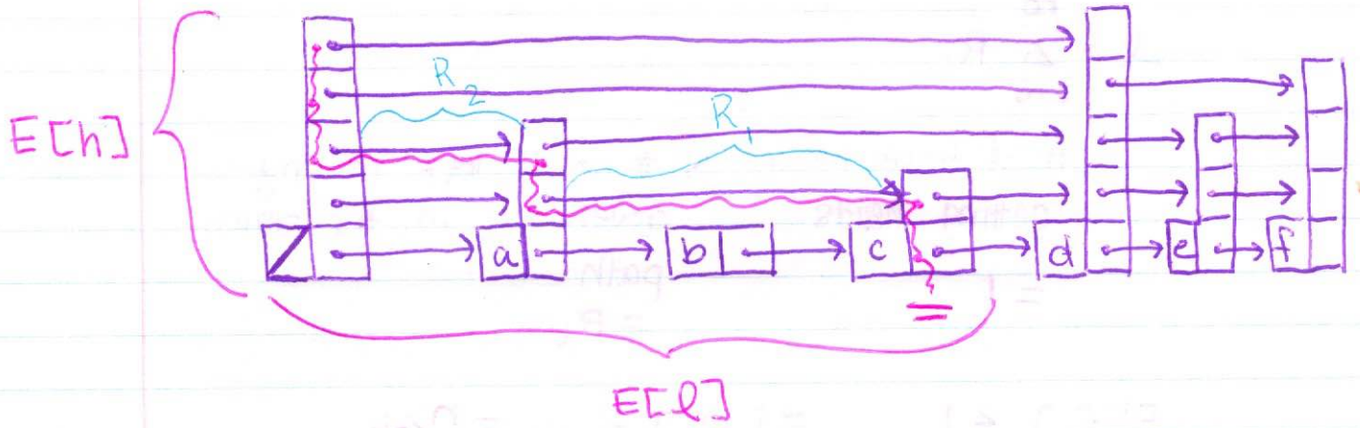
$$x^{\log_x n} = n$$

Fri, October 13
COMP 2402

SkipLists



E[search path] = E[h] + E[ℓ]

n = # of i such that L_i is not empty

$$I_i = \begin{cases} 1 \text{ if } L_i \text{ is not empty} & // \ n_i \geq 1 \\ 0 \text{ otherwise} & // \ n_i = 0 \end{cases}$$

$$h_i = \sum_{i=0}^{\infty} I_i$$

$$E[h] = E\left[\sum_{i=0}^{\infty} I_i\right] = \sum_{i=0}^{\infty} E[I_i]$$

$$\leq \sum_{i=0}^{\infty} n/2^i = n \sum_{i=0}^{\infty} 1/2^i = 2n$$

$$= \sum_{i=0}^{\lceil \log n \rceil} E[i] + \sum_{i=\lceil \log n \rceil + 1}^{\infty} E[I_i]$$

$$\leq \sum_{i=0}^{\lceil \log n \rceil} 1 + \sum_{i=\lceil \log n \rceil + 1}^{\infty} n/2^i$$

$I_i \leq 1$

$E[I_i] \leq 1$

$I_i \leq n_i$

$E[I_i] \leq E[n_i]$

$= \lceil \log n \rceil + (\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots)$

$= \log n + 1 + (1/2 + 1/4 \cdots)$

$i = \log n + 1 = \log n + O(1)$

$\rightarrow n/2^{\log n + 1}$

$= n/2^{\log n} \cdot 2^i$

$= n/n \cdot 1/2$

$= 1/2$

$i = \log n + 2$

$\rightarrow n/2^{\log n + 2}$

$= n/n \cdot 2^2 = 1/4$

R_i = # of horizontal steps taken in L_i

$$\ell = \sum_{i=0}^{\infty} R_i$$

# of tosses before getting heads $= 1$

$\geq$ # steps left in any given L_i in the search path $= R_i$

$E[R_i] \leq 1 \qquad E[R_i] \leq n_i = n/2^i$

$$E[\text{search path}] = E[h] + E[\ell]$$

$$= \log n + 2 + \sum_{i=0}^{\infty} E[R_i]$$

$$\leq \log n + 2 + \sum_{i=0}^{\lceil \log n \rceil} 1 + \sum_{i=\lceil \log n \rceil + 1}^{\infty} n/2^i$$

$$= \log n + 2 + \lceil \log n \rceil + (1/2 + 1/4 + 1/8 \cdots)$$

$$\leq \log n + 2 + \log n + 1 + 1/2 + 1/4 + 1/8 \cdots$$

$$= 2 \log n + O(1)$$