Localization and GPS

November 18, 2020

Location and Localizations

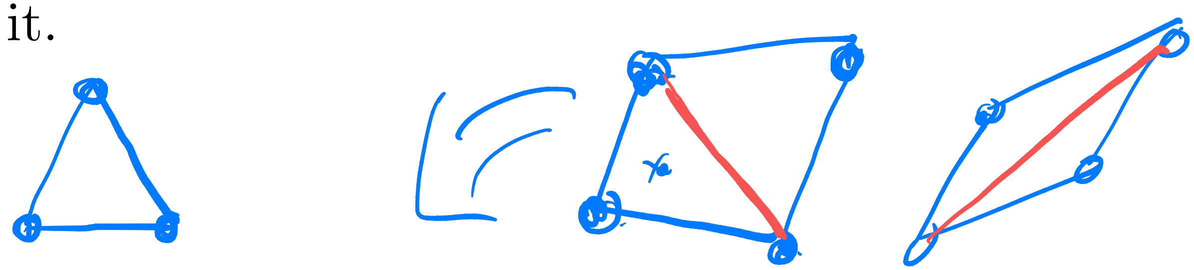
- Geographic location refers to a position on Earth.
  - Your absolute geographic location is defined by two coordinates: longitude and latitude.
  - For more accuracy you also need the height.
- Geographic Localization refers to algorithms for finding your geographic location.

(lon, lat)

(lon_t, lat_t)

Triangulation

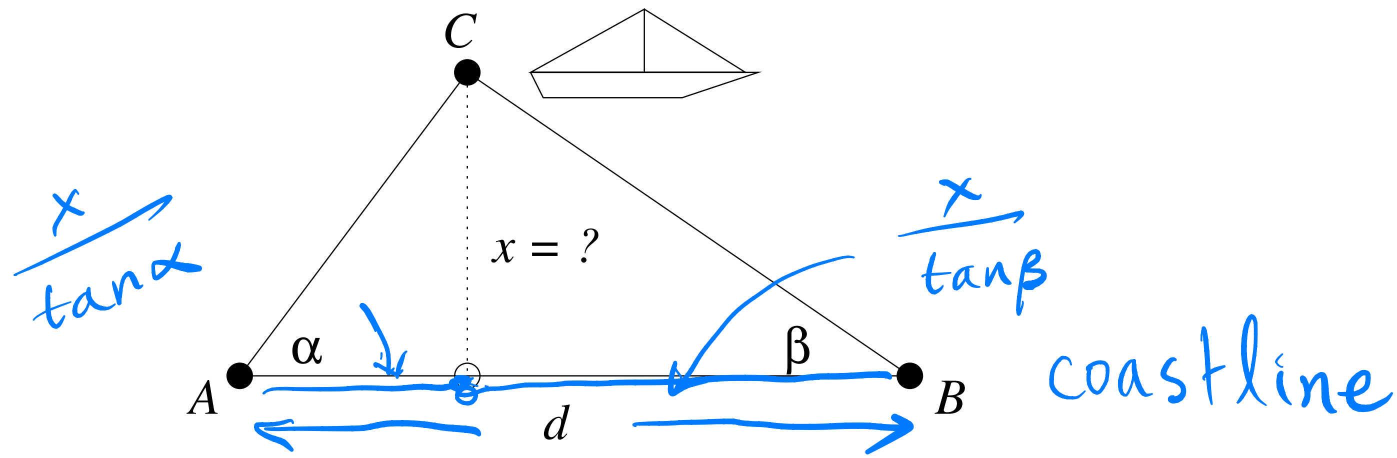
- Triangulation is the process of determining the location of a point by measuring angles to it from known points,
- It can also refer to the accurate surveying of systems of very large triangles, called triangulation networks.[a]
  - Surveying error is minimised if a mesh of triangles at the largest appropriate scale is established first, so that points inside the triangles can all then be accurately located with reference to it.



[a] Willebrord Snell in 1615-17, showed how a point could be located from the angles subtended from three known points, but measured at the new unknown point rather than the previously fixed points, a problem called re-sectioning.

Triangulation

- Assume a ship is being observed from two different locations.
- You want to measure its distance from coastline.



- Note that in the picture above
  - The coastline is formed by the line AB!
  - The coastline AB is perpendicular to the line formed by the observer and the ship!
  - You want to measure x.

trigonometry

Triangulation

- The unknown distance x can be computed from

$$d = \frac{x}{\tan\alpha} + \frac{x}{\tan\beta}$$

- It follows that

$$d = x\left(\frac{1}{\tan\alpha} + \frac{1}{\tan\beta}\right)$$

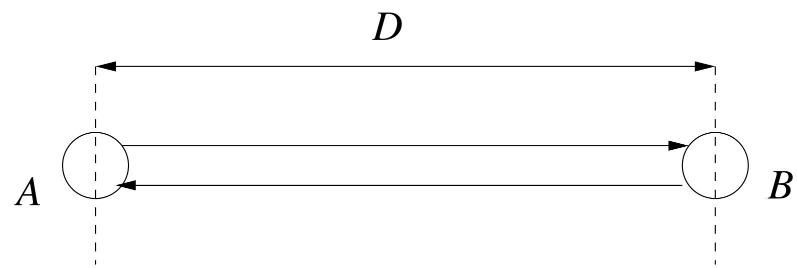
- Consequently

$$x = \frac{d}{\left(\frac{1}{\tan\alpha} + \frac{1}{\tan\beta}\right)}$$

- How do you compute α and β?
- How do you compute d?

Another Way to Measure the Distance

- Consider two sensors at unknown distance D.



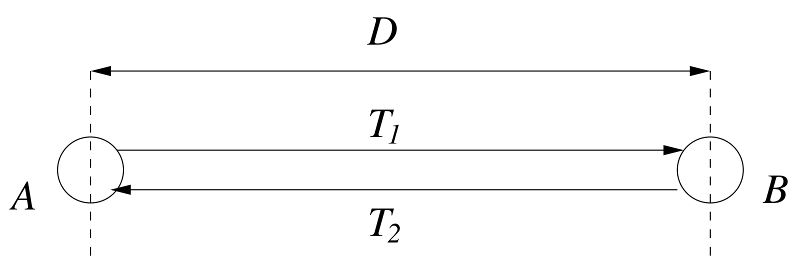
- **Algorithm**
  1. A sends a signal to B in medium1;
  2. B responds to A in a different medium2.
  3. Both A, B measure the roundtrip time, say T.
- From this they can determine the distance D!
- Why?

Another Way to Measure the Distance (1/2)

- Let v_1, v_2 be the propagation speeds in media medium1, medium2, respectively.

RTT

- Let T_1 (resp., T_2) be the time it takes from A to B (resp., B to A) in the first (resp., second) medium.



