

Question 5:

- Given that the algorithm compares the current element against either the stored max, or min in every iteration, and we start the iteration at $i=2$ (no comparison at $i=1$) we see that the total number of comparisons as a function of n is:

$$f(n) = n-1$$

Question 6:

- We can write the equation $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$ where $n \geq 1$ as:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}, \text{ Base Case: Prove for } n=1, \sum_{i=1}^1 i = \frac{1(1+1)}{2} = 1$$

Inductive Hypothesis: Assume the statement holds true for $k \in \{0, 1, \dots, n\}$

$$\text{Inductively } \sum_{i=1}^k i = \frac{k(k+1)}{2} \text{ when } n=k$$

Inductive Step: Prove for $k+1$ using the assumption above

$$\begin{aligned} \sum_{i=1}^{k+1} i &= \sum_{i=1}^k i + (k+1) = \frac{k(k+1)}{2} + (k+1) \\ &= \frac{k(k+1) + 2(k+1)}{2} \\ &= \frac{k^2 + k + 2k + 2}{2} = \frac{k^2 + 3k + 2}{2} \end{aligned}$$

$$\therefore \sum_{i=1}^{k+1} i = \sum_{i=1}^k i + (k+1) = \frac{(k+1)(k+2)}{2}, \text{ therefore this holds because when}$$

$i = k+1$, $n = k+1$ and $n+1 = k+2$. $\therefore \text{QED}$