ASSIGNMENT-2

A Varun Naik - EE22BTECH11004

Question 12.13.5.10 : A person buys a lottery ticket in 50 lotteries in each of which his chance of winning a prize is $\frac{1}{100}$. What is the prbability that he will win a prize a) atleast once b) exactly once c) atleast twice ?

Solution: Let X be number of winning prizes in 50 lotteies. The trials are Bernoulli trials. X has binomial distribution with n = 50 and p = $\frac{1}{100}$

$$q = 1 - p = 1 - \frac{1}{100} \tag{1}$$

$$q = \frac{99}{100} \tag{2}$$

$$\Pr(X = x) = {}^nC_x q^{n-x} p^x \tag{3}$$

$$= {}^{50}C_x \left(\frac{99}{100}\right)^{50-x} \left(\frac{1}{100}\right)^x \tag{4}$$

$a)\ \Pr(atleastonce) = \Pr(X \geq 1) \tag{5}$

$$= 1 - \Pr(X < 1) \tag{6}$$

$$= 1 - \Pr(X = 0) \tag{7}$$

$$= 1 - {}^{50}C_x \left(\frac{99}{100}\right)^{50} \tag{8}$$

$$= 1 - \left(\frac{99}{100}\right)^{50} \tag{9}$$

$b)\ \Pr(exactlyonce) = \Pr(X = 1) \tag{10}$

$$= {}^{50}C_1 \left(\frac{99}{100}\right)^{49} \left(\frac{1}{100}\right)^1 \tag{11}$$

$$= 50 \left(\frac{99}{100}\right)^{49} \left(\frac{1}{100}\right)^1 \tag{12}$$

$$= \frac{1}{2} \left(\frac{99}{100}\right)^{49} \tag{13}$$

$c)\ \Pr(atleasttwice) = \Pr(X \geq 2) \tag{14}$

$$= 1 - \Pr(X < 2) \tag{15}$$

$$= 1 - (\Pr(X = 0) + \Pr(X = 1)) \tag{16}$$

$$= (1 - \Pr(X = 0)) - \Pr(X = 1) \tag{17}$$

$$= \left(1 - \frac{99}{100}\right)^{50} - \frac{1}{2} \left(\frac{99}{100}\right)^{49} \tag{18}$$

$$= 1 - \left(\frac{99}{100}\right)^{50} \left(\frac{149}{100}\right) \tag{19}$$

$$= 1 - \left(\frac{149}{100}\right) \left(\frac{99}{100}\right)^{50} \tag{20}$$