- They can both measure the roundtrip time $T (= T_1 + T_2)$.
- So we have a system with two equations: v_1, v_2 are known and T_1, T_2 are unknown quantities:

$$\left\{ \begin{array}{rcl} T & = & T_1 + T_2 \\ v_1 T_1 & = & v_2 T_2 \end{array} \right\}$$

$D = v_1 T_1$

v_1, v_2, T known

Another Way to Measure the Distance (2/2)

- To solve the system observe that

$$\begin{aligned} T_1 + T_2 &= T \\ T_2 &= \frac{v_1}{v_2} T_1 \end{aligned}$$

- Substituting,

$$T_1 + \frac{v_1}{v_2} T_1 = T$$

- Therefore

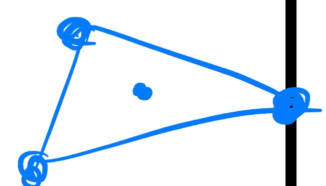
$$\begin{aligned} T_1 &= \frac{v_2 T}{v_1 + v_2} \\ T_2 &= \frac{v_1 T}{v_1 + v_2} \end{aligned}$$

- So, $D = v_1 T_1 = v_2 T_2$.

$$D = \frac{v_1 v_2 T}{v_1 + v_2}$$

Location Awareness and GPS

- Location awareness has proven to be an important component in designing communication algorithms in ad hoc systems.
- The current Global Positioning System (GPS) is satellite based and determines the position of a GPS equipped device using the radiolocation method.
- However, there are instances where devices may not have GPS capability either because the signal is too weak (due to obstruction) or integration is impossible.
- Adding to these the fact that such devices are easy to jam and there have been calls to declare GPS critical infrastructure.



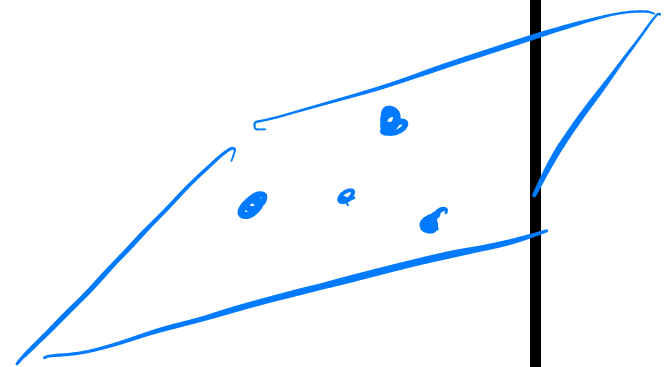
Modern Localization Techniques

- **Network-based** techniques utilize the service provider's network infrastructure to identify the location of the handset.
- **Handset-based** technology requires the installation of client software on the handset to determine its location.
- **Hybrid-based** techniques use a combination of network-based and handset-based technologies for location determination (e.g., assisted-GPS, which uses both GPS and network information to compute the location).

Various Techniques

- Cell Identification:
  accuracy depends on the known range of the particular network base station serving the handset at the time of positioning.
- Enhanced Cell Identification:
  similar to Cell Identification, but for rural areas, with circular sectors of 550 meters.
- Distance Based:
  TOA (Time of Arrival), TDOA (Time Difference of Arrival), AOA (Angle of Arrival).
- Assisted-GPS:
  uses an operator-maintained ground station to correct for GPS errors caused by the atmosphere/topography.
- Many more ...

Distance Based GPS Techniques

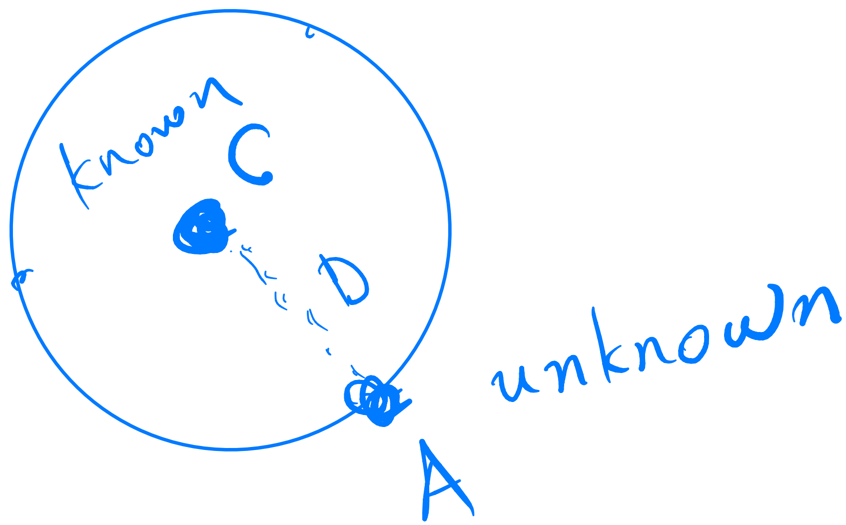


- Existing GPS techniques require line of sight propagation otherwise accuracy is affected.
  - TOA (Time of Arrival)
  - TDOA (Time Difference of Arrival).
  - AOA (Angle of Arrival)[a]
  - Signal Strength
- Three position aware neighbors are required to determine the location of a position unaware node, in a two dimensional model (e.g. latitude and longitude are determined).
- Four neighbors are required in a three dimensional model (e.g. altitude is determined as well).

[a] AOA won't discussed here

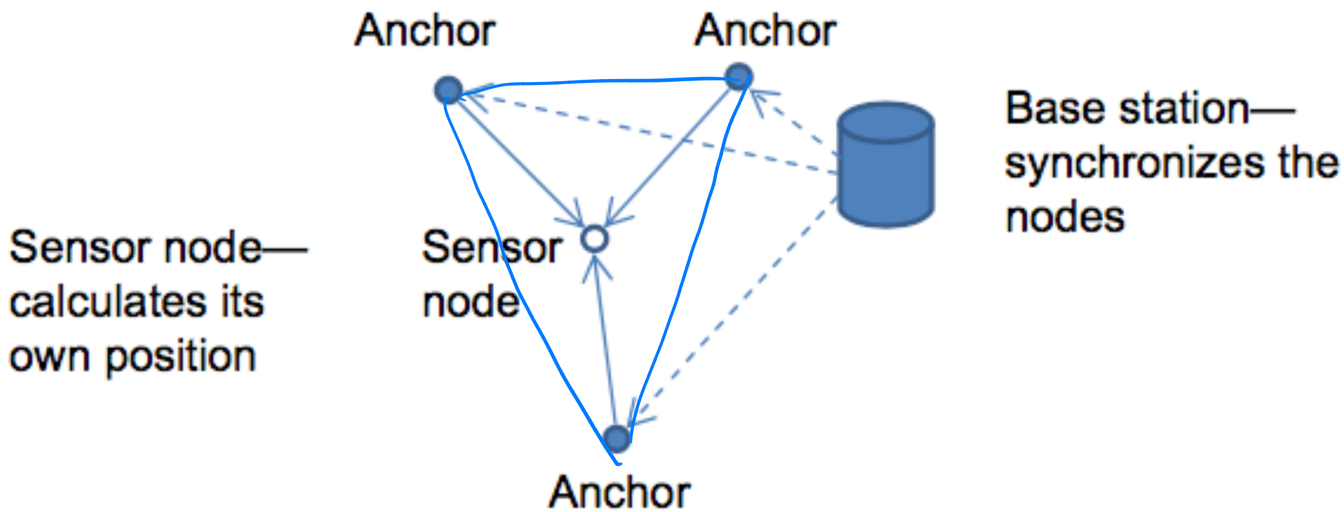
Common Features

- A sufficient number of nodes participate in a computation.
- Depending on the method: distances or angles are measured.
- Resulting system of equations is sufficient to determine locations.



