PRINCIPLES OF COMPUTER NETWORKS
COMP 3203

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Solve the problems below. Your answers do not have to be long, but they should be complete, precise, concise and clear. Write the solutions on your own and acknowledge your sources in case you used "library" material. Look in the course web page on how to avoid plagiarism and for submission details (where/when/how). Exercises marked with ($\star$) are usually more challenging. **NB:** some of the exercises may require material that will be covered in forthcoming lectures while others marked with ($\star$) may involve additional material not covered in class. It is preferred that you type your work using your favourite software package but you submit only in pdf. Two excellent and free packages are LATEX (for typesetting mathematics) and Ipe (for drawing pictures).

Assignment A

1 [10 pts]

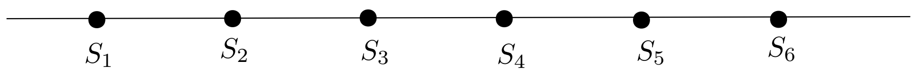
1. [**2 pts**] A sine wave is offset one-tenth of a cycle with respect to time zero. What is its phase in degrees and in radians?
2. [**2 pts**] Period (denoted by t) is the amount of time in seconds it takes a signal to complete a cycle. Frequency (denoted by f) of a signal is the number of periods in a second. As such one is the inverse of the other, i.e., $f = 1/t$. Express a period of 900 ms in microseconds.
3. [**2 pts**] Recall the "width" representation of a bit in the lecture on Performance. How "wide" (in seconds) is a bit on a 4000 $Gbps$ link?
4. [**2 pts**] What is the length (in centimeters) of a 10^{-9} sec wide bit in copper wire when the speed of propagation is 10×10^7 m/s?
5. [**2 pts**] Compute how many bits per sec are transmitted Video at a resolution of 720×480, 16 bytes per pixel, 60 frames per second.

2 [10 pts]

The transmission overhead overhead of a server (or a tandem system of servers) is the ratio of the number of additional bits added to a packet divided by the length of the original packet.

1. [**2 pts**] A server receives packets of length P bits and retransmits them by adding a header of length H bits. What is the transmission overhead of the server?

2. [**4 pts**] Consider a network of n servers in tandem. When server S_i receives a packet of length P bits it retransmits it by adding a header of length H_i bits

   A packet can enter the network only from S_1 and exit only from S_n. What is the resulting transmission overhead of the tandem network of n servers on a packet of length P bits?

3. [**4 pts**] Consider the tandem system described above and assume the i-th server's header has length 2^{-i} times the length of the original packet (i.e., the packet that entered at S_1). What is the resulting overhead of the tandem system? Give the formula.

3 [10 pts]

This exercise concerns Shannon's formula for the capacity of a channel.

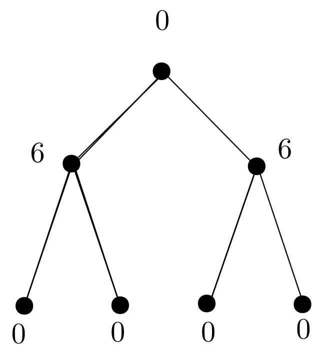
1. [**5 pts**] Consider the experiment of drawing at random from a deck of 100 lottery tickets (same probability for all tickets). What is the minimum length (in binary) of a message required to encode the outcome of the experiment.

2. [**5 pts**] A transmission channel has bandwidth $2,000$ Hz. If the noise level is $N = 20$ dB, what should the signal strength be (in dB) so that the channel capacity is at least $2,000$ bps?

4 [10 pts]

Initially each node v in a graph has n_v packets to deliver. A flooding algorithm at a node v can deliver one packet per neighbour only if its current value n_v is at least $\deg(v)$, where $\deg(\cdot)$ denotes the degree (number of links adjacent to it) of a node in the graph. Flooding is done synchronously by all the nodes.

1. [**6 pts**] What is the result of applying flooding to the graph below? Draw a tree diagram and label the vertices each with the number of packets it has.

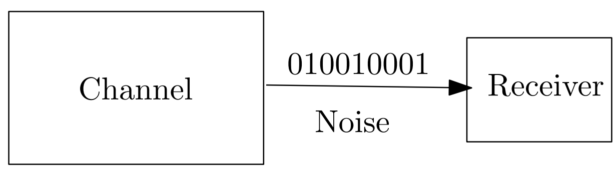


2. [**4 pts**] In how many iterations does the system stabilize? (5 points per question).

Note that links are duplex and the system stabilizes when all the values n_v no longer change.

5 [10 pts]

Consider a message source that generates the $0, 1$ symbols.



For each question below give the precise formula.

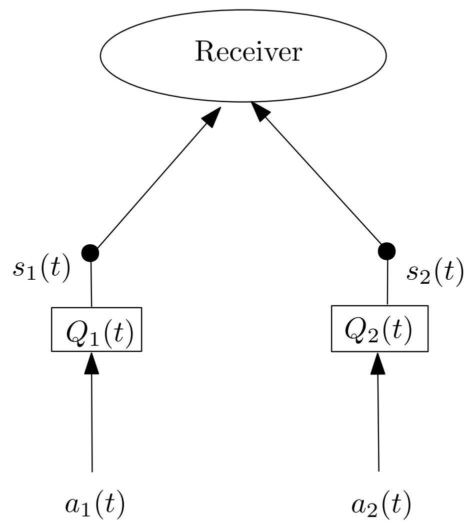
1. [**3 pts**] How many different messages of length m can be generated by the source and why?

