PRINCIPLES OF COMPUTER NETWORKS
COMP 3203

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Solve the problems below. Your answers do not have to be long, but they should be complete, precise, concise and clear. Write the solutions on your own and acknowledge your sources in case you used "library" material. Look in the course web page on how to avoid plagiarism and for submission details (where/when/how). Exercises marked with ($\star$) are usually more challenging. **NB:** some of the exercises may require material that will be covered in forthcoming lectures while others marked with ($\star$) may involve additional material not covered in class. It is preferred that you type your work using your favourite software package but you submit only in pdf. Two excellent and free packages are LATEX (for typesetting mathematics) and Ipe (for drawing pictures).

Assignment B

1 [10 pts]

Consider a line graph with n nodes labeled $1, 2, \ldots, n$ in the order they occur from left to right. Pairs of nodes $i < j$ communicate via shortest paths.

1. **[6 pts]** Given a node $1 \leq k \leq n$, for how many pairs of nodes i, j such that $i < j$ does the unique shortest path connecting them go through k?

2. **[2 pts]** Assume $n = 100$ nodes. Plot the number of unique shortest paths passing through k as a function of k.

3. **[2 pts]** For which k, is the number of paths passing through k above maximized?

2 [10 pts]

Consider a line graph with n nodes labeled $1, 2, \ldots, n$.

1. **[3 pts]** For a given $1 \leq d \leq n-1$, what is the number of pairs of nodes $i < j$ with hop distance d?

2. **[3 pts]** Give the formula for the average hop-distance between pairs of nodes?

3. **[4 pts]** Simplify the formula derived above and derive a closed formula for its exact value.

3 [10 pts]

A network has packet size L bits, and packet lifetime 2 min (i.e., after 2 min the numbering will restart). The network adds a sequence number that is S bits long to each packet. After running out of sequence numbers it restarts from the beginning.

1. **[3 pts]** How many packets with different sequence numbers can the network transmit in 2 minutes?

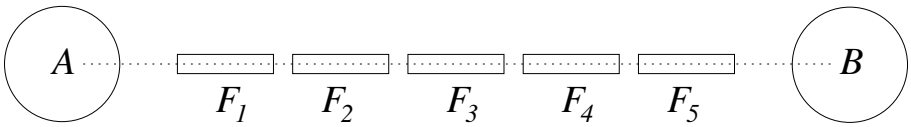
2. **[2 pts]** What is the max number of packets that the network can sent in 1 hour?

3. **[2 pts]** What is the max number of bits that can be sent in 1 hour?

4. **[3 pts]** A node is receiving all the packets and has buffer size of B bits. For given packet length L what is the max S so that the buffer does not overflow?

4 [10 pts]

In a siliding window protocol host A may send w frames to host B prior to receiving an ACK (the picture depcts two hosts and a sliding window size $w = 5$).



Suppose that packets are sent over a link with a loss probability p per-fragment and assume that losses are independent events.

1. **[3 pts]** What is the probability that an ACK will be sent by B after A has sent w frames?

2. **[2 pts]** What is the probability that an NAK will be sent by B after A has sent w frames?

3. **[3 pts]** What is the probability that exactly two frames were lost among the w frames that were sent?

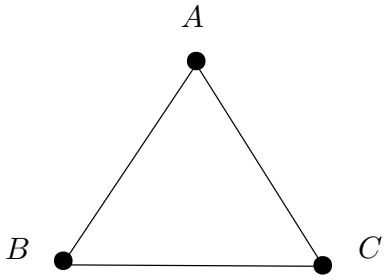
4. **[2 pts]** What is the probability that in a transmission of w packets the first $w-2$ fragments arrived correctly but the last two were lost?

5 [10 pts]

Sixteen stations $1, 2, \ldots, 16$ contend for a shared channel. Stations $2, 3, 4, 10, 12, 13, 16$ suddenly become ready to transmit at once. Give a picture of the tree and all the steps (collisions and successes) of the tree splitting algorithm until all given stations succeed to transmit.

6 [10 pts]

Three wireless hosts A, B, C are about to use the following CDMA sequences:



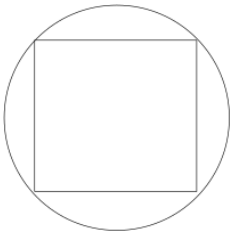
$$\mathbf{a} := 01110101$$
$$\mathbf{b} := 10101001$$
$$\mathbf{c} := 11100101$$

respectively. There are three possible pairs of CDMA sequences, namely $\{\mathbf{a}, \mathbf{b}\}$, $\{\mathbf{a}, \mathbf{c}\}$, and $\{\mathbf{b}, \mathbf{c}\}$. Give a complete explanation and proof of which among them is orthogonal and which one is not. **Hint:** Recall that you must convert $0 \to -1$ and $1 \to +1$ prior to compuing the inner products.

7 [10 pts]

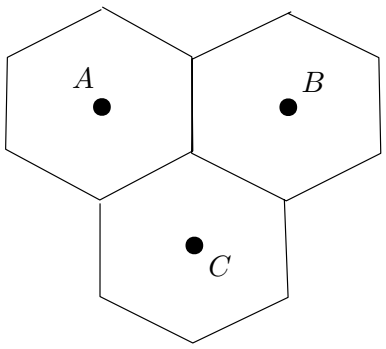
1. **[5 pts]**

   Inscribe a square S inside a disk D of diameter 2.

   A sensor is thrown at random inside the disk. What is the probability it will fall inside the disk but outside of the square?

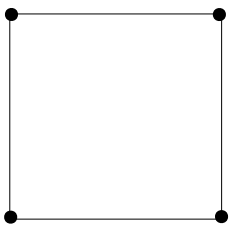
2. **[5 pts]** The figure below depicts three wireless base stations A, B, C and their corresponding hexagonal cells whose side has length 1.



What is the length of the perimeter of the triangle ABC?

8 [10 pts]

A network of four nodes is called complete if any two nodes can communicate directly with each other. Four sensors[1] with identical range (depicted below) are located on the vertices of a square with side equal to 1.



1. **[2 pts]** What is the min sensor range r required so that they form a complete network, if they are omnidirectional antennae?

2. **[3 pts]** If the sensors are directional antennae with angle $\phi = 0$, show how to orient the antennae so that the network is connected with $r = 1$. (**Note:** there may be more than one solution.)

3. **[2 pts]** If the sensors are directional antennae with angle $\phi = \pi/2$, show how to orient the antennae so that the network is complete. What is the min range r required?

4. **[3 pts]** Is there an orientation of the antennae with angle $\phi < \pi/2$ so that the network is complete?

[1]The range of a sensor is the radius of the circle it covers and its beamwidth of a sensor is the angle of the circular sector it covers in the plane.

9 [10 pts]

Consider a regular hexagon with side equal to 1. Assume you are allowed to place the antennae only on the vertices of the hexagon.

1. [**4 pts**] What is the minimum number of antennae of beam-width $2\pi/3$ and range 1 which are required so that each interior point of the hexagon is within range of an antenna? Illustrate (with a picture) and describe how to orient the antennae so that every point in the interior of the hexagon is within range of at least one antennae.

2. [**4 pts**] What is the minimum number of antennae of beam-width $\pi/3$ and range 1 which are required so that each interior point of the hexagon is within range of an antenna? Illustrate (with a picture) and describe how to orient the antennae so that every point in the interior of the hexagon is within range of at least one antennae.