

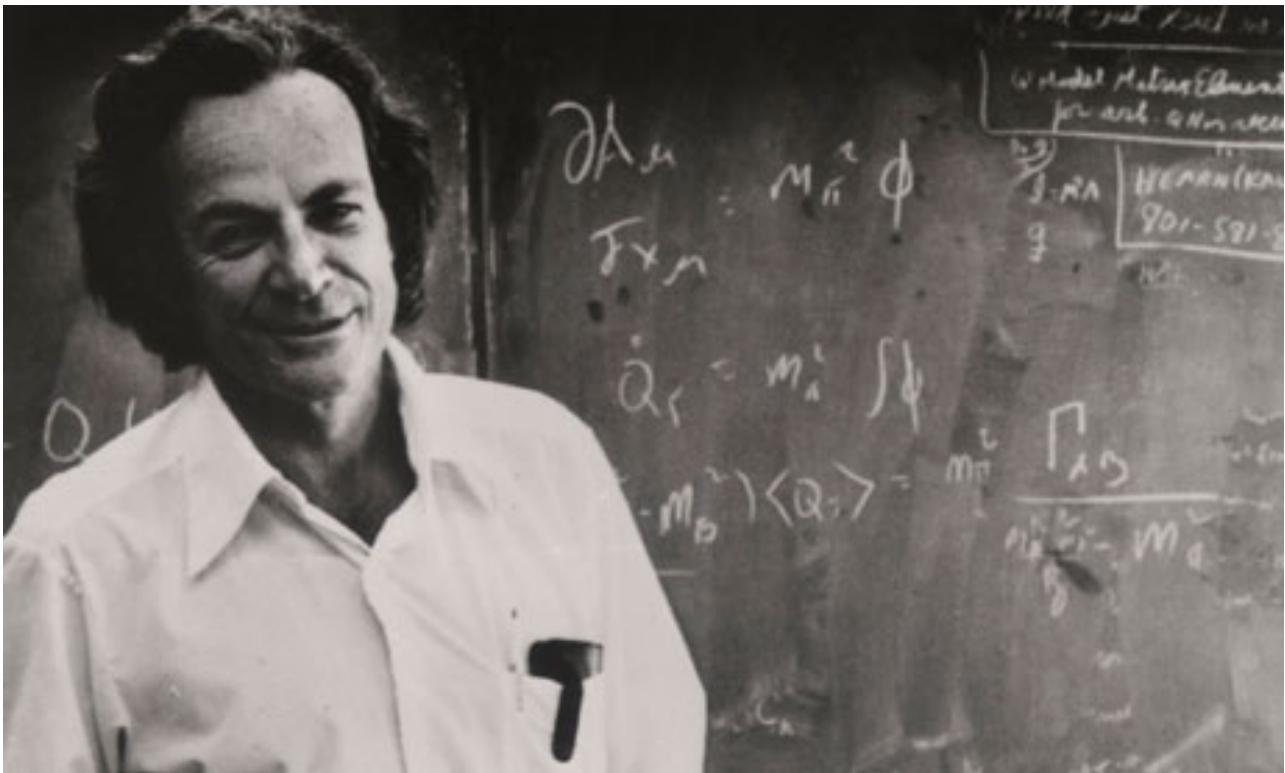
Volume 1

Feynman Hughes Lectures

Astronomy
Astrophysics
Cosmology

Oct. 1966-June 1967

*Notes taken & Transcribed by
John T. Neer*



Thanks for the
notes you made
on my astronomy
lectures.

Dick Feynman

At the completion of the year, I gave Feynman a copy of my notes which he appreciated and in return signed my FLP vol 1.

This picture of Feynman is as I remember him coming to the Research labs. Always casual in dress and always with chalk in hand

Lecture Organization Preface

When Feynman started this lecture series on astronomy as he referred to it, it really emphasized the astrophysics more than "classical" Astronomy. He did not have a definitive roadmap of his topics over the ensuing 40 weeks with 2 hours a session weekly.

As I have been reviewing and preparing these notes for release, I have been inserting more current findings, particularly for space observatories, to relate Feynman's lectures to today's "view" of the universe. Feynman's lectures covered a broad spectrum of topics which provide the reader with a rich foundation in the Astronomy, Cosmology and Astrophysics.

I understand and appreciate the significant advances in astrophysics and cosmology that have occurred since the time these notes were created and look forward to those whose knowledge and experience can add to the content herein.

Bookmarks are provided

	CHAPTER 1	Pages
	COSMOLOGY	1 - 59
	Models of the Universe Relativity and Its Application to Cosmology Curved Space-Time Gravitational Collapse	
	CHAPTER 2	60 - 111
	STELLAR STRUCTURE	
	Equilibrium Conditions of a Ball of Gas Mechanism of Energy Generation Nuclear Synthesis Carbon-Nitrogen Cycle β -decay URCA Process (pair production)	
	CHAPTER 3	112 - 171
	EVOLUTION OF THE STARS	
	Main Sequence Stars Population I and II Stars Production of the Higher Elements Solar Oblateness and Dicke's Theory Radio Galaxies Radio Emissions Synchrotron Radiation Quasars	
	CHAPTER 4	172 - 180
	SPECIAL LECTURE ON PAIR PRODUCTION	
	CHAPTER 5	181 - 204
	SOLAR SYSTEM	
	Cosmic Rays Earth's Magnetic Field Rotation of Mercury and Venus The Sun Structure Sunspots Solar Winds Theory of Corona	
	Mars Jupiter's Rotation	

INDEX

A

Absolute Time, 39
Absorption Coefficient, 149
Age of Universe (See Hubble Constant)
Angular
Distance (Diameter) 9, 13, 14, 23, 24
Separation, $d\theta$ 13, 17
Antiparticles, Theory of Existence 172, 180

B

Background Radiation (Universe) 126, 140.
147
Barred Galaxies, 4
Baryon, 142, 143
Beta Decay, 93, 103
Big Bang Theory, 7
Binding Energy, 101
Black Body Radiation 2, 62, 146
Black Stars, 58
Brightness (See Luminosity)
Brightness, Stellar, 2

C

Carbon 12 Production, 98
Carbon-Nitrogen Cycle, 94
Causality, 173, 174
Cepheid Variable Stars, 117
Charge Conjugation, 105
Complex Integration, 101-103
Conduction, 60
Convection 60, 64, 114, 115
Cosmic Rays, 181
Cosmological
Assumption 13, 18, 145, 141
Concepts 6, 124, 145, 146
Mirror, 125, 126, 145, 146
Coulombic Repulsion, 83
Curvature of Space 11, 43, 48

D

D'Alembertian, 135
Density, Critical, 12, 55, 56
Deuterium, 90, 91
Dicke's Theory 133-139

E

Earth, Magnetic Field
 $\epsilon(\tau)$, See Nebular Spacing
Einstein
Absolute Time, 39
Coefficients, 80
Cosmological Term, 29
Elevator Analogy, 34
Equivalence Law 34, 46, 134
Gravity, Theory of 32, 33, 45, 46, 135
Mass, Definition, 33
Motion Law of, 51
Relativity, Theory, 30
Electric Field, 156
Electromagnetic Mass, 36
Electron,
Gas, 77
K, Capture, 103
Radiating, 36, 148, 152
Scattering, 79
Elemental Length in Space-Time 55, 136
Elements
Higher than Iron, 122, 127
Proportion, 99
Elliptical Galaxies, 4
Energy Generation, 60, 82
Eotvos Experiment, 134
Europium, 122
Exclusion Principle, 74

F

Feynman-Gellmann Process (URCA), 103
Feynman Mean Curvature, 44
Fields,
Different types, 30
Energy, 30
Fourier Transform Theory 157-159
Frequency, ω
At Emission 14, 22, 23
At Reception, 14, 22, 23

G

Galactic, diameter, 9
Galaxies, 4
Gamma Ray Reactions, 102
Gas, Condensation 143, 145-146
 $g_{\mu\nu}$, Tensor, 40, 41
Effect on Time 50, 51
Collapse 56, 57
Geodesic, 43, 135-137
Gobular Clusters, 4
Gravitational
Attraction, 134
Collapse 56-57
Effect, 18
Field, 32, 134
Potential, 35, 69, 135, 138
Quadratic Form, 33
Red Shift, 37
Waves, 145

H

Harmonic Oscillator, 158
Helium Burning Process, 63, 92, 96
Hoyle's Theory, 28, 97
Hubble's
Constant, 12, 18
Theory, 29
Hydrogen Burning Process, 90
Hyperbolic Functions (Relativity)
16, 17, 26a

I

Inertia, 33, 134-136
Invariants, 105
Iron, 103
Isotopes, Unstable 127, 128

J

Jupiter Rotation 194-196

K

L

Lane-Emden Equation, 66
Least Action Principle,
Light Path 39, 52-54
Leptons, 142, 143
Lotentzi Time 8, 49
Luminosity 2, 9, 70, 73
Apparent 15, 24

M

Mach's Principle, 135
Magnitude, Stellar, 2
Main Sequence Stars, 3, 113, 114
Upper and Lower 114
Mass
Critical, 77
Electromagnetic, 36
Fraction, 60
Mechanical, 36
Newtonian, 33
Radius - Luminosity Ratio, 68
Maxwell Equations, 156
Maxwell - Boltzman Distribution, 83
Mercury's Orbit, Perturbations 134, 138-9
Rotations, 88

N

Nebular (Galactic) Distribution, 15, 27
 $\epsilon(\tau)$, spacing 15, 18, 21, 23, 24
Distance Unit ψ , 15, 16, 21, 23
Equations of Motion, 18
Neutron Capture 128, 129
Neutrino 90, 93, 142
Newtonian Mechanics
Leading to Relativity, 30
Mass 33
Nebular Motion 18
Nuclear
Processes (S and R) 100, 123, 127-130
Reactions, Beginning, 113
Synthesis, 93
Nucleons, 142

O

Oblateness, Sun 133, 138
Opacity, 60, 62, 70, 79-81
Optical Depth, 147
Oxygen Production, 95
Orion, 147, 151

P

Packing Fraction, 101
Pair
Production Annihilation 105-6, 179, 180
Penetration Distance, 85
Photon
In Calculating Nebular Distance 17, 22
In Gravitational Field, 38
Plasma Frequency, 150
Plasmon Decay, 105
Population, Stellar 2
Potential Barrier, 82
Pre-Main Sequence Stars 112

P

Pressure, 59
Gas 60, 62, 64, 69, 71, 72
Radiation, 62, 71, 72
Proper Time, 8
Emission, Reception 13, 21
Proton-Proton Processes
I, 90, 91
II, III, 92

Q

Quantum Mechanics 74, 75, 76
Quasars 5, 171
 q , Models of Universe 19, 20

R

Radar Astronomy, 186
Radioactive Elements 130-132
Radiation
Electron Acceleration 148-9
Radiogalaxies 5, 168-9
Radio Sources 140
Thermal 147-151
Radius of Universe 6
Reaction Rate 87-89
Red Giants 3, 116
Red Shift 3, 5, 7, 9, 19, 22, 23, 25
No Gravity 13
Defined 14
Change with Time 26
Relativity
Velocity 16
Bending of Light, 27, 28
General Theory, 30
Distance 40
Resonant Reactions 87
In Helium Process 96
Riemann Tensor, 45

S

Scalar Field 30, 31, 134
Scale Factor, Nebular 22
Scattering Amplitude 175, 176, 178
Scattering Cross Section 79, 87
Schrodinger's Equation 84
Schwarzchile 47
Self-Force 36
Solar System, 1, 184-196
Space
Curvature, 11
Analogue with Dug World 41-44
Space-Time 45
Increment 47-49
Flat 49

S

Spallation 94
Spectral Lines 63
Spiral Galaxies 4
Spontaneous Emission 80
Standard Model, 65
Clock 50
Star
Classification, 3
Equilibrium Equations 59, 60, 62, 71
Evolution 3, 112
Interior Structure 59
Mass 67
Mean Density
Novae 118
Population I, 100
Classification of 116-118
Population II
Classification of 119-121
Radius 66
Spectrum, 2
Surface 63
Statistical Mechanics, Nuclei Number, 102
Stellar Energy, Origin, 90
Stress, Tensor 137
Sun, 1, 189, 190, 198-204
Mass, 1
Gravitational Effect 48, 54c
Sunspots, 190
Synchrotron Radiation 152-167

T

Tensor 31, 134-137
Theta θ , Nebular Time 16, 21, 24
Tidal Effects 34
Time
Line, 8
Lorentz, 8
Increment, 40

U

Universe
Lumpiness 141-144
Oscillating Theory 144
Theories of Evolution 12, 27
Three Models 20
Uranium, 122
Urca Process

V

Vector Field 30, 31
Potential 31
Velocity of Light 32
Venus, 193, 188
Virial Theorem, 69

W

White Dwarfs 37, 74-78
World Lines, 174

w boson, 110

Y

Z

z, z (See Red Shift)

When Feynman was giving these lectures on astronomy, astrophysics and cosmology he was learning the material/subject matter as he was presenting it. In typical Feynmanism he went after the physics and then used the math language to explain the underlying physics.

On numerous occasions he would start off on the blackboard and work his way across until his mathematical development was not correct. He would stop and stare at what he had done then walk back to find out where he had made a mistake. The math was not coming out with the right physics which he somehow intuitively knew. Maybe it was a first order assumption; a near field or far field simplification or maybe an integral expansion error. To observe him stop and correct himself in real time was to observe a genius in action.

Clearly these lecture topics have been superceded by tremendous theoretical and experimental work in astrophysics and cosmology. I have attempted to insert some updated and related material that can be found readily on the web. Where I have inserted a wikipedia reference, I have attached their spherical logo. I would expect those who are engaged in the subject matter here will be able to contribute and expand on the new related material. I have included these "emended" additions without attempting in any way to correct or change Feynman's original lecture contents. A more professional errata/peer review would be of value but something that I leave to the reader & reviewers.

Wiki Logo
used with insert



Along the way I have elected to insert some imagery/pictures from current ground and space observatories where it seemed appropriate. Again, hot links to videos and other professional sources would enrich these notes.

REFERENCES

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CHANDRASEKHAR, PRINCIPLES OF STELLAR DYNAMICS
CHANDRASEKHAR, STELLAR STRUCTURE
ALLER, McLAUGHLIN - STELLAR STRUCTURE
STEIN - STELLAR EVOLUTION
STRUVE - ELEMENTARY ASTRONOMY

The references here were my supplement to the lectures. Feynman never called out a reading reference

I have updated my notes here in an attempt to insert some of the interesting experimental discoveries that have come about since this lecture series was presented in the '66-'67 time frame. Our venture into space have opened our perspective on the complexities of the universe and our place in it. Communication satellites caused Penzias and Wilson to "tune" into the microwave "noise" degrading our earliest global satellite services. Advanced electro-optical sensors and telescope, e.g. Hubble, opened our eyes to the wonders and mysteries of the universe. Sophisticated microwave receivers permitted us to tune in more precisely to the subtleties of earliest structures of the universe after the "Big Bang" was discovered.

Our earliest ventures in space for global communications and national security purposes unlocked the window into our understanding of Cosmology, Astrophysics and Astronomy. For all those who took up the exploratory and theoretical journey into the unknown, these notes might provide an interesting historical perspective on how far we have come in our understanding of our universe and our place in it. I would invite those engaged in such fascinating pursuits to emend these notes with their findings and thoughts. However, always remember my goal is to preserve the true Feynmanism in these 1966-67 lectures. Feynman had a lot of fun with these lectures and it was certainly a rare treat to sit in and capture his talks.

CHAPTER I

THE NATURE OF THE LECTURES

The lectures will start with the universe; covering the origin; moving on to the galaxies, stars, the sun, our solar system, and ending up on earth.

As a quick summary we will work our way out to the universe very swiftly, then collapse again going into detail. Thus some quick passing remarks will be made about each topic to be covered.

THE SOLAR SYSTEM

Approximately 98% of the total angular momentum is in the planets. It is speculated a cometary system fills the void between the stars or an estimated 2×10^{10} comets. This space has a density of 1 atom per cubic centimeter of space. A convenient measure of distance becomes a light year, i.e., 10^{16} meters.

A new system of exponential notation will be adopted in which 10^{16} m \rightarrow 1₁₆ m and is said to be 1 "co" 16 meters. A number like 10^{-2} m \rightarrow 1 'ack' 2 or 1₋₂ m. Thus we may multiply $2 \times 10^{10} \times 2.73 \times 10^{-3}$ as $(2)(2.73)_{-3} = 2.273_8$

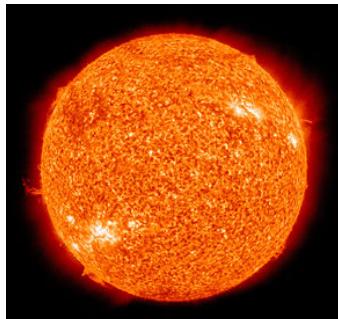
Feynman liked to create his own shorthand notations.

THE SUN

The mass of the sun is about 2_{30} gr and we define this to be 1 solar mass, i.e.,

$$2_{30} \text{ gr} = 1 \text{ SOLAR MASS}$$

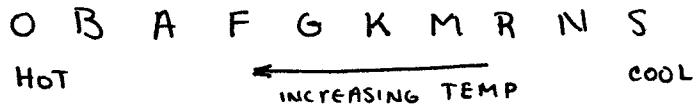
The sun has a period of 25 days. It radiates power at a rate of 4_{33} erg/sec or 4_{26} watts.



The STARS

A BRIGHTNESS OR LUMINOSITY IS DEFINED IN TERM OF MAGNITUDE; A HIGH MAGNITUDE, UNFORTUNATELY MEANS YOU CAN'T SEE THE DAMN THING. ONE MAGNITUDE IS ADOPTED TO BE $2\frac{1}{2}$, I.E., A FACTOR OF $2\frac{1}{2}$. OR A MAGNITUDE CORRESPONDING TO 2.5 REPRESENTS A FACTOR OF 10.

OTHER FACTORS RELATED TO THE LUMINOSITY ARE MASS AND THE RADIUS WHICH IS A MEASURE OF TEMPERATURE SINCE A LARGER AREA IS COOLER. THE SPECTRUM CORRESPONDING TO THE TEMPERATURE RANGE IS,



THE TEMPERATURE IS DETERMINED BY THE BLACK BODY RADIATION OF THE STARS,

$$Q = \sigma T^4$$

Q = ENERGY RADIATED

σ = CONSTANT.

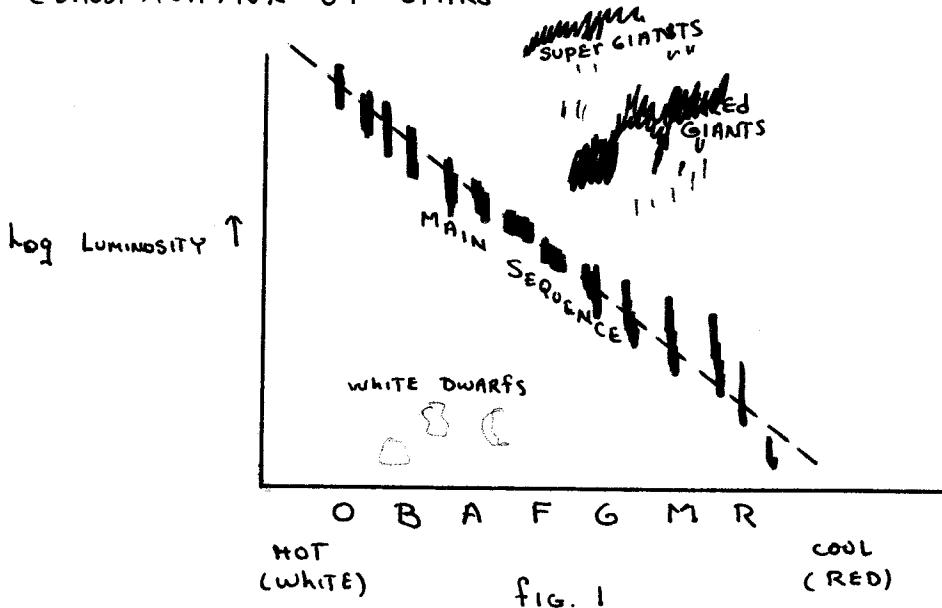
THE LUMINOSITY, L IS THEN

$$\begin{aligned} L &= 4\pi R^2 \times Q \\ &= 4\pi\sigma R^2 T^4 \end{aligned}$$

THUS ALL THREE FACTORS CAN BE SEEN TO BE RELATED; SO ONE CAN ALWAYS BE DERIVED FROM THE OTHER TWO. THE LEAST LUMINOUS STARS ARE ABOUT 1 MILLION TIMES FAINTER THAN THE SUN; WHILE THE BRIGHTEST IS ABOUT A MILLION TIMES BRIGHTER,

STARS ARE EITHER POPULATION I OR II DEPENDING ON THEIR DEVELOPMENT. POP. I'S ARE LATTER DEVELOPING AND USUALLY MADE IN THE SPIRAL ARMS OF NEBULAE.

CLASSIFICATION OF STARS



The EVOLUTION OF A STAR IS QUICKLY THE FOLLOWING: THE ENERGY OF GRAVITATION BECOMES KINETIC ENERGY AND THE STAR GETS HOT; IT ENTERS THE MAIN SEQUENCE IN THE LOWER BRIGHT AND MOVES UP. IT CONTINUES TO RADIATE ENERGY UNTIL THE TEMPERATURE GETS SO HOT THE NEUTRON-NEUTRON, PROTON-PROTON INTERACTIONS FORM DEUTERIUM AND WE HAVE FORMALLY A STAR. THE INITIAL MASS SIZE DETERMINES WHERE IT ENTERS THE MAIN SEQUENCE LINE AND HOW LONG IT WILL LAST. THE GIANTS DON'T LAST LONG; WHEN IT BURNS 15-20% OF ITS HYDROGEN IT BEGINS TO BURN HELIUM WHICH NECESSITATES A HIGHER TEMPERATURE - THE RESULT BEING ITS GETS BIGGER, COOLER OUTSIDE BUT HOTTER INSIDE AND IT MOVES OFF INTO THE RED GIANT REGIONS (HELIUM BURNER). WHAT HAPPENS AFTER IT RUNS OUT OF HELIUM IS NOT KNOWN. SOMEHOW THEY MIGHT WORK THEIR WAY DOWN TO THE WHITE DWARFS.

The STELLAR IS OBTAINED FROM BINARY STARS OF WHICH COMPRISE 10-20% OF ALL STARS BY USING KEPLER'S LAWS AND WORKING OUT THE PERIOD AND REDUCED MASS.

The SPEED OF RECESSION OR MOTION IS OBTAINED FROM THE DOPPLER SHIFT. THAT IS, AS THE EARTH RECEDES FROM A STAR OR VICE VERSA THE WAVELENGTH OF EMITTED LIGHT SHIFTS DOWNWARD AS THE VELOCITY INCREASES,

$$\frac{\Delta \lambda}{\lambda} = \frac{V_E}{C}$$

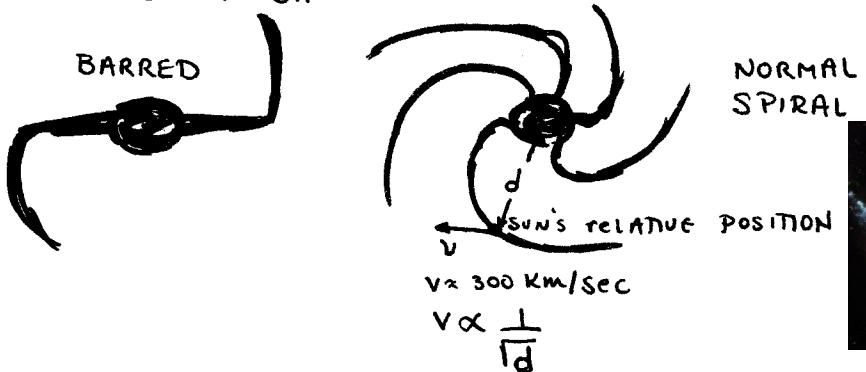
V_E = RECESSIONAL VELOCITY
OF THE EARTH

GALAXIES

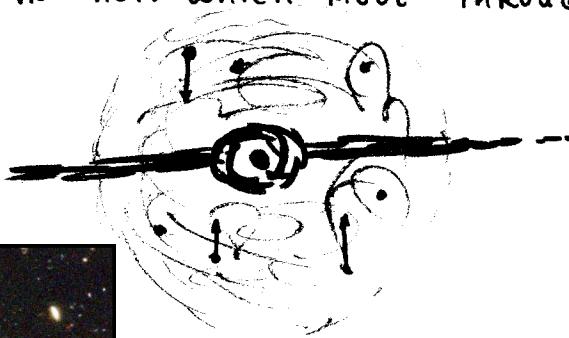
ON THE AVERAGE GALAXIES HAVE A MASS EQUIVALENT TO 10¹¹ SUN MASSES OR 2₄₁ GRAMS. GALAXIES ARE CLASSIFIED AS

1. ELLIPTICAL OR ELLIPSOIDAL IN CONTOUR
2. SPIRALS where THE VERTICAL DIMENSIONS RUN ABOUT 1/100¹ THE PLANAR DIAMETERS. THEY CAN BE EITHER NORMAL OR BARRED SPIRALS

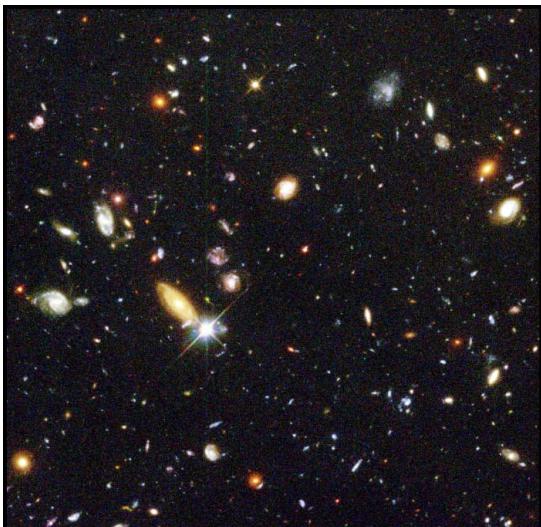
FIG. 2



THE HARD, DENSE, CENTRAL REGIONS ROTATE LIKE RIGID BODIES. THERE IS A DENSER CORE OF ABOUT 10 LIGHT YRS AND OUTER SPHEROIDS OF 200 L.Y. DIAMETER. THERE EXIST A HALO ABOUT THIS CORE MADE UP OF GAS, DUST, AND SMALLER GLOBULAR CLUSTERS IN THEM WHICH MOVE THROUGH THE SPIRALS



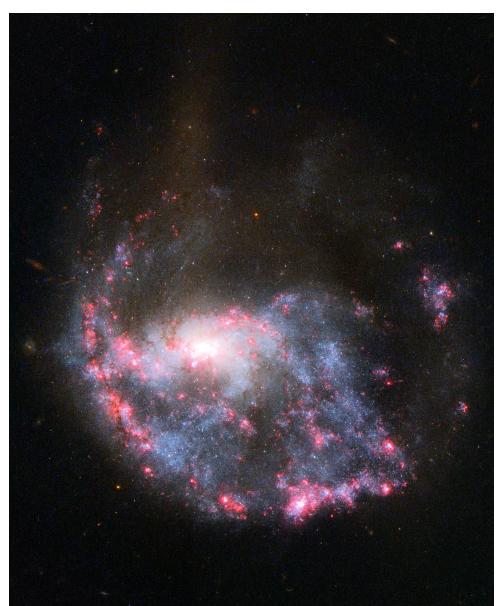
Farthest/oldest Galaxy



Hubble Deep Field

PRC96-01a · ST Scl OPO · January 15, 1996 · R. Williams (ST Scl), NASA

Galaxy in a nebulae-NCG 922



QUASARS

These ARE SOURCES OF STRONG RADIO EMISSION. They ARE SPECULATED TO BE WHAT IS LEFT OVER from SUPER NOVAE EXPLOSIONS IN which SYNCHROTRON RADIATION, POLARIZED LIGHT. They ARE QUASI- STELLAR RADIO GALAXIES. There IS A CENTRAL ELLIPTICAL, OBSERVABLE body SURROUNDED by TWO REGIONS of RADIATING EMISSION of The order of 130 WATTS or 3-10 masses of The SUN PER YEAR MUST be CONSUMMED ASSUMING $E=MC^2$ TO produce This energy. The configuration looks like:

Very interest Feynman sketch compared to the "real" thing seen on the right.



FIG. 4

There Are observable red shift from .3 to 2 where
 $\Delta\lambda = \frac{\lambda_{\text{observed}} - \lambda_{\text{emitted}}}{\lambda_{\text{emitted}}} = z$ corresponds
 TO A VELOCITY of .86c. This is, indeed, QUITE LARGE.

AGE of The UNIVERSE AS defined by HUBBLE'S CONSTANT, T_H

$$T_H \approx 10^{10} \text{ yrs} \pm 33\% \\ \approx 3.18 \text{ sec}$$

This is derived from The red shift

$$\frac{D}{v} = T_H = \pm$$

$$\Delta\lambda \rightarrow v$$

Cosmology

This is the problem of how it all began. To understand the question we must ask how the red shift began. Possibly the photons hit other photons during transit but this would require all photons to deflect the same. Another possible explanation is that in the past light was different and somehow light gets 'old'; hydrogen is continually changing with frequency me^2/hc may light does too. Or, maybe mass can change with time resulting in a change in Bohr orbits.

We will examine more closely the question of time varying constants by looking at the force between two protons. Let's compute the ratio between the electrostatic attraction and gravitational attraction,

$$\frac{e^2/R^2}{Gm_p^2/R^2} = \frac{e^2}{Gm_p^2} = 10^{36}$$

With the Hubble constant we define the radius of the universe to be

$$cT_H = 1_{10} 1_{16} = 1_{26} \text{ METER}$$

Now we ask how many protons in this volume:

$$V = \frac{4}{3}\pi r^3 = 1_{18} \text{ m}^3$$

Assume the density of protons is

$$\rho_p = 0.1 \frac{\text{PROTONS}}{\text{cm}^3} = 1_5 \text{ m}^{-3}$$

$$\therefore \rho V = 10^{83}$$

If we determine

$$\frac{T_H}{\text{COMPTON WAVELENGTH OF PROTON}} = \frac{T_H}{\frac{\lambda}{m_p c^2}} = \frac{1_{26}}{1_{16}} = 10^{92}$$

Now the force between electrons is

$$\frac{e^2}{Gm_e^2} = 10^{92}$$

Through the similarity of all these figures some have tried to imply a constancy of the universe. Out of this argument comes the relationship that

$$G \propto \frac{1}{T_H}$$

But this is a false deduction because a billion years ago the Earth would have a temp. of ocean about 97°C much too high to support life. This theory is not entirely nonsensical but it makes evolution a tight squeeze.

The Big Bang Theory

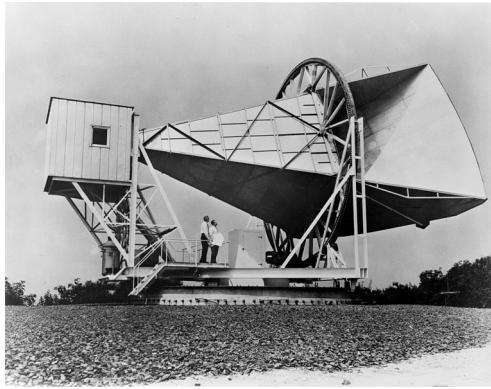
The consensus of modern day cosmology is that the nebulae are moving outward following a big bang. However, all creation laws are omitted and they don't solve the question where every thing came from. There is also one theory that claims the universe is perpetually expanding due to the H_0 creation which leads to new nebulae.

If, in fact, there was a big bang we might ask "What is the distribution of galaxies throughout space?" A basic cosmological assumption is that we are in no particular place of the universe and no matter where you look out in any part of the universe everything on the average is the same.

With the idea of a big bang it is possible to conjecture that all galaxies are moving out from some 'center point.' We can now ask how the red shift depends with distance. The only assumption made to begin with is that the galactic velocity is the same now as it was when they were shot out. This means we first will not consider any gravitational forces acting on the galaxies tending to slow them down. We further consider we're at rest. We first construct a time line,

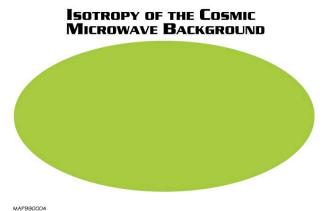
Experimental Cosmology-inserted here for a historical update-jtn

A year prior to this lecture series in 1966 the "big bang noise" was measured by Arno Penzias and Robert Wilson working for Bell Labs at the time. In the early days of geosynchronous communication satellites very large ground receiving antennas with low noise "frontends" were needed to detect the "signal" from the noise. It was unclear where a spurious noise was coming so the two went outside to measure the noise which seemed to come from all over the sky. This experiment was not motivated by pure science but rather investigating new communication technology. Here is a picture of the experimental facility using a large horn antenna:



What was measured was an "isotropic" microwave radiation that appeared to be "global" and appeared as a "monochromatic" picture of the sky:

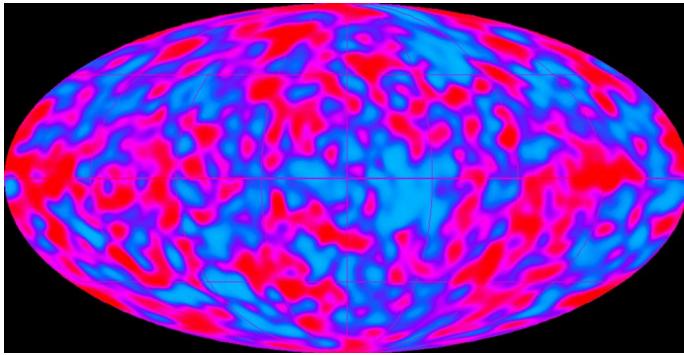
The 1978 Nobel Prize in Physics was awarded to Penzias & Wilson for their discovery. It was, however, Robert Dicke who explained the significance of their isotropic finding. Dicke was in search of the same experimental basis for the "Big Bang".



The experiment left the perplexing question: without any anisotropic structure left by this background radiation where were the "seeds" that gave birth to the galactic & stellar structure?

It was nearly 25 years later that NASA launched the COBE satellite to try to answer this question. This satellite was the first generation of advanced, highly sensitive differential microwave receivers followed by WMAP and more recently ESA's Planck satellite.

Most significant, however, was COBE's first results anxiously awaited for by the cosmological community. What COBE produced was a anisotropic image of the "unevenness" of the 3 deg K blackbody radiation:



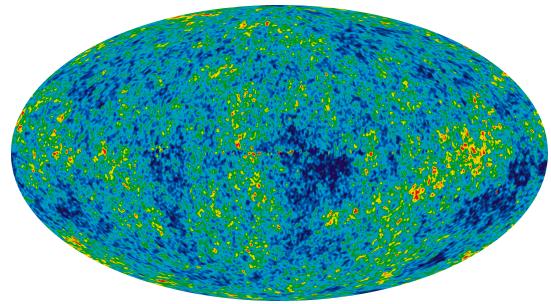
This expected finding but nevertheless a challenging satellite payload design at the time(late 80's) earning George Smoot and his NASA colleague, John Mather, the 2006 Nobel Prize in Physics.

On a side note here: As it turns out in 1962, George and I happened to be in the same high school physics class. Each of us pursued physics in our own way but found fascination and enjoyment with our choices & pursuits. George's interests had him apply space technologies to "look" outward and my focus was downward looking.

With the COBE success both NASA and ESA planned and deployed more advanced microwave sensors and detectors, specifically the NASA Wilkinson Microwave Anisotropy Probe, WMAP, and ESA's Planck satellite.

WMAP's improved microwave sensors and its unique operational orbit out at "L2" provided not only an improved map quality of the microwave background radiation that COBE detected but it also discovered other rather remarkable features of our universe. Here is an image of WMAP's improved anisotropic measurements:

WMAP provided the following important cosmological and astrophysical findings:
I include them here because of the significant implications of all of the findings on cosmology and our understanding of our universe.



WMAP's Top Ten- source: <http://map.gsfc.nasa.gov/>

NASA's Wilkinson Microwave Anisotropy Probe (WMAP) has mapped the Cosmic Microwave Background (CMB) radiation (the oldest light in the universe) and produced the first fine-resolution (0.2 degree) full-sky map of the microwave sky

WMAP definitively determined the age of the universe to be 13.75 billion years old to within 1% (0.11 billion years)

WMAP nailed down the curvature of space to within 0.6% of "flat" Euclidean, improving on the precision of previous award-winning measurements by over an order of magnitude

The CMB became the "premier baryometer" of the universe with WMAP's precision determination that ordinary atoms (also called baryons) make up only 4.6% of the universe (to within 0.2%)

WMAP's complete census of the universe finds that dark matter (not made up of atoms) make up 22.7% (to within 1.4%)

WMAP's accuracy and precision determined that dark energy makes up 72.8% of the universe (to within 1.6%), causing the expansion rate of the universe to speed up. - "Lingering doubts about the existence of dark energy and the composition of the universe dissolved when the WMAP satellite took the most detailed picture ever of the cosmic microwave background (CMB)." - Science Magazine 2003, "Breakthrough of the Year" article

WMAP has mapped the polarization of the microwave radiation over the full sky and discovered that the universe was reionized earlier than previously believed. - "WMAP scores on large-scale structure. By measuring the polarization in the CMB it is possible to look at the amplitude of the fluctuations of density in the universe that produced the first galaxies. That is a real breakthrough in our understanding of the origin of structure." - ScienceWatch: "What's Hot in Physics", Simon Mitton, Mar./Apr. 2008

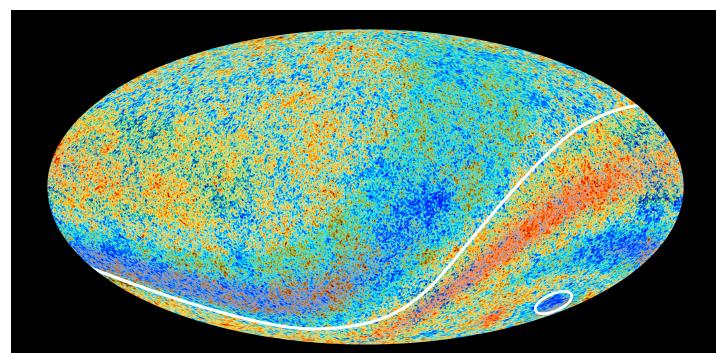
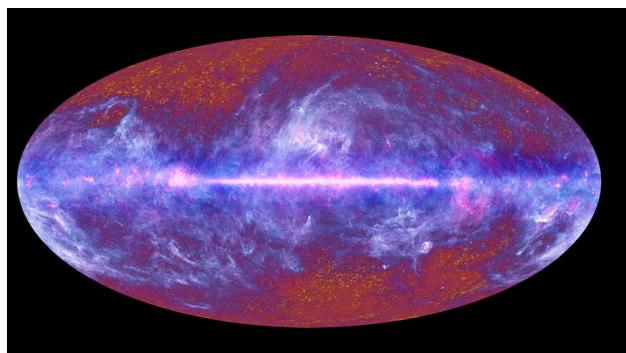
WMAP has started to sort through the possibilities of what transpired in the first trillionth of a trillionth of a second, ruling out well-known textbook models for the first time.

The statistical properties of the CMB fluctuations measured by WMAP appear "random"; however, there are several hints of possible deviations from simple randomness that are still being assessed. Significant deviations would be a very important signature of new physics in the early universe.

WMAP has put the "precision" in "precision cosmology" by reducing the allowed volume of cosmological parameters by a factor in excess of 30,000. The three most highly cited physics and astronomy papers published in the new millennium are WMAP scientific papers--- reflecting WMAP's enormous impact.

The Planck Satellite improved further on WMAP's performance & measurements:

http://www.esa.int/Our_Activities/Space_Science/Planck/Science_objectives



Back to the Feynman lectures in 1966:

CONSIDER WE ARE AT TIME T_0 AND AT SOME TIME IN THE PAST THE GALAXY WAS A POINT I AND TIME T_1 . THAT TIME T_1 IS GIVEN BY

$$T_1 = T_0 - \frac{D}{C}$$

$$\text{where } D = vt_1$$

SO EITHER

$$D = \frac{T_0 C}{1 + v/c}$$

OR

$$T_1 = \frac{T_0}{1 + v/c}$$

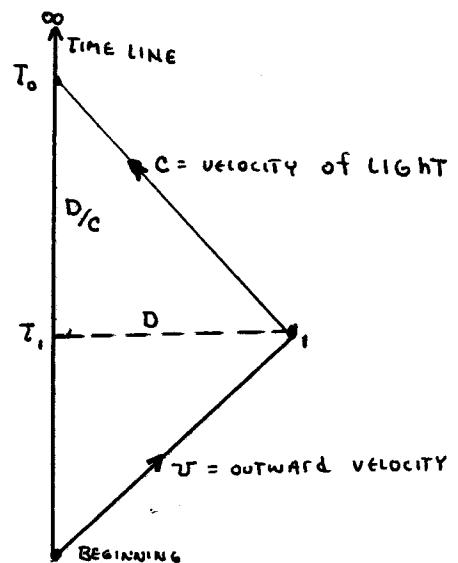


FIG. 5

The red shift is expressed as

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}} = f = 1 + z$$

for a moving, emitting source we have

$$1 + z = \sqrt{\frac{1+v}{1-v}} \Rightarrow (1+z)^2 = \frac{1+v}{1-v}$$

$$\text{or } v = \frac{(1+z)^2 - 1}{(1+z)^2 + 1}$$

$$\therefore D = \frac{T_0 C}{1 + \frac{C[(1+z)^2 + 1]}{(1+z)^2 - 1}} = \frac{T_0 C [(1+z)^2 - 1]}{(1+z)^2 - 1 + C[(1+z)^2 + 1]}$$

When we ask how old the universe, or that galaxy is at the observing time T_1 , the answer is not T_1 but rather,

$$T_1 = T_0 \sqrt{1 - v^2/c^2} \quad \text{which is the Lorentz Time}$$

$$\text{Thus the age } T_1 = \frac{T_0}{1 + v/c} \sqrt{1 - v^2/c^2}$$

where $v = v(z)$ can be found to get the age knowing the red shift.

$$\therefore \text{PROPER AGE AT Emitter} = \frac{\text{PROPER AGE OF RECEIVER}}{\text{red shift}} = \frac{\text{AGE}}{1+z}$$

To check this Theory we need more than one way to measure the distance, D . There are three currently.

1. SIZE - This method assumes all the galaxies observed are of the same size, on the average. This galactic diameter is of the order 100 million L.YR's. Thus by the parallax method

$$D_{fd} = \frac{L}{\delta\theta}$$

where

D_{fd} = angular diameter

$\delta\theta$ = angle subtended by L

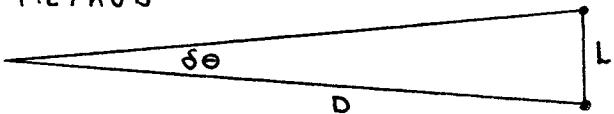


fig. 6

We can write

$$\frac{1}{(1+z)^2} = \frac{1-v}{1+v} = 1 + \frac{2v}{1+v}$$

$$\text{since } D = D_{fd} = \frac{v T_0}{1+v}$$

$$\frac{1}{(1+z)^2} = 1 - \frac{2 D_{fd}}{T_0}$$

But, most probably, $L = L(\tau)$ and gets smaller or larger.

2. LUMINOSITY - Consider luminosity is constant. Therefore, at a greater distance the same object would appear weaker.

$$\text{BRIGHTNESS observed} = \mathcal{L}_0 \cdot \frac{1}{D_B^2} \times \frac{1}{(1+z)^2}$$

where

\mathcal{L}_0 = LUMINOSITY

D_B = DISTANCE ON THIS SCALE

The light is observed to shift to the red.

$1+z$ is then a TIME SCALE RATIO

If \mathcal{L}_0 depends on AGE, this theory is shot. Without an evolution of GALAXY theory known this cannot be checked.

3. GALACTIC DISTRIBUTION: How MANY GALAXIES DO we see?

The NUMBER of GALAXIES EMITTED WITH MOMENTUM P IN
The RANGE $d^3 P$ IS

$$\frac{d^3 P}{2E}$$

This result is NOT INTUITIVELY OBVIOUS by results from
A LORENTZ TRANSFORMATION which RECESSITATES THIS
PARTICULAR FORM TO PRESERVE THE DISTRIBUTION PROBABILITY.

BUT $\frac{d^3 P}{2E} = \frac{P^2 dP}{2E} \cdot d\Omega$

where $d\Omega$ is THE SOLID ANGLE

BY RELATIVITY

$$E^2 = P^2 + M^2$$

$$\text{so } E dE = P dP$$

AND

$$\frac{d^3 P}{2E} = P dE d\Omega$$

SINCE $P = \frac{mv}{(1-\frac{v^2}{c^2})^{1/2}}$ $E = \frac{mc^2}{(1-\frac{v^2}{c^2})^{1/2}} \rightarrow dE = \frac{mv/c^2 dv}{(1-\frac{v^2}{c^2})^{3/2}}$

Hence $P dE = \frac{v^3 c^2 dv}{(1-v^2/c^2)^{3/2}}$

Thus

$$\begin{aligned} \text{The no. of galaxies} \\ \text{having recessional velocity} \\ v \text{ in range } dv &= \frac{4\pi \frac{v^2}{c^2} dv}{1 - \frac{v^2}{c^2}} \end{aligned}$$

$$\frac{4\pi}{c^2} \int_0^c \frac{v^2}{1 - \frac{v^2}{c^2}} dv$$

Now we consider the case of gravity and how it effects the recessional velocities. The consequence of a curved space is derived from this case.

FIRST off the curvature of space is NOT independent of the observer; the question of whether space is curved depends ^{on} who defines it. At a given time the sequence of points of the nebulae gives rise to a curved space but it is not necessarily spherical - it might be parabolic.

ASSUME WE STAND AT THE CENTER OF ANY ARBITRARY SPHERE OF RADIUS $R(T)$ WHICH THE SURROUNDING GALAXIES ARE MOVING OUTWARD ^{TOWARD} THE RIM WITH A VELOCITY v THAT STAYS CONSTANT WITH TIME.

ASSUME further the density is the same throughout the sphere. Because there are a lot of galaxies, i.e., a lot of mass inside, the outward motion is slowed down and the result is to curve back the nebulae.

FOR A GALAXY AT THE RIM MOVING WITH $v \ll c$ THE ACCELERATION IS

$$\ddot{R}_1(T) = - \frac{GM_{\text{INSIDE}}}{R_1^3(T)}$$

$$\text{where } M_1 = \rho R_1^3(T) \frac{4}{3} \pi$$

for a particle closer in, say at $R_2(T)$

$$\ddot{R}_2(T) = - \frac{GM_{\text{INSIDE}}}{R_2^3(T)}$$

$$\text{where } M_{\text{INSIDE}} = \rho \frac{4}{3} \pi R_1^3(T)$$

$$\text{or } \frac{M_1}{M_2} = \frac{R_1^3(T)}{R_2^3(T)}$$

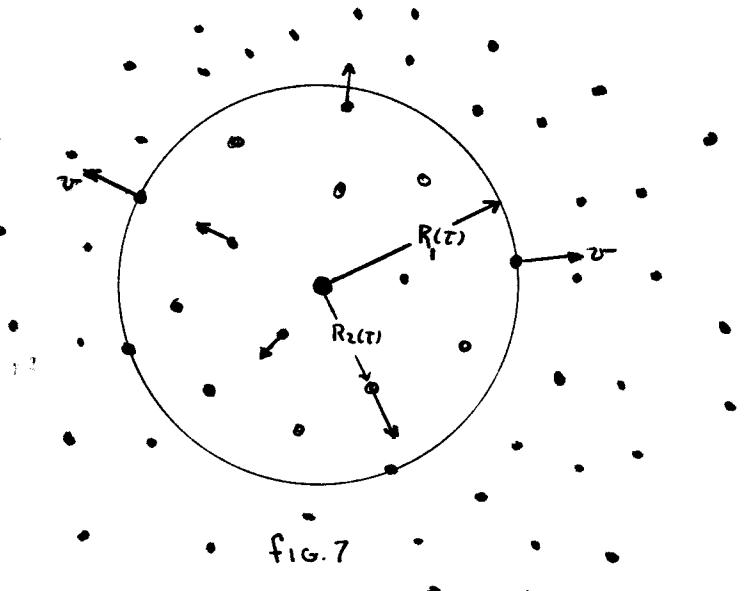


FIG. 7

The specific value of v is very crucial to the concept of the evolution of the universe. Depending on its value there are 3 possible universes,

1. If v is very large and the retarding gravitational slowdown is negligible, the galaxies will continually race out and the universe will be forever expanding.
2. The critical case is if v approaches 0 because the net gravitational pull just pulls hard enough to prevent the galaxies from escaping.
3. The catastrophic case is if v is less than v_c , the critical velocity, and the galaxies are sucked back to their origin.

We might ask 'What is the critical density?'

The velocity just needed to escape, v_{esc} is equal to the Hubble velocity of recession v_{rec}

$$v_{rec} = \frac{R}{T_h}$$

$$T_h = \text{Hubble constant} = 10^{10} \text{ yrs} = 3 \times 10^7 \text{ sec}^{-1}$$

Thus we equate the kinetic and potential energies

$$\frac{1}{2} v_{rec}^2 = \frac{GM_{\text{inside}}}{R}$$

$$M = \frac{4\pi}{3} R^3$$

$$\text{Thus } \frac{4\pi G \rho R^3}{3 R} = \frac{1}{2} \frac{R^2}{T_h^2} \Rightarrow$$

$$\rho \approx \frac{\Phi}{T_h^2 G}$$

GOING BACK OVER THE NOTES OF THE LAST LECTURE, CERTAIN POINTS NEED CORRECTING AND DESERVE A BETTER EXPLANATION. THE PROBLEM DEALS WHICH THE RED SHIFT FROM NEBULAE RECEIVING FROM US WITH A VELOCITY PROPORTIONAL TO THE DISTANCE BETWEEN US. WE MUST ASSUME THE CONSTANCY OF THE NEBULAR DENSITY TO PURSUE THE DISCUSSION. FURTHER, THE BASIC COSMOLOGICAL PREMISE IS USED, I.E., WHAT WE SEE WHEN LOOKING OUT IS WHAT SOMEONE OUT THERE SEES WHEN HE LOOKS OUT. AGAIN WE FIRST DISCUSS THE CASE OF NO GRAVITY AND THEN ADD IT TO OUR PICTURE.

RED SHIFT WITH NO GRAVITY

FIRST CONSTRUCT A PICTURE LIKE THE ONE ON PAGE 8:

WE ARE AT TIME T_0 ; THE LIGHT WAS EMITTED
A POSITION z_0 AND TIME τ_0 .

THE AGE OF THE NEBULA AT POSITION z_0 IS τ_0 .

THE ANGULAR DISTANCE $D\phi$ IS GIVEN BY

$$D\phi = v\tau_0 = c(T_0 - \tau_0)$$

$$\therefore \tau_0 = \frac{T_0}{1+v/c}$$

NOW WE ADOPT THE NOTATION OF C BEING UNITY THROUGHOUT THE ENSUING DISCUSSION.

SINCE $T_0 = \tau_0$, THE ABOVE BECOMES

$$\tau_0 = \frac{\tau_0}{1+v}$$

FROM WHICH WE GET,

$$D\phi = \frac{\tau_0 v}{1+v} = \frac{L}{\delta\phi}$$

WHERE $L, \delta\phi$ ARE SHOWN ON PAGE 9.

THUS THE PROPER TIME OF EMISSION $\tau_i = \tau_i(T_0)$. SO GIVEN τ_i , WE FIND τ_0 . THE TIME T_i IS A COMPUTED TIME WHICH HAS NO SIGNIFICANCE THE EITHER Emitter OR RECEIVER. WHILE $\delta\phi$ IS AN OBSERVABLE QUANTITY IN THE STRICTEST SENSE. TO FIND τ_i , WE USE A LORENTZ TIME TRANSFORMATION,

$$\tau_i = \sqrt{1-v^2} T_i = \sqrt{1-v^2} \frac{\tau_0}{1+v}$$

$$= \sqrt{\frac{1-v}{1+v}} \tau_0$$

$$\therefore \tau_0 = \sqrt{\frac{1+v}{1-v}} \tau_i \quad \begin{matrix} \text{RECEIVER'S} \\ \text{TIME} \end{matrix} \quad \begin{matrix} \text{EMISSION TIME} \end{matrix}$$

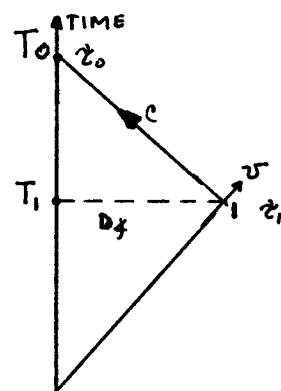


FIG. 8

Now, what about the red shift. Well, the Doppler effect of a receding source is,

$$\omega_{\text{REC}} = \sqrt{\frac{1-v}{1+v}} \omega_{\text{EMIT}}$$

ω = frequency of emitted & received light

Suppose the emitter shots out a light beam of a given spectral lines, i.e., given frequency; we ask what is the proper time between each node. As it is radiated toward O (us). That $\Delta \tau$ is given by

$$\Delta \tau = \sqrt{\frac{1+v}{1-v}} \Delta \tau_{\text{rec.}}$$

The red shift is defined as on page 8, i.e.,

$$1+z = \frac{\lambda_{\text{rec}}}{\lambda_{\text{emit}}} = \sqrt{\frac{1+v}{1-v}}$$

We shall henceforth speak of z where $z = 1+z$ since it is strictly an arbitrary value whether we add 1 or not to z .

$$z = \sqrt{\frac{1+v}{1-v}}$$

Therefore, in terms of frequency,

$$\omega_{\text{rec}} = \frac{1}{z} \omega_{\text{emitted}}$$

We can now find D_f ,

$$z^2 = \frac{1+v}{1-v} \quad \text{and} \quad 1 - \frac{1}{z^2} = \frac{2v}{1+v}$$

Since,

$$D_f = \frac{z_0 v}{1+v} \quad \text{Then}$$

$$D_f = \frac{z_0 (z^2 - 1)}{2 z^2}$$

LUMINOSITY

At the proper age t_0 the nebula had a certain luminosity \mathcal{L}_0 . What is the angular distance as seen from the nebula?

$$D'_f = \frac{t_0 v}{1+v}$$

This is similar to the previous expression for D_f , i.e.,

$$D_f = \frac{z_0 v}{1+v}$$

Thus we see the proper times are interchanged and the minus sign in the denominator of D'_f shows D'_f is bigger.

These two distances can be related

$$D'_f = \frac{t_0 v}{1-v} = \frac{V z_0}{1-v^2} = \frac{V z_0}{1+v} \sqrt{\frac{1+v}{1-v}} = D_f z$$

$$D'_f = D_f z$$

The result $D'_4 = D_4 z$ is unusual for it turns out to be the same relationship in an accelerating universe where the lines are curved. Now, the intensity seen by receiver is given as

$$I = \frac{L_0}{(D'_4)^2} \cdot \frac{1}{z} \cdot \frac{1}{z}$$

AS WE SEE IT
ENERGY THE OTHER GUY SEES FADING AWAY

PHOTONS WEAKER, i.e., ORDINARY DOPPLER EFFECT

Therefore, the APPARENT LUMINOSITY IS,

$$I = \frac{L_0}{D_4^2 z^4}$$

This CAN be TIED BACK TO THE TIME t_0 BY

$$D_4 = \frac{t_0 (z^2 - 1)}{2z^2}$$

$$I = \frac{4 L_0}{t_0^2 (z^2 - 1)^2}$$

NEBULAR DISTRIBUTION

WE DISCUSSED THIS QUESTION LAST TIME AS SEEN ON PAGE 10 BUT WE APPROACHED IT WRONG. THE QUESTION IS NOT HOW MANY GALAXIES WE SEE BUT REALLY HOW MANY THE LIGHT PHOTON PASSES WHEN GOING FROM PROPER TIME t_i TO t_0 . IT IS POSSIBLE TO MEASURE DISTANCE IN TERMS OF NEBULAE PAST NOT METER. THIS IS VISUALIZED IF THE PHOTON PICKED UP A PIECE OF DIRT FROM EACH NEBULA IT WENT BY AND THEN DUMPED ITS BAG OUT WHEN IT GOT TO US AND SAID 'I WENT BY 40,000 NEBULAE; KNOWING A UNIT SPACING WE COULD DETERMINE DISTANCE.'

WE ASSUME AGAIN THE NEBULAE WERE SPRAYED OUT EVENLY AND LOOK LIKE RAYS,

SUPPOSE THE SEPARATION BETWEEN ADJACENT NEBULAE IS DENOTED BY $\epsilon(t)$, i.e., SPACING VARIES WITH TIME. SPECIFICALLY IT VARIES LINEARLY,

$$\epsilon(t) = \epsilon_0 \frac{t}{a}$$

WHERE ϵ_0 WAS THE SPACING AT t_i .

THUS THE NUMBER OF NEBULAE PAST BY LIGHT AS IT GOES FROM $t_i \rightarrow t_0$ IS

$$\int_{t_i}^{t_0} \frac{dt}{\epsilon(t)}$$

$$\text{Therefore NO. PASSED} = \frac{a}{\epsilon_0} \ln\left(\frac{t_0}{t_i}\right) = \psi \frac{a}{\epsilon_0}$$

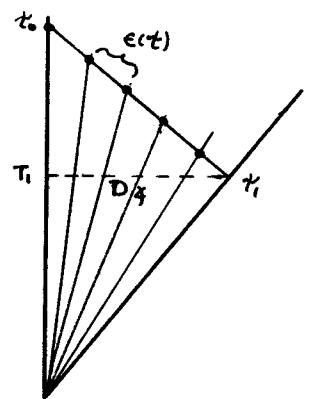


FIG. 9

AND ψ IS A MEASURE OF HOW FAR AWAY THE LIGHT IS FROM ITS SOURCE

$$\psi = \ln\left(\frac{t_0}{t_i}\right)$$

WE'LL MENTION NOW THAT $\psi = \theta_0 - \theta$, where $\theta = \ln(\frac{ct}{a})$. AS IT TURNS OUT IN THE LATTER NOTATION θ IS A TIME IN A DISTORTED SCALE. THIS WILL, HOPEFULLY, BE MORE CLEAR IN THE ACCELERATED CASE WHERE ψ ALSO = $\theta_0 - \theta$. AS ABOVE.

NOW THE RATIO TO SHIFT OF θ 'S, I.E., t_0/t , IS SIMPLY OUR RED SHIFT FACTOR Z SO THE FOLLOWING RESULT IS OBTAINED,

$$\psi = \ln z$$

UPON SOLVING FOR Z WE GET

$$z = e^\psi$$

NOW IF $z^2 = \frac{1+v}{1-v}$ THEN $v = \frac{z^2-1}{z^2+1} = \frac{1 - \frac{1}{z^2}}{1 + \frac{1}{z^2}}$

BUT $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^x(1 - e^{-2x})}{e^x(1 + e^{-2x})}$

AND SINCE

$$v = \frac{1 - z^{-2}}{1 + z^{-2}} = \frac{1 - e^{-2\psi}}{1 + e^{-2\psi}}$$

WE MAKE THE ASTONISHING DISCOVERY THAT

$$v = \tanh \psi$$

THIS RESULT IS STARTLING BECAUSE IT IS IDENTICAL TO THE RELATIVITY RESULT FOR TWO MOVING OBJECTS POSSESSING VELOCITIES $U \neq V$.

THEIR EFFECTIVE VELOCITY V' IS GIVEN BY THE RELATIVISTIC EQUATION,

$$V' = \frac{U+v}{1+uv}$$

BY LETTING

$$v = \tanh \omega \text{ AND } u = \tanh \lambda$$

$$V' = \frac{\tanh \omega + \tanh \lambda}{1 + \tanh \omega \tanh \lambda} = \tanh(\omega + \lambda)$$

TO UNDERSTAND THIS RESULT WE MUST THINK OF THE VELOCITY AS THE TIME RATE OF CHANGE OF THE DISTANCE. THAT IS, FOR SIMPLE THREE DIMENSIONAL SPACE WE KNOW $dy/dx = \tan \theta$ AND IF WE ROTATE THROUGH α THEN WE HAVE $\tan(\theta + \alpha)$.

THE ANALOGY IS THEN THAT WE HAVE MADE A ROTATION IN FOUR DIMENSIONAL SPACE. WE thus CORRELATE TANGENTS AND HYPERBOLE TANGENTS AS REPRESENTING THE TRANPOSITION FROM A EUCLIDEAN SPACE TO A CURVED SPACE.

WE HAVE THUS SHOWN A COMPLICATED CONCEPT OF RELATIVITY WITHOUT MENTIONING THE WORD AND WITHOUT THE MYSTERIOUS MATHEMATICS.

WE NOW RETURN TO THE QUESTION, "HOW MANY NEBULAE ARE THERE OUT TO ψ_i ?" WE MUST BE CAREFUL NOT TO JUMP TO ANY HASTY CONCLUSION, I.E., AS WE PAST ψ OF THEM RADIALLY WE DO NOT COMPUTE THE VOLUME AS $4/3\pi\psi^3$. IT WILL TURN OUT THE NUMBER OF NEBULAE IN $d\psi$ IS $\sinh^2\psi$.

THE ANGULAR SEPARATION $\delta\phi$ THAT WE SEE WHEN WE LOOK AT THE NEBULAE IS PROPORTIONAL TO THEIR SEPARATION AND INVERSELY PROPORTIONAL TO THE DISTANCE AWAY IT IS FROM US. THE FIGURE IS THE SAME AS ON PAGE 9. ONLY WE MUST BE CAREFUL TO KEEP OUR TIMES RIGHT; THE SEPARATION OF TWO NEBULAE IS $E(t)$ BUT EVALUATED AT TIME $t = \psi_i$. Thus

$$\delta\phi = \frac{E(\psi_i)}{D\psi}$$

NOW THE NUMBER OF PHOTONS BOUNCING IN ON US THROUGH THE SOLID ANGLE 4π IS GIVEN AS

$$\frac{4\pi}{(\delta\phi)^2}$$

NOW $E(\psi_i) \propto \psi_i$ AND $D\psi = \frac{z_0 v}{1+v}$

ACCORDING TO OUR DEFINITION OF z ON PAGE 13

$$z = \frac{z_0}{\psi_i}$$

SO

$$\frac{D\psi}{E(\psi_i)} \propto \frac{D\psi}{\psi_i} = \frac{z_0 v z}{z_0 (1+v)} = \frac{v}{1+v} \left(\frac{1+v}{1-v} \right)^{1/2} = \frac{v}{\sqrt{1-v^2}}$$

SINCE $v = \tanh \psi$

$$\frac{D\psi}{\psi_i} = \frac{\tanh \psi}{\sqrt{1-\tanh^2 \psi}} = \frac{\sinh \psi}{\cosh \psi} \times \cosh \psi = \sinh \psi$$

SINCE WE WANT $\left(\frac{D\psi}{\psi_i} \right)^2$ WE GET $\sinh^2 \psi$ AS A PROPORTIONATE FIGURE MEASURED WITH OUR NEBULAR YARDSTICK.

CHAPTER 3

GRAVITATIONAL EFFECT

WE NOW ERASE EVERYTHING WE DID BEFORE AND MAKE A NEW CALCULATION BECAUSE WE CAN NOT IGNORE GRAVITY AND SO WE MUST DO IT ALL AGAIN. THIS TIME THE RECESSIVE VELOCITY OF THE NEBULAE IT IS SLOWED DOWN BY THE GRAVITATIONAL PULL OF THE MATTER 'INSIDE' THE SPHERE WHICH TAKES IN THE NEBULA IN QUESTION. Thus ARE LINES ARE CURVED NOW AND WE ASK WHAT IS THE CURVE WHICH DESCRIBES THE MOTION?

AGAIN WE USE THE BASIC COSMOLOGICAL PREMISE THAT 'WHAT HAPPENS HERE HAPPENS THERE.' WITH THIS WE CAN PROJECT TO ALL NEBULAE THE LOCAL SPREADING FORMULAE WHICH WE OBTAIN BY OUR OBSERVATIONS. WE NEED $E(z)$ THE SPACING AS A FUNCTION OF TIME; THIS TURNS OUT TO BE NON-LINEAR. ONCE WE HAVE $E(t)$ WE CAN FIND SOMETHING OUT ABOUT $\psi = \psi(t_0, z_0)$.

HOW THEN DOES $E(t)$ VARY WITH TIME?

AGAIN WE CONSTRUCT A SPHERICAL PICTURE AS WE DID PREVIOUS (PAGE 11). BY GENERAL RELATIVITY WE CAN IGNORE ALL THE MATTER OUTSIDE THE RIM AND CONCERN OURSELVES ONLY WITH THE RETARDING EFFECT DUE TO THE INSIDES. THE EQUATION FOR THE ACCELERATION IS AGAIN

$$\ddot{r}(t) = -\frac{GM}{r^2}$$

WHERE WE THEN WOULD INTEGRATE FROM $t_0 \rightarrow t_1$

IF M REMAINS CONSTANT, WE FUDGE THE SOLUTION

$$r = r_x (\cosh \theta - 1) \quad t = a(\sinh \theta - \theta) \rightarrow \theta = \theta(z)$$

FOR REASONS OF FUTURE BENEFIT WE'LL DEFINE A SCALE FACTOR $R(t)$ TO BE

$$R(t) = a(\cosh \theta - 1)$$

SINCE,

$$dz = a(\cosh \theta - 1) d\theta$$

WE SEE

$$\frac{dr}{dt} = \dot{r} = \frac{r_x \sinh \theta}{a(\cosh \theta - 1)}$$

NOW IT TURNS OUT THE HUBBLE CONSTANT H IS DEFINED TO BE \dot{r}/r SO

$$H = \frac{\dot{r}}{r} = \frac{\sinh \theta}{a(\cosh \theta - 1)^2} = H(z)$$

SO IT TURNS OUT THAT THE VELOCITY IS NOT PROPORTIONAL TO THE DISTANCE OUT THE NEBULA GETS.

BUT, CONTINUING ON WE WANT \ddot{r} , I.E., $\frac{d\dot{r}}{dz}$. IT TURNS OUT TO BE

$$\ddot{r} = -\frac{r_x}{a^2(\cosh \theta - 1)^2} = -\frac{r_x^3}{a^2 r^2}$$

AND THIS THEN IS A SOLUTION TO THE EQUATION OF MOTION

IT FOLLOWS FROM THIS SOLUTION THEN, THAT

$$\frac{r_x^3}{a^2} = GM = G\rho V = G\rho \frac{4}{3}\pi r^3$$

OR

$$\frac{r_x^3}{a^2} = \frac{4}{3}\pi G\rho r^3 (\cosh \theta - 1)^3$$

Therefore

$$\frac{1}{a^2} = \frac{4\pi}{3} G\rho (\cosh \theta - 1)^3$$

THE PROBLEM IS NOW THAT WE DON'T KNOW THE DENSITY ρ . IF WE DID WE COULD TAKE ABOVE TO THE EXPRESSION JUST DERIVED FOR THE HUBBLE CONSTANT AND THEN GET A θ_0 AND a .

SO, WE GO OFF ON A DIFFERENT APPROACH AND LOOK AT THE CONSTANTS OF THE MOTION. FIRST WE MAKE A CONVENIENT REPRESENTATION, I.E., FIND THE QUANTITY $\frac{\ddot{r}r}{\dot{r}^2}$ AND EVALUATE IT AT t_0

$$\left. \frac{\ddot{r}r}{\dot{r}^2} \right|_{t_0} = \frac{1}{1 + \cosh \theta_0} = g$$

THIS IS A NICE DIMENSIONLESS QUANTITY.

CONSIDERING NOW THE ENERGY AS A CONSTANT OF THE MOTION FOR A NEBULA AT r_x WE HAVE

$$E = \frac{1}{2}\dot{r}^2 - \frac{GM}{r} = C = \frac{r_x^2}{2a^2}$$

where $GM = \text{CONSTANT} = \frac{r_x^3}{a^2}$

WE SEE ANOTHER NICE RATIO WOULD BE

$$\frac{GM}{(2E)^{3/2}} = a$$

WE STILL HAVEN'T HELPED OURSELVES THOUGH AND WE MUST STILL PLAY SOME MORE GAMES.

THE POTENTIAL ENERGY $- \frac{GM}{r}$ CAN BE EXPRESSED AS $\dot{r}(t)r$.

IF WE TAKE E AND FACTOR OUT A $\frac{1}{2}\dot{r}^2$ AND THEN MULTIPLY AND DIVIDE BY r^2 WE END UP WITH

$$E = \frac{1}{2}r^2 \left(\frac{\dot{r}}{r} \right)^2 \left[1 - 2\frac{\dot{r}r}{\dot{r}^2} \right]$$

STRANGELY ENOUGH IT TURNS OUT THIS EQUATION BECOMES

$$E = \frac{1}{2}r^2 H^2 \left[1 - 2g \right]$$

SINCE $H = \frac{\dot{r}}{r}$ AND $g = \frac{\dot{r}r}{\dot{r}^2}$

NOW WE KNOW H BUT NOT g , BECAUSE IT IS A FUNCTION OF ρ .
 BUT WE CAN SPECULATE ON WHAT HAPPENS TO THE ENERGY FOR
 DIFFERENT VALUES OF g . THERE ARE THREE CASES IN QUESTION:

- i. If $g < \frac{1}{2}$, THEN K.E > P.E
- ii If $g = \frac{1}{2}$, THEN K.E = P.E
- iii If $g > \frac{1}{2}$, THEN K.E < P.E.

CASE i CORRESPONDING TO THE CONSTANTLY EXPANDING UNIVERSE THEORY WHERE THE VELOCITIES CONTINUALLY INCREASE AND THE UNIVERSE NEVER QUITES.

CASE ii IS THE CASE WHERE THE PULL OF GRAVITY IS JUST RIGHT TO PULL THE NEBULAE UP AND IN THE LIMIT THEIR VELOCITY IS ZERO SO THE UNIVERSE QUITES EXPANDING BUT INFINITELY FAR OUT.

CASE iii THE CATASTROPHIC CASE WHERE GRAVITY PULLS EVERYTHING TOGETHER AGAIN AND WE HAVE A GLORIOUS ENDING. WHEN g TAKES ON THESE VALUES THE LINES GENERATED ARE CYCLOID WHICH CLOSE TOGETHER.

DIAGRAMMATICALLY CASE i & iii LOOK LIKE THE FOLLOWING:

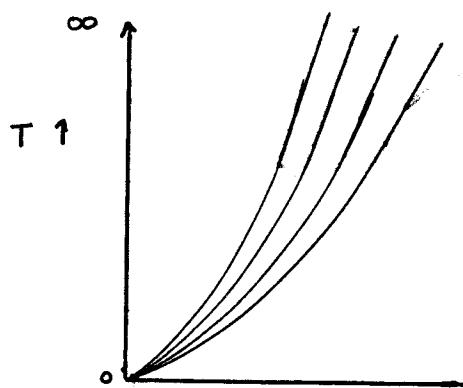


FIG. 10

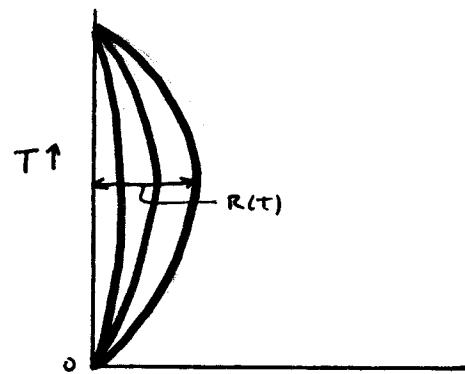


FIG. 11

ONE OF OUR OBJECTIVES IS TO FIND τ_i AND τ_o . SO WE'LL GET CRAFTY AND ASK WHAT IF $E(t)$ TOOK THE FORM,

$$E(t) = E_x (\cosh \theta - 1)$$

NOW THE DISTANCE IN NEBULAR UNITS IS $\frac{dt}{E(t)}$ SO TO GET THE NUMBER OF NEBULAE PASSED BY THE LIGHT WHICH WE KNOW IS $\psi^a E_x$ WE MUST INTEGRATE

$$\int_{\tau_i}^{\tau_o} \frac{dt}{E(t)} = \psi^a E_x$$

$$\text{BUT } dt = a(\cosh \theta - 1) d\theta$$

Therefore

$$\int_{\tau_i}^{\tau_o} \frac{a(\cosh \theta - 1) d\theta}{E_x (\cosh \theta - 1)} = \frac{a}{E_x} (\theta_o - \theta_i)$$

STRANGELY ENOUGH, OUR EARLIER DEFINITION OF ψ , I.E., $\psi = \theta_o - \theta_i$, IS PROVEN OUT. ANOTHER WAY OF EXPRESSING OUR RESULTS IS

$$\frac{\theta_{rec}}{\theta_o} = \frac{\theta_{emit}}{\theta_i} + \frac{\psi}{\text{DISTANCE}}$$

WE SEE THEN θ IS NOT IN A DISTORTED SCALE.
IF IT TURNS OUT THAT

$$\theta_{rec} - \theta_{emit} = \psi = \text{CONSTANT}$$

THEN ψ WILL REMAIN THAT VALUE FOREVER. THIS MEANS THAT AS WE LOOK AT TWO NEBULAE AND OBSERVE THEIR SPACING ψ WILL HAVE A CERTAIN VALUE. BUT SINCE THEY RECED RADIALLY THE SEPARATION IS FIXED. Thus ψ IS UNIQUE FOR ANY TWO NEBULAE BUT UNCHANGED FROM THEN ON.

WE WOULD ALSO LIKE AN EXPRESSION FOR τ_{rec} VS τ_{emit} . SINCE,

$$\tau_{emit} = \tau_i = a(\sinh \theta_i - \theta_i) = a[\sinh(\theta_o - \psi) - (\theta_o - \psi)]$$

$$\text{AND } \tau_o = a(\sinh \theta_o - \theta_o)$$

THEN IF WE KNOW θ_o WE GET τ_o AND τ_i

RETURNING TO THE RED SHIFT NOW WE WANT TO TIE THE FOLLOWING RELATIONSHIPS TOGETHER,

$$\Delta \tau_i \propto \Delta \tau_o$$

$$\Delta \theta_i \propto \Delta \theta_o$$

SINCE

$$\Delta \tau_i = a(\cosh \theta_i - 1) d\theta_i$$

AND

$$\Delta \tau_o = a(\cosh \theta_o - 1) d\theta_o$$

$$\frac{\Delta \tau_o}{\Delta \tau_i} = \frac{a(\cosh \theta_o - 1)}{a(\cosh \theta_i - 1)} = \frac{R(\tau_o)}{R(\tau_i)}$$

The ratio $\frac{R(t_0)}{R(t_1)}$ is a ratio between our scale factor which we defined earlier and this, in fact is the red shift z or the ratio of $\frac{\omega_{\text{emit}}}{\omega_{\text{rec}}}$. So we have now

$$z = \frac{\omega_{\text{emit}}}{\omega_{\text{rec}}} = \frac{R(t_0)}{R(t_1)} = \frac{\Delta t_0}{\Delta t_1}$$

And at last we connect z with the distance

This result is interesting in that it says that in the past the nebulae were closer together and now they are further apart in exactly the same ratio as the wavelength emitted is to the wavelength received. This is truly interesting.

Since the concept of the red shift is so important let's go back over it. This time we observe the photon is emitted with a certain frequency of emission ω_0 . This frequency experiences a little red shift as it arrives at the first nebula along its way to us. Then ω_1 is passed on to nebula 3, experiences another red shift, and goes on etc. So the photon experiences infinitely many small red shifts by the time we see it; they all add up to the red shift we see. Since the nebulae are so close we can integrate because there is no relativistic velocities involved.

So we now see the photon pass along its path which is curved in this case. We visualize it as the following:

At some time t_n the frequency is ω_n ; when going to t_{n+1} $\omega_n \rightarrow \omega_{n+1}$. We have the relationship between these two frequencies as

$$\omega_{n+1} = \omega_n(1-v)$$

where v is the difference in velocities from going to ω_{n+1} , i.e., relative velocity. $= \epsilon(t)$
we can differentiate this to get,

$$d\omega = -v \omega dn$$

Now we want to know what v is for one spacing. Well, we do know

$$v = \dot{\epsilon}(t)$$

Then

$$\frac{d\omega}{\omega} = \frac{\dot{\epsilon} dt}{\epsilon E(t)} = \frac{d\epsilon}{\epsilon}$$

This follows from our earlier result, $dn = \frac{dt}{E(t)}$

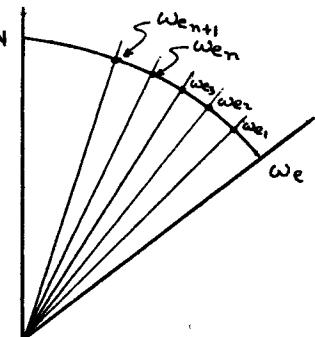


FIG. 12

SOLVING THIS EQUATION WE GET

$$\ln \omega = \ln e$$

OR

$$\frac{\omega_{\text{emit}}}{\omega_{\text{rec}}} = \frac{E_{\text{emit}}}{E_{\text{rec}}}$$

AND AGAIN THE SPACING AT EMISSION AND RECEPTION IS PRESERVED IN THE SAME RATIO WE SEE THE WAVELENGTH SHIFT.

HOW TO DETERMINE THE APPARENT DISTANCE D_A

TO ANSWER THIS QUESTION WE SEEK TO FIND THE ANGLE WHICH WE SEE BETWEEN TWO DISTANT OBJECTS. THIS TIME, HOWEVER, WE MUST CONSIDER THE LIGHT IS BENT TOGETHER BECAUSE OF THE GRAVITATIONAL PULL OF THE MATTER BETWEEN THE PATHS.

