Alžbeta Jurečková

1) Nech $f: V \to U$ je lin. zobrazenie konečnorozmerného vektorového priestoru. Potom $\dim(V) = \dim(\ker(f)) + \dim(\mathrm{Im}(f))$

$B: \mathbb{R}^3 \to \mathbb{R}^3$

$$B(x_1, x_2, x_3) = (x_1 + x_2 + x_3, x_1 - x_2, -x_1 + x_2)$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 1 & 0 \end{bmatrix}$$

$\ker(B)$: $\{\vec{x} \in V : f(\vec{x}) = 0\}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 0 & 2 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{matrix} -\frac{1}{2}a \\ -\frac{1}{2}a \\ a \end{matrix}$$

$$\ker(B) = \{(-\tfrac{1}{2}a, -\tfrac{1}{2}a, a) : a \in \mathbb{R}\}$$

$\mathrm{Im}(B)$: $\{\vec{y} \in U : f(\vec{x}) = \vec{y}$ pre $\vec{x} \in V\}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 1 & -1 & 0 & b \\ -1 & 1 & 0 & c \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 1 & -1 & 0 & b \\ 0 & 0 & 0 & c+b \end{array}\right] \sim \left[\begin{array}{ccc|c} 0 & 2 & 1 & a-b \\ 1 & -1 & 0 & b \\ 0 & 0 & 0 & c+b \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & 0 & b \\ 0 & 2 & 1 & a-b \\ 0 & 0 & 0 & c+b \end{array}\right] \quad c = -b$$

$$\mathrm{Im}(B) \{(a, b, -b),\ a, b \in \mathbb{R}\}$$

$\dim(\ker B) = 1$

$\dim(\mathrm{Im}(B)) = 2$

Báza $\ker(B) = \{(-\tfrac{1}{2}, -\tfrac{1}{2}, 1)\}$

$\langle (-\tfrac{1}{2}, -\tfrac{1}{2}, 1), (-\tfrac{1}{2}, -\tfrac{1}{2}, 1) \rangle = \frac{1}{4} + \frac{1}{4} + 1 = \frac{3}{2}$

ortonormálna báza $\ker(B) = \left(\sqrt{\tfrac{2}{3}}(-\tfrac{1}{2}, -\tfrac{1}{2}, 1)\right)$

Báza $\mathrm{Im}(B) = \{(\tfrac{1}{2}, 0), (0, \tfrac{1}{2})\}$

$\langle (\tfrac{1}{2}, 0), (0, \tfrac{1}{2}) \rangle = 0$

$\langle (\tfrac{1}{2}, 0), (\tfrac{1}{2}, 0) \rangle = \frac{1}{4}$

$\langle (0, \tfrac{1}{2}), (0, \tfrac{1}{2}) \rangle = \frac{1}{4}$

ortonormálna báza $\mathrm{Im}(B) = \{(1, 0), (0, 1)\}$

$$\begin{bmatrix} 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 \end{bmatrix}$$

$(0, 1, -1)$

$(1, 0, 0)$

$a(1, 0, 0) + b(0, 1, -1)$

$= a + b - b$