Nov 29
COMP 2402

# Sorting Algorithms

## MergeSort
- comparisons : $n \log(n)$ worst case
- in-place? no
- stable? yes

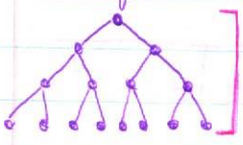
## QuickSort
- comparisons: $1.38 n \log(n) + O(n)$ expected
- in-place? yes
- stable? no

## HeapSort
- comparisons: $2n \log(n) + O(n)$ worst case
- in-place? yes
- stable? no

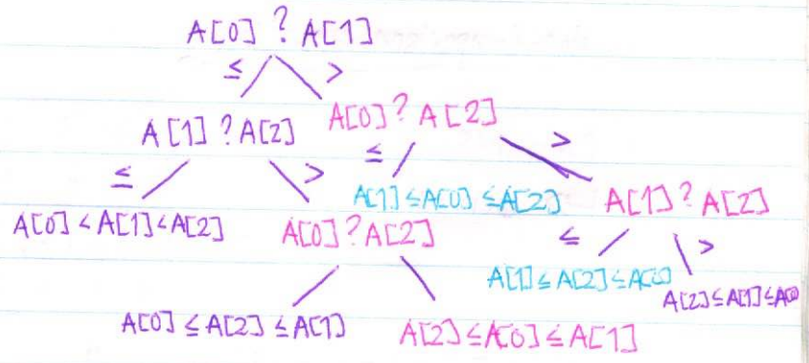
A:

| x | x | x |
|---|---|---|
| 0 | 1 | 2 |

$n!$ permutations of A
$n!$ leaves
↳ height $\geq \log(n!)$




m leaves $\leftrightarrow$ $\log(m) \leq h$




$\log(n!)$ is $\Omega(n \log n)$

$g = \Omega(f)$

upper: $\log(n!) = \log(n \cdot (n-1) \cdot (n-2) \cdots)$

$= \log(n) \cdot \log(n-1) \cdot \log(n-2) \cdots \log(1)$

$\leq \log n + \log n + \log n + \cdots \log(n)$

$= n \log n$

lower bound: $\log(n!) = \log(1) + \log(2) + \cdots + \log(n)$

$\geq \log(\frac{n}{2}+1) + \log(\frac{n}{2}+1) + \cdots \log(n)$ — cut out: $\log(1) \cdots \log(\frac{n}{2}-1)$

$\geq \log(\frac{n}{2}) + \log(\frac{n}{2}) + \cdots + \log(\frac{n}{2})$ — lowered everything to smallest term

$= n/2 \cdot \log(n/2)$

$= n/2 (\log(n) - \log 2)$

$= n/2 (\log(n) - 1)$

$= n/2 \log(n) - n/2$

$= \frac{1}{4} n \log n$

side bar:

$\log(n) \geq 2$ for $n \geq 4$

$1/4 \log(n) \geq 1/2$

$n/4 \log(n) \geq n/2$

$n/4 \log(n) - n/2 \geq 0$

$n/2 \log(n) - n/2 \geq n/4 \log n$

$\therefore$

$\frac{1}{4} n \log n \leq \log(n!) \leq n \log n$

## Non-Comparison-Based String

CountingSort ↳ $O(n+K)$

a: | 0 | 3 | 1 | 3 | 2 | 2 | 1 | 3 |

0 1 2 3 4 5 6 7 $K=4$

c: | 1 | 2 | 2 | 3 |

0 1 2 3 // number of times each number appears

```
for(i=0; i<n; i++)
    c[a[i]] ++
```

a: | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 3 |

Stable: c' | 1 | 3 | 5 | 8 |

b: | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 3 |

0 1 2 3 4 5 6 7

```
for(i=1; i<K; i++)
    c'[i] += c[i-1]
for (i=n-1 i>=0 i--)
    b[--c'[a[i]]] = a[i]
a=b
```

RadixSort

number sections we have to deal with

$K = w/d$

$d=4 \rightarrow$ ~~$K = 2^d = 16$~~

| | | | |
|---|---|---|---|
| 668 | 0010 | 1001 | 1100 |
| 3018 | 1011 | 1100 | 1010 |
| 1409 | 0101 | 1000 | 0001 |
| 3233 | 1100 | 1010 | 0001 |
| 714 | 0010 | 1100 | 1010 |

countingsort

CountingSort
0...K

Radix
$0 \ldots 2^K$