FOR THE CASE OF AN EXPANDING UNIVERSE, I.E., $\dot{R} > \frac{L}{2}$ THE LIGHT WILL COLLAPSING BE BENT BACK SO WE FIRST ASK HOW MANY NEBULAE THERE ARE OUT TO OUR OBJECT IN UNITS OF ψ . AS MENTIONED BEFORE IF WE KNOW THE ANGULAR SEPARATION OF TWO ADJACENT NEBULAE THE $\Delta\phi$ IS UNCHANGED WITH ψ BECAUSE THEY FLY OUT RADIALLY AS WE WATCH THEM. THIS MOTION PRESERVES THEIR ORIGINAL SPACING.

WE ORIGINALLY DEFINED THE PROPER DISTANCE IN THE FOLLOWING MANNER.

$$D_p = \frac{L}{\Delta\phi}$$

THIS RELATIONSHIP IS REPLACED WITH

$$D_p = \frac{E(t_i)}{\Delta\phi}$$

Thus $\Delta\phi = \Delta\phi(\psi)$ WHICH IS CONSTANT FOR EACH ψ AND ψ DEPENDS WHICH NEBULAE YOU PICK, NOT ON TIME. ONE WAY TO GET AN ESTIMATE ON $\Delta\phi$ IS THEN TO CONSIDER UNIFORM EXPANSION AND TAKE OUR EXPRESSIONS FOR R AND t , I.E.,

$$R = a(\cosh\theta - 1)$$

$$t = a(\sinh\theta - \theta)$$

IF WE LET THE EXPONENTIAL TERMS DOMINATE WE CAN APPROXIMATE $\Delta\phi$

$$\Delta\phi = \frac{E(t_i)}{D_p} = \sinh\psi$$

WE CONCLUDE THEN THE APPARENT DISTANCE OF THE OBJECT D_A IS

$$D_A = a(\cosh\theta_i - 1) \sinh\psi$$

WE CAN TIE THIS RESULT TO THE RED SHIFT BY OUR RESULT ON PAGE 21, I.E.,

$$z = \frac{R(t_0)}{R(t_i)} = \frac{\cosh\theta_0 - 1}{\cosh\theta_i - 1}$$

RETURNING TO THE LUMINOSITY ARGUMENT AGAIN WE KNOW THAT

$$D_f = R(t_i) \sinh(\theta_0 - \theta_i)$$

BECAUSE WE JUST PROVED IT SINCE $R(t_i) = a(\cosh\theta_i - 1)$ AND $\psi = \theta_0 - \theta_i$.

THE ORIGINAL INTENSITY ℓ_0 THAT THE NEBULA SENT OUT ITS LIGHT SPREADS OUT THROUGH SPACE IN ALL DIRECTION. AS THE Emitter SEEKS THE LIGHT FADE AWAY AS THE DISTANCE GROWS, THE APPARENT DISTANCE WHICH WE ARE AT AS HE SEES US IS D'_f . THIS DISTANCE WE EARLIER FOUND TO BE,

$$D'_f = \frac{R(t_0)}{R(t_i)} D_f = z D_f$$

SINCE THE ψ IS THE SAME OUR $\sinh \psi$ ARGUMENT IS THE SAME AND $E(t_i) \rightarrow E(t_0)$, WE CALCULATE THE APPARENT LUMINOSITY ℓ IS

$$\ell = \frac{\ell_0}{(D'_f)^2} \cdot \frac{1}{z} \cdot \frac{1}{z}$$

THIS IS THE SAME RESULT ON PAGE 15.

AND

$$\ell = \frac{\ell_0}{D_f^2 z^4}$$

TO CLEAR UP SOME OF THE CONFUSION WHICH HAS ARisen OVER THE PARTICULAR FORM OF THE NEBULAR SPACING, I.E.,

$$E(t) = E_x \frac{t}{a}$$

WHERE E_x IS SOME STANDARD NEBULAR SPACING AT TIME a . THE REASON WHY WE CHOOSE THE a FOLLOWS FROM OUR DISCUSSION ON PAGE 21. THERE WE SAW THAT ψ CONVENIENTLY EXPRESSED THE DIFFERENCE BETWEEN θ_{EMITTER} AND θ_{REC} .

SINCE WE HAVE THAT $d\eta = \frac{a d\psi}{E_x}$ AND ψ IS INDEPENDENT OF THE INFINITESIMAL SPACING $E(t)$. FURTHER WE ARGUED THAT $d\eta = \frac{dt}{E(t)}$ COMBINING THESE FACTOR WE HAVE

$$\frac{dt}{E(t)} = \frac{a d\psi}{E_x}$$

OR

$$\frac{dt}{d\psi} = \frac{a}{E_x} E(t)$$

BUT ALSO ON PAGE 21 WE HAD $E(t) = E_x (\cosh \theta - 1)$

THEN,

$$\frac{dt}{d\psi} = a (\cosh \theta - 1)$$

SINCE,

$$\theta = a (\sinh \theta - 1) \Rightarrow d\theta = a (\cosh \theta - 1) d\theta$$

WE HAVE

$$\frac{d\theta}{d\psi} = 1 \Rightarrow \psi = \theta_{\text{EMITTER}} - \theta_{\text{REC}}$$

We could have chosen to solve $\ddot{r}(\tau) = -\frac{GM}{r^2}$ different if we had taken,

$$r = r_x(1 - \cos\theta)$$

$$\tau = a(\theta - \sin\theta)$$

CONTINUING A SIMILAR DEVELOPMENT AS STARTED ON PAGE 18 we have

$$dr = r_x \sin\theta \quad d\tau = a(1 - \cos\theta)$$

$$\dot{r} = \frac{dr}{d\tau} = \frac{r_x \sin\theta}{a(1 - \cos\theta)}$$

$$H = \frac{\dot{r}}{r} = \frac{r_x \sin\theta}{a(1 - \cos\theta)^2} = H(t)$$

$$\ddot{r} = \frac{d\dot{r}}{d\tau} = \frac{d\dot{r}}{d\theta} \cdot \frac{d\theta}{d\tau}$$

$$\frac{d\dot{r}}{d\theta} = \frac{r_x}{a} \left[\frac{(1 - \cos\theta)\cos\theta - \sin\theta(\sin\theta)}{(1 - \cos\theta)^2} \right] = \frac{r_x}{a} \frac{(\cos\theta - 1)}{(1 - \cos\theta)^2}$$

$$\ddot{r} = \frac{d\dot{r}}{d\tau} = \frac{-r_x}{a^2(1 - \cos\theta)^2} = -\frac{r_x^3}{a^2 r^2}$$

WHAT WE WOULD REALLY LIKE OUT OF THIS CASE IS THE RELATIONSHIP OF THE RED SHIFT Z TO THE Θ_0 . AGAIN ANALOGOUS TO THE ARGUMENT ON PAGE 21.

$$Z = \frac{A\zeta_0}{A\zeta_1} = \frac{a(1 - \cos\theta_0)}{a(1 - \cos\theta_1)} = \frac{1 - \cos\theta_0}{1 - \cos(\theta_0 - \psi)}$$

where $\theta_1 = \theta_0 - \psi$

The NEXT LOGICAL QUESTION TO ASK About The red SHIFT IS, "How does IT CHANGE WITH TIME?" To START THIS TOPIC WE WRITE DOWN,

$$Z(\theta_0) = \frac{\omega_{\text{emit}}}{\omega_{\text{rec}}} = \frac{E(t_0)}{E(t_0 - \psi)} = \frac{\cosh \theta_0 - 1}{\cosh \theta_0 - 1}$$

where we MUST put IN $\theta_1 = \theta_0 - \psi$ SINCE we ARE CONCERNED ABOUT THE PRESENT OBSERVABLE red shift FOR AY GIVEN ψ , i.e., TWO ADJACENT NEBULAE WATCHED.

$$Z_\psi(\theta_0) = \frac{\cosh \theta_0 - 1}{\cosh(\theta_0 - \psi) - 1}$$

So what we WANT IS

$$\left. \frac{dz}{dt} \right|_0$$

which CAN be OBTAINED by

$$\frac{dz_0}{dt_0} = \frac{dz_0}{d\theta_0} \frac{d\theta_0}{dt_0} = \frac{1}{a(\cosh \theta_0 - 1)} \times \frac{dz}{d\theta_0}$$

$$\frac{dz(\theta_0)}{d\theta_0} = \frac{[\cosh(\theta_0 - \psi) - 1] \sinh \theta_0 - [\cosh \theta_0 - 1] \sinh(\theta_0 - \psi)}{[\cosh(\theta_0 - \psi) - 1]^2}$$

Then

$$\frac{dz}{dt} = \frac{1}{a[\cosh(\theta_0 - \psi) - 1]} \left[\coth \left(\frac{\theta_0}{2} \right) - \coth \left(\frac{\theta_0 - \psi}{2} \right) \right]$$

From This result we CAN determine The red shift ALWAYS HAS A NEGATIVE DERIVATIVE AND RAPIDLY CHANGING AT The BEGINNING BUT SLOWS DOWN AND COASTS TO 0 AT ∞
THIS RESULT EXPLAINS why THE LINES ON The LEFT FIGURE (PAGE 20) ARE CURVED INITIALLY but BECOME ASYMPTOTICALLY STRAIGHT After A LONG TIME. THIS IMPLIES A CONSTANTLY EXPANDING UNIVERSE

A SIMILAR ARGUMENT CAN be DEVELOPED
for $Z = \frac{1 - \cos \theta_0}{1 - \cos(\theta_0 - \psi)}$

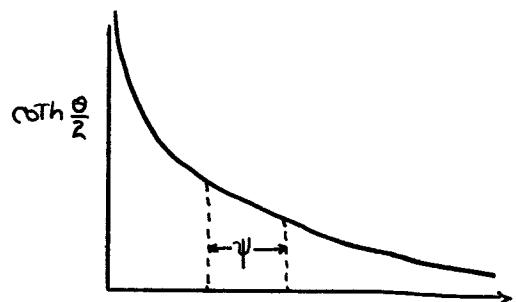


FIG. 13

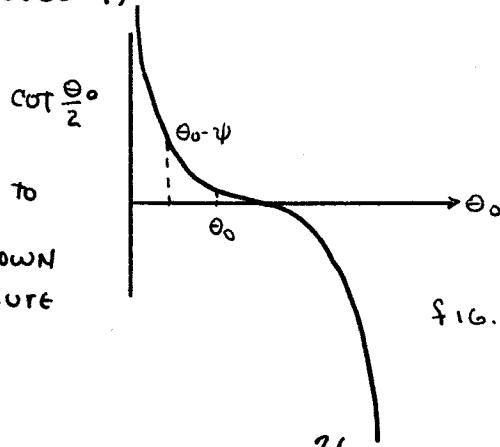


FIG. 14

This CORRESPONDS TO our COLLAPSING UNIVERSE AS SHOWN by The OTHER FIGURE ON PAGE 20.

The EXACT form of THE RATE OF CHANGE of THE REDSHIFT MIGHT BE A LITTLE PUZZLING AS TO THE MATHEMATICS SPECIFICALLY INVOLVED. SO WE WILL ATTEMPT TO CLARIFY IT here,

AS STATED,

$$\frac{dz}{dt} = \frac{1}{a(\cosh \theta_0 - 1)} \frac{dz}{d\theta_0}$$

$$\frac{d}{d\theta_0} \left[\frac{\cosh \theta_0 - 1}{\cosh(\theta_0 - \psi) - 1} \right] = \frac{\sinh \theta_0}{\cosh(\theta_0 - \psi) - 1} - \frac{\sinh(\theta_0 - \psi)(\cosh \theta_0 - 1)}{[\cosh(\theta_0 - \psi) - 1]^2}$$

$$\begin{aligned} \frac{dz}{dt} &= \frac{1}{a(\cosh \theta_0 - 1)} \left\{ \frac{\sinh \theta_0}{\cosh(\theta_0 - \psi) - 1} - \frac{\sinh(\theta_0 - \psi)(\cosh \theta_0 - 1)}{[\cosh(\theta_0 - \psi) - 1]^2} \right\} \\ &= \frac{1}{a[\cosh(\theta_0 - \psi) - 1]} \left\{ \frac{\sinh \theta_0}{\cosh \theta_0 - 1} - \frac{\sinh(\theta_0 - \psi)}{\cosh(\theta_0 - \psi) - 1} \right\} \end{aligned}$$

$$\text{Now } \cosh \theta_0 - 1 = 2 \sinh^2 \frac{\theta_0}{2}$$

$$\sinh 2\left(\frac{\theta_0}{2}\right) = 2 \sinh \frac{\theta_0}{2} \cosh \frac{\theta_0}{2}$$

SO THAT

$$\frac{\sinh \theta_0}{\cosh \theta_0 - 1} = \frac{2 \sinh \frac{\theta_0}{2} \cosh \frac{\theta_0}{2}}{2 \sinh^2 \frac{\theta_0}{2}} = \coth \frac{\theta_0}{2}$$

LIKewise

$$\frac{\sinh(\theta_0 - \psi)}{\cosh(\theta_0 - \psi) - 1} = \coth \frac{\theta_0 - \psi}{2}$$

FINALLY

$$\frac{dz}{dt} = \frac{1}{a[\cosh(\theta_0 - \psi) - 1]} \left[\coth \frac{\theta_0}{2} - \coth \frac{\theta_0 - \psi}{2} \right]$$

FORMALLY EXPANDING INTO EXPONENTIALS:

$$\frac{dz}{dt} = \frac{1}{a \left[\frac{1}{2} (e^{\theta_0 - \psi} + e^{-\theta_0 + \psi}) - 1 \right]} \left[\frac{e^{\theta_0/2} + e^{-\theta_0/2}}{e^{\theta_0/2} - e^{-\theta_0/2}} - \frac{e^{\theta_0 - \psi/2} + e^{-(\theta_0 - \psi)/2}}{e^{\theta_0 - \psi/2} - e^{-(\theta_0 - \psi)/2}} \right]$$

when $\theta_0 = 0 \Rightarrow \psi = 0$

$$\frac{dz}{dt} \Big|_{\theta_0=0} = \frac{1}{a \left[\frac{1}{2}(2) - 1 \right]} \left[\frac{2}{0} - \frac{2}{0} \right] = \infty$$

$\theta_0 = \infty \quad \psi = \text{CONSTANT}$

$$\frac{dz}{dt} = \frac{1}{a(\infty + 0)} [1 - 1] = 0 \quad \text{BECAUSE The denominator goes to infinity faster than the difference goes to 0.}$$

The no. of nebulae with a red shift below that corresponding to a certain ψ_N is simply,

$$\int_0^{\psi_N} \sinh^2 \psi d\psi$$

For the collapsing universe the no. of nebulae below ψ_N is

$$\int_0^{\psi_N} \sin^2 \psi d\psi$$

Since ψ is a function of θ_0 itself we conclude that there are many nebulae which we don't see, i.e., as time goes on we see more and more nebulae. This corresponds to the first integral. We might think of it this way - light must lift itself up and out to get away from its source. As we trace back along the ray we can go so far then lose it because the densely packed matter deflects the beam. As time goes on and everything spreads out, the density decreases, light is able to escape so we see more and more nebulae. If we wait infinitely long then, we will see all of them.

With the latter case about the results are more frightening because there is a finite time when we see all the nebulae and that's when they have collapsed together and then we don't care that we see them all.

We would now like to talk about the relativistic determination of light being bent by the sun, i.e., it is deflected twice as much as what Newtonian mechanics predicts. It turns out light is bent twice as much. We must determine how far apart two adjacent nebulae are for a given ψ and then follow the light back to us point by point as it is bent by the stuff between the rays.

The apparent angular separation $\Delta\phi$ is

$$\Delta\phi = \frac{L}{D_F}$$

L = separation at the proper time

$D_F = \sinh \psi \in(t_i)$ as found on page 23

replacing L by Y the height and θ_i by $\theta_0 - \psi$

$$\Delta\phi = \frac{Y}{\sinh \psi a [\cosh(\theta_0 - \psi) - 1]}$$

This differential can be solved and consequently we can deduce back to the initial points.

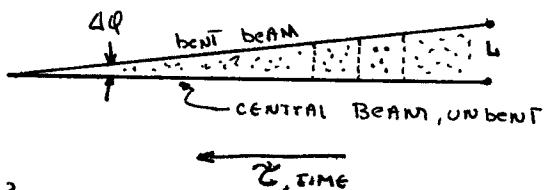


FIG. 15

Now we know the light energy $E(t)$ because we know its frequency (cf. p. 26). Thus the angular separation is -

$$\angle = \frac{P}{E}$$

where $P = \text{TRANSVERSE MOMENTUM}$

For short distances the slopes are small so we calculate the ordinary rate of change of the height with time to find,

$$\frac{dy}{dt} = -\frac{Py}{E}$$

and it should follow

$$\frac{dPy}{dt} = \frac{Gp(z)Y E(z)}{2}$$

where the force on the beam is due to the cylinder of matter contained between the line segments on the previous diagram.

If we take the cylinder to have radius y and density ρ and ignore matter outside we have by Gauss' theorem

$$G\pi y^2 \rho = \text{field } z\pi y$$

With the boundary conditions that $y=L$ at $t=0$ and $y=0$ at $t=t_0$, we should be able to solve the diff. eq's but we find, in doing so, we are off by a factor of 2. We must have

$$\frac{dPy}{dt} = \frac{Gp(z)Y E(z)}{2} \cdot 2$$

where $p(z) \propto E(z)^3$

This result follows from the realization that the energy and momentum combine to give a factor of twice the Newtonian energy. In this proof we assumed what we wanted to prove but did some hand waving because there is no simple way to get that 2 in there.

HOYLE'S THEORY

Hoyle proposed a steady-state model by assuming our variable $g = -\frac{\ddot{r}}{r^2} = -1$ in which the motion of matter is kinematically determined. Everything looks the same to him at all times because as the universe expands matter is created, nebulae evolved, and the uniformity preserved. This theory assumes the r is exponential with time so there is room for the new matter to be 'dropped' in. The luminosity must remain unchanged since the older nebulae are not brighter.

The trouble with this theory is that it is too precise and, therefore, not credible. It destroys the conservation of energy for the sake of explaining cosmology. It requires, further, matter exceed the speed of light. By saying the creation is so slow it is never observed the argument seems ridiculous.

Other Theories

When Einstein first discussed cosmology, he introduced a uniform negative pressure which had no cause. This he did because he had no red shift and the nebulae were motionless so to prevent the universe from collapsing he added a 'cosmological term' to the equation of motion as a negative pressure which pushes everything out thus saving the day.

The Hubble found the recessional relationships of velocity and red shift and Einstein was happy to see this term go. Some still cling to it though and deduce by it that the universe is oscillatory. That is, the pressure forces the universe out after it collapses.

In physical cosmology, the cosmological constant (usually denoted by the Greek capital letter lambda: Λ) is equivalent to an energy density in otherwise empty space. It was originally proposed by Albert Einstein as a modification of his original theory of general relativity to achieve a stationary universe. Einstein abandoned the concept after the observation of the Hubble redshift indicated that the universe might not be stationary, as he had based his theory on the idea that the universe is unchanging.[1] However, a number of observations including the discovery of cosmic acceleration in 1998 have revived the cosmological constant, and the current standard model of cosmology includes this term

In physical cosmology and astronomy, dark energy is a hypothetical form of energy that permeates all of space and tends to accelerate the expansion of the universe.[1] Dark energy is the most accepted hypothesis to explain observations since the 1990s that indicate that the universe is expanding at an accelerating rate. In the standard model of cosmology, dark energy currently accounts for 73% of the total mass-energy of the universe.[2]

Two proposed forms for dark energy are the cosmological constant, a constant energy density filling space homogeneously,[3] and scalar fields such as quintessence or moduli, dynamic quantities whose energy density can vary in time and space. Contributions from scalar fields that are constant in space are usually also included in the cosmological constant. The cosmological constant is physically equivalent to vacuum energy. Scalar fields which do change in space can be difficult to distinguish from a cosmological constant because the change may be extremely slow.



CHAPTER 3

THE GENERAL THEORY OF RELATIVITY AND HOW IT AFFECTS ASTRONOMY

IN ACTUALITY THE EFFECTS OF RELATIVITY UPON ASTRONOMY ARE REALLY SMALL DESPITE WHAT SOME PEOPLE BELIEVE. BUT WHAT IS THERE IS VERY IMPORTANT IN CERTAIN COSMOLOGICAL DISCUSSIONS AND THE INTERPRETATION OF GRAVITY.

THE PATH TO RELATIVITY STARTED, OF COURSE, WITH NEWTON AND HIS LAWS RELATING FORCES AND ACCELERATIONS BY MASSES; THEN THE NEXT IMPORTANT CONTRIBUTION WAS THE EXPLANATION OF ELECTRICITY VIA MAXWELL; THE MICHELSON MORLEY EXPERIMENT OF THE SPEED OF LIGHT; AND FINALLY THE RELATIVISTIC RELATIONSHIP BETWEEN ACCELERATED AND REST MASSES. THIS IS A BRIEF HISTORICAL OUTLINE.

IT PAVED THE WAY FOR AN ALTERATION OF NEWTON'S GRAVITY LAW IN SUCH A WAY THAT MAXWELL'S EQUATIONS REMAIN INVARIANT, I.E. THE NEW GRAVITY LAW WILL BE RELATIVISTICALLY INVARIANT. THE PROBLEM CAN BE SOLVED LABORIOUSLY NON-RELATIVISTICALLY BUT EINSTEIN DID IT SO ELOQUENTLY THAT PEOPLE DID NOT SEE THE OTHER SOLUTION FOR A LONG TIME.

WHEN LOOKING BACK ON PAST PHYSICAL DISCOVERIES WE OFTEN AS 'WHY DIDN'T HE THINK OF THAT?' BUT WITH EINSTEIN THE PROPER QUESTION IS 'HOW THE HELL DID HE THINK OF IT SO FAST?' THIS GREAT ARGUMENT IS SO PERFECT IT HAS NEVER BEEN USED TO SOLVE OTHER PHYSICAL PROBLEMS. THIS IS PERHAPS DUE TO THE GEOMETRICAL INTERPRETATION OF GRAVITY WHICH NO ONE CAN TRANSLATE INTO NON-GEOMETRICAL ARGUMENTS.

TO START THE THEORY SET FORTH BY EINSTEIN BY DISCUSSING FIELDS. THE FIELD ENERGY DUE TO TWO SOURCES S_1 AND S_2 IS GIVEN BY

$$E = \frac{S_1 S_2}{r_{12}^2}$$

WHERE r_{12} IS THE SEPARATION DISTANCE BETWEEN S_1 AND S_2 . NOW FIELDS ARE CLASSIFIED IN THE FOLLOWING MANNER:

SCALAR, ϕ
VECTOR, A^μ
TENSOR, $T_{\mu\nu}$
⋮
higher order
⋮
Tensors

THE VECTOR FIELD A^μ WE HAVE A GOOD KNOWLEDGE; THEY ARE THINGS LIKE THE VECTOR POTENTIAL IN ELECTROMAGNETIC FIELD THEORY. ALL THE OTHER FIELDS HAVE BEEN THEORIZIED BUT WE KNOW ONLY THE ONE FOR CERTAIN.

THE SCALAR FIELD HAS NO EXAMPLE AS MENTIONED AND WOULD NECESSITATE A CAUSE WITHOUT AN EFFECT BECAUSE THE VELOCITY OF MOTION WOULD BE EQUAL TO THE SPEED OF LIGHT.

The tensors are also theoretical concepts which have a possible interpretation which we will go into. First, however, we can express the relativistic nature of these fields in the following manner:

$$\text{SCALAR } S_1 = S_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$\text{VECTOR } S_1 = S_0 \cdot \mathbf{I}$$

$$\text{TENSOR } S_1 = \frac{S_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

From these equations which we assume to be true but can be derived the vector field displays the already known fact that like charges repel. By this property alone we throw out this field as describing gravity otherwise everything would be stuck together.

While we can argue that a gas in high motion motion has a weight proportional to its energy content which is a measure of the packing fraction we conclude the field would cause the system to weigh more since $E=mc^2$ etc. By observing the scalar field we see the field will get smaller with increased velocity. It must be thrown out as a possible representation of a gravitational field.

We are confronted with the first tensor field T_{uv} which gets heavier in just the right manner as prescribed by the above equation. Thus we have argued that the field source and energy content of the system are equal and lead to a gravitational field of the desired nature.

Where in E-M theory the charge is conserved and which leads to the invariance of the vector potential, i.e.,

$$A^i = A^i + \nabla$$

where ∇ is some gradient we add and preserve conservation. We expect an analogous conservation of source of energy to be represented something like

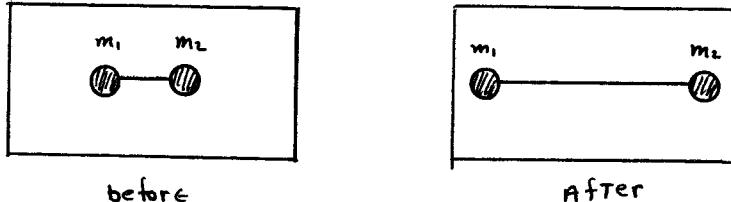
$$T^{i'}_{uv} = T_{uv} + \nabla_u h_{i'v} + \nabla_v h_{iu}$$

Thus the energy is a tensor.

But this is no real surprise because the energy density is a tensor of the 2nd rank and the field source can be related to the stress tensor.

WE NOW COME TO AN INTERESTING QUESTION THAT BEING THE RELATIONSHIP BETWEEN THE GRAVITATIONAL FIELD AND THE ENERGY CONTAINED IN THE FIELD.

IMAGINE A BOX WITH A DUMBBELL SUPPORTED BY A MASSLESS ROD WHICH IS THEN PULLED APART, I.E., ENERGY IS PUT INTO THE SYSTEM SO IF YOU LET GO THE BALLS WOULD ZAP TOGETHER GIVING OFF HEAT.



BEFORE THE STRETCH IF YOU WEIGH THE BOX YOU GET THE WEIGHT OF THE BALLS AND THE 'WEIGHT' OF THE GRAVITATIONAL FIELD DUE TO THE ATTRACTION OF THE TWO BALLS UPON ONE ANOTHER. NOW WHEN THEY ARE STRETCHED APART ENERGY IS PUT INTO THE SYSTEM AND THE SYSTEM SHOULD WEIGH MORE. THE ENERGY PUT IN MUST DO WORK ON THE GRAVITATIONAL FIELD SO IT IS FAIR TO ASK WHAT IS GRAVITY? - ENERGY OR MATTER - THE ANSWER TO THIS IS NOT IMMEDIATE SO WE'LL HAVE TO GO ON. WE MIGHT SUM UP THIS SECTION BY SAYING GRAVITY PRODUCES ITS OWN EFFECT.

EINSTEIN'S VIEWS

IN GENERALIZING NEWTON'S THEORY OF GRAVITY WE MUST ASSUME THE EFFECT ACTS INSTANTANEOUSLY WITH ITS CAUSE. BUT THE QUESTION IMMEDIATELY ARISES THAT THIS MEANS GRAVITY EXCEEDS THE SPEED OF LIGHT. BUT THIS CANNOT BE SO WE MUST ALTER THE GRAVITATIONAL FIELD THEORY TO TAKE ON A SIMILAR FORM AS TRAVELING ELECTROMAGNETIC WAVES WHICH PROPAGATE THROUGH SPACE AT THE VELOCITY C.

THE VELOCITY OF LIGHT C IS A VERY CRITICAL FUNDAMENTAL CONSTANT RELATING SPACE AND TIME. IT GIVES HOW MANY INCHES PASS BY PER TIME INTERVAL. NO MATTER WHAT SYSTEM OF UNITS YOU'RE IN THE INVARIANCE IS PRESERVED. WAVE PROPAGATION IS ONLY A CONSEQUENCE OF THIS INVARIANCE AND THUS SAYS THAT WE WON'T FIND ANOTHER C. IF LIGHT OR GRAVITY TRAVELED LESS THAN C, THEY WOULD LOSE THEIR INVARIANCE AND IF YOU SHAKED A SYSTEM AND OBSERVED IT STANDING STILL AND THEN FLYING BY, YOU WOULD GET TWO RESULTS. THIS CAN'T BE.

THE QUESTION IMMEDIATELY ARISES, "WHAT GOES AT THE SPEED OF LIGHT IN THE CASE OF GRAVITY?" HOW DO THE FORCES DEPEND ON THE VELOCITIES? IN ELECTRIC FIELD THEORY WE KNOW THE FORCE ON A CHARGE MOVING WITH A VELOCITY IN AN ELECTRIC AND MAGNETIC FIELD IS GIVEN BY,

$$\vec{F} = \vec{E}(x) + \vec{V} \times \vec{B}(x)$$

Where we must know the velocity so we can find one component as a linear sum of the velocity components, i.e.,

$$F_x = E_x + v_x B_{xx} + v_y B_{xy} + v_z B_{xz}$$

Now the gravitational analogue involves the usual Newtonian gravity field plus some linear terms in the velocity plus, then, some quadratic term. This takes the form,

$$G_x = F_x = C_x + v_x \beta_{xx} + v_y \beta_{xy} + v_z \beta_{xz} + v_x v_y \delta_{xyx} + v_y^2 \delta_{xxy} + \dots$$

where all told there would be nine terms as a consequence of gravity being a tensor.

WHAT EINSTEIN did then was to set out to find the laws of motion and the laws determining the coefficients.

WELL, HOW DID he DO IT? IN-STEAD of hit or miss guesswork type approach he set out to establish some principles which would serve as guide posts. He assumed, first, relativistic invariance and, secondly, the law must always be true.

HE BEGAN WITH THE THREE WAYS IN WHICH NEWTON DEFINED MASS,

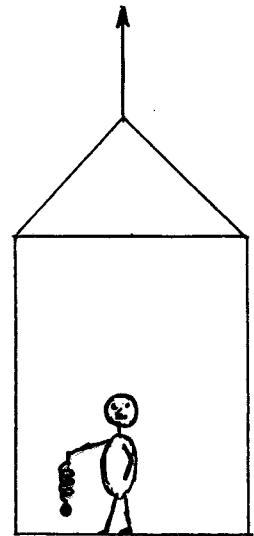
1. MASS IS MEASURED BY INERTIA, i.e., RESISTANCE TO MOTION
2. MASS IS THE REACTION TO A GRAVITATIONAL FIELD, i.e., IT IS WEIGHT
3. HOW IT PRODUCES A GRAVITATIONAL FIELD, i.e., THE TECHNIQUE OF MEASURE INTERACTING FIELD AND HOW MEASURE EARTH MASS WITH AID OF MOON.

TO NEWTON THESE SAID THE SAME THING BUT EINSTEIN QUESTIONED THEM AND ASKED IF IT IS TRUE FOR ALL TIMES. HE ASSUMED THAT TWO AND THREE WERE EQUIVALENT SINCE THE CAUSAL RELATIONSHIP BETWEEN ACTION AND REACTION FOLLOW FROM THEM. HE THEN ASSUMED 'ONE' WAS THE SAME AS THE OTHERS AND INFINITELY ACCURATE UNDER EVERY CASE. THUS THE PREVIOUS BOX ARGUMENT IN WHICH MORE ENERGY IMPLIED MORE INERTIA IMPLIED MORE WEIGHT WAS A VALID ARGUMENT. ALL he required was Lorentz invariance.

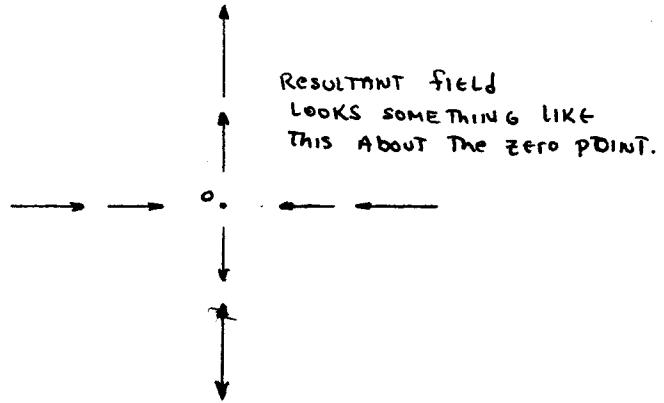
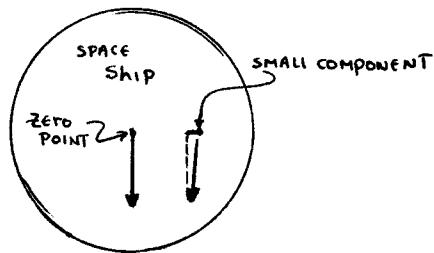
EINSTEIN PROCEEDED TO CHANGE HIS ARGUMENTATION BY CONSIDERING NON-UNIFORM MOTION. WHAT he did was to imitate gravity by inertia. WHAT THIS LEADS TO IS HIS FAMOUS PRINCIPLE OF EQUIVALENCE OF GRAVITATION AND INERTIA. THIS STATES THAT THERE IS NO WAY TO DISTINGUISH THE MOTION PRODUCED BY INERTIAL FORCES (ACCELERATION, RECOIL, CENTRIFUGAL FORCES, ETC) FROM MOTION PRODUCED BY GRAVITATIONAL FORCES.

HIS ARGUMENT CENTERED AROUND AN ACCELERATING ELEVATOR IN WHICH A BOY STANDS HOLDING A BALL ON THE END OF A SPRING. THE FORCE ON HIS LEGS IS PROPORTIONAL TO HIS MASS IF HIS LEGS DON'T GIVE IN. THE BALL ON THE SPRING GOES DOWN AS AN UPWARD FORCE IN THE SPRING ELONGATES. THUS IT IS LIKE THE BALL IS 'PULLED' DOWN WHEN IN FACT THE SYSTEM IS ACCELERATING.

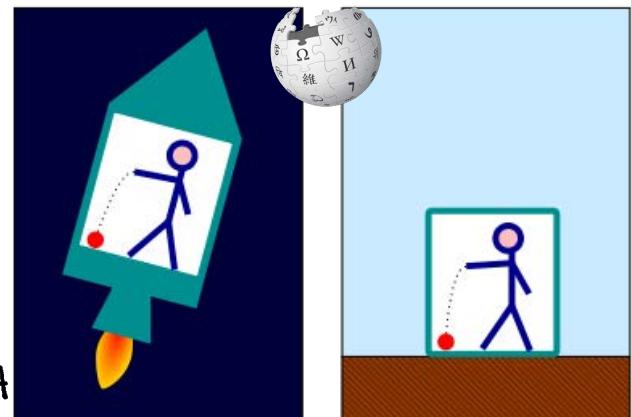
IN THIS WAY GRAVITY AND ACCELERATION ARE SHOWN EQUIVALENT. THAT IS TO SAY, MOTION BOTH UNIFORM AND NONUNIFORM CAN ONLY BE JUDGED WITH RESPECT TO SOME SYSTEM OF REFERENCE.— ABSOLUTE MOTION DOES NOT EXIST.



ANOTHER WAY TO LOOK AT THE SAME THING IS TO PICTURE A SPACESHIP ABOVE THE EARTH IN WHICH THE OCCUPANTS DON'T LOOK OUT TO SEE ALL THE OBJECTS GO BY. IF THERE WAS A BALL OF WATER IN THE SHIP IT WOULDN'T BE A BALL BUT RATHER AN ELLIPSOID. THIS WOULD BE A RESULT OF THE TIDES DUE TO THE EARTH. THAT IS, THE EARTH DOESN'T PULL ON THE WATER EQUALLY EVERYWHERE. IT IS POSSIBLE TO MAKE THE GRAVITATIONAL FIELD OF EARTH BE ZERO CHOOSING THE RIGHT ACCELERATION BUT THIS IS GOOD FOR ONLY ONE POINT. IF YOU MOVE AWAY A LITTLE BIT FROM THAT POINT OF 0 G'S, THE EARTH WILL HAVE A SMALL COMPONENT. IN EFFECT, A FIELD STILL EXISTS AROUND THAT POINT.



WE HAVE ONLY GOTTEN rid of THE LOCAL field BY THIS ARGUMENT.

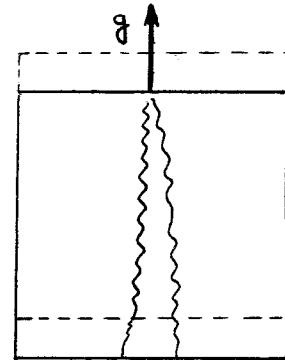


Now we ask another question, "WHAT HAPPENS TO LIGHT IN A UNIFORM FIELD?" THAT IS, WE DISREGARD THE SLIGHT FORCES OF THE CENTER AND THUS NEGLECT HIGHER ORDER EFFECTS. THEREFORE INERTIA AND REACTION ARE EQUAL LIKE. WE GO BACK TO THE ELEVATOR AND PUT A LIGHT THAT EMITS A DEFINITE FREQUENCY ON THE CEILING; THEN WE ASK WHAT WE SEE IF WE'RE LYING ON THE FLOOR AS THE ELEVATOR MOVE UPWARD WITH AN ACCELERATION, g . LET THE ELEVATOR HAVE SIDE, L .

SINCE IT TAKES A FINITE TIME TO RECEIVE THE PHOTON AT THE FLOOR THE FLOOR MOVES A SMALL DISTANCE TOWARDS THE SOURCE, OR WHERE IT WAS. AT A GIVEN INSTANT WHEN WE TAKE $v=0$, THE DOPPLER SHIFT WILL BE TO THE PURPLE AS A RESULT OF THE ACCELERATION.

THE TIME OF TRAVERSAL IS

$$T = \frac{L}{c}$$



THE VELOCITY OF THE BOTTOM RELATIVE TO THE TOP IS GIVEN BY

$$v_{rel} = g \frac{L}{c}$$

THE FREQUENCY DUE TO THE DOPPLER EFFECT IS

$$\omega = \omega_{emit} \left(1 + \frac{v_{rel}}{c} \right)$$

Therefore,

$$\omega_{rec} = \omega_{emit} \left(1 + \frac{gL}{c^2} \right)$$

WHERE WE CAN CALL gL THE GRAVITATIONAL POTENTIAL AT HEIGHT L
OR

$$\nabla \phi = g L$$

THEN, MORE ELOQUENTLY

$$\omega_{rec} = \omega_{emit} \left(1 + \frac{\nabla \phi}{c^2} \right)$$

$\nabla \phi$ HAS BEEN SUPPOSEDLY EXPERIMENTALLY CALCULATED BUT SINCE THE RESULTS ARE WITHIN \pm THE AMOUNT THEY WANT NOT TO ACCURATE. THE FIGURES ARE SOMETHING ~~like~~ LIKE 1mm/sec. THIS WAS DONE IN A 75 STORY ELEVATOR WHICH WAS SUBJECT TO CHANGES DUE TO TEMPERATURE GRADIENTS \neq IN THE BUILDING.

If A CHARGE IS ACCELERATING, THEN IT MUST RADIATE. SO IF WE THROW ELECTRON OR SOMETHING IN ELEVATOR WHAT HAPPENS?

BY THE THEORY OF RADIATING DM CHARGES DUE TO OSCILLATIONS, THE RATE OF ENERGY RADIATION IS

$$\frac{dW}{dt} = \frac{2}{3} \frac{e^2 (\ddot{x})^2}{c^3}$$

BUT WHEN THE CHARGE IS ACCELERATING THERE IS A DAMPING FORCE DUE TO RADIATION RESISTANCE GIVEN BY

$$F = \frac{2}{3} \frac{e^2}{c^3} \ddot{x}$$

THE WORK DONE AGAINST THIS FORCE IS $F\dot{x}$ OR

$$\frac{dW_r}{dt} = \frac{2e^2}{3c^3} \ddot{x} \dot{x} = \frac{2e^2}{3c^3} \left[\frac{d}{dt} (\ddot{x} \dot{x}) - \ddot{x}^2 \right]$$

IN A PERIODIC MOTION $\ddot{x} \dot{x}$ AVERAGES TO ZERO WHILE THE SECOND TERM IS ALWAYS POSITIVE BEING SQUARED. SO IF INTEGRATE \ddot{x}^2 WITH RESPECT TO TIME, IT IS POSSIBLE TO SHOW THAT THE CHARGE RADIATES ENERGY WHEN IT FIRST ACCELERATES AND WHEN IT STOPS BUT NOT WHILE IT ACCELERATES.

AN ASIDE HERE ALONG THESE LINES, DEAL WITH THE INERTIA COEFFICIENT BEING EQUAL TO THE MECHANICAL INERTIA. IF WE CALCULATE THE FORCE ON ONE PART OF A BALL OF CHARGE DUE TO OTHER PARTS WE GET THE SELF-FORCE,

$$F_s = \alpha \frac{e^2}{r_0 c^2} \ddot{x} - \frac{2e^2}{3c^3} \ddot{x} + \gamma \frac{e^2}{c^4} \ddot{x} + \dots$$

r_0 = RADIUS OF CHARGE

α, γ COEFFICIENTS DEPENDING UPON THE CHARGE DISTRIBUTIONS.

FOR A SPHERE $\alpha = 2/3$, THEREFORE WE HAVE IDENTIFIED AN ELECTROMAGNETIC MASS, M_{elec} TO BE

$$M_{elec} = \frac{2}{3} \frac{e^2}{r_0 c^2}$$

WHICH WHEN MEASURE THE MASS EXPERIMENTALLY MUST BE ADDED TO THE MECHANICAL MASS, I.E.,

$$M_{exp} = M_{mech} + M_{elec}$$

THUS WE SEE THAT THE WEIGHT OF THE FIELD MUST BE CONSIDERED.

NOTICEABLY PROBLEMS ARISE IF $r_0 \rightarrow 0$ THEN $F \rightarrow \infty$ AND THE CHARGE BLOWS UP BUT WE DEFINE POINCARÉ STRESS OR RUBBER BANDS TO HOLD EVERYTHING TOGETHER.

If we want to know what the difference in frequency is, say, at 12,000 miles up, we tie together a mess of elevators of length L and with little holes to let the light through. Thus we ignore relativistic velocities, and just sum up the little g 's. In the final result $\Delta\phi/c^2$ is still small.

But when light comes from the sun the distances become appreciable and the frequency shift more noticeable. If the sun emits a given na sodium line, the earth sees a shift corresponding to

$$1 - \frac{GM_{\text{sun}}}{R_{\text{sun}}^2 c^2}$$

where $\frac{GM_{\text{sun}}}{c^2} \approx 1/2 \text{ km}$

$$R_{\text{sun}} > 1.5 \text{ km}$$

and the shift is figured to be about $\frac{1}{400,000}$. This is a fairly reasonable value but it has been successfully observed yet mainly because we know what the natural frequency of sodium is.

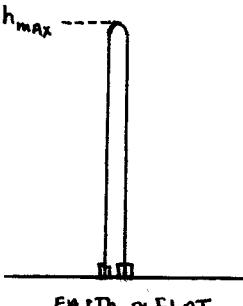
Now for white dwarfs where their mass $\propto M_{\text{sun}}$ but their radii are about that of the earth this shift is much greater. But hard to determine whether the shift is due to gravity or recession. Possibly double stars could help since they rotate about one another but often one star has visible spectral lines while the other is a continuous light so no help there.

PROBLEM: Consider you are on the earth and man says see how high you can go in one hour by my clock.

The traveling guy carries a watch which runs faster than the man on the ground. When he is at h_{max} , he looks purple to the guy on the ground for the above reasons. Thus we want to maximize the degree to which he is ahead, i.e., the higher the better. We won't let him get to c but instead make the following time correction

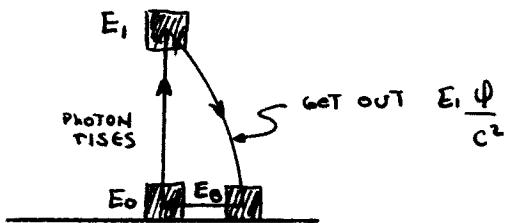
$$t_i = t_0 \sqrt{1 - \frac{v^2}{c^2}} \approx t_0 \left(1 - \frac{1}{2} \frac{v^2}{c^2}\right)$$

thus the traveler must go fast but not too fast or else he'll get into a red shift. So we use $w = w_{\text{emitter}} \left(1 - \frac{\Delta\phi}{c^2}\right)$, get the correct times and path of motion.



PROBLEM: A photon shoots up to a box which is in turn carried back down and the energy then released as it goes up again. At the end of the cycle if there is no net work energy should be conserved — is it?

LAST TIME WE MENTIONED AN INTERESTING PROBLEM - The idea of A PERPETUAL MOTION. IT GOES SOMETHING LIKE THIS: we have a box which emits a photon of energy E_0 which goes to a level at a higher gravitational potential than before; There it has an energy $E_1 = E_0(1 + \frac{\Phi}{c^2})$ because of the work it did to get up the potential Φ . Now we carry the box back down and do work $E_1\frac{\Phi}{c^2}$ to bring the photon to ground level again. It then has energy E_1 , because we got work out of it as we lower it. Thus the energy expended was E_0 and what we received back was $E_1 + \text{work done in lowering, i.e., } E_1\frac{\Phi}{c^2}$. Does the final energy EQUAL what we had to BEGIN with. The following is a diagram of ONE SUCH CYCLE:



where $E_B = \text{Energy coming back}$

$$= E_1 + E_1 \frac{\Phi}{c^2} = E_1 \left(1 + \frac{\Phi}{c^2}\right)$$

THE QUESTION IS: ?

$$E_0 = E_1 \left(1 + \frac{\Phi}{c^2}\right)$$

BEFORE GOING ON TO SOMETHING NEW\$, THE OTHER QUESTION I ASKED AT THE END MIGHT DESERVE A LITTLE MORE ATTENTION. WE WANTED TO MAXIMIZE THE TIME ON THE TRAVELER'S WATCH BY GOING UPWARDS TO GET TO A NEIGHBORING POINT OF WHERE HE STARTED.

THE TRAVELER OBSERVES TWO MOMENTS ON HIS CLOCK SEPARATED BY A TIME $d\tau$. WE RELATE THIS TIME TO THE PROPER TIME ON EARTH, dt . SINCE THE GUY IS HIGHER, HE LOOKS PURPLE BECAUSE OF GRAVITY SO,

$$dt = d\tau \left(1 + \frac{\Phi}{c^2}\right)$$

ALSO BECAUSE HE IS MOVING AT A GOOD CLIP, THE TIME DILATION ARISING FROM OUR APPROXIMATION IS

$$dt = d\tau + \frac{v^2}{2c^2} d\tau$$

SO THE PROPER TIME ON EARTH FOR THE INTERVAL IS

$$dt = d\tau + \frac{\Phi}{c^2} - \frac{v^2}{2c^2} d\tau$$

INTEGRATE d OVER $0 \leq t \leq T$

Our problem again is to get the traveling clock to be as late as possible. The time interval seen on the wrist watch is then

$$\begin{aligned} t_2 - t_1 &= \int_0^T \left[1 + \frac{\phi}{c^2} - \frac{v^2}{2c^2} \right] dt \\ &= T + \frac{1}{c^2} \int_0^T \left[\phi(x(t)) - \frac{\dot{x}^2(t)}{2} \right] dt \end{aligned}$$

so what we want is to make the bracket term an extremum since $\phi = g(x(t))$ we have

$$\Delta t = T + \int_0^T \left[g(x(t)) - \frac{1}{2} \dot{x}^2(t) \right] dt$$

We can solve this problem by the principle of least action. If we let $\bar{x}(t)$ be the orbit for which the path is extremum and $\eta(t)$ be a variation on that path such that the variation at the end points is zero, then

$$x(t) = \bar{x}(t) + \eta(t)$$

is the varied orbit and for first order changes in η this is zero. Therefore, we have,

$$\int_0^T \left[\underbrace{g \bar{x}(t)}_{\text{ACTUAL PATH}} - \frac{1}{2} \dot{\bar{x}}^2 + \underbrace{g \eta(t) - \dot{\bar{x}} \dot{\eta}}_{\text{VARIATED PATH}} \right] dt$$

INTEGRATING BY PARTS

$$\int_0^T \left[g \eta(t) + \ddot{\bar{x}} \eta \right] dt - \dot{\bar{x}} \eta \Big|_0^T$$

FINALLY WE MUST REQUIRE,

" 0 AT END POINTS

$$\ddot{\bar{x}} = -g \quad \text{for the integral to be zero}$$

Thus we see the orbit is a parabola just as it would be for a free falling body. This orbit makes the time on the traveling clock the greatest.

The consequence of this little exercise was very important to Einstein who seized it and concluded that since local clocks always measure the longest time, clock will vary under different circumstances; i.e., there is no absolute time. The rate of time progression depends on where you are in space and whose clock you use.

If there is no definite time, then because of the space-time relationship there is most likely no set distance, i.e., it changes with position. That is to say a ruler would shrink as it comes toward the earth while it would expand going the other way. We must have a coordinate system established which is invariant to position only that the proper time remains a maximum. Thus Einstein sought to replace Newton's simple $F=ma$ law.

Thus it is that scale lengths change with gravitational field as well as the usual relativistic contraction. We can represent the usual space-time interval by the four-vector,

$$dt^2 = dt^2 - dx^2 - dy^2 - dz^2 \\ = dt^2(1 - V^2)$$

$$dt = d\tau(1 - V^2)^{1/2}$$

where we have omitted the c^2 since we chose it to be unity

Now in a gravitational field this interval of time would be

$$dt^2 = d\tau^2 \left(1 + \frac{\Phi(x, y, z)}{c^2}\right) - \alpha dx^2 - \beta dy^2 - \gamma dz^2$$

This relation is good to the first order because of the approximation used in the gravitational potential. We note that in the first equation the four vector explicitly related the two time intervals. Now we are saying there might be some undetermined coefficients which have not been found and vary from place to place in space. In order to determine these coefficients accurately it would be necessary to be moving at the or near the speed of light in order to make a significant contribution to dt .

Einstein proposed to specify the coefficients as a function of position in an ingenious manner by describing the potential of gravity. He assumed, in fact, that the coefficients differ with position. In the most general case we can write the time interval as,

$$dt^2 = d\tau^2(1 + \Phi(x, y, z)) - g_{xx}dx^2 - g_{yy}dy^2 + g_{zz}dz^2 - 2g_{xy}dxdy - \dots$$

where at most we can have a tensor with 10 terms.

If we let $G_{tt} = (1 + \Phi(x, y, z))$, then we can express this time as

$$dt^2 = \sum_{\mu\nu} g_{\mu\nu} dx_\mu dx_\nu$$

The tensor elements $g_{\mu\nu}$ depending on where you are. In Newtonian worlds $g_{\mu\nu} = \delta_{\mu\nu}$.

So we have a real mess on our hand with 10 potentials all coupled together. So we must try to put some order to them. First we argue(?) the guy's form a symmetric tensor; if it were anti-symmetric, it would n't be any different than a lot of other tensors.

We need some geometrical interpretation in order to understand these guy's. That is because they are function of space and increase in importance as the density of matter increases. Said another way, the guy would be much greater around the center of a galaxy than it would be far for any object.

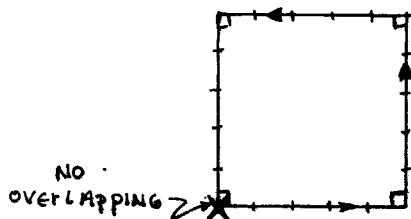
What Einstein argued was that as a consequence of $dt = \sqrt{g_{\mu\nu} dx^\mu dx^\nu}$ space-time could not be flat but rather represented as a curved space-time. In such a world matter would move on shortest-distance paths, i.e., dt is a minimum as $g_{\mu\nu}$ is a function of position. This is Einstein's motion law of gravity or something like that. This curved space is somewhat understood if we visualize a light beam shooting out and sooner or later it would return to its starting point. ~~where~~ we ignore the obvious question of how far around is it we are safe. The point being the trajectory follows a great circle route.

What then is the geometric interpretation of space. Let's limit our discussion, for the moment, to curved space only; we won't discuss time. Further let's assume a two dimensional and ask what meaning we can assign to curvature.

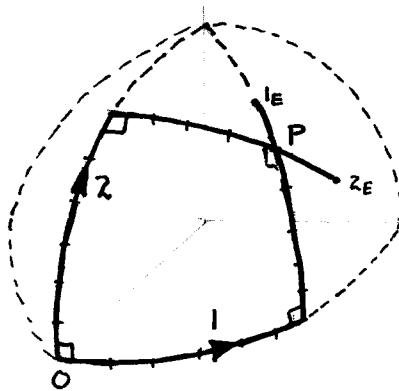
Imagine a gang of bugs live on the blackboard; call them "A". Suppose another group "B" live on a ball. We can have other 'worlds' like cylinders, saddles, donuts, etc. Assuming the bugs are a curious lot, they might like to know what the shape of their world is - is it flat, a ball, what? They cannot circum-navigate because they might be on a saddle; a technique must hold for all cases. There turns out to be several ways they could go about it.

METHOD 1. They could march out a given number of equal steps; mark off a right angle; go up the same distance; take another right; and see if finally they come back precisely to their origin.

For a flat surface we visualize immediately a square,



WHAT WOULD HAPPEN IF THE BUG TRIED THE SAME EXPERIMENT ON A BALL; WALKING EQUAL DISTANCES AND MAKING 3 RIGHT ANGLES, WHICH ARE DEFINED ON SPHERES.



IN THIS CASE THE BUG STARTS AT O; WALKS ON PATH 1 AND ENDS AT POINT Z_E. IF HE TOOK PATH 2 HE WOULD HAVE CROSSED PATH 1 AT P BEFORE ENDING AT Z_E. CLEARLY THE BUG MUST LIVE IN A CURVED WORLD.

METHOD 2 : BY LAYING OUT RESPECTIVE TRIANGLE ON THE SURFACE AND MEASURING THE SUM OF THE ANGLES. FOR A PLANE THEY TOTAL 180° ; FOR SPHERE THEY ARE MORE THAN 180° AND FOR A SADDLE LESS.

METHOD 3 : A MORE INTERESTING WAY CALLS UP A LARGE NUMBER OF BUGS TO MARCH OUT EQUIDISTANCES FROM A POINT AND JOINING HANDS TO FORM A CIRCLE. IF ANOTHER BUG THEN WALKED AROUND MEASURING THE PERIMETER, HE WOULD FIND THE CIRCUMFERENCE ALWAYS GIVEN IN INTEGRALS OF PI, I.E.,

$$\text{Circum} = 2\pi R$$

where R = NO. STEPS OUT, COMMONLY THE rADIUS

FOR A SPHERE WE HAVE SOMETHING DIFFERENT:

IN THIS CASE THE CIRCUMFERENCE

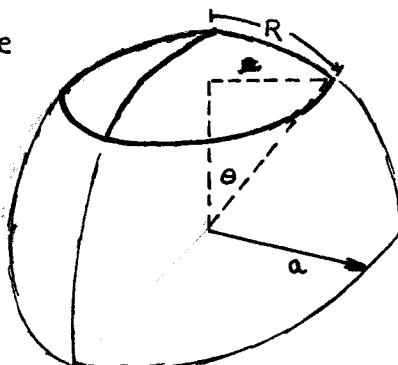
IS GIVEN BY $a = \text{radius of sphere}$

$$\begin{aligned} \text{Cir} &= 2\pi a \sin \theta \\ &= 2\pi a \sin \frac{R}{a} \end{aligned}$$

EXPANDING THE SINE INTO A SERIES TO FIRST 2 TERMS

$$\frac{\text{Cir}}{2\pi} = a \left(\frac{R}{a} - \frac{R^3}{3a^3} \right)$$

$$\frac{\text{Cir}}{2\pi} = R \left(1 - \frac{R^2}{3a^2} \right)$$



The result we have obtained is nice because it gives us an idea of how the curvature affects the circumference. We have the term $\frac{R^2}{a^2}$ which shows the effect of a small sphere, that being a larger correction factor. Feynman likes to picture curvature as a function of position as a dimpled face where in certain regions the bumps create a greater curvature.

Let's be more specific and define precisely what we mean by curvature; in our usage it will mean excess from the theoretically expected result. We can be more succinct mathematically

$$\frac{\text{ACTUAL CIRCUMFERENCE}}{2\pi R} = 1 - \text{Curvature} \times \text{AREA}$$

where we have taken $(1 - \frac{R^2}{3a^2})$ and rewrote it as $(1 - \frac{\pi R^2}{3a^2})$; specified πR^2 as the area and the curvature as $\frac{1}{3a^2}$. Significantly the curvature is inversely proportional to the radius squared

$$\text{Curv} \propto \frac{1}{a^2}$$

We need now some form of systematically identifying points in our space. In two dimensions the x-y axis work well. In three we can lay out a network of evenly spaced lines like the latitude and longitudinal lines on the earth. These grid lines need not meet at right angles, i.e.,

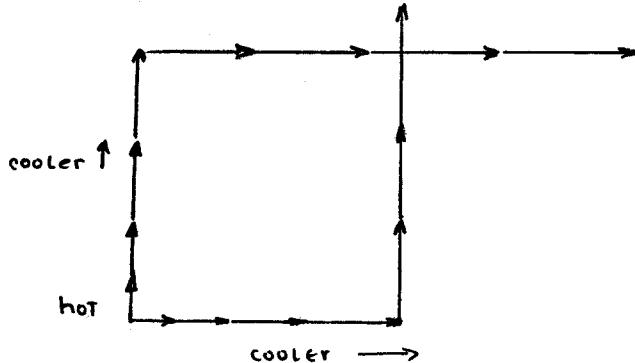
$$ds^2 = g_{xx} dx^2 + g_{yy} dy^2 + 2g_{xy} dx dy$$

In two dimensions we can pick the coordinates just right so

$$ds^2 = dx^2 + dy^2$$

If we had started with $x = x(u, v)$ and $y = y(u, v)$, then we would have had the general case in which of the 3 coefficients two would have been independent, i.e., corresponding to the two degrees of freedom permitted. With the 3 g 's then it is possible to determine the curvature

ANOTHER WAY OF THINKING OF THIS CURVED SPACE IS TO LET THE BUGS LIVE ON A HOT PLATE WHICH GETS HOTTER AS YOU MOVE NEARER THE CENTER. THUS RULERS WOULD EXPAND AS YOU GO OUT AND WE SHALL ASSUME THEY ALL EXPAND EQUALLY; EVEN THE BUGS THEMSELVES. SAY HE TAKES WHAT HE THINKS ARE 4 EQUAL STEPS LIKE HE DID ON THE BLACKBOARD AND SEE IF HE CAN END AT THE SAME POINT.



WE COULD DEVISE THE PERFECT TEMPERATURE LAW WHICH WOULD GIVE EXACTLY THE SAME EFFECT AS BEING ON A BALL AS THIS DIAGRAM SEEMS TO INDICATE.

THUS EVEN THOUGH WE ARE ON A FLAT PLATE THE EFFECT IS THE SAME AS BEING IN A CURVED SPACE; THE LENGTH OF THE RULER STILL VARIES WITH WHERE YOU ARE. SO SPACE IS CURVED FUNNY BECAUSE IT HAS A WEIRD EFFECT ON OUR MEASUREMENT.

WE WOULD EXPECT LENGTHS ON THE HOT PLATE TO TAKE THE FORM:

$$ds^2 = \text{Temp}(x,y) [dx^2 + dy^2]$$

WHERE $T(x,y)$ IS THE TEMPERATURE FUNCTION WHICH CORRESPONDS TO THE TWO g'S AND THE THIRD BEING EQUAL.

WHILE IN TWO DIMENSIONS THE CURVATURE COULD BE MEASURED BUT THE TWO CASES MENTIONED ABOVE COULD NOT BE SEPARATE. IN THREE DIMENSIONS THERE COULD BE 3 DIRECTIONS OF MAXIMUM CURVATURE WHICH WOULD DESCRIBE THE SHAPE. NOW WE TALK ABOUT THE ACTUAL RADIUS BEING

$$\text{ACTUAL RADIUS} = R + \frac{\text{MEASURED RADIUS}}{\text{AVERAGE CURVATURE}} \times \text{VOLUME}$$

FOR THE MOMENT ALL WE'LL SAY IS THAT THE AVERAGE CURVATURE, THE FEYNMAN MEAN CURVATURE, IS AN AVERAGE OF THE 3 MAXIMA. WE'LL SPEND MORE ON THIS LATTER.

NOW WE LET TIME JOIN SPACE AND TALK ABOUT FOUR DIMENSIONS. IMAGINE NOW YOU STAND IN ONE SPOT FOR AN HOUR; MOVE ONE FOOT; WAIT AN HOUR AND DO THE SAME EXPERIMENT AS BEFORE. WHAT WILL HAPPEN IS THAT THE TIMES WON'T BE THE SAME; THE PROPER TIMES ARE DIFFERENT. THIS MISMATCH IS THE SAME AS BEFORE.

THE PROBLEM NOW IS EVEN MORE COMPOUNDED BECAUSE THERE ARE FORTY CURVATURE COMPONENTS. EINSTEIN SOUGHT TO SOLVE TO QUESTIONS:

- 1) GIVEN THE G'S WHAT IS THE MOTION TO MAKE THE TIME AN EXTREMUM
- 2) WHAT DETERMINES THE POTENTIAL ϕ

HE SOUGHT AN ANALOGUE OF ACCELERATION = $-\nabla \phi$
AND FOR A GOOD FIT IN

$$\nabla^2 \phi = 4\pi G\rho$$

WHERE G = UNIVERSAL GRAVITATION CONSTANT
 ρ = DENSITY OF MATTER

HE DID NOT WANT TO HAVE THE G'S DEPEND ON ANY CHOICE OF COORDINATE SO HE HAD THE IDEA OF LETTING THEM EQUAL MATTER DENSITY. TO SPECIFY THE g's, WHICH REPRESENT THE CURVATURE, WE GO TO A BIG TENSOR COMPRISED OF VARIOUS DERIVATIVES OF g's PREVIOUSLY WORKED OUT BY RIEMANN.

$$R_{\mu\nu\rho} = \left(\frac{\partial^2 g}{\partial x^2} + \dots \left(\frac{\partial g}{\partial x} \right)^2 + \dots \frac{\partial g}{\partial x} \frac{\partial g}{\partial y} + \dots \right)$$

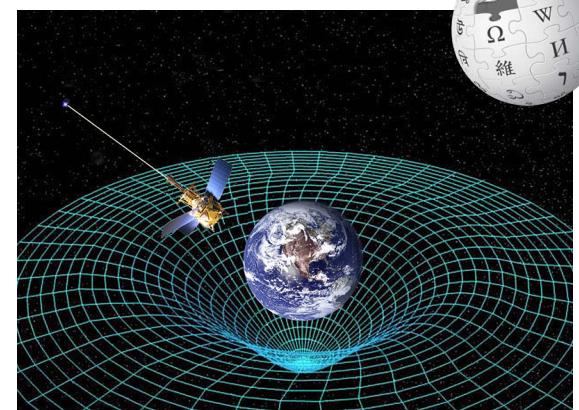
IN NEWTONIAN PHYSICS ENERGY DENSITY WAS INTERPRETED AS THE SOURCE OF GRAVITY. THE ENERGY DENSITY BEING PART OF THE FOUR TENSOR WHICH COMPLETELY DESCRIBES ELECTROMAGNETIC THEORY.

THE ANALOGUE OF THE AVERAGE CURVATURE BECAME, IN 4 DIMENSIONS, 10 MEAN CURVATURES, $T_{\mu\nu}$; THIS TAKES THE FORM OF A TENSOR OF SECOND ORDER DIFFERENTIALS. EINSTEIN GUessed THIS LAW

$$R_{\mu\nu} = 8\pi G T_{\mu\nu} = \text{EINSTEIN'S LAW OF GRAVITY}$$

HERE HE IGNORED ANY NONLINEARITIES IN $R_{\mu\nu\rho}$.

Gravity Probe B
Verifying "frame-dragging"



A WORD HERE NOT FROM THE LECTURE BUT FROM THE BOOK, GENERAL RELATIVITY AND A GRAVITATIONAL WAVES BY J. WEBER (INTERSCIENCE PUBLISHERS).

EINSTEIN'S IDEA THAT GENERAL LAWS OF NATURE SHOULD BE EXPRESSED BY EQUATIONS WHICH HOLD GOOD FOR ALL SYSTEMS OF COORDINATES FOLLOWED FROM A DISCOVERY OF MINKOWSKI. HE FOUND THAT TRANSFORMING FROM ONE INERTIAL FRAME TO ANOTHER MOVING WITH RELATIVE VELOCITY v CORRESPONDS TO THE ROTATION OF AXES IN A FOUR-DIMENSIONAL SPACE-TIME COORDINATE SYSTEM. THUS AN EVENT BECOMES A POINT IN SPACE-TIME, FOUR DIMENSIONAL SPACE.

THE SEPARATION BETWEEN TWO EVENTS OVER THE DIFFERENTIAL INTERVAL IS EXPRESSED AS WE ALREADY HAVE DONE,

$$ds^2 = dx^0^2 - dx^1^2 - dx^2^2 - dx^3^2$$

THAT IS

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

IF WE REWRITE THE FIRST EXPRESSION AS,

$$-ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

THEN THE ELEMENTS OF $g_{\mu\nu}$ ARE

$$g_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$g_{\mu\nu}$ IS CALLED THE METRIC TENSOR AND THE MATRIX RETAINS ITS FORM UNDER LORENTZ TRANSFORMATION.

AS AN EXAMPLE, THE CURVED TWO-DIMENSIONAL SPACE ON THE SURFACE OF A SPHERE IS DESCRIBED BY THE SQUARED LINE ELEMENT,

$$g_{\mu\nu} dx^\mu dx^\nu = r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2$$

$$g_{\mu\nu} = \begin{vmatrix} r^2 & 0 \\ 0 & r^2 \sin^2 \theta \end{vmatrix}$$

SINCE EINSTEIN ESTABLISHED THE EQUIVALENCE OF A GRAVITATIONAL FIELD TO AN ACCELERATED FRAME, HE NEEDED A CURVED-SPACE METRIC TO EXTEND RELATIVITY TO REGIONS WHERE GRAVITATIONAL FIELDS ARE PRESENT. THIS CURVATURE WAS REALIZED AS AN INTRINSIC PROPERTY, I.E., AT ANY POINT THE SAME VALUE IS OBTAINED IN EVERY COORDINATE SYSTEM. EINSTEIN'S THEORY OF GRAVITATION RELATES THE CURVATURE OF SPACE TO THE DISTRIBUTION OF STRESS AND ENERGY.

The proper time-space interval which we have been discussing, i.e.,

$$ds^2 = g_{\mu\nu} dx_\mu dx_\nu$$

has the property that it can always be rewritten as,

$$ds^2 = \delta_{\mu\nu} dx_\mu dx_\nu + h_{\mu\nu} dx_\mu dx_\nu$$

where $\delta_{\mu\nu}$ = KRONECKER DELTA

WHAT THIS EQUATION REPRESENTS IS WHAT WE DISCUSSED EARLIER ON PAGE 34. THERE WE TALKED ABOUT TIDAL FORCES, NON-UNIFORM MOTION AND CONCLUDED BY THE PROPER ACCELERATION WE COULD MAKE ONE POINT EXHIBIT UNIFORM MOTION BUT ALL THE SURROUNDING SPACE HAD A NEW POTENTIAL GRADIENT. WE HAVE SAID THE SAME THING ABOVE.

THE $\delta_{\mu\nu}$ CORRESPONDS TO OUR ACCELERATION AND THE $h_{\mu\nu}$ ARE THE NEW POTENTIAL GRAVITY SURROUNDING THE POINT. THUS EINSTEIN'S PRINCIPLE OF EQUIVALENCE REALLY MEANS THAT GRAVITY EQUALS SOME NEW GRAVITY PLUS AN ADDITIONAL ACCELERATION.

SCHWARZSCHILD FOUND A SOLUTION FOR ds^2 FOR THE CASE OF TIME INDEPENDENT $g_{\mu\nu}$ 'S AND IN SPHERICAL SPACE.

THE STARTING POINT IS THE SPHERICALLY SYMMETRIC SQUARED LINE ELEMENT IN OF FLAT SPACE-TIME:

$$ds^2 = c^2 f_0(r', \theta) dt^2 - f_r(r', \theta) dr'^2 + f_\theta(r', \theta) r'^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

BY CHOOSING A NEW COORDINATE r SUCH THAT $f_r(r', \theta) r'^2 = r^2$

$$ds^2 = c^2 f_0(r) dt^2 - f_r(r) dr^2 + -r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

IF WE SET $f_0 = e^\nu$, $f_r = e^\lambda$, THEN $g_{00} = -e^\nu$, $g_{11} = e^\lambda$ & $g_{22} = r^2$, $g_{33} = r^2 \sin^2 \theta$.

$$ds^2 = +e^\nu c^2 dt^2 - e^\lambda dr^2 + -r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

IT TURNS OUT (SEE WEBER REF. PP 56-60) TO SATISFY THE RESULTING DIFFERENTIAL EQUATIONS OF THE FORMS ON PAGE 45

$$e^{-\lambda} = e^\nu = 1 + \frac{K}{r}$$

WHERE K IS A CONSTANT WHICH IS DETERMINED FROM THE REQUIREMENT THAT NEWTON'S LAW OF GRAVITATION BE APPROACHED AT LARGE DISTANCES FROM THE MASS. TO ACCOMPLISH THIS

$$K = -\frac{2GM}{c^2 r}$$

Therefore,

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right) dt^2 - \frac{dr^2}{1 - \frac{2GM}{rc^2}} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

If the LINE ELEMENT between two points is given by the previous expression omitting all but,

$$ds = \frac{dr}{\sqrt{1 - \frac{2GM}{c^2 r}}}$$

Then we could find the RADIAL DISTANCE between two CIRCULAR ORBITS say r_1, r_2 AROUND THE SUN. WE WOULD SIMPLY HAVE

$$\text{RADIAL SEP.} = \int_{r_1}^{r_2} \frac{dr}{\sqrt{1 - \frac{2GM}{c^2 r}}}$$

This, however, is a horrible INTEGRAL TO EVALUATE SO TO THE FIRST ORDER,

$$\begin{aligned} \text{RAD. SEP.} &= \int_{r_1}^{r_2} \left(1 + \frac{2GM}{c^2 r} + \dots \right) dr \\ &= r_2 - r_1 + \frac{2GM}{c^2} \ln \frac{r_2}{r_1} \end{aligned}$$

Thus we see a CORRECTION appearing to the PREVIOUSLY KNOWN RESULT of just $r_2 - r_1$. The OTHER TERM is a CONSEQUENCE OF THE CURVATURE OF SPACE.

FOR THE SUN AS A CENTRAL MASS,

$$\begin{aligned} \frac{GM_{\text{SUN}}}{c^2} &= 6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg} \cdot \text{m}^2} \times 2 \times 10^{30} \text{kgm} \times \frac{1}{9 \times 10^{16} \frac{\text{m}^2}{\text{sec}^2}} \\ &\approx 1.5 \times 10^3 \text{m} \quad (\text{NT} = \frac{\text{kgm} \cdot \text{m}}{\text{sec}^2}) \end{aligned}$$

So the CORRECTIONS to RADII of the ORDER of THE EARTH'S SAY 10^9m , THE EFFECT of the CURVATURE IS SMALL.

The way in which we define space-time is confusing to some so let's go back over it again.

The world is characterized by events, i.e., flashes, etc. To locate such events it takes four numbers x, y, z, t which can be measured in any system. We assume that similar numbers ~~represents~~ represent events in succession or in close proximity. We then must be precise in establishing the distance between two events. The way we defined space-time is such that the elemental distance is given as,

$$ds^2 = g_{tt}(x, y, z, t) dt^2 + g_{xx}(x, y, z, t) dx^2 + g_{xt}(x, y, z, t) (dx dt)^2 + g_{xy}(x, y, z, t) (dx dy)^2 \dots$$

where the coefficients are functions of where you are.

Now we could have just as well chosen some other coordinate system, i.e.,

$$x' = f_1(x, y, z, t), \quad y' = f_2(x, y, z, t), \quad z' = f_3(x, y, z, t), \quad t' = f_4(x, y, z, t)$$

and subsequently found

$$ds^2 = g'_{\mu\nu} dx'^{\mu} dx'^{\nu}$$

as we did above

We observe then the labeling is purely arbitrary. But if we pick a particular system the $g_{\mu\nu}$'s might take on simple forms so we ^{then} decide to chose it. For instance, if there weren't any gravitational field, we would find a system for which

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

This is a flat space-time. This representation is not unique since our origin might move; there might be rotations; or we might make a Lorentz transformation which would preserve the form of the above, i.e.,

$$t' = \frac{t - vx}{\sqrt{1 - v^2}} \quad x' = \frac{x - vt}{\sqrt{1 - v^2}}$$

If we then have a world in which the $g_{\mu\nu}$'s are not all one, then there is just no best way to pick the coordinates, i.e., no special inertial frame and we are left loose as to our scale.

In this description of space-time t , the time, only orders events; it has no other significance. Time varies from place to place just like arc length does when you try to measure it. That is, to find ds we must know the angle $d\theta$ but more we must be explicit where you are,

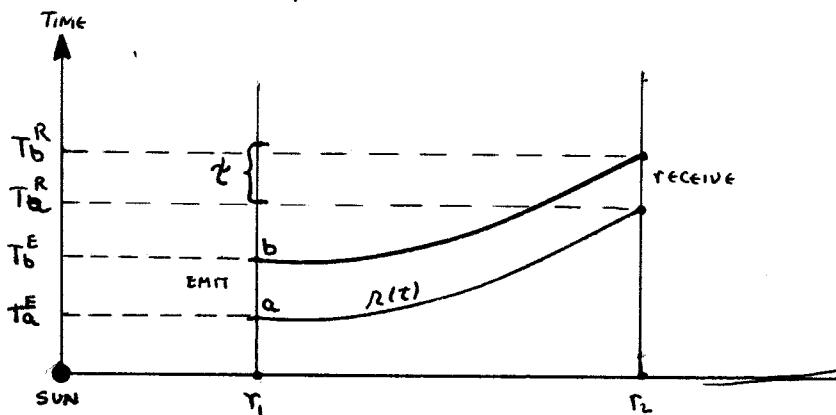
$$ds^2 = r^2 d\theta^2$$

so the arc seems longer at greater r for the same $d\theta$. Time is like this, in analogy.

LET'S EXAMINE THE FOLLOWING SITUATION:

SUPPOSE WE ARE A FIXED DISTANCE r_1 FROM THE SUN; TIME RUNS VERTICALLY. WE HAVE A STANDARD CLOCK, MAYBE CESIUM, WHICH HAS A TIME τ BETWEEN SUCCESSIVE CLICKS. WE HAVE TWO POINTS a AND b WHICH EMIT LIGHT TO A RECEIVER WHO IS ON ANOTHER TIME LINE AT ANOTHER DISTANCE r_2 SUCH THAT $r_2 > r_1$. WE WANT TO FIND THE TRAJECTORY OF LIGHT AS IT GOES FROM r_1 TO r_2 .

WE HAVE THE FOLLOWING PICTURE,



WITHOUT SOLVING FOR $R(t)$ WE CAN ESTABLISH A RELATIONSHIP BETWEEN THE INTERVAL EMITTED Δt^E AND THE INTERVAL RECEIVED Δt^R . WE REASON THAT THE TIME EMITTED AT a , T_a^E PLUS SOME CONSTANT TIME T EQUALS THE TIME RECEIVED T_a^R , I.E.

$$T_a^R = T_a^E + T$$

LIKewise at b ,

$$T_b^R = T_b^E + T$$

SO WE CAN FIND IMMEDIATELY

$$T_b^R - T_a^R = T_b^E - T_a^E$$

OR

$$\Delta t^R = \Delta t^E$$

THIS THEN RELATED THE DIFFERENCE IN THE TIME COORDINATES. HOW FAR APART ARE a AND b , AT EMISSION? HOW LONG DOES IT TAKE TO GO FROM T_a^E TO T_b^E . TO SOLVE THIS WE MUST USE OUR FORMULA FOR ds WHICH WE DEVELOPED LAST TIME, I.E.,

$$ds^2 = \left(1 - \frac{2m}{r}\right)dt^2 - \frac{dr^2}{\left(1 - \frac{2m}{r}\right)} - r^2(\sin^2\theta d\phi^2 + d\theta^2)$$

$$\text{where } m = \frac{GM}{c^2}$$

WHAT WE HAVE TO DO IS SOLVE ds^2 FOR ITS MINIMUM AND THAT WILL BE THE MOTION. THUS THE TIME ON THE TRAVELING CLOCK WOULD ALWAYS READ A MAXIMUM AS WE ARGUED PREVIOUSLY.

IF WE FIRST LOOK AT THE TIME DIFFERENCE ONLY, THAT IS, WE ARE AT A FIXED POINT IN SPACE SUCH THAT $dr = d\theta = d\phi = 0$, THEN THE TIME DIFFERENCE AND COORDINATE DIFFERENCE ARE GIVEN BY

$$ds = t^E = \left(1 - \frac{2m}{r_1}\right)^{1/2} \Delta t^E$$

WHERE t^E IS MEASURED BY THE CLOCK AT E.

SIMILARLY

$$t^R = \left(1 - \frac{2m}{r_2}\right)^{1/2} \Delta t^R$$

Thus

$$\frac{t^R}{t^E} = \left(\frac{1 - \frac{2m}{r_2}}{1 - \frac{2m}{r_1}} \right)^{1/2}$$

FOR SMALL M AND LARGE R WE CAN APPROXIMATE THIS BY,

$$\frac{t^R}{t^E} \approx 1 - \frac{m}{r_2} + \frac{m}{r_1}$$

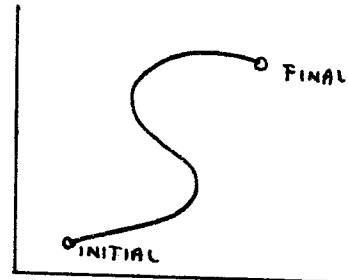
$$\approx 1 - \frac{GM}{c^2 r_2} + \frac{GM}{c^2 r_1} = 1 + \frac{\phi_R}{c^2} - \frac{\phi_E}{c^2}$$

THIS RESULT CORRESPONDS TO WHAT WE DISCOVERED EARLIER WHEN TALKING ABOUT THE GRAVITATIONAL RED SHIFT.

