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I pledge my honor that I have abided by the Stevens Honor System.

Point values are assigned for each question.   Points earned: ____ / 100, = ____ %

1. Find an upper bound for $f(n) = n^4 + 10n^2 + 5$. Write your answer here: $O(n^4)$ (4 points)

   Prove your answer by giving values for the constants c and n_0. Choose the smallest integral value possible for c. (4 points)

$$n^4 + 10n^2 + 5 \leq 2n^4$$
$$c = 2$$
$$n_0 = 11$$

2. Find an asymptotically tight bound for $f(n) = 3n^3 - 2n$. Write your answer here: $\theta(n^2)$ (4 points)

   Prove your answer by giving values for the constants c_1, c_2, and n_0. Choose the tightest integral values possible for c_1 and c_2. (6 points)

$$2n^3 \leq 3n^3 - 2n \leq 4n^3$$
$$c_1 = 2$$
$$c_2 = 4$$
$$n_0 = 2$$

3. Is $3n - 4 \epsilon \Omega(n^2)$? Circle your answer: yes / no. (2 points)

   If yes, prove your answer by giving values for the constants c and n_0. Choose the smallest integral value possible for c. If no, derive a contradiction. (4 points)

$$3n - 4 \geq n^2$$
$$for\ n = 3$$
$$3(3) - 4 \,! \geq 3^2$$

4. Write the following asymptotic efficiency classes in **increasing** order of magnitude.
   $O(n^2)$, $O(2^n)$, $O(1)$, $O(n \lg n)$, $O(n)$, $O(n!)$, $O(n^3)$, $O(\lg n)$, $O(n^n)$, $O(n^2 \lg n)$ (2 points each)

   $O(1)$,$O(logn)$,$O(n)$,$O(nlogn)$,$O(n^2)$,$O(n^2logn)$, $O(n^3)$, $O(2^n)$,$O(n!)$, $O(n^n)$

5. Determine the largest size *n* of a problem that can be solved in time *t*, assuming that the algorithm takes *f(n)* milliseconds. *n* must be an integer. (2 points each)

   a. *f(n)* = n, *t* = 1 second   1,000

   b. *f(n)* = $n \lg n$, *t* = 1 hour   204,094

   c. *f(n)* = n^2, *t* = 1 hour   1,897

d. $f(n) = n^3$, t = 1 day 442

e. $f(n) = n!$, t = 1 minute 7

6. Suppose we are comparing two sorting algorithms and that for all inputs of size n the first algorithm runs in $4n^3$ seconds, while the second algorithm runs in $64n \; lg \; n$ seconds. For which integral values of n does the first algorithm beat the second algorithm? $2 \leq n \leq 6$ (4 points)
   Explain how you got your answer or paste code that solves the problem (2 point):
   - I graphed the two functions and found their points of intersection.

7. Give the complexity of the following methods. Choose the most appropriate notation from among O, Θ, and Ω. (8 points each)

```
int function1(int n) {
    int count = 0;
    for (int i = n / 2; i <= n; i++) {
        for (int j = 1; j <= n; j *= 2) {
            count++;
        }
    }
    return count;
}
```

Answer: $\theta(nlgn)$

```
int function2(int n) {
    int count = 0;
    for (int i = 1; i * i * i <= n; i++) {
        count++;
    }
    return count;
}
```

Answer: $\theta(\sqrt[3]{n})$

```
int function3(int n) {
    int count = 0;
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= n; j++) {
            for (int k = 1; k <= n; k++) {
                count++;
            }
        }
    }
    return count;
}
```

Answer: $\theta(n^3)$

```
int function4(int n) {
    int count = 0;
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= n; j++) {
            count++;
            break;
        }
    }
    return count;
}
```

Answer: $\theta(n)$

```
int function5(int n) {
    int count = 0;
    for (int i = 1; i <= n; i++) {
        count++;
    }
    for (int j = 1; j <= n; j++) {
        count++;
    }
    return count;
}
```

Answer: $\theta(n)$