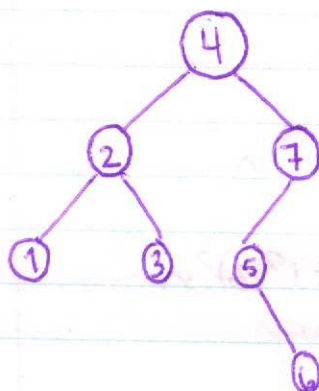


# Scapegoat ~~Space~~ Trees

NOV 10  
COMP 2402



$$O(\log n) \leq \log_{3/2} q$$

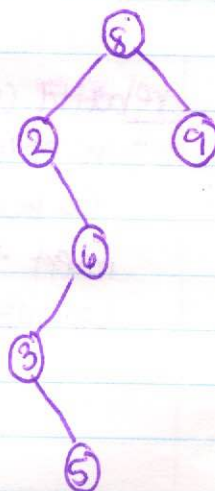
$n$  = # of nodes  
 $q$  = controller for size  
 $\hookrightarrow q = n$  initially

$$n \leq q \leq 2n$$

$$\text{height} \leq \log_{3/2} q$$

$$= \log_{3/2}(n) + \log_{3/2}(2)$$

$$= O(\log n) + O(1)$$



remove(x):

BST.remove(x) //  $O(\log(n))$

$n--$ ;

if ( $q > 2n$ )

rebuild(root) //  $O(n)$

$q = n$

$$n = 6$$

$$q = 6$$

$$n = 4$$

$$\log_{3/2}(q) = 4.4 \geq h$$

add(x)

BST.add(x) //  $O(\log(n))$

$n++$

$q++$

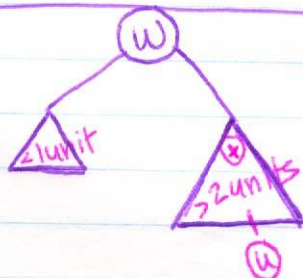
if ( $\text{depth}(u) > \log_{3/2} q$ )

$w = \text{findscapegoat}(u)$

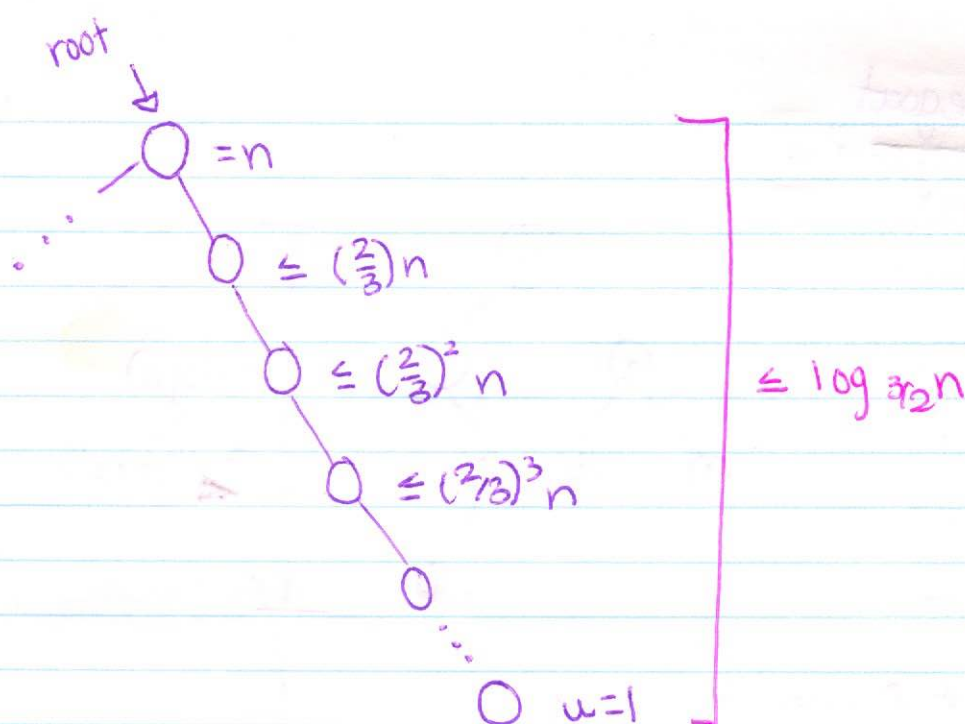
rebuild(w) //  $O(n)$

$w$  is a scapegoat  
if  $\text{size}(x)$

$$> \frac{2}{3} \text{size}(u)$$



Hilroy



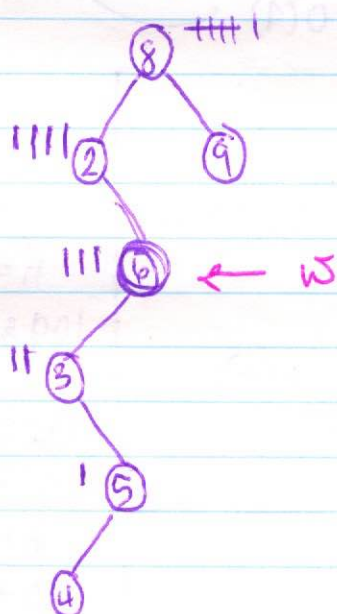
rebuild (w)

in order traversal @ w

↳ build a sorted array

insert the median element @ root

↳ repeat on left/right subtrees. //  $O(\text{size}(w))$

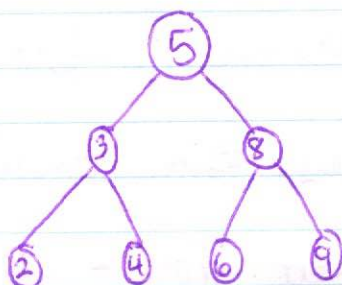


then: credits = 0  
after last rebuild @ w  
imbalance  $\leq 1$

Now: credits = 1  
imbalance = |left - right|  
 $\leq |2/3 \text{size}(w) - 1/3 \text{size}(w)|$   
 $= |1/3 \text{size}(w)|$



[2, 3, 4, 5, 6, 8, 9]



then  $q_0 = n_0$   
after last rebuild  
credits jar = 0

Now  $q > 2n_1 \rightarrow q - n > n_1$   
credits jar  $\geq \Delta n$   
 $= |n_0 - n_1|$   
 $= q - n_1$