OUR TASK NOW IS TO FIND A WAY TO DESCRIBE THE MOTION OF THE LIGHT; THEN DISCUSS THE PARTICULAR MOTION WHICH MAKES THE TIME A MINIMUM. FIRST WE GIVE $r = r(t)$, $\theta = \theta(t)$, AND $\phi = \phi(t)$ IN TERMS OF OUR COORDINATES. THUS WE WANT TO EXAMINE THE LAW OF MOTION IN A GRAVITATIONAL FIELD.

IF THE REAL MOTION IS GIVEN BY $\bar{r}, \bar{\theta}, \bar{\phi}$ THEN WE MUST HAVE

$$\int_{t_i, r_i, \theta_i, \phi_i}^{t_f, r_f, \theta_f, \phi_f} ds = \text{EXTREMUM}$$



THIS IS EINSTEIN'S LAW OF MOTION IN GRAVITY.

WE CAN WRITE THIS OUT MORE EXPLICITLY AS

$$\int_{\text{INITIAL}}^{\text{FINAL}} \left[\left(1 - \frac{2m}{r(t)} \right) - \frac{\dot{r}^2}{1 - \frac{2m}{r(t)}} - r(t) \left(\sin^2 \theta \sin^2 \phi + \dot{\theta}^2 \right) \right]^{\frac{1}{2}} dt$$

THIS INTEGRAL IS VALID FOR OUR CASE OF FREELY FALLING BODIES.
FOR NON-RELATIVISTIC VELOCITIES AND WEAK GRAVITATIONAL FIELD
WE CAN APPROXIMATE THIS INTEGRAL AS

$$\int \left[1 - \frac{m}{r(t)} - \frac{\dot{r}^2}{2} - \frac{r^2}{2} (\sin^2 \theta \dot{\phi}^2 + \dot{\theta}^2) \right] dt$$

$$\text{if we let } v^2 = \dot{r}^2 + r^2 \dot{\theta}^2 + r^2 \sin^2 \theta \dot{\phi}^2$$

WE FINAL SEE THAT THE ORBIT MUST SATISFY

$$\int \left[\frac{GM}{r(t)} + \frac{v^2}{2} \right] dt$$

WHERE WE HAVE OBTAINED THE USUAL NEWTONIAN ORBIT PLUS A CORRECTION TERM. SINCE THE BRACKET TERM IN THE TOP EQUATION DOES NOT EXPLICITLY CONTAINED t IN ANY COEFFICIENT, IT IS POSSIBLE TO ADD A CONSTANT T WITHOUT CHANGNG THE RESULTS. THIS WE ALREADY DID.

IT IS CHARACTERISTIC OF THE MOTION OF LIGHT FOR $ds=0$. IF WE ONLY CONCERN OURSELVES WITH LIGHT TRAVELING RADIALLY, $ds=0$ IMPLIES

$$(1 - \frac{2m}{r(t)}) dt^2 - \frac{dr^2}{(1 - \frac{2m}{r(t)})} = 0$$

or

$$\frac{dr(t)}{dt} = (1 - \frac{2m}{r(t)})$$

Then

$$\frac{dr(t)}{(1 - \frac{2m}{r(t)})} = dt$$

For LARGE $r(t)$ 'S AND SMALL GRAVITATIONAL EFFECTS

$$\int_{t_1}^{t_2} dt \approx \int_{r_1}^{r_2} \left(1 + \frac{2m}{r(t)} \right) dr = r_2 - r_1 + 2m \ln \frac{r_2}{r_1}$$

$$\therefore \Delta t = t_2 - t_1 = (r_2 - r_1) + 2m \ln \frac{r_2}{r_1}$$

Thus our IDEAL CLOCK HAS MEASURED ds WHICH IS NOT WHAT WE EXPECTED BUT RATHER HAS A CORRECTION TERM. THIS RESULT IS GOOD ONLY FOR OUR PARTICULAR CHOICE OF THE COEFFICIENT $(1 - \frac{2m}{r})$. WE MUST REALIZE THERE WAS NOTHING UNIQUE ABOUT THIS CHOICE ONLY THAT IT SIMPLIFIED CALCULATION.

We could have picked some other τ to be $r' = f(r)$. But now our definition of τ as being the circumference divided by 2π is no longer valid. Further we could have redefined coefficients in ds^2 by making a substitution like

$$\tau' = \sqrt{1 - \frac{2m}{\lambda}} \tau$$

By this choice we would have made $g_{rr} = 1$ but complicated the other g 's sufficiently to decide NOT TO SELECT this. So now we must decide what we want because we're always sacrificing something.

Now let's see how to calculate the orbit as best we can. We will make use of the variational principle to do so, i.e., we will add a small correction to r and ϕ so that the first order change in ds is zero. We will vary r and ϕ only because we will pick the orbit to be in a plane such that $\theta = 90^\circ$. This is arbitrary but it simplifies our calculations.

We will let,

$$r = \bar{r} + \rho$$

$$\phi = \bar{\phi} + \epsilon$$

and

$$S = \int_i^f \sqrt{\left(1 - \frac{2m}{\lambda(r)}\right) - \frac{\dot{r}^2}{1 - \frac{2m}{\lambda(r)}} - \bar{r}(t) \left(\bar{r}\dot{\theta}^2 + \dot{\phi}^2\right)} dt$$

becomes

$$S = \int_i^f \sqrt{\left(1 - \frac{2m}{\bar{r} + \rho}\right) - \frac{\dot{r}^2 + 2\bar{r}\dot{\rho} + \dot{\rho}^2}{1 - \frac{2m}{\bar{r} + \rho}} - (\bar{r} + \rho)^2 \left(\dot{\phi}^2 + 2\dot{\phi}\ddot{\phi} + \ddot{\phi}^2\right)} dt$$

For simplicity we'll define as the old stuff, \dot{S} to be

$$\text{old stuff} = \sqrt{1 - \frac{2m}{\bar{r}}} - \frac{\dot{r}^2}{1 - \frac{2m}{\bar{r}}} - \bar{r}^2 \dot{\phi}^2 = \dot{S}$$

If we first vary ϕ since it looks easier, then $\rho = 0$ and we get

$$S = \int_i^f \sqrt{\left(1 - \frac{2m}{\bar{r}}\right) - \frac{\dot{r}^2}{1 - \frac{2m}{\bar{r}}} - (\bar{r}^2 \dot{\phi}^2) - 2\bar{r}^2 \dot{\phi} \ddot{\phi}} dt \quad \begin{matrix} \text{for first order} \\ \text{effects in the} \\ \text{variation} \end{matrix}$$

$$= \int \dot{S} dt - \int \frac{\bar{r}^2 \dot{\phi} \ddot{\phi}}{\sqrt{1 - \frac{2m}{\bar{r}}}} dt$$

INTEGRATING THE LATER BY PARTS, i.e.

$$\text{LET } \frac{dV}{dt} = \frac{d\epsilon}{dt} \quad u = \frac{\bar{r}^2 \dot{\phi}}{\sqrt{1 - \frac{2m}{\bar{r}}}}$$

$$V = \epsilon \quad \frac{du}{dt} = \frac{d}{dt} \left(\frac{\bar{r}^2 \dot{\phi}}{\sqrt{1 - \frac{2m}{\bar{r}}}} \right)$$

WE OBTAIN

$$\int \frac{d}{dt} \left(\frac{\pi^2 \dot{\phi}}{r} \right) dt$$

AND FOR A MINIMUM WE MUST REQUIRE

$$\frac{d}{dt} \left[\frac{\pi^2 \dot{\phi}}{r} \right] = 0$$

which yields

$$\frac{\pi^2 \dot{\phi}}{ds/dt} = h = \pi^2 \frac{d\phi}{ds}$$

THIS IS THEN THE CONSERVATION OF ANGULAR MOMENTUM AND GIVES ONE OF THE CONSTANTS OF THE MOTION.

NOW IF WE VARY r AS $\bar{r} + \rho$ THEN IF EXPAND FOR SMALL ρ

$$S = \int_i^f \sqrt{\left(1 - \frac{2m}{\bar{r} + \rho}\right) - \frac{\dot{r}^2 + 2\bar{r}\dot{\rho} + \dot{\rho}^2 - (\bar{r} + \rho)^2 \dot{\phi}^2}{1 - \frac{2m}{\bar{r} + \rho}}} dt$$

$$ds = \int \frac{\frac{m}{\bar{r}^2} \rho + \frac{\dot{r}^2 m}{(1 - \frac{2m}{\bar{r} + \rho})^2} - \pi \dot{\phi}^2 \rho}{\sqrt{1 - \frac{2m}{\bar{r} + \rho}}} dt - \int \frac{\dot{\rho}/(1 - \frac{2m}{\bar{r}})}{\sqrt{1 - \frac{2m}{\bar{r} + \rho}}} dt$$

INTEGRATING THE LAST TERM BY PARTS AND SETTING THE TWO INTEGRANDS EQUAL SO $ds = 0$ WE GET,

$$\frac{d}{dt} \left[\underbrace{\frac{(d\bar{r}/dt)/(1 - \frac{2m}{\bar{r}})}{ds/dt}}_{\text{II}} \right] = - \frac{\frac{m}{\bar{r}^2} - \frac{\dot{r}^2 m}{\bar{r}^2}}{\underbrace{\left(1 - \frac{2m}{\bar{r}}\right)^2}_{\text{I}}} - \pi \dot{\phi}^2$$

THEN

$$\frac{dr^2}{ds^2} = - \frac{m}{r(ds/dt)^2} + r \left(\frac{d\phi}{ds} \right)^2$$

$$\text{but } \frac{d\phi}{ds} = \frac{h}{r^2} \Rightarrow \left(\frac{d\phi}{ds} \right)^2 = \frac{h^2}{r^4}$$

FINALLY

$$\frac{dr^2}{ds^2} = - \frac{m}{r(1 - \frac{2m}{\bar{r}} - \frac{\dot{r}^2}{1 - \frac{2m}{\bar{r}}})} + \frac{h^2}{r^3}$$

TO UNDERSTAND BETTER HOW THE VARIATIONAL PRINCIPLE WORKS, THE MISSING STEPS IN THE PREVIOUS CALCULATIONS WILL BE PUT DOWN.

STARTING WITH

$$S = \int_i^f \sqrt{\left(1 - \frac{2m}{\pi}\right) + \frac{\dot{\pi}^2}{1 - \frac{2m}{\pi}} - \pi^2 (\sin^2 \theta \dot{\phi}^2 + \dot{\theta}^2)} dt$$

FOR A VARIATION IN ϕ WITH $\theta = 90^\circ$, I.E.,

$$\phi = \bar{\phi} + \epsilon$$

WHERE $\bar{\phi}$ IS THE ACTUAL PATH AND ϵ IS THE VARIED PATH

IF WE DEFINE $\dot{\bar{s}}$ TO BE

$$\sqrt{\frac{\dot{\bar{s}}^2}{S}} = \sqrt{1 - \frac{2m}{\pi} - \frac{\dot{\pi}^2}{1 - \frac{2m}{\pi}} - \pi^2 \dot{\bar{\phi}}^2} = \dot{\bar{s}}$$

THEN

$$\begin{aligned} \delta S &= \int_i^f \sqrt{1 - \frac{2m}{\pi} - \frac{\dot{\pi}^2}{1 - \frac{2m}{\pi}} - \pi^2 \dot{\bar{\phi}}^2 - 2\pi^2 \dot{\bar{\phi}} \dot{\epsilon}} dt \quad \text{TO THE FIRST ORDER} \\ &= \int_i^f \sqrt{\dot{\bar{s}}^2 - 2\pi^2 \dot{\bar{\phi}} \dot{\epsilon}} dt = \int_i^f \sqrt{\dot{\bar{s}}^2} \left(1 - 2 \frac{\pi^2 \dot{\bar{\phi}} \dot{\epsilon}}{\dot{\bar{s}}^2}\right)^{1/2} dt \end{aligned}$$

EXPANDING THE SQUARE IN TERMS OF $\dot{\epsilon}$ AND HOLDING THE FIRST ORDER TERMS

$$\begin{aligned} \delta S &= \int_i^f \sqrt{\dot{\bar{s}}^2} d\tau - \int_i^f \frac{\pi^2 \dot{\bar{\phi}} \dot{\epsilon}}{\dot{\bar{s}}} d\tau \\ &= \int_i^f \dot{\bar{s}} d\tau - \int_i^f \frac{\pi^2 \dot{\bar{\phi}} \dot{\epsilon}}{\dot{\bar{s}}} d\tau \end{aligned}$$

AND WE REQUIRE FIRST ORDER VARIATIONS TO VANISH SO IF WE INTEGRATE THE SECOND INTEGRAND BY PART WE GET THE CONDITION

$$\frac{d}{d\tau} \left[\frac{\pi^2 \dot{\bar{\phi}}}{\dot{\bar{s}}} \right] = 0$$

which YIELD A CONSTANT OF THE MOTION

$$\frac{\pi^2 \dot{\bar{\phi}}}{ds/d\tau} = h = \pi^2 \frac{d\phi}{ds}$$

THE INTEGRATION BY PARTS IS CARRIED OUT BY LETTING

$$dv = \dot{\epsilon} dt \quad \text{AND} \quad u = \dot{\bar{\phi}}$$

WHICH GIVES $v = \epsilon$ AND $du = \frac{d\dot{\bar{\phi}}}{dt} dt$

$$\frac{\pi^2}{\dot{\bar{s}}} \int \dot{\bar{\phi}} \dot{\epsilon} dt = \frac{\pi^2}{\dot{\bar{s}}} \dot{\bar{\phi}} \epsilon \Big|_i^f - \frac{\pi^2}{\dot{\bar{s}}} \int \frac{d\dot{\bar{\phi}}}{dt} \epsilon dt \quad \text{WHICH GIVE } \frac{d}{d\tau} \left[\frac{\pi^2 \dot{\bar{\phi}}}{\dot{\bar{s}}} \right] = 0$$

SINCE $\epsilon(i) = \epsilon(f) = 0$ AND THE VARIATION MUST BE ZERO.

CONSIDERING THE VARIATION IN \bar{r} , I.E.,

$\bar{r} = \bar{r} + \rho$
WE FOLLOW THE SAME DEVELOPMENT AS BEFORE ONLY NOW WE HAVE,

$$\delta S = \int_i^f \sqrt{\left(1 - \frac{2m}{\bar{r} + \rho}\right) - \frac{\dot{\bar{r}}^2 + 2\bar{r}\dot{\rho} + \dot{\rho}^2}{1 - \frac{2m}{\bar{r} + \rho}} - (\bar{r} + \rho)^2 \dot{\phi}^2} dt$$

HOLDING ONLY TO FIRST ORDER VARIATIONS IN ρ , WE REWRITE

$$1 - \frac{2m}{\bar{r} + \rho} = 1 - \frac{2m}{\bar{r}(1 + \frac{\rho}{\bar{r}})} = 1 - \frac{2m}{\bar{r}} \left(1 + \frac{\rho}{\bar{r}}\right)^{-1} = 1 - \frac{2m}{\bar{r}} \left(1 - \frac{\rho}{\bar{r}}\right) \doteq$$

$$= 1 - \frac{2m}{\bar{r}} + \frac{2m\rho}{\bar{r}^2}$$

FROM THIS APPROXIMATION WE ARE LED TO THE NEXT APPROXIMATION

$$\frac{1}{1 - \frac{2m}{\bar{r} + \rho}} \doteq \frac{1}{1 - \frac{2m}{\bar{r}} + \frac{2m\rho}{\bar{r}^2}} \doteq \frac{1}{\left(1 - \frac{2m}{\bar{r}}\right)\left[1 + \frac{2m\rho}{1 - \frac{2m}{\bar{r}}}\right]} \doteq \frac{1}{1 - \frac{2m}{\bar{r}}} \left[1 - \frac{\frac{2m\rho}{1 - \frac{2m}{\bar{r}}}}{\bar{r}^2}\right]$$

$$= \frac{1}{1 - \frac{2m}{\bar{r}}} - \frac{2m\rho}{(1 - \frac{2m}{\bar{r}})^2 \bar{r}^2}$$

ALSO WE LET $(\bar{r} + \rho)^2 \doteq \bar{r}^2 + 2\bar{r}\rho$

AND IF WE DISREGARD $\dot{\rho}^2$ WE FINALLY OBTAIN for the TERM UNDER THE SQUARE ROOT

$$1 - \frac{2m}{\bar{r}} + \frac{2m\rho}{\bar{r}^2} - \left[\frac{1}{1 - \frac{2m}{\bar{r}}} - \frac{2m\rho}{(1 - \frac{2m}{\bar{r}})^2 \bar{r}^2}\right] \left[\dot{\bar{r}}^2 + 2\bar{r}\dot{\rho}\right] - \bar{r}^2 \dot{\phi}^2 - 2\bar{r}\rho \dot{\phi}^2$$

REARRANGING, WE GET,

$$\dot{s}^2 + \frac{2m\rho}{\bar{r}^2} + \frac{2m\rho \dot{\bar{r}}^2}{(1 - \frac{2m}{\bar{r}})^2 \bar{r}^2} - 2\bar{r}\dot{\phi}^2 \rho - \frac{2\dot{\bar{r}}\dot{\rho}}{(1 - \frac{2m}{\bar{r}})}$$

PUTTING THIS BACK INTO THE SQUARE ROOT, FACTORING OUT \dot{s}^2 , AND EXPANDING TO FIRST ORDER VARIATIONS WE GET

$$\delta S = \int_i^f \dot{s} dt + \int_i^f \frac{\frac{m}{\bar{r}^2} \rho + \dot{\bar{r}}^2 \frac{m}{\bar{r}^2} \rho - \bar{r} \dot{\phi}^2 \rho}{\dot{s}} dt - \int_i^f \frac{\dot{\bar{r}} \dot{\rho}}{(1 - \frac{2m}{\bar{r}}) \dot{s}} dt$$

SINCE THE VARIATION MUST BE ZERO, THE INTEGRATION OF THE LAST TWO TERMS AFTER PARTS MUST BE SET EQUAL TO ZERO,

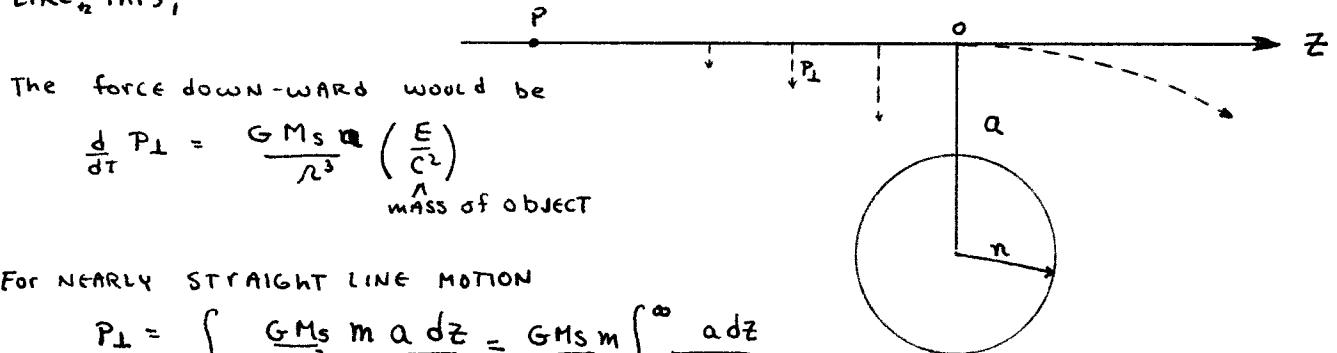
$$\frac{d}{dt} \left[\frac{\frac{d\bar{r}}{dt}}{(1 - \frac{2m}{\bar{r}}) \frac{ds}{dt}} \right] = -\frac{\frac{m}{\bar{r}^2} - \dot{\bar{r}}^2 \frac{m}{\bar{r}^2} (1 - \frac{2m}{\bar{r}})^{-2} - \bar{r} \dot{\phi}^2}{\frac{d\bar{s}}{dt}}$$

or

$$\frac{d\bar{r}}{ds^2} = -\frac{m}{\bar{r}(\frac{ds}{dt})^2} + \bar{r} \left(\frac{d\phi}{ds} \right)^2 = -\frac{m}{\bar{r}(\frac{ds}{dt})^2} + \frac{h^2}{\bar{r}^3}$$

The question arose how the classical bending of light was calculated and latter shown by Einstein to be twice what it should be.

If we consider an object of mass m , energy E and velocity v moving in a trajectory which will be considered essentially a straight line and such that its closest approach to the Sun is ' a ', then there will constantly be a downward component of force tending to bend it toward the Sun, say. So we have something that looks like this,



For NEARLY STRAIGHT LINE MOTION

$$P_{\perp} = \int \frac{GM_S m a}{v^3} \frac{dv}{dz} = \frac{GM_S m}{v} \int_{-\infty}^{\infty} \frac{adz}{\sqrt{a^2 + z^2}}$$

OR

$$P_{\perp} = \frac{2GM_S m}{v a}$$

NOW SINCE FOR SMALL ANGULAR deflections

$$\frac{P_{\perp}}{P_{\parallel}} = \theta$$

THEN IN GENERAL

$$\theta = \frac{2GM_S m}{v a P_{\parallel}}$$

while AT LOW VELOCITIES $P_{\parallel} = mv$ Then,

$$\theta = \frac{2GM_S}{a v^2}$$

Now if $v = c$, this calculation is off by a factor of two; where a now is the Sun's radius.

EINSTEIN showed the frequency of light at P was not the same at O as it moved through space which essentially is a medium of changing index of reflection arising from the curvature. He found

$$\theta = \frac{2MSG}{av^2} \left(1 + \frac{v^2}{c^2} \right)$$

Here is where Feynman starts in on "black stars(aka black holes) as he called them.

BEFORE GOING ON TO A NEW TOPIC THERE ARE A COUPLE OF PARADOxes THAT ARISE WHEN DISCUSSING GRAVITY. SO WEll SPEND A LITTLE TIME TALKING ABOUT THEM.

IN ALL CASES OF GRAVITATIONAL FIELDS WE TALKED IN TERMS OF THE ELEMENTAL LENGTH BETWEEN TWO EVENTS BEING

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \frac{dr^2}{\left(1 - \frac{2m}{r}\right)} - r^2 (\sin^2\theta \dot{\phi}^2 + \dot{\theta}^2)$$

SO FAR WE HAVE LIMITED OURSELVES TO GRAVITATION EFFECT OF THE FIRST ORDER, I.E., $1 - \frac{2m}{r}$ IS VERY NEARLY 1 FOR EVEN WHITE DWARFS WHERE AN OBSERVED GRAVITATIONAL RED SHIFT IS PERHAPS ONLY TEN PERCENT. SO WE DON'T HAVE ANY MYSTERY ABOUT OUR THEORY YET.

WHAT, THOUGH, HAPPENS WHEN THE RADIUS r IS OF THE ORDER $2m$ OR $\frac{2GM}{c^2}$? SPECIFICALLY WHEN THE RADIUS EQUALS $2m$. WHAT TROUBLES DOES THIS CAUSE IN THE APPARENT INFINITE TERM IN ds^2 ?

FIRST WE MUST RECALL THE ABOVE EXPRESSION IS GOOD OUTSIDE OF A STATIC SPHERICALLY SYMMETRIC MASS WHICH THUS HAS ONLY RADIAL MOTION OF THE MATTER INSIDE ITS BOUNDARY. SO WE ENCLOSE THE REGION OF MATTER JUST LIKE GAUSS' THEOREM DOES IN ELECTROMAGNETIC THEORY AND SAY OUTSIDE THERE IS NO MASS. HOWEVER, BY LOGICAL SEQUENCE WE REALIZE THAT ds^2 OF ABOVE IS NOT A SOLUTION INSIDE OUR BIG BALL.

TO OUR CURRENT UNDERSTANDING OF ASTRONOMY TO SAY WHAT HAPPENS WHEN $r < 2m$ SEEMS ONLY A RESULT OF ALGEBRAIC MANIPULATIONS SINCE NO STARS HAVE BEEN OBSERVED WITH RADII OF THIS ORDER. FOR INSTANCE, THIS RADIUS FOR THE SUN WOULD CORRESPOND TO SMASHING DOWN INTO A BALL 1.5 KILOMETERS IN RADIUS. SO WE ASK 'WHAT HAPPENS WHEN MATTER IS SQUEEZED LIKE THIS?'

IMAGINE WE TOOK A LARGE AMOUNT OF DUST, SAY COLD IRON FILINGS AND SPREAD THEM OUT OVER A BIG SPACE SUCH THAT THEIR MASS OF 10^8 SUNS WOULD BE SPHERICALLY SYMMETRICAL. FOR A BIG ENOUGH RADIUS WE HAVE NO TROUBLE; INITIALLY THEN WE ARE IN THE SAME POSITION AS WE HAVE ALREADY CONSIDERED, I.E., FIRST ORDER EFFECTS.

NOW IF THE MASS BEGINS TO FAIL TOGETHER UNDER THE PULL OF ITS OWN GRAVITY THERE MUST BE SOME CRITICAL DENSITY WHEN THE RADIUS EQUALS $2m = 2GM/c^2$ OR $m = GM/c^2$. SINCE R_{CRITICAL} IS RELATED TO P_{CRITICAL} , THE DENSITY BY

$$M = \frac{4}{3}\pi R_{\text{cr}}^3 \rho_{\text{cr}} \Rightarrow \rho_{\text{cr}} =$$

$$R_{\text{cr}} = \left(\frac{3}{4\pi} \frac{M}{\rho_{\text{cr}}}\right)^{1/3}$$

ALSO

$$R_{\text{CRITICAL}} = \frac{GM}{C^2}$$

Therefore EQUATING The two EQUATIONS

$$\left(\frac{3}{4\pi} \frac{M}{\rho_{\text{crit}}} \right)^{1/3} = \frac{GM}{C^2}$$

$$\rho_{\text{CRITICAL}} = \frac{3C^6}{4\pi G^3 M^2}$$

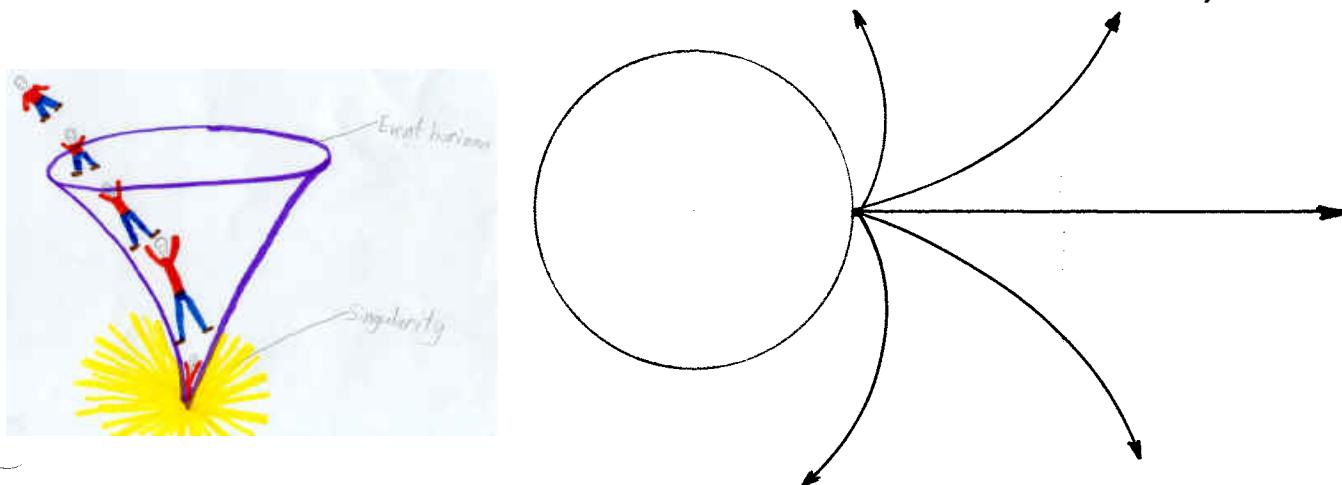
FOR THE SUN

$$\begin{aligned}\rho_{\text{CRITICAL}} &= \frac{3}{4\pi} \frac{(3 \times 10^8 \text{ m/sec})^6}{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{Kg} \cdot \text{m}^2} \right)^3} \times \frac{1}{(2 \times 10^{30} \text{ Kg} \cdot \text{m})^2} \\ &= \frac{3}{4\pi} \times \frac{729}{300 \times 4} \times \frac{10^{48}}{10^{-33} \times 10^{60}} \frac{\text{m}^6}{\text{Sec}^6} \times \frac{1}{\left(\frac{\text{Kg} \cdot \text{m} \cdot \text{m}^2}{\text{Sec}^2 \cdot \text{Kg} \cdot \text{m}^2} \right)^3} \text{ Kg} \cdot \text{m}^2 \\ &\approx 1.5 \times 10^{20} \frac{\text{Kg} \cdot \text{m}}{\text{m}^3}\end{aligned}$$

BUT WE WON'T WORRY ABOUT SUCH DENSITIES BUT RATHER PICK A NICE REPRESENTATIVE DENSITY OF OUR IRON FILLING UNIVERSE TO BE MAYBE 1 GRAM/CC. WITH THIS WE CAN THEN FIND THE MASS M OF THE BALL. BUT IF WE LET THIS BALL START TO FALL IN ON ITSELF, IT WILL GENERATE HEAT, AND THUS RADIATE ENERGY. THIS RADIATED MASS, HOWEVER, DOES NOT SOLVE OUR APPARENT INFINITY. THIS POSSIBILITY ~~DOES~~ MIGHT SUGGEST SOME UNKNOWN FORCE FOR IS SUBSEQUENTLY GENERATED TO BUCK THE GRAVITATIONAL COLLAPSE.

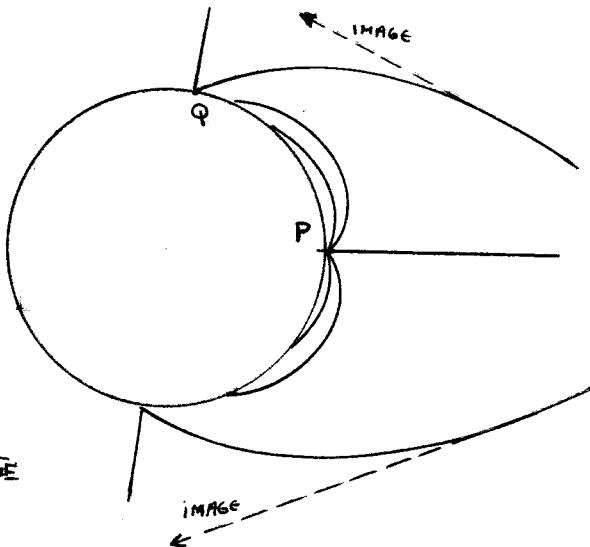
IF WE RODE ONE OF THESE IRON FILINGS IN, WE COULDN'T DISTINGUISH WHEN WE PASSED THE CRITICAL RADIUS. CERTAIN TIDAL FORCES WOULD BE EXERTED ON US BUT WE COULD LOCALLY REMOVE THESE AS WE HAVE DISCUSSED. BUT WITH THE VOLUME WE ARE DEALING ~~WITH~~ THESE ARE HIGHER ORDER EFFECTS SO WE ARE SAFE TO SAY NO APPARENT JOLT OR RUMBLINGS ARE FELT AT THAT CRITICAL POINT - EVERYTHING KEEPS RIGHT ON GOING DOWN.

AS THE BALL SHRINKS IT WILL EMIT LIGHT FROM ITS SURFACE AND TRAVELS RADIALLY BUT IT GETS SLOWED DOWN AND BENT AROUND. THUS THE LIGHT LOOKS REDDER AND REDDER. WE VISUALIZE THE RAYS LOOKING LIKE,



WE WOULD THEN EXPECT THAT EVENTUALLY LIGHT WOULD BE BENT BACK TO THE SURFACE IN SUCH A WAY THAT AS WE LOOK AT THE BALL IT WOULD APPEAR TO BE GETTING BIGGER AND BIGGER WHILE IT GOT REDDER AND REDDER. A SKETCH MIGHT MAKE THIS CLEARER,

AT SOME PROPER TIME TO THE MAN ON THE IRON FILING HE WILL CEASE TO EMIT LIGHT BECAUSE IT WILL ALL FALL BACK TO THE SURFACE. SO THE OBSERVER AT O SAYS HE DISAPPEARS. BUT FROM THE INSIDE HE HAS A FINITE TIME AT WHICH HE STOPS EMITTING. MORE PROPER THAN SAYING THE MASS DISAPPEARS AS O LOOKS OUT THERE IS AN EXPONENTIAL COLLAPSE OVER AN ~~FINITE~~ INFINITE TIME PERIOD.



IMAGINE OUR FRIEND DANCING ON THE IRON FILING. TO O THE GUY DANCES SLOWER AND SLOWER UNTIL EACH MOTION TAKES ALMOST AN INFINITE TIME TO EXECUTE. FINALLY WHEN LIGHT INVERTS ITSELF; IT LOOKS TO O LIKE THE GUY WAS FROZEN IN AN ENDLESS MOVEMENT. Thus ONLY TO THE MAN AT Q & P IS THERE ANYTHING UNIQUE ABOUT TIME, SPECIFICALLY WHEN HE PASSES INTO THE REGION OF $R < m$. SINCE LIGHT FROM THE PAST MAY STILL REACH HIM AT P; HE IS UNFORTUNATELY TRAPPED TO HIS DOOM SINCE HE CAN'T SIGNAL FOR HELP. Now our iron filing friend MUST FACE THE PROBLEM OF R APPROACHING ZERO AT WHICH POINT THE TIDAL FORCES WOULD BECOME INFINITE AND HE WOULD BE SMASHED TO DEATH, SUPPOSING HE GOT THAT FALL IN THE FIRST PLACE. This, indeed, is a problem which we must somehow reason ourselves out of.

THAT'S REALLY NOT TOO HARD TO SOLVE; IF WE MAKE A TRANSFORMATION WHICH REMOVES THE SINGULARITY AT $R=0$. BUT WE'RE JUST GOING TO GIVE OURSELVES HEADACHES SOMEWHERE ELSE SO THAT'S NOT A GOOD IDEA. SOME DREAMERS HAVE CONCEIVED A SQUASHED BALL SO THAT THE ENERGY ARISING FROM THE GRAVITATIONAL CONTRACTION WOULD BE SPIT OUT AT THE EQUATOR IN SOME CENTRIFUGAL FORCE MANNER. THAT REQUIRES A LOT OF ENERGY FOR A SQUIRT TO SAVE THE DAY AND SO THIS ISN'T TOO MUCH OF A THEORY.

There IS ANOTHER THEORY UNIQUELY CALLED THE ALRIGHT-JACK THEORY. This STATES THAT SINCE THE OBSERVER O CAN'T PREDICT WHAT HAPPENS BEYOND A CERTAIN POINT why worry about the OTHER GUY'S PROBLEM AS LONG AS I'M 'ALRIGHT-JACK!' Well, THIS ARGUMENT SORT OF BEATS AROUND THE BUSH BY NOT ANSWERING ANYTHING.

THE QUESTION OF WHETHER THERE IS A PROPER TIME INSIDE THE SPHERE IS A TRICKY ONE AND DOES NEED SOME THOUGHT.

IT would be possible to, perhaps, verify the existence of one of these 'BLACK STARS' if it were a companion to another star which is visible and which together makes up a binary system. If a black star were effecting the rotation of another visible star, we might find it by observing the perturbed orbit and calculating the mass needed to account for the errors. See figure below

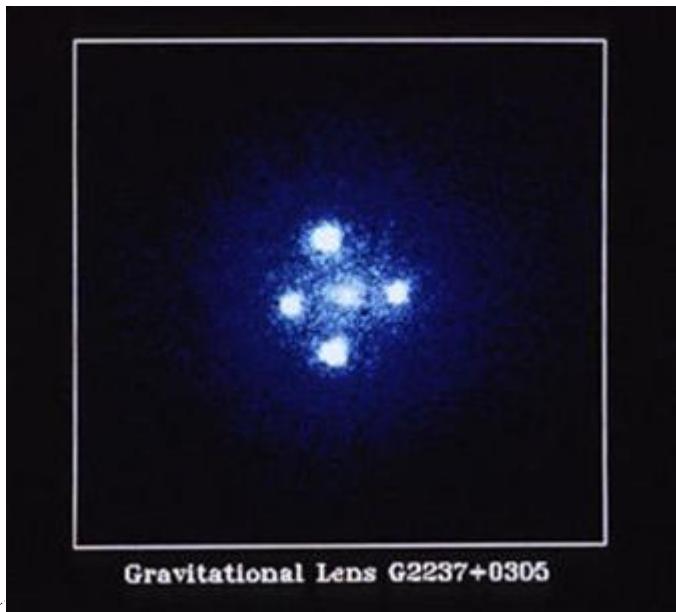
Of course an easier way would be to use the gravitation lens effect that Einstein described. But to find another light source behind the dark star so its light passed close by the unseen mass enough to be bent is highly improbable but not impossible.

Note: Lensing has been observed by Hubble-see below

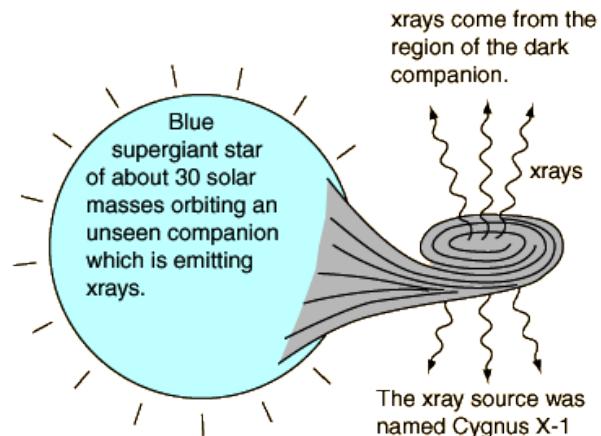
SOME BRIGHT THINKING ASTRONOMERS HAVE CLAIMED THAT IF A BLACK STAR WAS A MASS GREATER THAN SOMETHING LIKE 1.44 SUN MASSES, THEN UNDER NORMAL PRESSURES NOTHING WOULD PREVENT THE COLLAPSE OF THE STAR. THESE THEORIST CLAIM IS AN ATTEMPT TO GET DOWN TO THE 1.44 EQUILIBRIUM CONDITION. BUT AGAIN THIS ARGUMENT IS POOR BECAUSE IT SAYS THE EXPLOSIONS ARE CAUSED BY THE MASS BEING GREATER THAN 1.44. IN OTHER WORDS GOD IS CAUSING THE EXPLOSION ~~SO~~ SO WE WON'T BE WORRIED.

WELL, THAT'S ALL I WANT TO SAY ABOUT GRAVITY AND COSMOLOGY WHICH IS MORE THAN WHAT I INTEND BUT - TO GO BACK TO THE OUTLINE, GALAXIES ARE TO BE DISCUSSED NEXT. SINCE, HOWEVER, VERY LITTLE IS UNDERSTOOD ABOUT GALACTIC THEORY WE WILL PUT THIS TOPIC ASIDE AND GO TO THE DISCUSSION OF STARS AND TALK ABOUT THE THEORY OF THE INTERNAL CONSTITUTION OF STARS.

Hubble image



Gravitational Lens G2237+0305



Since black holes by their very definition cannot be directly observed, proving their existence is difficult. The strongest evidence for black holes comes from binary systems in which a visible star can be shown to be orbiting a massive but unseen companion. The indirect evidence for the black hole Cygnus X-1 is a good example of the search for black holes.

CHAPTER 4

INTERIOR STRUCTURE OF STARS

THERE ARE ALL SORTS OF STARS: PULSATING, SUPER NOVAE, MAIN SEQUENCE, RED GIANTS, WHITE DWARFS, ETC. AS IT TURNS OUT WE KNOW MORE ABOUT WHAT'S INSIDE THESE THINGS THAN WE DO ABOUT THE STRUCTURE OF THE EARTH AND THE MOON.

SO HOW DO WE HOPE TO STUDY ABOUT THE INSIDE OF THE SUN WHEN ALL WE SEE IS ITS SURFACE. WE CAN START BY USING THE THEORY OF MATTER CONTRACTING, GENERATING HEAT, AND RADIATING ENERGY SIMILARLY AS WE JUST DID. SINCE ABOUT 1860 THE ANALYSIS HAS BEEN CARRIED OUT WITH A GAS BALL ANALOGY. THAT IS, TO STUDY THE STABLE CONFIGURATION WHICH PREVAILS BECAUSE OF SOME BEAUTIFUL TEMPERATURE DISTRIBUTION INSIDE IT. SO WE WILL STUDY THE SUN BY CONSIDERING AT EVERY DEPTH THE MATTER BEHAVES LIKE A PERFECT GAS. WE THEN MUST HAVE A RELATIONSHIP BETWEEN PRESSURE, DENSITY AND TEMPERATURE. SUCH AN EQUATION EXISTS IN THE FORM,

$$\frac{P_{\text{gas}}}{\rho} = \mu RT$$

WHERE μ IS THE MOLECULAR WEIGHT, T IS TEMPERATURE, ρ IS THE DENSITY, AND P THE PRESSURE DUE TO GASES.

IF WE CONSIDER THE SUN APPROXIMATELY SPHERICALLY SYMMETRICAL THEN WE MUST HAVE

$$\frac{dP}{dr} = \frac{G \rho(r) M(r)}{r^2} \quad \text{where } P = P_{\text{gas}} + P_{\text{radiation}}$$

SO THE FORCES BALANCE OUT ON TWO LAYERS AND WHERE $M(r)$ IS THE MASS INSIDE OF r AND IS JUST THE NEWTONIAN GRAVITY EFFECT. SO WE DON'T BOTHER WITH ANY RELATIVISTIC MASSES.

ALSO WE KNOW,

$$\frac{dM}{dr} = 4\pi r^2 \rho$$

IF WE COULD CONNECT P AND ρ , WE WOULD BE DONE BUT WE HAVE THE TEMPERATURE DEPENDENCY COMING IN WHICH IS ALSO HEIGHT DEPENDENT.

WE CAN FIND THE ENERGY GENERATION DUE TO RADIATION IN TERMS OF THE LUMINOUS FLUX FLOWING OUT OF r , I.E., LET

$$L(r) = \text{ENERGY LEAVING AT } r$$

WHERE

$$L(R) = L_0 = \text{TOTAL LUMINOSITY OF THE STAR}$$

WE CAN NEXT CALCULATE,

$$\frac{dL(r)}{dr} = 4\pi r^2 \epsilon(r) \rho$$

where $\epsilon(r)$ THE RATE OF ENERGY GENERATION PER UNIT MASS AT RADIUS r .
GENERALLY $\epsilon(r)$ IS A FUNCTION OF DENSITY, TEMPERATURE AND COMPOSITION,
 $\epsilon = \epsilon(\rho, T, \mu)$

THIS ENERGY GENERATION IS FROM THE THEORY OF NUCLEAR ENERGY GENERATION.
WE WILL CONSIDER THE CHEMICAL COMPOSITION TO BE CONSTANT AND NOT VARY
WITH THE RADIUS FROM THE CENTER.

There MUST ALSO BE A TEMPERATURE GRADIENT RESULTING FROM THE HEAT
FLOW SO WE RELATE THE TEMPERATURE TO THE LUMINOUS FLUX BY,

$$\frac{dP_{rad}}{dr} = \frac{K \rho L(r)}{4\pi r^2 c}$$

where $P_{rad} = \frac{1}{3} \alpha T^4$ AND IS THE PRESSURE DUE TO RADIATION

AND K IS THE OPACITY AND MEASURES HOW HARD IT IS FOR THE HEAT
TO WORK ITS WAY OUT OF THE BALL. GENERAL,

$$K = K(T, \rho, \mu)$$

THE INTRODUCTION OF THIS TERM NECESSITATES ANOTHER THEORY OF RADIATED HEAT TRANSFER.

NOW WE ASK IF OUR FORMULAE ARE SUFFICIENT TO SOLVE THE EQUATIONS AND
THEN COMPLETELY DESCRIBE THE BEHAVIOR ON THE GUTS OF THE STAR. SO WE
WANT TO KNOW HOW THE HEAT IS TRANSFERRED. WHENEVER A TEMPERATURE
GRADIENT EXISTS IN A BODY THERE IS A NET TRANSFER OF HEAT OR THERMAL
ENERGY FROM THE WARMER TO THE COLDER REGIONS AS THE BODY SEEKS EQUILIBRIUM.
WE HAVE JUST DESCRIBED HEAT TRANSFER BY CONDUCTION.

ALSO, THERE IS A TRANSFER OF HEAT THROUGH THE ACTION OF MOVING
FLUID OR GASEOUS MATERIAL. THIS IS CONVECTION AND FREE OR NATURAL
CONVECTION IS WHERE THE MOTION IS PRINCIPALLY DUE TO GRAVITY ACTING
ON THE DENSITY DIFFERENCES DUE TO FLUID EXPANSION.

WHILE CONDUCTION IS NOT SIGNIFICANT IN STELLAR PROCESSES, CONVECTION
MOST CERTAINLY IS. ON THIS CONCLUSION WE MUST REALIZE WE HAVE NOT
CONSTRUCTED A SATISFACTORY THEORY OF CONVECTION TO ALLOW US TO
SOLVE THE EQUATIONS.

WE WILL HAVE TO TAKE A CLOSER LOOK AT THESE PROCESSES TO
EXAMINE THEIR NATURE AND INTERDEPENDENCIES. WE NEED TO KNOW
SUCH THINGS AS HOW THE OPACITY DEPENDS ON THE COMPOSITION;
HOW $\epsilon(r)$ VARIES WITH TEMPERATURE, ETC. WE'LL STAR THERE
NEXT TIME.

LAST TIME WE WERE DISCUSSING THE STUFF INSIDE THE STARS AND HOW IT GOT OUT. WE SAW THE PRESSURE GRADIENT WAS GIVEN AS

$$\frac{dP(r)}{dr} = -\frac{GM(r)}{r^2} \rho(r)$$

where

$$P = P_{\text{GAS}} + P_{\text{RADIATION}} = \text{TOTAL PRESSURE}$$

NOW WE CAN FIND THE P_{GAS} FROM THE GAS EQUATION

$$PV = NRT$$

where $V = \text{VOLUME}$ $R = \text{UNIVERSAL CONSTANT}$ $T = \text{TEMPERATURE}$

REALIZING THAT THE NUMBER OF MOLECULES $N = \frac{1}{\mu} \rho V$, THE PRESSURE IS GIVEN AS

$$P = \frac{RT\rho}{\mu}$$

where $\rho = \text{DENSITY} = 1/V$ AND $\mu = \text{MEAN MOLECULAR WEIGHT}$

NOW IF WE HAVE A MIXTURE OF TWO ELEMENTS, SAY, U_1 AND U_2 , THEN LET x AND $1-x$ BE THE RESPECTIVE FRACTION PRESENT IN 1MOLE. Thus

$$P_1 V = x RT$$

$$P_2 V = (1-x) RT$$

BY THE LAW OF PARTIAL PRESSURES. THEN THE NUMBER OF GRAMS IS

$$x U_1 + (1-x) U_2$$

SO THAT,

$$\rho = \frac{x U_1 + (1-x) U_2}{V}$$

SO OUR MEAN MOLECULAR WEIGHT, $\bar{\mu}$, IS DEFINED TO BE,

$$\bar{\mu} = x U_1 + (1-x) U_2$$

THE MEAN MOLECULAR STUFF IN THE STARS IS GROUPED INTO THREE PARTS, HYDROGEN, HELIUM, AND ALL THE OTHER ELEMENTS (O, Na, Mg, Si, K, Ca, Fe). WE CAN THEN ASSOCIATE A MASS FRACTION, LIKE WE DID ABOVE, TO EACH OF THESE PARTS SUCH THAT

$$x = \text{FRACTION OF HYDROGEN}$$

$$y = " " HELIUM$$

$$1-x-y = " " \text{ OTHER STUFF}$$

WE CHOOSE THIS SPECIFICATION BECAUSE THERE IS SO MUCH HYDROGEN AND HELIUM THAT THE OTHER PART CONSTITUTES REALLY A SMALL FRACTION OF THE STAR'S MASS. THIS FRACTION WILL BECOME ^{IMPORTANT} LATTER AS WE DISCUSS THE CREATION THROUGH FUSION OF THE HIGHER ELEMENTS.

SINCE 1 ELECTRON AND ONE NUCLEUS MAKE UP A HYDROGEN ATOM, WE SAY IT HAS A MEAN MOLECULAR WEIGHT OF 2. FOR HELIUM WHICH HAS 2 ELECTRONS, 1 NUCLEUS, AND 4 MASS UNITS, WE SAY IT HAS A $U = \frac{3}{4}$. THEN FOR THE HIGHER ELEMENTS THE MASS IS ROUGHLY TWICE THE CHARGE SO WE HAVE $U = \frac{1}{2}$. THE USUAL WAY TO DENOTE U IS BY

$$U = \frac{1}{2x + \frac{3}{4}y + \frac{1}{2}(1-x-y)} = \frac{2}{1+3x-\frac{1}{2}y}$$

THIS TOTAL MEAN MOLECULAR WEIGHT IS A GOOD APPROXIMATION WHILE IT IS SLIGHTLY TEMPERATURE DEPENDENT.

THE RADIATION PRESSURE, P_{RAD} IS GIVEN AS ARISING FROM BLACK BODY RADIATION WHICH, OF COURSE, IS NOT THE CASE

$$P_{\text{RAD}} = \frac{1}{3} \alpha T^4$$

WHERE α IS SOME CONSTANT

LAST TIME WE STATED THAT WITH THE TWO EQUATIONS OF

$$\frac{dP(r)}{dr} = -\frac{GM(r)}{r^2} \rho(r) \quad \text{AND} \quad \frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

IF WE HAD A RELATIONSHIP BETWEEN PRESSURE AND DENSITY, ^{WE} WOULD BE THROUGH. BUT WE NOW HAVE THE TEMPERATURE ENTERING INTO OUR DEFINITION OF ρ SO WE MUST KNOW THE METHOD OF THE HEAT FLOW. TO DO THIS WE SAW WE WERE LED TO TWO MORE EQUATIONS INVOLVING THE OPACITY AND ENERGY GENERATION, I.E.,

$$\frac{dP(r)}{dr} = -\frac{K(T, \rho) \rho L(r)}{4\pi r^2 c} \quad \frac{dL(r)}{dr} = 4\pi r^2 \epsilon(r) \rho(r)$$

WHERE THE TERM $\frac{dL(r)}{dr}$ ARISES FROM THE ENERGY PRODUCTION BETWEEN r AND $r+dr$.

WE CAN ESTABLISH SOME RATHER LOOSELY INTERPRETED BOUNDARY CONDITION AT THE CENTER AND AT THE SURFACE, I.E.,

$$\text{AT CENTER} \quad P=P_c \quad T=T_c \quad \rho=\rho_c \quad L=L_c=0$$

$$\text{AT SURFACE} \quad P(R_0)=0 \quad T(R_0)=T_0 \quad \rho=\rho(R_0)=0 \quad L(R_0)=L_0$$

WHERE L_0 = TOTAL AMOUNT OF ENERGY RADIATION FROM THE STAR AND TO SAY $\rho(R_0)=0$ DEPENDS ON HOW SERIOUS WE TAKE THE LAWS AS WE SHALL SEE BECAUSE IT IS HARD TO DEFINE THE SURFACE AND CONSEQUENTLY THE SURFACE FROM THE ATMOSPHERE

THE OPACITY IS DEFINED TO BE BIGGER FOR MORE OPAQUE MATERIAL WHICH IMPLIES A GREATER TEMPERATURE GRADIENT. THE ENERGY GENERATION $\epsilon(r)$ IS A RESULT OF NUCLEAR GENERATION.

The whole question of defining the surface leads to a great deal of confusion. The atmosphere is defined in terms of the mean free path of a photon and how far it makes it through the layers of gas.

The reason why we see spectral lines from the star arises from the fact that throughout the gaseous atmosphere there are isotherms, levels of constant temperature. When a light of certain wavelength penetrates these levels, excitation occurs when the temperature is just right to cause absorptions. So we actually see various layers in the atmosphere.

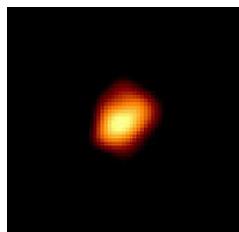
However, for the main sequence stars we don't worry too much about the exact position of the surface because the flux which we calculated to radiate from the star was integrated over the range of R . Since we obtain, say, 99% of L_0 a few hundred meters or so from the surface, the total contribution to the net flux out is so small that we ignore it. So we should really talk about some effective temperature of the surface which from the black body radiation says,

$$\frac{L_0}{4\pi R_0^2} = \sigma T_{\text{eff}}^4$$

This T_{eff} is what the astronomers measure and with a calculation of L_0 and the distance to the object as we determine earlier in the course we can find R_0 . So the effective temperature is just a measure of how much stuff comes out be unit area.

There are a class of stars which this approximation would ^{not} work for and that's the red giants. For these stars that have become helium burners and condensed to a hard dense core but in the process blew out a large amount of matter. Since its total radius R_0 might be a 1000 times that of the sun, its T_{eff} is lower because of the above relationship. At any rate we could still carry out the differentiation and find still perhaps 95% of the mass in the hard core but the extra 5% is so spread out that we really miss the radius in this case.

As an aside, the question of what happens after the helium is gone is an interesting one. To burn the higher element with higher ionization potentials they have to get hotter and hotter. What happens is when the helium is gone is that chaos sets in our better yet ignorance. That is, we don't know what happens. White dwarfs are so dense that they are white even though they are cooling off, i.e., the radius is so small say of the order of the earth and mass of the sun, that they put out a lot of energy per unit area. The answer of how the red giants turn into white dwarfs is a fairy tale will tell later.



NOW WHAT ABOUT CONVECTION? IF THE STAR HAS A HOTTER CENTER AND A COOLER SURFACE SO THAT THE TEMPERATURE FALLS AS THE MATERIAL MOVES OUT, DOESN'T THIS MEAN WE HAVE TO HAVE CONVECTION? THE SOLUTION OR ANSWER TO THIS IS NOT AS SIMPLE AS THE CASE OF HOT AIR RISING AND COOL AIR FALLING.

Suppose we have a box of MATTER AT LEVEL P_1 , i.e., CONSTANT pressure AND we carry it TO LEVEL P_2 . AS The pressure is reduced The MATTER EXPANDS. The IMPORTANCE QUESTION is how the TEMPERATURE IT HAS THERE COMPARE WITH THE MATTER THAT IS ALREADY THERE. IS IT COOLER or hotter THAN the NEIGHBORING STUFF AT P_2 ? If IT IS hotter, IT will float There AND if IT IS LESS DENSE IT WILL GO ON UP. SO THE QUESTION OF CONVECTION IS CONSIDERED BY EXAMINING THE LOCAL TEMPERATURES.

IF WE CONSIDER $P_{\text{AD}} = 0$, THEN THE pressure of THE GAS FOR ADIABATIC EXPANSION IS GIVEN by

$$P = \rho^\gamma$$

where $\gamma = 5/3$ for A MONATOMIC NON-RELATIVISTIC GAS

TO DETERMINE HOW MUCH THE TEMPERATURE WILL CHANGE WITH PRESSURE WE FIND

$$\frac{P}{\rho} = \rho^{\gamma-1} \approx T = P^{\frac{\gamma-1}{\gamma}}$$

CONSIDER AGAIN OUR TWO PRESSURE LEVELS,
THERE WILL BE A CRITICAL CONDITION AT WHICH CONVECTION WILL OCCUR AND THAT IS,

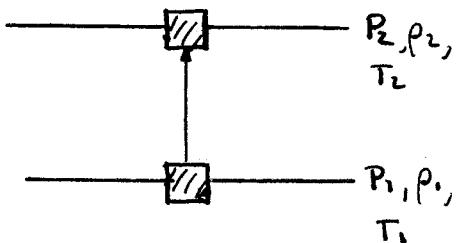
$$\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^\gamma = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma-1}{\gamma}}$$

DIFFERENTIALLY,

$$\frac{dP}{P} = \frac{\gamma-1}{\gamma} \frac{dT}{T}$$

OR

$$\frac{\frac{1}{P} \frac{dP}{dR}}{\frac{1}{T} \frac{dT}{dR}} \leq \frac{\gamma-1}{\gamma} \quad \text{AS THE CONVECTION CONDITION.}$$



WHEN THE LESS THAN INEQUALITY HOLDS THE CONDITION IS STABLE. THAT IS, IF T_2 IS HIGHER THAN T_1 THERE WON'T BE CONVECTION.

CONVECTION THEN OCCURS IN ALL THE MAIN SEQUENCE STARS. BUT EVEN A LITTLE CONVECTION MEANS THE HEAT DISTRIBUTION IS DRAMATICALLY CHANGED. WE WILL ASSUME FOR CONVECTIVE REGIONS WHERE THE FORCE NEEDED TO OVERCOME THE VISCOS DRAG OF THE MATERIAL IS NOT TOO GREAT, THE MACHINERY INVOLVED IN CARRYING THE HEAT IS NOT IMPORTANT.

THE STANDARD MODEL

AS WE MENTIONED BEFORE, IF WE HAD A RELATIONSHIP ^{DONE} BETWEEN THE PRESSURE AND DENSITY, OUR TASK WOULD BE ESSENTIALLY ^{DONE}. ALL NEED BE CALCULATED WOULD BE THE FIRST PAIR OF EQUATIONS ON PAGE 62.

WELL, WITH SOME GUESS WORK, I.E., USING SOMEONE ELSE'S SUPPOSITION LET,

$$P = A \rho^{(1+\frac{1}{n})} \quad \text{where } A \text{ IS SOME CONSTANT}$$

AND LET THIS DISTRIBUTION HOLD THROUGHOUT THE STAR. THIS WILL BE OUR MODEL WHICH WE CAN USE TO EXAMINE THE DIFFERENTIAL EQUATIONS. THIS EQUATION WILL HOLD ~~NOT~~ FOR THE CONVECTIVE REGIONS AND EVEN FOR WHITE DWARFS IT IS ALMOST A GOOD GUESS. THIS IS CALLED THE EDDINGTON MODEL TO LAUD PRAISE OR BLAME IF NEED BE.

SO TO BEGIN WITH FROM,

$$\frac{dP}{dr} = - \frac{GM(r)}{r^2} \rho(r)$$

WE FIND

$$M(r) = - \frac{r^2}{G \rho(r)} \frac{dP}{dr}$$

THEN FROM $\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$ WE GET

$$\frac{d}{dr} \left[- \frac{r^2}{G \rho(r)} \frac{dP}{dr} \right] = + 4\pi r^2 \rho(r) G$$

WHERE THE PRESSURE IS THE ABOVE EXPRESSION,

$$P = A \rho^{(1+\frac{1}{n})}$$

THE CONSTANT A BEING DEFINED AS THE VALUE OF $P/\rho^{(1+\frac{1}{n})}$ AT THE CENTER, I.E.,

$$A = \frac{P_c}{\rho_c^{(1+\frac{1}{n})}}$$

IF WE NOW ASSUME FOR SIMPLICITY, THE DENSITY VARIES AS

$$\rho = \rho_c \Theta^n \quad \text{where } \Theta = \Theta(r)$$

THEN

$$P = A \rho_c \Theta^{n+1}$$

AND WE OBTAIN

$$\frac{d}{dr} \left[\frac{r^2 A \rho_c \Theta^{n+1}}{\rho_c \Theta^n G} \frac{dP}{dr} \right] = - 4\pi r^2 G \rho_c \Theta^n$$

This can be simplified to,

$$\frac{P_c(n+1)}{4\pi P_c G} \frac{d}{dr} \left(r^2 \frac{d\theta}{dr} \right) = -\theta^n$$

If we introduce the new variables,

$$r = a\xi \quad a = \left[\frac{4\pi P_c^2 G}{P_c(n+1)} \right]^{1/2} = \left[\frac{P_c(n+1)}{4\pi P_c^2 G} \right]^{1/2}$$

where 'a' has the dimension of LENGTH

So we now have,

$$\frac{1}{\xi^2} \frac{d}{d\xi} \left(\xi^2 \frac{d\theta}{d\xi} \right) = -\theta^n$$

This equation is called the LANE-EMDEN EQUATION of index n . The equation governs the density distribution in any region where r and 'a' are given by the above values. The equation must hold throughout the mass and be subjected to the following boundary conditions,

$$\theta = 1 ; \quad \frac{d\theta}{d\xi} = 0 \quad \text{AT } \xi = 0$$

Once n , the index, is fixed the solution to the differential equation is obtained. The results hold for many stars where the standard model holds or is a good approximation.

The LANE-EMDEN function can be constructed from a series expansion near the origin, i.e., a series of the form,

$$\theta = 1 + a\xi^2 + b\xi^4 + \dots$$

This series must satisfy the above boundary condition; so there is no ξ term since $d\theta/d\xi = 0$ at the origin. Thus we only get even powers of ξ . Upon substitution into the L-E equation and equating coefficients,

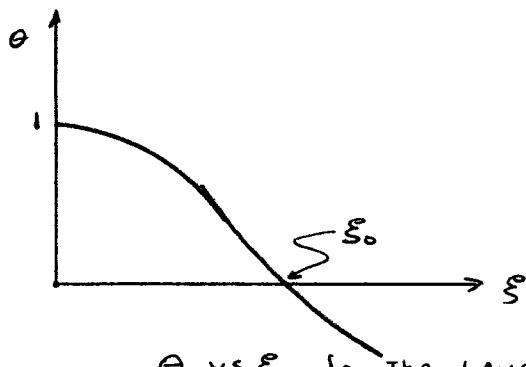
$$\theta = 1 - \frac{1}{6}\xi^2 + \frac{n}{120}\xi^4 + \dots$$

In all this discussion of the guts of the stars there are three quantities we are most interested in: the radius, mass, and luminosity. We can easily define the radius R to be simply

$$R = a\xi_0 = \left[\frac{P_c(n+1)}{4\pi P_c^2 G} \right]^{1/2} \xi_0$$

where ξ_0 defines the first zero of θ_n . And thus is associated with the radius of the star.

A representative graph of θ vs. ξ might be:



Θ vs ξ for the LANE-EMDEN FUNCTION

To find the mass $M(R)$ we use the differential equation for $M(r)$ expressed as a function of ξ , i.e., $M(\xi)$

$$\begin{aligned} M(\xi) &= \int_0^{\alpha \xi_0} 4\pi r^2 \rho(r) dr \\ &= \int_0^{\alpha \xi_0} 4\pi a^2 \xi^2 \rho_c \theta^n a d\xi \end{aligned}$$

SINCE

$$\theta^n = -\frac{1}{\xi^2} \frac{d}{d\xi} (\xi^2 \frac{d\theta}{d\xi})$$

$$M(\xi) = -4\pi a^3 \rho_c \int_0^{\alpha \xi} \frac{d}{d\xi} (\xi^2 \frac{d\theta}{d\xi}) d\xi$$

Then,

$$M(\xi_0) = -4\pi a^3 \rho_c \xi_0^2 \frac{d\theta}{d\xi_0}$$

SUBSTITUTING BACK OUR TOTAL EXPRESSION FOR a

$$M(\xi_0) = -4\pi \rho_c \left[\frac{P_c(n+1)}{4\pi \rho_c^2 G} \right]^{3/2} \xi_0^2 \frac{d\theta}{d\xi_0}$$

AND

$$M(\xi_0) = - \left[\frac{(4\pi)^2 P_c (n+1)}{G} \right]^{3/2} \frac{\xi_0^2}{\rho_c^2} \frac{d\theta}{d\xi_0}$$

WE NOW SEE THE MASS OF THE STAR IS ONLY DEPENDENT UPON THE CENTRAL PRESSURE P_c AND INDEPENDENT OF THE CRITICAL PRESSURE, ρ_c . ALSO WE MUST KNOW THE SLOPE OF Θ VS ξ_0 AT ξ_0 .

FOR A GIVEN n WE CAN DEFINE SOME AVERAGE DENSITY $\bar{\rho}$ OF THE STAR TO BE

$$\bar{\rho} = \frac{M(R)}{\frac{4\pi}{3} R^3}$$

OR

$$\bar{\rho}(\xi_0) = \frac{M(\xi_0)}{\frac{4\pi}{3} a^3 \xi_0^3}$$

$$\bar{\rho}(\xi_0) = -\frac{3}{\xi_0^2} \rho_c \left(\frac{d\theta}{d\xi_0} \right)$$

FROM THE LAST EXPRESSION ρ_c CAN BE EXPRESSED IN TERMS OF THIS MEAN DENSITY, I.E.,

$$\rho_c = - \frac{\xi_0^2 \overline{\rho_c}}{3 \left(\frac{d\theta}{d\xi_0} \right)}$$

DEVIATING A MOMENT FROM OUR MASS, RADIUS, LUMINOSITY STUDY, WE SHOULD NOTE WHAT A MESS WE'VE GOT. IT'S A MESS BECAUSE OF ALL THE INTERDEPENDENCIES. IF WE CHOOSE TO ELIMINATE ANY ONE VARIABLE, WHICH WE CAN DO, THERE IS STILL A DIFFICULT TASK TO UNTWINE THE DIFFERENTIAL EQUATIONS. WE HAVEN'T REALLY DEALT WITH THE OPACITY AND ENERGY GENERATION FACTORS WHICH FURTHER COMPLICATE MATTERS BECAUSE OF THEIR DEPENDENCIES ON TEMPERATURE, PRESSURE, DISTANCE, ETC. IF WE ELIMINATE, FOR INSTANCE ρ_c , WE THEN DON'T KNOW ρ_c .

LOOKING INTO THE CENTRAL PRESSURE A LITTLE DEEPER, WE RECALL FROM PAGE 61 THAT $P = RT\mu/\mu$. AT THE CENTER WE HAVE SOME CENTRAL TEMPERATURE T_c SO THAT IF WE CONSIDER MOST OF THE PRESSURE IN THE STAR ARISES FROM THE GAS AT THE CENTER, I.E., $P_{\text{RADIATION}}$ IS NEGIGIBLE. THE OUR VARIABLE 'a' BECOMES,

$$a = \left[\frac{RT_c(n\tau)}{4\pi G \mu \rho_c} \right]^{1/2}$$

SINCE ξ_0 IS JUST SOME NUMERICAL FACTOR RESULTING FROM THE L-E EQUATION FOR A GIVEN τ , WE CAN SUCK ALL THE CONSTANTS UP INTO THIS TERM AND FIND,

$$R \propto a \propto \left(\frac{T_c}{\mu \rho_c} \right)^{1/2}$$

SIMILARLY WE CAN OBTAIN,

$$M(R) \propto \frac{T_c^{3/2}}{\rho_c^{1/2} \mu^{3/2}}$$

ELIMINATING ρ_c FROM THE ABOVE TWO EXPRESSIONS,

$$\frac{M}{R} \propto \frac{T_c}{\mu}$$

THIS MASS-RADIUS RATIO IS USEFUL BECAUSE OBSERVATIONALLY IT CAN BE DETERMINED AND THUS GIVE THE CENTRAL TEMPERATURE OF THE SUN. EDDINGTON WAS, THIS WAY, ABLE TO PREDICT THE INTERIOR OF THE SUN HAD A T_c OF 20,000,000°.

FROM THIS RATIO WE SEE SOMETHING INTERESTING IF WE SIMPLY MULTIPLY BY G , THE UNIVERSAL GRAVITATION CONSTAANT,

THAT IS, WE GET,

$$\frac{GM}{R} \propto \frac{T_c}{\mu}$$

WHERE WE RECALL GM/R TO BE THE GRAVITATIONAL POTENTIAL OF THE STAR. IF WE DO THIS THEN WE CAN SAY THAT THE GRAVITATIONAL POTENTIAL ENERGY EQUALS THE THERMAL ENERGY. IF THE STAR IS IN EQUILIBRIUM, I.E., THE ENERGY GENERATION $E(R)$ IS ZERO AND THE OPACITY K IS INFINITE SO NOTHING LEAKS OUT WE CAN APPLY THE VIRIAL THEOREM OF THERMODYNAMICS. THIS THEOREM IS CONCERNED WITH THE AVERAGE KINETIC ENERGY OF THE PARTICLES IN A SYSTEM. WHEN THE GAS BALL IS IN EQUILIBRIUM AND THE PRESSURE IS DUE TO KINETIC ENERGY AND P_{rad} IS NEGIGIBLE, THERE IS ONLY A LOT OF MOLECULES ATTRACTING EACH OTHER AND HOLDING THEMSELVES TOGETHER. SINCE THE FORCES ARISING ARE ONLY FROM INTERACTING PAIRS OF PARTICLES, WE CAN WRITE THIS FORCE AS A GRADIENT OF SOME POTENTIAL ENERGY. WE HAVE THEN WHEN THE ATTRACTION ONLY DEPENDS ON THE SEPARATION DISTANCE \bar{r} ,

$$\overline{K.E} = \left\langle \sum_{i,j} \bar{r}_i \cdot \nabla_i (r_{ij}) \right\rangle_{ave.}$$

WHERE \bar{r}_i IS THE POSITION VECTOR OF THE i^{th} PARTICLE AND $\nabla_i r_{ij}$ THE FORCE ARISING ON THE i^{th} PARTICLE FROM ALL THE OTHER PARTICLES.

WHEN THE POTENTIAL IS OF THE $\frac{1}{R}$ FORM, THE GRADIENT OF THIS GIVES $-\frac{\bar{r}}{R^3}$ SO $\bar{r} \cdot -\frac{\bar{r}}{R^3}$ YIELD $-\frac{1}{R}$ WHICH IS AGAIN THE POTENTIAL ENERGY V .

WE NOW KNOW THAT

$$\overline{K.E} = -\langle V \rangle$$

THIS GIVES US SOME IDEA HOW MUCH WORK IT TAKES TO COMPRESS THE GAS COMPARED TO HOW MUCH IT TAKES TO DE COMPRESS IT.

RETURNING NOW TO THE MASS-RADIUS-LUMINOSITY DISCUSSION, WE WANT TO SET UP SOME PRELIMINARY EVALUATION OF THE LUMINOSITY. IT IS REALLY PRELIMINARY BECAUSE OF ALL THE UNCERTAINTIES INVOLVED IN THE EVALUATION. IF WE HAD THE TEMPERATURE DISTRIBUTION IN THE STAR AND KNEW THE OPACITY, THE LUMINOSITY COULD BE FOUND FROM

$$\frac{dP(R)}{dR} = -K \frac{\rho L(R)}{4\pi R^2 c}$$

RECALLING THAT THE OPACITY WAS A FUNCTION OF THE DENSITY, TEMPERATURE AND CHEMICAL COMPOSITION, EVERYTHING LOOKS IMPOSSIBLE. INDEED, IF IT WEREN'T FOR A MIRACLE THAT GAVE THE PRESSURE THE VALUE OF,

$$P = \rho^{1+\frac{1}{m}}$$

ALL WOULD BE LOST.

WITH THIS MODEL EDDINGTON WAS ABLE TO FIND, SOMEHOW, A RELATION FOR THE OPACITY,

$$\kappa = \frac{K_0 \rho}{T^{3.5}} g$$

WHERE THE g IS SOME FACTOR ABOUT EQUAL TO ONE THAT IS INSERTED BECAUSE THE FORMULA IS NOT RIGHT WITHOUT IT.

FOR RADIATIVE EQUILIBRIUM WE HAD THE EQUATION

$$\frac{dP_{\text{rad}}}{dr} = - \frac{L(r) K \rho}{4\pi r^2 c}$$

SINCE $P_{\text{rad}} = \frac{1}{3} \alpha T^4$ WE GET

$$\frac{1}{3} \alpha \frac{dT^4}{dr} = - \frac{K_0 \rho g^2}{T^{3.5}} \frac{L}{r^2}$$

FOR OUR PRELIMINARY EVALUATION WE ASSUME THAT L IS PROPORTIONAL TO THE TOTAL LUMINOSITY AND THE TEMPERATURE T^4 IS PROPORTIONAL TO THE TEMPERATURE AT THE CENTER T_c^4 . FROM THESE ASSUMPTIONS WE FIND

$$T_c^4 \sim \frac{K_0 \rho_c^2}{T_c^{3.5}} \frac{L}{R}$$

RECALLING FROM PAGE 68 THAT $R \sim (\frac{T_c}{M \rho_c})^{1/2}$ OR $\rho_c \sim \frac{T_c}{M R^2}$ THEN,

$$T_c^4 \sim \frac{K_0 T_c^2 L}{M^2 R^5 T_c^{3.5}} \sim \frac{K_0}{M^2} \frac{L}{T_c^{1.5}} \frac{1}{R^5}$$

OR

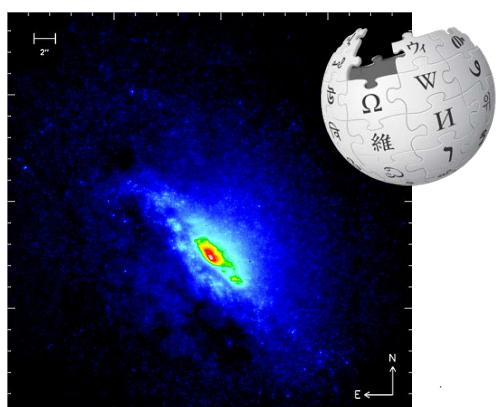
$$L \sim \frac{M^2}{K_0} T_c^{5.5} R^5$$

WE CAN NOW ELIMINATE T_c BECAUSE WE SHOWED $\frac{M}{R} \sim \frac{T_c}{M}$ AND WE FIND GET AS AN APPROXIMATE EXPRESSION FOR L ,

$$L = \frac{M^{7.5}}{K_0} \frac{M^{5.5}}{R^{0.5}}$$

THIS EXPRESSION TELLS US THAT THE LUMINOSITY IS VERY SENSITIVE TO THE CHEMICAL COMPOSITION. ALSO IT THEN FOLLOWS IT IS FAIRLY SENSITIVE TO THE MASS. THIS EXPRESSION IS NOT TOO BAD FOR A ROUGH CALCULATION. EMPIRICALLY THE LUMINOSITY GOES AS THE FOURTH POWER OF THE MASS WHILE, OF COURSE, THE MEAN MOLECULAR WEIGHT IS NOT KNOWN. ANYWAY OUR GUESS THAT STARS ARE BIG GAS BALLS LOOKS RIGHT.

Image of galaxy NGC 4945 showing the huge luminosity of the central few star clusters, suggesting there are 10 to 100 supergiant stars in each of these, packed into regions just a few parsecs across.



EDDINGTON HAD A LITTLE TRICK HE DEVISED TO SOLVED THE FOUR DIFFERENTIAL EQUATIONS WE HAVE BEEN DISCUSSING. THE TRICK INVOLVED AN ANSATZ, I.E., AN ASSUMPTION THAT'S OKAY. COLLECTING THE FOUR FUNDAMENTAL EQUATIONS OF STELLAR STRUCTURE WE HAVE,

(i). THE EQUATION OF HYDROSTATIC EQUILIBRIUM,

$$\frac{dP}{dr} = - \frac{GM(r)\rho}{r^2} \quad \text{where } P = P_{\text{gas}} + P_{\text{rad.}} \\ = \frac{\rho RT}{M} + \frac{1}{3}\alpha T^4$$

(ii). THE EQUATION FOR CONSERVATION OF MASS,

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

(iii). THE LUMINOSITY EQUATION,

$$\frac{dL(r)}{dr} = 4\pi r^2 \epsilon(r)$$

(iv). THE EQUATION FOR RADIATIVE EQUILIBRIUM,

$$\frac{dP_{\text{rad}}}{dr} = - \frac{K\rho L(r)}{4\pi r^2}$$

WHAT EDDINGTON DID WAS TO OBSERVE THE OBVIOUS SYMMETRY OF THESE EQUATIONS. IT WOULD BE NICE IF $L(r) \sim M(r)$ THEN EQUATIONS ii AND iii WOULD BE ANALOGOUS AS WOULD i AND iv. BY INTRODUCING THE QUANTITY $\eta(r)$ AS,

$$\eta(r) = \frac{L(r)}{M(r)} / \frac{L_0/M_0}{}$$

WHERE L_0 AND M_0 ARE THE LUMINOSITY AND MASS OF THE STAR WE CAN GET SOME INTERESTING RESULT. $\eta(r)$ CAN BE INTERPRETED AS THE ENERGY GENERATION PER GRAM OF GOOP INSIDE RADIUS r AS COMPARED TO THE WHOLE STAR. FROM THIS RELATIONSHIP EQUATION iv BECOMES,

$$\frac{dP_{\text{rad}}}{dr} = - \frac{M(r)\eta(r)K(r)(L_0/M_0)\rho}{4\pi r^2}$$

IF BY ANOTHER STREAK OF GOOD LUCK $\eta(r)K(r) = \text{CONSTANT}$, WE CAN MAKE A BETTER COMPARISON WITH (i). FIRST WE LET β EQUAL THE NUMBER FRACTION OF ~~MATTER~~ THE TOTAL PRESSURE DUE TO GAS AND $1-\beta$ THE PRESSURE FROM RADIATION, I.E.,

$$\beta P = P_{\text{gas}}$$

$$(1-\beta)P = P_{\text{rad}}$$

WHERE $\beta \approx 1$ FOR MOST STARS.

