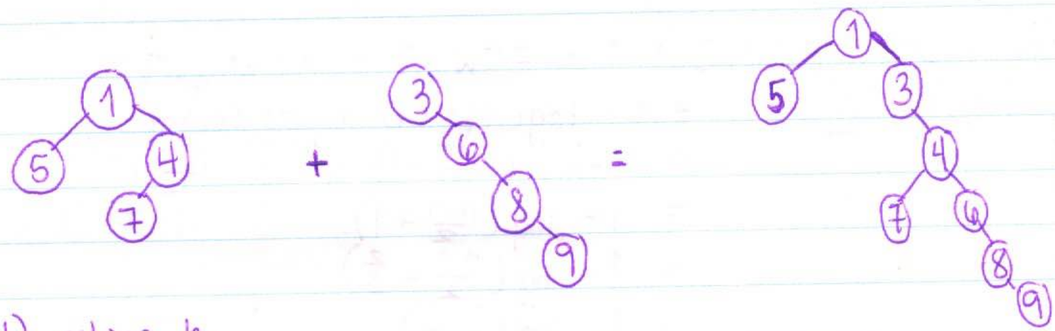
NOV 22
COMP 2402

Meldable Heaps
→ Randomized heap

meld (h_1, h_2)

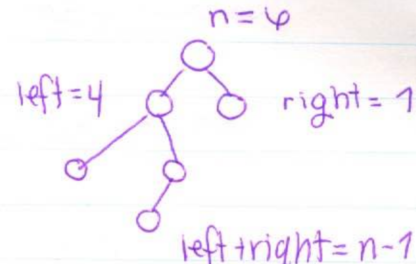


if (h_1 == null) return h_2
if (h_2 == null) return h_1
if ($h_1.x > h_2.x$) ~~return~~ meld (h_2, h_1)
flip a coin
if (heads)
    h_1.right = meld (h_1.right, h_2)
else
    h_1.left = meld (h_1.left, h_2)

- Random walk : a path through a binary tree.
  → The expected length of a random walk in a binary tree of size n ($E[W_n]$) is $\leq \log(n+1)$

* will clean up later (correction)

Proof: $E[W_n] \le \log(n+1)$, by induction on n



Base case: $n = 1$

$W_1 = 1$

$\log(1+1) = \log_2(2) = 1$ ✓

Assume for all $n' < n$, that $E[W_{n'}] \le \log(n'+1)$

maximized left = right = $\frac{n-1}{2}$

$$
\begin{aligned}
E[W_n] &= 1 + E[W_{left}] + E[W_{right}] \\
&\le 1 + \log(left+1) + \log(right+1) \quad \text{// by induction} \\
&\le 1 + \tfrac{1}{2}\log\left(\tfrac{n-1}{2}+1\right) + \tfrac{1}{2}\log\left(\tfrac{n-1}{2}+1\right) * \\
&\le 1 + \log\left(\tfrac{n-1}{2}+1\right) \\
&= 1 + \log\left(\tfrac{n-1}{2}+\tfrac{2}{2}\right) \\
&= 1 + \log\left(\tfrac{n+1}{2}\right) \\
&= 1 + \log(n+1) - \log_2(2) \\
&= 1 + \log(n+1) - 1 \\
&= \log(n+1)
\end{aligned}
$$

$$
\begin{aligned}
meld(h_1, h_2) &= E[W_{n_1}] + E[W_{n_2}] \\
&\le \log(n_1+1) + \log(n_2+1) \\
&\le 2\log(n+1)
\end{aligned}
$$