Comp 3203 Assignment 3

1\.

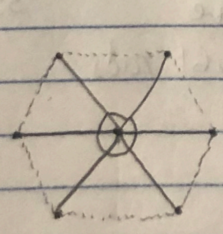
1. We traversed the faces A, B, C, D (Let A, B, C, D represent the faces traversed)

   A = edges 6, 15, 14, 29, 12
   B = edges 29, 18, 31, 19, 13
   C = edges 24, 23, 22, 21, 20, 31
   D = edges 25, 26, 28, 27, 21, 22, 23

2. Left-hand rule from S → T : 12, 29, 13, 19, 31, 20, 21, 22, 21, 27, 28
   Path from S → T : 12, 13, 19, 20, 27, 28

3. Right-hand Rule from S → T : 6, 15, 14, 29, 18, 31, 24, 23, 22, 23, 25, 26
   Path from S → T : 6, 15, 14, 18, 24, 25, 26

2\. As all the sensors in the network have a range of 1, if we visualize our outer sensors as connected equillaterals, and all sensors including C being omnidimensional, we can see how many sensors would fit around C to be just in range of each other

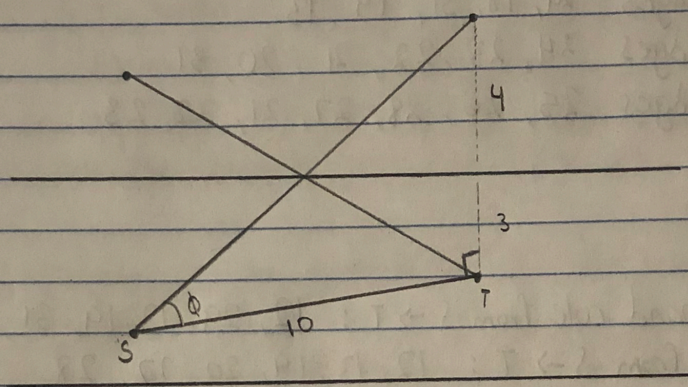


Given that equillaterals have interior angles of 60° we know that there needs to be 6 sensors around C for them to be in range of each other as:

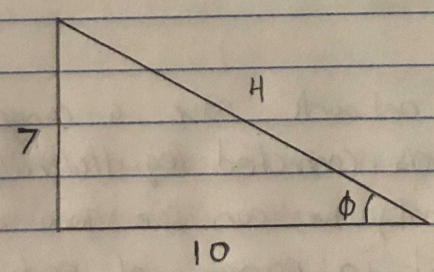
$360°/60° = 6$

∴ for the sensors to not be in range of each other k must be ≤ 5

3. We are given the horizontal distance of 10 between S and T, as well as the distance of S from W_2 as 1, and T from W_2 as 2. Because of this we know S from $W_1 = 4$ and T from W_1 as 3. Using this information we can create the virtual image



Now that we have two side lengths and a right angle triangle we can calculate the desired angle



$\tan \phi = \frac{O}{A}$ $\qquad \phi = \tan^{-1}\left(\frac{7}{10}\right)$

$\tan \phi = \frac{7}{10}$ $\qquad \phi = 0.61$ radians

Or

34.99 degrees

$\cong$

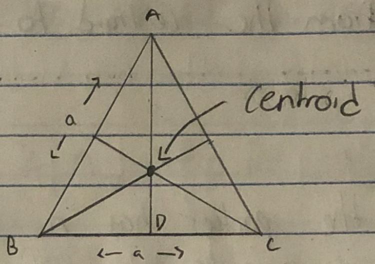
35 degrees

Therefore the angle the ray should leave to reach T after one reflection is 0.61 rads or 35 degrees

4.

1. The point in the triangle furthest from all three corners is the centroid as it is an equilateral triangle. This point is the furthest from all three corners as it is the point where the median of the corners meet. As it is an equillateral triangle this intersection would be equidistant from all three corners.

For Example:



2. Using pythagoreas theorem we know that the heigte =

$$\sin\frac{\pi}{3} = \frac{AD}{a}$$

$$AD = \sin\frac{\pi}{3}a$$

$$= a\sqrt{3}/2$$

Let the height be D

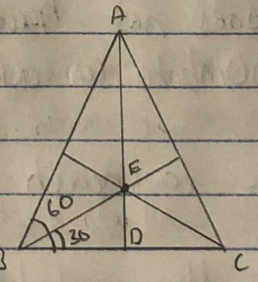
Let the centroid be E

We know the centroid is 2/3's of a given median

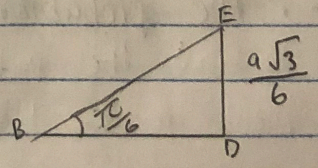
$$\therefore \text{ the line } ED = AD/3$$

$$= \frac{a\sqrt{3}}{2} \div 3$$

$$= \frac{a\sqrt{3}}{6}$$



Now looking at triangle EBD,



$$\sin\frac{\pi}{6} = \frac{O}{H} = \frac{ED}{EB}$$

$$\sin\frac{\pi}{6} = \frac{\frac{a\sqrt{3}}{6}}{EB}$$

$$\therefore EB = \left(\frac{a\sqrt{3}}{6}\right) \Big/ \frac{1}{2}$$

$$= \frac{a\sqrt{3}}{3}$$

$\therefore$ The distance of this point from the corners of the triangle is $\frac{a\sqrt{3}}{3}$

3. Given that the centroid is equidistance from all the points, the minimum radius to cover every point in the plane is the distance from the centroid to the vertices, or: $\frac{a\sqrt{3}}{3}$

5.