Now dividing (i) by iv,

$$\frac{dp_n}{dp} = \frac{KL(n)}{4\pi c G M(n)} = \frac{\bar{Kn} L_0}{4\pi c G M_0}$$

INTEGRATING from $r=0$ to R AND REQUIRING $p_n=0$ AT $r=R$

$$p_n = \frac{\bar{Kn}}{4\pi c G} \frac{L_0}{M_0} P$$

where \bar{Kn} is THE AVERAGE OVER THE STAR

NOW USING $p_n = (1-\beta) P$

$$(1-\beta) = \frac{\bar{Kn} L_0}{4\pi c G M_0}$$

OR

$$\bar{Kn} = (1-\beta) 4\pi c G \frac{M_0}{L_0}$$

THE QUANTITY $\eta(r)$ HAS BEEN DEFINED SO $\eta(R)=1$ AND Thus DECREASES AS YOU MOVE OUT FROM THE CENTER, WHILE THE OPACITY GETS BIGGER AS YOU GO OUT BECAUSE THERE'S MORE GOOP IN THE WAY FOR THE ENERGY TO GET THROUGH. SO OUR EXERCISE HAS BEEN REASONABLE IN THAT IT TELLS US THAT MOST OF THE ENERGY COMES FROM THE JUICE AT THE CENTER.

GOING A LITTLE FURTHER ALONG THIS SAME LINE WE CAN WRITE

$$P_{\text{GAS}} = \frac{\beta}{1-\beta} P_{\text{RAD}}$$

RECALLING THAT $P_{\text{GAS}} = \frac{\rho R T}{\mu}$ AND $P_{\text{RAD}} = \frac{1}{3} a T^4$ SO THAT

$$\rho \frac{R T}{\mu} = \left(\frac{\beta}{1-\beta} \right) \frac{1}{3} a T^4$$

OR

$$\rho = \frac{\mu}{R} \left(\frac{\beta}{1-\beta} \right) \frac{1}{3} a T^3$$

SINCE $P \sim T^4$ AND $\rho \sim T^3$ WE CAN FIND A PRESSURE DENSITY RELATIONSHIP,

$$\frac{P}{\rho} \sim P^{4/3} \Rightarrow P \sim \rho^{4/3}$$

IF WE RECALL OUR STANDARD MODEL FORMULA OF $P = \rho^{(1+\frac{1}{n})}$ WE SEE FOR $n=3$ THE TWO RELATIONS ARE THE SAME. Thus for $n=3$ WE CAN LOOK UP THE TABLES AND THUS GET THE LUMINOSITY. IN THIS WAY WE SUCCEED IN EVALUATING L . IN ADDITION FROM T_c WE CAN FIND P_{RAD} . AT THE CENTER WE CAN GUESS AT β . From M/R^3 ρ_c CAN BE FOUND AND THEN T_c ABSOLUTELY.

NOW LET US SUPPOSE,

$$K = K_0 \rho T^{-3-S} \quad \text{where } S \sim \frac{1}{2}$$

ALSO LET

$$E(R) \sim \epsilon_0 T^\gamma \rho^\alpha$$

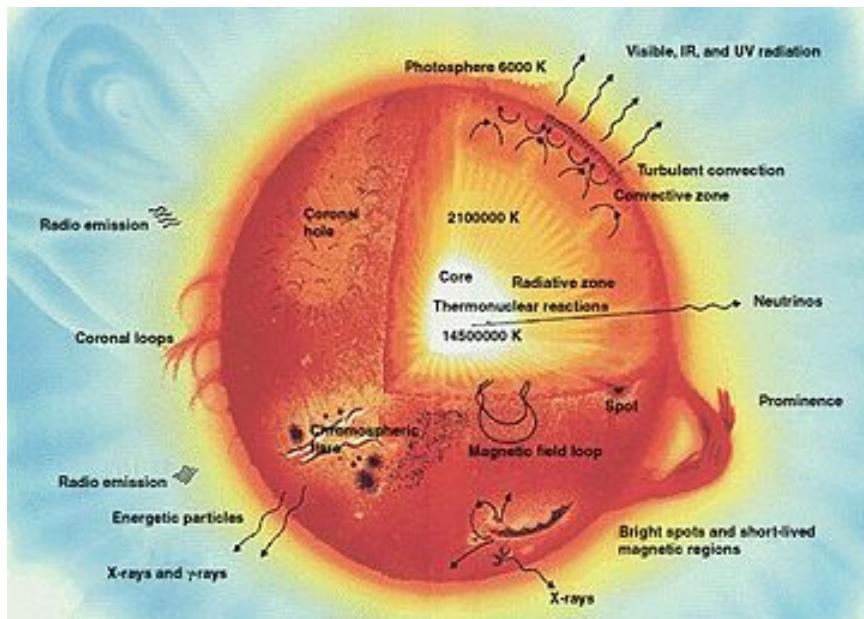
SOLVING THE EQUATIONS AGAIN WE GET,

$$L = (\text{constant}) \frac{1}{K_0} \frac{M^{5+S}}{R^S} \mu^{7+S}$$

WHERE THE CONSTANT IS A FUNCTION OF α, γ, S

THE QUESTION WOULD ARISE NOW IF IT HASN'T ALREADY, "WHAT ARE WE DOING?" WE, OF COURSE, ARE NOT OUT TO TEST THEORIES IN ASTRONOMY BUT RATHER TO UNDERSTAND WHAT IS GOING ON OUT THERE. BY GATHERING EMPIRICAL DATA ON STARS NEARBY WE CAN MEASURE THEIR MASS AND RADIUS FAIRLY WELL, ESPECIALLY IF THE STAR IS BINARY. FROM THIS DATA THE LUMINOSITY CAN BE DETERMINED AND SUBSEQUENTLY AN IDEA ABOUT THE MOLECULAR COMPOSITION CAN BE GUessed INTELLIGENTLY. BY HAVING SOME PHYSICAL THEORIES TO CORRELATE WITH THIS DATA FURTHER STARS CAN BE MEASURED AND ANALYZED BY A PROCESS SIMILAR TO MATHEMATICAL INDUCTION, I.E., WE PROVE OUR FORMULAE OUT FOR THE SIMPLEST CASES THEN PROJECT ITS VALIDITY TO LARGE NUMBERS. THUS WE HOPE TO EXPLAIN OUR OBSERVATIONS.

Indeed, it is a great accomplishment of mankind to have such a firm grip on the interior composition of stars millions and billions of miles from us when, in fact, we know practically nothing about the ground beneath us.



The White Dwarfs

EARLIER IN OUR COURSE WE SAID WE WANTED TO FIND OUT HOW RED GIANTS BECAME WHITE DWARFS. TO DO THIS ANALYSIS WE MUST KNOW WHAT IS GOING ON INSIDE THESE HOT STARS. WE MENTIONED THAT THEY ARE ACTUALLY DYING STARS WHICH HAVE BEEN COMPRESSED TO RADII ON THE ORDER OF THE EARTH BUT HAVE A MASS COMPARABLE TO THE SUN. FOR THESE STARS ANY RESIDUAL HEAT ESCAPING FROM THE SURFACE RADIATES OUT OVER SUCH A SMALL AREA THAT THE THING LOOKS HOTTER THAN HELL. BUT REMEMBER ITS BURNER HAS BEEN EXHAUSTED AND IT IS ACTUALLY COOLING OFF!

SINCE THE TEMPERATURE IS SO HIGH, I.E., OF THE ORDER OF 10^7 DEGREES, THE PRESSURE IS SO HIGH THAT $P = \rho^{1/3} n$ IS NO LONGER A SATISFACTORY MODEL. WE STILL HAVE EQUATIONS i AND ii BUT WE MUST FIND AN EXPRESSION FOR THIS HIGHLY COMPRESSED MATTER. SINCE THE REAL THERMAL RADIATION IS NEAR THE SURFACE, WE CAN DISCUSS THE DWARFS AS CONSISTING OF DEGENERATE GAS AT ZERO DEGREES SQUEEZED BY ITSELF AND HELD TOGETHER BY ITS OWN GRAVITY. THE IMPORTANT CONCEPT WE MUST DESCRIBE IS THIS QUANTUM SQUEEZE SQUEEZE RATHER THAN RANDOM MOTION OF THE PARTICLE AS IN OUR GASEOUS THEORY.

THE MATERIAL WE ARE NOW DEALING WITH INVOLVES THE HEAVIER ELEMENTS BECAUSE TO BE A WHITE DWARF ALL THE HYDROGEN AND HELIUM MUST BE BURNED UP. FOR THE HIGHER ELEMENTS WE CAN SAY THERE ARE TWO MASS UNITS PER ELECTRON. WE WANT TO CALCULATE THE EQUATION OF STATE UNDER THIS BIG SQUEEZE.

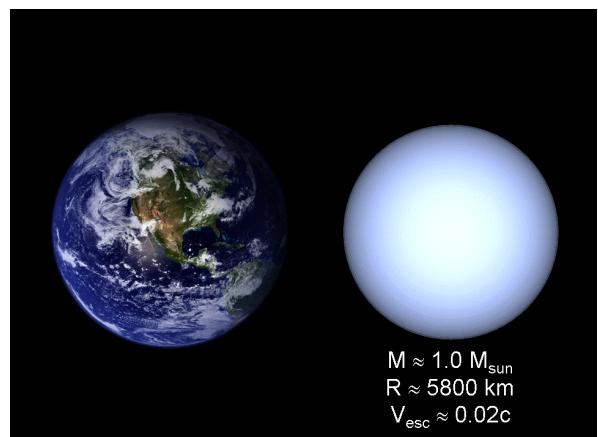
ACCORDING TO THE EXCLUSION PRINCIPLE ONLY TWO ELECTRONS CAN BE PUT INTO THE SAME STATE CORRESPONDING TO THE TWO POSSIBLE SPINS. ONCE THE STATE IS FILLED, THE ELECTRONS GO TO THE HIGHER LEVELS UNTIL THE VARIOUS SHELLS ARE COMPLETE. BUT THE STRUCTURE OF THE ATOM IS UTTERLY DESTROYED BY THE PRESSURE AND EVERYTHING IS IONIZED INTO FREE ELECTRONS AND NUCLEI.

WE MUST CONSIDER THE QUANTUM-MECHANICS OF JAMMING AN ELECTRON INTO A BOX OF DIMENSIONS a, b AND c . ASSOCIATED WITH THE WAVE PROPERTIES OF THIS PARTICLE IS A MOMENTUM p ,

$$p = k k$$

WHERE k IS THE WAVE NUMBER, $\frac{2\pi}{\lambda}$ AND λ IS THE WAVELENGTH SINCE THE PARTICLE IS FREE AS THE ENERGY IS KINETIC SO

$$E = \frac{p^2}{2m} = \frac{k^2 k^2}{2m}$$



WE WON'T WORRY ABOUT THE NUCLEAR FORCES BECAUSE THE ELECTRONS GENERATE ALL OF THE PRESSURE SINCE THE NUCLEUS CANNOT BE COMPRESSED. ONLY AT ENORMOUS PRESSURES WOULD NUCLEAR FORCES BE SIGNIFICANT.

FOR A SPACE CUBE A WAVE FUNCTION CAN BE CONSTRUCTED OF THE THREE COMPONENT WAVE FUNCTIONS IN THE X, Y, AND Z DIRECTIONS. WE REQUIRE THE WAVE TO VANISH AT THE WALLS SO

$$\psi = \sin k_x x \sin k_y y \sin k_z z$$

where $k_x a = n_x \pi$

$k_y b = n_y \pi$

$k_z c = n_z \pi$

THIS WAVE FUNCTION IS THEN A SOLUTION TO SCHRODINGER'S EQUATION,

$$-\frac{\hbar^2}{2m} \nabla^2 \psi = E\psi$$

THAT IS

$$E = \frac{\hbar^2}{2m} (k_x^2 + k_y^2 + k_z^2) = \frac{p^2}{2m}$$

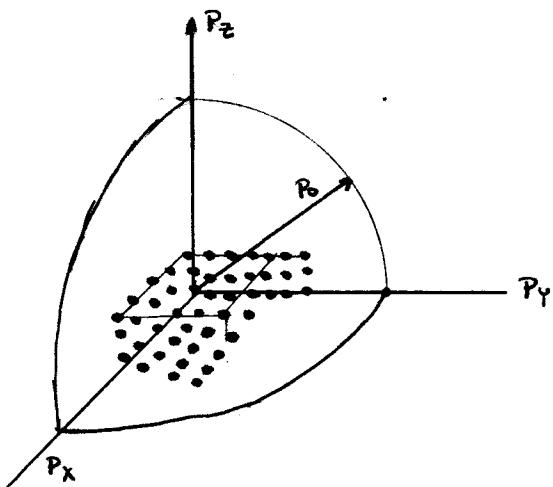
AND WE CHOSE TO DEFINE $E(p)$ TO BE THE TOTAL ENERGY,

$$E(p) = \frac{p^2}{2m}$$

THUS EACH MODE EQUALS A SINGLE STATE. TO FILL ALL THE MODES REQUIRES A LOT OF ELECTRONS IN FACT N OF THEM. WHAT ENERGY DO WE HAVE TO GO TO? ALL THE MODES ARE OCCUPIED WHEN THE LAST ONE TO BE SQUEEZED IN HAS THE SAME P AS ITS SUCCESSOR. LET p_0 BE THIS FINAL MOMENTUM. THEN,

$$N = \text{NUMBER OF MODES WHOSE } p^2 < p_0^2 \text{ OR } E(p) < E_0(p_0)$$

WHAT WE HAVE IS A MOMENTUM SPACE INSIDE A SPHERE OF RADIUS p_0 WITH EACH POINT CORRESPONDING TO A p . THESE POINTS FORM A RECTANGULAR LATTICE WITHIN THIS SPHERE SINCE ONLY CERTAIN MOMENTA OR ENERGIES ARE PERMISSIBLE. THE POINTS ARE SEPARATED BY $\frac{k\pi}{a}$, $\frac{k\pi}{b}$, AND $\frac{k\pi}{c}$. WHAT WE HAVE IS REPRESENTED IN THE FOLLOWING DRAWING:



The TOTAL NUMBER of MODES IN ONE OCTENT of The sphere IS,

$$\frac{N}{2} = \frac{1}{8} \cdot \frac{4}{3} \pi P_0^3 \cdot \frac{1}{\frac{\hbar\pi \cdot \hbar\pi \cdot \hbar\pi}{a \cdot b \cdot c}} = \frac{4}{3} \pi P_0^3 \cdot \frac{V_{OL}}{(2\pi k)^3}$$

Accounts for
TWO SPINS

Therefore,

$$\frac{\text{No. of ELECTRONS}}{\text{UNIT VOLUME}} = n = \frac{N}{V}$$

UNDER THE EXTREME COMPRESSIONS IN THE WHITE DWARFS n MIGHT RISE 10^6 TIMES WHICH MEANS P_0 INCREASE BY ABOUT 100 OR THE ENERGY BY 10,000 VOLTS WHICH IS CONCLUSIVE EVIDENCE THAT EVERYTHING IS IONIZED.

HOW DO WE GET THE EQUATION OF STATE? WE NEED THE TOTAL AMOUNT OF ENERGY OF THE ELECTRONS IN THE BOX FOR EACH MODE $E(P_0)$.

$$\text{TOTAL ENERGY, } U = \int_0^{P_0} E(p) 4\pi p^2 dp \cdot \frac{V}{(2\pi k)^3} \cdot 2$$

$$\text{SINCE } E(p) = \frac{p^2}{2m}$$

$$\begin{aligned} U &= \frac{4}{5} \frac{\pi}{2m} P_0^3 \cdot P_0^2 \cdot \frac{V}{(2\pi k)^3} \cdot 2 \\ &= \frac{P_0^2}{2m} \cdot \frac{3}{5} \left[\frac{4\pi}{3} \frac{P_0^3 \cdot V}{(2\pi k)^3} \cdot 2 \right] \end{aligned}$$

Therefore,

$$U = \frac{3}{5} \left(\frac{P_0^2}{2m} \right) N$$

THIS SIMPLY GIVES OF $3/5$ TIMES THE AVERAGE ENERGY. FROM ABOVE WE SEE $P_0 \sim V^{-1/3}$ SO THAT

$$U \sim V^{-2/3}$$

SINCE WE WOULD LIKE TO FIND THE PRESSURE, WE CAN DO SO SINCE,

$$P = - \left. \frac{dU}{dV} \right|_N = \frac{2}{3} \frac{U}{V}$$

WHERE THE PRESSURE IS EVALUATED OVER A CONSTANT NUMBER OF PARTICLES. Thus we find,

$$P = \frac{2}{3} \left(\frac{3}{5} \frac{P_0^2}{2m} \right) \cdot n$$

BUT

$$P_0 = \left[\frac{3(2\pi k)^3}{8\pi} \right]^{1/3} n^{1/3}$$

SO THAT,

$$\begin{aligned} P &= \frac{2}{3} \cdot \frac{3}{5} \cdot \frac{1}{2m} \left[\frac{3}{8\pi} \right]^{2/3} 4\pi k^2 n^{2/3} \cdot n \\ &= \frac{\pi k^2}{5m} \left(\frac{3}{\pi} \right)^{2/3} n^{5/3} \end{aligned}$$

NOW TO GET FROM n TO THE NUCLEAR DENSITY WE USE

$$\rho = (2M_p)n$$

where M_p IS THE MASS OF THE PROTON

SO WE GET THAT,

$$P = \frac{\pi \hbar^2}{5m} \left(\frac{3}{\pi}\right)^{2/3} \left(\frac{1}{2M_p}\right)^{5/3} \rho^{5/3}$$

RECALLING OUR MODEL $P = \rho^{(1+\frac{1}{n})}$ IF $n = 3/2$ THEN WE HAVE ANALOGOUS PRESSURE DENSITY RELATIONSHIPS AND WE COULD LOOK UP IN THE TABLES $n = 3/2$ TO FIND OUT WHAT HAPPENS.

WHEN THE ENERGY BECOMES RELATIVISTIC, I.E., THE MOMENTA DOES WE INSERT $E(P) = [(mc^2)^2 + P^2 c^2]^{1/2} - mc^2$ IN THE INTEGRAL FOR U AND WHEN E IS OF THE ORDER OF mc^2 THEN THE PRESSURE GOES AS $\rho^{4/3}$.

THE CASE WHEN $P = \rho^{4/3}$ IS INTERESTING BECAUSE THEN THE STAR IS NEUTRALLY STABLE. THAT IS, WHEN IT IS SQUEEZED TO ONE-HALF ITS SINCE THE ENERGY RISE AND GRAVITATIONAL POTENTIAL ATTRACTION BALANCE EACH OTHER SO THE STAR JUST SITS THERE. WHEN $P = \rho^{5/3}$, THE PRESSURE RISES TOO FAST AND THE BALL WILL BOUNCES OUT. THUS FOR POWERS HIGHER THAN $5/3$ THE STUFF IS STIFF. WE'LL TAKE A LOOK AT THE CAUSE OF THE POWER BETWEEN $4/3$ AND $5/3$ MORE CLOSELY. THIS IS MORE CRITICAL BECAUSE THE STAR CAN'T MAINTAIN ITSELF UNDER ITS OWN GRAVITATIONAL ATTRACTION WITHOUT BREAKING UP.

SINCE WE ARE DEALING WITH A COMPLETELY DEGENERATE ELECTRON GAS IN THE DISCUSSION OF WHITE DWARFS, WE MUST USE THE FOLLOWING DENSITY RELATIONSHIP,

$$\rho = \mu_e n_e$$

WHERE μ_e IS THE MEAN MOLECULAR WEIGHT PER FREE ELECTRON AND IS NOT THE SAME μ AS WE USED IN THE THEORY OF GASEOUS STARS.

ALSO n_e IS THE NUMBER OF ELECTRONS PER UNIT VOLUME THAT WE FOUND ON THE PREVIOUS PAGE, I.E.,

$$n_e = \frac{8\pi P_0^3}{3(2\pi\hbar)^3}$$

THE MOMENTUM P_0 IS THE THRESHOLD VALUE WE USED EARLIER. IF THE STAR IS RELATIVISTIC SO $P \propto \rho^{4/3}$ WITH THE ABOVE VALUE FOR P , IT TURNS OUT (CHANDRA SEKHAR P.422) THAT THE MASS ASSUMES A FINITE LIMIT,

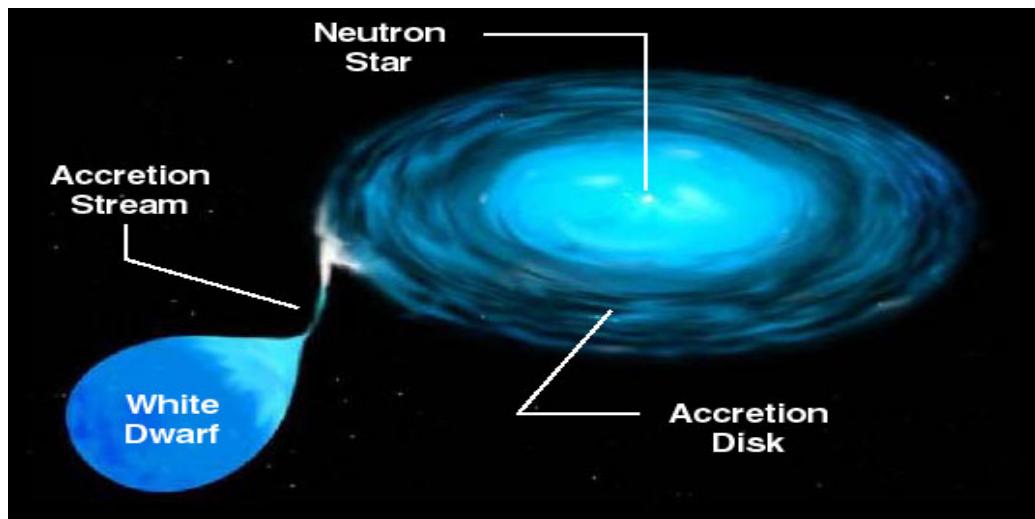
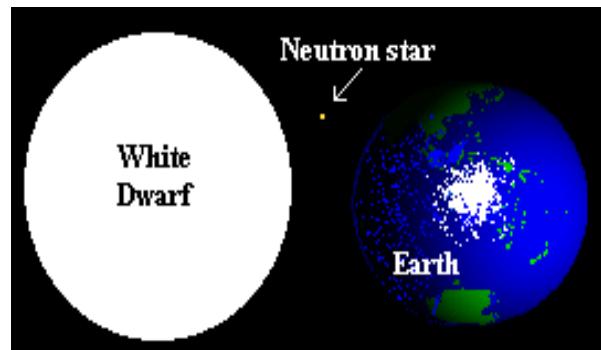
$$M_c = \frac{5.75}{(\mu_e)^2} M_\odot$$

WHERE M_\odot IS THE MASS OF THE SUN.

If a dwarf had this mass it would really be in a critical state. For if someone spit on the mass, the added weight would cause the star to collapse under its own weight and never stop. Some theories claim that it goes beyond the relativistic limit and falls through a hole in space but we won't go into that! See below

For less critical cases we can discuss for a moment what happens when we start to pile up a bunch of cold material like the Earth. If we kept piling on rocks, the added weight would cause compression of what was under it. As it got more massive, the radius would not grow drastically because of the compression. This happens when the size is comparable to Jupiter which is about the maximum radial size a cold object can get without emitting heat from nuclear generation. There are, or could be, heavier ones but not bigger.

Feynman alluded to but did not pursue the transition and further collapse of the white dwarf to the neutron star state



OPACITY

So far we have put off an explanation of how the opacity is determined. Along with the determination of the energy generation factor, this is the most important and most difficult part of stellar theory. We return to the main sequence star to first find the opacity for a gas.

Recalling again $P_{\text{gas}} = \frac{RTP}{\mu}$, $P_{\text{rad}} = \frac{1}{3} \alpha c T^4$ we remember P_{rad} is mechanically not important but it is in the determination of the luminosity, $L(R)$, i.e.,

$$\frac{dP_{\text{rad}}}{dr} = - \frac{K\rho L(r)}{4\pi r^2}$$

OR,

$$L(r) = + \frac{4\pi r^2}{K\rho} \left(- \frac{dp}{dr} \right)$$

Thus there is a certain rate of heat flowing out of the star because of the temperature gradient $(-\frac{d\ln \alpha c T^4}{dr})$ existing between the interior and the surface.

Therefore, we must consider the method of photon diffusion as the heat tries to escape through the material of certain ρ , μ , and T . That is, what are the mechanisms that limit the photon from zooming through at the speed of light like a neutrino can do. The two processes we will discuss are

- (i) SCATTERING
- (ii) ABSORPTION - RE-EMISSION

FIRST, ELECTRON SCATTERING. The hot stars, i.e., $T \sim 10^7$ K, electron scattering is a very important absorptive process. According to classical theory of electro magnetic waves, an accelerated electron emits radiation of intensity

$$\frac{2e^2}{3c^2} (\ddot{x})^2 = \frac{2e^4}{3m^2c^3} E^2 = \frac{8\pi}{3} \left(\frac{e^2}{mc^2} \right)^2 \left(\frac{c}{4\pi} E^2 \right)$$

The electron draws this energy from the incident radiation. In the incident E-M waves (i.e., radiation), half of the energy is electric and half magnetic, and the electric and magnetic fields are at right angles so that the Poynting flux for the wave is $\frac{c}{4\pi} E^2$. Therefore, the scattering per electron is given by

$$\sigma = \frac{8\pi}{3} \left(\frac{e^2}{mc^2} \right)^2 \approx 10^{-25} \text{ cm}^2$$

Since the mass of the particle is in the denominator, σ for all other particles than electron is much less so we concern ourselves with only electron during scattering.

SECONDLY, ABSORPTION - RE-EMISSION : DURING THIS PROCESS THE ATOM IS NOT COMPLETELY IONIZED SO THAT IT HAS SOME ENERGY LEVELS LEFT. A PHOTON IS ABSORBED TO PUT THE ATOM IN ONE OF THESE HIGHER STATES. LATER THE ATOM TURNS AROUND AND EMITS THE ABSORBED PHOTON

IMAGINE WE HAVE AN ATOM IN ITS GROUND STATE, SAY LEVEL OR STATE m . THE PROBABILITY THAT THE ATOM WOULD ABSORB A QUANTUM OF RADIATION $h\nu_m$ THAT WOULD PUT THE ATOM IN THE HIGHER STATE n IS DENOTED BY B_{mn} . THE PROBABILITY B_{mn} IS CALLED THE EINSTEIN COEFFICIENT OF INDUCED EMISSION. THUS THE NUMBER OF ATOMS GOING TO THIS HIGHER LEVEL PER CENTIMETER PENETRATION OF THE PHOTON, OR THROUGH ITS MEAN FREE PATH IS GIVEN BY,

$$\frac{\text{No. going up}}{\text{cm}} = B_{mn} N_m$$

WHERE N_m IS THE NUMBER OF ATOMS PER UNIT VOLUME IN THE STATE m . BUT THEN THE PROBABILITY THAT AN ELECTRON IN THE EXCITED STATE n EMITS THE SAME QUANTUM OF ENERGY $h\nu_m$ AND FALLS DOWN TO LEVEL m IS GIVEN BY A_{nm} . A_{nm} IS CALLED THE EINSTEIN COEFFICIENT FOR SPONTANEOUS EMISSION. WHEN THE NUMBER OF ATOMS GOING DOWN IS LESS THAN THE NUMBER GOING UP, THE CONDITION FOR NET ABSORPTION IS SATISFIED. THE RATIO BETWEEN THE POPULATIONS OF STATES m AND n IS GIVEN BY

$$\frac{N_n}{N_m} = \frac{e^{-E_n/kT}}{e^{-E_m/kT}} = e^{-(E_n - E_m)/kT} = e^{-h\nu_m/kT}$$

EXPRESSING THEN THE ABSORPTION OF RADIATION AS

$$B_{mn} N_m - B_{mn} N_n$$

$$\text{but } N_n = N_m e^{-h\nu/kT}$$

SO THAT

$$B_{mn} (1 - e^{-h\nu/kT})$$

IF WE INTERPRET $B_{mn} N_m$ AS THE OPACITY ARISING FROM DIRECT ABSORPTION WE MAY WRITE THE TOTAL OPACITY AS

$$K = K_{ab} (1 - e^{-h\nu/kT}) + K_{\text{SCATTERING}}$$

THUS, WE SEE THAT ~~THE~~ THE REAL ABSORPTION IS MADE UP OF TWO PARTS PRIMARILY DUE TO DIRECT ABSORPTION. WE HAVE OUR RESULT UPSIDE-DOWN, HOWEVER, CONVENTIONALLY TO OPACITY IS DEFINED AS

$$\frac{1}{K} = \left\langle \frac{1}{K_{ab} (1 - e^{-h\nu/kT}) + K_s} \right\rangle$$

WHERE WE FIND THE MEAN-OPACITY AND THUS HAVE A VALUE TO PUT IN THE EXPRESSION,

$$L(r) = \frac{4\pi r^2}{K\rho} \left(-\frac{dP_{\text{rad}}}{dr} \right)$$

ANOTHER WAY TO DEFINE OPACITY IS IN TERMS OF THE FREQUENCY ν . THIS TIME WE DEFINE THE MEAN MASS OPACITY COEFFICIENT AS

$$\frac{1}{K'} = \frac{\int_0^\infty \frac{1}{K(\nu)} \frac{\partial I(\nu, T)}{\partial T} d\nu}{\int_0^\infty \frac{\partial I(\nu, T)}{\partial T} d\nu}$$

where $I(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$

THE INTEGRATION BEING OVER ALL POSSIBLE FREQUENCIES.

WITHIN THE STELLAR MASS THERE ARE THREE METHODS BY WHICH RADIATION IS ABSORBED:

- (i). THE BOUND-FREE TRANSITION OR PHOTO-ELECTRIC EFFECT. HERE A QUANTUM OF RADIATION OF ENERGY THAT IS GREATER THAN THE BINDING ENERGY OF AN ELECTRON SETS THE ELECTRON FREE. THERE IS CONTINUOUS ABSORPTION IN THE B-F TRANSITION; WHEN THE ENERGY IS GREATER THAN THE BINDING ENERGY.
- (ii). THE FREE-FREE TRANSITION. A FREE ELECTRON MOVING IN A HYPERBOLIC ORBIT UNDER THE FORCE OF ATTRACTION OF AN ION MAY ABSORB A QUANTUM OF ENERGY AND BE ACCELERATED. SINCE A QUANTUM OF ANY ENERGY CAN BE ABSORBED, IN F-F TRANSITION, CONTINUOUS ABSORPTION RESULTS.
- (iii). THE BOUND-BOUND TRANSITION OR BREMSSTRAHLUNG. THIS IS WHERE SOLITARY ELECTRONS CAN INTERACT WITH RADIATION BY CHANGING THE DIRECTION AND FREQUENCY OF THE INCIDENT BEAM OF RADIATION; THIS IS SCATTERING.



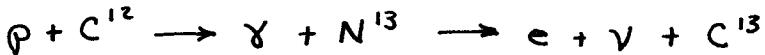
Opacity is the measure of impenetrability to electromagnetic or other kinds of radiation, especially visible light. In radiative transfer, it describes the absorption and scattering of radiation in a medium, such as a plasma, dielectric, shielding material, glass, etc. An opaque object is neither transparent (allowing all light to pass through) nor translucent (allowing some light to pass through). When light strikes an interface between two substances, in general some may be reflected, some absorbed, some scattered, and the rest transmitted (also see refraction). Reflection can be diffuse, for example light reflecting off a white wall, or specular, for example light reflecting off a mirror. An opaque substance transmits no light, and therefore reflects, scatters, or absorbs all of it. Both mirrors and carbon black are opaque. Opacity depends on the frequency of the light being considered. For instance, some kinds of glass, while transparent in the visual range, are largely opaque to ultraviolet light. More extreme frequency-dependence is visible in the absorption lines of cold gases. Opacity can be quantified in many ways; for example, see the article mathematical descriptions of opacity

CHAPTER 5

ENERGY GENERATION

WE'LL NOW MOVE ON TO THE AREA OF WHERE THE STARS GET THE ENERGY TO KEEP GOING. WE WILL THEN TALK ^{ABOUT} THE ENERGY GENERATION FACTOR INTRODUCED EARLIER AND WHICH WE RECALL WAS $E(\rho, T, \mu)$. THAT IS, IT IS A FACTOR OF DENSITY, TEMPERATURE, AND CONSTITUTION. THE ANSWER TO THE QUIRY IS OBVIOUSLY - NUCLEAR REACTION.

AN EXAMPLE OF A POSSIBLE REACTION, WOULD BE THE CONSEQUENCE OF A PROTON SMASHING INTO A CARBON ATOM, I.E.,



where γ = GAMMA RADIATION
 ν = NEUTRINO

AN INTERESTING RESULT FOLLOWS IF WE CALCULATE THE ENERGY A PROTON WOULD HAVE AT THE SUN'S INTERIOR, I.E., AT ABOUT $10,000,000^{\circ}\text{K}$. AT THIS TEMPERATURE kT WOULD BE ABOUT 2000 eV SINCE 1 eV IS ABOUT EQUAL TO $11,000^{\circ}\text{K}$. WE CAN NOW SHOW FOR ENERGIES OF THIS ORDER THAT NUCLEAR REACTIONS CANNOT OCCUR; SOLELY BECAUSE THE PROTON CANNOT GET CLOSE ENOUGH TO THE CARBON DUE TO THE REPULSIVE FORCES. THE ENERGY AT CLOSEST APPROACH IS

$$E = \frac{Ze^2}{R_c}$$

where R_c IS THE CRITICAL RADIUS OR CLOSEST APPROACH
 If we do the math right, R_c turns out to be about

$$R_c \approx 3.5 \times 10^{-10}\text{ cm}$$

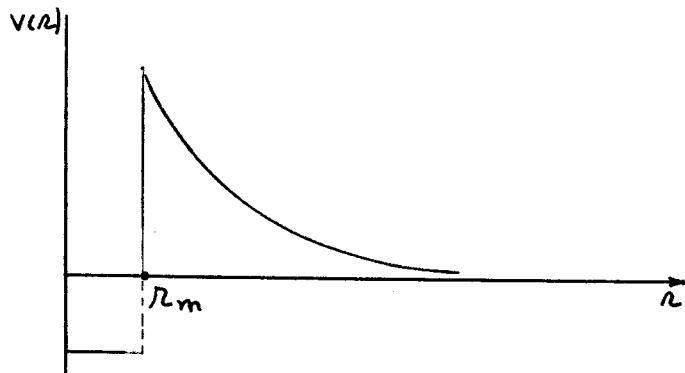
Now for a nucleus, the radius is of the order of

$$r = A^{1/3} 1.6 \times 10^{-13}\text{ cm}$$

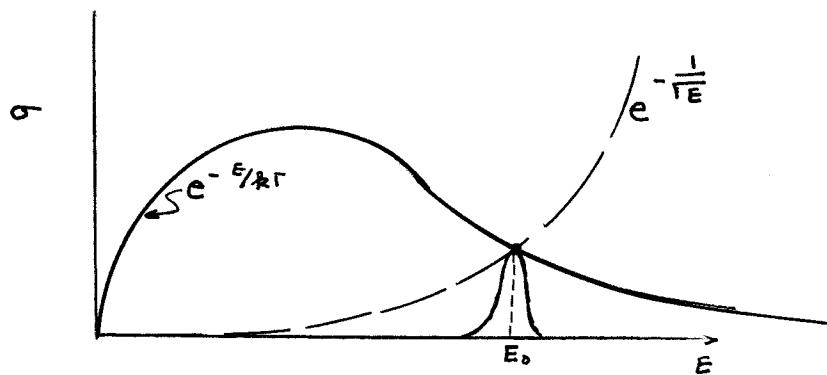
where A is a function of the stuff inside, i.e., the atomic weight
 BUT ANYWAY WE SEE THAT R_c IS ABOUT $1000r$ SO THE REPULSION IS SO GREAT THAT THE REACTION COULD NEVER GET STARTED.

So for sometime the early stellar ^{theorists} were leary of describing nuclear reaction but were certain the phenomena must occur. It was only with the advent of quantum mechanics that an answer was found. Through Q-M it was realized that there is a finite probability, i.e., the Boltzmann distribution $e^{-E/kT}$ that a proton would have the right energy to get everything going.

ONCE THE PARTICLE APPROACHES THE NUCLEUS, IT MUST OVERCOME OR PENETRATE THE POTENTIAL BARRIER DUE TO THE COULOMBIC REPULSION. THIS POTENTIAL RISES SHARPLY WITH DECREASE RADIAL DISTANCE, DOWN TO THE NUCLEAR RADIUS WHERE IT DROPS TO A CONSTANT NEGATIVE VALUE,

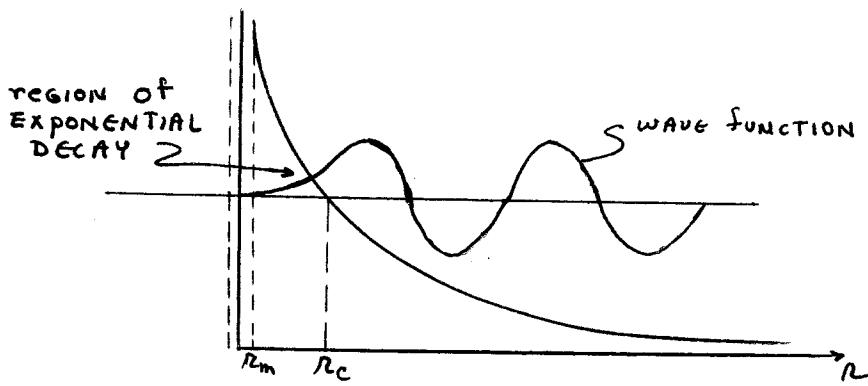


THERE WILL THUS BE SOME PROBABILITY FACTOR GIVEN BY THE REACTION CROSS-SECTION FOR THE COLLISIONS. WE CAN GET AN IDEA OF THE CROSS-SECTION, σ , AS A FUNCTION OF ENERGY AND ON THE SAME GRAPH GIVE THE MAXWELL-BOLTZMANN ENERGY DISTRIBUTION $e^{-E/kT}$



WHAT WE CAN SEE FROM THIS DRAWING, WHICH IS NOT TOO CLEAR, IS THAT MOST OF THE REACTION OCCURS RIGHT AROUND SOME E_0 , I.E., THE TWO CURVES ARE MULTIPLIED TOGETHER. FOR ENERGIES OF THE ORDER OF 1000 kT WE ARE TOO FAR OUT ON THE $e^{-E/kT}$ TO GIVE ANY CONTRIBUTION. AT MUCH LOWER ENERGIES, SAY $5-10 \text{ kT}$ WE REACH A REGION OF RESONANCE OR SIGNIFICANT CONTRIBUTIONS TO REACTION.

WHAT WE MUST DO IS TO SEE HOW THE WAVE FUNCTION OF THE PROTON TAILS OFF EXPONENTIALLY AS IT COMES INTO THE REGION OF THE NUCLEUS, I.E., AS IT GOES BEYOND THE CRITICAL RADIUS. TO HELP UNDERSTAND THE NEXT DRAWING MIGHT HELP,



So we write down Schrödinger's EQUATION,

$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = E\psi$$

The LAPLACIAN OPERATOR CAN be written for A SPHERICALLY SYMMETRIC OBJECT AS

$$\frac{1}{r^2} \frac{d}{dr} (r^2 \frac{d\psi}{dr})$$

or AS A MORE CONVENIENT form

$$\frac{1}{r} \frac{d^2}{dr^2} (r\psi)$$

Thus we have,

$$\frac{\hbar^2}{2m} \frac{d^2}{dr^2} (r\psi) = (V-E)(r\psi)$$

$$\text{where } V-E = \frac{Ze^2}{r} - \frac{Ze^2}{r_c}$$

so we write

$$\frac{d^2}{dr^2} (r\psi) = \frac{2m}{\hbar^2} Ze^2 \left(\frac{1}{r} - \frac{1}{r_c} \right) (r\psi)$$

Now MAKE THE SUBSTITUTION,

$$e^{-\varphi(r)} = r\psi$$

$$\text{so THAT } \frac{d}{dr} (r\psi) = -\varphi'(r) e^{-\varphi(r)} \text{ AND } \frac{d^2}{dr^2} (r\psi) = \left[\varphi''(r) + \varphi'^2(r) \right] e^{-\varphi(r)}$$

Then Schrödinger's EQ. becomes

$$-\varphi''(r) + \varphi'^2(r) = \frac{2m}{\hbar^2} Ze^2 \left(\frac{1}{r} - \frac{1}{r_c} \right)$$

WE CAN GO ONE STEP further AND LET

$$u^1 = \varphi'$$

Then

$$-u^1 + u^2 = \frac{2m}{\hbar^2} Ze^2 \left(\frac{1}{r} - \frac{1}{r_c} \right)$$

SO WE HAVE INTRODUCED A SLIGHT NON-LINEARITY IN The EQUATION by reducing it FROM A SECOND TO first order differential EQUATION.

Over the range of r where we are concerned, i.e., for very good approximation we may use

$$U = \sqrt{\frac{2mZe^2}{\hbar^2}} \sqrt{\frac{1}{r} - \frac{1}{R_c}}$$

where

$$U' = -\sqrt{\frac{2mZe^2}{\hbar^2}} \cdot \frac{1}{2r^2} \left(\frac{1}{r} - \frac{1}{R_c} \right)^{-1/2}$$

so that

$$-U' + U^2 = \frac{2mZe^2}{\hbar^2} \left(\frac{1}{r} - \frac{1}{R_c} \right) \text{ becomes}$$

$$+ \sqrt{\frac{2mZe^2}{\hbar^2}} \frac{1}{2r^2} \frac{1}{\sqrt{\frac{1}{r} - \frac{1}{R_c}}} + \frac{2mZe^2}{\hbar^2} \left(\frac{1}{r} - \frac{1}{R_c} \right) = \frac{2mZe^2}{\hbar^2} \left(\frac{1}{r} - \frac{1}{R_c} \right)$$

Recalling now our substitution

$$\frac{dU(r)}{dr} = \phi(r)$$

We can write the function as

$$r\psi \propto \exp \left[- \int_{R_m}^{R_c} \sqrt{\frac{2mZe^2}{\hbar^2}} \sqrt{\frac{1}{r} - \frac{1}{R_c}} dr \right]$$

Finally, the penetration can be expressed as the probability of the amplitude squared at R_m over that at R_c or

$$P = \left| \frac{(r\psi)_{r=R_m}^2}{(r\psi)_{r=R_c}^2} \right|^2$$

The r , radius, has a place in the amplitude because it is a measure of penetration distance. Simply enough then,

$$(r\psi)^2 \propto \exp \left[-2 \int_{R_m}^{R_c} \sqrt{\frac{2mZe^2}{\hbar^2}} \sqrt{\frac{1}{r} - \frac{1}{R_c}} dr \right]$$

According to the WKB approximation we can express the wave function as

$$\psi = \exp \left(\frac{i}{\hbar} S \right)$$

where S is a function of x and is a phase factor. The physical significance of S is that its rate of change of with position ds/dx is equal to the mean momentum or

$$S = \int_{x_0}^x p dx \quad \text{where } p = \sqrt{2m(E-V)}$$

Then

$$\psi = \exp \left[\frac{i}{\hbar} \int \sqrt{2m(E-V)} dx \right]$$

S is more generally approximated by a power series in \hbar , i.e.,

$$S = S_0(x) + \frac{\hbar}{2} S_1(x) + \frac{\hbar^2}{2!} S_2(x) + \dots$$

This approximation is good only for slowly changing wavelengths by assuming that the wave function is not changed much from the form it would have if V were constant. Thus we have similar results if we make the substitutions so we are about right.

The INTEGRAL INVOLVING THE FACTOR $\sqrt{\frac{1}{R} - \frac{1}{R_c}}$ SHOWS AS R_m APPROACHES ZERO THE $\frac{1}{R}$ TERM DOMINATES IN SUCH A WAY THAT ONCE YOU MAKE IT AS FAR AS TO R_m IT DOESN'T TAKE THAT MUCH MORE ENERGY TO GO A LITTLE FURTHER. SO WE WANT TO EVALUATE

$$I = \int_0^{R_c} \sqrt{\frac{1}{R} - \frac{1}{R_c}} dR$$

MAKE THE SUBSTITUTION

$$R = R_c \sin^2 \theta \quad \text{AND} \quad dR = 2R_c \sin \theta \cos \theta d\theta$$

THEN

$$I = \int_0^{\pi/2} R_c 2 \sin \theta \cos \theta \frac{1}{R_c} \sqrt{\frac{1}{\sin^2 \theta} - 1} d\theta$$

$$I = \int_0^{\pi/2} 2\sqrt{R_c} \frac{\sin \theta \cos \theta}{\sin \theta} \sqrt{\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta} - \sin^2 \theta} d\theta$$

$$= \int_0^{\pi/2} 2\sqrt{R_c} \cos^2 \theta d\theta = 2R_c \left[\frac{\theta}{2} - \frac{1}{4} \sin 2\theta \right]_0^{\pi/2} = \sqrt{R_c} \frac{\pi}{2}$$

SO OUR PENETRATION FACTOR $r\psi$ NOW TAKES THE FORM

$$\exp \left[-\frac{\pi \sqrt{2mZe^2/R_c}}{\hbar} \right] = \text{PENETRATION FACTOR}$$

WE COULD CHOOSE TO WRITE THIS IN TERMS OF ENERGY BY RECALLING

$$\frac{ze^2}{R_c} = E \quad \text{OR} \quad R_c = \frac{ze^2}{E}$$

THEN

$$\exp \left[-\frac{\pi \sqrt{2mZe^2Ze^2}}{\hbar \sqrt{E}} \right]$$

IF WE EXPRESSED THE ENERGY AS

$$E = \frac{1}{2}mv^2$$

THEN

$$\exp \left[-\frac{2\pi}{\hbar} \frac{ze^2}{v} \right]$$

WHERE v IS THE RELATIVE VELOCITY OF THE TWO THINGS COMING TOGETHER.

DIMENSIONALLY WE CAN CHECK OUR RESULT BY REWRITING IT AS

$$e^{-\frac{2\pi Ze^2}{\hbar c} \cdot \frac{c}{v}} = e^{-\frac{2\pi z}{137} \left(\frac{c}{v} \right)}$$

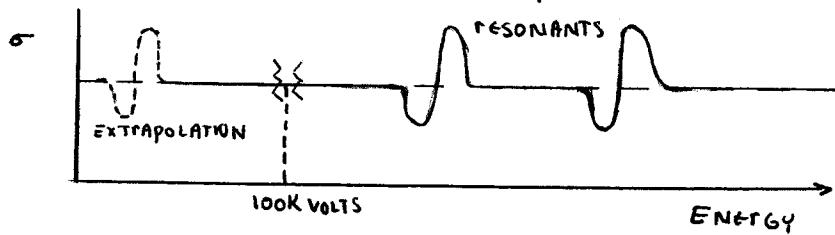
IT IS WORTHWHILE NOTING THAT IF $\frac{ze^2}{kv}$ IS MUCH GREATER THAN ONE, CLASSICAL MECHANICS IS SUFFICIENT TO DESCRIBE THE REACTION BUT IF IT IS MUCH LESS THAN ONE, WE MUST GO TO QUANTUM MECHANICS TO DESCRIBE THE SCATTERING. A HISTORICAL NOTE Bohr JUST HAPPENED TO HAVE WORKED WITH THE CASE WHERE THE FACTOR WAS MUCH GREATER THAN ONE AND MIRACULOUSLY GOT THE RIGHT SCATTERING RESULTS

FINALLY WE CAN WRITE A SCATTERING CROSS-SECTION for The reACTION AS

$$\sigma(E) = e^{\frac{-2\pi Z_1 Z_2 e^2}{\hbar v}} \cdot \pi \lambda^2 \cdot f(E)$$

where $F(E)$ IS SOME SMOOTH FUNCTION OF ENERGY AND $\pi \lambda^2 f(E)$ IS THE RATE OF THE reACTION MODIFIED BY THE PENETRATION FACTOR TAKING INTO CONSIDERATION THOSE WITH PROPER ENERGY.

This factor is so small that we cannot observe it experimentally. During the life of the sun, for instance, the reaction occurs often for it to be significant. In the lab the scattering cross-section is measured at high energies and by taking out the amount reduced by the penetration factor it is possible to get an idea of what is happening. There are some resonances at certain higher energies but in the range of lower energies which we can't directly work with to see the scatter there might also be a resonance. If there is, then $F(E)$ is not varying as we figured and we have to correct with another approximate. A graph might look like,



We are at a little loss at this point because the theory of nuclear reaction does not suffice to explain how the function $f(E)$ behaves. We must know more as to which reaction predominate because in the sun we know a proton-proton interaction give tiny $f(E)$'s while proton-carbon twelve give big $f(E)$. Considering a mixture of these reaction complicates the theory.

FROM KINETIC Theory we can write the rate of reaction occurring as,

$$\text{RATE OF REACTION} = \nu \sigma(E) e^{-E/kT}$$

where ν is the mutual velocity of the colliding particles; $\sigma(E)$ is the reaction cross-section occurring at the given energy value and $e^{-E/kT}$ gives the percentage of particles with the right energy to undergo reaction.

This reaction rate can be also written in terms of an integral over the frequencies or energy and simplified by collecting all the constant and other garbage in a nice function $F(E)$, i.e.,

$$\text{RATE} = (\text{factors}) \int \nu \sigma(\nu) e^{-E/kT} \nu^2 d\nu$$

Inserting the value for $\sigma(\nu)$, we can write

$$\text{RATE} = \int F(E) e^{-\beta E} e^{-\frac{b}{kE}} dE$$

$$\text{where } \beta = \frac{1}{kT}$$

Since the reaction primarily occurs around the bump, i.e., at E_0 as seen on page 83, we would like to know where this is. Thus the problem is to find the maximum of the integrand. If we consider $F(E)$ to be nice and smooth we will just evaluate it at E_0 to get the constant $F(E_0)$ which we now take outside. So we are left with maximizing the exponential factor,

$$\beta E + \frac{b}{kE}$$

This argument has a minimum at E_0 which makes the exponent maximum. We can approximate E_0 by a Gaussian curve near the minima and then perform the integral.

Thus we differentiate the argument with respect to E and set equal to zero,

$$\frac{d}{dE} \left(\beta E + \frac{b}{kE} \right) = 0$$

which gives upon evaluating at E_0

$$\beta = \frac{b}{2(E_0)^{3/2}}$$

Since we want the product βE_0 we can write,

$$\beta E_0 = \frac{b^{2/3}}{2^{2/3}} \beta^{1/3}$$

OR

$$\beta E_0 = \frac{b}{2E_0^{1/2}}$$

SINCE WE ARE ONLY CONCERNED ABOUT FIRST ORDER FLUCTUATIONS
FROM THE POINT E_0 WE WRITE

$$E = E_0(1+x)$$

INSERTING THIS INTO $\beta E + b E^{-1/2}$ AND EXPANDING $(1+x)^{-1/2}$ WE FIND

$$\beta E_0(1+x) + \frac{b}{\sqrt{E_0}} \left(1 - \frac{1}{2}x + \frac{3}{8}x^2 + \dots \right)$$

SINCE $\beta = \frac{b}{2\sqrt{E_0}}$, WE SEE THAT FIRST ORDER ERRORS CANCEL TO LEAVE

$$\frac{3}{2} \frac{b}{\sqrt{E_0}} + \frac{3}{8} \frac{x^2}{\sqrt{E_0}}$$

RETURNING NOW TO THE INTEGRAL WITH THE ABOVE ESTIMATION ON
THE EXPONENTIAL TERM ABOUT E_0 WE SEE

$$\text{RATE} = F(E_0) e^{-\frac{3}{2} \frac{b}{\sqrt{E_0}}} \int e^{-\frac{3}{8} \frac{b}{\sqrt{E_0}} x^2} E_0 dx$$

$$\text{where } dE = E_0 dx$$

THIS INTEGRAL IS NOW IN THE GAUSSIAN FORM AND CAN BE EVALUATED TO

$$\sqrt{\frac{8\pi}{3b} \sqrt{E_0}} e^{-\frac{3\beta E_0}{8}} \delta(E-E_0)$$

WHERE THE DELTA FUNCTION $\delta(E-E_0)$ SHOWS THAT EVERYTHING
HAPPENS AT E_0

The ENERGY POINT E_0 IS GIVEN BY

$$E_0 = \left(\frac{b}{2\beta}\right)^{\frac{1}{3}}$$

OR AS OTHERS WRITE

$$E_0 = 1.22(Z_1^2 Z_2^2 A T^2)^{1/2} \text{ keV}$$

$$\text{where } A = \frac{A_1 A_2}{A_1 + A_2} \text{ THE REDUCED ATOMIC WEIGHTS}$$

THIS THEN IS THE LAW USED TO ANALYZE THE REACTION RATE FOR STATS.

Here starts two important topics that Feynman tore into as only he could

THERE ARE TWO CLASSIC QUESTIONS WHICH ARE ASKED ABOUT STELLAR STRUCTURE:

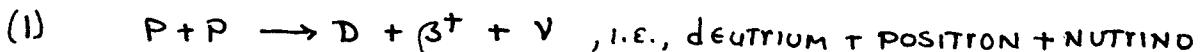
- (i) WHAT IS THE ORIGIN OF THE SOLAR ENERGY
- (ii) WHAT IS THE ORIGIN OF THE PROPORTION OF THE ELEMENTS AND ISOTOPES

EITHER ALL THE CURRENT PROPORTIONS RESULTED DURING SOME GIGANTIC EXPLOSION AT THE TIME OF THE UNIVERSE'S CREATION OR THE PROPORTIONS ARE BEING MAINTAINED BY THE FORMATION OF NEW STARS.

THERE IS EVIDENCE THE HEAVIER ELEMENTS ARE BEING SYNTHESIZED BY SOME STARS. FOR A CERTAIN CLASS OF STARS, CALLED THE GIANT S-TYPE STARS, SHOW AMOUNTS OF TECHNETIUM IN ITS ATMOSPHERE. SINCE THE LONGEST LIVED ISOTOPE OF TECHNETIUM HAS A HALF-LIFE TIME OF 2×10^5 YEARS WHICH IS LESS THE AGE OF THE STARS, IT MUST BE MANUFACTURED BY THE STAR. THE ACTUAL PROCESSES FOR THIS SYNTHESIZING ARE FOR; FOR THE ABOVE PROCESS IT IS THE NEUTRON CAPTURE PROCESS.

WE WILL BEGIN THE DISCUSSION OF THESE PROCESSES BY TALKING ABOUT THE HYDROGEN BURNING PROCESS SINCE MOST OF THE STAR STUFF IS HYDROGEN IN THE STAR'S YOUTH. WE CALL THIS THE PROTON-PROTON (PP) CHAIN. THIS IS, PERHAPS, THE MOST IMPORTANT REACTION FOR MAIN SEQUENCE STARS OR STARS WITH THE TEMPERATURE ABOUT THAT OF THE SUN.

THE CYCLE STARTS OUT AS TWO PROTONS HITTING EACH OTHER,



THE NEUTRINO IS LOST ENERGY BECAUSE IT IS SHOT OUT WITHOUT LOSING ITS ENERGY DURING ANOTHER COLLISION. THE THEORETICAL RATE OF THIS REACTION IS GIVEN BY

$$\text{RATE} = \lambda_{PP} \frac{H^2}{2}$$

where $H^2 = n_1 n_2$

AND $\lambda = \langle \sigma v \rangle$ Thus $\lambda = \lambda(\text{Temp.})$

SINCE σ IS SO SMALL FOR THIS REACTION, IT IS NOT OBSERVED EXPERIMENT BUT IN THE SUN THE SLAMMING AROUND OF PROTON IS SO FREQUENT THAT THE REACTION CROSS-SECTION IS IMPORTANT.

THE ABOVE REACTION IS NOT VERY STABLE BECAUSE IT IS HIGHLY IMPROBABLE THAT THE PROTON FINDS ITSELF IN THE GROUND STATE OF DEUTERIUM. ALSO, IF THIS WERE THE ONLY REACTION, THE STAR WOULD SOON TURN UP AND TURN INTO A DEUTERIUM MASS.

SINCE WE OBSERVE NO DEUTRIUM WE ASSUME EITHER

- (i). THERE MUST BE A TINY MATRIX ELEMENT OF BETA DECAY
- (ii). THERE IS A CHANCE OF NUCLEAR PENETRATION

WHAT (i) MEANS IS THAT THERE IS A SMALL PROBABILITY THAT THE DEUTRON, i.e., THE DEUTERIUM NUCLEUS UNDERGOES TRANSFORMATION INTO STABLE SPECIES BY EMISSION OF EITHER ELECTRONS OR POSITRONS. THESE TWO PROCESSES DEFINE BETA DECAY.

WE INTERPRET (ii) TO MEAN THAT A PROTON CAN PENETRATE INTO THE DEUTRON AND CAUSE A NUCLEAR REACTION,



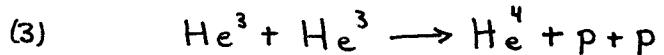
THIS PROCESS HAPPENS VERY EASILY AND THE RATE IS GIVEN BY

$$\text{RATE} = \lambda_{PD} H \cdot D$$

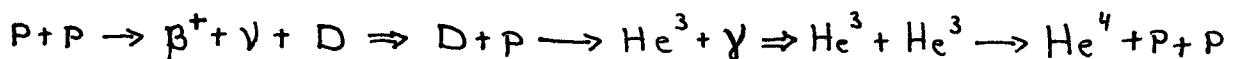
WHERE γ IS A GAMMA OF RADIATION, i.e., HEAT

THE ABOVE TWO PROCESSES ARE COMMON TO THE FOLLOWING POSSIBLE SUBSEQUENT REACTIONS.

WE COULD HAVE THE REACTION OF THE He^3 'S COLLIDING, i.e.,



AND WE SEE WE ARE BACK TO THE TWO PROTON REACTION. THE CYCLE ENDS HERE WITH 7 MeV'S OF ENERGY BEING RELEASED WITH THE NEUTRINO CARRYING AWAY ABOUT 1.9% OF THAT FIGURE. THIS PROCESS HAS THUS BEEN CALLED PROTON-PROTON I AND IS SUMMARIZED AS



THIS PROCESS IS SLOW AND THUS DETERMINES ESSENTIALLY THE COMPLETE RATE OF REACTION. THAT IS, THE OTHER PROCESSES, TO BE DISCUSSED HAPPEN VERY FAST. FOR THOSE INTERESTED, THE FEYNMAN METHOD OF THINKING ACOUSTICALLY WILL HELP TO GET A FEEL FROM WHAT IS GOING ON. THIS PROCESS GOES - BLURP-BLURP-BLURP WHILE THE SUBSEQUENT ONES GO RUR-R-R-R-R! IF THINKING THAT WAY CAN GET FEYNMAN TO HIS PROMINENCE, IT CAN GET YOU THROUGH NUCLEAR REACTIONS.

THESE ARE A SET OF DIFFERENTIAL EQUATIONS WHICH CAN BE WRITTEN FOR THE RATE OF CHANGE OF THE DIFFERENT SUBSTANCES. FOR DEUTERIUM,

$$\frac{dD}{dt} = \lambda_{PP} \frac{H^2}{Z} - \lambda_{PD} H \cdot D$$

WHERE H AND D REPRESENT THE CONCENTRATIONS PRESENT.

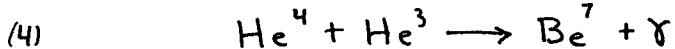
FOR STEADY BURNING, I.E., WHERE THE DEUTERIUM CHANGE IS ESSENTIALLY ZERO AND WE CAN SOLVE FOR THE RATIO OF D/H, I.E.,

$$\frac{\lambda_{PP}}{2\lambda_{PD}} = \frac{D}{H}$$

SINCE $\lambda = \lambda(\text{Temp})$ AT THE 20,000,000 °K OF THE SUN THIS NUMBER IS ABOUT 10^{-18} .

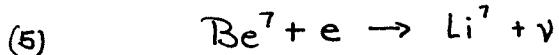
WE CAN SAFELY SAY THAT THERE IS NO DEUTERIUM IN THE SUN OR FOR PRECISENESS ONE PART IN 10^{-18} . WE SHALL NOTE THAT THE RATIO OF D/H FOR SEA WATER IS ABOUT $\frac{1}{2} \times 10^{-4}$. THE NATURE OF THIS MISMATCH IS NOT KNOWN. SINCE THE VIOLATION IS HERE ON EARTH WE MUST HAVE LOST A LOT OF HYDROGEN SOMEWAY.

WE NOW GO ON TO CONSIDER ANOTHER POSSIBLE REACTION, THAT BEING WITH THE PRODUCTION OF He^4 THE POSSIBILITY OF COLLIDING WITH AN He^3 ,

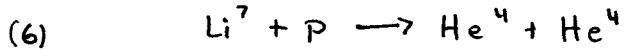


RECALLING OUR FIRST TWO STEPS INCLUDE (1) AND (2) WE SEE ANOTHER GAMMA OF ENERGY IS LOST. AS THE CONCENTRATION OF He^4 BUILDS, REACTION (4) IS MORE PROBABLE THAN (3) BECAUSE IT IS HARDER FOR TWO He^3 'S TO FIND EACH OTHER THAN FOR A He^3 TO FIND A He^4 .

BUT BERYLLIUM SEVEN ISN'T VERY STABLE AND UNDERGOES BETA DECAY TO GIVE,



THIS REACTION IS WEAK BUT LITHIUM SEVEN INTERACTS WITH A PROTON,

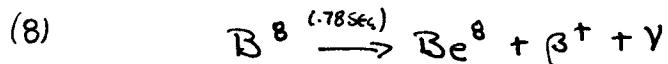


THIS IS A VERY FAST REACTION CONSIDERING THE HIGH CONCENTRATION OF PROTON IN THE STAR.

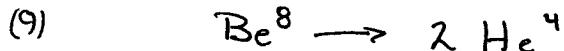
IF WE GO BACK TO STEP (4) AND CONSIDER A PROTON COLLIDING WITH A Be^7 , WE SEE THAT,



BUT BORON EIGHT IMMEDIATELY UNDERGOES DECAY,



THEN, FINALLY



THIS IS CALLED PROTON-PROTON III PROCESS WHILE THE STEPS 4-6 DESCRIBE THE P-P II PROCESS. P-P III LOSES 27.3% OF THE ENERGY PRODUCED TO THE NEUTRINO WHICH HAS SUCH A SMALL (ABOUT ZERO) CROSS-SECTION THAT IT ESCAPES. AMONG THESE PROCESSES P-P II IS MOST IMPORTANT.

So far we have only discussed the proton-proton chain which occurs in young stars. During this process neutrinos are liberated which carry away a small amount of energy. We would like to have a good neutrino detector then we could "see" the center of the star since the shoot right through the mass.

The rate of liberation of neutrinos is so high that at the Earth's surface the flux of incoming neutrinos from the Be^7 decay is

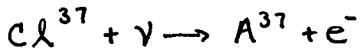
$$\Phi(\text{Be}^7) = 10^{10} \text{ per cm}^2/\text{sec}$$

And for the Boron decay

$$\Phi(\text{B}^8) = 2 \cdot 10^7 \text{ per cm}^2/\text{sec}$$

which is an enormous amount of neutrinos.

Some people have suggested observing these neutrinos by observing the reverse beta decay caused by low energy neutrinos from the sun hitting a barrel of carbon tetrachloride with a little chlorine uniting with a neutrino to form Argon 37 + an electron,



Most of the neutrinos would be observed or come from the P-P III reaction because they have the highest energy even though the above flux ratio says the P-P II reaction produces about a 1000 times as many at the Earth. Even with this large flux only about 4×10^{-35} atoms of Cl^{37} per sec/per atom would undergo this decay. We would need then about 100,000 gallons of carbon tetrachloride to significantly measure this bombardment.

Not only would this experiment be extremely costly but its results would be highly probabilistic because we would only get one result so it couldn't be checked. Such spurious effects as gamma rays and other such things could introduce large errors.

The problem of the century is to devise a neutrino detector capable of great accuracies.

NUCLEAR SYNTHESIS

In the above reaction we start with hydrogen and get out helium. With this reaction it is impossible to get out of this cycle because there is no stable element number 5. It is possible for He^3 and He^4 to combine to Li^7 but that is very unstable and quickly collapses back to He^4 . Maybe $\text{He}^4 + \text{He}^4$ yield Be^8 but Be^8 quickly decays to 2 He^4 so there is little possibility of getting out of the cycle here.

Question: Where do the higher elements come from? Hydrogen burning alone can't explain the process by which the higher elements are formed?

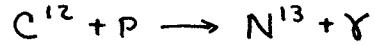
The PROBLEM OF HOW THE HIGHER ELEMENTS WERE FORMED WAS A TRUE MYSTERY DISAPPOINTMENT TO THE THEORIST. THE THEORY OF HYDROGEN BURNING JUST CAN'T EXPLAIN HOW THESE ELEMENTS WERE FORMED. IF WE CALCULATE THE RATIO OF THESE INTERMEDIATE ELEMENTS OR ISOTOPES Li^6 , Li^7 , Be , B^8 TO THE CONCENTRATION OF HYDROGEN WE FIND DUE TO THE EXTREMELY QUICK REACTION THE RATIO IS VIRTUALLY ZERO.

IT IS A MIRACLE THAT THEY EXIST AT ALL AS RARE AS THEY ARE. THERE SEEMS TO BE MORE IN THESE PROPORTIONS THAN CAN BE UNDERSTOOD. THE CURRENT BELIEF IN THE "LARGE" PROPORTIONS OF THE ELEMENTS FOUND ON EARTH SEEMS TO BE DUE TO BOMBARDMENT OF GAMMA RAYS SUCH THAT BITS AND PIECES OF MATTER ARE KNOCKED OFF OF HIGHER ELEMENTS (THIS PROCESS IS CALLED SPALLATION). WHERE THOSE HIGHER ELEMENTS CAME FROM IS QUESTIONABLE AND THE ARGUMENT IS NON-SEQUITOR (I.E., IT BEGS THE QUESTION) UNLESS WE ACCEPT HOYLE'S THEORY - TO BE DISCUSSED.

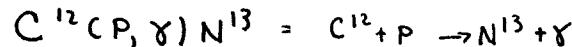
IF THE P-P REACTION WAS THE ONLY ONE, WE WOULD BE IN A LOT OF TROUBLE. MAYBE THE JUNK ON EARTH IS THE RESULT OF SOME STAR THAT EXPLODED AND SPIT THE HEAVIER CRAP OUT TO GIVE US THE PROPORTIONS WE NOW SEE. BUT EVERY ELEMENT PRESENTS ITS OWN PROBLEM AND THEY SEEM TO FORM A REAL CHALLENGE TO FIGURE OUT HOW THEY WERE FORMED. WE'LL COME BACK TO THE QUESTION OF SYNTHESIS A LITTLE LATER. NOW WE TURN TO ANOTHER NUCLEAR REACTION -

CARBON - NITROGEN CYCLE

AS A SIDE REACTION TO THE C-N CYCLE WE WILL DISCUSS THE C-N-O CYCLE OR "BI"-CYCLE AS SOME RATHER CORNLY SAY. WE CONSIDER A COLLISION OF CARBON 12 NUCLEUS WITH A PROTON TO FORM NITROGEN 13 PLUS A GAMMA RAY,

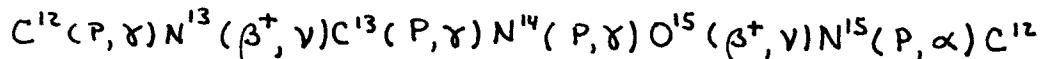


IN A SHORTHAND NOTATION WE SHALL USE,



TO MEAN THE SAME THING.

THE REACTION ONCE STARTED CONTINUES ON IN THE FOLLOWING MANNER,



THE RESULT OF THIS WHOLE CYCLE IS TO PRODUCE BACK THE CARBON NUCLEUS C^{12} PLUS FOUR PROTONS WHICH COMBINE TO FORM A HELIUM NUCLEUS. Thus C^{12} SERVES AS A CATALYST FOR THE PRODUCTION OF HELIUM. RIGHT NOW WE WON'T WORRY WHERE THE C^{12} CAME FROM.

THIS REACTION WAS PROPOSED BEFORE THE PROTON-PROTON CYCLE SO IT WAS NOT KNOWN WHERE THE ENERGY CAME FROM TO GET IT STARTED. ALSO IT IS DIFFICULT TO CALCULATE WHICH REACTION IS DOMINANT.

Another side bar about incompetence in the workplace. Note it is 2 years later that the Peter Principle was published by Laurence Peter, then at USC.

WHILE WE ARE WANDERING ASIDE IT MIGHT BE A POINT OF INTEREST TO POINT OUT THE CLUE TO UNDERSTANDING EVERYTHING. INEVITABLY WHEN A MAN OF COMPETENCE ENTERS INTO A LINE OF BUSINESS THERE IS SOME BLOCKHEAD ABOVE HIM WHO ~~WE~~ HAS TO BE AN INCOMPETENT IDIOT. ~~HOW DID~~ HE GET THERE? IF THE ~~LESSER~~ MAN EXHIBITS HIS SKILLS HE SOON WILL BE PROMOTED TO A POSITION OF HIGHER STATURE AND RESPONSIBILITY. AS HIS COMPETENCE INCREASES SO DOES HIS IMPORTANCE UNTIL HE TOO IS PROMOTED OUT OF HIS OWN LIMITS AND HE BECOMES THE BUNGLING FOOL HE ONCE SCORNED BY THEN, HOWEVER, HE DOESN'T LOOK AROUND AND IS LESS CRITICAL OF HIS PEERS AND SUPERIORS.

CONTINUING ON WITH THE CARBON-NITROGEN CYCLE WE MUST NOTE THAT FOR THIS CYCLE TO BEGIN REQUIRES THE PRESENCE OF CARBON. WHERE DID IT COME FROM? IT ISN'T A STABLE PRODUCT OF THE P-P CYCLE YET IT IS PRESENT.

AFTER THE HYDROGEN IS EXHAUSTED THE STAR GOES THROUGH A GRAVITATIONAL CONTRACTION WHICH RESULTS IN A MUCH HIGHER TEMPERATURE AND DENSITY, I.E., ABOUT 10^8 K AND 10^5 GM/CC RESPECTIVELY. THESE ARE THE RED GIANTS. AT THESE EXTREME CONDITIONS HELIUM IS FUSED WITH ITSELF TO FORM CARBON. THE REACTION WAS PROPOSED BY SALPETER

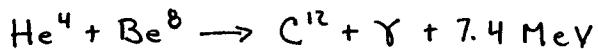


NOT ONLY DOES THIS REACTION CONSUME A LOT OF ENERGY BUT ALSO THE BERYLLIUM IS VERY UNSTABLE IN ITS GROUND STATE AND DISINTEGRATES BACK INTO TWO HELIUM ATOMS IN ABOUT 10^{-14} SEC. SO HOW DO WE GET CARBON?

THERE IS A STRONG RESONANCE IN THE $\text{He}^4 + \text{He}^4$ REACTION AT ABOUT 310 KEV SUCH THAT THE HELIUM PIDDLES AROUND LONG ENOUGH FOR THE Be^8 CONCENTRATION TO BUILD UP ENOUGH SO THAT

$$\frac{\text{Be}^8}{\text{He}^4} \approx 10^{-9}$$

WHILE 10^{-9} OF SOMETHING ISN'T MUCH OF ANYTHING IT IS NOT NOTHING. SO, THERE IS THE POSSIBILITY OF THE FOLLOWING REACTION,



SALPETER ESTIMATED ABOUT 1 IN 10^{10} NUCLEI IN A STAR IS A Be^8 IN DYNAMICAL EQUILIBRIUM IN THE ABOVE PROCESS.

TO SAY A REACTION CONSUMES ENERGY OR IS ENDOTHERMIC MEANS, IN THE CASE ABOVE, THAT THE BOUND STATE OF THE BERYLLIUM ATOM IS -95 KEV BELOW THE IONIZATION POTENTIAL WHERE THE ELECTRONS WOULD BE FREE.

Hoyle, in proposing his steady state theory, argued that the initial stellar matter was hydrogen. As such it would be extremely unlikely that the resulting temperatures could not possibly create enough Be^8 to unite with He^4 to create C^{12} and thus get the higher elements going. Hoyle proposed that there must be an energy level at 7.360 MeV of even parity and spin 0 for a decay to $\text{C}^{12} + \gamma$. His theory rested upon this point otherwise he couldn't explain how the other elements were formed. Having announced this to the nuclear physicists who wouldn't believe ~~it~~ they had missed such a level in spectroscopic examination of carbon, they searched and found a level at 7.625 MeV of the right spin and parity. This then explained how the heat kept building in the red giant.

Hoyle had theorized that during nuclear reaction of the first generation star, i.e., the hydrogen burners some of the heavier elements were synthesized. Second generation stars which congealed from the remains of a first generation contained these higher elements but burnt, primarily, helium. But the cycle is repeated and the heavier elements get more plentiful. But Hoyle had to get the heat from somewhere to get the C^{12} formed and he was correct in his prediction.

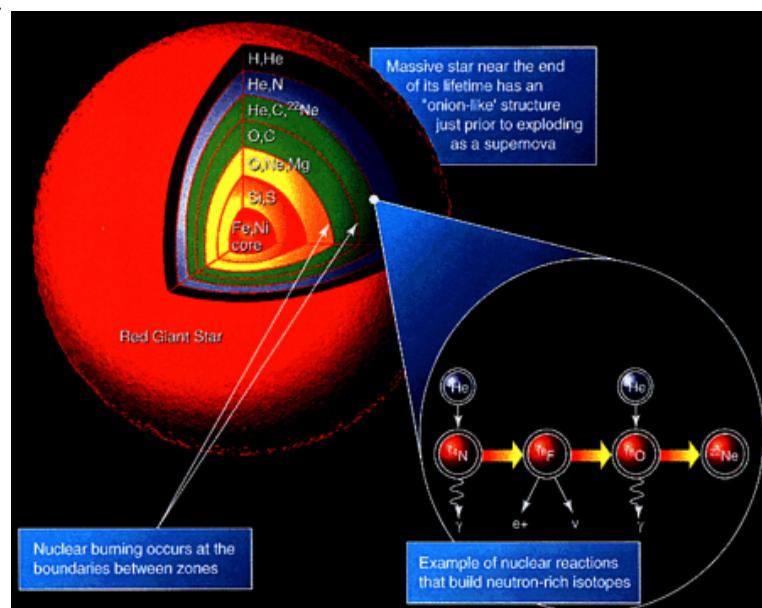
However, he unfortunately abandoned his theory when exaggerated results concerning quasars led him to believe his theory was inconsistent. With a more careful study he wishes now that he hadn't made his statement. For not only does his theory stand as a milestone in astro-physics but it represents the power of careful thinking and reflecting the properties of the universe into the atom.

Feynman on an accidental universe?

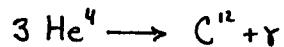
WHAT WE REALLY NEED TO STUDY are the MIRACLES of the MYSTERIES in the UNIVERSE. The CHARACTER of the UNIVERSE depends on TOO MANY "ACCIDENTS." IT SEEMS UNLIKELY THAT GOD GAVE HOYLE AN ENERGY LEVEL SO HIS THEORY WORKED. OTHER "ACCIDENTS" LIKE THE MASS of THE PROTON BEING A TINY bit SMALLER than the NEUTRON HAVE FAR REACHING CONSEQUENCES AS TO THE NATURE of our UNIVERSE. JUST HOW MUCH of WHAT WE SEE DEPENDS ON ACCIDENTS?

Stellar nucleosynthesis refers to the assembly of the natural abundances of the chemical elements by nuclear reactions occurring in the cores of stars. Those stars evolve (age) owing to the associated changes in the abundances of the elements within. Those stars lose most of their mass when it is ejected late in the stellar lifetimes, thereby enriching the interstellar gas in the abundances of elements heavier than helium. For the creation of elements during the explosion of a star, the term supernova nucleosynthesis is used. The goal is to understand the vastly differing abundances of the chemical elements and their several isotopes as a process of natural history.

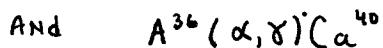
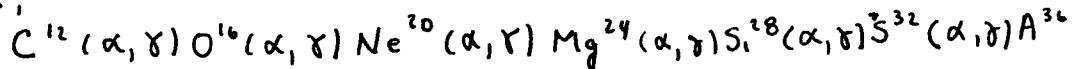
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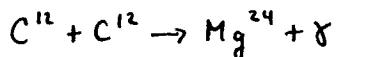
IT TURNS OUT THE REACTION, $\text{He}^4 + \text{Be}^8 \rightarrow \text{C}^{12} + \gamma$, IS A DARN HARD WAY TO MAKE C^{12} BUT IT IS MADE ANYWAY. PERHAPS THERE IS ANOTHER WAY IT CAN GET GOING AND, INDEED, THERE IS:



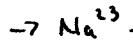
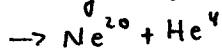
THAT IS, IN THE CORE OF RED GIANTS THREE HELIUMS GET TOGETHER FASTER THAN A HELIUM CAN FIND A SCATTER BERYLLIUM ATOM TO SMASH AGAINST AND FOR C^{12} . THIS REACTION TENDS TO PROMOTE THE PRODUCTION OF C^{12} WHICH CAN THEN UNDERGO A WHOLE STRING OF SUCCESS REACTIONS,



SO WE HAVE RUN THE GAMUT FROM C^{12} TO Ca^{40} BY THIS α -PROCESS. IF THE STAR BURNS IT HELIUM UP, IT MIGHT UNDERGO ANOTHER GRAVITATIONAL COLLAPSE AND HEAT STILL HIGHER. IF IT DOESN'T BLOW UP, CARBON NUCLEI CAN COLLIDE AND FORM,



OR



OTHER REACTIONS WITH OXYGEN, AND NEON NUCLEI WITH CARBON COULD FORM ALL THE ELEMENTS UP TO IRON. IF THE STAR BECAME UNSTABLE IN THE INTERIM IT COULD EXPLODE AS HOYLE PREDICTS AND SPRAY THE SPACE WITH THE HEAVIER ELEMENT WHICH COULD COLLECT TO FORM THE HIGHER GENERATION STARS AS HE CALLS THEM.

