Examples/Questions see also TA Francois' Session different example sources

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Independence

when are two events independent?

$\Pr(A \cap B) = \Pr(A) \cdot \Pr(B)$

Imagine you select two cards consecutively from a complete deck of playing cards. Are the two selections independent?

What is A what is B?

Now, suppose you were to select two cards "with replacement" by returning your first card to the deck and shuffling the deck before selecting the second card. Are the two selections independent?

Selected examples from web-sources

- Two friends are playing billiards, and decide to flip a coin to determine who will play first during each round. For the first two rounds, the coin lands on heads. They decide to play a third round, and flip the coin again. What is the probability that the coin will land on heads again?
- First, note that each coin flip is an independent event.
- For any coin flip, there is a 1/2 chance that the coin will land on heads. Thus, the probability that the coin will land on heads during the third round is 1/2.

example

- When flipping a coin, what is the probability of getting tails 5 times in a row?
- Recall that each coin flip is independent, and the probability of getting tails is 1/2 for any flip. Also recall that the following statement holds true for any two independent events A and B:
- $Pr(A \cap B)=Pr(A) \cdot Pr(B)$
- Finally, the concept of independence extends to collections of more than 2 events. We use a procedure ... then product rule.
- Therefore, the probability of getting tails 5 times in a row is: $(½)^5$ = 1/32

To show independence

- Two events are independent if any of the following are true:
  - $Pr(A|B)=Pr(A)$
  - $Pr(B|A)=Pr(B)$
  - $Pr(A \cap B)=Pr(A) \cdot Pr(B)$

So, you just need to show one of the above to be true.

Why? it the proof e.g., of $Pr(B|A)=Pr(B)$

uses:

If $Pr(B) > 0$ then $Pr(B|A)= Pr(A \cap B)/Pr(B)$

cards

Probability = # of favourable outcomes / total # of possible outcomes

What is the probability that the drawn card is Ace ?

# of favourable outcomes

= 4

total # of possible outcomes

= 52

Probability(first card drawn being an ace)

= 4/52 = 1/13

A 4 digit PIN is selected. What is the probability that there are no repeated digits?

- There are 10 possible values for each digit of the PIN (namely: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9), so there are $10 \times 10 \times 10 \times 10 = 10^4 = 10000$ total possible PINs.
- To have no repeated digits, all four digits would have to be different, which is selecting without replacement. We could either compute $10 \times 9 \times 8 \times 7$, or notice that this is the same as the permutation ${}_{10}P_4 = 5040$.
- nPr = n!/ (n-r)!

  where ***n*** is the number of things to choose from, and we choose ***r*** of them, no repetitions, order matters.

A 4 digit PIN is selected. What is the probability that there are no repeated digits?

- The probability of no repeated digits is the number of 4 digit PINs with no repeated digits divided by the total number of 4 digit PINs. This probability is
- ${}_{10}P_4$ $/10^4$=5040/10000=0.504

Compute the probability of randomly drawing five cards from a deck and getting exactly one Ace.

- Thus we use combinations to compute the possible number of 5-card hands, ${}_{52}C_5$ (simpler to type notation for 52 choose 5). This number will go in the denominator of our probability formula, since it is the number of possible outcomes.
- there are four Aces and we want exactly one of them, there will be ${}_4C_1$ ways to select one Ace;
- 48 non-Aces and we want 4 of them, there will be ${}_{48}C_4$ ways to select the four non-Aces.
- P(one Ace)= ${}_4C_1 \; {}_{48}C_4 \; / \, {}_{52}C_5$ = 778320/2598960≈0.299

Probability of getting a Queen, King, Ace

Think it through yourselves:

Hint: How many ways can you choose a Queen?

Then, we only have to choose the suit of the King and Ace as they are fixed.

....

Now ask yourself what is the probability of getting three arbitrary, but consecutive cards? that includes ay Ace 2 3 (the Ace can be high and low) . so 2,3,4 3,4,5 ... Queen, King, Ace and Ace 2, 3

Then, ask yourself what if we want those not to be all from the same suit similarly to having a straight vs. having a flush or a royal flush.

A coin is thrown 3 times. What is the probability that at least one head is obtained?

- **Sol:** Sample space = [HHH, HHT, HTH, THH, TTH, THT, HTT, TTT]
  Total number of ways = 2 × 2 × 2 = 8.

Favourable Cases = 7

P (A) = 7/8

or

P (of getting at least one head) = 1 – P (no head)⇒ 1 – (1/8) = 7/8

What is the probability of getting a sum of 7 when two dice are thrown?

Solution:

Total number of ways = 6 × 6 = 36 ways.

Favorable cases = (1, 6) (6, 1) (2, 5) (5, 2) (3, 4) (4, 3) --- 6 ways.

P (A) = 6/36 = 1/6

What is the probability of getting a 5 when a die is rolled and probability of not getting 5?

- Total Number of possible outcomes while rolling a die is 6. The number of ways, that a 5 occurs while rolling a die is 1. Hence we can calculate the Single Event Probability using the formula :
- **Probability of event A that occurs, P(A) = |A|/ |S| Probability of event A that does not occur, P(A') = 1 - P(A)**
- P(A) = 1/6 =0.167 Hence, the single event probability is 0.167 Probability of event A that does not occur, =1 - 0.167 = 0.833.

a pair of dice Calculate the probability of getting odd numbers and even number together and the probability of getting only odd number. Find the conditional probability?