Lateration

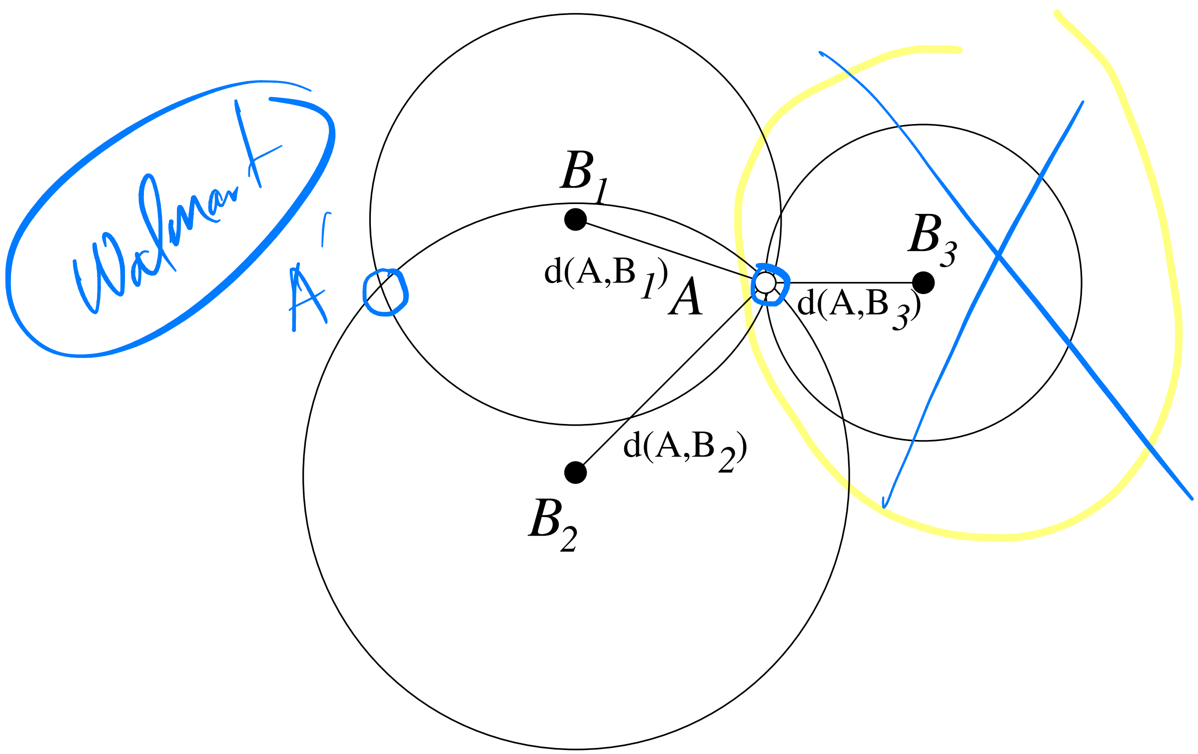
- Use a number of fixed anchor nodes at known positions:



- Anchors are synchronized to emit a signal at the same time.

The TOA Technique

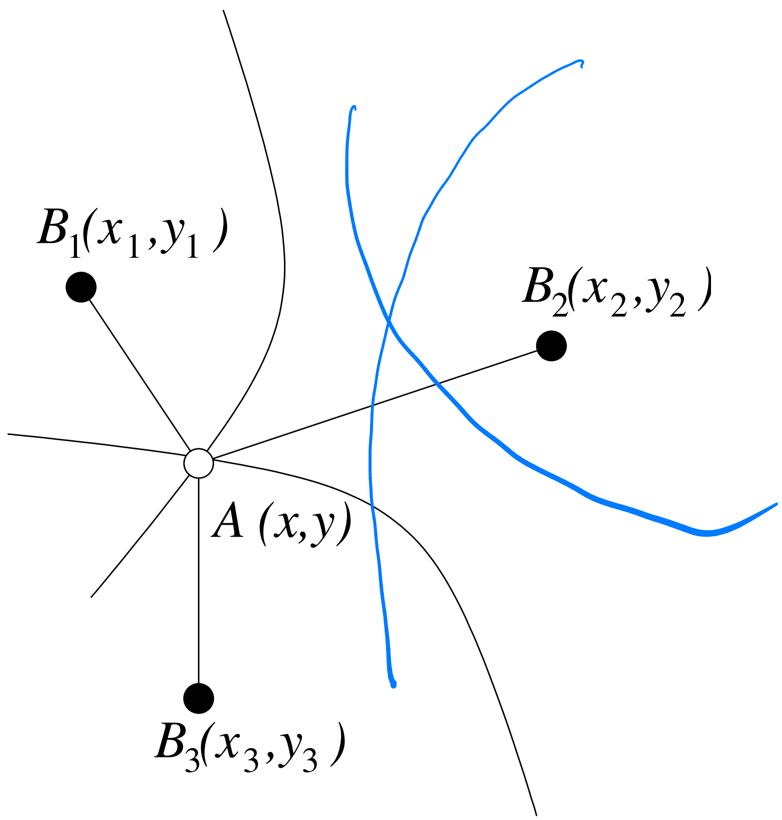
- "Vehicle" A computes its distance from fixed stations B_1, B_2, B_3, resp.. A lies on circles centered at B_1, B_2, B_3.



- A sensor at A not equipped with a GPS device can determine its position from the positions of its three neighbors B_1, B_2, B_3.

The TDOA Technique

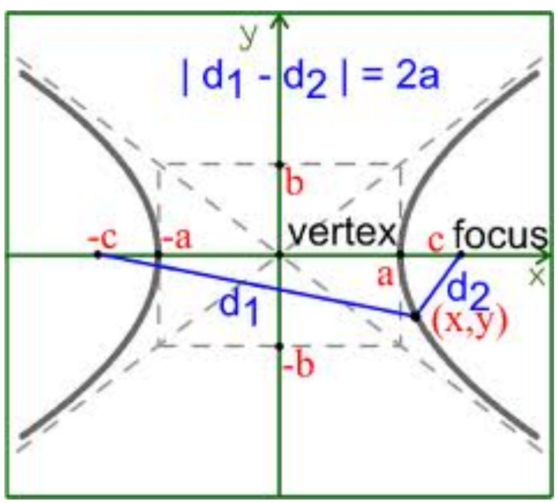
- Time difference $|t_1 - t_2|$ of arrivals t_1 and t_2 of signals from B_1 and B_2, respectively, are measured by "vehicle" A.

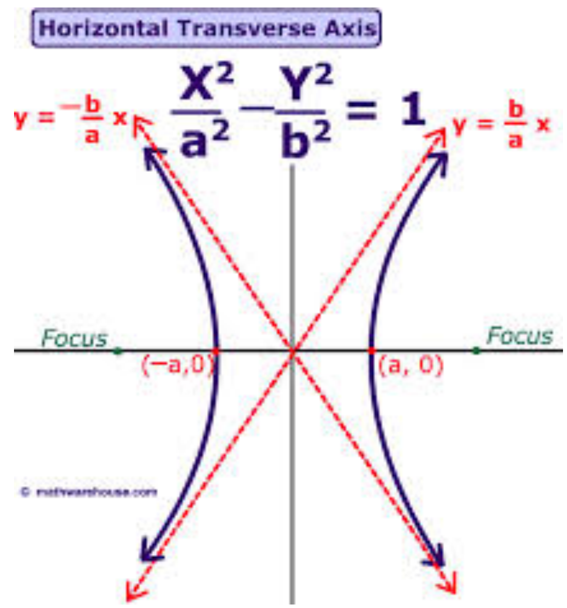


- A lies on the hyperbola with foci the pair B_1 and B_2.