FOR A TYPICAL STAR OF RELATIVE SUN MASS, IF THE ENERGY CONSUMPTION RATE IS:

$$M_\odot \sim 2 \frac{\text{ERGS}}{\text{GM/SEC}}$$

THEN IT IS PRIMARILY A PROTON-PROTON REACTOR.

IF THE RATE IS HIGHER BUT OF SIMILAR MASS

$$M_\odot \sim 10^3 \frac{\text{ERGS}}{\text{GM/SEC}} \quad \rho \sim 10^{4.5} \quad T \sim 10^8$$

THE RATES OF REACTION WILL BE GREATLY ACCELERATED.

WHILE FOR A MASS AND RATE OF

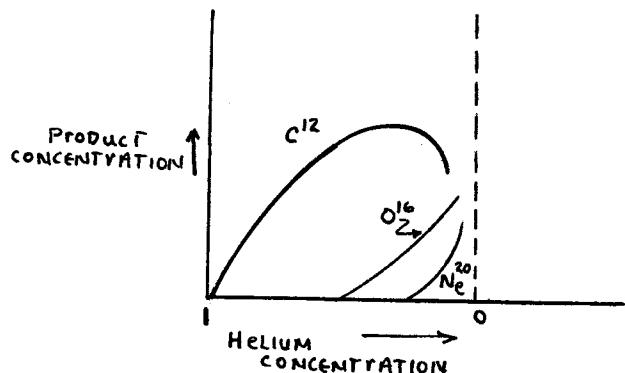
$$10M_\odot \sim 10^7 \frac{\text{ERGS}}{\text{GM/SEC}} \quad \rho \sim 10^{2.5} \quad T \sim 2.5 \times 10^8$$

WE ARE IN THE HELIUM-RED GIANT PHASE.

CHAPTER 6

PROPORTIONS OF THE ELEMENTS IN A STAR

THE NEXT QUESTION TO ASK IS IF THE HIGHER ELEMENTS ARE PRODUCED, IN WHAT PROPORTIONS DO THEY APPEAR. TO SHOW THE EMPIRICAL DISTRIBUTION WE WILL PLOT C^{12} , O^{16} AND Ne^{20} AGAINST THE HELIUM CONCENTRATION, THAT IS, WE WILL EXHIBIT HOW THE FLUCTUATION OF THE HIGHER ELEMENTS TUN AS THE HELIUM IS SLOWLY CONSUMED.



CURVES SHOWING RELATIVE CONCENTRATION OF C^{12} , O^{16} , AND Ne^{20} AS A FUNCTION OF THE HELIUM CONSUMPTION.

THE ABOVE GRAPH IS AT BEST AN INTELLIGENT GUESS AS TO THE ACTUAL PROPORTIONS. ONE PROBLEM, AS DISCUSSED A LITTLE EARLIER, IS AT LOW TEMPERATURES THE $C^{12}(\alpha, \gamma)$ RATE IS NOT WELL-KNOWN; SO WE DON'T REALLY KNOW HOW FAST IT GETS GOING. ALSO THE HIGHER ELEMENTS ARE FORMED VERY SLOWLY DUE TO THEIR CONCENTRATION AND AS THE HELIUM BURNS OUT IT IS HARD TO PREDICT WHAT PROPORTIONS THE OTHER ELEMENTS WILL ASSUME AS THEY START TO REACT WITH EACH OTHER - PROVIDED THE STAR REMAINS STABLE.

OF A LITTLE MORE INTEREST. THE PROPORTION OF THE ELEMENTS IN THE SUN ARE ABOUT

C^{12}	N_e^{14}	O^{16}	Ne^{20}	Mg^{24}
5.5	1.0	9.6	5	0.3

THE PROPORTION OF Ne^{20} TO O^{16} PRESENTS A PROBLEM IN EXPLAIN WHY WE SEE SO MUCH NEON. IN ORDER TO GET ESSENTIALLY A 2:1 RATIO THE TEMPERATURE MUST BE VERY HIGH TO HAVE O^{16} GO OVER TO Ne^{20} . BUT Ne^{20} HAS A LARGE CROSS-SECTION AND WOULD GO RIGHT ON TO MAGNESIUM WHICH ODDLY ENOUGH IS ~~ABOUT~~ ABOUT 17% LESS CONCENTRATED THAN Ne^{20} . ONE SOURCE OF THE INCONSISTENCY MIGHT BE THAT THE Ne^{20} ABUNDANCE IS HARD TO DETERMINE RELATING TO THE OTHERS BECAUSE IT HAS A HIGH IONIZATION AND HARD TO SEE SPECTROSCOPICALLY.

SO WE HOPE THE Ne^{20} CONCENTRATION IS WRONG OR ELSE WE HAVE A LOT OF EXPLAINING TO DO.

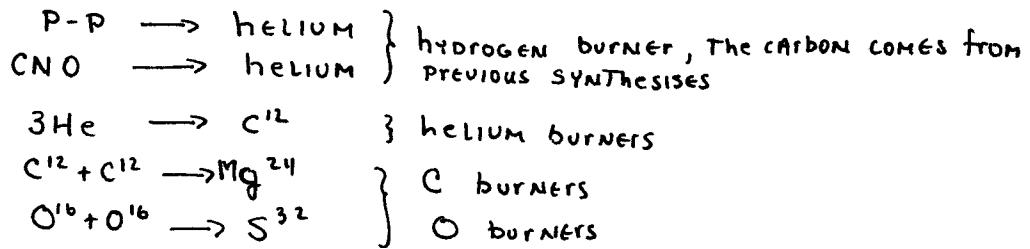
WE HAVE PRETTY WELL COVERED THE ELEMENT PROPORTIONS OR PRODUCTION. WITH THE REACTIONS MENTIONED ON THE PREVIOUS PAGE AS THE HELIUM BURNS OUT AND THE TEMPERATURE APPROACHES 10^9 °K THE HIGHER ELEMENTS BECOME POSSIBLE, I.E., UP TO IRON. BUT AS THE TEMPERATURE RISES, THE NEUTRINO PRODUCTION BECOMES MORE IMPORTANT AND WE NOW TURN TO IT.

FIRST, HOWEVER WE MIGHT MENTION THAT WE HAVE BECOME ENgrossed IN A SIDETRACK FROM OUR MAIN GOAL. WE SET OUT TO SOLVE THE DIFFERENTIAL EQUATIONS OF A STAR AS EARLIER DESCRIBED BY SEEKING TO FIND EXPRESSIONS FOR THE OPACITY AND ENERGY GENERATION. TO THIS END WE HAVE BECOME BOGGED DOWN IN THE NUCLEAR REACTIONS AND SYNTHESIS INSIDE STARS. WE WILL GET BACK TO THE PROBLEM OF SOLVING THOSE EQUATIONS IN A WHILE BUT NOW WE WILL STAY ON OUR SIDETRACK.

AS AN OUTLINE TO DATE WE CAN COLLECT THE NUCLEAR SYNTHESIS AS FOLLOWS

- (1). He^4 (helium) WAS OBTAINED BY THE HYDROGEN BURNING PROCESS WHILE He^3 WAS AN INCOMPLETE PROCESS OF THE P-P CHAIN
- (2). DEUTERIUM, LITHIUM, BERYLLIUM, AND BORON WE BY-PASSED DURING THIS P-P PROCESS AND WE SO UNSTABLE THAT THEY COULD NOT HAVE RESULTED FROM THE INTERNAL COOKING OF A STAR. WE PROPOSE SOME NON-THERMAL PROCESS IS INVOLVED WHICH STIRS THE GOOP UP TO THE SURFACE; PERHAPS, THE SOLAR FLARES OR EXPLOSIONS GENERATE HIGH ENERGY PARTICLES WHICH BOMBARD HELIUM AND CREATE SUCH THINGS AS LITHIUM,
- (3). C^{12} , O^{16} THE NEXT TWO MOST ABUNDANT ELEMENTS COME FROM HELIUM BURNING. FROM THIS PROCESS WE CAN ALSO GET O^{18} & Ne^{20}

THESE CONVENTIONAL NUCLEAR PROCESS ARE



WE RECALL WE ONCE TALKED ABOUT POPULATION I AND II STARS WHERE THE I'S WE CLASSIFIED AS THOSE IN THE ARMS OF SPIRAL GALAXIES AND THE II'S WERE IN THE GLOBULAR CLUSTERS. THE II'S ARE VERY POOR OF THE HEAVIER ELEMENTS AND WERE MADE WHEN HYDROGEN WAS PREVALENT. WHILE THE I'S ARE YOUNGER AND RICHER IN THE HIGHER ELEMENT. THERE IS SOME SUPPORT IN HOYLE'S EXPLANATION THAT THE I'S WERE A RESULT OF A CONDENSATION FOLLOWING AN EXPLOSION OF A II. BUT THE SPIRAL ARMS MIGHT HAVE BEEN FORMED DIFFERENTLY OR THE CORE OF THE SPIRAL COULD HAVE SQUIRTED OUT THE DUST AND CRAP IN SOME UNBEKNOWN FASHION. THERE ARE NOTHING BUT QUESTIONS HERE. WE JUST DON'T UNDERSTAND HOW THE STUFF GOT OUT FROM THE INSIDE OF THE STARS.

- WE MISSED STEPS 4 AND 5 AND THEY ARE
- (4) Ne^{20} , Na^{23} , Mg^{24} , Al , Si^{28} FROM CARBON BURNING
 - (5) Mg , Al , Si , P , S^{32} FROM OXYGEN BURNING

SO WHAT NEXT?

WHAT ABOUT THE ABUNDANCE OF ELEMENTS AROUND IRON? CAN THE ABUNDANCE BE EXPLAINED BY SOME INTERNAL STELLAR PROCESS OR PERHAPS IS IT DUE TO AN EXPLOSION. AFTER HELIUM HAS BEEN BURNED WE ARE LEFT WITH PRIMARILY Si^{28} AND S^{32} WITH Si BEING VERY ABUNDANT AFTER THE OXYGEN BURNING. WE HAVE TO STUDY SOME NUCLEAR PROPERTIES.

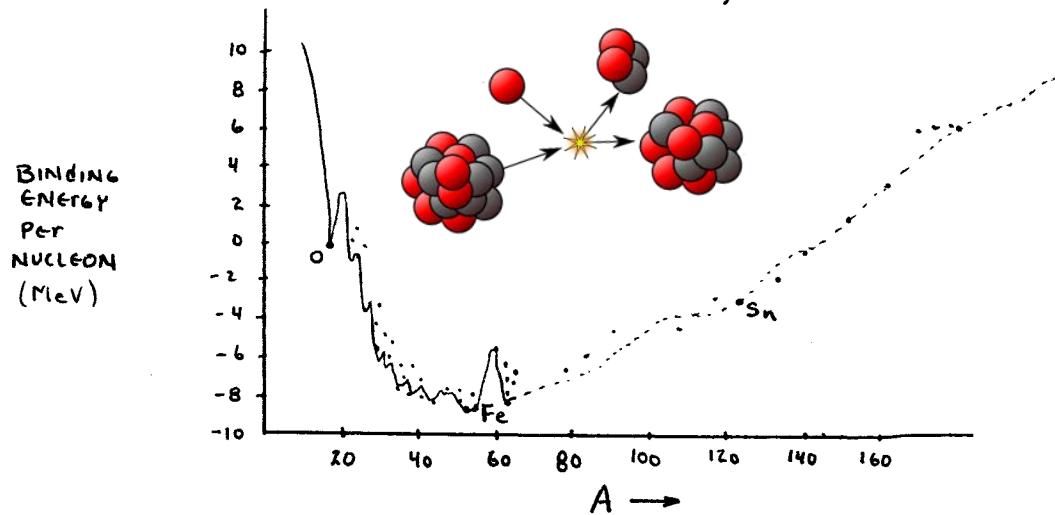
WE WANT TO PLOT THE PACKING FRACTION AS A FUNCTION OF A TO GIVE THE VARIATION OF THE AVERAGE BINDING ENERGY PER NUCLEON. THE BINDING ENERGY OF NUCLEUS IS A MEASURE OF HOW MUCH ENERGY IT TAKES TO KNOCK IT INTO PROTONS AND NEUTRONS. THIS BINDING ENERGY IS RELATED TO THE DIFFERENCE OF MASS OF THE NUCLEUS AND THE SUM OF THE NEUTRONS AND PROTONS, I.E., THE MASS DEFECT ΔM . THIS, BY EINSTEIN, GIVES US THE ENERGY

$$B = \Delta M c^2$$

EXPANDED OUT IN TERMS OF THE MASS OF A NEUTRAL ATOM $M(Z, N)$ WHERE Z EQUALS THE PROTONS AND N THE NEUTRONS AND PROTON AND NEUTRON MASSES m_p AND m_n RESPECTIVELY,

$$B(Z, N) = [m_p Z + m_n N - M(Z, N)] c^2$$

THE PACKING FRACTION IS THEN B/A WHERE A IS THE NUMBER OF NUCLEONS. THE VALUES m_p AND m_n ARE 1.00759 AND 1.00898 RESPECTIVELY TO MAKE THE PLOT WE GET SOMETHING LIKE,

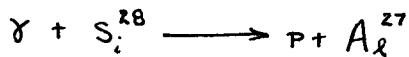
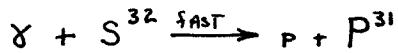


SO HOW DO WE EXPLAIN ALL THOSE DOTS? AS WE MIGHT EXPECT AS HELIUM RUNS OUT AND THE BURNER TURNS OFF, THERE IS NO PRESSURE TO HOLD THE STAR UP AND IT WILL COLLAPSE WHICH CAUSES THE TEMPERATURE TO RISE TO THE POINT THAT MAGNESIUM AND SILICON CAN REACT WITH THEMSELVES TO FORM STILL HIGHER ELEMENTS. BUT THE COULOMB POTENTIALS OF THESE NUCLEI ARE SO HIGH THAT SOMETHING ELSE MUST SHRELY HAPPEN BEFORE THE TEMPERATURE GETS HOT ENOUGH SO THE REACTIONS CAN GO.

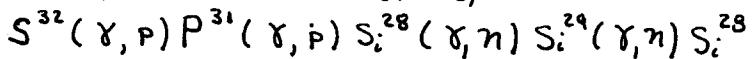
Binding energy is the mechanical energy required to disassemble a whole into separate parts. A bound system typically has a lower potential energy than its constituent parts; this is what keeps the system together—often this means that energy is released upon the creation of a bound state.



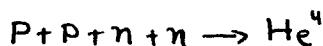
And, in fact, hot gamma ray reactions get going to cause the following,



Whereupon, the ~~energetic~~ ^{ENERGETIC} GAMMAS ~~PROTON~~ PROTON quickly eats the phosphorous and we eventual get back stable silicon 28,



We thus generate a lot of protons and neutrons which can bang into other nuclei and eventual form helium



The temperature is around 2.5×10^9 °K for gamma ray reactions.

If the temperature goes on up to 3×10^9 °K then all these chains like $Si^{28}(\gamma, p) Al^{27}$ get enormously complicated. The net result is the decay period of neutrons being 3×10^3 sec. or about one hours says things happen pretty fast. If we go on up in temperatures, further reactions become endo-thermic or they absorb energy as shown by a decrease in binding energy.

But the reaction rates are so fast and furious protons can exist in equilibrium with the ~~the~~ nuclei. So we have to shift our analysis to one of statistical mechanics to explain how the proton goes around trying to find its lowest energy level.

That is, to get the number of nuclei with A, Z at a given temperature T ,

$$N(A, Z) = G(A, Z) \frac{(2\pi M kT)^{3/2}}{\hbar^{3/2}} A^{3/2} e^{-E(A, Z)/kT} e^{-\mu_p Z/kT} e^{-\mu_n (A-Z)/kT}$$

where M = mass of nucleus

μ_p, μ_n = ^{DENSITY} mass of proton, neutron

$E(A, Z)$ = nuclear energy

$G(A, Z)$ = spin factor

where we can find the concentration of proton to be,

$$n_p = 2 \frac{(2\pi M_p kT)^{3/2}}{\hbar T} e^0 e^{-\mu_p / kT}$$

The nuclear energy is what we discussed earlier and called $B(M, Z)$

$$E(A, Z) = C^2 [-M(A, Z) + Z M_p + (A-Z) M_n]$$

ACTUALLY THE BOTTOM OF OUR PACKING PLOT IS NOT TOO WELL DEFINED AND IT IS A TRADE OFF AS TO WHICH ELEMENT AMONG SUCH THINGS AS Fe^{54} , Fe^{56} , Ni^{56} OCCUPIES THAT MINIMUM VALUE. THIS MINIMUM VALUE IS NOT TEMPERATURE DEPENDENT AS WOULD BE ANTICIPATED BUT RATHER GOES WITH THE CONCENTRATION OF PROTONS AND NEUTRONS, n_p, n_n .

If, in fact, $n_p = n_n$, then the tendency is towards Nickel. We expect to reach this equal concentration if we start with Si^{28} . Why? Because γ REACTIONS DO NOT CHANGE THE CHARGE NOR DO WE LOSE ANYTHING BUT SOME ENERGY DURING β -DECAY.

WE DO NOT UNDERSTAND ~~where~~ where the elements to the right of the minimum come from. They have a positive binding energy and could not have have been formed by star cookery.

ANOTHER PROBLEM IS FOR INCREASED ENERGY OR TEMPERATURE Fe^{56} COLLAPSES BACK TO HELIUM SO WHAT HAPPENS AFTER THE HOUR? IS EVERYTHING THAT IS SO WELL CONSTRUCTED DESTROYED BY A CATASTROPHIC EXPLOSION?

TO ADD FURTHER MISAPPREHENSION TO THIS WHOLE CONFUSED AND PROBLEM RIDDLED THEORY TO GUYS NAMED FEYNMAN AND GELLMANN INTRODUCED A NEW THEORY ON NEUTRINO PROCESSES WHICH THREW A MONKEY WRENCH IN THIS ASTROPHYSICIST'S THEORY.

BETA-DECAY

THE FIRST PROCESS WE WILL MENTION IS ONE WE HAVE BEEN DISCUSSING but will define and make a little clearer here. MOST OF THE ARTIFICIALLY PRODUCED RADIOACTIVE NUCLEI UNDERGO TRANSFORMATION INTO STABLE SPECIES BY EMISSION OF EITHER ELECTRONS OR POSITRONS - COLLECTIVELY THIS IS β -DECAY. If A NUCLEUS IS UNSTABLE BECAUSE IT HAS TOO MANY NEUTRONS, IT WILL EJECT AN ELECTRON. If IT HAS TOO MANY PROTONS, IT EJECTS A POSITRON. The former are stabilized by conversion of A NEUTRON INTO AN ELECTRON (which leaves the nucleus) AND A PROTON. The latter undergo conversion of A PROTON INTO A NEUTRON PLUS A POSITRON, which is emitted from the nucleus. BOTH EJECTED POSITRON AND ELECTRONS HAVE ALL POSSIBLE VELOCITIES.

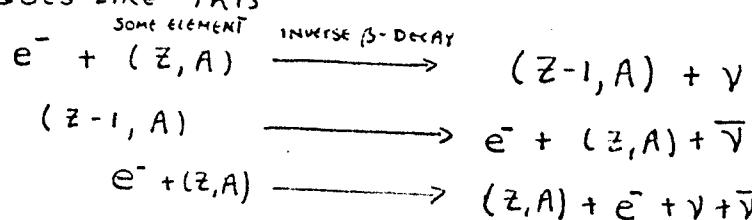
IT IS POSSIBLE TO ACHIEVE STABILITY ANOTHER WAY AND THAT IS BY HAVING AN ELECTRON ENTER INTO THE NUCLEUS AND REACT WITH A PROTON TO FORM A NEUTRON. THE ENERGY FORMED IN THIS PROCESS IS LIBERATED BY A NEUTRINO WHICH HAS A ZERO REST MASS. DECAY OF THIS KIND IS CALLED ELECTRON OR K-CAPTURE SINCE THE ELECTRON CAPTURED BY THE NUCLEUS GENERALLY COMES FROM THE INNER-MOST ELECTRON ORBIT, THE K-LEVEL.

AN EXAMPLE OF ELECTRON CAPTURE,



URCA - PROCESS

The second process which we discuss is that theorized by Feynman and Gell-Mann in 1957. The process is called the $u\bar{u} + d\bar{d}$, $K\bar{K}$ process. The process goes like this



This process says an electron is driven into the nucleus and it is called the URCA (The Gambling Capital of the world, in Brazil, at the time of the writing) process because either way the process goes it losses a little energy. But it is a way to generate neutrinos and anti-neutrinos without any fundamental changes occurring. But the neutrinos carry away a certain amount of energy, again the game goes that the guts can't hold up the star; it begins to collapse and get hotter. This gets the photons going better; more electrons get driven into the nucleus and poof there goes a supernova or something. However, this process is overshadowed by an alternative one.

~~NEW~~ This ~~wrong~~ process $e^- + e^+ \rightarrow \nu + \bar{\nu}$ produces a strange effect and that is to limit the time which it could occur. That is, the cross section for this reaction is so small $\left[\sigma = 1.4 \times 10^{-45} \left(\frac{c}{v_{\text{relative}}} \right) \left(\frac{\omega^2 - 1}{\text{center of mass energy}} \right) \text{cm}^2 \right]$ that the positron generated by the protons as mentioned on the previous page at a given temperature ~~thus~~ possess a high rate of energy loss to the neutrinos, i.e.,

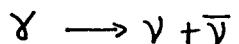
$$\begin{aligned} \frac{dM\nu}{dt} &= 4.6 \times 10^{15} (T \times 10^9)^2 \frac{\text{erg}}{\text{cm}^3/\text{sec}} \text{ for } T > 3 \times 10^9^\circ \\ &= 5 \times 10^{18} \end{aligned}$$

At temperatures of the order of $3 \times 10^9^\circ \text{K}$ the time to empty the mess of its thermal energy is,

$$\frac{\text{Thermal Energy}}{\text{Rate of } \nu \text{ Emission}} = 2 \text{ hours}$$

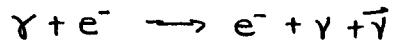
There must be an implosion or chaos or something screwy going on. This process has shook up a few astro-theorists but went unnoticed for awhile. We have to go further though to seek some sort of credible explanation. There are a couple more processes which we can still discuss as part of this URCA process.

The process involving a similar $\gamma + \bar{\gamma}$ production results from a plasmon decay, i.e.,



NOTE: we have previously used γ to denote a gamma ray; here we give it the meaning of a fundamental particle the plasmon. This process will go at the right plasma frequency.

Also, there is the possibility of the plasmon interacting with an electron in the following way,



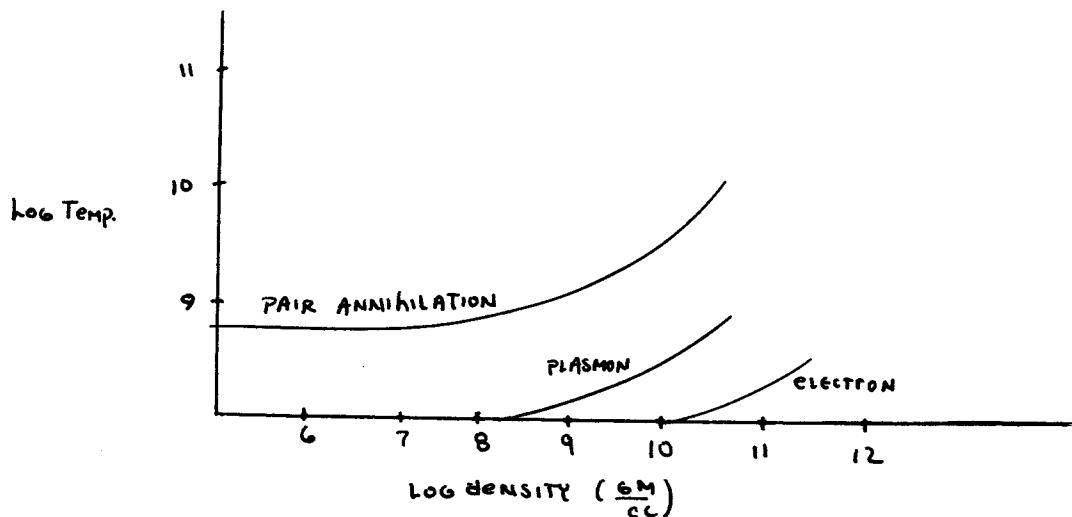
The two component neutrinos are not invariant under spatial reflexion. That is, the neutrino spin is anti-parallel to its momentum and while the spin or angular momentum remains unchanged under a spatial reflexion, the momentum changes sign. The neutrino ~~is~~ remains unchanged only if we perform simultaneous spatial reflexion and charge conjugation. Thus the parity conservation law is violated by these particles.

A charge conjugation simply means particle-anti-particle flipping of notation. Like a positron to an electron. Interesting to note the neutrino has no electrical charge, will not interact with an electro-magnetic field, a spin of $1/2$, and rest mass of zero. But it still does a lot for its "nothingness"

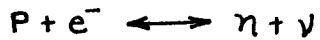
SUMMARIZING OUR PAIR PRODUCTION



We can plot the rates of pair production to see which is dominant and we get,

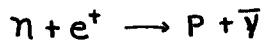


Our problem is to explain why process (i) is favored since experimentally the cross-section is so small we cannot observe it. However beta-decay is ~~far~~ understood to a high degree today since the CP, charge and parity violation has been straightened out. But the existence of normal beta decay is not clear. That is to say the direction of beta decay

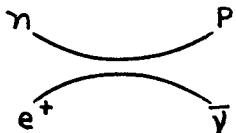


depends on the energy of the system. More explicitly, Fermi originally postulated β -decay was analogous to electromagnetic radiation and as such the ~~E-M~~ E-M radiation is due to a time-dependent interaction between the radiation system (the atom) and its surrounding E-M field. The interaction leads to an exchange of energy between the system and the field.

The above reaction can be rewritten differently in terms of the anti-particles but the reaction is the same



Thus we can think of the reaction as occurring in the following schematic diagram,



That is there is a transition from a neutron to a proton while simultaneously there is a transition of a positron into an anti-neutrino. While we don't know why the reaction goes we can describe its existence in terms of a transition amplitude of the two reactions just mentioned.

To express this rate of beta decay arising by a certain transition amplitude between these particle we will adopt a shorthand notation. If the beta decay was rewritten still another way, remembering to make a charge conjugation when we take the particle from one side to the other, i.e.,



we write

$$\alpha'(e^- \bar{\nu})(\bar{P} n)$$

The α' is a coefficient which we will discuss in a minute. The terms in parenthesis represent the amplitudes (with little factors omitted which tell whether the spin of the neutron and proton is up or down but spin in both are the same) of β -decay and this amplitude does depend on the spin direction. However the world is so constructed that the Hamiltonian of this reaction would have an amplitude such that it describes a unidirectional transition so we must add the hermitian conjugate of the above reaction to permit real expectation values to be obtained.

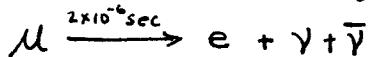
We thus preserve the hermiticity of the HAMILTONIAN by writing it as

$$a' [(e^- \nu)(\bar{p}N) + (\bar{N}P)(\bar{\nu}e)]$$

where the coefficient is the same in both processes.

At the time of the URCA process publication there were other weak decay processes which had to be considered for a complete description of the fundamental processes. Unfortunately, the existence of other weak interaction led to an inherent coupling between the processes which completely complicated the mathematical description.

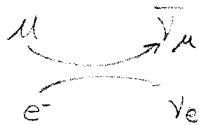
There is present a mu-meson or muon decay which goes as,



The neutrino and anti-neutrino evolved are associated with the mu-meson and electron respectively, i.e.,



We can rewrite the equation as

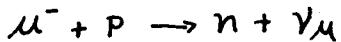


The combined Hamiltonian analogously to the above reaction

$$a' [(e^- \nu)(\bar{\nu}_\mu \mu) + (\bar{\mu} \nu_\mu)(\bar{\nu} e)]$$

It is noted that the neutrinos are not the same and somehow they "remember" where they came from. That is the neutrino "knows" it is a mu-meson neutrino and the anti-neutrino knows it is an electron anti-n.

We have still another mode of transition and that is for a negative muon. It is called muon capture; the μ^- is captured by a nucleus like the K-capture. This process is,

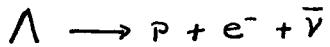


The transition amplitude can be written as

$$a'' [(\bar{\mu} \nu)(\bar{p}N) + (\bar{N}P)(\bar{\nu}_\mu \mu)]$$

where again the anti-neutron remembers it is a mu-anti neutrino. Note that all the coefficients a', a, a'' are different so far but we will soon discuss their interrelationships.

Still another set of decay mode has been predicted involving the Lambda (Λ) particle. One such decay is as follows:



SUMMARIZING THOSE REACTION RATES

$$b[(e^- \gamma)(\bar{p} \Lambda) + (\bar{\Lambda} p)(\bar{\nu} e)]$$

$$b'[(\bar{\mu} \nu_\mu)(\bar{p} \Lambda) + (\bar{\Lambda} p)(\bar{\nu}_\mu \mu)]$$

$$b''[(\bar{N} p)(\bar{p} \Lambda) + (\bar{\Lambda} p)(\bar{p} N)]$$

IS THIS CORRECT?

THESE EQUATIONS PLUS THE OTHER THREE CONSTITUTE ALL OF BETA DECAY.
THE PROBLEM IS THERE ARE ALL COUPLED TOGETHER. THIS COUPLING COMES
SOME FROM THE COEFFICIENTS WHICH ARE NOT ALL EQUAL BUT FOUND TO BE,

$$\alpha' = .97\alpha$$

$$\alpha'' = \alpha' \text{ WITHIN } \pm 1 \text{ OR } 2\%$$

$$b = (.24)\alpha \pm 10\%$$

$$b' \approx b \pm 10\%$$

$$b'' \approx b' \text{ PRETTY WELL}$$

SO WHAT DO WE DO?

WELL, WE SEE WE CAN EXPRESS ALL THE HAMILTONIANS IN TERMS OF α , I.E., WRITING THE FIRST PART OF THE HAMILTONIAN,

$$\alpha[(e^- \gamma)(\bar{\nu}_\mu \mu)] \quad (1)$$

$$.97\alpha[(\bar{e} \gamma)(\bar{p} N)] \quad (2)$$

$$.97\alpha[(\bar{\mu} \nu)(\bar{p} N)] \quad (3)$$

$$(.24\alpha)[(\bar{e} \gamma)(\bar{p} \Lambda)] \quad (4)$$

$$(.24\alpha)[(\bar{\mu} \nu_\mu)(\bar{p} \Lambda)] \quad (5)$$

$$(.24\alpha)[(\bar{N} p)(\bar{p} \Lambda)] \quad (6)$$

NOW WE SEE WHERE THE PROBLEM REALLY IS AND THAT IS THE COUPLING BETWEEN THE REACTIONS. COUPLING SUCH AS EQUATION (1) AND (2) AND (3) AND (5) ETC. TO UNCOUPLE THESE EQUATIONS WE GUESS AT A FORM SOMETHING LIKE THE FOLLOWING FOR THE TRANSITION AMPLITUDE PROPORTIONAL TO $J^* J$

$$\alpha \left\{ (e^- \gamma) + (\bar{\mu} \nu_\mu) + [\bar{p} (.97N + .24\Lambda)] \right\} \left\{ (\bar{\nu} e^+) + \bar{\nu}_\mu \mu + [(.97\bar{N} + .24\bar{\Lambda}) p] \right\}$$

TO SEE WHY THIS WAS CHOSEN LET'S EXPAND IT LIKE WE WOULD

$$(a+b+c)(d+e+f)$$

where $J = \sum (\bar{B} A) = (\bar{e} \gamma) + (\bar{N} p) + (\bar{\mu} \nu) + (\text{STRANGE PARTICLE})$

THE CROSS TERM $(\bar{p} N)(\bar{e} \gamma)$ GIVES N DECAY, $(\bar{\nu}_\mu)(\bar{e} \gamma)$ GIVES MU DECAY,
AND $(\bar{\Lambda} \mu)(\bar{N} p)$ GIVES MU CAPTURE.

THE PRODUCTS LOOK SOMETHING LIKE,

$$\begin{aligned}
 & a(e^- \gamma)(\bar{\nu}e^+) + a(e^- \gamma)(\bar{\nu}_\mu \mu) + .97a(e^- \gamma)(\bar{N}P) + .24a(e^- \gamma)(\bar{\Lambda}P) \\
 & a(\bar{\mu}\nu_\mu)(\bar{\nu}e^+) + a(\bar{\mu}\nu_\mu)(\bar{\nu}_\mu \mu) + .97a(\bar{\mu}\nu_\mu)(\bar{N}P) + .24a(\bar{\mu}\nu_\mu)(\bar{\Lambda}P) \\
 & .97a(\bar{P}N)(\bar{\nu}e) + .24a(\bar{P}\Lambda)(\bar{\nu}e) + .97a(\bar{P}N)(\bar{\nu}_\mu \mu) + .24a(\bar{P}N)(\bar{\nu}_\mu \mu) \\
 & a(.97\bar{P}N + .24\bar{P}\Lambda)(.97\bar{N}P + .24\bar{\Lambda}P) = \\
 & (.97)^2 a(\bar{P}N)(\bar{N}P) + a(.97)(.24)(\bar{P}N)(\bar{\Lambda}P) + a(.97)(.24)(\bar{P}\Lambda)(\bar{N}P) + a(.24)^2(\bar{P}\Lambda)(\bar{\Lambda}P)
 \end{aligned}$$

SO WE HAVE A REAL MESS - WELL, YES TO THE EXTENT WE HAVE IMPLIED THE EXISTENCE OF SOME REACTION COUPLING BY TRYING TO SIMPLIFY THE DECAY HAMILTON IN OUR INITIAL FORM. SUCH REACTIONS AS $a(e^- \gamma)(\bar{\nu}e)$ AND $a(\bar{\mu}\nu_\mu)(\bar{\nu}_\mu \mu)$ ARE NOW IMPLIED TO GO DEPENDENT ON THE AMPLITUDE "a". EXPERIENCE TELLS US (?) WE CAN SUBSTITUTE THE REACTION MY whenever we have $e^- \gamma$. BUT THIS DOESN'T SOLVE OUR PROBLEM.

BY IMPLYING SUCH REACTIONS AS $a(e^- \gamma)(\bar{\nu}e)$ MUST OCCUR IN ASTAR IT IS WITH A BENIGN DISLIKE for ~~for~~ CURRENT STELLAR AND NUCLEAR PROCESSES THAT WE DO SO. THE PROBLEM IS - HOW DO WE KNOW THE MONKEY WRENCH IS IN THE STAR AS WE HAVE JUST THROWN IT THERE. THE ASTRONOMER YELLS TO FEYNMAN TO GO BACK TO HIS CRAZY THINKING AND LEAVE HIS WORLD ALONE. BUT EVEN THIS DOESN'T DISCOURAGE A CONGENIAL GUY LIKE FEYNMAN so he goes back to his paper and says, ALL RIGHT YOU GUYS YOU DON'T KNOW WHAT YOU SEE BUT I DO. IF YOU DON'T SEE MY REACTION, WELL IT MIGHT NOT REALLY EXIST BUT THAT ONLY MEANS THE SIMPLICITY OF ~~WITH~~ WRITING THE HAMILTONIAN IS IMPAIRED. SO I'LL TRY SOMETHING DIFFERENT.

THE PROCESSES PREDICTED ARE NOT ADMISSIBLE AS THEY STAND BECAUSE PARITY AND CHARGE ARE NOT CONSERVED. WHILE TOGETHER CP IS CONSERVED, THE WHOLE SCHEME LOOKS BAD. IF WE COULD INVENT A WAY WHICH THE CHARGE SIGN COULD FLIP AROUND WE WOULD BE ~~for~~ ALRIGHT. IF WE REFLECT THE WHOLE PROCESS IN A CHARGE-PARITY-TIME (CPT) MIRROR WE MIGHT HAVE SOMETHING. THE CONSTANT PHASE FACTOR WE INSERT NEED NOT BE REAL SO FOR EACH PROCESS WE INSERT A CORRESPONDING PHASE, i.e.,

$$a \left\{ (e^- \gamma) + e^{i\theta} (\bar{\nu}_\mu \mu) + [\bar{P} (.97e^{iA} N + .24e^{i\omega\Lambda})] \right\} \left\{ (\bar{\nu}e^+) + e^{-i\theta} (\bar{\nu}_\mu \mu) + [(.97e^{-iA} \bar{N} + .24e^{-i\omega} \bar{\Lambda})P] \right\}$$

BUT WE COULD JUST AS WE GO THROUGH AND CHANGE THE PHASE RELATION ON μ , N AND Λ TO NULL OUT THE EFFECTS OF θ , A , ω RESPECTIVELY SO WE DON'T DO ANYTHING.

Suppose we let loose of the idea that we want a product of these reaction for our HAMILTONIAN and examine what would happen to changing the phase on just one equation, say (3). We could write

$$a [e^{i\theta} (\bar{\mu} \nu_\mu) (\bar{p} N) + e^{-i\theta} (\bar{N} p) (\bar{\nu}_\mu \mu)]$$

WELL, WHAT DID WE DO? WE ESSENTIALLY CHANGED THE WAVE FUNCTION OF THE ANTI-MUON, $\bar{\mu}$ BY A FACTOR θ . BUT THIS MESSES UP EQUATION(1)

$$a [(\bar{e} \gamma) (\bar{\nu}_\mu \mu) + (\bar{\mu} \nu_\mu) (\bar{\nu} e^+)] \quad (1)$$

AND WE STILL DON'T HAVE MUCH SO IT'S TIME TO QUIT AND THINK MORE ABOUT THIS.

WELL, THERE IS AN UNHAPPY ENDING TO THE TALE I WAS TELLING LAST WEEK; THAT MEANS I HAVE BEEN UNABLE HOW TO GET A SATISFACTORY RESULT.

WE THEORIZED THE EXISTENCE OF A ($e\gamma$) INTERACTION IN β -DECAY BUT WE HAVE FORGOTTEN TO MENTION A COUPLE OF THINGS. WE TRIED TO WRITE THE COMPLETE PROBABILITY CURRENT AS THE PRODUCT OF THE SUM OF THE TWO TRANSITION AMPLITUDES, I.E., THE WAVE FUNCTION AND ITS CONJUGATE AND GOT SOMETHING THAT LOOKS LIKE

$$a (\bar{e} \gamma + \bar{\mu} \gamma + \bar{p} N + \bar{p} \Lambda) (\bar{\nu} e + \bar{\nu} \mu + \bar{N} p + \bar{\Lambda} p)$$

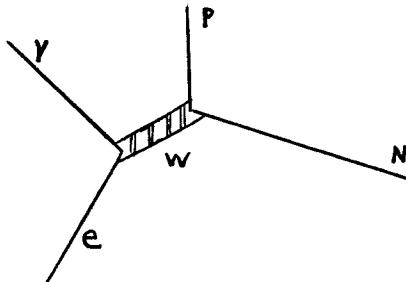
TO A SIMPLIFIED APPROXIMATION.

WE TRIED VARIOUS PHASE CORRECTIONS; THAT DIDN'T DO ANYTHING. THEN WE LOOKED AT THE SIX INDIVIDUAL PROCESSES AND TRIED TO CHANGE THINGS SO THE COUPLING IS REDUCED BY INSERTING SOME COMPLEX i 'S OR SOME OTHER STUFF.

ANOTHER APPROACH, WHICH I FORGOT TO MENTION, HAS TO DO WITH THE THEORIZING OF A PARTICLE DENOTED BY W WHICH COUPLES ALL THE CURRENTS AND EXPLAINS THEIR BEHAVIOR, I.E. WE WRITE

$$(\bar{e} \gamma + \bar{\mu} \gamma + \bar{p} N + \bar{p} \Lambda) W$$

WHERE WE DEPICT THE INTERACTION IN A FEYNMAN DIAGRAM,



SINCE W HAS ^{NOT} BEEN OBSERVED OR ANY SUBSTANTIAL EVIDENCE DISCOVERED TO SUPPORT ITS EXISTENCE, WE DON'T FIND MUCH HELP HERE.

Worth inserting here is more on Feynman's introduction of the "W" coupling force

In today's understanding this is the W-boson which was followed by the Z Boson.
From Wikipedia:



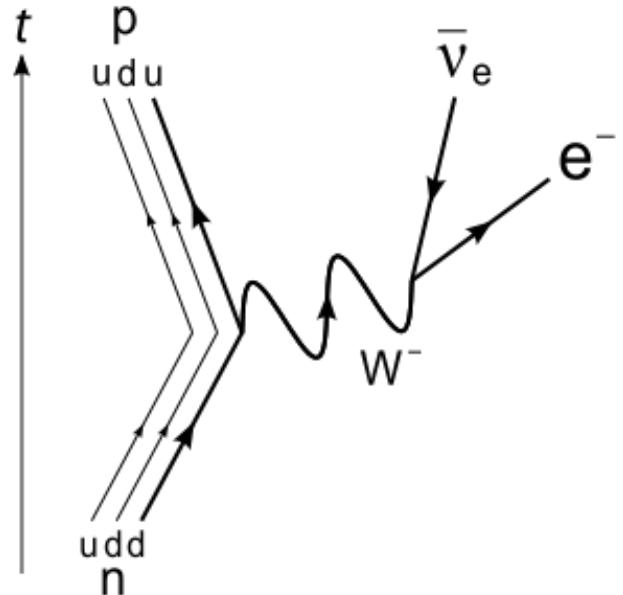
Following the spectacular success of quantum electrodynamics in the 1950s, attempts were undertaken to formulate a similar theory of the weak nuclear force. This culminated around 1968 in a unified theory of electromagnetism and weak interactions by Sheldon Glashow, Steven Weinberg, and Abdus Salam, for which they shared the 1979 Nobel Prize in Physics.[6] Their electroweak theory postulated not only the W bosons necessary to explain beta decay, but also a new Z boson that had never been observed.

W bosons

The W bosons are best known for their role in nuclear decay. Consider, for example, the beta decay of cobalt-60, an important process in supernova explosions.

It is worth pointing out here that this Feynman discussion of the need for the "W Boson" was in early 1967 in advance of the "1968 ...weak interactions by Glashow, Weinberg and Salam.

Interesting Feynman came in from this nuclear synthesis perspective which is a different path than from unifying electromagnetism. Typical Feynman like his sum over all histories approach to quantum theory.



ALSO WHEN WE WROTE THE (PN) INTERACTION WE IMPLY SOME SMALL SCATTERING EFFECT DURING β -DECAY. BECAUSE PARITY IS VIOLATED IN β -DECAY WE MUST EXAMINE NUCLEAR TRANSITIONS IN WHICH A VERY SMALL PART OF THE NUCLEON FORCES ARE NOT PARITY CONSERVED. THAT IS, WE LOOK AT THE NUCLEI LEVEL FOR TRANSITIONS WHICH ARE IMPOSSIBLE IF PARITY IS CONSERVED.

AS AN EXAMPLE CONSIDER A STATE OF SPIN 2 EVEN PARITY BUT IS NOT A PURE STATE AND 1 PART IN 10^7 IS SPIN 2 ⁽⁺⁾ ODD PARITY. IF THE PARITY IS A LITTLE COCKEYED A GAMMA RAY WILL BE EMITTED AS IT MAKES A TRANSITION TO A SPIN ZERO EVEN PARITY STATE. IF PARITY IS VIOLATED WE SHOULD BE ABLE TO DETECT WHICH WAY THE GAMMA IS CIRCULARLY POLARIZED. THE WHOLE THING IS NOT CLEAR AND EVEN IF THIS THING EXISTS IT'S NOT CLEAR HOW SIGNIFICANT IT IS TO (PN) TRANSITION.

I TRIED TO: ONE, ESTABLISH SOME BEAUTIFULLY SIMPLE OR NATURAL FORM OF THE TRANSITION AMPLITUDE AND, TWO, TO CHECK THESE SMALL MATRIX ELEMENTS. I COULD NEITHER FIND A SIMPLE FORM OR A NUMERICAL CHECK. THUS WE CONCLUDE THERE MUST BE A CROSS TERM.

CHAPTER 7

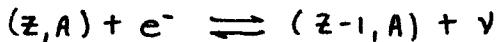
NEUTRINO PROCESSES

H.Y. CHIU

NEUTRINOS ARE IMPORTANT BECAUSE THEY ACT AS ENERGY SINKS BECAUSE THEIR MEAN FREE PATH IS ABOUT 1000 LIGHT YEARS. THUS THE STAR IS TRANSPARENT TO THE NEUTRINO.

The Urca Process

The LOSS OF ENERGY DUE TO ORDINARY β DECAY IS CALLED THE URCA PROCESS.



THE ELEMENTS WHICH CONTRIBUTE MOST TO THE URCA PROCESS ARE Ce^{35} AND S^{32}

NEUTRINO BREMSSTRAHLUNG

THE THEORY OF FEYNMAN AND GELL-MANN DESCRIBES THE WEAK INTERACTIONS AS PRODUCED BY A CURRENT WHICH INTERACTS WITH ITSELF. THE CURRENT,

$$J = (e\bar{\nu}_e) + (p\bar{n}) + (\mu\bar{\nu}_\mu)$$

INTERACTING WITH ITSELF PRODUCES THE REACTIONS,

$$\begin{aligned} n &\rightarrow p + e^- + \bar{\nu}_e \\ \bar{n} &\rightarrow e^- + \bar{\nu}_e + \bar{\nu}_\mu \\ (p, n) &(p, n)^+ \\ (\mu, \bar{\nu}_\mu) &(\mu, \bar{\nu}_\mu)^+ \\ (p, n) &(\mu, \bar{\nu}_\mu)^+ \\ (e, \bar{\nu}_e) &(e, \bar{\nu}_e)^+ \end{aligned}$$

THE SQUARE TERM PROCESS $(e, \bar{\nu}_e)(e, \bar{\nu}_e)^+$ BECOMES IMPORTANT WHEN THE THERMAL ENERGY kT IS OF THE ORDER OF THE ELECTRON MASS, I.E., $m \sim 0.5 \text{ MeV}$, $T \sim 7 \times 10^9 \text{ K}$. ALSO $(\mu\bar{\nu}_\mu)(\mu\bar{\nu}_\mu)^+$ IS IMPORTANT WHEN kT IS ABOUT THAT OF MUON, $M_\mu \sim m_\mu \sim 106 \text{ MeV}$ AND $T \sim 10^{12} \text{ K}$ BECAUSE THE PAIRS ARE IN EQUILIBRIUM WITH THE PHOTON FIELD.

PHOTO-NEUTRINO PROCESS

FOR A REAL PHOTON,

$$\gamma + e^- \rightarrow e^- + \bar{\nu}_e + \nu_e$$

PAIR ANNIHILATION PROCESS ($T > 10^9 \text{ K}$)

THE ANNihilation OF AN ELECTRON-POSITRON PAIR INTO NEUTRINOS.

THE PAIRS ARE IN THERMODYNAMIC EQUILIBRIUM WITH THE PHOTONS BECAUSE THE TIME SCALE OF PAIR PRODUCTION AND ANNihilation TO PHOTONS IS MUCH SHORTER THAN THAT OF ANY NEUTRINO PROCESS

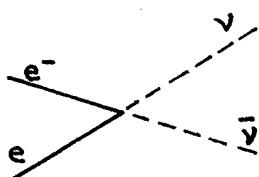
PLASMA PROCESS ($\text{HIGH } \rho, \text{ TEMP } < 10^9 \text{ K}$)

A FREE PHOTON CANNOT DECAY INTO NEUTRINOS BECAUSE THE DECAY OF A ZERO MASS PARTICLE INTO TWO PARTICLES IS FORBIDDEN BY ENERGY-MOMENTUM CONSERVATION. IN AN ELECTRON GAS PHOTONS MAY APPEAR TO HAVE A REST MASS. THE RELATION BETWEEN THE FREQUENCY ω AND THE WAVE NUMBER k IS FOR $\omega > \omega_0$ (THE PLASMA FREQ.)

$$\hbar^2 \omega^2 = \hbar^2 \omega_0^2 + k^2 c^2$$

IN THIS CASE A PHOTON MAY DECAY INTO A NEUTRINO-ANTI-NEUTRINO PAIR.

THE PAIR PROCESS IS IMPORTANT IN STELLAR COLLAPSE AND PLASMA PROCESS IN THE CREATION OF WHITE DWARFS.



EVOLUTION OF STARS

WE RETURN, ONCE AGAIN, TO OUR MAIN LINE OF DISCUSSION AND THAT WAS TO SOLVE THE FOUR DIFFERENTIAL EQUATIONS WE FIRST TOOK UP ON PAGES 59. AND 60. WE NEEDED A KNOWLEDGE OF THE OPACITY K AND ENERGY GENERATION $E(r)$ IN ORDER TO COMPLETE OUR DISCUSSION.

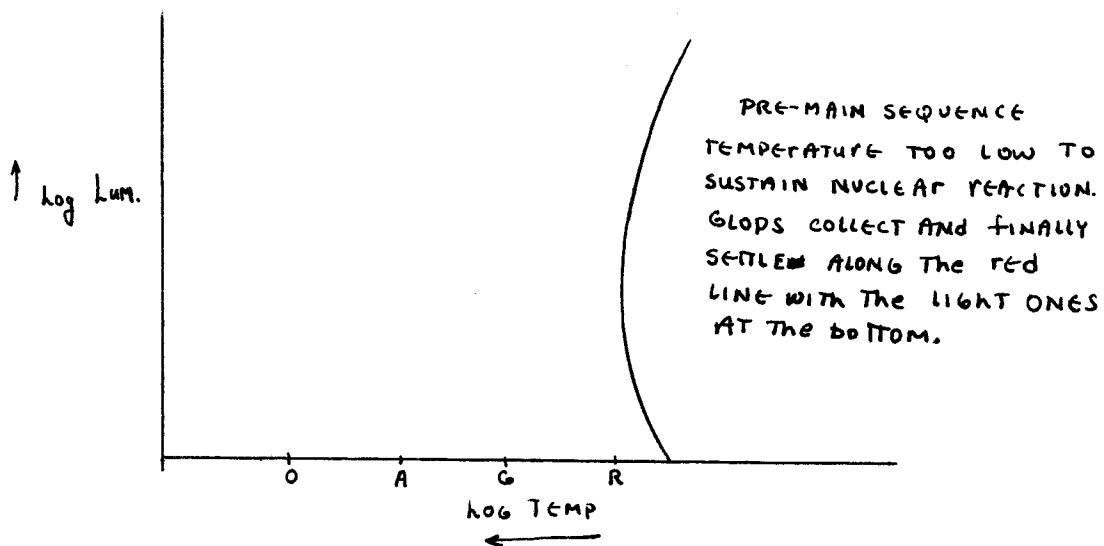
SO TO START OFF IN AN UNCERTAIN WAY AS TO HOW THE GAS COLLECTS WE MAKE TWO ASSUMPTIONS:

- (i). THE STAR DOES NOT ROTATE
- (ii). THE STAR DOES NOT LOSE MASS

THE FIRST ASSUMPTION COULD LEAD TO QUITE AN IMPORTANT OMISSION BECAUSE A ROTATING STAR CANNOT BE IN EQUILIBRIUM. THIS MEANS CONVECTIVE REGIONS EXIST WHICH STIRS THE GOOP UP FROM THE CENTER AND MAKES A MESS OUT OF OUR CALCULATIONS.

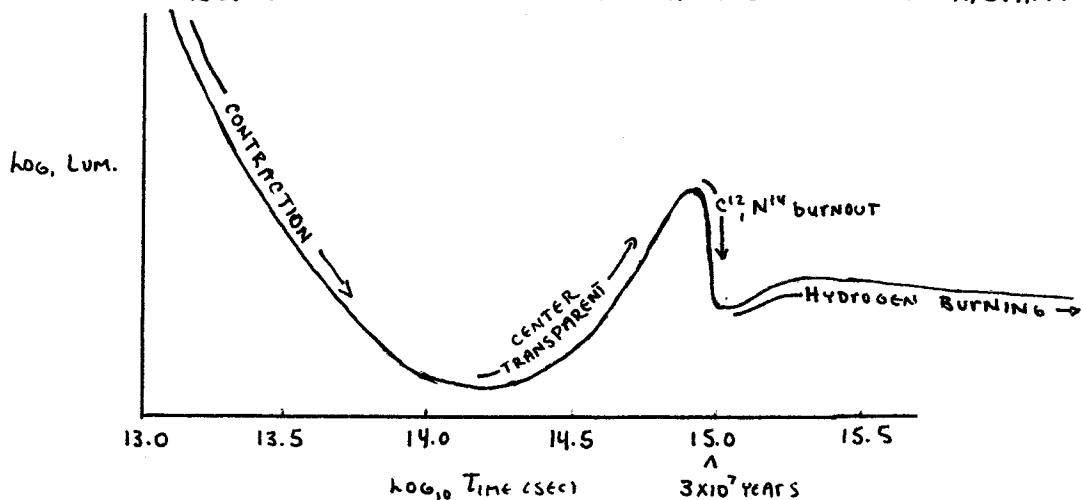
WE BEGIN THEN WITH A SPHERICALLY SYMMETRIC GLOP OF GAS WHICH FALLS IN UNDER GRAVITATIONAL FORCE AND HEATS UP UNDER THE COMPRESSING FOR. THE FIRST PROBLEM IS THAT THIS CONTRACTION IS VERY TIME DEPENDENT AND HOW LONG THE STUFF FLOATS AROUND IS ANY BODY'S GUESS. FURTHER, WE EXCLUDE FROM OUR CONSIDERATION DYNAMICALLY VARYING STATES, I.E., AT A GIVEN TIME WE CONSIDER THE STAR TO BE IN HYDROSTATIC EQUILIBRIUM.

THE STAR COLLAPSES UNTIL THE COMPRESSION STOPS BECAUSE OF # THE TEMPERATURE RISE AND THE MASS BOUNCES AND SETTLES DOWN INTO THE PRE-MAIN SEQUENCE STAGE. THE TEMPERATURE IS TOO LOW FOR NUCLEAR REACTION SO IT FLOATS AROUND BUT AFTER A WHILE HEAT LEAKS OUT. THE CONSEQUENCE IS TO ALLOW THE STAR TO COLLAPSE CAUSING A HEATING AND A HIGH THERMAL GRADIENT AND OPACITY RESULT. IT IS NOW ABOVE ADIABATIC CONDITIONS AND, THEREFORE, WILL CONVECT THROUGHOUT THE STAR. BUT A SATISFACTORY CONVECTION THEORY IS NOT KNOWN TO EXPLAIN THIS PRE-MAIN SEQUENCE STAGE WHERE NO REACTIONS HAVE YET BEGUN. THE FIRST STAGE OF THE STAR CAN BE DEPICTED IN THE FOLLOWING H-R (HERTZ-SPRUNG RUSSELL DIAGRAM).



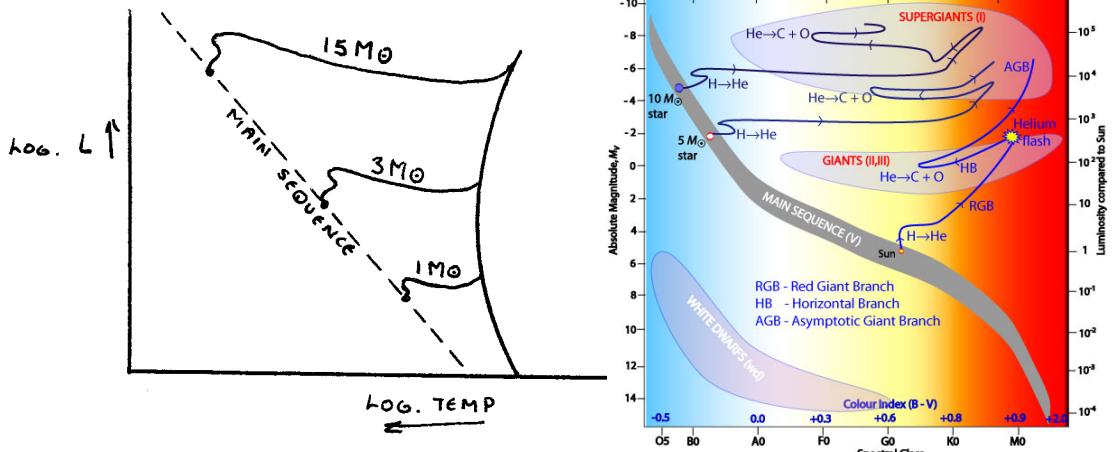
As the center gets hotter due to the contraction or compressive forces, it becomes transparent. The big temperature gradient no longer exists and the convection ceases. The temperature at the surface, however, rises because the radius shrinks.

Nuclear reactions begin and the carbon present begins to burn, $C^{12}(p,\gamma)C^{13}$ and the nitrogen part also goes, $N^{14}(p,\gamma)O^{15}$. The nitrogen burning is very slow. The cycle gets stuck. But as the C^{12} burns the opacity lowers, more heat escapes, the center becomes more convective and will burn what C^{12} remains at a faster and faster rate. The temperature rises. And we start the hydrogen (P-P) cycle and the star becomes a main sequence star. This occurs about 5×10^7 years and about 99% of the energy is from this nuclear reaction and the gravitational effect is essentially gone. This is for a star of mass about that of the sun. The sequence of events can be shown in the following diagram:



The nature of the surface depends to a large degree on the convective regions below and this becomes very complicated. The changes in chemical composition (opacity), temperature gradients, etc play an important part in this process but we just don't know that much about them. We do know that when convection starts the readjustment of the matter requires gravitation work so the star "cools" as seen during C^{12} burnout. The P-P reaction commences after the convection stops, i.e., when there are no high temperature gradients left.

For more massive stars the C^{12} burning continues and the core remains convective so that the distribution of stars "falling" into the main sequence depends on its mass and the reconstruction of the process and actual observed and plotted main sequence stars is amazingly close,



The DISTRIBUTION OF STARS ALONG THE MAIN SEQUENCE IS NOT SENSITIVE TO COMPOSITION AWAY from the LINE AS IT IS ALONG IT, I.E., SMALL FLUCTUATION IN COMPOSITIONS WOULD MOVE THE STAR UP OR DOWN THE LINE A LOT BUT IT WON'T GET OFF OF IT VERY FAR.

ONCE THE STAR LANDS ON THE MAIN SEQUENCE IT IS RELATIVELY CALM AS IT BURNS ITS HYDROGEN AND SITS THERE FOR ABOUT 10^{10} YEARS UNLESS IT IS A VERY MASSIVE STAR. NOW WE DISCUSS THE DIFFERENCE.

MAIN SEQUENCE STARS

WE SEPARATE THESE STARS INTO TWO GROUPS:

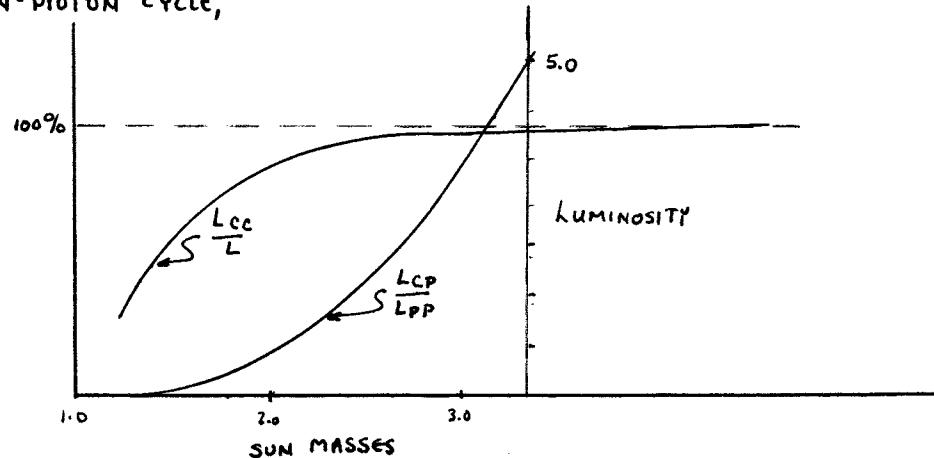
- (1). UPPER MAIN SEQUENCE - CARBON-OXYGEN-NITROGEN CYCLE PREVALENT
- (2) LOWER MAIN SEQUENCE - PROTON-PROTON CYCLE IMPORTANT

STRUCTURALLY THESE STARS ARE DIFFERENT AS SEEN BY THE REACTIONS OCCURRING. THE LOWER MAIN SEQUENCE STARS LIKE THE SUN HAVE A MUCH LOWER CORE TEMPERATURE AND MUCH HIGHER CENTRAL DENSITIES WITH THE CORE IN RADIATIVE EQUILIBRIUM IN THE P-P PROCESS. THERE IS ALSO AN OUTER CONVECTIVE SHELL OUTSIDE THE CORE. THE CHEMICAL COMPOSITION WILL VARY WITH TIME AND ALSO RADIAL OUTWARD SINCE THE RATE OF BURNING HYDROGEN DEPENDS ON TEMPERATURE WHICH DECREASES OUTWARD.

THE UPPER MAIN SEQUENCE STARS, ON THE OTHER HAND, HAVE A CONDUCTIVE CONVECTIVE CORE DURING THE CNO CYCLE SO IT IS WELL STIRRED UP. THE OUTSIDE IS IN RADIATIVE EQUILIBRIUM. THESE STARS HAVE HIGHER LUMINOSITY THAN THE LOWERS SO IT BURN HYDROGEN FASTER AND THUS DON'T LAST AS LONG.

A GOOD THEORY OF CONVECTION IS NEEDED TO UNDERSTAND THE MIXING ACTION. A PARAMETER CALLED THE MIXING LENGTH HAS BEEN CONCEIVED TO EXPLAIN HOW DEEP EACH LITTLE CONVECTIVE CELL IS AND HOPEFULLY EXPLAIN HOW SUCH THINGS AS LITHIUM GET TO THE SURFACE OF THE SUN. PRESUMABLY THE COOLER STARS HAVE THICKER CONVECTIVE CORES. BUT AN "A" TEMPERATURE STAR MIGHT HAVE A FAST ROTATION AND ANGULAR MOMENTUM SPITS OUT MATTER WITH HIGH ANGULAR VELOCITY AND ALL SORTS OF STUFF.

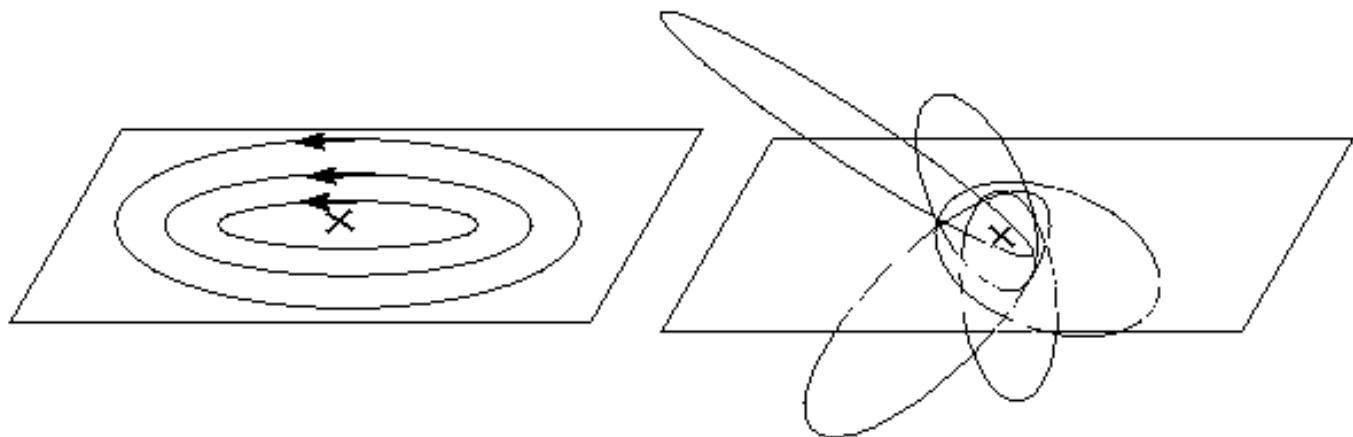
WE CAN GIVE A ROUGH IDEA HOW MUCH ENERGY THE CONVECTIVE CORE PRODUCES IN PROPORTION TO THE REST OF THE STAR AND ALSO SHOW THE RATIO OF THE ENERGY PRODUCTION OF THE CARBON CYCLE TO THE PROTON-PROTON CYCLE,



The curve S_{Lcc}/L shows the core is responsible for nearly 90% of the energy production while the carbon and proton cycle are equal in energy power generated at $2M_\odot$. For stars of mass $15M_\odot$ nearly half the mass is in the convective core.

The hydrogen is consumed and a helium core is formed. The energy generation in the core stops, the temperature gradient cannot be maintained. An outer hydrogen shell forms but the rapid change in structure causes the star to move away from the main sequence and it enters the red giant phase as a helium burner.

The next lecture begins with population I and II stars.



Population I stars: ordered motion.
Circular orbits in the disk plane;
younger, more metal-rich.

Population II stars: random motion.
Eccentric orbits passing through disk plane; older, more metal-poor.

TODAY WE ARE GOING TO TAKE A LONG TOUR OF THE HEAVENS AND DESCRIBE THE CHARACTER OF THE STARS AS SEEN BY OBSERVERS TODAY. THIS FOLLOWS FROM A DISCUSSION I HAD WITH JESSE GREENSTEIN THIS MORNING SO THE MATERIAL IS NOT WELL DIGESTED, BUT I'LL LEARN MORE ABOUT IT AS TIME GOES ON.

WE DIVIDE THE STARS INTO TWO CATEGORIES - POPULATION I AND II:

POPULATION I CONSIST OF THOSE STARS IN THE SPIRAL ARMS. THEY ARE YOUNGER AND HEAVIER IN THE HIGHER METALS. THEY ARE SECOND GENERATION AND SURROUND US AND PROVIDE A GOOD SOURCE OF MATERIAL FOR OBSERVATIONS.

POPULATION II THESE ARE THE STARS COMPRISING THE GLOBULAR CLUSTERS. THEY PASS "VERTICALLY" THROUGH THE GALACTIC PLANE AND ARE CALLED HIGH VELOCITY FIELD STARS. THESE MOVE BY ~~AS~~ AND ~~ARE~~ ARE OUR SOURCE OF DATA.

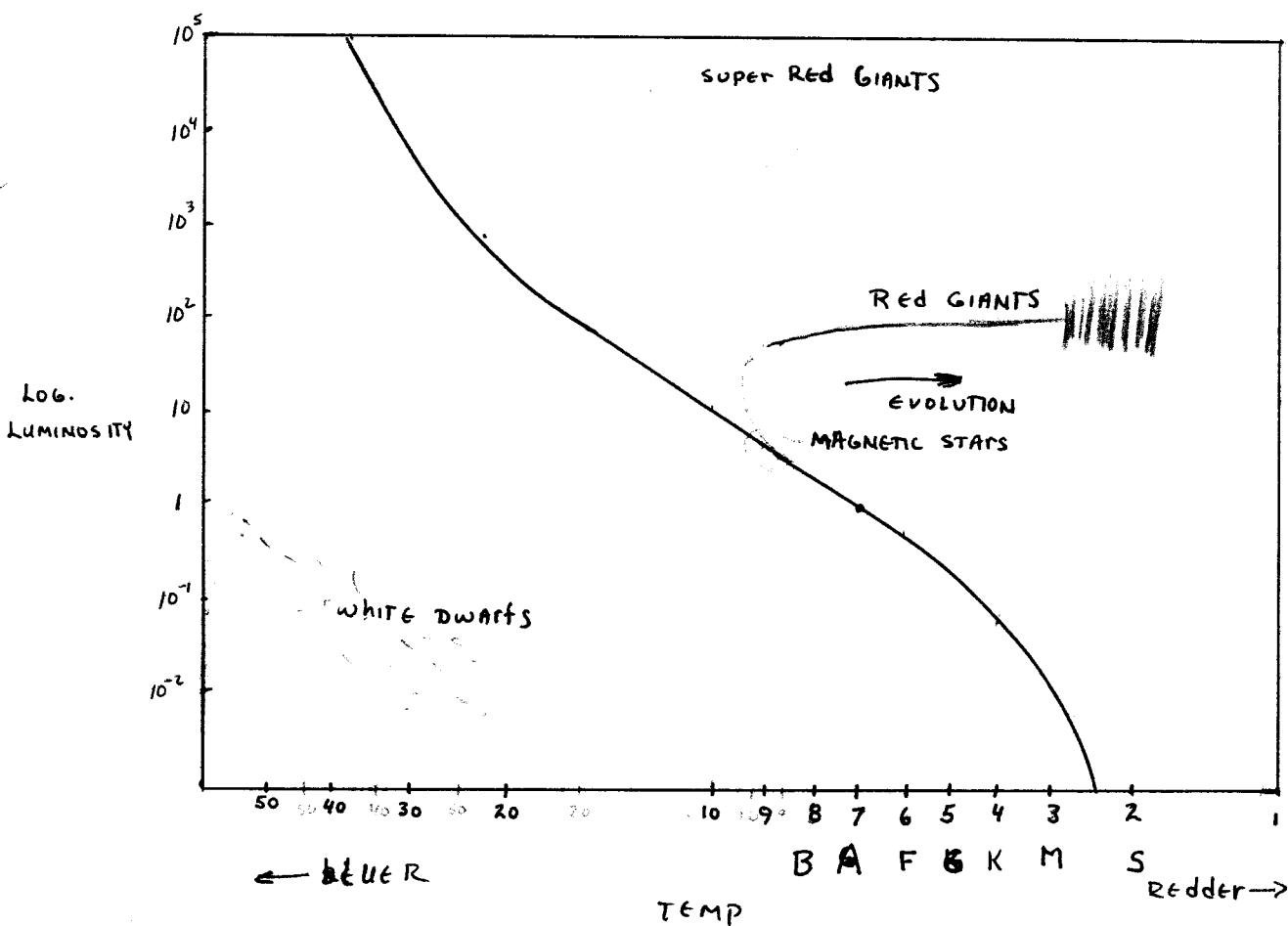
WE ~~DO~~ BEGIN WITH THE POP. I'S AND CATEGORIZE THEM IN THE FOLLOWING WAY:

- (1). MAIN SEQUENCE - THESE WE HAVE PRETTY WELL DISCUSSED ALREADY.
- (2). RED GIANTS - THESE ARE MASSIVE STARS WHICH HAVE BROKEN OFF THE MAIN SEQUENCE AND SPREAD OUT VERTICALLY (SEE NEXT PAGE) AND THE SPREAD IN LUMINOSITY DEPEND ON THE SPLIT IN COMPOSITION:
 - (a). THE M STARS ARE NORMAL AND FOLLOWING THE STELLAR EVOLUTION TO THE RIGHT
 - (b). S STARS HAVE HEAVY ELEMENTS IN ODD PROPORTIONS LIKE LANTHANUM TO IRON IS ABOUT A 100 TIMES THAT IN THE SUN. WE COME BACK TO EXPLAIN THIS LATTER.
 - (c). THE C STARS ARE THE CARBON RICH STARS, I.E., C¹³ IN PARTICULAR AND C¹³/C¹² IS ABOUT 1/2 FOR THE HIGHEST CONCENTRATION AND 1/100 FOR THE OTHER BUT 1/100 IS STILL A LOT OF C¹³ TO HAVE AROUND. ON EARTH ITS ABOUT 1/10,000.

THE S-STARS ARE UNUSUAL IN THAT A CERTAIN AMOUNT OF TECHNETIUM HAS BEEN OBSERVED IN THE ATMOSPHERE. SINCE Tc HAS A HALF-LIFE OF 10⁵ YEARS IT MUST BE MADE IN THE CORE NOW AND BE CARRIED TO THE SURFACE IN SOME UNKNOWN FASHION.

THE MASSES OF THE STARS ARE ABOUT 4 M_⊕

- (3). SUPER-GIANTS THESE HAVE MASS 10-20 THAT OF THE SUN
- (4). WHITE DWARFS - IF THESE ARE REMNANTS OF BURNED OUT STARS THEN WE MIGHT EXPECT SOME CHARRIED EMBERS DOWN BELOW THE H-R DIAGRAM AS COOL ^{BLUE}-~~STARS~~ STARS.
- (5). ONE ODD BALL CLASS OF STARS ARE THE MAGNETIC STARS. THESE HAVE STRONG MAGNETIC FIELDS WHICH EXCITE OR ACCELERATE PARTICLES TO VELOCITIES NECESSARY FOR NUCLEAR REACTION.



THAT IS A BRIEF RUN DOWN OF THE HAPPY STARS NOW WE WANT TO DISCUSS THE REAL ODD BALLS - THE VARIABLES. WE CLASSIFY THEM INTO THE FOLLOWING GROUPS:

(1). CEPHEID VARIABLES - THESE ARE VERY PERIODIC IN THEIR FLUCTUATION AND OSCILLATION IN PERIODS FROM $1\frac{1}{2}$ - 70 DAYS. NO MASS IS KNOWN FOR SURE BUT IT IS BELIEVE TO BE FROM 3-10 MO. THE PERIOD OF THESE OSCILLATIONS IS GIVEN BY

$$P, \text{ period} = \frac{Q \text{ constant}}{\sqrt{\rho, \text{ density}}}$$

BY A DIMENSIONAL ANALYSIS WE CAN ARGUE THIS RELATIONSHIP. IF THE STAR IS ^{under}SFERICAL MODE OF OSCILLATION THE NORMAL MODE OF VIBRATION HAS A FREQUENCY ABOUT EQUAL TO THE SPEED OF LIGHT WITH

$$P = \frac{R, \text{ radius}}{\text{velocity of sound}}$$

BUT THE VELOCITY CAN BE ESTIMATED FROM THE KINETIC ENERGY EQUALING THE GRAVITATIONAL POTENTIAL ENERGY SO

$$v^2 = \frac{GM}{R}$$

Thus,

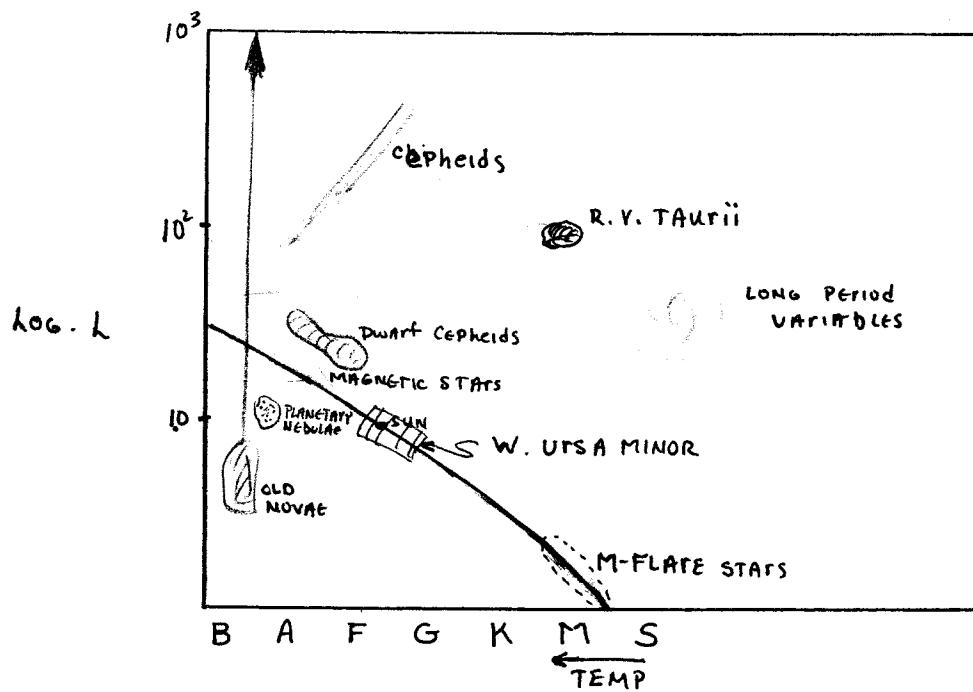
$$P = \frac{R^{3/2}}{\sqrt{GM}} = \frac{1}{\sqrt{M/R^3}} = \frac{1}{\sqrt{\rho}}$$

WE DON'T MEAN THE WHOLE STAR IS PULSATING BUT RATHER THE OUTER SHELLS ARE MAKING THE REAL MOVEMENT. IT IS LIKE SNAPPING A WHIP; THE STATION IS INVARIANT ALONG THE RADIALS SO THE TIP MUST MOVE A LOT SINCE THERE IS SO LITTLE MASS TO DISTURB.

THE STARS ARE THOUGHT TO BE BURNING HELIUM IN LAYERED SHELLS. AND SEEM TO BE WELL OBSERVED BUT ONE CEPHEID STOPPED COMPLETELY ABOUT FOUR YEARS AGO AND HASN'T KICKED UP YET. SO DO THEY START AND STOP? IS THERE A DRIFT IN FREQUENCIES OF OSCILLATION IF THEY VARY IN THE DIRECTION OF AN EXPANDING UNIVERSE, I.E., TOTAL HIGHER VELOCITIES.

