COMP 2804: Assignment 3

Due Date: Sunday, March 14th at 11:59PM

School of Computer Science   Carleton University

Your assignment should be submitted online on cuLearn as a single .pdf file. No late assignments will be accepted. You can type your assignment or you can upload a scanned copy of it. Please, use a good image capturing device. Make sure that your upload is clearly readable. If it is difficult to read, it will not be graded!

Question 1 [10 marks]

Consider a set of $n \geq 3$ people sitting in a circle around a campfire. What is the probability that three particular people, call them A, B, and C, sit together contiguously? Justify your answer.

Question 2 [10 marks]

Consider a set of $n \leq 12$ people. What is the probability that two or more people in the set share the same birth month? Assume there is an equal probability of being born in each month. Justify your answer.

Question 3 [10 marks]

Provide an example of a probability space and events where the events are pairwise independent, but not mutually independent. Try to make your sample space as small as possible. Show that your example is correct.

Question 4 [10 marks]

Suppose you draw cards with replacement from a standard 52 card deck. First, you draw one card. If it is a face card (i.e. Jack, Queen, or King), you draw one more card. If it is not a face card, you draw two more cards. Compute the probability that you draw exactly one face card and exactly one ace. Assume that the card is replaced after each draw. Justify your answer.

Question 5 [10 marks]

Consider a three player game where players take turns rolling a standard six-sided die. The winner is the first player who rolls a six. What is the probability the player who rolls first wins the game? What is the probability the player who rolls second wins the game? What is the probability the player who rolls third wins the game? Justify your answer.

Question 6 [10 marks]

Consider the set $X = \{a, b, c, d, e, f, g, h\}$. Suppose we choose a random 5-element subset of X, call it Y. Define the events:

- A is the event a and b are in Y

- B is the event g or h is in Y

Compute $P(A|B)$ and $P(B|A)$. Justify your answer.

Question 7 [10 marks]

Consider a scenario where we flip a coin 8 times. Define the events:

- A is the event the first flip is Heads

- B is the event exactly four flips are Heads and exactly four flips are Tails

Are the events A and B are independent? Justify your answer.

Question 8 [5 marks]

Prove or disprove the following: Event A does not depend on whether event B occurs or not if and only if A does not depend on whether event $\overline{B}$ occurs or not.

Question 9 [10 marks]

We have two events A and B. A: D.T. is an idiot with probability 0.6. B: D.T. is a crook with probability 0.7. We also know that neither is true with probability 0.25.

- What is the probability that D.T. is an idiot or a crook, but not both?

- What is the conditional probability that D.T is a crook, given that he is not an idiot.

End of Assignment 3.