The TDOA Technique: Why it works

- Since the speed is known it is equivalent to measure difference in time and difference in distance!
- One measures the difference of arrivals

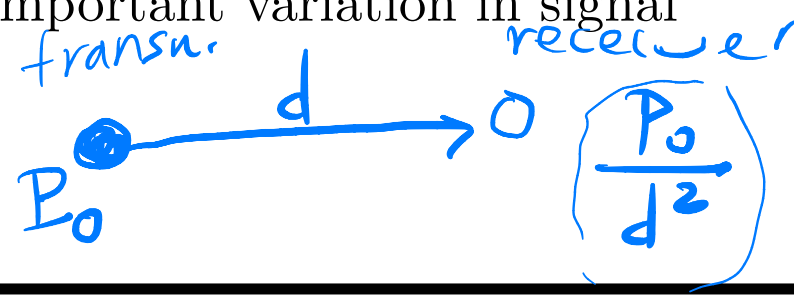




- This leads to a hyperbola!

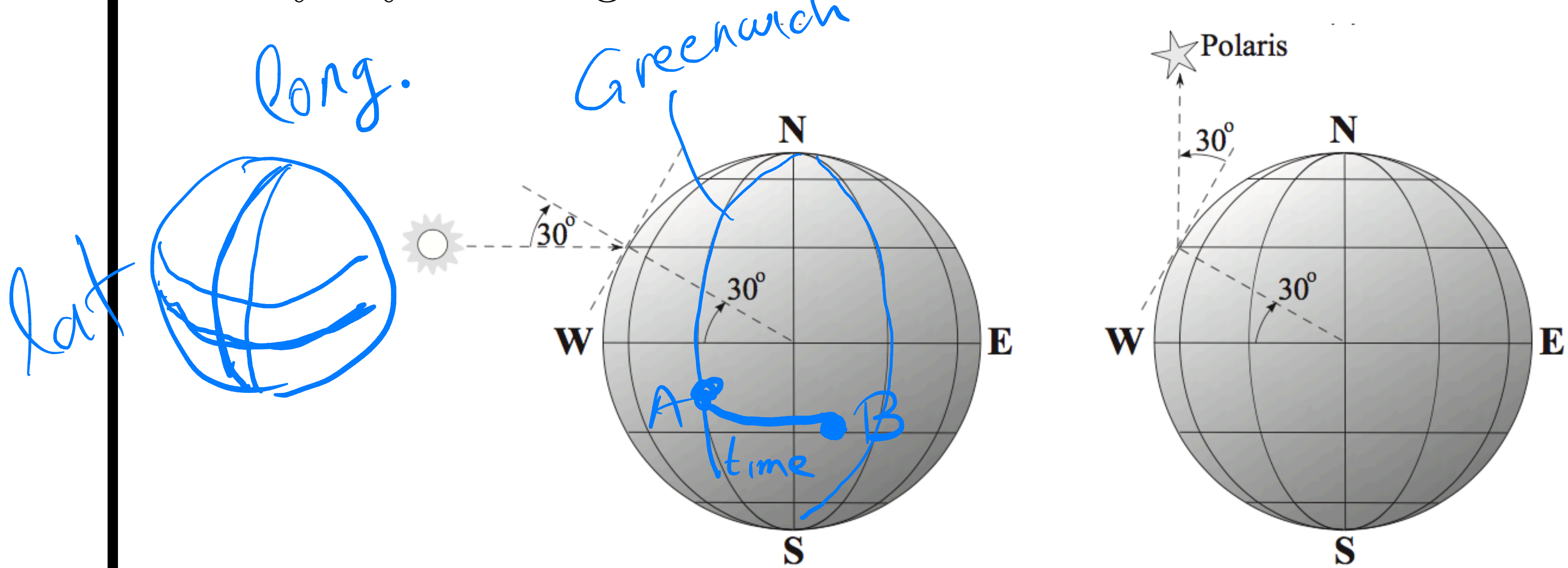
The Signal Strength Technique

- The signal strength based technique exploits the fact that a signal loses its strength as a function of distance.
  - Given the power of a transmitter and a model of free-space loss, according to the formula $\frac{P}{d^2}$ a receiver can determine the distance traveled by a signal.
  - If three different such signals can be received, a receiver can determine its position in a way similar to the TOA technique.
- The main criticism about the accuracy of the technique
  - is due to transmission phenomena such as multi path fading and shadowing that cause important variation in signal strength.



Finding your Latitude!

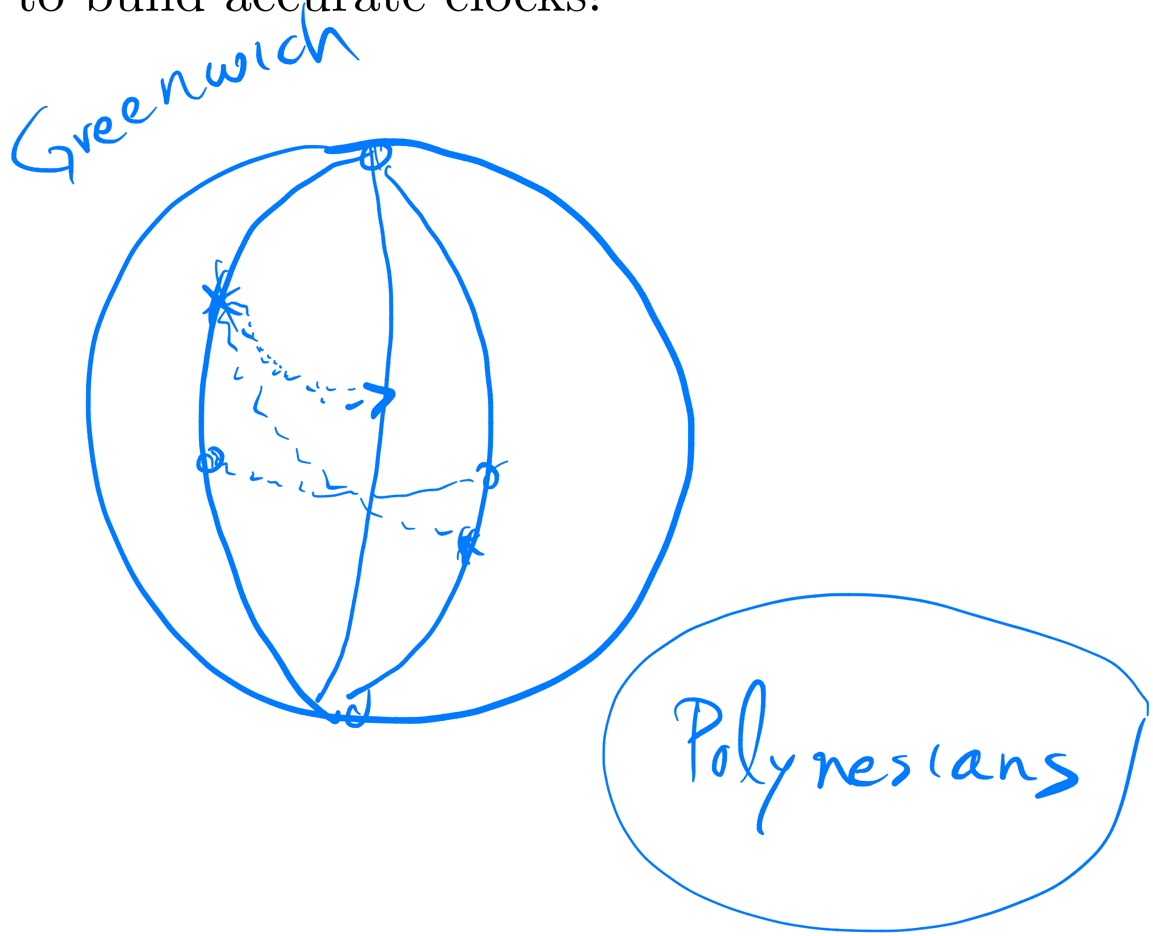
- Determining latitude from the sun or the Pole Star: Measured by ray shooting to the sun or to the Pole Star.



