

COMP 2804: Assignment 4

Due Date: Wednesday, April 14th, at 11:59PM

School of Computer Science

Carleton University

Your assignment should be submitted online on cuLearn as a single .pdf file. No late assignments will be accepted. You can type your assignment or you can upload a scanned copy of it. Please, use a good image capturing device. Make sure that your upload is clearly readable. If it is difficult to read, it will not be graded!

Question 1 [10 marks]

Consider two random variables X and Y . If X and Y are independent, then it can be shown that $E(XY) = E(X)E(Y)$. In this exercise, you will show that the converse of this statement is, in general, not true.

Let X be the random variable that takes each of the values 1, 0, and 1 with probability $1/3$. Let Y be the random variable with value $Y = X^2$.

- Prove that X and Y are not independent.
- Prove that $E(XY) = E(X)E(Y)$.

Question 3 [10 marks]

The Ottawa Senators and the Toronto Maple Leafs play a best-of-seven series: These two hockey teams play games against each other, and the first team to win four games wins the series. Assume that

- each game has a winner (thus, no game ends in a tie),
- in any game, the Sens have a probability of $3/4$ of defeating the Leafs,
- the results of the games are mutually independent.

Determine the probability that seven games are played in this series.

Question 4 [10 marks]

Prove that, for any graph G with m edges, the sequence X_1, X_2, \dots, X_m of random variables in the proof of Theorem 7.1.1 is pairwise independent. Give an example of a graph for which this sequence is not mutually independent.

Question 5 [10 marks]

Prove that Theorem 7.5.4 also holds if G is not connected.

Question 6 [10 marks]

Let K_5 be the complete graph on 5 vertices. In this graph, each pair of vertices is connected by an edge. Prove that K_5 is not planar.

Question 7 [10 marks]

Suppose we have a graph $G = (V, E)$ with m edges. Prove that there exists a partition of V into three subsets A, B, C such that there are $\frac{2m}{3}$ edges between these subsets (i.e. between A and B, between B and C, or between A and C).

Question 8 [10 marks]

Suppose we randomly draw two integers from the range $[1, n]$ with uniform probability. Define X to be the value of the first integer drawn; define Y to be the value of the second integer drawn. Define $Z = |X - Y|$. Compute $E(Z)$.

End of Assignment 4.