2. [**3 pts**] How many, among messages of length m, have k or more 1 symbols and why?

3. [**4 pts**] Consider a message source that generates the 0 symbol with probability 0.8 and the 1 symbol with probability 0.2. With what probability does the source generate the sequence 111111111 and why?

6 [10 pts]

Consider the two-queue system in the slides on mathematical modelling and analysis.



Packets arrive at the queues processed at the servers and sent to the receiver.

1. [**3 pts**] Assume the arrival rate at $a_i(t)$ is $25i$ bits per sec, for $i = 1, 2$. However, the second server $s_2(t)$ is faulty and loses 10% of its bits in processing. What is the arrival rate per sec at the receiver?

2. [**3 pts**] Assume the arrival rate at $a_i(t)$ is $25i$ bits per sec, for $i = 1, 2$. Server 1 pays a cost of 3 units per bit transmitted while server 2 pays 5 units per bit transmitted. What is the payment per bit at the receiver?

3. [**4 pts**] Assume the arrival rate at time t is given by $a_i(t)$ (this a function of t), for $i = 1, 2$. For each $i = 1, 2$ at each time slot a packet which arrived reaches the server s_i with probability p. What is the total expected number of arrivals at time t by the receiver?

7 [10 pts]

Consider the material on queues discussed in class. Recall that we have shown that the probability dinners overlap is $\frac{7}{16}$.You observe a particular person. What is the likelihood of seeing this person at dinner 9 or more times in 12 days, assuming that their arrival times are in fact random. Give the formulas and calculate the probability numerically.

8 [10 pts] ($\star$)

A set U of n clients/servers is labeled $1, 2, \ldots, n$. A quorum Q is defined as a nonempty subset of the set $\{1, 2, \ldots, n\}$. A coterie C is defined as a set of quorums which satisfies the following two properties: 1) **nonempty intersection:** any two quorums have nonempty intersection, and 2) **minimality property:** there are no two quorums such that one is a strict subset of the other. Clients may ask any of the servers for information using quorums from a given coterie C. Answer the questions below by providing in each case a brief explanation.

1. **[3 pts]** Consider an arbitrary client/server system $U = \{1, 2, \ldots, n\}$ and any coterie C on U. Two clients $i, j \in U$ want to exchange data packets but they don't have a common server: i selects a quorum $Q_i \in C$ and j a quorum $Q_j \in C$ and ask all servers in their respective sets to enable communication. Can they find a common server? Which one?

2. **[3 pts]** Consider the sets $\mathcal{C}_1 = \{\{1\}, \{1, 2\}, \{1, 2, 3\}, \ldots, \{1, 2, 3, \ldots, n\}\}$, and $\mathcal{C}_2 = \{\{i\} : i = 1, 2, \ldots, n\}$. Is $\mathcal{C}_1$ a coterie? Is $\mathcal{C}_2$ a coterie? Explain in detail!

3. **[4 pts]** Consider the sets $\mathcal{C}_1 = \{\{1, 2, 3, \ldots, n\} \setminus \{i\} : i = 1, 2, \ldots, n\}$, and $\mathcal{C}_2 = \{Q \subseteq \{1, 2, 3, \ldots, n\} : |Q| = \lceil \frac{n+1}{2} \rceil\}$. Is $\mathcal{C}_1$ a coterie? Is $\mathcal{C}_2$ a coterie? Explain in detail!

9 [10 pts]

Let $Q(R,K)$ be the set of processors in row R and column K. Is the set $\{Q(R,K) : R \text{ is row}, K \text{ column}\}$ a coterie?

Continued from Exercise 8.

1. **[3 pts]** Consider n clients/servers placed at the vertices of a square $\sqrt{n} \times \sqrt{n}$ grid. The server labeled (i, j) occupies the i-th row and j-th column. ~~Is the set $\{R, K\}$ consisting of any one row R and any one column K a quorum~~? Explain. Draw a picture of the grid with the n servers a row and column.

2. **[3 pts]** Consider the sets of clients/servers $U_1 = \{1, 2, 3\}$, and $U_2 = \{4, 5, 6\}$. Show that $C_1 = \{\{1, 2\}, \{2, 3\}, \{3, 1\}\}$ and $C_2 = \{\{4, 5\}, \{5, 6\}, \{6, 4\}\}$ are coteries in their respective client/server systems. Explain.

3. **[4 pts]** Consider the set of clients/servers $U_1 \cup U_2$. Is the set

$$\{\{1, 2\}, \{2, 4, 5\}, \{2, 5, 6\}, \{2, 6, 4\}, \{4, 5, 1\}, \{5, 6, 1\}, \{6, 4, 1\}\}.$$

a coterie? Prove or disprove. Explain.