- At equinox, if the sun is due south at noon, a measured altitude of 60 (with the sun 30 from zenith) means that the latitude of the observer is 30.
- Measured altitude of Pole Star equals observer's latitude.

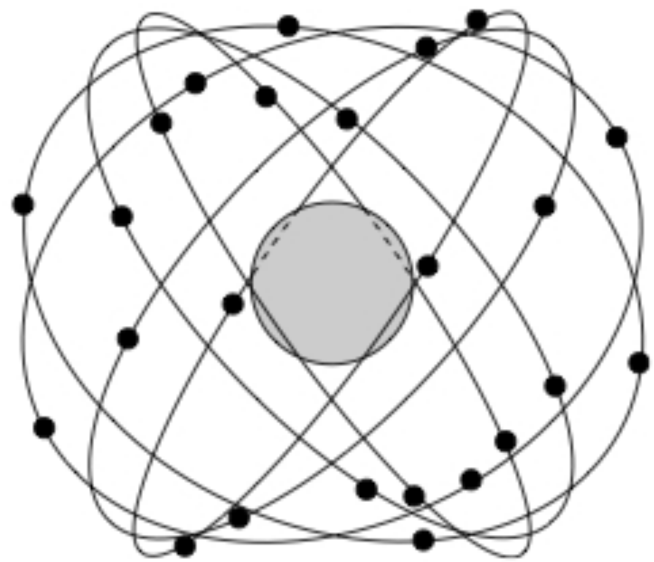
Finding your Longitude!

- That's not so easy!
- It was necessary to build accurate clocks!



GPS Satellite System

- Completed in July 1995 by the US Defense Department, and authorized for use by the general public.



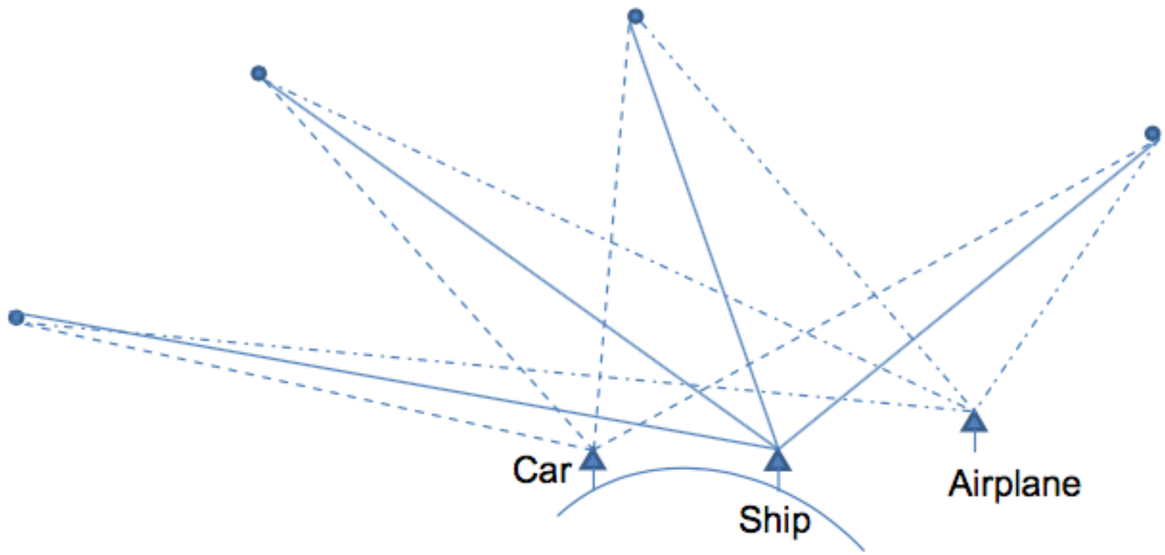
- When first deployed, it consisted of 24 satellites designed such that at least 21 would be functioning 98% of the time.
- There are several more GPS systems available today with varied accuracy in performance.

Uses of GPS

- Transportation
- Surveying
- Location Based services
- Map making
- Sports

GPS in Transportation

- Use of GPS in transportation

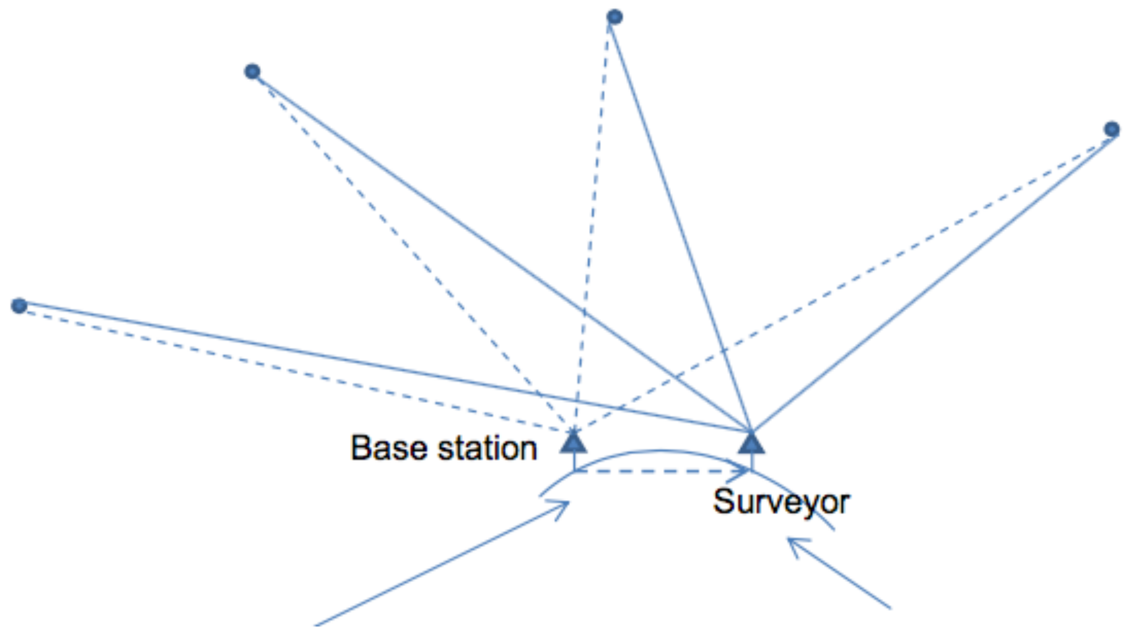


- Driving and other transportation uses–using devices installed in aircraft, cars, trucks, and ships.

