frac{99}{100} \quad (2)$$

$$\Pr(X = x) = {}^nC_x q^{n-x} p^x \quad (3)$$

$$= {}^{50}C_x \left(\frac{99}{100}\right)^{50-x} \left(\frac{1}{100}\right)^x \quad (4)$$

$$a) \Pr(\text{atleastonce}) = \Pr(X \geq 1) \quad (5)$$

$$= 1 - \Pr(X < 1) \quad (6)$$

$$= 1 - \Pr(X = 0) \quad (7)$$

$$= 1 - {}^{50}C_0 \left(\frac{99}{100}\right)^{50} \quad (8)$$

$$= 1 - \left(\frac{99}{100}\right)^{50} \quad (9)$$

$$b) \Pr(\text{exactlyonce}) = \Pr(X = 1) \quad (10)$$

$$= {}^{50}C_1 \left(\frac{99}{100}\right)^{49} \left(\frac{1}{100}\right)^1 \quad (11)$$

$$= 50 \left(\frac{99}{100}\right)^{49} \left(\frac{1}{100}\right)^1 \quad (12)$$

$$= \frac{1}{2} \left(\frac{99}{100}\right)^{49} \quad (13)$$

$$c) \Pr(\text{atleasttwice}) = \Pr(X \geq 2) \quad (14)$$

$$= 1 - \Pr(X < 2) \quad (15)$$

$$= 1 - (\Pr(X = 0) + \Pr(X = 1)) \quad (16)$$

$$= (1 - \Pr(X = 0)) - \Pr(X = 1) \quad (17)$$

$$= \left(1 - \frac{99}{100}\right)^{50} - \frac{1}{2} \left(\frac{99}{100}\right)^{49} \quad (18)$$

$$= 1 - \left(\frac{99}{100}\right)^{50} \left(\frac{149}{100}\right) \quad (19)$$

$$= 1 - \left(\frac{149}{100}\right) \left(\frac{99}{100}\right)^{49} \quad (20)$$