

ASSIGNMENT-2

A Varun Naik - EE22BTECH11004

Question 12.13.5.10 : A person buys a lottery ticket in 50 lotteries in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize a) atleast once b) exactly once c) atleast twice ?

Solution: Let X be number of winning prizes in 50 lotteries. The trials are Bernoulli trials. X has binomial distribution with $n = 50$ and $p = \frac{1}{100}$

$$c) \Pr(X \geq 2) = 1 - \Pr(X < 2) \quad (14)$$

$$= 1 - F_X(1) \quad (15)$$

$$= \left(1 - \frac{99}{100}\right)^{50} - \frac{1}{2} \left(\frac{99}{100}\right)^{49} \quad (16)$$

$$= 0.0894 \quad (17)$$

TABLE 0
PARAMETERS FOR CDF

parameter	value
n	50
p	$\frac{1}{100}$
q	$\frac{99}{100}$

$$q = 1 - p = 1 - \frac{1}{100} \quad (1)$$

$$q = \frac{99}{100} \quad (2)$$

$$p_X(k) = \Pr(X = k) \quad (3)$$

$$p_X(k) = {}^nC_k q^{n-k} p^k \quad (4)$$

$$= {}^{50}C_k \left(\frac{99}{100}\right)^{50-k} \left(\frac{1}{100}\right)^k \quad (5)$$

The Cdf for the following pmf :

$$F_X(k) = p_X(0) + p_X(1) + p_X(2) + \dots + p_X(k) \quad (6)$$

$$F_X(k) = \sum_{i=0}^k {}^5C_i \left(\frac{99}{100}\right)^{50-i} \left(\frac{1}{100}\right)^i \quad (7)$$

$$a) \Pr(X \geq 1) = 1 - \Pr(X < 1) \quad (8)$$

$$= 1 - F_X(0) \quad (9)$$

$$= 1 - {}^{50}C_0 \left(\frac{99}{100}\right)^{50} \quad (10)$$

$$= 0.394 \quad (11)$$

$$b) \Pr(X = 1) = {}^{50}C_1 \left(\frac{99}{100}\right)^{49} \left(\frac{1}{100}\right)^1 \quad (12)$$

$$= 0.3055 \quad (13)$$