COMP 2804: Assignment 4

Due Date: Wednesday, March 31st, at 11:59PM

School of Computer Science — Carleton University

Your assignment should be submitted online on cuLearn as a single .pdf file. No late assignments will be accepted. You can type your assignment or you can upload a scanned copy of it. Please, use a good image capturing device. Make sure that your upload is clearly readable. If it is difficult to read, it will not be graded!

Question 1 [10 marks]

Suppose we have a bag with 5 balls in it, numbered 1 to 5. Consider the drawing two balls from the bag without replacement. Define the random variable X to be the absolute difference between the values of the two balls. Compute the distribution function for X.

Question 2 [10 marks]

Suppose we draw three cards from a shuffled standard 52 card deck (without replacement). Define the random variable H to be the number of hearts in the three card hand. Define the random variable K to be the number of kings in the three card hand. Show whether or not K and H are independent random variables.

Question 3 [10 marks]

Suppose X and Y are independent random variables on a probability space (S, P). Show that $E(X \cdot Y) = E(X) \cdot E(Y)$.

Question 4 [10 marks]

Consider a game of chance where you roll two fair six-sided dice 10 times. Suppose you win x dollars every time the sum of the two dice is greater than or equal to 10 (assume you win zero dollars otherwise). What is the expected amount of money you win at the end of this game?

Question 5 [10 marks]

Consider a game of chance where you roll two fair six-sided dice. Suppose you win the game once you roll such that the sum of the two dice is greater than or equal to 10. What is the expected number of turns for this game (consider rolling the two dice once to be a turn)?

Question 6 [10 marks]

Consider a game where two players select elements from the set X = a, b, c, d, e, f, g, h. We consider it a "win" if the two Players select the same element.

Suppose Player A and Player B use the following strategy: each player independently selects an element from X with uniform probability.

What is the expected number of "wins" if Player A and Player B play this game n times?

Question 7 [10 marks]

Suppose X is a random variable on a probability space (S, P). Show that $E(E(X)) = E(X)$.

Question 8 [10 marks]

Consider a die that is not fair. When rolled you obtain: 1 or 2 or 3 with probability 1/12 (each) or 4, 5, or 6 with probability 1/4 (each).

- Is this a valid probability distribution?
- What is the expected value of a roll?

Question 9 [10 marks]

On a car assembly line, out of 100 cars 3 are defective and need additional work.

1. What is the probability that the first defective car is the 5^{th} inspected?
2. What is the probability that the first defective car is among the first 10 cars?
3. What is the probability that the first defective car is the 5^{th}, 6^{th}, 7^{th}, ..., or 10^{th} car?

For items 2 and 3 do NOT compute each of the 10 (6, respectively) probabilities explicitly.

End of Assignment 4.