COMP 2804: Assignment 1

Due Date: Sunday, February 7th at 11:59PM

School of Computer Science Carleton University

Your assignment should be submitted online on cuLearn as a single .pdf file. No late assignments will be accepted. You can type your assignment or you can upload a scanned copy of it. Please, use a good image capturing device. Make sure that your upload is clearly readable. If it is difficult to read, it will not be graded!

Question 1 [10 marks]

Argue concisely and mathematically correct as to why at least two students registered in COMP 2804 will receive the same letter grade (A+, A, $\cdots$, F). We have 238 students in COMP 2804.

Question 2 [10 marks]

How many words built by using each of the letters A, B, C, D, E, F, G and H exactly once, contain the string ABC as a (consecutive) substring? Ex. DEFABCGH contains ABC, but not ACBDEFGH. (Note that e.g. ABCABCDEFGH is not a legal word as it contains some letters twice).

Question 3 [5 marks]

Order the following functions by which one is bigger for large n:

$n!, n$, n^n, $\log n$, n^2, $2n$.

You do not have to give proofs for this.

Question 4 [10 marks]

How many bitsrings of length n exist which start with 00 and end in 11?

Question 5 [10 marks]

A ternary number is a number made using in each of its positions exactly one of the numbers 0, 1, or 2. Ex.: 220111 is a ternary number. How many ternary numbers are there of length n? Prove your result.

Question 6 [10 marks]

How many bitstrings are there of length 2n that contain exactly n 0s and n 1s? Prove your result.

Question 7 [10 marks]

How many alphabetic strings (strings containing letters only) of length n start with a vowel or end with a consonant? For example, ABBE, BCD, AXYZ all start with a vowel or end with a consonant; DCBA does not. Assume Y is always a consonant. Prove your result.

Question 8 [10 marks]

How many alphanumeric strings (strings containing letters or numbers) of length n start with a letter and contain at least one number? For example, A1010, B10BB are valid; 1010BB, CDEF are not. Prove your result.

Question 9 [10 marks]

Consider the set A = $\{a, b, c, d, e, f, g, h\}$. How many permutations of this set exist such that a appears before b? Note that a is not required to appear immediately before b. For example, $abcdefhg$, $acdefghb$, $cdaefbgh$ are valid permutations; $bacdefgh$, $bcdefgha$ are not valid.

Question 10 [5 marks]

Simplify the following expression: $\sum_{k=0}^{n} 2^k \binom{n}{k}$.

Question 12 [10 marks]

How different six digit numbers can be made by rearranging the following digits: 0, 1, 1, 2, 2, 3? Note a six digit number cannot begin with a zero. For example, 101232, 231120 are valid six digit numbers, 021312 is not valid.

Question 12 [10 marks]

Recall that a strictly descreasing sequence is a sequences for which each element is strictly smaller than the prior element. Show that we cannot have a strictly descreasing sequence of n positive integers starting at m, for $n > m$. For example, we cannot have a strictly descreasing sequence of 20 positive integers starting at 10.

Question 13 [10 marks]

Suppose we have a set of n identical balls in a bag. How many ways can m players choose balls from the bag such that the first player gets at least k balls. Assume $k \leq n$.

Question 14 [10 marks]

Show the following identity is true: $k\binom{n}{k} = n\binom{n-1}{k-1}$.

End of Assignment 1.