- (2). DWART CEPHEIDS - THESE ARE FASTER PULSATING, 0.2 DAYS. IT IS NOT KNOWN WHERE THEY FALL EVOLUTIONARY WISE. IF THEY ARE MORE DENSE, THEY WOULD HAVE SHORT PERIODS FOR THE ABOVE REASON.
- (3). THE LONG-PERIOD VARIABLES TAKE 150-450 DAYS PER CYCLE AND THESE ARE NOT PERFECTLY REGULAR IN THEIR PULSATING.
- (4). RV TAURI - THESE ARE SEMI-REGULAR OR QUITE IRREGULAR WITH 50-150 DAYS IN A PERIOD. THESE HAVE SHOCK OSCILLATIONS, I.E., THE OUTER SHELL COMES DOWN ON THE INNER STUFF AND SET UP SHOCK WAVES THAT ROLL UP AND DOWN THROUGH THE STAR.
- (5). W. URSA MINOR - THESE ARE 6 HOURS IN PERIODICITY AND ARE BINARY STARS IN CONTACT ROLLING ABOUT EACH OTHER.
- (6). THE OLD NOVAE ARE STARS THAT FLARE UP TO MAY BE 10^6 TIME THE LUMINOSITY OF THE SUN THEN PETER OUT. THEY ARE UNDERSTOOD TO BE OLD DOUBLE OR BINARY STARS IN CONTACT ROTATING AT VERY HIGH FREQUENCIES; IN FACT THEY ARE JUST FIZZING. THE PERIODS ARE A FEW MINUTES, I.E., THEIR RADII ARE SO SMALL THE FREQUENCIES ARE VERY FAST. WHILE MENTIONING BINARY STARS THERE ARE BINARY WHITE DWARFS - ONE WITH A PERIOD OF 4 HOURS 39 MINUTES. ANOTHER WITH AN EXCEPTIONALLY REGULAR (1 PART IN 10^{-7}) PERIOD OF 1 MINUTE MEASURED OVER SEVERAL YEARS. IT IS BELIEVED TO HAVE A SLIGHT BUMP ON ITS SURFACE AND OSCILLATES ABOUT THE PERTURBATION LIKE A PENDULUM.
- (7). FLARE STARS - SIMILAR TO NOVAE FLARE UP TO A FACTOR OF 50 BEFORE PETERING OUT IN A MATTER OF 4 HOURS. ACTUALLY THEY ARE CALLED M-FLARE STARS AND WHY COOL STARS DO THIS IS NOT KNOWN.

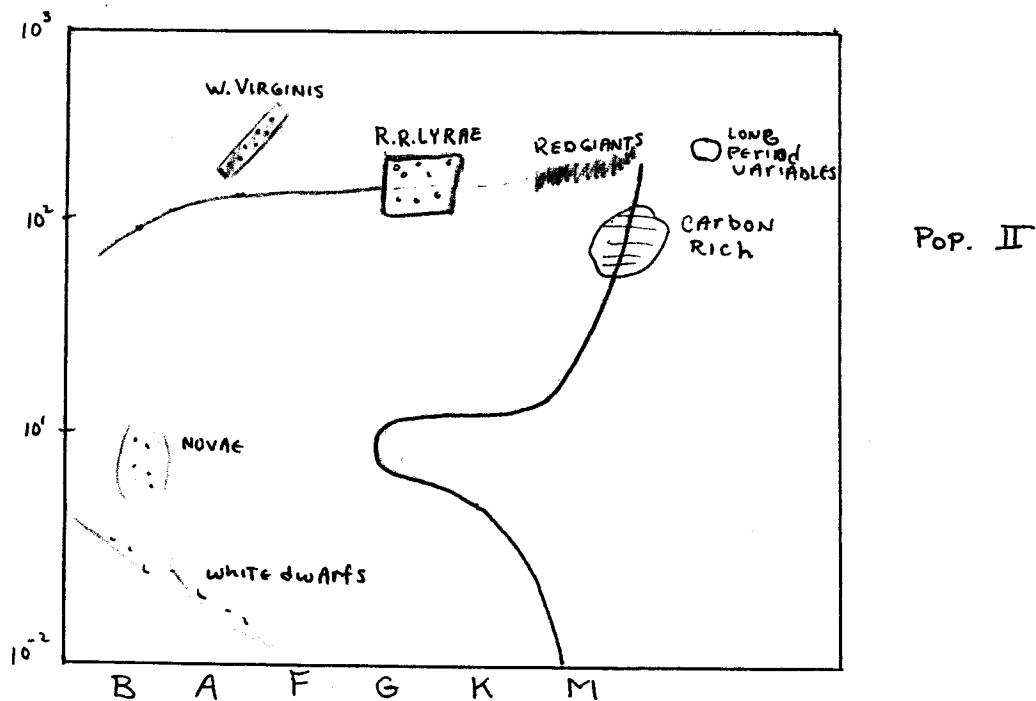
ANOTHER CLASS AS THE PLANETARY NEBULAE WHICH HAVE VERY HOT CENTERS LIKE A LITTLE WHITE DWARF SURROUNDED BY GAS. THE FREQUENCIES ARE DIFFERENT IN THIS STAR AND IT MIGHT BE A PLACE TO CHECK FOR THE EXISTENCE OF THE $(\bar{e}r)(e\bar{v})$ TERM IN THE β -DECAY.



SKETCH SHOWING
RELATIVE POSITION
OF VARIABLE STARS
FOR POP. I

POPULATION II STARS

AS WE MENTIONED EARLIER THESE STARS ARE MAINLY FOUND IN THE GLOBULAR CLUSTERS. THEY ARE OLDER AND SUBSEQUENTLY THE HIGHER PORTIONS OF THE MAIN SEQUENCE HAVE LONG SINCE BURNED OUT. A ROUGH SKETCH OF THE SCATTERING OF THESE STARS ACROSS A H-R DIAGRAM LOOKS LIKE,



The doubling in luminosity, i.e., two values at about K° in temperature is rather common but mysterious as to what the evolution process is through this phase.

The red giants burn off the main sequence and more or less aggregate in a cluster. These stars are metal deficient; the ratio of metals to hydrogen is a 100 times less than the sun. They are generally low rotation and there are no close binary stars. Even binaries of large separation are rare.

It is estimated that 80% of the stars are Pop. II while in our vicinity it is the other way in favor of the Pop. I's since we are in a spiral arm.

The stars on the previous diagram are classified in the following groups:

1. The carbon rich and, therefore, rich in heavy elements.
2. W. Virginis stars are really Pop II Cepheids. They are periodic except that they are a factor of 3 or so less intense than Pop I's of the same period.
3. R.R. Lyrae - The complete dynamic calculations of their pulsation are known. They have been worked out on a computer and they fall close to the mathematical values. It is not known though if they are incoming or outgoing helium burners. There is a gap which we can't explain as yet.
4. The red novae are rare and they might consist of some old Pop I's.
5. L.P. variables have different distributions as a function of period.

Finally, only one planetary nebula is known in Pop. II. See below

All masses are of the order of $1 - .75 M_{\odot}$ because the heavy ones have burned out. The $.75 M_{\odot}$ stars are the most numerous stars in the universe; they just sit there and burn and burn and burn. In this group there are no known stars above $10^3 L_{\odot}$ where Pop. I novae can flare up to as high as $10^5 - 10^6 L_{\odot}$.

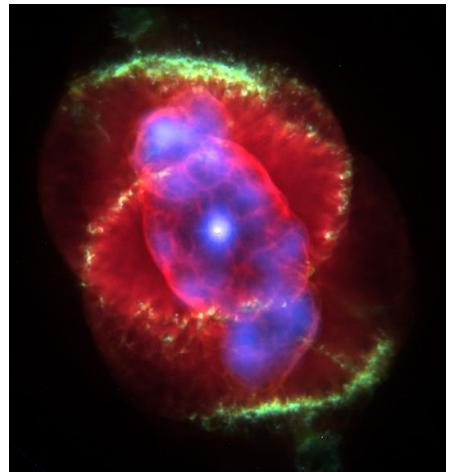
SUPER NOVAE

The last group we want to mention are the supernovae. These blow up to about $10^6 L_{\odot}$. They happen about once every 400 years and we are due since that last one occurred about 1550 which Tycho Brahe observed. There are no pre-supernovae observed like Pop. I novae that only reach $10^6 L_{\odot}$ and fall back in a cluster.

There are two types of supernovae:

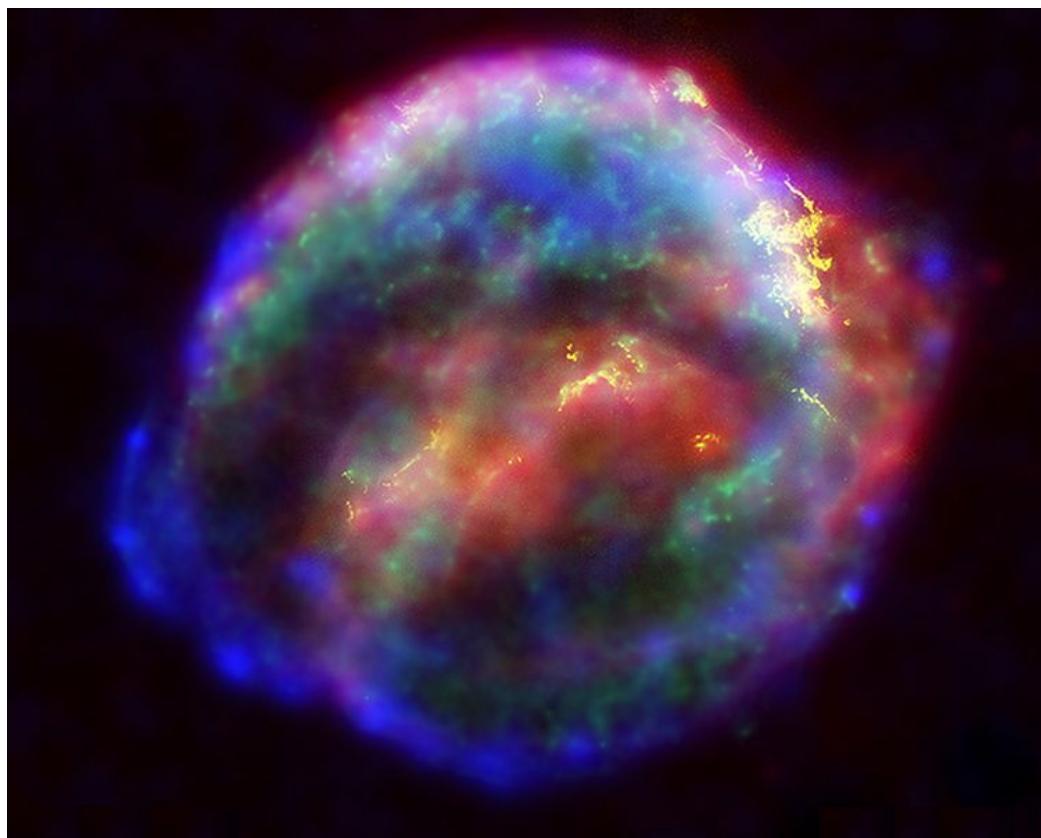
- TYPE 1 which occur in Pop. II stars
- TYPE 2 which occur in Pop I stars

"Cat's Eye" Planetary Nebula-Hubble image



TYPE 2 SUPERNOVAE ARE NOT IDENTIFIED WITH ANY REGION ON THE H-R DIAGRAM. THEIR ORIGIN IS UNKNOWN. THEY RANGE FROM $10^{8.5} - 10^{10}$ L₀ WHEN THE BLOW UP AND LAST ABOUT A ~~WEEK~~ MONTH. THE POWER LIBERATED IN THE FORM OF LIGHT IS OF THE ORDER OF 10^{50} ERGS. THE AMERICAN EXPERTS CLAIM THAT $10^{52} - 10^{54}$ ERGS ARE LIBERATED IN THE ACTUAL EXPLOSION TO GET ALL THAT LIGHT OUT BUT THE RUSSIANS AND FEYNMAN DISAGREE BECAUSE THEY TALK ABOUT COLLECTIVE ELECTRON MOTION AND DON'T NEED THE EXTRA POWER IN EXPLAINING SYNCHROTON RADIATION. BUT IF WE ASSUME SAY 10^{54} ERGS OF POWER GOES UP IN SMOKE, THIS IS EQUIVALENT TO $1M\odot c^2$, I.E., CONVERTING THE SUN INTO RAW ENERGY.

TYPE 1 POP II SUPERNOVAE REACH 10^{11} L₀ WHICH CORRESPONDS TO THE ORDER OF MAGNITUDE OF THE TOTAL LUMINOSITY OF OUR GALAXY. THIS LARGE MASS IN THE FORM OF RADIATION CAUSES THE LIGHT TO BE BENT AND IT TAKE ABOUT 54 DAYS FOR IT TO BLOW ITSELF OUT. THERE ARE EMISSION LINES WHICH DISCLOSES SOME UNKNOWN SUBSTANCE CALLED X WHICH IS CLOSE TO HYDROGEN. THE REMNANTS OF A SUPERNOVAE CAN BE SEEN IN THE FORM OF THE CRAB NEBULA WHICH BLEW UP IN 1054 AND RECORDED BY THE CHINESE. BUT THE EUROPEANS WERE SO FAR INTO THE DARK AGES NOT ONE SCRIBE RECORDED IT.



Multiwavelength X-ray, infrared, and optical compilation image of Kepler's supernova remnant, SN 1604.

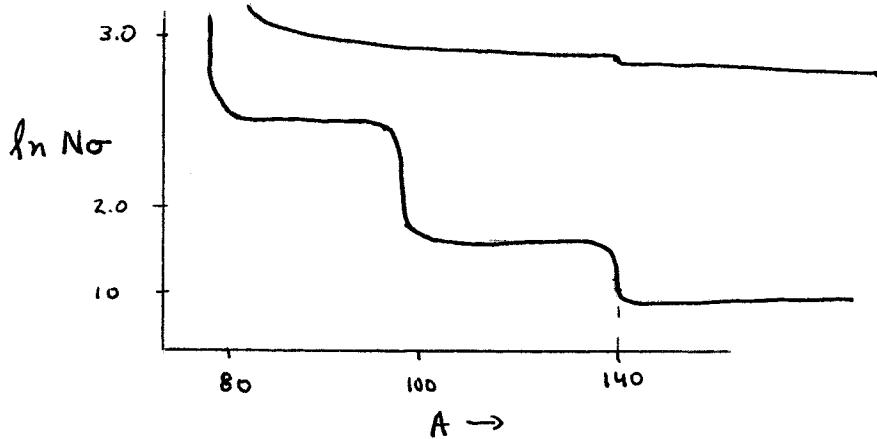
ONE IDEA PRESUPPOSES THE EXISTENCE OF IRON IN THE STELLAR ATMOSPHERE. THEN SOME OTHER PROCESS PRODUCES NEUTRONS, SAY $C^{13}(d, n)O^{16}$ WHICH ARE VERY SLOW. BUT A LOT OF THEM HIT THE IRON. THEY ARE ABSORBED AND THE ATOMIC NUMBER STARTS TO CLIMB. WILL BETA-DECAY INSURE THE CHARGE NUMBER CORRESPONDENCE. IT IS REASONABLE TO ASSUME THEN AFTER A LONG ENOUGH PERIOD OF TIME WE COULD BUILD UP TO URANIUM. IT IS BELIEVED THAT COLLAPSING CORES PRODUCE MILLIONS OF NEUTRONS AND THIS PROVIDES THE NECESSARY SUPPLY.

HOWEVER, THE ELEMENT EUROPIUM HAS A HIGH CROSS SECTION AREA FOR NEUTRONS AND WE WANT TO SAY THAT WE WOULD HAVE FEW OF THIS ELEMENT IF CAPTURES NEUTRONS SO FAST IT IS CONSTANTLY CHANGING TO SOMETHING ELSE. THE RATE AT WHICH THIS PROCESS TAKES PLACE IS GIVEN BY

$$R = \sigma \eta N$$

WHERE N IS THE NUMBER OF NEUTRONS CROSSING A UNIT AREA EACH SECOND THAT IS, IT IS A NEUTRON FLUX AND THUS PROPORTIONAL TO $1/\sigma$, THE CROSS SECTION OF THE ABSORBING SPECIES WHILE η IS A NUMBER DENOTING THE NUMBER OF NUCLEI OF ABSORBING ELEMENT PER CC.

IT IS ILLUSTRATING TO SHOW THIS DAMNING UP EFFECT IF N_σ IS PLOTTED AGAINST THE ATOMIC NUMBER:



THERE IS DEFINITELY A BUMP AT 140, I.E., WHERE THE NEUTRONS PILE UP AND WAIT A WHILE BEFORE THEY GO AROUND. THUS THE ELEMENTS BEYOND THE POINT FALL OFF IN N_σ . AT 140 THERE ARE TIGHTLY BOUND NUCLEI WHICH CAUSES THIS PHENOMENA. THUS A LOGICAL DEDUCTION THAT DIFFERENT PLACES IN THE UNIVERSE WOULD SHOW DIFFERENT DEGREES OF BUMP. WITH THE OLD SECTIONS SHOWING A SMALL BUMP SINCE THEY HAD A LONG TIME TO WORK THOUGH. indeed, THIS IS THE CASE. therefore, THE NEUTRON BOMBARDMENT IDEA SEEMS TO HOLD WATER.

TODAY REPRESENTS ESSENTIALLY THE LAST LECTURE ON STELLAR STRUCTURES. THAT IS TO SAY MY FINITE CAPACITY TO DIGEST AN INFINITE AMOUNT OF MATERIAL HAS BEEN REACHED. WHILE I AM IN THE PROCESS OF FINDING OUT WHAT SOME OF THE REALLY ENTHUSIASTIC THEORIES ABOUT STELLAR EVOLUTION ARE, I WILL PROCEED ALONG OUR PATH ORIGINALLY OUTLINED WITH A FEW MODIFICATION DUE TO SOME UPDATED KNOWLEDGE WHICH I WILL ACQUIRE.

Nuclear synthesis of elements higher than iron

ONE LAST AREA WE WOULD LIKE TO DISCUSS DEALS WITH NUCLEAR SYNTHESIS AND HOW THE HEAVIER THAN IRON ELEMENTS ARE FORMED. WE TALKED ABOUT THE CNO CYCLE AND HOW IT IS DIFFICULT TO MEASURE THESE RATIOS. ALSO HOW SOME STARS HAVE TOO MUCH OR TOO LITTLE C¹² FROM THE THREE HYDROGEN BURNING PROCESS. HOW LITHIUM 6 AND 7, BERYLLIUM, AND BORON COULD NOT REMAIN IN EQUILIBRIUM AND YET THEY ARE OBSERVED IN THE STELLAR SPECTRA. ALSO HOW THE IRON PEAK SEEMS TO BE IN EQUILIBRIUM AND PERHAPS A PREDICTED $e^- \rightarrow e^-$ REACTION DISSIPATES ENERGY TO MAINTAIN THE EQUILIBRIUM. WE SAW THE MAXIMUM ABUNDANCE OF ANY ELEMENT IS IRON BECAUSE IT HAS THE HIGHEST BINDING ENERGY PER NUCLEON. SINCE WE HAVE CLEARED UP ALL OF THESE CRITICAL PROBLEMS WE WILL MENTION HOW THE HEAVIER ELEMENTS ARE FORMED BECAUSE THE EXPERTS CLAIM THAT IRON WOULD NATURALLY DECAY BACK TO HELIUM.

Spallation

THE PROCESSED BELIEVED TO BE THE CAUSE OF THE HIGHER ELEMENTS IS SPALLATION. THIS INVOLVES HIGH ENERGY PROTONS OR NEUTRONS BOMBARDING THE ELEMENTS AND KNOCKING OR CHIPPING OFF LITTLE PIECES. THAT IS LIKE A 100 Mev PROTON HITTING AN OXYGEN ATOM TO FORM Li⁶, Li⁷ PLUS OTHER DEBRIS. THIS IS NOT A FISSION PROCESS BUT RATHER LIKE ITS NAME, IMPLIES A KNOCKING OFF. THIS IS REALLY COSMIC RAY BOMBARDMENT. EXPERIMENTALLY WE CAN PRODUCE A SPALLATION EFFECT AND GET A MEASURABLE QUANTITY OF Li⁶, Li⁷, Be. THE RATIO OF Li⁶, Li⁷/Be IS AT BEST ABOUT 20:1 WITH HIGH Mev PROTONS. SINCE IN A STAR Li⁶ WOULD BE EATEN RIGHT AWAY BY AN ALPHA PARTICLE, IT IS EXPECTED THAT THE DETECTABLE RATIO IN STARS WOULD NEVER EXCEED 20:1. BUT TWO STARS HAVE BEEN OBSERVED WHICH CONTRADICT THIS STATEMENT. SO IT IS VERY HARD TO MEASURE THE QUANTITIES OF Li⁶ AND Li⁷ SINCE THEIR SPECTRAL LINES LIE SO CLOSE TO ONE ANOTHER.

BUT WHERE DOES THE URANIUM COME FROM? WE USE THE ENERGY NECESSARY TO BIND IT TOGETHER BUT HOW WAS IT FORMED? FROM A VOLCANO MAYBE? CERTAINLY WHATEVER THE PROCESS IT COULD NOT HAVE BEEN AN EQUILIBRIUM ONE.

was

SINCE THAT ABOUT TAKES CARE OF MY CURRENT KNOWLEDGE WE WILL HAVE TO GO TO SOMETHING ELSE AND START BY ASKING FOR ANY QUESTION. WE MIGHT BRIEFLY MENTION THE PATH WE ORIGINALLY SET FOR OURSELVES AS TO WHAT WE ARE GOING TO COVER YET. THE AREAS ARE

- (1). GALACTIC DYNAMICS
- (2) RADIO EMISSION, HOW THEY ARE EMITTED, SYNCHROTRON RADIATION
- (3) COSMIC RAYS
- (4). RADIO GALAXIES AND QUASARS
- (5). SOLAR SYSTEM, PLANET, SUN, AND THE "NOTHINGNESS" OF SPACE

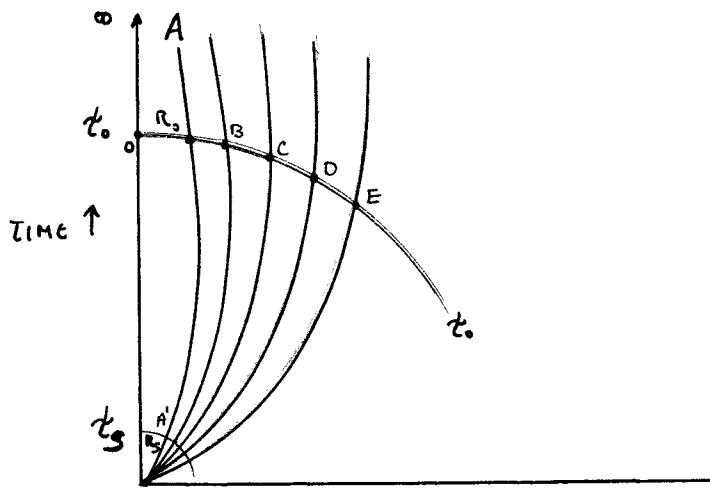
TOPICS 2, 3 AND 4 WILL BE DEALING WITH PLASMAS OR ACTUALLY A MAGNETO HYDRODYNAMICS DISCUSSION.

SOMEONE BROUGHT UP THE QUESTION OF COSMOLOGICAL THEORIES WHICH REMINDS ME OF ONE INTERESTING POINT WHICH I FAILED TO MAKE AT THE TIME. WE TALKED ABOUT THE COSMOLOGICAL EXPANSION FROM SOME EARLIER TIME AND NOW EVERYTHING IS FLYING APART. THIS PRESUPPOSES SOME TIME PERIOD WHEN THE UNIVERSE WAS PACKED SO TIGHTLY THAT IT WOULD BE POSSIBLE FOR PHOTONS TO EXIST IN EQUILIBRIUM. IF WE ACCEPT THE BASIC COSMOLOGICAL ASSUMPTION THAT EVERYTHING LOOKS THE SAME HERE AS IT DOES ANYWHERE ELSE, WE WOULD LIKE TO SHOW THAT THERE REMAINS A RESIDUAL DISTRIBUTION OF PHOTONS IN THE EXACT SAME PROPORTION KNOW AS THERE WAS AT A PREVIOUS PROPER TIME. THAT IS TO SAY, AFTER THE EXPLOSION THAT SQUIRTED EVERYTHING OUT, AS THE DENSITY REDUCED AND THE TEMPERATURE COOLED AND THE PHOTONS STOPPED INTERACTING, WE ESTABLISHED SOME BLACK BODY RADIATION TEMPERATURE T_s AND PROPER TIME t_s . THE SPACING BETWEEN THE NEXT NEAREST GALAXY IS GIVEN BY R_g . AT TIME t_0 NOW THERE IS A CORRESPONDING TEMPERATURE T_0 GIVEN BY THE RATIO,

$$\frac{T_0}{T_s} = \frac{R_0}{R_s}$$

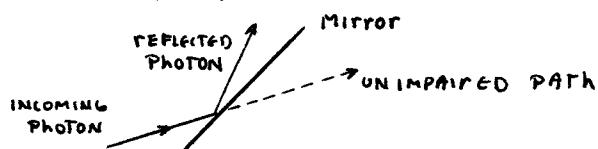
THIS EQUATION DOES NOT SAY ANYTHING PROFOUND BUT IT DOES EXPRESS IN A FURTHER SUCCINCT FASHION A VERY COMPLICATED PHENOMENA. IT STOPS HOW THE EXPANSION OF THE UNIVERSE IS RELATED TO THE OVER ALL PHOTON INTERACTION TENDING TO CAUSE A UNIFORM BACKGROUND OF BLACK-BODY RADIATION THROUGHOUT ALL SPACE. "CBR"

WE MIGHT PICTURE THIS PHENOMENA AS WE DID EARLIER IN THE COURSE BY DRAWING A PROPER TIME CURVE SHOWING THE NEBULAR MOTION FROM SOME PRESUPPOSED SINGULARITY OR ORIGIN. BY DEFINING OUR REFERENCE POINT AT t_s WE AVOID THE DIFFICULT PROBLEM OF EXPLAIN THE REAL INITIAL CONDITION. TODAY WE MIGHT THINK OF R_0 AS THE DISTANCE FROM US TO ANDROMEDA. THE NEXT GRAPH MIGHT HELP:



THE DISTANCE R_0 AT PROPER TIME t_0 , NOW, SHOULD BE EQUAL ACROSS THE ADJOINING GALACTIC SPACINGS $A \rightarrow B \rightarrow C \rightarrow D \dots$ WHILE THE CORRESPONDENCE AT t_s IS THAT $O \rightarrow A$ OR R_0 IS THE SAME PROPORTIONALLY AS $s \rightarrow A'$ OR R_s AS THE DECREASE IN OVERALL BLACK BODY RADIATION T_0/T_s .

SUPPOSE WE ONLY CONSIDER A LOCAL PHENOMENA. SUPPOSE WE HAD A MIRROR AT t_0 PLACED ON ANDROMEDA. THE PHOTONS MOVING THROUGH THAT POINT COME IN ALL DIRECTIONS WITH EQUAL PROBABILITY. THIS MEANS THAT IF A MIRROR WAS THERE EACH REFLECTION WOULD CORRESPOND TO A LOST PHOTON, I.E.,



IF THE MIRROR IS THEN ALLOWED TO EXPAND, THE PHENOMENA IS ANALOGOUS TO A STANDING WAVE PATTERN IN AN EXPANDING BOX. THAT IS, IF WE CONSIDER THE NUMBER OF QUANTA IN THE BOX, ONLY THE MODES CHANGE. THE WAVE NUMBER k CHANGES BY A SCALE FACTOR TO A NEW VALUE

$$k \rightarrow \alpha k$$

THE DISTRIBUTION OF PHOTONS PER GIVEN WAVE NUMBER k IS GIVEN BY THE BOLTZMANN DISTRIBUTION, FERMI-DIRAC

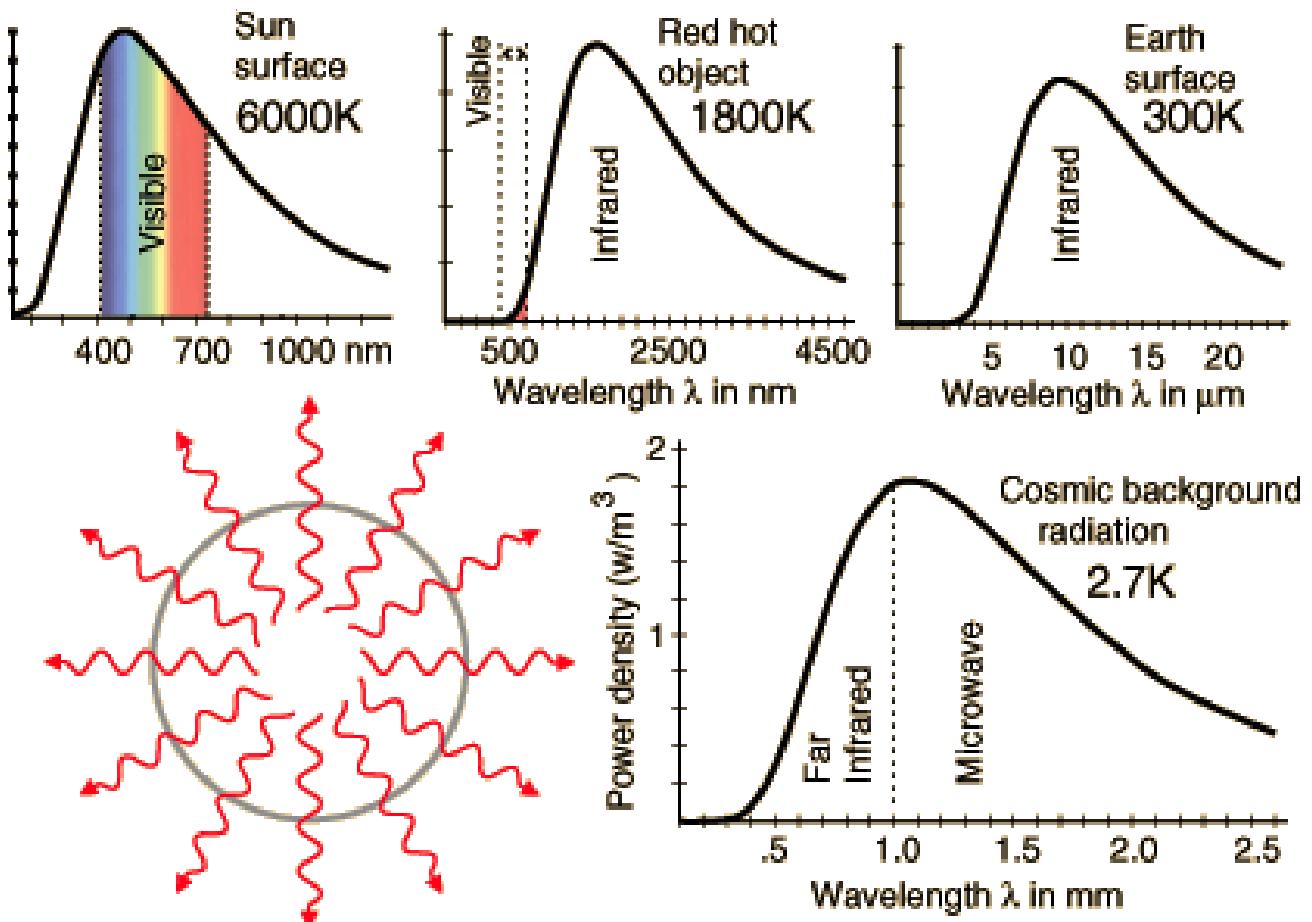
$$n_k = \frac{1}{e^{\frac{h\nu}{kT}} + 1} = \frac{1}{e^{\frac{E}{kT}} + 1}$$

WHERE THE ENERGY OF THE PHOTON = $h\nu = hkc$
Thus AT THE NEW WAVE NUMBER

$$n_{\alpha k} = \frac{1}{e^{\frac{\alpha hkc}{kT}} + 1}$$

THE MIRROR IS NOW ASSUME TO BE A BLACK BODY AT AN ESTIMATED TEMPERATURE OF 5°. AT THIS LOW TEMPERATURE THE ENERGY IS PROPORTIONAL TO ν^2 . WE ASSUME A UNIFORM BACKGROUND NOISE CORRESPONDING TO ABOUT A 3°K BLACK BODY RADIATION. THAT IS, A 3° BLACK BODY DISTRIBUTION. AS FAR AS OUR CRUDE MEASUREMENTS GO THIS SEEMS TO FIT THE BELIEF THAT THIS IS THE TEMPERATURE OVER ALL SPACE.

This special topic on black body radiation and arriving at the 3 deg K approximation is a strong validation of Feynman's insight into a cosmological theory that was just emerging with the Penzias-Wilson experiment described above.



CHAPTER 8 NUCLEAR SYNTHESIS

TODAY I AM GOING TO BACK TO NUCLEAR SYNTHESIS, HOPEFULLY, TO FINISH UP THIS DISCUSSION BEFORE GOING ON. THIS IS THE LECTURE I SHOULD HAVE PREPARED LAST TIME.

AS YOU RECALL THE PROBLEM IS TO EXPLAIN HOW THE ELEMENTS BEYOND IRON ARE formed. THERE ARE A NUMBER OF THEORIES WHICH SEEM TO PROPORT TO UNDERSTAND THE PROCESS. WE WILL ONLY DESCRIBE TWO THEORIES. THE IDEA, AS WE BRIEFLY MENTIONED, HAS TO DO WITH NEUTRON BOMBARDMENT. IF THE BOMBARDMENT IS SLOW ENOUGH THE ELEMENTS WITH ADD NEUTRONS UNTIL THEY UNDERGO A β -DECAY AND THUS JUMP TO THE NEXT Z-NUMBER. IF THE PROCESS GOES FOR A LONG PERIOD WE CAN SYNTHESIZE MOST OF THE ELEMENTS.

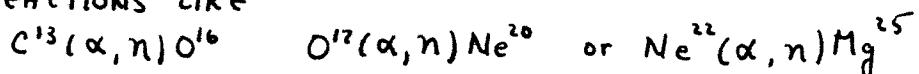
WE SKETCH A SMALL PART OF THE ISOTOPE TABLES AND EXAMINE HOW MOVEMENT CAN OCCUR UP THE TABLE IN ATOMIC WEIGHT.

N = NUMBER OF NEUTRONS →												
										121	123	
										57	43	
⁵¹ Sb												
⁵⁰ Sn	112 1.02		114 .69	115 .38	116 14.3	117 S,T 7.6	118 S,T 0	119 S,T 8.5	120 S,T 32.5	121 27 hrs	123 48	
⁹⁹ In		113 4.2	114 MS	115 S,T 95.8	116 13 SEC							
⁴⁸ Cd	110 1240	111 12.8	112 28.8	113 12.3	114 28.8	115 54 hrs.	116 7.6					

THE RED NUMBERS INDICATE THE PERCENTAGE OF THE STABILIZER ISOTOPE.

LET'S FOLLOW CADMIUM 110. SUPPOSE IT IS BOMBARDDED BY A SLOW NEUTRON FLUX AND IT KEEP ADDING A NEUTRON UNTIL IT REACHES Cd¹¹⁵ WHICH IS UNSTABLE. THE DENSITY OF NEUTRONS DURING THIS SLOW S-PROCESS IS ESTIMATED TO BE ABOUT 10^7 cm^{-3} . THIS IS A LOT OF NEUTRONS AND THE LOGICAL QUESTION TO ASK IS WHERE THEY COME FROM.

IT SEEMS LIKELY THAT THEY ARE LEFT OVER FROM THE CARBON CYCLE AND HIGHER REACTIONS LIKE



THE ORIGIN OF THESE REACTIONS APPEARS TO BE OCCURRING IN THE INTERIOR OF RED GIANTS BECAUSE IN A SPECIAL GROUP OF THESE STARS WHICH ARE METAL RICH WE SEE TECHNETIUM AND OTHER HIGHER ELEMENT. SO A DENSITY OF 10^7 DOES NOT SEEM TOO HIGH FOR THE PROCESSES TO PRODUCE INSIDE RED GIANTS

This process goes until it reaches an unstable isotope. At that time β -decay will occur faster than another neutron can be captured so we jump to the next element, say Indium 115. The process continues; a neutron is captured; β -decay follows; neutron capture lets us go through Tin 121 which decays to Antimony 121, etc., etc.

We are quick to note that there are a lot of blank spaces on either side of this dotted line. That is, some, or perhaps many, elements cannot be made in this way. We would require an extremely long time to let neutrons trickle past Cd¹¹⁶ to start higher cycles. But if the density of neutrons was high enough so that β -decay occurs before capture, the whole process is altered. There exists then two processes which occur to create the majority of the higher elements:

- (1) slow or S-process where all the β -decays go as expected
- (2). rapid or r-process where the neutron flux is so fast β -decay does not have time to occur at all.

The elements are labeled as to their means of production and we see that the r-process fills in the lower portion of our table, the neutron rich side. Some element can be synthesized by both, i.e., the total proportion has parts from both the r's process. On the proton rich side or upper portion the percentages are very low.

The exact reason for these proton rich elements seems to be tied up in another process called the p-process which is not very well understood. These elements subject to gamma rays loose a neutron and leave behind excesses of protons. We have no idea where this process occurs or when.

However, the S-process will not let us reach Uranium because when it gets up to Lead and gets caught in a cycle. It will continue through Bismuth 209 but Bismuth will undergo alpha emission followed by neutron capture, β -decay and around again. Bismuth is the most massive stable element.

To further understand this neutron capture process we would like to talk in terms of the neutron cross-section. The cross-section of a process is defined as the probability that the process occurs if the incident beam consists of a single particle and the target contains one nucleus per unit area.

WE CAN DEFINE A QUANTITY

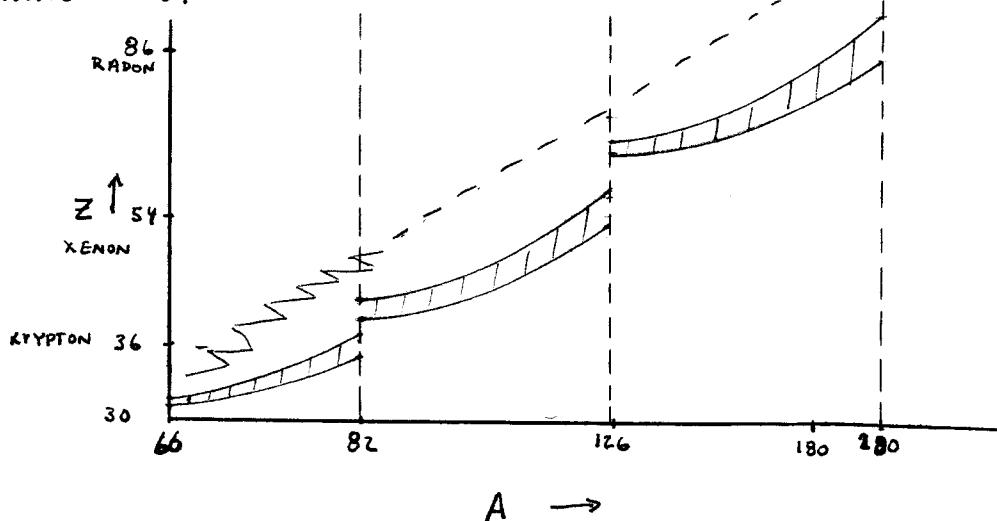
$$\sigma_{nN} = \text{constant}$$

THEN AS A LOCAL RULE for NEUTRON CAPTURE BECAUSE IT IS NOT TRUE FOR A LONG PERIOD OF TIME. IN FACT WE SHOWED LAST TIME HOW A GRAPH OF σ_{nN} VERSUS "A" LOOKS AND POINTED OUT VARIOUS POINTS OF FLUCTUATIONS. IT WOULD BE NICE TO HAVE SEVERAL ISOTOPES OF ONE ELEMENT & FORMED BY THE S-PROCESS SO WE CAN CHECK TO SEE IF WE UNDERSTAND WHAT IS GOING ON. IT TURNS OUT SAMARIUM 148 AND 150 ARE PROBABLY PRODUCED THIS WAY AND IN FACT IT TURNS OUT THAT THE PRODUCT NON FOR NEUTRONS AT TEMPERATURE 30 KILOVOLTS ARE FAIRLY CLOSE, I.E. $2,930 \pm 540$ AND $2,770 \pm 535$ RESPECTIVELY.

HOWEVER, THE PROBABILITY OF CAPTURE VARIES IN A CHARACTERISTIC MANNER WITH THE DIFFERENCE BETWEEN THE ENERGY E OF THE COLLIDING SYSTEMS AND THE ENERGY E_A OF THE RESONANCE LEVEL. THAT IS, σ FLUCTUATE AS A FUNCTION OF ENERGY AND HITS A RESONANCE WHEN ITS ENERGY MATCHES A LEVEL OF THE NUCLEUS. AS THE ATOM BECOMES MORE EXCITED, ON THE AVERAGE, THE SEPARATION BETWEEN LEVELS DECREASES WITH INCREASING ENERGY OF EXCITATION. THE TIGHTER THE BINDING THE LESS THE SPACING. WHILE AT THE SAME TIME THE LEVEL WIDTHS DECREASE WITH INCREASING MAGNITUDE OF THE ELECTROSTATIC BARRIER. THE ELECTROSTATIC BARRIER INHIBITS THE EMISSION OF PROTONS.

R- PROCESS

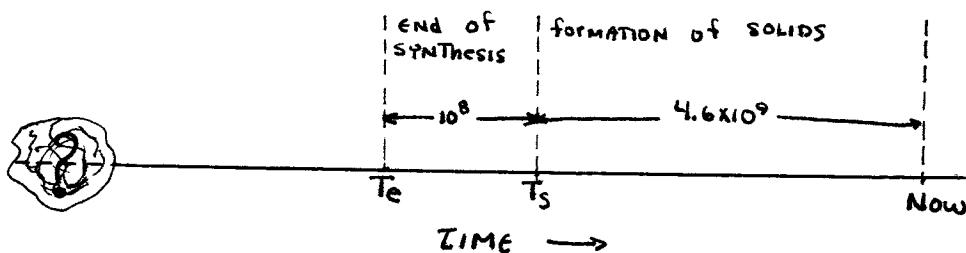
TURNING NOW TO THE R- PROCESS WE FIRST CONSIDER THE ABUNDANCE OF NUCLEI FORMED BY THE R- PROCESS. A PLOT OF Z VS. ATOMIC NUMBER LOOKS SOMETHING LIKE:



The DELAYS AT THE NUMBERS 82, AND 126 ARE DIRECTLY RELATED TO THE NEUTRON BOMBARDMENT. THESE NUMBERS ARE RECOGNIZED AS THE MAGIC NUMBERS FOR CLOSED SHELLS. AT ATOMIC NUMBER 82 THE NEXT INCOMING NEUTRON IS KNOCKED OUT BY A GAMMA. β -DECAY OCCURS TO INCREASE THE Z NUMBER, I.E., THE PROTON NUMBER BUT THE NUCLEUS STAYS AT 82. THIS PROCESS INCREASES THE PROTON COUNT NOT THE NEUTRON TOTAL UNTIL IT REACHES A POINT WHERE THE NEXT NEUTRON CAN BE HELD AND THE PROCESS GOES ON. THIS PROCESS GOES ON PAST XENON BUT THE NUCLEI ARE PROGRESSIVELY WEAKLY BOUND AND FINALLY AT THE UPPER END SAY AROUND RADON FISSION OCCURS AND THE PARTICLES SPLIT AND BEGIN THE CLIMB AGAIN. THE ENERGY GIVEN UP BY THESE UNSTABLE ELEMENTS TRYING TO GET DOWN TO A STABLE CONFIGURATION FOR INSTANCE LEAD IS THE STORE ENERGY RELEASED IN BOMBS, NUCLEAR POWER PLANTS, ETC. THE QUESTION IS WHERE THE ENERGY ORIGINALLY CAME FROM TO CREATE THESE UNSTABLE ISOTOPES. PERHAPS, THE MOST LIKELY PLACE IS ~~IS~~ DURING THE EXPLOSIONS OF SUPER NOVAE. WE JUST DON'T KNOW WHERE A NEUTRON FLUX OF THE DENSITY (ABOUT 10^{24} cm^{-3}) COULD EXIST TO PRODUCE THIS NUCLEAR SYNTHESIS.

RADIOACTIVE ELEMENTS

AN INTERESTING PROBLEM ARISES CONCERNING THOSE ELEMENTS WITH LONG HALF LIVES. FOR INSTANCE, BOTH IODINE 126 WITH HALF-LIFE 17 MILLION YEARS AND PLUTONIUM 244 OF 82 MILLION YEARS AND VIRTUALLY EXTINCT ON EARTH. BUT IODINE DECAYS INTO XENON AND XENON 129 IS FOUND IN ROCKS CONTAINING I^{129} . SINCE ALMOST ALL THE IODINE IS GONE IT COULD NOT HAVE BEEN FORM MUCH MORE THAN 3×17 MILLION YEARS BEFORE THE MEASUREMENT, I.E., NOW. WHAT WE NOW CAN ESTIMATE IS WHEN THIS NUCLEAR SYNTHESIS STOPPED AND FORMATION OF SOLIDS BEGAN. BY MEASURING Xe^{129}/I^{129} DIRECTLY NOW, NOT ON EARTH BUT IN METEORS, WE ARRIVE AT A FIGURE OF 4.6×10^9 YEARS AS A TIME OF SOLID FORMATION. WITH SOME MUMBLING A PERIOD OF 10^8 YEARS HAS BEEN GUessed AS TO THE POINT OF SYNTHESIS CESSION. A TIME PLOT OF THE ABOVE STATEMENTS WOULD LOOK LIKE:



The other radioactive element can give more clues to affixing the proper times to our scale but the question is wide open and very confused. Elements like U^{235} (7.18), U^{238} (4.519), Thorium (14.39), and Rhenium (40g) would be interesting to study and see what concentrations they occur relative to each other. For instance, the formation of U^{235} and U^{238} is by the R-process. But U^{238} will undergo alpha emission to get back to 238. By guessing the many different ways this decay could occur, it is believed that the ratio of the concentrations is

$$U^{235}/U^{238} \approx 1.65 \pm .3$$

while Thorium to U^{238} is about the same

$$Th/U^{238} \approx 1.65 \pm .3$$

What can we do with these figures? Well, we know the ratio now to be about,

$$U^{235}/U^{238} \Big|_{\text{now}} \approx .00723$$

Let's work backward over 4.6 billion years at the time the solids were forming. Arriving at a figure of

$$U^{235}/U^{238} \Big|_{4.69} = .31$$

This is not 1.65 as we guess above but it certainly is not more. If it was this would mess everything up.

We can, at least, say this was about the last moment when something could be synthesized but it is not where it had to be made. U^{238} could have been made long before this. The exact proportions of the production say at the instant of first formation and the last fleeting moment when the conditions were favorable could vary a great deal. Maybe 80% of U^{238} was made 139 years ago and the ~~rest~~ rest 4.69 years ago. We are left at a difficult and confusing point to try and get information.

Since Thorium has a longer period, the contributions in the last few minutes of production would not matter much. But the ration of Th/U now is not known very well. On Earth it is estimated to be 3.8 which gives a time of 2.39 when the solids were formed-

A more interesting case still is the decay of Rhenium 187 into Osmium 187 because it takes 40 billion years which is believed to be older than the universe. If we had the ratio Os/Re we could calculate back to when it was made and be fairly certain that we had all the production at that time.

The problem here is that some of the osmium is formed by the S-process. So that we must subtract this S-process formed osmium from the osmium we know we have. Thus we can find the osmium due to disintegration

$$\text{Os}^{187}_{\text{DIS}} = \text{Os}^{187} - \left(\frac{\sigma(186)}{\sigma(187)} \right) \text{Os}^{186} \text{ (BY S ONLY)}$$

Since there is no rhenium isotope made by the S-process we have no way to calibrate the osmium process. So we have to measure millions of σ 's for each isotope and calculate exactly how much osmium is formed by the S-process.

CHAPTER 9

FEYNMAN ON -

DICKE ON THE OBLATENESS OF THE SUN

SINCE THE DEMAND IS TO DISCUSS DICKE AND AS I PROMISED LAST WEEK BUT TOTALLY FORGOT, I WILL PROCEED TO TELL WHAT I KNOW ABOUT HIS THEORY. SINCE I DIDN'T ATTEND HIS FIRST LECTURE AT CAL TECH WHEN HE EXPLAINED HIS EXPERIMENT, I CANNOT CRITICIZE THAT AND FOR THAT I APOLOGIZE.

THE CLAIM IS THAT THE SUN POSSESS AN OBLATENESS PERPENDICULAR TO ITS AXIS OF ROTATION. THE ASYMMETRY OF 1 PART IN 10,000 MEANS IT IS SQUASHED TOO MUCH. EXPERIMENTALLY IT IS VERY DIFFICULT TO LOOK AT THE SUN AND SEE THIS OBLATENESS BECAUSE OF SOLAR FLARE, DIFFICULTY IN DISCERNING THE REAL SURFACE, ETC. WHAT DICKE DID WAS TO TECHNICALLY OBSCURE THE SUN WITH A CIRCULAR MASK. THEN TO GO AROUND THE EDGE AND LOOK FOR ANY SECOND HARMONICS IN BRIGHTNESS. ANY ASYMMETRY IN BRIGHTNESS WOULD BE RELATED TO EQUI-POTENTIAL GRAVITY REGION AND THUS TO THE OBLATENESS.

THE RESULTING CONSEQUENCE OF THIS SQUASHED SUN IS THAT MERCURY, IN MAKING A ~~SQ~~ SWING AROUND THE SUN, DOESN'T COME BACK TO THE ORIGINAL POINT. WHEN ALL THE PERTURBING FORCES ON MERCURY ARE CONSIDERED, E.G., VENUS, EARTH, ETC., A RESULT OF 41.7 SECONDS OF AN ARC REMAIN UNEXPLAINABLE IN ANSWERING WHY MERCURY PRECESSES ABOUT IT PERIHELION SOME 539' ARC-SEC/CENTURY. THE NUMBER 41.7 IS THE FEYNMAN NUMBER SINCE I REMEMBER THIS FROM 25 YEARS AGO AND CAN'T VERIFY IT RIGHT NOW. AT ANY RATE, THE GENERAL THEORY OF RELATIVITY, AS EINSTEIN PROPOSES PREDICTED A VARIATION IN MASS WITH VELOCITY FOR MERCURY THAT GAVE A CORRECTION FACTOR OF 43" SECONDS OF ARC. THUS ASTROMERS NUMBERED A LITTLE AND SAID 1.3 ARC-SEC IS A GOOD AGREEMENT.

BUT NOW DICKE SAYS THAT AS A RESULT OF THE SUN'S OBLATENESS A QUADRUPOLE IS ESTABLISHED WHICH CAUSES A ~~SMALL~~ CORRECTION OF 4 ARC-SEC SO THAT THE REMAINING PERIHELION SHIFT IS 37.7". THIS IS GETTING TO BE A NOT SO GO ~~GOOD~~ AGREEMENT AND, PERHAPS, SOMETHING IS WRONG SOMEWHERE-EITHER IN EINSTEIN'S THEORY OR IN DICKE'S.

NOW A COUPLE OF REMARKS - FIRST, IT IS VERY DIFFICULT TO TAKE A MEASUREMENT OF BRIGHTNESS AND OBLATENESS. THE QUESTION OF WHETHER EQUI-BRIGHTNESS CORRESPONDS TO EQUI-PRESSURE OR EQUI-POTENTIAL IS HARD TO ANSWER.

SECONDLY, THE QUESTION OF HOW THE ~~EXACT~~ PERTURBATIONS DUE TO EACH PLANET AS CORRECTIONS TO THE TOTAL ERROR IS IN DOUBT. THAT IS, HOW ACCURATELY IS 41.7 KNOWN SINCE IT WAS DONE SOME FORTY YEARS. THIS FIGURE OF 539 SECONDS OF ARC IS A TYPICAL ASTRONOMICAL NUMBER, I.E., THERE ISN'T ANY $539 \pm$ SOME ERROR. ASTROMERS ARE NOT USE TO WORKING WITH ERRORS AND IF YOU ASK ME THIS WHOLE SUBJECT STINKS. TO DISCUSS DEVIATIONS OF 4 ARC-SECONDS WHEN 539 HAS NO ERROR VALUE IS DOWN RIGHT INSANE. IF, IN FACT, THE SUN HAS THIS EFFECT ON MERCURY THEN THE OBLATENESS EFFECT ON ALL THE OTHER PLANETS MUST BE CAREFULLY CALCULATED. I WOULD RECHECK ALL THESE CALCULATIONS VERY CAREFULLY FIRST BEFORE MAKING A THEORY TO EXPLAIN SOMETHING WHICH YOU'RE NOT SURE IS IN ERROR IN THE FIRST PLACE.

DICKE, however, is in the unusual situation of experimentally checking a theory, he himself, has ~~written~~ written down. I have a good deal of reliability in Dicke as a very careful experimentalist and theorician. He looks at everything and is not fooled by what has gone before. It is his own hard luck if he is on a wild goose chase like I think he is.

So Dicke worked on a new theory of gravity and to explain why he thinks gravity is made up of a mixture of tensor and scalar fields in a proportion of 1-s : s respectively we go back a little to Einstein's theory. In that theory Einstein postulated that gravitational and inertial masses are ~~a~~ strictly proportional. Remember our discussion of the elevator analogue in which we explained his principle of equivalence. The force of gravity is then proportional to the mass, the quantity which is fundamentally a measure of inertia. It is a measure of how hard it is to hold something which is going around in a circle. This force is exactly proportional to the mass with great precision; if it were not exactly proportional there would be some effect by which inertia and weight would differ. The absence of such an effect has been checked with great accuracy by an experiment done first by Eötvös in 1909 and, notably, by Dicke more recently. The accuracy of this experiment and equivalence is about 1 part in 10¹⁰.

The Eötvös experiment involves a swing mass and measures the ratio of gravitational to centrifugal force ~~for~~ and compares their ratio for different material. The question is: in the early evening as the sun is setting and we're on roller skates why don't we just coast off towards the sun. Well, it's simple - the earth is falling with us and in a proportional amount. Since roller skates are hard to make, let's put two objects of equal inertia at the end of a long quartz rod hung on a perfect pivot. If the inertias are equal and the gravitational attractions unequal, the rod would twist around until it lined up pointing toward the sun. But the centrifugal force balances the gravitational attraction so well that we say they are equal for all intent and purpose.

If gravity consists of two parts or field in a mixture of 1-s : s where s is a ~~proportion~~ proportionality factor. s could be as much as 20%, i.e., a scalar field contribution of that much. But if s is picked right we can explain that 4 arc-sec by the relation,

$$(1 - \frac{4}{3}s) = \frac{\text{correct}}{43 \text{ arc-second}}$$

I think in which case s is about 0.10.

HOWEVER, THE DEFLECTION OF LIGHT BY THE SUN IS PROPORTIONAL ONLY TO THE TENSOR FIELD CONTRIBUTION (I-S). THE PRECISE AMOUNT OF DEFLECTION IS AN AREA OF EXPERIMENTATION WHICH WILL HAVE TO RECHECK. BUT I'LL BET HE DOESN'T FIND ANY SUCH FIELD CONTRIBUTION AND I'M GLAD HE'S ON THE CHASE NOT ME.

MACH'S PRINCIPLE IS ALSO INVOLVED IN THIS DISCUSSION BECAUSE IT ASSERTS THAT THE INERTIAL PROPERTIES OF MATTER ARE DETERMINED BY THE PROPERTIES OF MATTER IN THE UNIVERSE. THIS IMPLIES THAT THERE IS NO ABSOLUTE FRAME OF REFERENCE SINCE TWO UNIVERSES ONE WITH A LOT OF NEBULAE AND THE OTHER WITH A FEW WOULD SHOW DIFFERENT AMOUNTS OF CENTRIFUGAL FORCE ON AN OBJECT ROTATING. LOCALLY THE SUN AND THE GALAXY ARE SOURCES OF ANISOTROPY OF MATTER BUT NO EFFECT WAS MEASURED BETTER THAN ONE PART IN 10^{10} . THAT IS TO SAY INERTIA IS GENERATED OR CAUSED BY SOMETHING ELSE. IF WE COULD FIND THE DENSITY OF THIS BACKGROUND MATTER CAUSING ALL THE INERTIA WE WOULD FIND S. DICKE ASSERTS THAT MACH'S PRINCIPLE REQUIRES THAT ANISOTROPIES BE UNOBSERVABLE BECAUSE ALL INSTRUMENTS HAVE THE SAME ISOTROPY.

THE PROBLEM RELATED TO MACH'S PRINCIPLE IS WHETHER OR NOT THE MOTION OF THE WATER INSIDE A SPINNING PALE DEPENDS ON THE MOTION OF THE RELATIVE NUMBER OF NEBULAE. WE KNOW THE WATER SPLASHES UP AGAINST THE SIDE BUT WHAT HAPPENS IN A SPACE VOID OF MATTER; DOES IT STILL RISE IN A PARABOLIC SHAPE?

TO ANSWER THIS QUESTION WE CONSIDER A COSMOLOGICAL THEORY IN WHICH THERE ARE NO NEBULAE. IN THIS SPECIAL CASE EINSTEIN'S THEORY OF GRAVITATION WHICH HE WROTE AS

$$R_{\mu\nu} = T_{\mu\nu}$$

WHERE $R_{\mu\nu}$ IS A FUNCTION OF THE GRAVITATIONAL POTENTIAL AND ALL ITS DERIVATIVES $g_{\mu\nu}, \frac{\partial g_{\mu\nu}}{\partial x}, \frac{\partial^2 g_{\mu\nu}}{\partial x^2}, \dots$

AND $T_{\mu\nu}$ IS THE STRESS TENSOR OF THE MATTER OUTSIDE THE REGION OF INTEREST

OR WE CAN WRITE

$$\square^2 \phi = \rho_{\text{matter}}$$

WHERE $\square^2 =$ THE D'ALEMBERTIAN OPERATOR $= \frac{\partial^2}{\partial t^2} - \nabla^2$

ϕ IS THE GRAVITATION POTENTIAL

ALSO WE RECALL THE ELEMENTAL LENGTH IN FOUR-VECTOR NOTATION IS

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

If we have no matter so that $T_{\mu\nu} = 0$, then the solution to $R_{\mu\nu}$ takes the form

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

or $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$

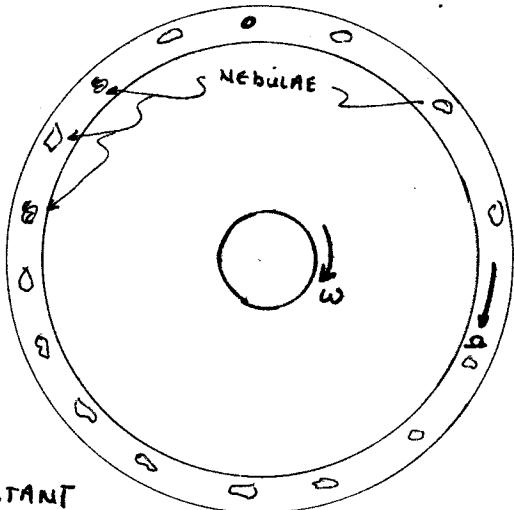
This line element describes a flat space and as a consequence of this form matter has inertia thus we expect the pale to experience inertia.

If we consider a pale at the center of a shell made up of nebulae that are rotating at ~~some~~ some angular velocity ω , then in order for the water to stay flat it must turn with the same ω . But the pale turns at a rate less than ω ; it is an ω which is proportional to ω with some $\frac{Mm}{R}$ factors reducing it. The centrifugal force then depends on the relative angular velocity between the two systems.

The difficulty in understanding this problem is in finding a solution to the Einstein equation for $T_{\mu\nu} = 0$ and satisfying the correct boundary condition. Beyond the shell of matter the metrical tensor $g_{\mu\nu}$ is assumed to approach $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ or simply 1 but this is wrong. $g_{\mu\nu}$ should go to 0.

To see this we consider an analogue in electromagnetic theory. From the Maxwell equations we know $\nabla \cdot \vec{E} = \rho$ and $\nabla \times \vec{E} = 0$ etc. If there aren't any charges so that $\nabla \cdot \vec{E} = 0$, we have a solution in which we have a constant electric field in the z -direction, say. But to have a uniform field in the z -direction implies we must have charges farther out or below causing the field. If we have a solution then for which $g_{\mu\nu}$ does not vanish the nebulae must be charged since we cannot attribute the field to empty space. What, in fact, might be the case is a large shell farther out causing the inertia effect.

What we are missing is a way to predict inertia from physical properties. So far we haven't stated our law right. That is to say the equation $\nabla \cdot \vec{E} = \rho$ does not contain the statement that all fields come from matter so we can't argue from the differential equations alone. While Einstein permits inertia to exist when matter is not present, Maxwell permits a field to exist when no charge is around only if the postulate that the field comes from some charges.



The STRESS ENERGY TENSOR $T_{\mu\nu}$ IS THE SOURCE OF GRAVITATION FIELD. THIS TENSOR CONSISTS OF AN ENERGY CONTRIBUTION WHICH WE CALL ρ PLUS T_{ii} WHICH IS THE DEFORMATION STRESS, ---, T_{ij} THE FLOW MOMENTUM, ETC. WE CAN SHOW THE MAIN CONTRIBUTION TO THE GRAVITY FIELD COMES FROM T_{44} OR

$$\square^2 g_{\mu\nu} \sim T_{\mu\nu}$$

REDUCES TO

$$\square^2 g_{44} \sim T_{44}$$

SUPPOSE THAT

$$\square^2 \phi = \text{FIELD PRODUCED BY THE TRACE OF } T_{\mu\nu}$$

OR

$$\square^2 \phi = T_{44} - T_{11} - T_{22} - T_{33}$$

IF WE CONTAIN A SMALL GLOB OF STUFF IN A VOLUME AND PERMIT ALL SORTS OF STRESS CONTRIBUTION, SQUEEZING, SQUASHING, TWISTING, ETC, WE CAN REPLACE THE CONTRIBUTIONS BY VOLUME INTEGRALS TO DETERMINE THE REALIZE STRENGTH OF THE ENERGY CONTENT, I.E.,

$$\int T_{44} dVOL = \int T_{11} dVOL + \int T_{22} dVOL + \int T_{33} dVOL$$

WHERE $\int T_{44} dVOL$ IS DUE TO THE MASS

IT WILL SUFFICE TO SHOW $\int T_{11} dVOL = 0$ AND THE REST CAN BE WORKED OUT SIMILARLY.

If we make a cut through the glob PERPENDICULAR TO THE X-AXIS, THEN T_{11} corresponds to the force per unit area IN THE X-DIRECTION. SINCE THE OBJECT IS STANDING THERE THE TOTAL UPPER FORCE = 0. SO WE HAVE

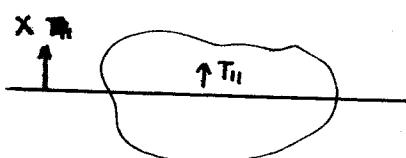
$$\int F_1 d\text{Area} = 0$$

INTEGRATING AGAIN OVER THE HEIGHT dx

$$\int F_1 d\text{Area} dx = 0$$

WE STILL GET 0. NOW WE HAVE THE VOLUME INTEGRAL = 0 FOR T_{11} . SO WE CONCLUDE THAT T_{44} IS THE ONLY SOURCE FOR GRAVITY AND THERE IS NO INTERNAL CONTRIBUTION. I THINK I HAVE PROVED THIS IN GENERAL.

IT IS POSSIBLE EVEN THAT THE GRAVITATION CONSTANT IS VARYING WITH THE AGE OF THE UNIVERSE IN WHICH CASE WE HAVE MORE TO WORRY ABOUT. THAT IS, DICKE DOES. HIS WORK IS A GOOD IDEA BECAUSE IT CHECK SOME OF OUR OLD FIGURES MENTIONED ABOVE. SO WHILE I THINK HE IS WRONG I ADMIRE HIS CONVICTION.



For reference what stimulated Feynman's talk and response above

DICKE'S THEORY
(SCIENTIFIC AMERICAN
MARCH 1967)

if PART (8%) of The ADVANCE of MERCURY'S PERIHELION CAN BE ATTRIBUTED TO THE OBLATENESS of THE SUN, IT WILL BE NECESSARY TO MAKE A SPECIFIC MODIFICATION IN THE GENERAL Theory of RELATIVE AS PROPOSED by EINSTEIN. TO EINSTEIN GRAVITY IS NOT A force but A CONSEQUENCE of THE CURVATURE of SPACE. GRAVITY IS ASSOCIATED WITH THE PRESENCE OF A TENSOR FIELD, WHICH IS THE METRIC TENSOR of THE RIEMANNIAN GEOMETRY of CURVED SPACE.

DICKE PROPOSED TO ADD A FORCE COMPONENT ASSOCIATED WITH A SCALAR FIELD, I.E., A FIELD WHICH EACH POINT IN SPACE HAS A SPECIFIC PHYSICAL QUALITY TO WHICH A SINGLE VALUE CAN BE ASSIGNED. IN THIS THEORY THE ADVANCE OF MERCURY'S PERIHELION IS LESS THAN IN EINSTEIN'S BY ABOUT TEN PERCENT, OR BY THE FACTOR $1 - 4S/3$ WHERE S IS THE FRACTION OF A BODY'S WEIGHT DUE TO THE SCALAR INTERACTION.

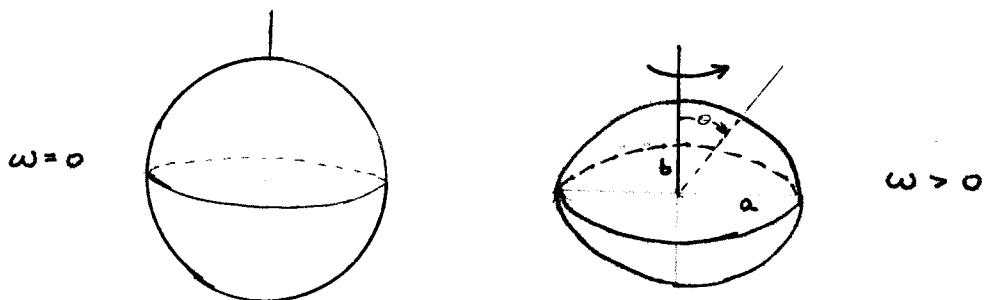
AS A RESULT OF DICKE'S WORK, HE CONCLUDES THE CORE OF THE SUN MUST BE ROTATING MUCH FASTER THAN THE SUN'S SURFACE. TO GET THE DESIRED FLATTENING IT MUST ROTATE ONCE IN ABOUT 1.8 DAYS

MACHE'S PRINCIPLE

"WHEN A BODY ROTATES RELATIVELY TO THE FIXED STARS, CENTRIFUGAL FORCES ARE PRODUCED; WHEN IT ROTATES RELATIVELY TO SOME DIFFERENT BODY NOT RELATIVE TO THE FIXED STARS, NO CENTRIFUGAL FORCES ARE PRODUCED." THAT IS, ROTATION OF A BODY RELATIVE TO THE FIXED-STAR SYSTEM IS EQUIVALENT TO A ROTATION OF FIXED STARS ABOUT THE BODY.

PERTURBATION OF MERCURY'S ORBIT - H.Y. Chiu

The rotation of a sphere will make it bulge in the center



In order that a sphere in rotation be gravitationally bound, the gravitational force per unit mass at a radius must be greater than the centrifugal force:

$$\frac{V^2}{R} \leq \frac{GM}{R^2}$$

$$V = \omega R \quad \rightarrow \quad R^3 \omega^2 \leq GM$$

$$\omega^2 \leq \frac{GM}{R^3}$$

for SUN

$$= \frac{(7 \times 10^{-8})(2 \times 10^{33})}{(7 \times 10^{10})^3} = 40 \times 10^{-8}$$

$\omega < 6.3 \times 10^{-4}$ rad/sec for the sun to be rotationally stable, for the solar rotation period

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{26 \times 10^5} = 2.5 \times 10^{-6}$$
 rad/sec

$$\frac{\omega}{\omega_{\max}} \approx \frac{1}{50}$$

The solar rotation frequency is not very much smaller than the maximum frequency at which the sun could rotate and remain gravitationally bound.

Expanding the gravitational potential of a non-symmetric or oblate object in terms of the Legendre polynomials

$$\Phi(n, \theta) = \Phi(r) + P_2(\cos \theta) \Phi_2(n) + \dots$$

where θ is defined as the polar angle.

Such an oblate object has its perihelion rotate at a frequency

$$\omega_p = \frac{6\pi}{5} \epsilon \left(\frac{R}{r}\right)^2 \times (\text{geometrical factor})$$

(This work is attributed to Brouwer)

The perihelion is the point on the orbit which is closest to the attraction center and $\epsilon = \frac{b-a}{a}$ is the eccentricity of the central body

The factors which contribute to the rotation of the perihelion of Mercury are as follows

CONTRIBUTION FROM	MERCURY ARC-SECOND
MERCURY	0.025 ± 0.00
VENUS	277.856 ± 0.68
EARTH	90.038 ± 0.08
MARS	2.536 ± 0.00
JUPITER	153.584 ± 0.00
SATURN	7.302 ± 0.01
URANUS	0.141 ± 0.00
NEPTUNE	0.042 ± 0.00
MOON	—
GENERAL PRECESSION OF THE EQUINOX	5025.645 ± 0.5
TOTAL EXPERIMENTALLY OBSERVED Difference	5557.1 ± 0.8 5599.7 ± 0.4 42.6 ± 0.9

The lower value $42.6 - 0.9 = 41.7$ would be the Feynman number.

EINSTEIN'S THEORY OF GENERAL RELATIVITY PREDICTS AN ADVANCE IN PERIHELION IN ADDITION TO THOSE JUST GIVEN OF,

$$\frac{6\pi GM}{R_{\text{PLANET}} R_{\odot}^2} \frac{1}{1-e^2} = 43'' .03 \pm 0.01$$

e is the eccentricity of the planetary orbit.

THE GRAVITATIONAL PERIHELION ADVANCE MAY BE WRITTEN AS

$$\omega_g = 6\pi \times 2 \times 10^{-8}$$

WHILE THE ADVANCE CALCULATED FROM ω' DUE TO THE SUM BEING ASYMMETRICAL IS

$$\omega_a = 6\pi \times 6 \times 10^{-4} e$$

TO ACCEPT GENERAL RELATIVITY THEN $e \ll 10^{-4}$ DESPITE THE RELATIVELY HIGH VALUE OF THE ROTATIONAL FREQUENCY.

SO IF DICKE CLAIMS TO HAVE FOUND AN OBLATENESS OF 10^{-4} HE IS FORCED TO ABANDON GENERAL RELATIVITY TO EXPLAIN THIS PERTURBATION.

FROM LECTURES IN THEORETICAL PHYSICS - ASTROPHYSICS AND WEAK INTERACTIONS
1963 V.2 BRANDEIS

CHAPTER 10

RADIO GALAXIES

WE NOW MOVE TO A NEW TOPIC AND DISCUSS THE SOURCES OF RADIO EMISSION IN THE UNIVERSE. FIRST WE WILL DISCUSS THE CURRENT EXPERIMENTAL SITUATION AND THEN DISCUSS SOME THEORIES ALONG WITH A TALK ABOUT COSMIC RAYS.

UNFORTUNATELY OUR ATMOSPHERE IS OPAQUE TO EVERYTHING EXCEPT VISIBLE LIGHT, I.E., THAT MINUTE PORTION OF THE SPECTRUM WHICH OUR EYES ARE SENSITIVE TO. THIS IS, INDEED, SOMETHING TO BE GRATEFUL FOR AND GOD DID A GOOD JOB HERE. SO WE DON'T HAVE A CLEAR 'VIEW' OF THESE RADIO TRANSMISSIONS DUE TO THIS BLOCKAGE. THERE ARE, HOWEVER, SOME "WINDOW" IN THE SPECTRUM WHICH ALLOWS US TO LOOK THROUGH AND SEE WHAT'S GOING ON OUT THERE. ONE SUCH WINDOW IN THE RADIO WAVE REGION LIES BETWEEN 1CM TO 30 CM. SMALLER TRANSPARENCIES IN THE SKY EXIST AT OTHER WAVELENGTHS BUT FORTUNATELY THEY ARE NOT WIDE. OTHERWISE WE COULD HEAR EUROPEAN RADIO BECAUSE THE IONOSPHERE WOULD BE A LEAKY REFLECTOR.

SO WHAT DO WE SEE OR HEAR? WHERE ARE THE SOURCES OF RADIO EMISSION?

- (1). FIRST, THE SUN ACTS AS A 'NOISY' SOURCE OF RADIO EMISSION. THE PATTERNS ARE VERY COMPLICATED AND AREN'T VERY INTERESTING
- (2). THE MOON; NOT MUCH TO SAY HERE
- (3). MARS AND JUPITER. JUPITER IN PARTICULAR BECAUSE THERE ARE SOME VERY INTERESTING THUNDERSTORMS GOING ON UNDER ITS ATMOSPHERE. I'LL TRY TO TALK ABOUT THIS LATTER IN THE COURSE. BUT THESE ARE SOURCES OF BLACKBODY RADIATION AND REALLY AREN'T INTEREST TO OUR TOPIC
- (4). NEXT, THOUGH, IS THE GALAXY ITSELF. THIS WE WILL DEVOTE MORE TIME TO IN A MINUTE.
- (5). FINALLY, THE ALL INCOMPASSING GROUP - THE EXTRA-GALACTIC SOURCES.

RETURNING TO THE GALAXY THERE ARE A COUPLE OF TOPIC HERE THAT WE NEED TO DISCUSS. THERE ARE, SCATTERED THROUGHOUT THE GALAXY, SMALL SPOTCHES OR POINT SIZE SOURCES OF RADIO EMISSION. WHILE AT THE SAME TIME THERE IS A GENERAL BACKGROUND RADIATION WHICH VARIES ALL OVER THE SKY. THE INTENSITY VARIES GREATLY WHICH THE COLDEST REGIONS CORRESPONDING TO A BLACKBODY AT 80° KELVIN AND THE HOTTEST, I.E., IN THE AREA NEAR THE CENTER OF THE GALAXY OF NEAR 1000° K. IN ADDITION TO THESE SOURCES THERE ARE THERMAL AND NON-THERMAL SOURCES. ORION IS AN EXAMPLE OF THE FORMER AND THE REMNANT OF SUPERNOVAE LIKE THE CRAB NEBULAE FOR THE LATTER. ALSO FLARE STARS LIKE SOLAR OUTBURST OF THE SUN WHICH SEND OFF PULSES OF ELECTROMAGNETIC RADIATION HAVE BEEN OBSERVED.

THERE ARE SPECTRAL LINE OBSERVED IN CLOUDS OF HYDROGEN. THESE EMISSION ARE DUE TO SPIN-FLIPS IN THE HYDROGEN ATOMS WHICH CAN OCCUR NATURALLY.

SPECIAL UNPREPARED TALK ON THE LUMPINESS OF THE UNIVERSE

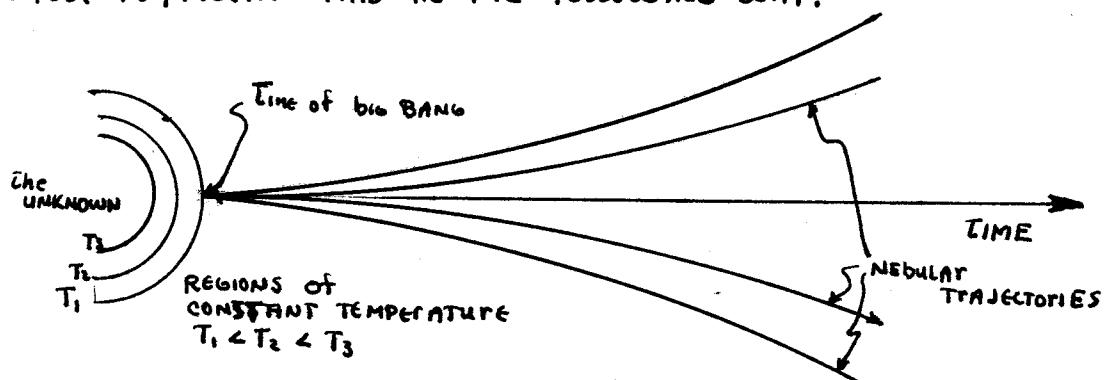
...or where does does the anisotropic (I.e. Lumpiness) come from?

This topic was brought by one of the attendee by request of Feynman who has been having trouble learning about synchrotron radiation. So off on a tangent - but a very interesting one - we go.

The question was brought up in regards to a special lecture at Cal Tech in which the lumpiness property of the universe, i.e., the clustering of galaxies can be connected with a cosmological theory. If the universe was lumpy now, was it lumpy during the Big Bang if, in fact, there was a Big Bang? Will it get more lumpy as time goes on?

To begin with all models of the universe seem to agree on one point - some time in the past the universe was closer together and it is currently spreading out. We accepted the cosmological assumption that the universe is fairly constant so that if we were a thousand light years from here, we would see the same things - dust, stars, galaxies, hydrogen, helium, etc. Since then we discussed the way stars generate new elements by starting with just a mass of gas. The question arises as to what the mixture was at the time of creation (using that word carefully) to give the current distribution of hydrogen, helium, etc. Just to assume an expanding universe model does not necessarily fix the initial mixture because of all the nuclear reactions, s-processes, etc which we have mentioned and which can change the constitution of the universe all the time. We could make a rather artificial assumption - in the beginning there was hydrogen but this is not a well-founded assumption although it might be true.