Emergence: 5G

GPS in Surveying

- Base station GPS receives satellite signals and hands them to a base station radio transmitter that broadcasts them.



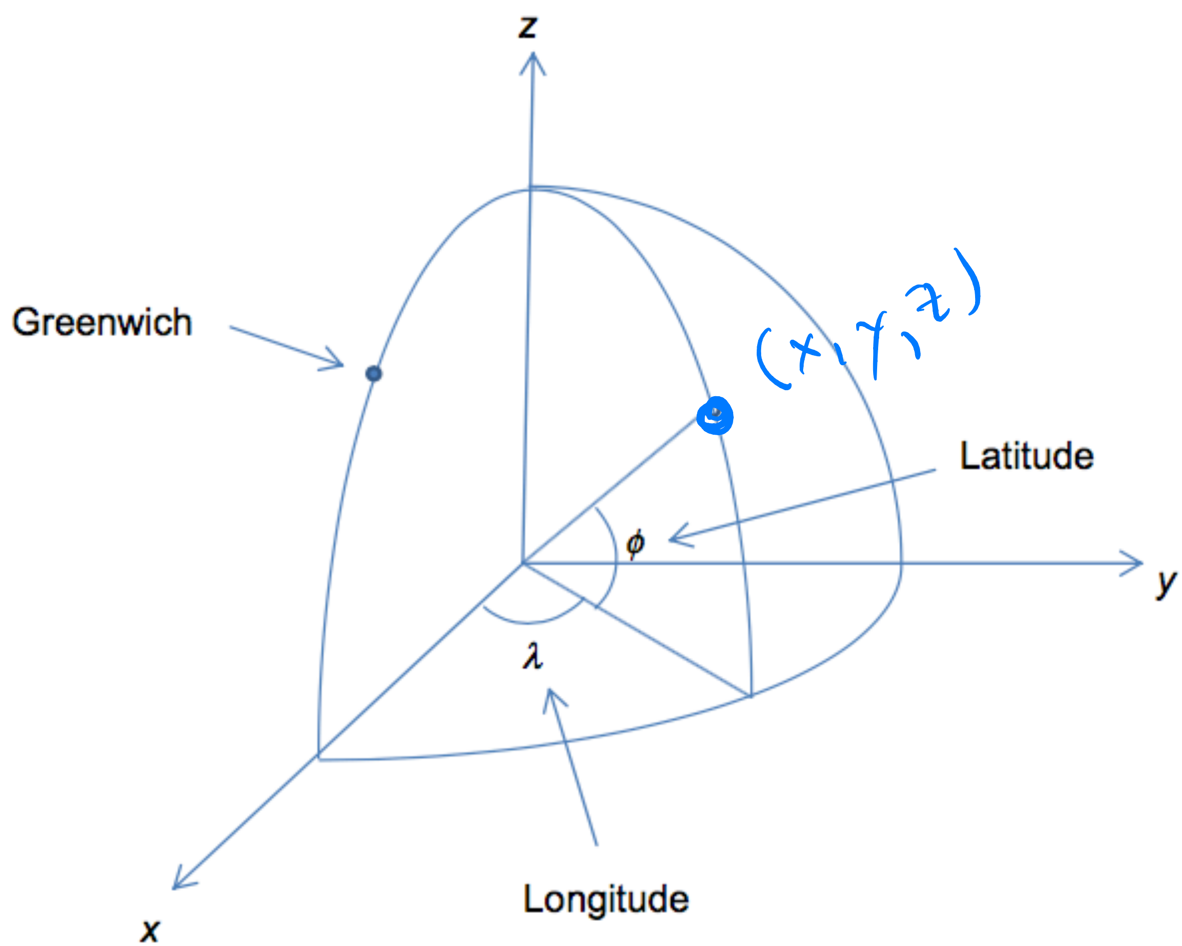
- Surveyor carries GPS antenna (for receiving satellite signals). Also carries a backpack containing a receiver connected to the antenna, as well as a radio receiver and radio (for receiving the base station's signals).

Satellites

- In 2005 the system had 32 satellites, of which at least 24 are to be functioning while the others are ready to take over in case a satellite fails.
- Satellites positioned 20,200 km from the surface of the Earth.
- Distributed across 6 orbital planes, each tilted at an angle of 55 degrees to the equatorial plane.
- At least 4 satellites per orbital plane, roughly equidistant from each other.
- Each satellite completes a circular orbit around the Earth in 11 hours and 58 minutes.
- Satellites are situated such that at any moment and at any location on Earth we may observe at least 4 satellites.

Geographic Coordinates

- Cartesian (x, y, z) and spherical (R, ϕ, λ) coordinates



Earth-Centered, Earth Fixed (ECEF) Coordinate System

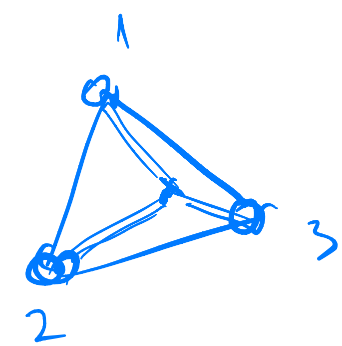
- In practice the coordinate system used is geocentric but has fixed axes with respect to the Earth, and the axes rotate with Earth.
- It is a rotating frame of reference.
- The coordinates of any point on the Earth's surface are fixed.
- This coordinate system is called the ECEF frame.

How does the receiver calculate its position?

UE

- Assume clocks of the receiver and all the satellites are perfectly synchronized.
- Receiver calculates its position through triangulation.
- The basic principle of triangulation methods is to determine where a person (object) is located by using some knowledge relating the position of the person (object) with respect to reference objects whose positions are known.
- In the case of the receiver of the GPS, it calculates its distance to the satellites, whose positions are known.

- Sattelites are rotating
- A UE localizes itself w.r. to the satellites.

