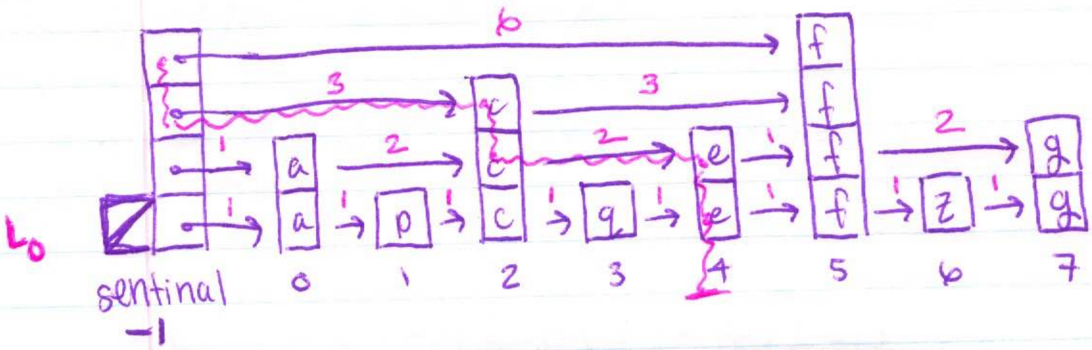
Oct 11, 2017
COMP 2402



get (5)

$j = -1$

$j + 6 = -1 + 6 \geq 5 \geq 5$ ↓

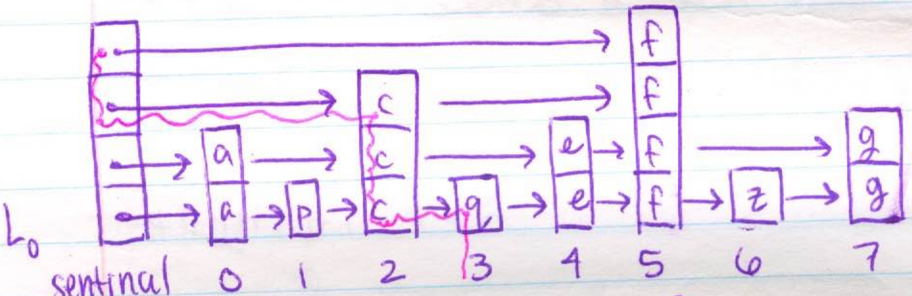
$-1 + 3 = 2 < 5, \ j = 2$ →

$2 + 3 = 5 \geq 5$ ↓

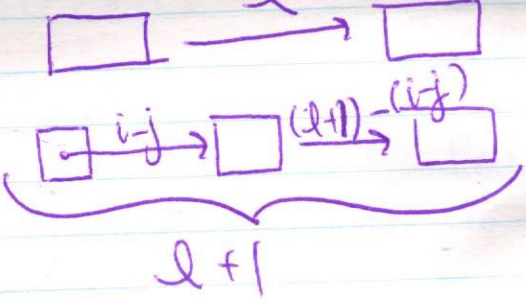
$2 + 2 = 4, \ j = 4$ →

$4 + 1 = 5 \geq 5$ ↓

$4 + 1 = 5 \geq 5$ ↓



add (4, x)



Random Variable

$$E_{[X]} = \sum_x x \cdot Pr\{X=x\}$$

Dice

$$E_{[\;]} = 1 \cdot 1/6 + 2 \cdot 1/6 + 3 \cdot 1/6 + 4 \cdot 1/6 + 5 \cdot 1/6 + 6 \cdot 1/6$$

$$= 3.5$$

Indicator Variable

$$E_{[I]} = 0 \cdot Pr\{I=0\} + 1 \cdot Pr\{I=1\}$$

$$= Pr\{I=1\}$$

$$E_{[A+B]} = E[A] + E[B]$$

$$E\left[\sum_{i=a}^{b} x_i\right] = \sum_{i=a}^{b} E[x_i]$$

X = # times flip a coin until you get tails

H, H, T $x=3$

T $x=1$

HHHHHHT $x=7$

$$E_{[x]} = \sum_{i=1}^{\infty} i \cdot Pr\{x=i\}$$

$$= \sum_{i=1}^{\infty} i \cdot \left(\tfrac{1}{2}\right)^i$$

$Pr\{x=1\} = 0.5$

$Pr\{x=2\} = 0.25$

$Pr\{x=3\} = 0.125$

$Pr\{x=i\} = (1/2)^i$

$$I_i = \begin{cases} 1 & \text{if we make ith coin toss?} \\ 0 & \text{otherwise} \end{cases}$$

$X = \sum_{i=1}^{\infty} I_i$

$E[X] = E[\sum_{i=1}^{\infty} I_i]$

$= \sum_{i=1}^{\infty} E[I_i]$

$= \sum_{i=1}^{\infty} Pr\{I_i = 1\}$

$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \cdots$

$= 2$

$Pr\{I_1 = 1\} = 1$

$Pr\{I_2 = 1\} = 1/2$

$Pr\{I_3 = 1\} = 1/4$

$Pr\{I_4 = 1\} = 1/8$

$\vdots$

$Pr\{I_i = 1\} = (\frac{1}{2})^{i-1}$

let $s = 1 + 1/2 + 1/4 + 1/8 + \ldots$

$1/2 s = 1/2 + 1/4 + 1/8 + \ldots$

$s - 1/2 s = 1$

$1/2 s = 1$

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n_i = # of items in $\mathcal{L}_i$

eg.) $n_0 = n$

$I_{ij} = \begin{cases} 1 \text{ if node } j \text{ appears in } L_i \\ 0 \text{ otherwise} \end{cases}$

$E[n] = ?$

$E[I_{ij}] = Pr\{I_{ij} = 1\}$

$= (\frac{1}{2})^i$

$E[n_i] = \sum_{j \in L_0} E[I_{ij}]$

$E[n_i] = E[\sum_{j \in L_0} I_{ij}]$

$= \sum_{j \in L_0} (\frac{1}{2})^i = n/2^i$

$$E[\text{size of skiplist}] = E[n_0 + n_1 + n_2 + \cdots]$$

$$= E\left[\sum_{i=0}^{\infty} n_i\right]$$

$$= \sum_{i=0}^{\infty} E[n_i]$$

$$= \sum_{i=0}^{\infty} n/2^i$$

$$= n \cdot \sum_{i=0}^{\infty} \left(\tfrac{1}{2}\right)^i$$

$$= n \cdot \left(1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} \cdots\right)$$

$$= 2n$$