The problem we face then is what assumption do we start with? If we go along with the majority and assume everything was more tightly packed together in the past, what can we say? The real difficulty is that we can only pack material so tight that it will eventually explode; what occurs before that point is the real question. The only safe assumption is then that we must start with an assumption. Alright, we've double-talked enough; let's assume that we can start with homogeneous mixture at some very high temperature T . Going further back in time only raises that temperature. We can diagrammatically represent this in the following way:



This homogeneity allows us to understand the expanding radiation as that from a black body source. At any given instant we can specify the following three conditions:

TEMPERATURE, NUMBER OF NUCLEONS, WHAT THE NEUTRON RATIO IS
PER CC. PROTON

If we know the first two numbers we could calculate the point of thermodynamic equilibrium before the temperature goes too low, i.e., after expansion starts. So let's suppose at temperature T_1 the goop is in thermal ~~equilibrium~~ equilibrium. As expansion progresses the mixture cools and lumps form. But the real beauty of this picture is that we know the temperature and density of neutrons we can find out how much hydrogen and helium there was. From a calculation of this kind it has been found that the ratio of hydrogen to helium is of the order of 20-40 %

$$\frac{H}{He} = 20-40\% \text{ INITIALLY}$$

This initial condition is good agreement for population II stars; this we have checked with the R.R. LYRAE VARIABLE STARS. The agreement for other elements such as carbon, nitrogen, and oxygen is not so good but these concentration are easily altered by nuclear reactions. For most astronomers a 30% concentration ratio is fine - except for some small blue stars which display a spectra from their atmosphere which hasn't any helium. The suggestion has been made that this is due to a gravitational settling out of helium but this seems wrong. However, intuition is not always the best guide when discussing the size and time periods involved.

There arises a problem when trying to work with the three quantities enumerated above. That being, the possibility of pairs existing in equilibrium in the super goop. By pairs we mean positron-electron, neutrino-antineutrino, etc. Thus we have the two way beta decay



which shows more clearly that N/p is not such a good ratio to pick since excesses of $\bar{\nu}$ or \bar{e} would tend to drive the reaction in the characteristic direction. More fundamental then would be to calculate the number of baryons per cc and how many ℓ leptons to anti-leptons there are, where baryons are the class of particles consisting of nucleons and hyperons (unstable particles with mass 2,181 times that of an electron). And the generic terms leptons and anti-leptons referred are used for muons, electrons, mu neutrinos, electron neutrinos and their anti-particles.

So our new quantities are

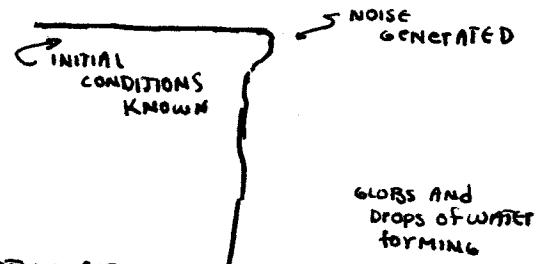
TEMPERATURE, NUMBER OF BARYONS, RATIO OF $\frac{\text{LEPTONS}}{\text{ANTI-LEPTONS}}$ PER G.C.

The latter ratio being equal when the number of electrons = the number of positrons and the same for neutrinos and anti-neutrinos. Fortunately there exists a number baryons over anti-baryons, i.e., nucleons over anti-nucleons. Fortunately, that is, because if it were the other way there wouldn't be any matter in the universe, just propagating light! Thus, we realize another fundamental miracle of design. ~~This~~.

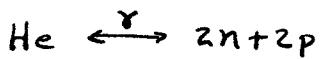
Therefore, it is the choice number of leptons which determines the ratio N/P. If the number of leptons = number of anti-leptons we get the 20-40% figure.

Well, getting back to the lumpiness problem, suppose one region on the edge of this primordial ball cools a little faster than the surrounding junk. Then globular regions would form and become more dense. The result being a lower expansion rate. Since gravity would tend to magnify these lumps, we expect the universe to be more lumpy today. One of the problems we are confronted with here is how to determine the galaxy size and whether or not there is a lumpiness at all. We haven't analyzed the phenomena of gas condensation very well. If the gas never becomes unstable, galaxies would never form. What is necessary is enough noise or background fluctuation to disturb this expansion and cause gravitational condensation to begin. While the specific noise pattern might determine the form of condensation it does not fix the size and character of the resulting masses.

An analogy is a waterfall in which we look up at and watch the water coming over. Before the edge of the cliff the initial conditions can be well stated as to flow velocity, depth of water, etc. As the water starts over the break point noise is generated which eventually breaks and shakes the smooth flow up into a globular flow and finally a mist. If we were to ask the what the chances are of having a particular drop end up on our nose given its initial condition we immediately realized the intermediate noise has little to do with a particular drop landing on us. So even specifying the initial conditions would not clearly define the answer as to how the current structure of the universe, say, got from the past.



As the temperature cools and the helium reaction occurs with high energy gamma rays being emitted, i.e.,



There is a specific concentration at equilibrium,

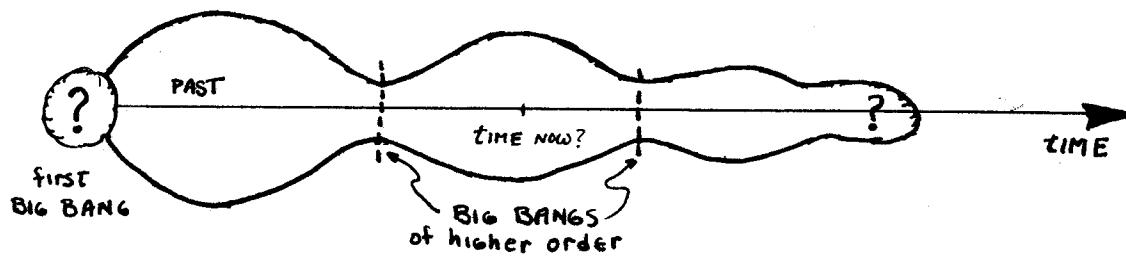
$$\frac{[\text{He}]}{[\text{H}]^2 [\text{P}]^2} = e^{-E_{\text{helium}} - E_{\text{nucleons}} / kT}$$

This example breaks down at low temperatures but for equilibrium there is pure helium at zero degrees, which, after expansion, freezes helium in its original proportions. Also, there is a γ -ray decoupling occurring as the material becomes transparent to these γ -rays. (Note: if this section is confusing to you, it's because it was confusing to me.)

IS THE UNIVERSE OSCILLATING?

Another interesting question involves the oscillating universe and whether it will ever end, i.e., damp itself out. A fellow named Tolman believes because entropy is always increasing the radius as a function of time shows a kinematic dependence. But, in fact, the radius of the universe is determined by relativity and I would guess it would slow up and eventually stop oscillating. It seems that the radiation dissipation following compression is a dynamic condition like a piston pushing on some gas, not a kinematic one as Tolman believes. The problem involves an irreversible process where energy conservation does not hold. Consider the brain-teaser for sophomore physics students of the chain dropping on a ~~chain~~ table. What is the force on the table due to the periodic pounding motion of the chain links as they successively hit the table and impart a small amount of momentum to it. If the chain drops with velocity v in a gravity free region, what is the resulting speed of the table? As we mentioned energy conservation does not work because the links bounce back a little bit and this action must be considered.

A picture of our oscillating model might look like:



WE CONCLUDE, OR REASON, THE MOTION OF THE RADIUS NEED NOT BE SMOOTH. GRAVITY CAN ACT TO CAUSE NOISE AND DISTURB THE GAS. AS THE WAVES PROPAGATE THROUGH THE MEDIA, LOCAL AREAS OF CONDENSATION CAN RESULT WHICH ACT AS COLLECTING CENTERS. AFTER THE FIRST OR PRIMARY BIG BANG THE WHOLE SYSTEM COULD BECOME SHAKY. SPURIOUS GRAVITATIONAL WAVE VIBRATIONS STIR UP THE MEDIA AND AS TIME PROGRESS SLOW UNCOUPLE WITH MATTER. THAT IS, TODAY GRAVITY WAVES PENETRATE RIGHT THROUGH MATTER. HOWEVER, THE EXACT POINT IN TIME WHEN LIGHT AND GRAVITY BECOMES DECOUPLED NEED NOT COINCIDE.

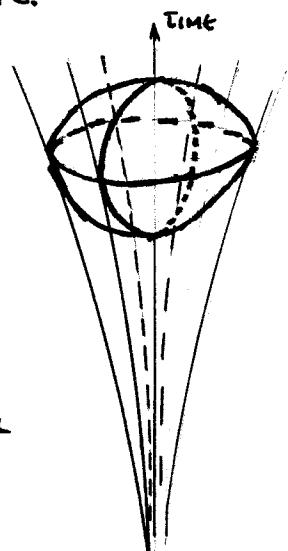
The Cosmological Mirror

WE HAVE MENTIONED BEFORE A VERY NIFTY ANALOGY FOR UNDERSTAND THE COMPLEX PHENOMENA OCCURRING IN THE UNIVERSE. IT IS EASY TO GET LOST IN THE PHYSICS UNDERLYING THE UNIVERSE. SUCH AREAS OF INTEREST OF RELATIVITY, METRIC TENSORS, SCHWARZCHILD SINGULARITY, GRAVITONS, ANTI-NEUTRINOS, NUCLEAR REACTIONS, RADIATION PROPAGATION ETC., ETC., BECOME A TANGLE MESS IN NO TIME AT ALL.

LET'S MAKE USE OF OUR HOMOGENEOUS MODEL OF THE UNIVERSE AND SEE WHAT WE CAN DO WITH IT. FIRST, AS WE RECALL, IF WE WENT OUT TO A NEAR-BY GALAXY AND PASTED A MIRROR ON IT, THE NUMBER OF PARTICLES INCIDENT ON EITHER SIDE WOULD STATISTICALLY BE THE SAME. ALRIGHT THEN, LET'S NOT GO OUT TOO FAR, SAY TO A RADIUS OF TEN NEBULAR UNITS. BY THIS WE SIMPLY MEAN TEN TIMES THE AVERAGE SPACING BETWEEN NEBULAE. PUT A GREAT BIG SPHERICAL MIRROR THERE.

WHAT WE HAVE IN EFFECT IS SOMETHING LIKE THE DRAWING HERE - WITH A LITTLE IMAGINATION ADD TO PUT IT IN FOUR DIMENSIONS.

BY NOT ATTEMPTING TO TAKE SUCH A MONSTROUSLY LARGE VOLUME THE RELATIVISTIC EFFECTS OF GRAVITY CAN BE IGNORED. HOWEVER, AS WE RECALLED FOR A SIMILAR ARGUMENT FOR A SPHERE OF MATTER AND ASKED WHAT HAPPENS AND SAID WE COULD IGNORE THE STUFF OUTSIDE THE BOUNDARY, WHAT'S INSIDE MUST BE CONSIDERED. SO WE STILL MUST EXAMINE THE GRAVITATIONAL WAVES ZIPPING ABOUT BUT REALIZING RELATIVISTIC EFFECTS BECOME SMALL.



BY THUS MAKING CURVATURE AN UNIMPORTANT PART OF OUR UNDERSTANDING OF WHAT GOES ON INSIDE THE SPHERE WE CAN RESORT TO A NEWTONIAN EXPLANATION OF THE SYSTEM. ONE WE ASSUME A HOMOGENEOUS MIXTURE OF JUNK, THE ENTIRE HISTORY OF THAT MATERIAL IS REFLECTED IN WHAT HAPPENS INSIDE THE SPHERE. IT IS IMPORTANT TO FIX THE BOUNDARY CONDITIONS OF THE PRESSURE AT THE WALL, I.E., DOES THE PRESSURE INSIDE EXERT AN ACCELERATING FORCE TO THE WALL TO CAUSE EXPANSION? THE ANSWER BEING YES BUT BECAUSE OF THE FINITE MECHANICAL DENSITY INSIDE THE EXPANSION CANNOT PROCEED AT AN INFINITE RATE.

IN THIS WAY WE HAVE COMPRESSED THE WHOLE UNDERSTANDING OF THE UNIVERSE TO A SIMPLE AND WORKABLE ANALOGY OF THE NATURE OF A HOT GAS IN A BOX. (IF SOME OF THE HOT GAS BLOWING AROUND IN COSMOLOGICAL CIRCLES WOULD BE DIVERTED TO A UTILIZED PURPOSE OF UNDERSTANDING THIS SIMPLE CASE, PERHAPS WE WOULD UNDERSTAND MORE ABOUT WHERE WE CAME FROM AND HOW). BY CAREFULLY EXAMINING THE INHOMOGENEITIES WHICH OCCUR IN THE EXPANSION WE SHOULD BE LED TO A BETTER KNOWLEDGE OF THE UNIVERSE.

SINCE THE PRESSURE AND VOLUME OF THE GAS ARE RELATED BY THE SIMPLE LAW

$$PV^\gamma = \text{CONSTANT}$$

WE FIND THAT

$$\text{TEMPERATURE} \approx V^{-\gamma+1}$$

FOR A PROTON GAS $\gamma = 4/3$ SO THE TEMPERATURE GOES AS $1/\text{LENGTH OF A SIDE}$. SIMPLE ENOUGH. FOR INTERSTELLAR GAS A GOOD GUESS FOR γ IS $5/3$. SO THAT

$$\frac{T_2}{T_1} = \frac{V_2^{-\gamma+1}}{V_1^{-\gamma+1}} = \frac{V_2^{-2/3}}{V_1^{-5/3}}$$

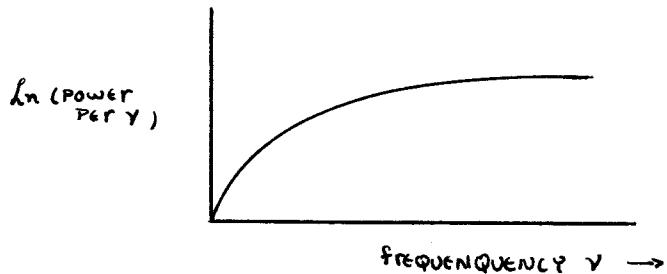
TO SEE HOW GOOD THIS APPROXIMATION IS AT THE EARTH RADIATION DENSITY IS GIVEN AS ABOUT 300° . FOR A STAR AT THE DISTANCE OF ONE LIGHT YEAR OR 3×10^17 SEC WHILE ~~EMITS~~ WHICH GETS THE LIGHT FROM OUR SUN WHICH IT SEES AS A BLACK BODY AT TEMPERATURE T EMITTING A DENSITY OF LIGHT $\sim T^4$ THE POWER GOES AS $1/r^2$ THE RADIUS TO IT. FROM WHICH REASONING FEYNMAN ESTIMATES ABOUT A 1° RADIATION DENSITY.

Radio Emission

WE WANT TO TALK MORE TECHNICALLY ABOUT THE ACTUAL SOURCES AND ORIGIN OF RADIO EMISSION. WE WILL FIRST DISCUSS BRIEFLY THERMAL OR BLACKBODY EMISSION AND MORE EXTENSIVELY SYNCHROTRON RADIATION.

FROM THERMAL SOURCES LIKE OIION NEBULAE THE SPECTRUM OF LIGHT EMITTED HAS THE DISTRIBUTION $\frac{v^3 dv}{e^{hv/kT} - 1}$. AT LOW FREQUENCIES WE GET A DISTRIBUTION

CORRESPONDING TO $\hbar T v^2 dv$. INSTEAD OF FLUX UNITS WE ASSOCIATE A TEMPERATURE THAT THE ANTENNA "SEES" WITH THE RADIATION. IF WE PLOT THE LOGARITHM OF POWER PER UNIT FREQUENCY VERSUS FREQUENCY, WE GET THE FOLLOWING CHARACTERISTIC CURVE:



THE EXPLANATION OF THE SHAPE OF THE CURVE IS THAT THE GAS CAN BOTH RADIATE AND ABSORB. SINCE THERE ARE A LOT OF ELECTRONS RUNNING AROUND WHICH ARE ACCELERATED, THEY NATURALLY EMIT RADIATION. THE ABSORPTION CROSS SECTION PER ATOM TIMES THE NUMBER OF ATOMS EQUALS ONE OVER THE MEAN FREE PATH. IF THE MATERIAL IS MANY MEAN FREE PATHS LONG, SAY L , WHICH CORRESPONDS TO THE OPTICAL DEPTH τ (TAU). WHERE WE DENOTE THE OPTICAL DEPTH TO MEAN THE NUMBER OF ABSORBED WAVELENGTHS THE THING IS DEEP.

WHAT HAPPENS FOR DIFFERENT τ 'S? IF $\tau \approx \infty$, THEN THE RADIATION FROM THE WALL IS PROPORTIONAL TO $\hbar T v^2 dv$, I.E., THE GAS AT TEMPERATURE T EMITS LIKE A BLACKBODY. FOR FINITE τ THE BLACK BODY RADIATION IS REDUCED BY AN AMOUNT $e^{-\tau}$, I.E.,

$$\hbar T v^2 dv (1 - e^{-\tau})$$

FOLLOWING THIS FURTHER WE CAN DEFINE A SOURCE TEMPERATURE, WHICH IS NOT REALLY A TEMPERATURE AT ALL BUT RATHER UNITS OF STRENGTHS OF EMISSION, AS A FUNCTION OF THE ELECTRON PATH TEMPERATURE (?)

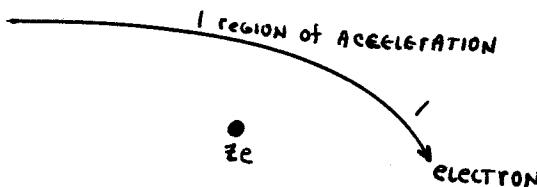
$$T_s = T_{el} (1 - e^{-\tau})$$

OUR IMMEDIATE PROBLEM IS TO FIND OUT WHAT κ IS. TO DO THIS WE WANT TO DETERMINE HOW MUCH EMISSION COMES FROM A VERY THIN LAYER OF GAS. AFTER THAT CALCULATED WE CAN THEN FIND ΔL A THICK LAYER. THE EMISSION IN THIS SMALL LAYER IS THUS GIVEN BY

$$\kappa T v^2 dv [dt]$$

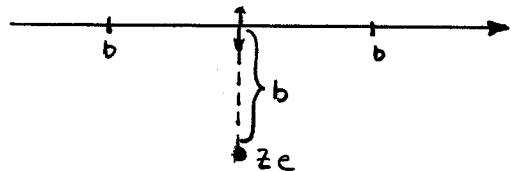
IN ORDER TO SPECIFICALLY CALCULATE HOW MUCH Δt THERE IS FOR A GIVEN AMOUNT OF MATERIAL WE MUST CALCULATE THE ABSORPTION COEFFICIENT BY USING THE EMISSION PHENOMENA. REMEMBER WE TALKED ABOUT A SIMILAR PROCESS WHEN DISCUSSING OPACITY, EINSTEIN COEFFICIENTS, ETC.

WHY DOES THE ELECTRON EMIT RADIATION. FOR OUR DISCUSSION CLASSICAL THEORY WILL SUFFICE. WE ASSUME A COULOMB POTENTIAL IN WHICH A MOVING CHARGE IS ACCELERATED BY THE ATTRACTION INTO A PARABOLIC ORBIT:



WE CAN GET AN ANSWER WHICH IS ABOUT 95% RIGHT BY A SIMPLIFIED CASE OF THE ABOVE ACCELERATION. IF THE CHARGE ze AND ELECTRON ARE SO FAR AWAY THAT THE PATH OF THE ELECTRON REMAINS ESSENTIALLY A STRAIGHT LINE. IF WE ASSUME A SEPARATION DISTANCE OF b FOR THE POINT OF MINIMUM APPROACH THEN THE ACCELERATION CAN BE GIVEN AS

$$a = \frac{ze^2}{b^2 m}$$



THE RATE OF ACCELERATION OF THE CHARGE IS GIVEN BY

$$\text{RATE OF RADIATION} = \frac{ze^2}{3c^3} a^2 = \frac{ze^2}{3c^2} \left(\frac{ze^2}{b^2 m} \right)^2$$

ASSUMING THE MAJOR ACCELERATION OCCURS WITH A $2b$ DIAMETER OF THE POINT OF MAXIMUM ACCELERATION, THE TIME OF COLLISION IS GIVEN BY

$$\text{TIME} = \frac{2b}{V_{\text{vel}}} \sim \frac{1}{V_{\text{freq}}}$$

The ACCELERATION THEN CAUSES A GAUSSIAN LIKE RADIATION CURVE AROUND THE MAXIMUM POINT, I.E., IT FORMS A LITTLE BUMP. SINCE THE FREQUENCY OF EMISSION IS INVERSELY PROPORTIONAL TO THE TIME OF EMISSION, WE FIND

$$\frac{2db}{\nu} = \frac{dv}{\nu^2} \rightarrow dv = \frac{\nu}{2b^2} db$$

THE EMISSION OCCURRING IN dv EQUALS THE PROBABILITY db OF b BEING IN db TIMES THE ENERGY EMITTED, I.E., THE AREA WHICH HAS TO BE HIT

$$\text{ENERGY IN } dv = 2\pi b db \times \left[\frac{2b}{\nu} \times \frac{e^6 z^2}{m^2 c^3 b^4} \right] = \frac{8\pi e^6 z^2}{m^2 c^3} \frac{dv}{\nu^2}$$

WE ARE TRYING TO GET THE SPECTRUM INDEPENDENT OF ν THE FREQUENCY. TO DO THIS WE WILL HAVE TO GO FURTHER. ACTUALLY THE NUMBER OF COLLISIONS PER SEC PER UNIT VOLUME PER $d\nu$ IS GIVEN AS A FUNCTION OF ELECTRON, ION NUMBERS AND ELECTRON VELOCITY

$$\text{NO. COLLISIONS PER SEC} = N_{el} N_{ion} v_{el} 2\pi b db$$

FOR NEUTRAL MATERIAL $N_{ion} \cdot Z = N_{el}$. AND WE GET

$$\frac{\text{ENERGY IN } dv}{\text{SEC-VOL}} = \frac{N_{el}^2 e^2 z}{m^2 c^3 \nu} dv$$

REALIZING THAT MANY ELECTRONS HAVE DIFFERENT VELOCITIES WE AVERAGE TO GIVE A MEAN VELOCITY

$$\bar{v} = \sqrt{\frac{kT}{m}}$$

WHAT THIS WHOLE THING IS ALL ABOUT IS THAT I AM TRYING TO UNDERSTAND THE ABSORPTION COEFFICIENT FOR $Z=1$ FOUND IN THE KRAUS BOOK, I.E.,

$$\chi = 9.8 \times 10^{-13} N_{\text{per cubic meter}}^2 T_{el}^{-3/2} \nu^{-2} [19.8 + \ln(T^{3/2}/\nu)]$$

WE WANT TO COMPARE THIS TO $kT \nu^2 dv [\Delta t]$ WHERE $\Delta t = K \Delta x$.

IT IS POSSIBLE THEN TO EQUATE OUR ENERGY EMISSIONS FORMULAE AND SOLVE FOR K TO, HOPEFULLY, GET SOMETHING LIKE THE ABOVE, I.E.

ACCORDING TO FEYNMAN:

$$\frac{4\pi kT \nu^2 dv}{(2\pi)^3} \chi = \frac{N_{el} e^6 z}{m^2 c^3 \nu} dv$$

(EITHER THIS IS COPIED WRONG OR ELSE IT IS NOT IMMEDIATELY OBVIOUS WHAT χ IS).

IN FACT, WE AREN'T QUITE THROUGH.

IN OUR CASE THE FREQUENCY CANNOT GO TOO HIGH OR ELSE b GETS SO SMALL THAT THE WHOLE THING IS NO GOOD ANY MORE. WE CAN, HOWEVER, CALCULATE THE MINIMUM b TO GIVE US THE MAXIMUM FREQUENCY BY EQUATING KINETIC AND POTENTIAL ENERGY AT b_{\min} ,

$$kT = \frac{ze^2}{b_{\min}} \rightarrow \frac{kT}{ze^2} = \frac{1}{b_{\min}}$$

$$\text{BUT } b = \frac{v}{2\sqrt{\nu}} \text{ or } v_{\max} = \frac{2\sqrt{\nu}}{2b_{\min}}$$

$$v_{\max} = \frac{kT \sqrt{\nu}}{2ze^2}$$

RECALLING $\bar{\nu} = \sqrt{\frac{kT}{m}}$ WE GET A CHARACTERISTIC CUT-OFF POINT $\frac{\nu}{T^{3/2}}$

$$\text{LOW CUT-OFF} = \frac{\nu}{T^{3/2}}$$

WHILE A UPPER CUT-OFF RESULTS FROM A b_{\max} GIVEN BY THE SHIELDING DISTANCE OF THE NUCLEUS

$$b_{\max} = \left(\frac{kT}{4\pi e^2 N} \right)^{1/2}$$

Therefore

$$v_{\min} = \frac{\bar{\nu}}{2b_{\max}} = \frac{1}{2} \sqrt{\frac{kT}{m}} \sqrt{\frac{4\pi e^2 N}{kT}} = \left(\frac{4\pi e^2 N}{m} \right)^{1/2}$$

WHICH A FEW MIGHT RECOGNIZE AS THE PLASMA FREQUENCY ω_{pl} . SO OUR APPROXIMATION ISN'T ANY GOOD BELOW THE PLASMA FREQUENCY.

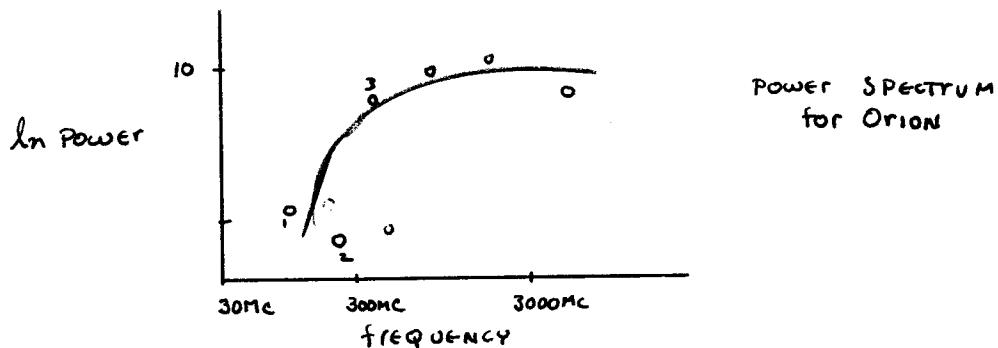
FOR A TYPICAL, TYPICAL BECAUSE EVERYONE HAS AGREED TO ACCEPT IT AS TYPICAL, GAS $T_{el} = 10^4 \text{ }^{\circ}\text{K}$ WE GET $\nu \sim 10^3 \text{ Mc}$ SO THAT THE LOGARITHM IN THE EQUATION FOR K IS ABOUT -7 AND THUS $19.8 - 7 \sim 12.8$. SO FINALLY WE CAN DETERMINE K . AND FOR MOST MATERIALS THIS OPTICAL DEPTH THEN DEPENDS ON $1/\nu^2$.

FINALLY THEN FOR A GIVEN THICKNESS OF GAS RADIATION GOES AS

$$\nu^2 (1 - e^{-\frac{l}{\nu^2}}) dy$$

FOR LOW VALUES OF ν WE GET AN OPTICALLY THICK MEDIA, I.E., RADIATION $\sim \nu^2 dy$. WHILE AT HIGH FREQUENCIES IT GOES AS A CONSTANT TIMES dy . SEE OUR PREVIOUS DIAGRAM FOR ORION. THE AGREEMENT IS PRETTY GOOD.

IT IS INTERESTING TO LOOK A LITTLE MORE CLOSELY AT THE CURVE WE SKETCHED FOR ^{ORION} NO OTHER REASON THAN TO SHOW HOW ASTRONOMICAL DATA IS STRETCHED TO FIT THEORETICAL PREDICTION.



WHAT HAPPENS AT POINT 2 IS SOMEWHAT CONFUSING? WHY DOES IT DIP BELOW POINT 1? WE CAN MENTION, THOUGH, THE FLATNESS IS DUE TO THE INCREASING TRANSPARENCY. WE CAN ~~EASILY~~ ESTIMATE THE ELECTRON DENSITY KNOWING THE TEMPERATURE, SAY 10^9 , AND THICKNESS AS OBSERVED BY THE TELESCOPE. FROM SUCH CALCULATIONS THEY ARRIVE AT A FIGURE OF A FEW THOUSAND PER CC NEAR THE CENTER TO ABOUT TEN PER CC. AT THE EDGE. FROM THIS DENSITY WE GET RADIO EMISSIONS

AS A SHORT PHILOSOPHICAL ASIDE IN REGARDS TO THE QUESTION OF EXPLAINING THAT CURVE ABOVE, IT SEEMS THAT PHYSICS ~~CANNOT~~ IS A PERPETUAL DISCUSSION INVOLVING THE MOST UNRELIABLE POINTS.

TO, MORE OR LESS, WRAP THIS SECTION UP WE LIST SOME SUPER NOVAE REMNANTS GIVING SOME OF THE ASTRONOMICAL DATA COLLECTED

OBJECT	FLUX 10^{26} WATTS CM ² CPS	freq MC	DISTANCE PAR-SEC	TYPE SUPER- NOVAE	AGE YRS.	SPECTRAL INDEX, n $\frac{dV}{V}$
CASSIOPEIA A	11 ₃	178	3.4 ₃	II	265	—
PUPPIS A	7 ₃	100	1.1 ₃	II	—	.27
CRAB	1 ₃	1000	1.1 ₃	I	911	—
HB 21	.5 ₃	100	2.3	II	—	—
IC 443	.2 ₃	178	2.3	II	—	—
CYGNUS	.3 ₃	100	.77 ₃	II	—	.28
TYCHOS SN	.13 ₃	178	.36 ₃	II	50,000	.4-.5
KEPLERS	.08 ₃	100	1 ₃	II	393	.6
				I	361	.7

ALL THESE STARS ARE TRANSPARENT AND EMIT A LITTLE LIGHT.
CASSIOPEIA A IS THE STRONGEST SOURCE IN THE SKY, OUTSIDE OF THE SUN.

CHAPTER 11

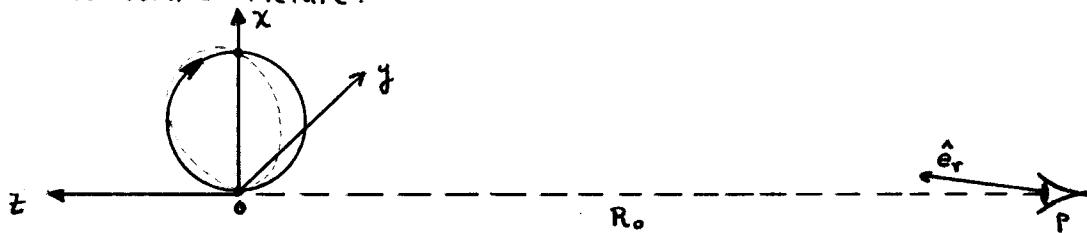
Theory of Synchrotron Radiation (SEE LECTURES ON PHYSICS, VOL I, FEYNMAN)

To begin discussing the theory of emission we seek to find the spectral distribution of an object going around in a circle. To accomplish this we must solve Maxwell's equations but the question is where to start.

The electric field produced by a moving charge seen from far away has two components one going as one over that distance square and the other going as one over the distance. If we get far enough away from the charge, the last dying piece is fading as $1/R$. Physically then the situation is easy to describe.

Imagine the electron carries a little light which has no Doppler shifting. Let's paint a picture of how it looks to us. As our dot moves around, the acceleration of the image point then equals the strength of the field. If we are looking directly in the direction rotation, i.e., we look at the edge of a hoop and can't tell its a circle, what do we see? To us the electron appears to move towards us and then away. If we tilt the plane of the circle so we look at an angle we, in fact, see an ellipse. To us it is like the charge goes slow coming towards us along the bottom of the circle and then goes like mad over the top - think of a roller-coaster. The tremendous accelerations are similar. The reason we see this variation in motion involves the retarded time involved in receiving the signals. This we must go into more deeply.

Imagine the following picture:



If the direction of observation from an observer at point P is denoted by \hat{e}_r in spherical coordinates, then electrodynamics says a moving charge produces an electric field given by

$$\vec{E} = -\frac{e}{c^2} \frac{d^2}{dt^2} \hat{e}_r$$

Imagine a plane some unit distance from the eye where the coordinates in that spot are given by

$$x'(t), y'(t)$$

as we see it.

To determine the E_x' and E_y' electric field components we must determine

$$E_x' = -\frac{e}{c^2 R_0} \frac{d^2 x'}{dt^2}, \quad E_y' = -\frac{e}{c^2 R_0} \frac{d^2 y'}{dt^2}$$

where R_0 is the distance to the source.

WE MUST BE VERY CAREFUL TO SPEAK ABOUT THE CORRECT TIME RELATIONSHIPS BECAUSE IT IS VERY IMPORTANT IN DETERMINING WHAT WE SEE. WE SHALL DENOTE THE PROPER TIME OF THE EMISSION, I.E., THE POINT THE CHARGE SPITS OUT ITS RADIATION AS τ (TAU). AT THE MOMENT τ THE COMPONENTS OF THE CHARGE POSITION ARE GIVEN BY $x(\tau)$, $y(\tau)$, AND $z(\tau)$. SO WE MUST CONNECT OUR TIME OF OBSERVATION t WITH THE MOMENT OF EMISSION τ AND WHEN WE DO SO THE COMPONENT $z(t)$ DOES NOT EFFECT THE RETARDATION. THE DELAY IN TIME OCCURS OVER THE DISTANCE R_0 WHICH MEANS A TIME OF R_0/c TO GET THE LIGHT FROM POINT O TO POINT P BUT THIS ISN'T TOO INTERESTING BECAUSE IT JUST SHIFTS THE ORIGIN OF t BY A CONSTANT. THE IMPORTANT CONTRIBUTION COMES IF $z(\tau)$ IS A LITTLE FARTHER BEYOND O SO THE TIME DELAY FOR THE SIGNAL TO REACH O IS $z(\tau)/c$. PROPERLY THEN, WE HAVE

$$t = \tau + \frac{R_0}{c} + \frac{z(\tau)}{c}$$

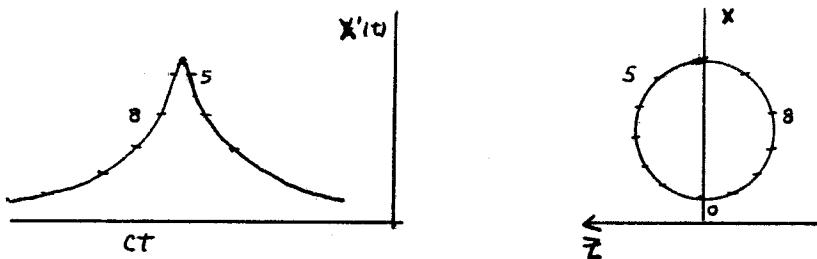
AND REMEMBERING WE ARE IN THE PLANE OF MOTION

$$x'(t) = x(\tau) \quad \text{AND} \quad y'(t) = y(\tau)$$

FINALLY, DROPPING R_0/c AND WORKING IN UNITS OF $c=1$

$$t = \tau + z(\tau)$$

TO WORK OUT THE APPARENT MOTION OF THE ELECTRON WE SEE FROM OUR EQUATION THAT IF THE MOTION OF THE ELECTRON IS CIRCULAR, THEN THE APPARENT MOTION IS A TRANSLATION ALONG OUR LINE OF SIGHT A DISTANCE $c\tau$. WHAT WE "SEE" THEN IS A CURVE CALLED A HYPOCYCLOID WITH CUSPS OCCURRING WHEN IT APPEARS THAT THE CHARGE IS MOVING TOWARDS US.



SO OUR PICTURE REDUCES TO A MACHINE GUN SPRAYING OUT BULLETS IN ALL DIRECTION BUT WE GET IT WORSE WHEN THEY ARE COMING RIGHT AT US! THAT IS, THE ELECTRIC FIELD APPEARS TO PULSE ONLY WHEN THE CHARGE MOVES TOWARDS US. WE SEE IT OVER A LONG PERIOD OF ITS TIME IN A SHORTER PERIOD OF OUR TIME DUE TO THIS RETARDATION. THE NET RESULT IS A COMPRESSION IN TIME.

FROM HERE WE WANT TO ESTABLISH HOW MUCH LIGHT ENERGY THERE IS IN THE LIGHT; WHERE THE ENERGY IS PROPORTIONAL TO E^2 . FOR SIMPLICITY WE WILL WORK WITH ONE PULSE IN WHICH THE TOTAL ENERGY IS

$$\int d\tau \frac{E^2}{4\pi}$$

WE WANT TO FIND THE FOURIER TRANSFORM OF THIS EQUATION, I.E.,

$$\int \frac{d\omega}{2\pi} \frac{E^2(\omega)}{4\pi}$$

$$\text{where } E(\omega) = \int E(t) e^{i\omega t} dt$$

FOR CIRCULAR MOTION AND OBSERVATIONS TAKEN FROM THAT PLANE THE POWER PER $\frac{d\omega}{2\pi}$ IS $\frac{1}{4\pi} [E(\omega)]^2 = \frac{\omega^2}{4\pi} [|X(\omega)|^2 + |Y(\omega)|^2]$

WHERE WE HAVE TO SPECIFY THE TWO DIRECTIONS OF POLARIZATION.
WITH THE MOTION IN THE X, Z PLANE AND DEFINING $\theta=0$ TO BE THE POINT AT WHICH THE RADIATION IS SPIT OUT, I.E., AT THE TOP OF THE CIRCLE,

$$X(t) = a \cos \nu t$$

$$Y(t) = 0$$

$$Z(t) = a \sin \nu t$$

WHERE a IS THE RADIUS AND ν THE FREQUENCY ANGULAR VELOCITY.

SO WE THEN HAVE

$$X'(t) = a \cos \nu t$$

$$\text{AND } t = \theta + a \sin \nu t \rightarrow dt = d\theta(1 + a \nu \cos \nu t)$$

THE FOURIER TRANSFORM OF $X'(t)$ BEING

$$X(\omega) = \int e^{i\omega t} X'(t) dt$$

SUBSTITUTING,

$$X(\omega) = \int e^{i\omega(\theta + a \sin \nu t)} a \cos \nu t dt(1 + a \nu \cos \nu t)$$

THE EXACT SOLUTION TO THIS INTEGRAL, WHICH CONTAINS THE SIDE BAND OF THE FREQUENCY MODULATION, IS VERY DIFFICULT TO GET AND INVOLVES BESSSEL FUNCTIONS. NOTICING, HOWEVER, THAT THE DIFFERENTIAL IS THAT OF THE EXPONENT WE TRY INTEGRATING BY PARTS

$$\begin{aligned} \text{LET } u &= a \cos \nu t & dv &= e^{i\omega(\theta + a \sin \nu t)} dt(1 + a \nu \cos \nu t) \\ du &= -a \nu \sin \nu t dt & v &= \frac{1}{i\omega} e^{i\omega(\theta + a \sin \nu t)} \end{aligned}$$

$$X(\omega) = \frac{a \cos \nu t}{i\omega} e^{i\omega(\theta + a \sin \nu t)} + \int \frac{v}{i\omega} e^{i\omega(\theta + a \sin \nu t)} a \sin \nu t dt$$

SINCE $a \cos \nu t = 0$, AFTER ONE CYCLE IT CUTS OUT, WE HAVE

$$X(\omega) = \int e^{i\omega(\theta + a \sin \nu t)} a \sin \nu t dt$$

WHICH WE COULD HAVE OBTAINED BY DIFFERENTIATING WITH RESPECT TO "a",

$$X(\omega) = \frac{av}{(i\omega)^2} \int \frac{de}{da} e^{i\omega(\theta + a \sin \nu t)} dt$$

WE WON'T WORK OUT THE OPPOSITE FOURIER TRANSFORM RIGHT NOW BUT IT IS

$$X(\omega) = \frac{av}{(\omega)^2} \frac{d}{da} \left[\int e^{-i\omega(t-a\sin\omega t)} dt \right]$$

SINCE WE ONLY GET A MAJOR CONTRIBUTION DURING A VERY SHORT MOMENT,
THE INTEGRAL CAN BE APPROXIMATED BY

$$\int e^{-i\omega(1-av)t - i\omega av^3 \frac{t^3}{6}} dt$$

REMEMBER WE ARE LIMITING OURSELVES TO ONE PULSE OTHERWISE WE WOULD
GET IN TROUBLE WITH BIHARMONICS.

IN OUR CONSIDERATION THE VELOCITY OF THE ELECTRON ABOUT THE CENTER
IS CLOSE TO THE SPEED OF LIGHT OR

$$av \sim c$$

SYNCHROTRON RADIATION

LAST TIME WE WERE TRYING TO FIND OUT WHAT THE FREQUENCY SPECTRUM OF RADIATION EMITTED BY A RELATIVISTIC CHARGED PARTICLE IN INSTANTANEOUSLY CIRCULAR MOTION IS. TODAY I WANT TO TRY TO DEVELOP THE SAME EQUATION FOR THE ENERGY DISTRIBUTION IN A DIFFERENT WAY. WE WILL STAY WITH CLASSICAL ELECTRODYNAMICS, I.E., MAXWELL'S EQUATIONS; USE A LITTLE KNOWN PROPERTY OF VECTORS AND FINALLY EMPLOY FOURIER TRANSFORMS TO GO INTO THE K-SPACE REPRESENTATION.

The four MAXWELL EQUATIONS ARE

$$\nabla \cdot \vec{E} = \rho/\epsilon_0 \quad (i)$$

$$\nabla \cdot \vec{B} = 0 \quad (ii)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (iii)$$

$$\nabla \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{j}/\mu_0 \quad (iv)$$

NOW WE USE THAT ANY VECTOR IN GENERAL CAN BE WRITTEN AS THE SUM OF TWO PARTS - A LONGITUDINAL AND TRANSVERSE ONE, I.E., LET \vec{A} DENOTE A VECTOR THEN

$$\vec{A} = \vec{A}_L + \vec{A}_{TR}.$$

Such That,

$$\nabla \cdot \vec{A}_{TR} = 0 \quad \text{AND} \quad \nabla \times \vec{A}_L = 0$$

WE EMPHASIZE THE GENERALITY OF THIS PROCEDURE. SINCE THE ELECTRIC FIELD IS A PERFECTLY GOOD VECTOR,

$$\vec{E} = \vec{E}_L + \vec{E}_{TR}$$

FROM (i) WE SEE

$$\nabla \cdot \vec{E} = \nabla \cdot \vec{E}_L + \nabla \cdot \vec{E}_{TR} = \rho/\epsilon_0$$

OR

$$\nabla \cdot \vec{E}_L = \rho/\epsilon_0$$

A COMPLEMENTARY EQUATION NECESSARY TO UNIQUE DETERMINE THE LONGITUDINAL COMPONENT OF \vec{E} IS

$$\nabla \times \vec{E}_L = 0$$

THUS \vec{E}_L IS DETERMINED BY THE GRADIENT OF SOME POTENTIAL WHICH DEPENDS DIRECTLY ON THE CHARGE CONCENTRATION AS PER ELECTROSTATICS AND INVERSELY AS THE DISTANCE FROM THE CHARGE

$$\vec{E}_L = -\nabla \phi$$

where

$$\phi = \frac{\text{charges by ELECTROSTATICS}}{R}$$

IF WE EXAMINE THE LONGITUDINAL PART OF THE MAGNETIC FIELD, WE DISCOVER A VERY UNUSUAL THING, NAMELY, THERE ISN'T ANY LONGITUDINAL PART; THE MAGNETIC FIELD IS ALL TRANSVERSE.

To show this we use EQUATION (ii)

$$\nabla \cdot \bar{B} = 0 = \nabla \cdot \bar{B}_L + \nabla \cdot \bar{B}_{Tr}.$$

but $\nabla \cdot \bar{B}_{Tr} = 0$ IN GENERAL Thus

$$\nabla \cdot \bar{B}_L = 0$$

SINCE $\nabla \times \bar{B}_L = 0$, IT IS NECESSARY THAT $\bar{B}_L = 0$. Therefore

$$\bar{B} = \bar{B}_{Tr}.$$

NOW LET'S SEE WHAT HAPPENS WITH EQUATION (iii), $\nabla \times \bar{E} = \frac{\partial \bar{B}}{\partial t}$

$$\nabla \times \bar{E}_{Tr} + \nabla \times \bar{E}_L = \frac{\partial \bar{B}_{Tr}}{\partial t}$$

OR

$$\nabla \times \bar{E}_{Tr} = \frac{\partial \bar{B}_{Tr}}{\partial t}$$

FROM EQ. (iv) WE SEE THAT

$$\nabla \times \bar{B}_{Tr} \neq \frac{\partial \bar{E}_{Tr}}{\partial t} \neq \frac{\partial \bar{E}_L}{\partial t} = \bar{j}_L + \bar{j}_{Tr}$$

IF WE WANT WE COULD COMPARE LONGITUDINAL AND TRANSVERSE COMPONENTS AND EQUATE THEM. BUT LET'S TAKE THE DERIVATIVE AND GET

$$\nabla \times \frac{\partial \bar{B}_{Tr}}{\partial t} + \frac{\partial^2 \bar{E}_{Tr}}{\partial t^2} + \frac{\partial^2 \bar{E}_L}{\partial t^2} = \frac{\partial \bar{j}_L}{\partial t} + \frac{\partial \bar{j}_{Tr}}{\partial t}$$

USING OUR JUST DERIVED FACT THAT $\frac{\partial \bar{B}_{Tr}}{\partial t} = \nabla \times \bar{E}_{Tr}$ AND SEPARATING OUT WE FIND

$$\nabla \times (\nabla \times \bar{E}_{Tr}) + \frac{\partial^2 \bar{E}_{Tr}}{\partial t^2} = \frac{\partial}{\partial t} \bar{j}_{Tr}$$

RECALLING THE VECTOR IDENTITY

$$\nabla \times \nabla \times \bar{A} = \nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}$$

BUT $\nabla \cdot \bar{A}_{Tr} = 0$ Thus

$$\nabla \times (\nabla \times \bar{E}_{Tr}) = -\nabla^2 \bar{E}_{Tr}$$

WE ARE LEFT WITH THE VERY INTERESTING RESULT THAT AFTER WE RID OURSELVES OF ELECTROSTATICS THE ELECTRIC FIELD IS GENERATED ENTIRELY BY THE TRANSVERSE CURRENT,

$$-\nabla^2 \bar{E}_{Tr} + \frac{\partial^2 \bar{E}_{Tr}}{\partial t^2} = \frac{\partial \bar{j}_{Tr}}{\partial t}$$

NOW IT IS ALSO TRUE FOR ANY VECTOR DESCRIBED IN POSITION SPACE THERE CORRESPONDS A FOURIER TRANSFORM OF THAT VECTOR IN \vec{k} OR WAVE-NUMBER SPACE,

$$\bar{A}(\vec{R}, t) = \int_{-\infty}^{\infty} \bar{A}(\vec{k}, t) e^{i\vec{k} \cdot \vec{R}} \frac{d^3 k}{(2\pi)^3}$$

THE ACTION OF DIFFERENTIATING $\bar{A}(\vec{R}, t)$ WITH RESPECT TO \vec{R} IS THE SAME AS MULTIPLYING THE INTEGRAL BY $i\vec{k}$ EACH TIME,

TAKING THE LAPLACIAN OF $\vec{E}(\vec{k}, t)$ IN THE \vec{k} th MODE WE GET

$$(-i\vec{k})^2 c^2 \vec{E}_{tr} + \frac{\partial^2 \vec{E}_{tr}}{\partial t^2} = \frac{d}{dt} \vec{j}_{tr}$$

OR $\vec{k}^2 c^2 \vec{E}_{tr} + \frac{\partial^2 \vec{E}_{tr}}{\partial t^2} = \frac{d}{dt} \vec{j}_{tr}$ DROPPING THE \vec{k} th MODE NOTATION

Thus far we have made no simplifications; we have ~~failed~~ talked in general terms if you let me get by with the instantaneous longitudinal electric field which no one questioned so I'll go on. We can note, however, that the solution to the final form the equation above is just that of a harmonic oscillator driven by a force corresponding to the time derivative of the transverse current.

We can take the Fourier transform of \vec{j}_{tr} if we can take it of \vec{j} , i.e.

$$\vec{j}_{tr}(\vec{k}, t) = \int_{-\infty}^{\infty} \vec{j}(\vec{k}, t') e^{i\vec{k} \cdot \vec{R}} \frac{d^3 \vec{R}}{(2\pi)^3}$$

Now \vec{j}_{tr} is perpendicular to \vec{j} . But \vec{j} can be written in the funny notation

$$\vec{j} = \frac{\vec{k} \cdot \vec{j}}{|\vec{k}|^2} \vec{k}$$

or \vec{j} points in the same direction as \vec{k} . This implies \vec{j}_{tr} is perpendicular to \vec{k}

The solution of the driven harmonic oscillator for the k th mode of the transverse electric field in the \vec{k} -space, i.e., the component always normal to the direction of propagation is given as

$$\vec{E}_k(\vec{k}, t) = \frac{1}{\omega} \int_{-\infty}^t \sin \omega(t-t') \frac{d}{dt'} \vec{j}_k(\vec{k}, t') dt'$$

That is, we used the general solution to the equation

$$\ddot{x} + \omega^2 x = Y(t)$$

WITH

$$X(t) = \frac{1}{\omega} \int_{-\infty}^t \sin \omega(t-t') Y(t') dt'$$

Where the time t is the time we want the field, since the observation occurs at a much latter time than the event itself since it takes 100's of light years to reach us, we can, for all practical purposes take that time to be infinity so our integration runs from $-\infty$ to $+\infty$.

If we make the substitution,

$$\sin \omega(t-t') = \frac{e^{i\omega(t-t')}}{-e^{-i\omega(t-t')}}$$

$$\vec{E}_k(\vec{k}, t) = \frac{1}{2i\omega} \int_{-\infty}^{\infty} \left[e^{i\omega(t-t')} - e^{-i\omega(t-t')} \right] Y(t') dt'$$

where $Y(t') = \frac{d}{dt'} \vec{j}_k(\vec{k}, t')$

Rewriting

$$\vec{E}(K, t) = \frac{e^{i\omega t}}{2i\omega} I_1 + \frac{e^{-i\omega t}}{2i\omega} I_2$$

IN THIS FORM WE CLEARLY SEE WE HAVE DONE SOMETHING WRONG BECAUSE THE POWER WHICH GOES AS \vec{E}^2 CONTAINS INTERFERING TERMS LIKE $I_1 I_2$ WHICH MEANS WE HAVE OUTGOING AND INCOMING WAVES INTERFERING.

WELL, IF YOU'LL PERMIT ME TO CHEAT AND WRITE DOWN WHAT I WANT, I.E., WITH A FEW FACTORS MISPLACED

$$\vec{E}_K(t) = \frac{e^{i\omega t}}{2i\omega} \int_{-\infty}^{\infty} e^{-i\omega t'} \frac{d}{dt'} j(K, t') dt'$$

WHAT THEN IS THE CURRENT FOR MOVING ELECTRON GIVEN BY THE POSITION VECTOR $\vec{Q}(t)$ AND COMPONENTS $X(t), Y(t), Z(t)$. WE NOTE THE CURRENT IS ZERO EXCEPT WHERE THE ELECTRON IS, I.E., WE HAVE A DELTA FUNCTION. THE CHARGE DENSITY IN IK SPACE IS

$$\rho(\vec{K}, t) = \int_{-\infty}^{\infty} \rho(\vec{R}, t) e^{i\vec{K} \cdot \vec{R}} d^3 R$$

$$\text{but } \rho(\vec{R}, t) = g \delta(\vec{R}(t) - \vec{Q}(t))$$

SO WE SIMPLY HAVE
$$\rho(\vec{K}, t) = g e^{i\vec{K} \cdot \vec{Q}(t)}$$

TO FIND THE CURRENT DENSITY WHICH IS SIMPLE THE CHARGE DENSITY TIMES THE VELOCITY OF THE CHARGE

$$\vec{j}(\vec{K}, t) = g \vec{Q}(t) e^{i\vec{K} \cdot \vec{Q}(t)}$$

IT IS AMAZING HOW SIMPLE ELECTRODYNAMICS CAN BE MADE IF ONLY WE WOULD TEACH FOURIER TRANSFORM THEORY FIRST. BECAUSE ONCE WE GO TO IK-SPACE WE ELIMINATE THE NEED TO TALK ABOUT RETARDED WAVE AND ALL THAT JUNK. IF WE NOW INTEGRATE $\vec{E}_K(t)$ BY PART BY LETTING $u = e^{-i\omega t}$ AND $dv = \frac{d}{dt'} j(K, t') dt'$

$$\begin{aligned} \vec{E}_{ik}(t) &= \frac{e^{i\omega t}}{2i\omega} \left[j(K, t) e^{-i\omega t} \Big|_{-\infty}^{\infty} + i\omega \int_{-\infty}^{\infty} e^{-i\omega t'} j(K, t') dt' \right] \\ &= \frac{e^{i\omega t}}{2} \int_{-\infty}^{\infty} e^{-i\omega t'} \vec{j}(K, t') dt' \end{aligned}$$

SINCE $\frac{e^{i\omega t}}{2}$ IS JUST A PHASE FACTOR, WE CHOSE TO IGNORE IT.

WE FINALLY END UP WITH AN INTEGRAL OF THE FORM

$$\vec{E}_K(t) = \int_{-\infty}^{\infty} e^{-i\omega t'} \vec{Q}(t') e^{i\vec{K} \cdot \vec{Q}(t')} dt'$$

THE VELOCITY VECTOR BEING NORMAL TO THE DIRECTION OF PROPAGATION \vec{K} . THIS IS THE SAME TYPE OF INTEGRAL AS WE HAD BEFORE.

WE NOW HAVE THE ELECTRIC FIELD AS A FUNCTION OF \vec{K} . IF WE FIX THE ORBIT AND VARY \vec{K} , WE ASK WHAT THE RADIATION IS. FIRST, WE WANT TO DESCRIBE CIRCULAR MOTION; THAT MEANS THE POSITION COMPONENTS IN THE X, Z DIRECTIONS ARE GIVEN BY

$$a_z = a \sin \nu t$$

$$a_x = a \cos \nu t$$

THE DIRECTION OF PROPAGATION \vec{k} IS GIVEN BY

$$\vec{k} = (k_z, k_x, k_y, t)$$

SINCE THE MOTION IS RESTRICTED TO THE X, Z PLANE $k_y = 0$ AND THE RADIATION OCCURS AT $t=0$. WE HAVE THEN,

$$\vec{k} = (k_z, k_x, 0, 0)$$

SUCH THAT

$$\omega^2 = k_x^2 + k_z^2$$

IF WE TILT THE PLANE OF MOTION THROUGH A VERY SMALL ANGLE θ , THEN

$$k_z = \omega \cos \theta \quad k_x = \omega \sin \theta$$

TO A GOOD APPROXIMATION

$$k_z = \omega(1 - \frac{\theta^2}{2}) = \omega - \frac{\omega \theta^2}{2} \quad \text{AND} \quad k_x = \omega \theta$$

THE COMPONENTS OF \vec{k} BECOME $(\omega - \frac{\omega \theta^2}{2}, \omega \theta, 0, 0)$.

SINCE WE MUST HAVE THE FACTOR

$$\vec{k} \cdot \vec{a}(t)$$

WE SEE

$$\vec{k} \cdot \vec{a}(t) = k_z a \sin \nu t + k_x a \cos \nu t$$

THE ELECTRIC FIELD FOR A PARTICULAR WAVE NUMBER, I.E., IN THE K^{TH} MODE IS GIVEN BY

$$E_K(t) = \int_{-\infty}^{\infty} e^{-i\omega t} e^{ik_z a \sin \nu t} e^{ik_x a \cos \nu t} a \nu \sin \nu t dt$$

DROPPING THE t' NOTATION. ALSO WHERE WE USED THE VECTOR VELOCITY COMPONENT IN THE X DIRECTION ONLY.

HOWEVER, THE MAIN CONTRIBUTION TO THE INTEGRAL OCCURS WHEN $\theta=0$ AND UNDER THAT CONDITION $k_z = \omega$, $k_x = 0$. SO WE GET

$$E_K(t) = \int_{-\infty}^{\infty} e^{-i\omega t} e^{+i\omega a \sin \nu t} a \nu \sin \nu t dt$$

FOR VERY HIGH FREQUENCIES, ω , THE EXPONENTIAL OSCILLATES SO RAPIDLY THAT THE INTEGRAL DOESN'T GIVE US MUCH. WHEN THE CIRCULAR FREQUENCY ν IS VERY HIGH, THEN WE HAVE THIS LARGE ω . SO WE MUST REQUIRE THAT THE OSCILLATIONS BE SMALL BUT AT THE SAME TIME GET A HIGH ω . TO DO THIS LET'S EXPAND $\sin \nu t$ FOR SMALL VALUES OF νt , I.E.,

$$\sin \nu t = \nu t - \frac{\nu^3 t^3}{6}$$

SO THAT

$$E_{IK}(t) = \int_{-\infty}^{\infty} e^{-i(\omega - \omega_{av})t} e^{-i\omega \frac{v^3}{6} t^3} V \sin \nu t dt$$

BUT ω_{av} = VELOCITY ON THE ORBIT, v WHICH IS ALMOST THE SPEED OF LIGHT. THE VELOCITY CAN BE WRITTEN IN TERMS ~~OF~~ OF γ WHERE $E = \gamma mc^2$ AS

$$v = 1 - \frac{1}{2\gamma^2}$$

THUS $\omega - \omega_{av} = \omega - \omega + \frac{\omega}{2\gamma^2} = \frac{\omega}{2\gamma^2}$. IF WE IGNORE THE OTHER MUCH SMALLER EXPONENTIAL, WE SEE $\frac{\omega}{2\gamma^2}$ THAT EVEN WHEN γ IS VERY LARGE MAYBE 2000 FOR 1 BeV, ω CAN BE VERY LARGE SO THAT THE OSCILLATIONS ARE SMALL AND THUS GET A CONTRIBUTION FROM THE INTEGRAL.

FOR EASE OF HANDLING LET'S CONSIDER THE INTEGRAL

$$I = \int_{-\infty}^{\infty} e^{-i\omega t + iK_z \alpha \sin \nu t} dt$$

WHERE WE CHOSE TO IGNORE THE TERM $e^{iK_z \alpha \cos \nu t}$ BECAUSE FOR $\nu t \approx 0$ WE ONLY HAVE $e^{iK_z \alpha}$ WHICH IS JUST A PHASE DIFFERENCE.

WITH THIS INTEGRAL WE CAN DEFINE THE ELECTRIC FIELD AS

$$E_{IK}(t) = v \frac{\partial I}{\partial K_z}$$

THUS WE ONLY HAVE TO WORRY ABOUT WHAT I TURNS OUT TO BE THEN TAKE THE DERIVATIVE WITH RESPECT TO K_z .

AGAIN FOR SMALL ANGLES νt

$$I = \int_{-\infty}^{\infty} e^{-i(\omega - K_z \alpha v)t} e^{-iK_z \alpha \frac{v^3}{6} t^3} dt$$

LET $P = \frac{iK_z \alpha v^3}{2}$ AND $\lambda = \omega - K_z \alpha v$ SUCH THAT

$$I = \int e^{-i\lambda t} e^{-iPt^3/3} dt$$

THIS INTEGRAL IS A BESSEL FUNCTION OF ORDER $1/3$ OR SOMETHING LIKE THAT. SINCE THE MATHEMATICS CAN BE WORKED OUT FROM TABLES, I WOULD LIKE TO WORK OUT AN ASYMPTOTIC APPROXIMATION TO UNDERSTAND THE MEANING OF THE INTEGRAL.

WE WOULD LIKE TO KNOW WHERE THE PHASE VARIES THE SLOWEST, I.E., WHERE DOES ITS MINIMUM OCCUR. THUS LET'S DIFFERENTIATE THE EXPONENT WITH RESPECT TO TIME TO GET

$$\lambda + PT_0^2 = 0$$

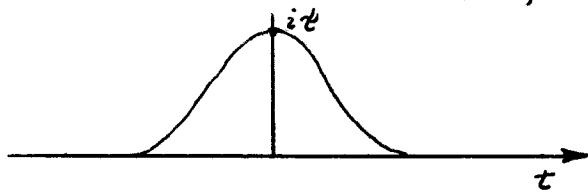
OR

$$T_0 = \sqrt[3]{-\lambda/P} = i\psi$$

WHERE $\psi = \sqrt{\lambda P}$

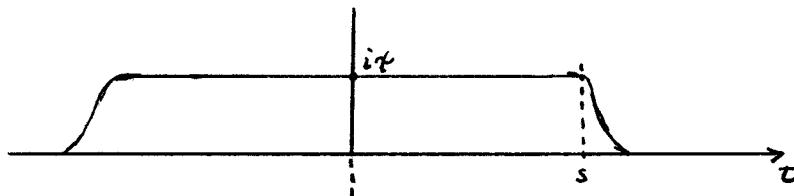
SO WE LEARN A VERY ANNOYING THING, I.E., THE PHASE NEVER VARIES SLOWLY OVER t ! WE JUST GOT THROUGH TRYING TO FIND THIS POINT AND NOW WE LEARN IT LIES ON THE COMPLEX TIME AXIS. BUT WE AREN'T THROUGH YET.

SUPPOSE WE TAKE A CONTOUR SO THAT WE TRAVEL ALONG THE TIME AXIS AND THIS GOES UP TO INCLUDE THE POINT $i\tau$, E.G.



LET ME CHEAT AGAIN AND TAKE A DIFFERENT CONTOUR; THIS TIME WE HAVE THE POINT WHERE THE OSCILLATION OCCURS AT $-i\tau$ AND WE FOLLOW THE PATH

$$t = -i\tau + s \quad \rightarrow \quad dt = ds$$



where the DISTANCE MEASURED ALONG THE FLAT IS s .

WE NEED THEN THE TERM $-i\lambda t - \frac{iP\tau^3}{3}$

$$-i\lambda t = -i\lambda(-i\tau + s) = -\lambda\tau - i\lambda s$$

$$\begin{aligned} -\frac{iP\tau^3}{3} &= -\frac{iP}{3}(-i\tau + s)^3 = -\frac{iP}{3} [(-i\tau)^3 + 3(-i\tau)^2 s + 3(-i\tau)s^2 + s^3] \\ &= +\frac{P\tau^3}{3} + iP\tau^2 s - \tau Ps^2 - \frac{iPs^3}{3} \end{aligned}$$

Therefore,

$$I = \int_{-\infty}^{\infty} e^{-i\lambda\tau - i\lambda s + \frac{P\tau^3}{3} + iP\tau^2 s - \tau Ps^2 - \frac{iPs^3}{3}} ds$$

NOTE: SINCE $t = \sqrt{\lambda/P}$ WHICH IMPLIES $t^2 = \lambda/P$, WE FIND $-i\lambda s + iPt^2 s = -i\lambda s + i\lambda s = 0$; OR THE FIRST ORDER TERM IN s CANCELS OUT. IF WE NEGLECT THE s^3 TERM, THE REMAINING SECOND ORDER TERM IS A GAUSSIAN.

FOR $\tau \gg 1$ WE GET AN ASYMPTOTIC SOLUTION OF THE FORM

$$I = \frac{\sqrt{4\pi}}{\Gamma(1/4)} e^{-\lambda\tau} e^{+P\tau^3/3}$$

$$\text{OR } I = \frac{\sqrt{4\pi}}{\lambda^{1/4} P^{1/4}} e^{-\frac{2}{3} \frac{\lambda^{3/2}}{P^{1/2}}}$$

THE ELECTRIC FIELD \vec{E} CAN NOW BE DETERMINED REALIZING THAT λ AND P ARE FUNCTIONS OF $1/K_z$. THE ANSWER FEYNMAN GOT WAS

$$E = \frac{\sqrt{1}}{4} \frac{\sqrt{4\pi}}{\lambda^{5/4} P^{1/4}} e^{-\frac{2}{3} \frac{\lambda^{3/2}}{P^{1/2}}}$$

JTN Note: I don't recall this attempt to help Feynman and don't know if this correct-treat it accordingly

KNOWING THE OUTCOME OF THE DAY'S LECTURE AND THAT DR. FEYNMAN WAS NOT GETTING THE ANSWERS HE WANT I MIGHT SPECULATE THAT HERE IS A TROUBLE SPOT. ACCORDING TO OUR FORMULAE FOR E , I.E.,

$$\bar{E}_K = \nu \frac{d}{dK_2} I$$

AND REMEMBERING $\lambda = \omega - IK_3 \alpha v$ $P = IK_3 \alpha v^3/2$ IF WE PUT THOSE INTO THE EXPRESSION FOR I WE GET

$$I = \frac{\sqrt{4\pi}}{(\omega - IK_3 \alpha v)^{5/4} (IK_3 \alpha v^{3/2})^{1/4}} \exp \left[-\frac{2}{3} \frac{(\omega - IK_3 \alpha v)^{3/2}}{(IK_3 \alpha v^{3/2})^{1/2}} \right]$$

THIS THEN MUST BE DIFFERENTIATED WITH RESPECT TO IK_2 . I'LL CHECK IT LATTER BUT IT DOESN'T LOOK LIKE IT GIVES THE \bar{E} FOUND BY DR. FEYNMAN.

AFTER A SHORT RECALL, I THINK I HAVE IT. SINCE THE OSCILLATIONS MUST BE SMALL, AS STATED, TO GET A SIGNIFICANT CONTRIBUTION TO THE INTEGRAL WE REQUIRED THAT $\lambda = \omega - IK_3 \alpha v \approx 0$. SO WE ONLY DIFFERENTIATE $\lambda^{-5/4}$ WITH RESPECT TO IK_2 BECAUSE λ GIVES THE BIGGEST CONTRIBUTION. SO

$$\frac{dI}{dK_2} = -\frac{1}{4} \frac{\sqrt{4\pi}}{\lambda^{5/4} P^{1/4}} e^{-\frac{2}{3} \frac{\lambda^{3/2}}{P^{1/2}}} \frac{d\lambda}{dK_2} = +\frac{\alpha v \sqrt{4\pi}}{4 \lambda^{5/4} P^{1/4}} e^{-\frac{2}{3} \frac{\lambda^{3/2}}{P^{1/2}}}$$

Thus OBTAINING FOR E ,

$$\bar{E}_K = \frac{\alpha v^2}{4} \frac{\sqrt{4\pi}}{\lambda^{5/4} P^{1/4}} e^{-\frac{2}{3} \frac{\lambda^{3/2}}{P^{1/2}}}$$

I'LL TRY TO EXAMINE THE EFFECT OF THE EXTRA FACTOR αv ON THE FINAL RESULT DR. FEYNMAN OBTAINED.

CONTINUING WITH THE LECTURE NOTES, WE CAN GET AN APPROXIMATE EQUATION FOR λ IN THE FOLLOWING WAY,

$$\lambda = \omega - IK_3 \alpha v$$

$$\text{where } IK_3 = \omega \left(1 - \frac{\Theta^2}{2}\right) \text{ AND } \alpha v = v = 1 - \frac{1}{2} \frac{1}{\gamma^2}$$

$$\lambda = \omega - \omega \left(1 - \frac{\Theta^2}{2}\right) \left(1 - \frac{1}{2} \frac{1}{\gamma^2}\right) = \omega - \omega \left(1 - \frac{1}{2} \frac{1}{\gamma^2} - \frac{\Theta^2}{2} + \frac{\Theta^2}{4} \frac{1}{\gamma^2}\right)$$

NEGLECTING $\Theta^2/4\gamma^2$ WE GET

$$\lambda = \omega - \omega + \frac{\omega}{2} \left(\frac{1}{\gamma^2} + \Theta^2\right)$$

or

$$\lambda = \frac{\omega}{2} \left(\Theta^2 + \frac{1}{\gamma^2}\right)$$

TO ESTIMATE P FOR INTERESTING FREQUENCIES WHERE $\alpha v \approx 1$

$$P = IK_3 \frac{\alpha v \gamma^2}{2} \approx IK_3 \frac{\gamma^2}{2}$$

BUT TO A GOOD APPROXIMATION $IK_3 = \omega \left(1 - \frac{\Theta^2}{2}\right) \approx \omega$

Therefore

$$P \approx \frac{\omega \gamma^2}{2}$$

SINCE THE POWER GOES AS E^2 WE HAVE

$$E_K^2 = \frac{V^2 4\pi}{16 \lambda^{5/2} p^{1/2}} \exp \left[-\frac{4}{3} \frac{\lambda^{3/2}}{p^{1/2}} \right]$$

UPON SUBSTITUTION

$$E_K^2 = \frac{4\pi V^2}{16} \frac{\exp \left[-\frac{4}{3} \left(\frac{\omega}{2} \right)^{3/2} \left(\theta^2 + \frac{1}{\gamma^2} \right)^{3/2} / \left(\frac{\omega}{2} \right)^{1/2} V \right]}{\left(\frac{\omega}{2} \right)^{5/2} \left(\theta^2 + \frac{1}{\gamma^2} \right)^{5/2} \left(\frac{\omega}{2} \right)^{1/2} V}$$

WHICH SIMPLIES TO

$$E_K^2 = \frac{2\pi V}{\omega^3} \frac{1}{\left(\theta^2 + \frac{1}{\gamma^2} \right)^{5/2}} \exp \left[-\frac{2}{3} \frac{\omega}{V} \left(\theta^2 + \frac{1}{\gamma^2} \right)^{3/2} \right]$$

TO RID THE EQUATION OF V WE MULTIPLY BY a/a SO THAT aV CAN BE REPLACED BY 1 AND USE THE IDENTITY

$$\frac{1}{\gamma} = a \quad \text{THE RADIUS OF THE ORBIT}$$

$$E_K^2 = \frac{2\pi V}{\omega^3} \frac{1}{\left(\theta^2 + \frac{1}{\gamma^2} \right)^{5/2}} \exp \left[-\frac{2}{3} \frac{\omega}{\gamma} \left(\theta^2 + \frac{1}{\gamma^2} \right)^{3/2} \right]$$

WHEN $\theta=0$ THE POWER IS GIVEN BY

$$\frac{2\pi V}{\omega^3} \frac{1}{\gamma^5} \exp \left[-\frac{2}{3} \frac{\omega}{\gamma^4} \right] = \frac{2\pi \gamma^6}{\omega^3} \exp \left[-\frac{2}{3} \frac{\omega}{\gamma^4} \right]$$

DR. FEYNMAN CHOSE TO WRITE ω/γ AS $a\omega$ SO THAT $\omega/\gamma^4 = \omega a/\gamma^3$. FROM WHICH AN w_{crit} WAS DEFINED AS

$$w_{\text{crit}} = \frac{\gamma^3}{a} \approx \gamma^3 V$$

WHERE THE RADIATION CUTS OUT. IT IS THE FREQUENCY WE OBSERVE. THIS MEANS THAT λ IS γ^3 SHORTER THAN THE DIAMETER OF THE ORBIT. FOR A 1 Bev ELECTRON $\gamma = 2000$ SO γ^3 IS PRETTY SMALL. CONSIDERING $\frac{wa}{\gamma^3} \ll 1$ THEN THE POWER RADIATED WOULD GO, ACCORDING TO FEYNMAN AS,

$$\frac{\gamma^4}{\omega^3}$$

AND WHEN $\frac{wa}{\gamma^3} \gg 1$ IT WOULD TEND TO GO AS $\frac{\gamma^4}{\omega^3} e^{-\frac{2}{3} \frac{wa}{\gamma^3}}$.

AS IT TURNED OUT DR. FEYNMAN DECIDED HE MADE THE WRONG APPROXIMATION SINCE THE ANSWER WASN'T COMING OUT.

THE ERROR GOES BACK TO THE INTEGRAL

$$I = \int_{-\infty}^{\infty} e^{-\lambda t - i\lambda s + \frac{ps^3}{3} + ips^2} dt - \int_{-\infty}^{\infty} e^{-\lambda t - i\lambda s + \frac{ps^3}{3} + ips^2} dt$$

IN ESTIMATING THE RELATIVE IMPORTANCE OF THE LAST TWO TERMS

$$-\left(\frac{ps^2}{3} + ips^3 \right)$$

THERE MUST BE A CRITICAL S VALUE OCCURRING AT THE INFLECTION POINT OF THE CURVE JOINING THE T AXIS AND THE LINE S.

If $tP S_{cr}^2 = 1$ then $P S_{cr}^3 \ll 1$. Rewriting as $\frac{P}{(tP)^{3/2}} \ll 1 \rightarrow \frac{P^2}{\sqrt{tP^3}} \ll 1$
 OR $P \ll \lambda^3$. The INTEGRAL $\int e^{-tP^2 - tP^3} dx$ IS AN AIRY INTEGRAL. The
 CONTRIBUTE TO THE POWER OF $e^{-tP^3/3}$ IS SIMPLY A NUMBER AND IS THEREFORE
 IGNORED FOR THIS ANALYSIS, i.e.

$$\frac{1}{P^{''3}} \left| \int e^{-tx^3/3} dx \right|^2 = \text{NUMBER}$$

The question which has some meaning is what is the spectrum we see. This depends on the spectrum of the electron and what we have done so far does not give us the answer. We need the energy distribution for the electron.

The spectrum of synchrotron radiation goes as $\frac{dw}{w^{0.7}}$. If we reverse our argument and ask what the distribution w^{α} of electron energy must be to give this, we might get out of our entanglement with the mathematics. Had we worked this theory out right the energy spectrum of the electrons should go as

$$n(E) = \frac{dE}{dE^\alpha}$$

From which we expect

$$w^{(1-\alpha)/2} dw$$

This means we need an α of 2.4 to get 0.7. Amazingly enough we see in cosmic rays an α of 2.8 which is not bad so we argue the same process which tends to accelerate electron to synchrotron frequencies will likewise accelerate protons. It seems then we are on the right track to understanding this process.

However, to tell what the energy spectrum of an electron in a gas, or more specifically in a beam generated by that gas, is very difficult because we must know the probability of escape at some energy. But the theory of escape is not independent of the energy distribution in a cube of gas so I consider the value of $\alpha = 2.4$ not a real brilliant fact yet. At the same time it is worth emphasizing that all radio sources show nearly the same index. Whatever is causing this radiation appears to progress at a fairly equal pace. I propose as a problem of the first importance to compute α to see if it is really a universal constant.

The model is a gas of churning, sloping, slurping fields, charges, dirt, and other junk all in turbulent motion and this requiring what the electrons do in that mess. If we start with long waves will the gas churn down to short waves or vice versa. This is actually statistical magnetohydrodynamics. Working with scalable models like the magnetic field variations, perhaps we can figure out the real nature of synchrotron radiation.

The power contain or released that is during this radiation is sizeable. In fact the total energy in the magnetic field and in the electron density is so great that something seems to be the matter. The three primary sources of synchrotron radiation need more energy than what we see.

In quasars, galactic centers and a spot in the crab nebulae all possess extremely high power outputs. Even flare stars generate enormous amounts of energy which we don't understand either.

DURING THIS WHOLE DISCUSSION WE ASSUMED THE ELECTRON RADIATED BY ITSELF, I.E., NOT INTERACTING WITH ANY OF ITS SURROUNDING ELECTRONS. SUPPOSE WE PUT TWO ELECTRONS CLOSE TOGETHER; SO CLOSE, IN FACT, ONE IS ON TOP OF THE OTHER AND GOES AROUND A CIRCLE JUST LIKE before. THEN THE MAGNETIC FIELD IS TWICE AS STRONG WHICH MEANS THE EMISSION BECOMES FOUR TIMES AS GREAT. THEN IF A LUMP OF n ELECTRONS WERE ALL IN PHASE THE FIELD WOULD BE n^2 . IT SEEMS POSSIBLE THAT THESE STRONG RADIATIONS ARE THE RESULT OF COHERENT MOTIONS OF LUMPS OF ELECTRONS WHICH ARE ACCELERATED BY THE MAGNETIC FIELD. THE LUMPINESS WOULD BE ENHANCED AS THE LUMPS ARE PUSHED TOGETHER.

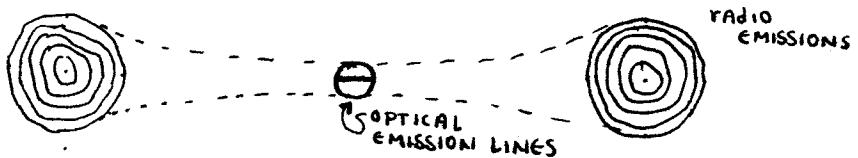
EXTRAGALACTIC SOURCES OF RADIO EMISSION

TODAY I WANT TO TALK ABOUT EXTRA GALACTIC SOURCES OF RADIO EMISSIONS WHICH FALL INTO TWO CLASSES, WHICH I CAN'T REMEMBER THE DIFFERENCE CAUSE ITS BEEN THREE WEEKS SINCE I PREPARED THIS STUFF AND MY MEAN LIFE FOR REMEMBERING THIS STUFF IS JUST ABOUT THAT LONG. IN FACT, I'M TRYING TO MAKE THIS PRESENTATION APPEAR SO POOR THAT EITHER I'LL GET TIRED OR ELSE CHANGE TO A NEW TOPIC; MY ABILITY TO DIGEST THIS MATERIAL AND PRESENT IT IN AN UNDERSTANDABLE FASHION IS DWINDLING.

AT ANY RATE THE TWO CLASSES ARE, PRESUMABLY, (1) RADIO GALAXIES AND (2) QUASARS. IT IS CURRENTLY BELIEVED, OR ARGUED MORE STRONGLY, THAT QUASARS ARE VERY FAR AWAY FROM RECEEDING AT TREMENDOUS VELOCITIES. THE INTERESTING QUESTION CAN THE BE SO BIG; ARE THEY CLOSER BY? IF THEY ARE THEN THERE ARE SOME VERY FUNDAMENTAL PROBLEMS RESULTING FROM AND EXPLANATION OF THE PRESUMED TREMENDOUS RED SHIFT. IT IS INTERESTING TO NOTE ALSO THAT NO BLUE SHIFTS ARE OBSERVED, I.E., NONE OF THESE QUASARS ARE HEADING TOWARDS US. THEY SEEMED TO HAVE ALL PASSED US BY A LIKE A SPEEDING COP ON THE WAY TO A WRECK.