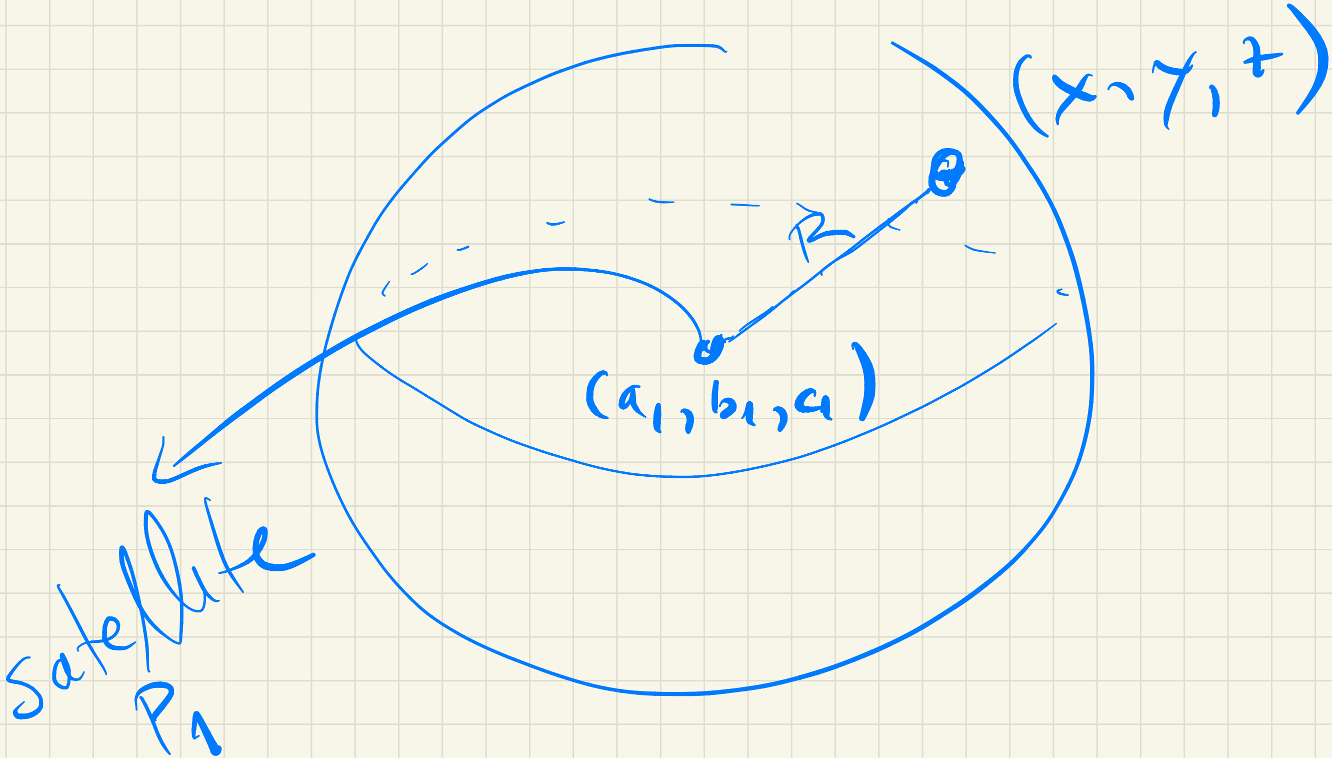


Receiver Measurements (1/2)

- The receiver measures the time t_1 it takes for the signal emitted from satellite P_1 to reach it.
- Given that the signal travels at the speed of light c, the receiver can calculate its distance from the satellite as $r_1 = ct_1$.
- The set of points situated at a distance r_1 from the satellite P_1 forms a sphere S_1 centered at P_1 with radius r_1.
- So we know that the receiver is on S_1. Consider these points as defined in a Cartesian coordinate system.
- If (x, y, z) is the unknown position of the receiver and (a_1, b_1, c_1) the known position of the satellite P_1 then (x, y, z) must satisfy the equation describing points on the sphere S_1,

$R = ct_1$

$$\underbrace{(x - a_1)^2 + (y - b_1)^2 + (z - c_1)^2}_{\text{square of Euclidean dist.}} = r_1^2 = \underbrace{c^2 t_1^2}_{R^2}. \tag{1}$$



(x, y, z)
R
(a_1, b_1, c_1)
satellite
P_1

Receiver Measurements (2/2)

- This piece of information is insufficient to determine the precise position of the receiver.
- But the receiver can repeat the same procedure with two more satellites: P_2, P_3 having positions (a_2, b_2, c_2) and (a_3, b_3, c_3).

$$(x-a_2)^2 + (y-b_2)^2 + (z-c_2)^2 = r_2^2 = c^2 t_2^2. \qquad (2)$$

and

$$(x-a_3)^2 + (y-b_3)^2 + (z-c_3)^2 = r_3^2 = c^2 t_3^2. \qquad (3)$$

- Equations (1, 2, 3) are a system of 3 equations with 3 unknowns.

$$(x-a_1)^2 + (y-b_1)^2 + (z-c_1)^2 = r_1^2 = c^2 t_1^2 \quad (1)$$
$$(x-a_2)^2 + (y-b_2)^2 + (z-c_2)^2 = r_2^2 = c^2 t_2^2 \quad (2)$$
$$(x-a_3)^2 + (y-b_3)^2 + (z-c_3)^2 = r_3^2 = c^2 t_3^2. \quad (3)$$

$$(x-a_1)^2-(x-a_2)^2 = (a_1-a_2)(2x-a_1-a_2)$$

$$(y-b_1)^2-(y-b_2)^2 = (b_1-b_2)(2y-b_1-b_2)$$

$$(z-c_1)^2-(z-c_2)^2 = (c_1-c_2)(2z-c_1-c_2)$$

Reducing the System

- The equations of this system are quadratic, not linear, which complicates the solution.
- We can replace the system by an equivalent system obtained by replacing the first equation by the difference (1) – (3) and the second equation by the difference (2) – (3) and by keeping the third equation

$$
\begin{aligned}
2(a_3 - a_1)x + 2(b_3 - b_1)y + 2(c_3 - c_1)z &= A_1, && (4)\\
2(a_3 - a_2)x + 2(b_3 - b_2)y + 2(c_3 - c_2)z &= A_2, && (5)\\
(x - a_3)^2 + (y - b_3)^2 + (z - c_3)^2 &= r_3^2 = c^2 t_3^2, && (6)
\end{aligned}
$$

where

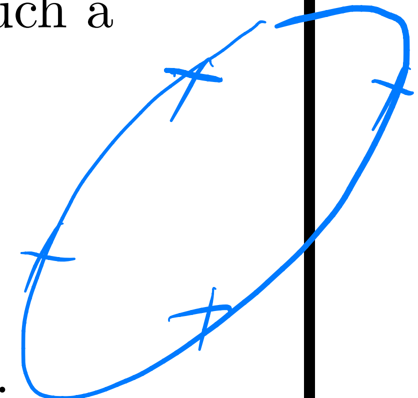
$$
\begin{aligned}
A_1 &= c^2(t_1^2 - t_3^2) + (a_3^2 - a_1^2) + (b_3^2 - b_1^2) + (c_3^2 - c_1^2),\\
A_2 &= c^2(t_2^2 - t_3^2) + (a_3^2 - a_2^2) + (b_3^2 - b_2^2) + (c_3^2 - c_2^2).
\end{aligned}
$$

Non-Linearity

- By orbital design the satellites have been placed in such a manner that no three

$$(a_1, b_1, c_1), (a_2, b_2, c_2), (a_3, b_3, c_3)$$

will ever fall along a line.



