ASSIGNMENT-2

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Question 12.13.5.10 : A person buys a lottery ticket in 50 lotteries in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize a) atleast once b) exactly once c) atleast twice ?

Solution: Let X be number of winning prizes in 50 lotteries. The trials are Bernoulli trials.X has binomial distribution with n = 50 and p = $\frac{1}{100}$

$$q = 1 - p = 1 - \frac{1}{100} \tag{1}$$
$$q = \frac{99}{100} \tag{2}$$
$$p_X(k) = \Pr(X = k) \tag{3}$$
$$p_X(k) = {}^nC_k q^{n-k} p^k \tag{4}$$
$$= {}^{50}C_k \left(\frac{99}{100}\right)^{50-k} \left(\frac{1}{100}\right)^k \tag{5}$$

The Cdf for the following pmf :

$$F_X(k) = p_X(0) + p_X(1) + p_X(2) + ... + p_X(k) \tag{6}$$
$$= {}^{50}C_0 \left(\frac{99}{100}\right)^{50} + {}^{50}C_1 \left(\frac{99}{100}\right)^{49} \left(\frac{1}{100}\right) + ... + {}^{50}C_k \left(\frac{99}{100}\right)^{50-k} \left(\frac{1}{100}\right)^k \tag{7}$$
$$F_X(k) = \sum_{i=0}^{k} {}^5C_i \left(\frac{99}{100}\right)^{50-i} \left(\frac{1}{100}\right)^i \tag{8}$$

$$a) \Pr(X \geq 1) = 1 - \Pr(X < 1) \tag{9}$$
$$= 1 - F_X(0) \tag{10}$$
$$= 1 - {}^{50}C_0 \left(\frac{99}{100}\right)^{50} \tag{11}$$
$$= 1 - \left(\frac{99}{100}\right)^{50} \tag{12}$$
$$= 0.394 \tag{13}$$

$$b) \Pr(X = 1) = {}^{50}C_1 \left(\frac{99}{100}\right)^{49} \left(\frac{1}{100}\right)^1 \tag{14}$$
$$= 50 \left(\frac{99}{100}\right)^{49} \left(\frac{1}{100}\right)^1 \tag{15}$$
$$= \frac{1}{2} \left(\frac{99}{100}\right)^{49} \tag{16}$$
$$= 0.3055 \tag{17}$$

$$c) \Pr(X \geq 2) = 1 - \Pr(X < 2) \tag{18}$$
$$= 1 - F_X(1) \tag{19}$$
$$= 1 - \left({}^{50}C_0 \left(\frac{99}{100}\right)^{50} + {}^{50}C_1 \left(\frac{99}{100}\right)^{49} \left(\frac{1}{100}\right)\right) \tag{20}$$
$$= \left(1 - \frac{99}{100}\right)^{50} - \frac{1}{2}\left(\frac{99}{100}\right)^{49} \tag{21}$$
$$= 1 - \left(\frac{99}{100}\right)^{49} \left(\frac{149}{100}\right) \tag{22}$$
$$= 1 - \left(\frac{149}{100}\right) \left(\frac{99}{100}\right)^{49} \tag{23}$$
$$= 0.0894 \tag{24}$$