- The total number of possible outcomes of rolling a dice once is 6. Hence, the total number of outcomes for rolling a dice twice is (6x6) = 36.
- The probability of getting an odd and even number is 18 and the probability of getting only odd number is 9. i.e., $|A| = 18$ $|B| = 9$
- We can calculate the multiple event probability using the above formulae as well as these:
- **Probability that both the events occur $P(A \cap B) = P(A) \times P(B)$**
- **Probability that either of event occurs $P(A \cup B) = P(A) + P(B) - P(A \cap B)$**
- **Conditional Probability $P(A \mid B) = P(A \cap B) / P(B)$**

a pair of dice

- Multiple Event Probability are as follows: $P(A) = 4 / 6 = 0.667$.
- **Hence, the Probability that event A occurs is 0.667** $P(B) = 5 / 6 = 0.833$.
- **Hence, the Probability that event B occurs is 0.833** $P(A') = 1 - P(A) = 1 - 0.667 = 0.333$.
- **Hence, the Probability that event A does not occur is 0.333** $P(B') = 1 - P(B) = 1 - 0.833 = 0.167$.
- **Hence, the Probability that event B does not occur is 0.167** $P(A \cap B) = P(A) \times P(B) = 0.667 \times 0.833 = 0.556$.

a pair of dice

- **Hence, the Probability that both the events occurs is 0.556** $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.667 + 0.833 - 0.556 = 0.944$.
- **Hence, the Probability that either of event occurs is 0.944** $P(A \mid B) = P(A \cap B) / P(B) = 0.556 / 0.833 = 0.667$. **Conditional probability of A given B is 0.667**

Life expectancy

- 82 170 of 100 000 children live 40 years and 37 930 of 100 000 children live 70 years. Determine the probability of a 40 years old person to live 70 years.
- A – live 70 years, P(A) = 0,3793
  B – live 40 years, P(B) = 0,8217
- The probability equals 46%

$$A \cap B = 0,3793$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A/B) = \frac{0,3793}{0,8217}$$

$$P(A/B) = 0,46$$

$$P(A/B) = 46\%$$

Three dice are rolled together. What is the probability as getting at least one '4'?

Sol: Total number of ways = 6 × 6 × 6 = 216.

Probability of getting number ‘4’ at least one time
= 1 – (Probability of getting no number 4)

= 1 – (5/6) x (5/6) x (5/6) = 91/216

A problem is given to three persons P, Q, R whose respective chances of solving it are 2/7, 4/7, 4/9 respectively. What is the probability that the problem is solved?

Probability of the problem getting solved = 1 – (Probability of none of them solving the problem)

$$P(P)=\frac{2}{7}\Rightarrow P(\overline{P})=1-\frac{2}{7}=\frac{5}{7},\ P(Q)=\frac{4}{7}\Rightarrow P(\overline{Q})=1-\frac{4}{7}=\frac{3}{7},\ P(R)=\frac{4}{9}\Rightarrow P(\overline{R})=1-\frac{4}{9}=\frac{5}{9}$$

Probability of problem getting solved = 1 – (5/7) x (3/7) x (5/9)
= (122/147)

Find the probability of getting two heads when five coins are tossed.

Number of ways of getting two heads $= {}_5C_2 = 10$.

Total Number of ways $= 2^5 = 32$

P (two heads) = 10/32 = 5/16

From a pack of cards, three cards are drawn at random. Find the probability that each card is from different suit.

Total number of cases = ${}_{52}C_3$

One card each should be selected from a different suit.

The three suits can be chosen in ${}_{4}C_3$ ways

The cards can be selected in a total of $({}_{4}C_3)$ x $({}_{13}C_1)$ x $({}_{13}C_1)$ x $({}_{13}C_1)$

Probability = ${}_{4}C_3 \times ({}_{13}C_1)^3 / {}_{52}C_3 = 4 \times (13)^3 / {}_{52}C_3$

Fifteen people sit around a circular table. What is the probability of two particular people sitting together?

15 persons can be seated circularly in 14! ways.

No. of ways in which two particular people sit together is $13! \times 2!$

The probability of two particular persons sitting together $13!2! / 14! = 1/7$

Three bags contain 3 red, 7 black; 8 red, 2 black, and 4 red & 6 black balls respectively. 1 of the bags is selected at random and a ball is drawn from it. If the ball drawn is red, find the probability that it is drawn from the third bag.

Let E1, E2, E3 and A are the events defined as follows.

E1 = First bag is chosen

E2 = Second bag is chosen

E3 = Third bag is chosen

A = Ball drawn is red

There are three bags and one of the bags is chosen at random, so P (E1) = P(E2) = P(E3) = 1 / 3

If E1 has already occurred, then the first bag has been chosen which contains 3 red and 7 black balls.

cont'd

The probability of drawing 1 red ball from it is 3/10. So, P (A/E_1) = 3/10, similarly $P(A/E_2)$ = 8/10, and $P(A/E_3)$ = 4/10.

We are required to find $P(E_3/A)$ i.e. given that the ball drawn is red, what is the probability that the ball is drawn from the third bag by Baye's rule.

$$= \frac{\frac{1}{3}\times\frac{4}{10}}{\frac{1}{3}\times\frac{3}{10}+\frac{1}{3}\times\frac{8}{10}+\frac{1}{3}\times\frac{4}{10}} = \frac{4}{15}.$$