- Using the system of Equations (4), (5), and (6) and linear algebra, this ensures that at least one of the 2×2 determinants

$$\begin{vmatrix} a_3 - a_1 & b_3 - b_1 \\ a_3 - a_2 & b_3 - b_2 \end{vmatrix}, \begin{vmatrix} a_3 - a_1 & c_3 - c_1 \\ a_3 - a_2 & c_3 - c_2 \end{vmatrix}, \begin{vmatrix} b_3 - b_1 & c_3 - c_1 \\ b_3 - b_2 & c_3 - c_2 \end{vmatrix}$$

is not zero.

- In fact, if all three determinants were zero, then the vectors (depicted in the determinants) would be collinear, implying that the three points (i.e., satellites) P_1, P_2, P_3 fall on a line.

Cramer's Rule

Solution (1/2)

- Using Cramer's Rule in linear system (4), (5), and (6), we see

Determinants from Linear Algebra

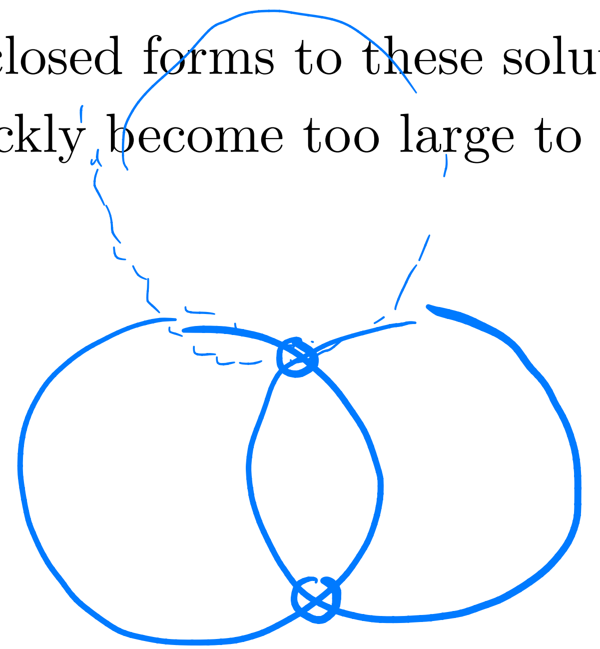
$$x = \frac{\begin{vmatrix} A_1 - 2(c_3 - c_1)z & 2(b_3 - b_1) \\ A_2 - 2(c_3 - c_2)z & 2(b_3 - b_2) \end{vmatrix}}{\begin{vmatrix} 2(a_3 - a_1) & 2(b_3 - b_1) \\ 2(a_3 - a_2) & 2(b_3 - b_2) \end{vmatrix}}$$

$$y = \frac{\begin{vmatrix} 2(a_3 - a_1) & A_1 - 2(c_3 - c_1)z \\ 2(a_3 - a_2) & A_2 - 2(c_3 - c_2)z \end{vmatrix}}{\begin{vmatrix} 2(a_3 - a_1) & 2(b_3 - b_1) \\ 2(a_3 - a_2) & 2(b_3 - b_2) \end{vmatrix}}$$

- Substituting x, y into Equation (3) yields a quadratic equation in z, which we solve to find the two solutions z_1, z_2.

Solution (2/2)

- Back-substituting z for the values z_1 and z_2 into the two above equations yields the corresponding values x_1, x_2, y_1, y_2.
- We could easily find closed forms to these solutions, but the formulas involved quickly become too large to offer any insight or convenience.



Relativistic Effects (1/3)

- Calculations relating to special relativity (SR) and general relativity (GR) effects have to be carried out.
- The speed of the satellites is sufficiently large that all of the calculations must be adapted to account for the effects of special relativity.
- The clocks on the satellites are traveling very fast compared to those on Earth.
- SR theory predicts that these clocks will run slower than those on Earth.
- The satellites are in relatively close proximity to the Earth, which has significant mass.
- GR predicts a small increase in the speed of the clocks on board the satellites.

Relativistic Effects (2/3)

- While the ECEF frame is useful for navigation, many physical processes are easier to describe in the inertial reference frame.
- A point in the inertial frame is denoted by cylindrical space-time coordinates (t, r, ϕ, z).
- The point in ECEF is denoted by (t', r', ϕ', z').
- The coordinates are related to one another as follows:

$$t = t', r = r', \phi = \phi' + \omega_E t', z = z',$$

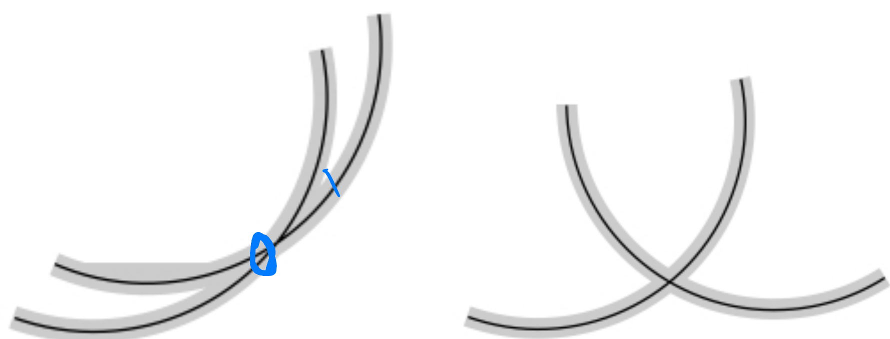
where ω_E is the uniform angular velocity of the Earth.

Relativistic Effects (3/3)

- The velocity of a satellite clock is relatively small and the gravitational fields near the Earth are relatively weak: Both these aspects, however, cause significant relativistic effects.
- The fundamental concept upon which GPS is based is that the speed of light, c, is constant.
- Satellites contain clocks stable to about 4 ns over one day. At the speed of light, a 1 ns error is about 30 cm. If speed of light varied then a GPS measurement would be out by ≥ 30 cm.
- Calculations, take account of the gravitational fields near the Earth due to the Earth's own mass. The relevant expression in the amended version of the solution of Einstein's field equations involves a number of components, including the Earth's quadrupole moment coefficient and centripetal potential.

Many Other Issues

1. Layers that surround the Earth: Ionosphere, Troposphere.
   - Refraction, Reflection effects.
2. Satellites and receivers may not be perfectly in sync.
   - Satellites are laid out such that no four of them that are visible from a given point on the Earth will ever lie in the same plane. Use extra equation to correct clock offsets!
3. Which satellites should I choose if I can see more than four?
   - Choose the spheres that minimize the errors, i.e., that intersect each other at as large an angle as possible



Exercises[a]

1. The power of a signal attenuates according to the inverse cubic law $P(d) = P(0)/d^3$, where $d > 0$ is the distance, $P(d)$ is the power at distance d, and $P(0)$ is its power at the start. How far can a signal reach if its power at distance d has to be at least 1/8 its power at the start?

2. Due to the presence of obstacles, the power of a signal attenuates according to the inverse ath power law $P(d) = P(0)/d^a$, where $d > 0, a > 1$ is the distance, $P(d)$ is the power at distance d, and $P(0)$ is its power at the start. If the power at distance $d = 1$ is 8, up to what distance d is the power of the signal at least 1/10 its power at the start?

3. Two stations located at A and B transmit wireless signals simultaneously and against each other. The signal at station A

[a] Do not submit!

has speed u and the signal at station B has speed v. Determine the point at which the two signals collide.

(a) Do the same exercise as above when the signals are transmitted with a time difference $\Delta t > 0$.