FIN251 Fall 2022
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Laboratory Exercise for Group Project II
Constant Expected Return Model, Monte Carlo Simulation and Bootstrapping
Due: Thursday, Nov. 16th

Readings

- Lecture notes on Constant Expected Return Model and bootstrapping.

Programs and Data

- CerModelExamples.r
- bootStrap.r

Instructions: In this lab you will use R to

- Estimate parameters of the constant expected return (CER) model, compute standard errors and confidence intervals and test various hypotheses about the parameters and assumptions of the model.
- Perform Monte Carlo simulations of CER model.
- Perform bootstrapping of CER model estimates.

Exercises

The following questions require R. Copy and paste all statistical results and graphs into a MS Word document while you work, and add comments and answer all questions in this document. Please do not turn in the assignment without comments!

In this lab, you will analyze continuously compounding monthly return data on the stock or asset that you are interested in. The script file walks you through all of the computations for the lab. You do not need to show the R commands in your lab write up. The Monte Carlo and bootstrapping questions will build on these results. You will use functions from the boot, PerformanceAnalytics, tseries, and zoo packages. Remember to install these packages before you load them into R.

1. Consider the constant expected return model (CER)

$$\begin{aligned} R_{it} &= \mu_i + \varepsilon_{it}, \quad t = 1, \ldots, T = 60. \\ \varepsilon_{it} &\sim iid \quad N(0, \sigma_i^2) \\ cov(\varepsilon_{it}, \varepsilon_{jt}) &= \sigma_{ij} \end{aligned}$$

where R_{it} denotes the continuously compounded return on asset i, i = your stock, Vanguard long term bond index fund Instl. (VBLLX), Fidelity Magellan stock mutual fund (FMAGX). Follow the commands in the R script file to answer the following questions.

- a) Using sample descriptive statistics, give estimates for the model parameters $\mu_i, \sigma_i^2, \sigma_i, \sigma_{i,j}$ and ρ_{ij}.
  - The R functions together with the R function apply() can be used to compute these estimates from the matrix object ret.mat.
- b) For each estimate of the above parameters (except σ_{ij}) compute the estimated standard error. That is, compute $SE(\hat{\mu}_i), SE(\hat{\sigma}_i), SE(\hat{\sigma}_i^2)$ and $SE(\hat{\rho}_{ij})$. Show the estimates with the corresponding SE values underneath. Briefly comment on the precision of the estimates.
  - The formulas for these standard errors were given in class, and are given in the lecture notes on the constant expected return model
- c) Using the technique of Monte Carlo simulations, create 1000 simulated data sets of size $T = 60$ from the CER model using as true parameters the estimate parameters for FMAGX: $\hat{\mu}_{FMAGX} = -0.0008, \quad \hat{\sigma} = 0.06716, \quad \hat{\sigma}^2 = 0.00451$. Use set. seed(123) to initialize the random number generator.
  - For each of the 1000 data sets, compute $\hat{\mu}, \hat{\sigma}^2, \hat{\sigma}$ using the R functions.
  - Create histograms for the 1000 values of$\hat{\mu}, \hat{\sigma}^2, \hat{\sigma}$. Are the centers of these histograms close to the true values given above? Do the distribution look normal?
  - Compute the average and standard deviation of $\hat{\mu}, \hat{\sigma}^2, \hat{\sigma}$ across 1000 data sets. How close are the Monte Carlo averages of $\hat{\mu}, \hat{\sigma}^2, \hat{\sigma}$ to their true values? How close are the Monte carlo standard deviations to the analytic standard error estimates of $\hat{\mu}, \hat{\sigma}^2, \hat{\sigma}$ computed from the actual data from part (b) above?
- d) **If you are using the statistics you have obtained so far to write your term paper, in which section you should put the statiscis? Provide your itemized summary of the term paper. The summary must include "the section of the summary statistics you have obtained above".**

2. Bootstrapping the CER model estimates

- a) For each estimate of your stock model parameters $\mu_i, \sigma_i^2, \sigma_i$, and ρ_{ij}, compute the estimated standard error using the bootstrap with 999 bootstrap replications. That is compute $SE(\hat{\mu_i}), SE(\hat{\sigma}_i), SE(\hat{\sigma}_i^2)$ and $SE(\hat{\rho}_{ij})$. Compare the bootstrap standard errors to the analytic standard errors you computed in Question 1 part (b).

- b) For each asset, compute estimates of the 5% value-at-risk based on an initial \$100,000 investment. Use the bootstrap to compute $SE(\widehat{VaR}_{0.05})$ values as well as 95% confidence intervals. Briefly comment on the accuracy of the 5% VaR estimates.