1. Given that the packet has n bits, and we assume errors are independant events, $\therefore$ if p is our probability of bit error, then $1-p$ would be our probability that our bit does not have an error. Now applying this to our packet length gives us $(1-p)^n$

Although if we took this as our final answer as this number is based on the probability that the bit does not have an error, this will lead us to a decreasing number as our packet length n increases

$\therefore$ to reach our final answer we must subtract 1.

$\therefore$ the probability the packet of length n has an error is

$$1-(1-p)^n$$

2. To find the maximum possible packet length so the probability the packet has an error of at most ϵ we set up an inequality:

$\epsilon \leq 1-(1-p)^n$, Next bring the 1 over and log both sides

$\log_{(1-p)}(\epsilon - 1) \leq \log_{(1-p)}(1-p)^n$, $\log_{(1-p)}(1-p)^n = n$

$\log_{(1-p)}(\epsilon - 1) \geq n$, flip the inequality as $0 < 1-p < 1$

$\therefore$ The maximum packet length such that the probability a packet has an error is at most ϵ is $\lceil \log_{(1-p)}(1-\epsilon) \rceil$

6. To solve we first need to choose 1 station: $\binom{k}{1}$
Given that the stations choose 1 frequency at random: $\frac{1}{n}$

We want the probability that the rest of the stations $(k-1)$ talk at a frequency different than the one chosen $\left(\frac{n-1}{n}\right)$

$\therefore$ probability $= \binom{k}{1}\left(\frac{1}{n}\right)\left(\frac{n-1}{n}\right)^{(k-1)}$

7.

1. A buffer of initial size C_o, increasing at a rate of r per time unit (Let t represent time units, where $t > 0$) gives us the formula:

$r^t(C_o)$ for the buffer size at a given time unit

$\therefore$ The buffer will overflow at:

$$C \leq r^t(C_o) \qquad \log_r\left(\frac{C}{C_o}\right) \leq \log_r r^t$$

$$\frac{C}{C_o} \leq r^t \qquad \log_r\left(\frac{C}{C_o}\right) \leq t$$

In other words, the buffer will overflow when $t \geq \log_r\left(\frac{C}{C_o}\right)$

2. As the warning message will send at p% of C we now want to see at what t will the function surpass Cp (Assuming p is already in decimal form)

$$\therefore \quad Cp \leq r^t(C_o) \qquad \log_r\left(\frac{Cp}{C_o}\right) \leq \log_r r^t$$

$$\frac{Cp}{C_o} \leq r^t \qquad \log_r\left(\frac{Cp}{C_o}\right) \leq t$$

$\therefore$ a error message will be sent when $t \geq \log_r\left(\frac{Cp}{C_o}\right)$

8.

1. Given that there are m nodes in the leftmost column and m nodes in the rightmost column, under the assumption that nodes cannot hop more than a distance of 1 on their path, then:

m nodes traversing
m nodes being traversed to
$\therefore$ at least $m \times m$ or m^2 paths through the m nodes in the middle

2. Again, assuming a hop distance of 1 for nodes, if there are m nodes that are each sending m messages, then even if each node in the LHS only used each node in the middle once, then the nodes still would have m paths through it, as:

m nodes sending m messages $= m^2$ through middle
m^2 messages split evenly among m nodes

$$= \frac{m^2}{m} \Rightarrow = m$$

And in the case where some nodes have no paths through them, the nodes that do have paths will have more than m. For example: m^2 messages, $m-1$ available middle nodes to pass through, then one node must have $m+1$ paths.

3. Similar to the answer for part 2, there is still m nodes m nodes creating paths but now only L non faulty nodes in M

$\therefore$ m^2 messages split evenly among L nodes

$$= \frac{m^2}{L}$$

$\therefore$ there is a node that has at m^2/L paths routed through it

9.

1. To find the percentage of the packet is occupied by headers, we must first find the size of the packet after undergoing n applications of protocols

Let L be the length of the packet before any protocols

$\therefore$ size of the packet after n applications $= L + h^n$
Given $L + h^n$ is 100% of the packet
Let x be the % occupied by protocol headers

$\therefore \; 100\% = L + h^n$
$\quad\; x\,\% = h^n$

$\frac{100\%}{x\%} = \frac{L + h^n}{h^n}$ gives us the same ratio

Now solve:

$$\frac{100\%}{x\%} = \frac{L + h^n}{h^n} \qquad x\% = \frac{100(h^n)}{L + h^n}$$

$$\frac{x\%}{100\%} = \frac{h^n}{L + h^n}$$

2. Triple the length of the original packet gives us $3L$

$\therefore \; 3L \leq L + h^n$
$3L - L \leq h^n$

$2L = h^n$
$\log_h 2L = n$