RADIO GALAXIES

CONSIDERING FIRST RADIO GALAXIES, WE CAN EXPLAIN IN SOME DETAIL THEIR PHYSICAL NATURE. USUALLY (BECAUSE IN ASTRONOMY THERE ARE ALWAYS PLEADING EXCEPTIONS) RADIO GALAXIES CONSIST OF DOUBLE REGIONS OF STRONG RADIO SOURCES EQUIDISTANTLY SPACED, MORE OR LESS, AWAY FROM AND INTENSE BY SMALL OPTICAL OBJECT. FOR A PARTICULAR GIANT D TYPE GALAXY² THERE IS FROM A VERY PREDOMINANT DUST LANE OR LINE EXTENDING ACROSS THE CENTRAL OBJECT WHICH IS BASICALLY AN ELLIPTICAL GALAXY. A PICTURE OF A TYPICAL RADIO GALAXY MIGHT BE AS FOLLOWS



THE TWO OUTER REGIONS OF RADIO EMISSIONS ARE AMAZINGLY SIMILAR IN OVERALL STRUCTURE AND EMISSION NATURE; I.E., THEIR SPECTRAL PATTERNS ARE ESSENTIALLY THE SAME. THESE REGIONS ARE UTTERLY TRANSPARENT; THERE ARE NO SOURCES OF VISIBLE LIGHT THERE ONLY OF RADIO EMISSIONS.

THESE ARE, HOWEVER, STRONG EMISSION LINES IN THE OPTICAL SPECTRUM FROM THE CENTRAL OBJECT. THERE IS AN OBSERVED POLARIZATION OF A FEW PERCENTS & IN THE OUTER REGIONS WHICH SEEM TO LEND SOME CREDIBILITY TO THE BELIEF THAT SYNCHROTRON RADIATION IS GOING ON THERE. ALSO THE RADIO INTENSITIES OF THESE OUTER REGIONS ARE OF THE ORDER OVER THE WAVELENGTH RANGE OF ABOUT 100.

ABOUT THE ONLY WAY TO DESCRIBE THESE SOURCES IS TO LIST THEM ONE AT A TIME AND GIVE SOME OF THE VITAL STATISTICS ABOUT IT. THERE WILL APPEAR SOME NUMERICAL INCONSISTENCIES ARISING IN THE REPORTED DATA AS RECORDED BY DIFFERENT PEOPLE. THIS, HOWEVER, SIMPLY REFLECTS THE CURRENT STATE OF THE ART OF ASTRONOMICAL MEASUREMENTS AND TOGETHERNESS IN REPORTING THE DATA. I SHALL LIST THE LARGER, MORE CAREFULLY INVESTIGATED OBJECTS WHICH FALL INTO THIS CATEGORY.

THE RED SHIFT Z IS THAT RECORDED BY THE ANTENNAE AND GIVES A RELATION FOR THE RECESSIONAL VELOCITY V IF WE ASSUME NO GRAVITATIONAL RETARDATION OF

$$z + 1 = \sqrt{\frac{1+v/c}{1-v/c}} = \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}}$$

THE SIZE DIMENSIONS, I.E., THE DIAMETER AND SEPARATION OF THE TWO OUTER OBJECTS ARE ALSO GIVEN IN 1000'S PARSECS (1 PC = 3.26 LY). THE LOG OF THE ABSOLUTE LUMINOSITY IS GIVEN IN ERGS/SEC. THE OPTICAL MAGNITUDE MEASURED ABSOLUTELY, I.E., IF THE LIGHT FROM THE OBJECT WAS BROUGHT TO A DISTANCE OF 30 LIGHT YEARS FROM US IS GIVEN. THE LATTER TWO QUANTITIES BEING RATHER NOMINAL QUANTITIES OR BETTER THEY'RE NOT NECESSARILY RIGHT FOR THE WAY WE CHOOSE TO MEASURE THEM. WE ASSUME THAT

$$v_{\text{recess.}} = H \cdot \frac{\text{distance}}{\text{Mpc}}$$

where H = Hubble's constant
as a standard

$$100 \text{ km/sec/Mpc} = \text{Hubble's constant}$$

FOR SMALL V'S THE RED SHIFT IS PROPORTIONAL TO THE VELOCITY. WHEN WE GET RED SHIFTS OF A $\frac{1}{2}$ OR HIGHER WE START GETTING INTO SOME TROUBLE.

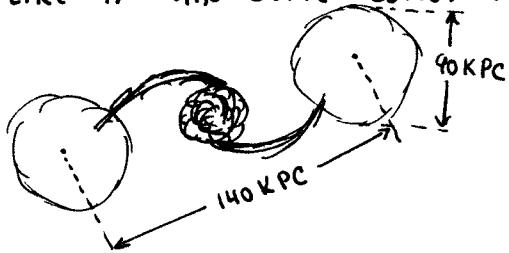
radio source	distances Mpc	z	size (1000pc) diameter (x separation)	log $ L $ ergs/sec	optical mag. absolute	light ergs/sec
CYGNUS A	170	.057	17 (x 100)	44.8	-21.1	6×10^{43}
CENTAURUS A	4	.003	{ 3.5 (x 8.3)	41.8	-21.3	4×10^{43}
FORNAX A	17	.006	2120 (x 240)	41.8	-21.8	10^{44}
3C 33		.060	92 (x 140)	41.8	-20.9	4×10^{43}
3C 295		.461	10 (x 200)	42.8	-20.1	
			5 (x 15)	450		
HERCULES A		.154	90 (x 330)	44.2		
VIRGINIS A		.004	13 (x 0.7)	47.7	-20.5	
3C 338		.030	32	41	-21.6	

SOME PECULIARITIES OF THESE SOURCES ARE THE FOLLOWING:

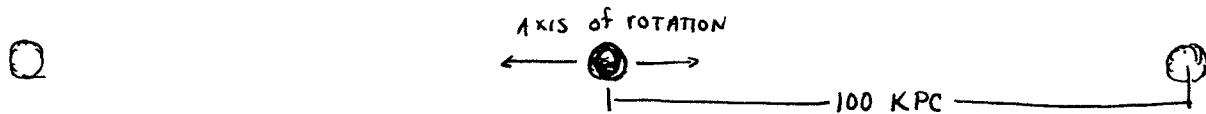
FOR CENTAURUS A IT LOOKS LIKE,

The little splotches look like places where the galaxy shot out some crap and might be early in its development.

FORNAX A LOOKS LIKE IT HAS SOME COMET TAILS COMING OUT OF IT:

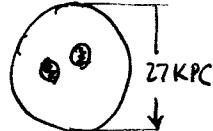


3C33 IS STRANGE OR ATYPICAL IN THAT THE RADIO SOURCES ARE ESSENTIALLY POINT SOURCES DIRECTED, IT IS BELIEVED, RADIALLY ALONG THE AXIS OF ROTATION,



3C295 IS THE MOST DISTANT KNOWN RADIO GALAXY RECORDED EXCLUDING QUASARS. THE DISTANCE OUT DEPENDS ON WHICH COSMOLOGICAL MODEL YOU PICK.

VIRGINIS A HAS TWO SMALL OPTICAL GALAXIES INSIDE A CLOUD OF RADIO EMISSION.



THIS COULD BE A NORMAL RADIO GALAXY WHICH WE ARE VIEWING FROM THE END BUT THAT DOESN'T CONSIDER THE TWO OBJECTS INSIDE.

3C338 IS SIMILAR TO VIRGINIS A, I.E., IT IS A SOURCE WITH NO DOUBLE REGIONS OF RADIO EMISSIONS

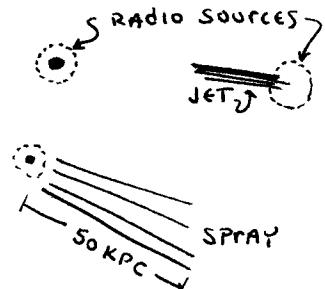


THE QUESTION OF OBSCURATION ARISES IN INTERPRETING WHAT WE SEE ON THE PHOTOGRAPHIC PLATES. PERHAPS, A BIG CLOUD FLOATS AROUND CUTTING DOWN THE INTENSITY FROM THESE SOURCES. BUT IT IS INTERESTING TO NOTE THAT THE OPTICAL PRECISION OF DETECTING THESE SOURCES IS MUCH BETTER THAN WITH RADIO WAVES BUT AFTER FINDING THE REGIONS OF RADIO EMISSIONS THE ASTRONOMERS WERE INSTRUCTED TO REEXAMINE THE PLATES MORE CLOSELY. THE WHOLE PROBLEM SEEMS TO BE ONE OF SELECTION, I.E., WHAT ARE THE SENSITIVITIES OF THE RECEIVING ANTENNE.

SUPPOSE IT WAS SENSITIVE TO A CERTAIN INTENSITIES AND A SCATTERING OF A 100 STARS HAS 90 WITH LUMINOSITY MAGNITUDE 41, 9 WITH 44 AND 1 WITH 46. SINCE THE FARTHER WE GO THE WEAKER THE INTENSITY SO NOT ALL OF THESE OBJECTS MIGHT BE CLASSED THE SAME.

NOW WE MENTION A COUPLE OF QUASARS

	z	DIST MPC	RADIO	OPTICAL
3C 27.3	.158	474	3×10^{44}	4×10^{45} ERG/SEC
3C 48	0.367	1100	5×10^{44}	1×10^{45}



THE NAME QUASARS OR QUASI- STELLAR RADIO SOURCES SEEM TO REFER TO THOSE OBJECTS IN WHICH THE RADIO SOURCES ARE ESSENTIALLY POINT SOURCES AND BEYOND THE RESOLUTION OF THE ANTENNA. WHILE, ON THE OTHER-HAND, RADIO GALAXIES HAVE VERY LARGE AREAS OF RADIO EMISSIONS.

SOME QUASARS HAVE BEEN OBSERVED WITH A RED SHIFT OF 2.2 OR

$$z+1 = 3.2 = \sqrt{\frac{1+v/c}{1-v/c}} \Rightarrow v = \frac{9}{11} c \approx .82 c$$

THESE OBJECTS SEEM TO HAVE MORE BLUE LIGHT THAN NORMAL GALAXIES. SOME ARE WEAK AND HAVE LOW RADIO INTENSITIES TO THE POINT WE CAN'T RECORD ANY EMISSIONS. SOME OF THE CRAZY ESTIMATES OF THE SIZE OF THESE OBJECTS GO DOWN TO A LIGHT WEEK WHICH IS PRETTY DAMN SMALL WHEN YOU CONSIDER THE POWER THESE THINGS ARE PUTTING OUT. IT'S POSSIBLE, PERHAPS, THE SYNCHROTRON PROCESS IS NOT UNDERSTOOD WELL ENOUGH AND THAT IT IS MORE EFFICIENT THAN PREDICTED. OR MAYBE SOMETHING ELSE IS CAUSING THESE TREMENDOUS RED SHIFTS - GRAVITY, PERHAPS? BUT THERE IS NO CURRENT MODEL WHICH COULD ACCOUNT FOR THESE LARGE SHIFT. MAYBE SUPERNOVAE GO OFF AND THEN DECAY DOWN WITH NO APPARENT REGULARITY. SOME RUMOR WENT AROUND RADIO WAVE RAYING BUT THIS IS BELIEVED DUE TO SYNTILLATIONS IN THE WAVES DUE TO ATMOSPHERIC OR PLANETARY ^{INTERF} INTERFERENCE. ANY CRAZY IDEA IS THAT AS THE GALAXY SPINS AROUND IT WINDS UP THE MAGNETIC FIELD LINES UNTIL THEY SNAP AND RELEASE ALL THEIR ENERGY IN SOME ERUPTIVE MANNER.

TO MEASURE THESE RED SHIFTS OBSERVED ABSORPTION LINES FROM THESE OBJECTS HAVE BEEN CORRELATED WITH THE KNOWN ELEMENTS. CONSIDERING THE FANTASTIC RED SHIFT IT WOULD BE EXCEEDINGLY DIFFICULT TO CLAIM A CERTAIN LINES WAS THE MAGNESIUM 2 LINE⁺ SHIFTED THROUGH 2956.58 Å OR SOMETHING STUPID LIKE THAT. THE IDEA GETS WORSE WHEN ONLY THREE, TWO OR MAYBE EVEN ONLY ONE LINE IS RECORDED FROM WHICH WE CLAIM TO KNOW THE RED SHIFT. IT IS FIRST NECESSARY TO ASCERTAIN WHETHER THE CONDITION WITHIN THE OBJECT, TEMPERATURE, COMPOSITION, ETC ARE SUCH THAT THE PRODUCTION OF Mg²⁺ OR WHATEVER IS EVEN POSSIBLE AND THIS, IN ITSELF, IS ALMOST IMPOSSIBLE TO DO AS WE SOUGHT EARLIER IN THE COURSE. THE FARTHER AWAY THE OBJECT IS THE FEWER THE MEASUREABLE LINES BECAUSE THE INTENSITIES ARE LESS; ANOTHER PROBLEM TO FACE. SO, IN FACT, THE VERY IDENTITY OF QUASARS IS QUESTIONABLE, PERHAPS THEY ARE NORMAL RADIO GALAXIES IN DIFFERENT STAGES OF EVOLUTION; MAYBE, THEY ARE FAST RECEESSING GALAXIES OF VERY PECULIAR NATURE.

CHAPTER 12

"ONE MAN'S ELECTRON IS ANOTHER MAN'S POSITRON"

A LECTURE ON ANTI-PARTICLES

here is a fascinating side trip down one of Feynman's favorite topics

This LECTURE IS ABOUT ANTI-PARTICLES - WHY THEY EXIST AND WHAT THEY ARE. THE PARTICLES FOLLOW AS A CONSEQUENCE, A NECESSARY ONE, WHEN COMBINING A DISCUSSION OF QUANTUM MECHANICS AND RELATIVITIES PLUS SOME OTHER THINGS WHICH WE'LL SOON DISCUSS. THE ONLY THEORIES WHICH EVOLVE FROM THIS LINE OF REASONING AND MAKE ANY SENSE ALL HAVE IN COMMON THE FOLLOWING CHARACTERISTICS:

- (1). THERE IS NO WAY TO AVOID PAIR-PRODUCTION, I.E., THE EXISTENCE OF ANTI-PARTICLES
- (2). THE RELATION BETWEEN SPIN AND STATISTICS; THAT BEING, SPINS OF $\frac{1}{2}$ ARE ASSOCIATED WITH BOSE STATISTICS AND WHOLE INTEGRAL SPIN REQUIRE FERMI STATISTICS
- (3). THE RESULTING ANTI-PARTICLES SATISFY CERTAIN LAWS WHICH COMPLETELY DETERMINE THEIR BEHAVIOR; MORE IMPORTANT THE LAWS ARE THE SAME FOR THE REAL PARTICLES. WE CALL THIS, SUCCINCTLY, THE C = PT INVARIANCE. TO THINK OF ANTI-PARTICLES WE CAN IMAGINE A SITUATION WHERE WE TAKE A MOTION PICTURE OF SOME ORDINARY PARTICLE AN ELECTRON, PERHAPS, THEN REVERSE THE MOVIE AND LOOK AT IT THROUGH A MIRROR. WHAT WE SEE IS EXACTLY HOW AN ANTI-ELECTRON OR POSITRON WOULD BEHAVE.
- (4). THERE ARE NO ARBITRARY FUNCTIONS APPEARING IN THE HAMILTONIAN. FOR INSTANCE, NON-RELATIVISTICALLY THE HAMILTONIAN IS GIVEN BY

$$H = \frac{\hbar^2 \nabla^2}{2m} + V(r)$$

WHERE THE ADDITION FUNCTION $V(r)$ IS THE POTENTIAL FUNCTION OF THE PROBLEM. THIS HOWEVER IS NOT QUITE TRUE AND WE CAN ADD A DELTA FUNCTION OF THE PARTICLE'S POSITION, E.G., $a\delta^3 r$. SO WE HAVE IN EFFECT A LOCAL INTERACTION AS OPPOSED TO THE USUAL POTENTIAL. THIS HAS ANOTHER MEANING AND IT INVOLVES THE WORLD-LINE SEPARATING THE PAST AND THE FUTURE. THE IDEA ASSOCIATED WITH AN ADDITIVE FUNCTION HAS THE EFFECT OF SHIFTING THAT LINE A LITTLE SO THAT THE TRANSITION BETWEEN THE PARTICLE'S PAST AND FUTURE NOW HAS A FINITE, BUT SMALL, WIDTH WHICH MUST BE CONSIDERED AS A REGION OF POSSIBLE EVENTS.

- (5) LASTLY, WE RUN INTO ALL SORTS OF DIVERGENT DIFFICULTIES WHERE ANSWERS TEND TO CROP UP THAT GO TO INFINITY.

THIS IS QUITE A STRAIGHT JACKET TO HAVE AROUND US BUT, AS YET, NO ONE HAS COME UP WITH A SENSIBLE THEORY WHICH DIDN'T HAVE THESE PROBLEMS CROPPING UP. FOLLOW LOGICALLY.

There has been a lot of attention given to proving that these are the only restrictions which plague us since we can't seem to get around them. The claim is that we can, in fact, show that the first four difficulties result from the combination of quantum mechanics, relativity, plus some other junk. The CPT invariance has been experimentally proven after having shown the violation of CP invariance could be overcome by the combination CPT thus disproving, as it was, the erroneous idea that time was invariant in the CP picture.

We would like to know how to prove that these statements exist as I have written them. The proof is really so obscure that is terribly hard to read. I would like to at least show the essence of the proof, i.e., why we can prove anything at all.

To begin the discussion we must add to our basic starting point, Q-M + Relativity a very far reaching hypothesis - the idea of causality. In addition we have some passive assumptions which result from Q-M which introduces the idea of associating an amplitude with an event. Thus we say the sum of all the probabilities, i.e., the square of the amplitudes must equal 1. Further, all the energy states must be greater than that in vacuo (the lowest energy state). This we add 'cause some bonehead will raise a stink if we don't like an axiom of Euclid where he failed to make clear the profound statement that a line joining two points lying on opposite sides of another line bisect that line! So we might summarize our basic ingredients will be;

$$\text{QUANTUM MECHANICS} + \text{RELATIVITY} + \left\{ \begin{array}{l} \text{CAUSALITY,} \\ \text{LOCALNESS} \end{array} \right. + \left\{ \begin{array}{l} \sum \text{PROBABILITIES OF ALL ALTERNATIVES} = 1 \\ \text{ALL ENERGY STATES} > \text{VACUUM} \end{array} \right.$$

+ some other things which don't help much = 5 problems

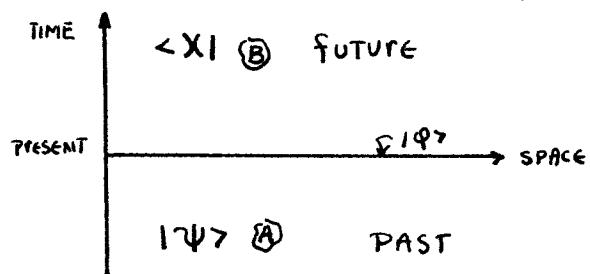
Causality

The idea of causality means that what happens in the past can not be changed by what happens in the future. The condition known as the present carries the information from the past into the future. In quantum mechanics the messenger of this information is the present state of the system denoted by $|n\rangle$ (the amplitude); the dependence is on what happened at an earlier time. If the amplitude of some future event is denoted by $\langle \lambda |$, say, then the probability of that event occurring is denoted by

$$\langle \lambda | n \rangle$$

where $\langle \lambda |$ is free to change but $|n\rangle$ is not because it already happened. That is causality.

WE CAN REPRESENT THE TRANSITION OF EVENTS FROM THE PAST TO THE FUTURE BY DRAWING A SPACE-TIME DIAGRAM WHERE THE STATE $|\Psi\rangle$ REPRESENTS THE PRESENT,



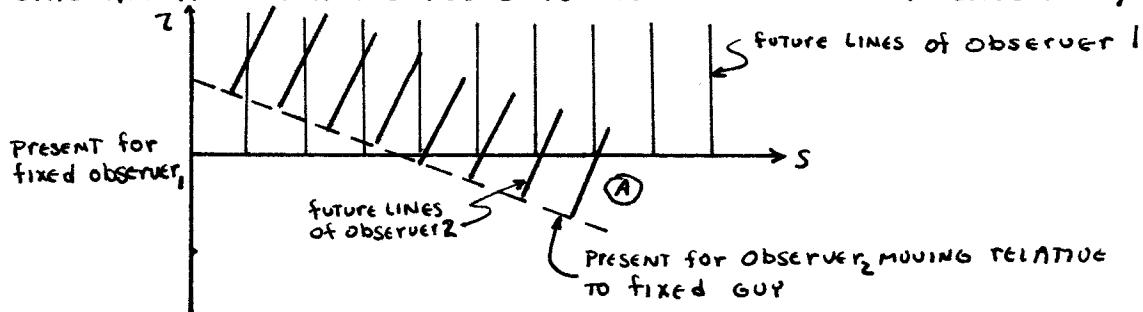
THE FORMULA WHICH RELATES ALL PHENOMENA OF THE WORLD OF PAST TO THE FUTURE VIA THE PRESENT IS

$$\langle X | \text{TIME PASSES} | \Psi \rangle = \sum_{\phi} \langle X | B | \phi \rangle \langle \phi | A | \Psi \rangle$$

WHERE A AND B REPRESENT THE TWO EVENTS AND $|\Psi\rangle$ AND $\langle X |$ THEIR AMPLITUDES RESPECTIVELY.

THIS EXPRESSES THE COMBINATION OF CAUSALITY AND QUANTUM MECHANICS. WE MUST USE THIS RELATIONSHIP.

NOW WHY IS IT POSSIBLE TO DEDUCE A LOT OF STUFF FROM THIS SEEMINGLY INNOCENT EQUATION. WELL, IT'S BECAUSE THE PRESENT MOMENT IS NOT A RELATIVISTICALLY HAPPY IDEA. FOR SOME OBSERVER IN A MOVING SYSTEM THE IDEA OF PAST-FUTURE PRESENT AND FUTURE ARE NOT THE SAME AS OURS. THE EVENTS ARE NOT SIMULTANEOUSLY OBSERVED BY THE TWO GUYS. BUT SINCE THE EVENTS ARE THE SAME THEY MUST SEE OR ^{GIVE} ~~GET~~ THE SAME RESULTS. HOW ARE THESE TWO OBSERVERS RELATED? LET'S DRAW ANOTHER SPACE-TIME DIAGRAM IN WHICH THE TWO GUYS FOLLOW DIFFERENT WORLD-LINES;



THE ABOVE DRAWING, PERHAPS, MAKES IT CLEARER WHAT I AM SAYING. EVENT A OCCURS IN OBSERVER 1'S PAST WHILE OBSERVER 2 HASN'T EXPERIENCED IT YET SINCE IT'S IN HIS FUTURE. SINCE THE EVENT IS THE SAME, I.E., IT IS UNCHANGED, THEY MUST SEE THE SAME THING OCCUR. IT IS A CONSEQUENCE OF THESE INTERSECTING WORLD LINES THAT ANTI-PARTICLE EVOLVE AS THE ONLY METHOD OF SALVAGING A SATISFACTORY EXPLANATION OF WHAT THE TWO GUYS SEE.

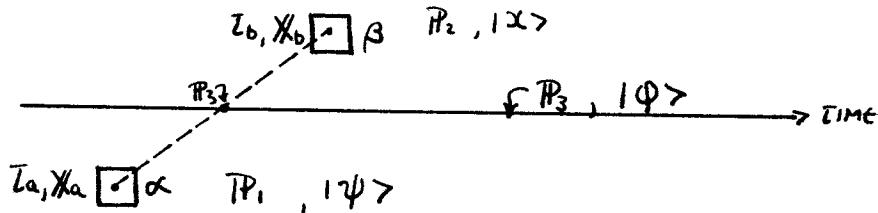
To make the proof as easy as possible we will assume that in the world the only event occurring are particles being scattered by a potential. Furthermore, the particles are conserved so for everyone getting scattered we don't get 2 or 5 or 10 out. We will assume, then, no pair production and show this leads to an inconsistent answer and we must ultimately accept the reality of pair production. Also we will leave out spin for the discussion.

If we have some scattering center held at some potential which is located in space time by x, t , such that incoming particles of specific momentum p_i are scattered and have resulting momentum p_i' , the way they are scattered is described by

$$\alpha \underbrace{e^{i(E_i t - p_i \cdot x)}}_{\text{OUTGOING WAVE}} \underbrace{e^{-i(E_i' t - p_i' \cdot x)}}_{\text{INCOMING WAVE}}$$

where $E^2 = \sqrt{p^2 + m^2}$ or specifically $E_0 = \sqrt{p_0^2 + m^2}$ and α is characteristic of the scattering center where for small centers it is independent of the momentum and potential (if it is the same as seen by the two observers).

Our picture of two such scattering events occurring in the past at t_a, x_a which has momentum p_i and amplitude $|\psi\rangle$ and in the future at t_b, x_b with momentum p_i' and amplitude $|x\rangle$ are connected by the present with a measurable momentum p_3 and amplitude $|\phi\rangle$



The combined scattering amplitude is thus given by

$$\text{SCATTERING AMP.} = \sum_{p_3} \alpha \beta e^{i(E_i t_b - p_i \cdot x_b)} e^{-i[E_3(t_b - t_a) - p_3 \cdot (x_b - x_a)]} e^{-i(E_i t_a - p_i \cdot x_a)}$$

If we require this event to be relativistically invariant the product $\alpha \beta$, which is independent of position, must be normalized to 1 particle per unit volume. However, in the moving system the volume becomes compressed and the normalization increases by a factor of the square root of the thing being squashed up. The volume element $d^3 p$ is not independent of the factor and is normalized by dividing by $2E$, the 2 entering by force of habit. Summing relativistically then we require

$$\sum' \rightarrow \int \frac{d^3 p}{(2\pi)^3 2E}$$

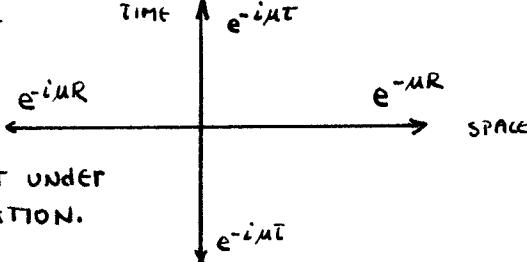
The SCATTERING AMPLITUDE BECOMES

$$\beta \propto e^{+i\vec{P}_i \cdot \vec{x}_b} N(\vec{x}_b - \vec{x}_a) e^{+i\vec{P}_i \cdot \vec{x}_a}$$

where the function N is a function of the distance and time separating the two events

$$N(\vec{x}, t) = \int \frac{d^3 \vec{p}}{(2\pi)^3 2\sqrt{\mu^2 + \vec{p}^2}} e^{-i(\sqrt{\mu^2 + \vec{p}^2} t - \vec{p} \cdot \vec{x})}$$

THE SOLUTION TO THIS INTEGRAL IS VERY COMPLICATED AND IS OF THE FORM OF A BESSSEL FUNCTION WHICH BEHAVES IN THE FOLLOWING LIMITING CASE IN SPACE TIME



THIS FUNCTION IS INVARIANT UNDER A LORENTZ TRANSFORMATION.

SO FAR WE HAVE BEEN DISCUSSING THE CASE WHERE THE SECOND EVENT OCCURRED AFTER THE FIRST, $\vec{t}_b > \vec{t}_a$. IF THE SECOND EVENT, B, HAPPENED EARLIER THAN A WE MUST START ALL OVER AGAIN

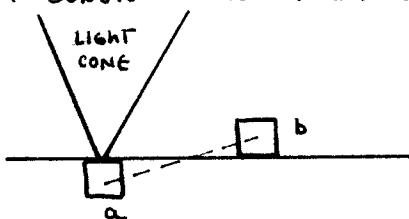
$\alpha \square$

$\beta \square$

IN THIS CASE THE SCATTERING AMPLITUDE TAKES THE FORM

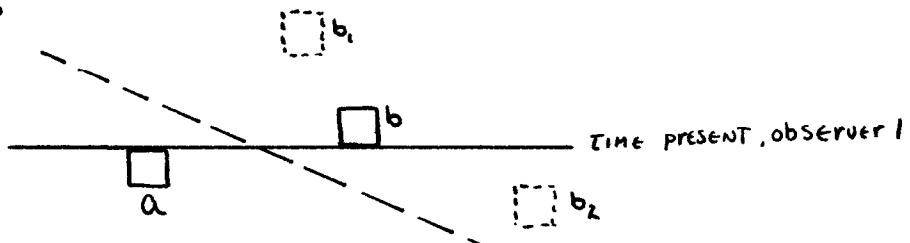
$$\beta \propto e^{+i\vec{P}_i \cdot \vec{x}_a} N(\vec{x}_b - \vec{x}_a) e^{-i\vec{P}_i \cdot \vec{x}_b}$$

WHERE WE HAD TO SHIFT THE \vec{x}_a AND \vec{x}_b AROUND BUT THIS MOVE MAKES THE FUNCTION DISCONTINUOUS. TO UNDERSTAND WHAT HAPPENS WE'LL ONLY CONSIDER THE $\vec{P}_i \cdot \vec{x}_a$ TERM FOR THE CASE OF $\vec{t}_b > \vec{t}_a$ AND $\vec{t}_b < \vec{t}_a$.



WE CONSIDER THE SCATTERING AREA b TO LIE OUTSIDE OF THE LIGHT CONE OF a. BUT, ODDLY, ENOUGH THERE IS A SCATTERING AMPLITUDE OF A PARTICLE REACHING b WHILE AT THE SAME TIME RESTRICTING THE LIGHT TO PARTICLE TO THE SPEED OF LIGHT. SEEMS STRANGE THAT THIS COULD HAPPEN APPARENTLY SIMULTANEOUSLY. IF THE FUNCTION N WAS 0 IN THIS CASE, IT WOULD BE ALL RIGHT BUT IT'S NOT.

HOW CAN THIS BE? LET'S CALL UP OUR SECOND OBSERVER AND TELL HIM TO COME BY AND OBSERVE THE SAME CASE WE JUST DREW. HERE'S WHAT WE WOULD HAVE,

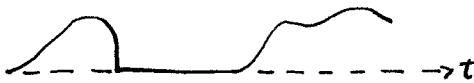


FIRST, IF WE HAD A SCATTER FROM a TO b_1 ; NO PROBLEM; THIS IS THE CASE WE DESCRIBED BEFORE. BOTH HAVE a IN THEIR PAST AND b_1 IN THEIR FUTURE. BUT WHAT ABOUT $a \rightarrow b$. TO OBSERVER 1 b IS OCCURRING SIMULTANEOUSLY WITH EVENT a YET HOW CAN THE PARTICLE GET THERE IN 0 TIME? WELL, OBSERVER 2 SEES NO DIFFICULTY WHATSOEVER. BOTH a AND b ARE IN THE ~~PAST~~ HIS PAST AND FUTURE AND HE HAS NO PROBLEM WITH INSTANTANEOUS ACTION. EVEN FOR THE CASE $a \rightarrow b_2$, OBSERVER 2 STILL HAS NO DIFFICULTY IN EXPLAINING THE EVENT YET TO OBSERVER 1 EVENT b_2 IS IN HIS PAST! WHO IS RIGHT?

WELL, WE MUST RECALL A THEOREM FROM MATH WHICH SAYS ANY FUNCTION $f(t)$ CAN BE REPRESENTED BY

$$f(t) = \int_{-\infty}^{\infty} e^{-i\omega t} \phi(\omega) d\omega$$

EVEN IF $f(t)$ IS DISCONTINUOUS LIKE



WE CAN STILL EXPRESS $f(t)$ IN THIS INTEGRAL FORM. ALSO BOTH ϕ AND f CAN BE COMPLEX FUNCTIONS. IF WE LIMIT OURSELVES TO JUST POSITIVE ω 'S (FREQUENCIES) WE ONLY CUT DOWN THE NUMBER OF FUNCTIONS. BUT THE TROUBLE LIES IN THAT WE CAN'T HAVE BOTH THE REAL AND IMAGINARY PART EQUAL TO ZERO OVER A FINITE PERIOD OF TIME.

Therefore, TO FIND $N(x,t) \neq 0$ OVER A FINITE TIME PERIOD WE HAVE NO WAY OUT. OBSERVER 2 MUST RECORD THE EXPECTED RESULT SO WE CONCLUDE WE LEFT SOMETHING OUT FOR OBSERVER 1 SO THAT HE CAN EXPLAIN WHAT HAPPENS WHEN $T_b < T_a$. WE NEED SOME WAY TO EXPAND

$$\alpha \beta e^{i p_2 \cdot x_b} N(x_b - x_a) e^{-i p_1 \cdot x_a}$$

FOR THE INTERMEDIATE STATE (AS OBSERVER 2 SEES IT) OF THE FORM

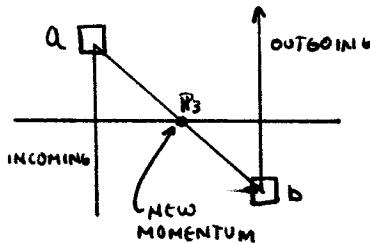
$$\langle x | b | \psi \rangle \langle \phi | a | \gamma \rangle$$

WRITING OUT EXPLICITLY AGAIN THE EXPRESSION FOR THE SCATTERING AMPLITUDE,

$$\alpha \beta e^{i(E_2 t_b - p_2 x_b)} e^{-i[E_2 L(t_b - t_a) - \vec{p}_3 \cdot (\vec{x}_b - \vec{x}_a)]} e^{-i(E_1 t_a - p_1 \cdot \vec{x}_a)}$$

WE SEE WHEN $t_b < t_a$ THE FORM OF THIS EQUATION CAN ONLY BE
KEPT IF E_2 IS NEGATIVE DURING THE INTERMEDIATE STATE.

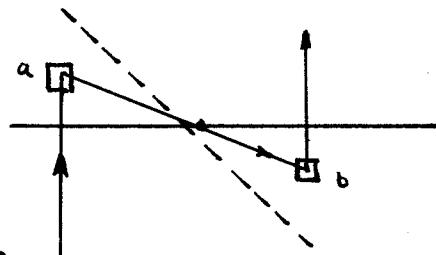
BUT, ALAS, WE ARE SAVED BECAUSE IN A SPACE-LIKE REGION THE
FUNCTION $N(\vec{x}_b - \vec{x}_a) = N^*$ ITS COMPLEX CONJUGATE AND WE THUS
CAN WRITE $e^{+iE_2(t_b - t_a)}$ AGAIN. BUT IN SO DOING WE HAVE CREATED
A NEW PARTICLE γ WITH ENERGY $E_3 = \sqrt{m^2 + p_3^2}$



PAIR PRODUCTION IS THUS NECESSARY FOR RELATIVISTIC INVARIANCE.

IF WE LOOK ONCE AGAIN AT OUR OTHER OBSERVER'S POINT OF VIEW
WE REALIZE THAT HE CAN QUITE EASILY
WRITE DOWN THE USUAL AND EXPECTED
SCATTERING AMPLITUDE. WE CAN THUS
MAKE A VERY PROFOUND STATEMENT;

THE AMPLITUDE OF PAIR PRODUCTION IS
EXACTLY THE SAME AS THE AMPLITUDE OF
SCATTERING AS SEEN BY THE MOVING OBSERVER.



THE SCATTERING LAWS THUS COMPLETELY DETERMINE PAIR PRODUCTION.

ANOTHER WAY OF WRITING THIS IS TO DEFINE AN OPERATOR ASSOCIATED
WITH THE EFFECT OF SCATTERING j SO THE PRODUCT OF THE TWO EVENTS
IS SIMPLY

$$j(x_a, t_a) j(x_b, t_b) = j(a) j(b)$$

IN THE OTHER COORDINATE SYSTEM WE HAVE $j(b) j(a)$ SO THAT THE
EQUALLY ABOVE CAN BE WRITTEN

$$j(b) j(a) = j(a) j(b)$$

PROVIDED A AND b ARE SEPARATED BY A SPACE REGION.
WHICH WE CAN RE-WRITE IN COMMUTATOR FORM

$$[j(a), j(b)] = 0$$

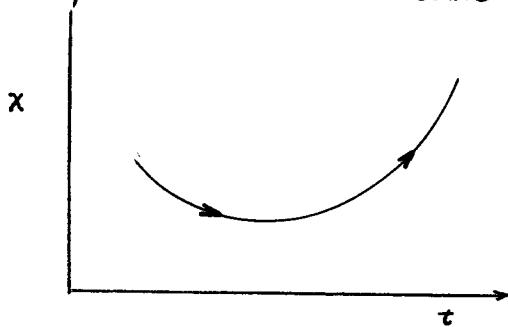
THIS THEN CONSTITUTES THE BASIC ASSUMPTION OF CAUSALITY THAT THE
OPERATORS COMMUTE OUTSIDE THE LIGHT CONE. IN Q-M THE COMMUTATOR
OF TWO OBSERVABLES VANISHES ONLY IF THEY ARE KNOWN SIMULTANEOUSLY;
THAT'S OUR ANALOGUE.

FEYNMAN'S THEORY

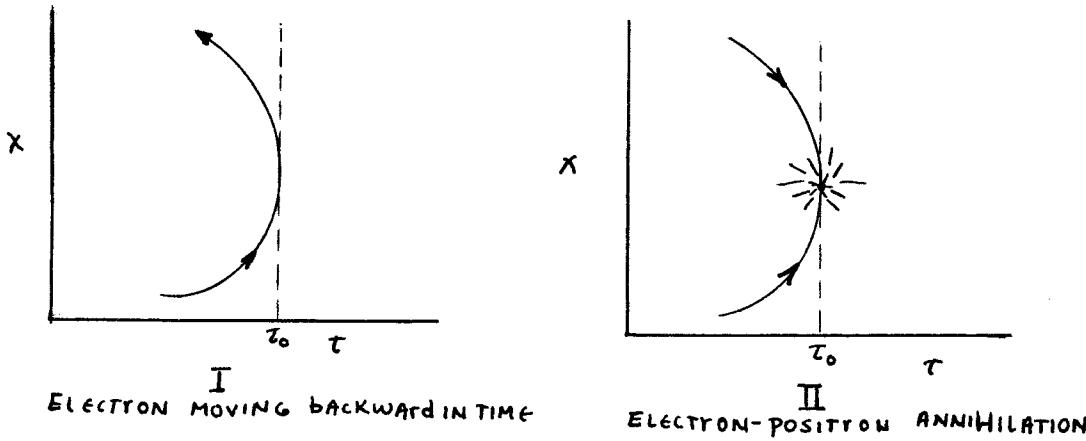
CLASSICALLY, AN ELECTRON CAN MOVE IN EITHER DIRECTION ALONG THE X AXIS BUT IT MOVES ONLY IN THE DIRECTION OF INCREASING TIME. FOR A FREE ELECTRON,

$$P = \pm \sqrt{mE}$$

CLASSICALLY WE ONLY CONSIDER THE + SIGN
THE MOTION CAN BE DEPICTED IN THE FOLLOWING DIAGRAM,



BUT $P = -\sqrt{mE}$ IS ANOTHER VALUE OF MOMENTUM BUT WHAT DOES IT MEAN IT HAVE A NEGATIVE VELOCITY. WE CAN DESCRIBE THE MOTION BY SAYING THE ELECTRON IS MOVING BACKWARD IN TIME. A DIAGRAM OF THE SITUATION IS



IN I THE ELECTRON APPEARS TWICE FOR $t < t_0$ BUT NEVER APPEARS FOR $t > t_0$. THE PROCESS LOOKS VERY MUCH LIKE TWO PARTICLES COMING TOGETHER AND ANNIHILATING. THE ELECTRON MOVING BACKWARD IN TIME BEHAVES EXACTLY AS A POSITRON MOVING FORWARD IN TIME.

WHAT DOES THIS ALL MEAN?

WHEN THERE IS PAIR-CREATION OF AN ELECTRON AND ITS ANTI-PARTICLE THE POSITRON WHICH IS EXTREMELY SHORT-LIVED. IT IMMEDIATELY COLLIDES WITH ANOTHER ELECTRON, BOTH ARE ANNIHILATED AND OFFGOES A GAMMA RAY. 3 SEPARATE PARTICLES, A POSITRON AND TWO ELECTRONS SEEM TO BE INVOLVED. FEYNMAN'S THEORY CLAIMS THERE IS ONLY ONE, THE ELECTRON.

WHAT WE OBSERVE AS A POSITRON IS SIMPLY AN ELECTRON MOMENTARILY BACK IN TIME. BECAUSE OUR TIME IN WHICH WE OBSERVE THE EVENT RUNS UNIFORMLY FORWARD WE SEE THE TIME-REVERSED ELECTRON AS A POSITRON. WE THINK THE POSITION VANISHES WHEN IT HITS ANOTHER ELECTRON, ~~AS A POSITRON~~ BUT THIS IS JUST THE ORIGINAL ELECTRON RESUMING ITS FORWARD TIME DIRECTION.

- REVERSE BETA DECAY WOULD ENTAIL COLLISION OF AN ELECTRON, PROTON AND ANTI NEUTRINO SHOT FROM SAY THE FAR REACHES OF SPACE TO ONE POINT.
- ONLY IN HUMAN CONSCIOUSNESS, IN THE ^{ONE} WAY PROCESS OF ~~OF~~ OUR MINDS CAN TIME HAVE A UNIDIRECTIONAL MOTION.
- STATISTICAL LAWS PROVIDE THE MOST FUNDAMENTAL WAY TO DEFINE THE DIRECTION OF TIME, I.E. IRREVERSIBLE PROCESSES WHICH DECREASE ENTROPY ARE EXPLAINED STATISTICALLY. THIS, HOWEVER, IS NOT STRICTLY TRUE.
- CERTAIN WEAK-INTERACTIONS ARE APPARENTLY NOT TIME-REVERSIBLE.
- CPT THEOREM:
 - (i) REVERSE CHARGE AND MATTER BECOMES ANTI-MATTER, E.G. A STONE IS AN ANTI-STONE
 - (ii) REVERSE PARITY AND THE WHOLE STRUCTURE WOULD GENERATE ITS MIRROR IMAGE BUT STILL EXIST. YANG AND LEE, HOWEVER, FOUND A VIOLATION HERE IN WEAK INTERACTIONS. HOWEVER, SYMMETRY RESTORED BY REFLECTING THE EVENT IN A CP MIRROR.
 - BUT, TOO, OTHER WEAK INTERACTIONS VIOLATE CP-SYMMETRY, I.E., NOT ALL DUPLICATES EXIST.
 - (iii) REVERSE TIME AND CP VIOLATORS ARE SYMMETRIZED IN A CPT MIRROR.
 - TIME REVERSAL IN AN INTEGRAL PART OF RELATIVITY.
SOME PARTICLE EVENTS ARE BELIEVED TO GO ONLY IN ONE TIME DIRECTION
- TO PRESERVE THERMAL EQUILIBRIUM REGIONS OR POCKETS EXIST WHERE ENTROPY IS INCREASING AND OTHERS WHERE IT IS DECREASING (I.E., FROM DISORDER TO ORDER). THE "BIG BANG" WAS A MOMENT OF COLOSSAL ENTROPY DECREASE. BUT MORE TO TIME REVERSED THAN ENTROPY DECREASE.
- TIME IS RELATIONAL OR RELATIVE LIKE UP AND DOWN, RIGHT AND LEFT. WITH NO ABSOLUTE TIME ARROW OUTSIDE AN EVENT WE CANNOT FIX ITS DIRECTION.
- A TIME-REVERSED GALAXY WOULD NOT BE SEEN SINCE LIGHT WOULD BE RADIATING TOWARDS IT. THE MEMORIES OF OBSERVERS IN TWO GALAXIES WOULD BE RUNNING BACK OPPOSITE. IF YOU TALK TO A GUY, HE WOULD INSTANTLY FORGET IT BECAUSE THE EVENT WOULD INSTANTLY BECOME PART OF HIS FUTURE RATHER THAN HIS PAST.

SPIN AND STATISTICS

If the two parameters α, β are not pure scalars but rather functions of E_3 and P_3 then we have to identify two sets α', β' and α, β . As an example, if $\alpha = E_3$ then

$$N(x, t) = \int \frac{d^3 p}{(2\pi)^3 2\sqrt{\mu^2 + p^2}} E_3 e^{-i(E_3 t + \mathbf{p} \cdot \mathbf{x})}$$

We can get around this integral if we take

$$\cancel{\alpha} N(x, t) = i \frac{\partial}{\partial t} \int \frac{d^3 p}{(2\pi)^3 2\sqrt{\mu^2 + p^2}} e^{-i(E_3 t + \mathbf{p} \cdot \mathbf{x})}$$

Without going into a lot of detail the other parameter α' is given by the negative of α or in this case $-E_3$, more generally $\alpha' = \alpha(-E_3, -P_3)$

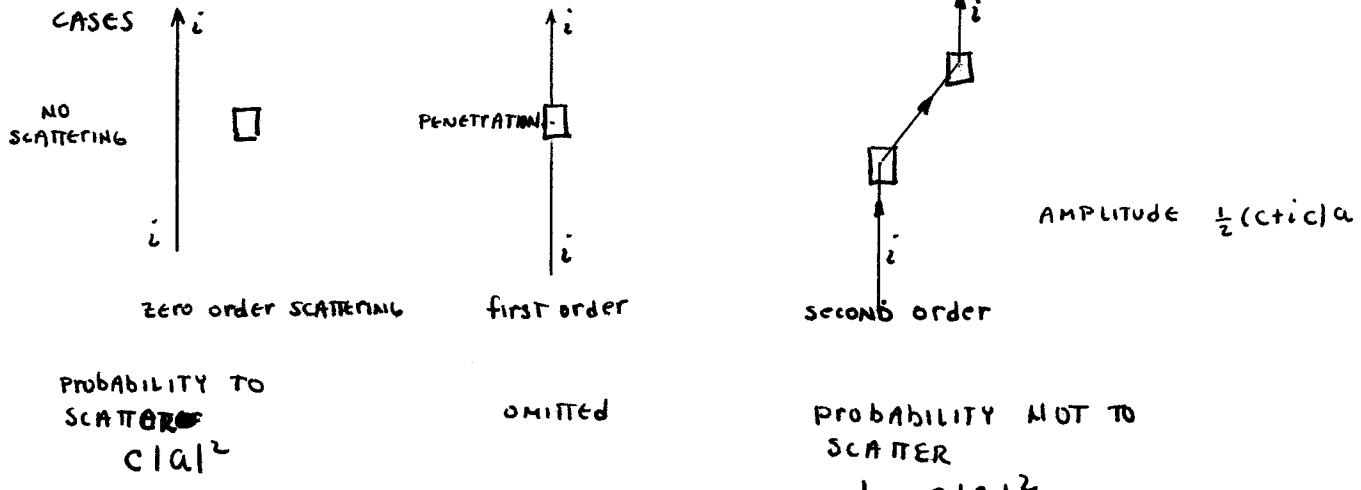
The

The problem mentioned at the beginning #2 does not follow immediately from our equation for the scattering amplitude. In fact, the explanation is very elaborate but I have worked on this problem and this is the best I can do — for that I apologize. Again we choose a negative approach to show that things won't work out unless we use Bose statistics for spin $1/2$, i.e., the sum of the probabilities is not 1 turns out to be the real problem.

Since we now have pair production established, i.e., in the sense of virtual production we must allow for the possibility of real pair production where an electron and positron are produced.

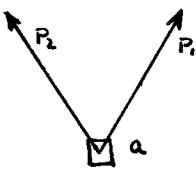
We will denote the amplitude to scatter by a and the probability of scattering by $|a|^2$. Since we can't allow the future to depend on the past the sum of the probabilities must be one. Herein, the trouble lies. It doesn't always equal one.

Considering first an electron starting out in some initial state i and ending up in the same state we have to consider three possible cases



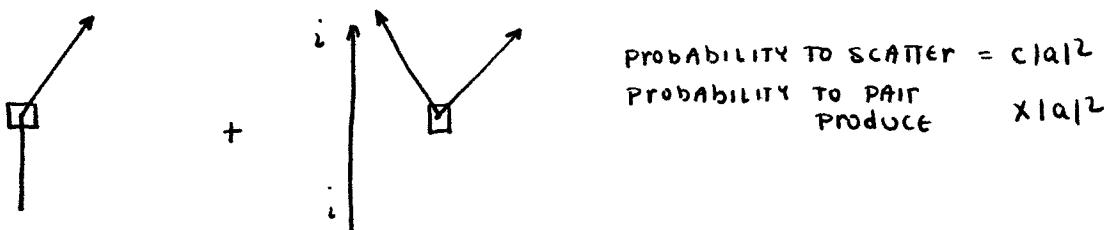
We see immediately the probability will only add to one if the constants are the same.

IN THE RELATIVISTIC CASE IT WILL ONLY BE WITH THE HELP OF STATISTICS THAT EVERYTHING WORKS OUT FOR US. TO BEGIN WE CONSIDER A VACUUM IN WHICH WE HAVE REAL PAIR PRODUCTION. THE AMPLITUDE TO CREATE A PAIR IS PROPORTIONAL TO THE POTENTIAL a SO THE PROBABILITY TO MAKE A PAIR IS $X|a|^2$ WHERE X IS SOME CONSTANT.

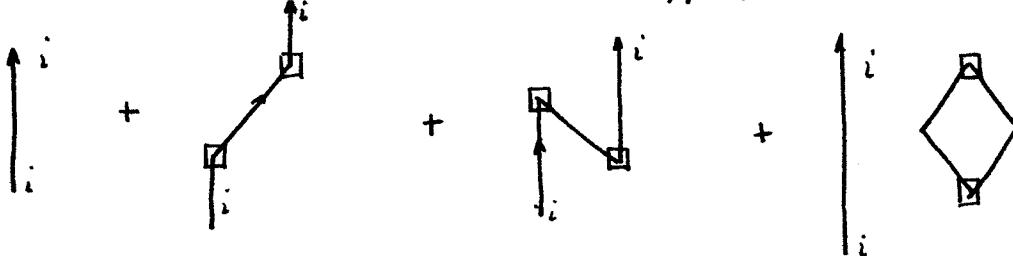


WE START WITH NOTHING AND GO INTO NOTHING, THAT IS, PAIR ANNIHILATION LEADS US BACK INTO A VACUUM. THE AMPLITUDE FOR NOTHING TO HAPPEN MUST BE 1. THE AMPLITUDE FOR THIS PRODUCTION-ANNIHILATION IS $1 - \frac{1}{2}(X+iX')a$ SO THE PROBABILITY TO GET NOTHING IS $1 - X|a|^2$.

WE MUST CONSIDER ALL POSSIBLE EVENTS; THEY ARE THE FOLLOWING FOR AN ELECTRON IN STATE i TO FIRST ORDER SCATTERING



NOW CONSIDERING THE CASE WHEN NOTHING HAPPENS WE HAVE FOUR POSSIBILITIES:

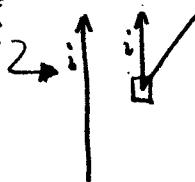


$$\begin{array}{cccc} \text{AMPLITUDE} & 1 & -\frac{1}{2}(c+iC')|a|^2 & -\frac{1}{2}(d+iD')|a|^2 \\ \text{of SCATTERING} & & & -\frac{1}{2}(X+iX')|a|^2 \end{array}$$

$$\text{PROBABILITY NOTHING HAPPENS} = 1 - c|a|^2 - d|a|^2 - X|a|^2$$

$$\text{PLUS About CASE} \quad \frac{+c|a|^2}{+d|a|^2} \quad + X|a|^2$$

BUT $d \neq 0$ SO WE HAVE A PROBLEM. d ITSELF = PAIR PRODUCTION IN STATE i . BOSE STATISTICS STRAIGHTENS THIS PROBLEM OUT BECAUSE IT STATES THAT THE ELECTRON IS NOT NECESSARILY THE SAME IN STATE i . THE ACTUAL PROOF OF HOW THIS STRAIGHTENS OUT THE PROBLEM IS QUITE DIFFICULT AND I APOLOGIZE AGAIN FOR NOT HAVE AN EASY PROOF.



WE WANT TO EMPHASIS IN CLOSING THAT FOR POSITIVE FREQUENCIES WE CAN'T HAVE SPREADING FASTER THAN THE SPEED OF LIGHT SO ONE MAN'S ELECTRON IS ANOTHER MAN'S POSITION AND THAT'S ALL THERE IS TO IT.

CHAPTER 13

A LECTURE ON THE SOLAR SYSTEM AND OTHER UNRELATED PHENOMENA.

TODAY I AM GOING TO TALK ABOUT THE SOLAR SYSTEM. BETTER YET, JUST BITS AND PIECES OF THINGS I HAVE PICKED OUT OF THE SEEMINGLY INFINITE AMOUNT OF MATERIAL ON THE SUBJECT TO BOIL IT DOWN TO A LECTURE. WHAT THE HELL AM I SUPPOSE TO DO? NOTHING SEEMS ORGANIZABLE. EACH WEEK IT GETS HARDER AND HARDER TO MAKE A SENSIBLE LECTURE SO NEXT WEEK WE'LL HAVE A MOVIE - ON THE SUN - A GOOD ONE NEVERTHELESS AND PERHAPS IT WILL GIVE ME A CHANCE TO GET AHEAD OF YOU GUYS.

COSMIC RAYS

A WHILE BACK WE BRIEFLY DISCUSSED COSMIC RAYS (THOSE NOTES HAVE NOT BEEN TRANSLATED YET IN ANTICIPATION OF THIS LECTURE; HOWEVER, DURING THE INTERIM THEY SEEM TO HAVE BEEN HIDDEN AND MAY NEVER BE RECOVERED). THEY HAVE BEEN SEEN TO CONTAIN SUCH THINGS AS LITHIUM, BERYLLIUM AND BORON PROBABLY BY THE SPALLATION PROCESS MENTIONED BEFORE. IT APPEARS TO OUR BEST MEASUREMENTS THAT THE COSMIC RAY BOMBARDMENT IS COMPLETELY ISOTROPIC AT THE EARTH'S SURFACE. FURTHER WE ARE NOT MOVING THROUGH THE COSMIC RAYS BECAUSE THERE IS NO OBSERVED RAIN EFFECT (I.E., A POUNDING FROM THE FRONT AS WE RUN INTO THEM). IF THERE ARE CLOUDS OF COSMIC RAYS, THEN THEY ARE MOVING WITH THE SAME RELATIVE MOTION WITH THE WHOLE GALAXY.

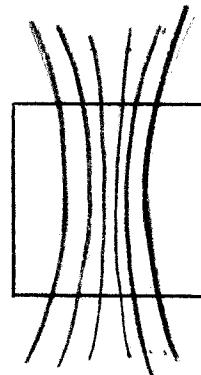
THE EXISTENCE OF VARIOUS MATERIALS IN THESE COSMIC RAY CLOUDS IS DETERMINED BY THE SCATTERING OF STARLIGHT THROUGH THEM. IN FACT THE EXISTENCE OF MAGNETIC FIELD IN THE GALAXY LEADS TO A POLARIZATION OF THE LIGHT AS IT PASSES THROUGH. THIS IS A RESULT OF IRON AND OTHER DUST PARTICLES WITH THEIR INTRINSIC SPIN CREATING LITTLE EDDY CURRENTS SUCH THAT THEY LINE UP AND CREATE SCATTERING CENTERS. IT HAS BEEN MEASURED (IN TYPICAL ASTRONOMICAL FASHION) THAT THE FIELD HAS A STRENGTH OF ABOUT 10^{-6} GAUSS. FOR THE OBSERVED FARADAY EFFECT TO OCCUR IT IS SPECULATED THAT A HIGHER FIELD, SAY 5×10^{-6} GAUSS IS NEEDED. SO MAYBE ITS 10^{-6} ; MAYBE 7×10^{-6} WHO KNOWS. THE ESSENTIAL FEATURE OF THIS MEASURING SEEMS TO BE LACKING. THAT IS, THEY ONLY FIND THIS EFFECT WHERE THE CIRCULAR POLARIZATION IS THE GREATEST. THIS MEANS WE SEE THE STRONGEST FIELDS FIELD. BUT HOW CAN WE COMPARE RESULTS WHEN ONE GUY LOOKS AT ONE STAR A CERTAIN WAY AND ANOTHER GUY LOOKS AT ANOTHER STAR AND USES ANOTHER METHOD? IT'S ALL RIDICULOUS! THERE IS NO CONSISTENT PATTERN FOLLOWED WHEN THIS DATA IS BEING RECORDED. WE DON'T REALLY KNOW IF THE FIELD IS, IN FACT, ALL OVER OR NOT.

THE RECORDED ENERGY OF THESE RAYS AT THE EARTH IS OF THE ORDER OF A BILLION ELECTRON VOLTS AND GREATER. THIS MEANS THE ORBIT OF THE PARTICLES IS VERY SMALL EVEN FOR A FIELD ON THE ORDER OF 10^{-6} GAUSS. IT IS ABOUT 10^9 METERS IN DIAMETER. THUS THE MOTION IS VERY TIGHTLY WOUND AROUND THE FIELD LINES AND IS SUBSEQUENTLY BOUND TO THE LINES RESTRICTING THE MOTION - THEY DON'T ESCAPE VERY EASILY.

WITH THE VALUES JUST MENTIONED WE ARE ABLE TO CALCULATE THREE THINGS:

- (1). THE TOTAL ENERGY IN THE COSMIC RAYS
- (2). THE TOTAL ENERGY IN THE MAGNETIC FIELD ($B^2/8\pi$)
- (3). THE TOTAL ENERGY IN THE CLOUD MOTION AS IT CHURNS AROUND.

IT TURNS OUT ALL THREE ARE ABOUT THE SAME ORDER OF MAGNITUDE WHICH HAS LEAD SOME TO CONCLUDE SOME MECHANISM OF EQUIPARTITION TO ACCOUNT FOR THIS. ONE WAY TO THINK OF THE MECHANISM IS TO PICK A UNIT CUBE OF SPACE ENCLOSING SOME MAGNETIC FIELD LINES. IF THE CUBE IS SQUASHED LATERALLY THE LINES GET CLOSER AND THE FIELD GETS BIGGER. WHEN THE CUBE IS COMPRESSED FROM THE TOP AND BOTTOM, THE FIELD GETS WEAKER AS THE LINES ARE FORCED OUTWARD. THERE ARE, THUS, TWO OUT OF THREE POSSIBLE MOTIONS WHICH TEND TO INCREASE THE ENERGY. HOWEVER THE CONTRACTION CANNOT PROCEED UNCHECKED BECAUSE EVENTUALLY THE ENERGY IN THE FIELD CAUSES THE COMPRESSION TO STOP AND USUALLY REVERSE. THE WINDING UP OF THESE FIELD LINES IS LIKE PUTTING SOME RED FOOD COLORING IN A BATCH OF TAFFY AND THEN PULLING IT OUT. THE LINES OF RED WILL BECOME CLOSER AND CLOSER AS THE MIXING CONTINUES. ANOTHER ANALOGY IS PUTTING PAINT IN A BUCKET AND STIRRING IT IN.



THE ENERGY IN THE COSMIC RAYS CAN BE DISSIPATED OVER LONG PERIODS OF TIME. IT IS CALCULATED, OR ESTIMATED, THAT THE RAYS HIT ABOUT $3 \text{ GM}/\text{CM}^2$ OF MATTER BEFORE THEY REACH US. AT SUCH A COLLISION RATE IT WOULD ONLY TAKE ABOUT A FEW MILLION YEARS BEFORE IT LOSES ALL ITS ENERGY. THEREFORE, THE RAYS COULDN'T HAVE BEEN GOING LONG BEFORE THEY REACHED US. THIS MEANS THERE MUST BE A SOURCE OF COSMIC RAYS WITHIN THE GALAXY. IT IS SPECULATED THAT THE SUPER-NOVAE COULD BE A POSSIBLE LOCATION OF THE ORIGIN OF COSMIC RAYS SINCE THEY GENERATE SUCH TREMENDOUS POWER AS THE BLOW UP. EVEN CONSIDERING THEIR OCCURRENCE IS ABOUT ONE EVERY HUNDRED YEAR THEY STILL GENERATE ABOUT FIFTY TIMES AS MUCH ENERGY NEEDED TO ACCOUNT FOR THE COSMIC RAY ENERGY. WE DON'T KNOW HOW THEY ARE PRODUCED AND THUS DON'T UNDERSTAND MUCH ABOUT THEM.

THESE COSMIC RAY CLOUDS THAT WE'VE BEEN TALKING ABOUT HAVE BEEN OBSERVED ALL OVER THE GALAXY. THEY ARE 10 PARSECS ACROSS AND ABOUT 30 PARSECS APART. THE MASS OF CRAP IN A TYPICAL CLOUD IS ABOUT $5000 M\odot$. THEY POSE A PROBLEM TO US IN TERMS OF EXPLAINING HOW THEY CAN HAVE SUCH A BIG MASS AND STAY LUMPY. GRAVITY ALONE WON'T SUFFICE. IT IS SPECULATE THAT AT THE LOW FIELD INTENSITIES INVOLVED THE OVERALL INSTABILITY OF THE CRAP GENERATES FORCES WHICH ADD TO THAT OF GRAVITY. ALSO THEY MOVE MORE OR LESS RANDOMLY AT A SPEED OF $5 \text{ KM}/\text{SEC}$.

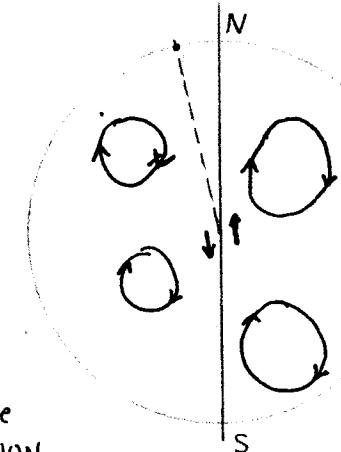
THE EARTH'S MAGNETIC FIELD

WHILE I'M ON MAGNETIC FIELDS, THE EARTH HAS A VERY INTERESTING ONE. THE FIELD IS OBSERVED TO CHANGE RAPIDLY DUE TO CURRENTS IN THE OUTER REGION SURROUNDING EARTH. THERE IS NO KNOWN FIELD INSIDE THE EARTH. NOW MOST OF THE FLUCTUATIONS IN THE FIELD ARE BLAMED ON THE SUN, AS YOU ALL KNOW. SOLAR FLARES, THE SUN SPOTS AND OTHER FREAK PHENOMENA DISTURB THE CHARGE DISTRIBUTION IN THE SURROUNDING SPACE THUS CAUSING THE CHANGES. WE ARE ALSO AWARE THAT THE TRUE MAGNETIC POLE IS NOT AT THE TRUE NORTH POLE BUT IS ABOUT 10° OFF SOMEWHERE IN CANADA.

IN PRINCIPLE, WE SHOULD BE ABLE TO TELL IF THE SOURCES OF THE FIELD ARE INSIDE OR OUTSIDE THE EARTH. THAT IS, BY WORKING WITH THE MAXWELL EQUATIONS. BUT A STRANGE THING, WHILE IT KEEPS THE GENERAL NORTH-SOUTH ORIENTATION, IT REVERSES IT DIRECTION ABOUT ONCE EVERY 100,000 YEARS OR SO. THEY CAN FIND THIS OUT STUDYING VARIOUS ROCK FORMATIONS AND OBSERVING HOW THEY SETTLED. THE FIELD IS PERPETUALLY CAUGHT UP IN THE EARTH'S CORIOLIS MOTION AND OTHER TWISTING UP EFFECTS OF THE EARTH. THE FIELD IS ALWAYS INCREASING AND IS LIKE A SELF-EXCITED DYNAMO. THAT IS, IT IS NATURE'S OWN DYNAMO.

BUT IT IS AN IRREGULAR DYNAMO AT BEST. THE PROBLEM OF REVERSING THE POLARITY OF THE POLES REALLY DESTROYS THE THEORY I WAS WORK ON. THE MODEL I ARRIVED AT DOESN'T WORK BECAUSE IT IS TOO SYMMETRICAL. THE PROBLEMS INTRODUCED BY THE LIQUID CORE (IF IT IS LIQUID), THE ASYMMETRICAL SHAPE OF THE PLANET, AND ALL THE ROCK MOTIONS, ETC MAKE IT VERY DIFFICULT TO FABRICATE A GOOD WORKING MODEL. WHILE I THINK MY THEORY IS RIGHT, I CAN'T WORK IT OUT. THE CIRCULATIONS OF CURRENTS WITHIN THE EARTH ARE UNSTEADY. AS THEY ADD TOGETHER THE FIELD BUILDS TO THE POINT WHEN THE CURRENTS REACH THE SURFACE AND SOMEHOW CAUSE THE FLIP IN POLARITY. THE RANDOMNESS OF THE DISTRIBUTION OF CURRENTS IS WHAT MAKES THE PROBLEM SO DIFFICULT. YET, THE SYMMETRY AND PERFECTNESS OF THE DIPOLE FIELD OUTSIDE THE EARTH PRESENTS A REAL MYSTERY AS TO HOW ALL THE INTERNAL DIPOLES - OCTOPOLES, SHOCKTOPOLIES ADD UP.

THE PURSUIT OF A SOLUTION OF THE UNDERSTANDING OF THE EARTH'S FIELD ~~SEEM~~ IS A CLEAR EXAMPLE OF WHAT SCIENCE IS. THAT IS, WHERE WE START OUT TO SOLVE A PROBLEM, NEVER DO, BUT MAKE TREMENDOUS CONTRIBUTIONS TO OUR WEALTH OF KNOWLEDGE ON THE WAY.



THE SOLAR SYSTEM

FINALLY WE GET DOWN TO THE TOPIC OF THE DAY AND DISCUSS SOME OF THE PROPERTIES OF THE SOLAR SYSTEM. I WILL BE GRABBING FOR THINGS TO SAY THAT ARE INTERESTING BECAUSE IT WOULD STARTLE TO MANY PEOPLE WHEN I TELL YOU THERE IS A SUN ABOUT WHICH THE PLANETS TURN. THE PLANETS LYING ESSENTIALLY IN A PLANE WITH SLIGHT DEVIATIONS.

THE PLANETS ARE LOCATED BY A THING CALLED BODE'S RELATION WHICH ASSIGNS THE EARTH'S DISTANCE FROM THE SUN AS UNITY; THE OTHER DISTANCES ARE FOUND BY THE RELATION

$$0.4 + (0.3 \times 2^n)$$

PLANET	BODE'S RELATION
MERCURY	$0.4 + (0.3 \times 2^{-1}) = 0.5$
VENUS	$0.4 + (0.3 \times 2^0) = 0.7$
EARTH	$0.4 + (0.3 \times 2^1) = 1.0$
MARS	$0.4 + (0.3 \times 2^2) = 1.6$
ASTEROIDS	$0.4 + (0.3 \times 2^3) = 2.8$
JUPITER	$0.4 + (0.3 \times 2^4) = 5.2$
SATURN	$0.4 + (0.3 \times 2^5) = 10.0$
URANUS	$0.4 + (0.3 \times 2^6) = 19.6$
NEPTUNE	$0.4 + (0.3 \times 2^7) = 38.8$
PLUTO	$0.4 + (0.3 \times 2^8) = 77.2$

PLUTO IS A LITTLE ODD IN THAT ITS ORBIT COMES INSIDE THAT OF NEPTUNE. PLUTO WAS DISCOVERED AFTER CAREFUL PERTURBATIONS OF NEPTUNE'S ORBIT WHICH LEAD TO THE PREDICTION OF PLUTO; IT WAS FOUND IN 1938. IT WAS AT ONE TIME A SATELLITE OF NEPTUNE - MAYBE.

WE ONCE MENTIONED THAT ABOUT 98% OF THE TOTAL ANGULAR MOMENTUM OF THE SOLAR SYSTEM IS CONCENTRATED IN THE PLANETS; IN FACT, 60% IS IN JUPITER ALONE. ALL THE PLANETS ARE GOING AROUND THE SUN IN THE SAME DIRECTION AND EACH SPINS ABOUT ITS OWN AXIS. THIS INCLUDES MERCURY AND VENUS WHICH WE ONCE THOUGHT TO ALWAYS HAVE THE SAME FACE ~~OF~~ TOWARD THE SUN. THAT IS, THEY ROTATE ABOUT THEIR AXIS ONCE EVERYTIME THEY REVOLVE ABOUT THE SUN. URANUS IS UNUSUAL IN THAT ITS AXIS OF ROTATION IS TILTED 95° TO THE ECLIPTIC AND, IN FACT, IS SPINNING THE WRONG WAY (RETROGRADE DIRECTION). MORE RECENTLY IT HAS BEEN OBSERVED THAT VENUS POSSESSES A RETROGRADE MOTION AND ITS AXIS IS AT A LATITUDE ANGLE OF $-85^\circ \pm 2^\circ$. MERCURY ROTATES ABOUT ITS AXIS ONCE EVERY 59 DAYS WHILE REVOLVING ABOUT THE SUN IN 88 DAYS.

THEN THERE ARE THE ASTEROIDS AND METEORS. THE METEORS BEING LEFT OVER CRAP FROM COMETS. WHENEVER WE PASS THROUGH THESE THINGS WE CALL IT A METEOR SHOWER AND IF A ROCK SURVIVES THE ATMOSPHERE, WE CALL IT A METEORITE.

COMETS



MOST OF THE COMETS WE OBSERVE HAVE HIGHLY ELLIPTIC ORBITS WHICH CARRY THEM OUT BEYOND JUPITER. SOME OF THEM, HOWEVER, GO OUT SO FAR THAT IT TAKES THEM ALMOST A MILLION YEARS OR SO TO COME BACK. IN FACT, IT IS HARD TO SAY WHETHER OR NOT THEY ARE ACTUALLY DEGENERATE PARABOLAS. IT IS THEORIZED THAT AS THEY COME IN TOWARD THE SUN THEY ARE DEFLECTED BY A PLANET (MOST LIKELY JUPITER) AND LOSE ENERGY RESULTING IN A CAPTURE. CURRENTLY THERE ARE NO KNOWN COMETS WITH A HYPERBOLIC ORBIT.

THE COMET HAS A SOLID NUCLEUS "UP FRONT" WHICH VERY SMALL, ONLY ABOUT A KILOMETER IN DIAMETER. AT A DENSITY OF ABOUT 5×10^4 PARTICLES PER cm^3 . SURROUNDING THE NUCLEUS IS A MASS OF GAS CALLED THE COMA IN WHICH NEARLY A THOUSAND DIFFERENT ABSORPTION LINES OF VARIOUS MOLECULES AND ATOMS HAVE BEEN OBSERVED. LIGHT IS EMITTED BY A FLORESCENCE RERADIATION WHERE THE BANDS RESULT FROM A RESONANT OF VIBRATION INTO AN UPPER LEVEL. THAT IS, SUNLIGHT HITS THE MOLECULES AND EXCITES THEM TO HIGHER ENERGY LEVELS WHICH, IN TURN, RERADIATE THE ENERGY AS THEY "FALL" BACK TO THEIR GROUND STATE. HOWEVER THE SUN HAS CERTAIN FRAUNHOFER LINES WHICH LEAD TO GAPS IN THE SPECTRA FROM THE COMETS. THESE LINES ARE PRODUCED BY THE ABSORPTION OF RADIANT ENERGY WHEN THE OUTER ELECTRONS OF THE ATOMS COMPOSING THE ATMOSPHERIC GASES OF THE SUN JUMP TO HIGHER ENERGY LEVELS. SO THE MISSING LINES CAN BE EXPLAINED. EVEN TO THE POINT THAT ALL THE PIECES OF THE PUZZLE FIT SO WELL TOGETHER AND HAVE REPRODUCED THE COMET SPECTRA ON COMPUTERS THAT WE THINK WE KNOW WHAT WE ARE TALKING ABOUT FOR ONCE.

MOST OF THE MATERIAL FOUND IN COMETS ARE NOT VERY STABLE; THIS IS WHY SUNLIGHT EASILY DECOMPOSES THEM. SOME OF THE MORE PRUDENT THINGS OBSERVED ARE: OH, NH, CN, CH, C₂, C₃, C₄H, NH₂, NA, Fe, CO₂⁺, CO⁺. THE RADIATION PRESSURE OF THE SUNLIGHT IS BELIEVED TO BE THE REASON WHY THE MOLECULES ARE DRIVEN INTO THE TAIL AS WE CALL IT.

RADAR ASTRONOMY

A MORE RECENT TECHNIQUE FOR MAKING ASTRONOMICAL MEASUREMENT IS BY SENDING A PULSE OF RADIATION TO AN OBJECT AND MEASURING NOT ONLY THE DELAY INVOLVED IN GETTING IT BACK BUT ALSO THE FREQUENCY SHIFT (DOPPLER SHIFT). THUS WE CAN MEASURE BOTH THE DISTANCE TO THE OBJECT AND ITS RELATIVE VELOCITY. THIS METHOD HAS BEEN APPLIED TO VENUS, THE MOON, MARS, MERCURY AND JUPITER. BEST SUCCESS HAS BEEN OBTAINED FROM VENUS AND THE MOON.

THIS TECHNIQUE CAN EVEN BE USED TO "EXPLORE" THE PLANET BY SCANNING THE SURFACE AND RECORDING THE TOTAL REFLECTION INTENSITY. CERTAIN SURFACE FEATURES CAN BE OBSERVED IN THIS WAY. ROTATIONAL MOTION CAN BE CHECKED USING THE DOPPLER SHIFT ARISING FROM THE PLANET'S MOTION TOWARDS AND AWAY FROM US CAUSING DIFFERENT DELAYS OVER THE SURFACE.

WE CAN CHECK THE PLANETARY ORBITS AND MEASURE THE SCALE OF THE SOLAR SYSTEM. THE ASTRONOMICAL UNIT IS THE EARTH'S ORBIT RADIUS WHICH IS NOW FIXED AT $149,596,000 \pm 1000$ KM OR 99.005 LIGHT SEC.

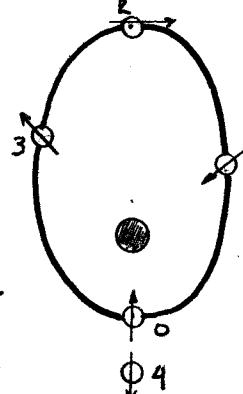
ROTATION OF MERCURY

LAST TIME WE WERE TALKING ABOUT RADAR ASTRONOMY AND HOW IT WAS USED TO MAKE ROTATIONAL MEASUREMENTS OF MERCURY AND VENUS. TODAY SINCE I DON'T HAVE THE MOVIE I PROMISED YOU I WILL CONTINUE THIS DISCUSSION BECAUSE IT IS VERY INTERESTING.

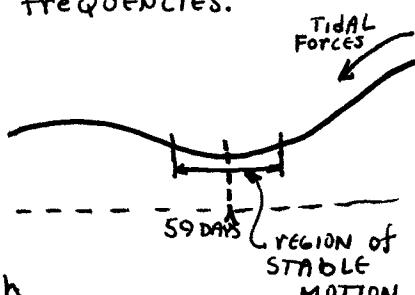
WE RECALL THAT THE PERIOD OF REVOLUTION AROUND THE SUN IS 88 DAYS WHILE ROTATION ABOUT ITS AXIS OCCURS ONCE EVERY 59 DAYS. IT TURNS OUT THAT 59 IS ALMOST $\frac{2}{3}$ OF 88 DAY. SO CLOSE IN FACT THAT IT HAS LEAD TO A THEORY WHICH SEEMS TO EXPLAIN MERCURY'S STRANGE BEHAVIOR.

MERCURY'S ORBIT IS VERY ECCENTRIC MUCH MORE SO THAN ANY PLANET. THERE IS THUS A PERIGEE AND APOGEE ^{other} WHICH IS QUITE PRONOUNCED. FURTHER MERCURY ITSELF IS LOPSIDED AND ASSUMES AN ELLIPSOIDAL SHAPE WITH THREE UNEQUAL AXES. THERE IS THUS TIDAL FORCES CREATED IN THE BODY WHICH ARE GENERATED BY THE SUN TENDING ~~TO~~ TURN THE QUADRUPOLE MOMENT TOWARD THE SUN. ASSUME THE PLANET STARTS OUT AT ITS ~~PERIGEE~~ POSITION 0 ON THE SIDE DIAGRAM WITH ITS LONG AXIS POINTING TOWARD THE SUN.

IF WE FOLLOW THE PATH WE SEE BY THE TIME MERCURY REACHES IT APOGEE IT HAS GONE THROUGH $\frac{3}{4}$ REVOLUTIONS AND HAS ITS LONG AXIS PERPENDICULAR TO THE TIDAL FORCE DIRECTED RADIALLY TOWARD THE SUN. THIS SEEMS TO CONTRADICT WHAT WE SAID ABOUT THE LONG AXIS ALWAYS POINTING TOWARD THE SUN - BUT NOT REALLY. WHILE THE MINIMUM ENERGY IS WHEN THE LONG AXIS POINTS TOWARD THE SUN THE MEAN TIDAL FORCE ON THE QUADRUPOLE IS STILL POSITIVE. BY THE TIME IT COMES BACK TO POSITION 4 THE PLANET HAS GONE THROUGH $1\frac{1}{2}$ ROTATIONS WITH ITS LONG AXIS STILL POINTING TOWARD THE SUN. MOREOVER THE MOTION IS STABLE AND DYNAMICAL SENSIBLE. HOW CAN THIS BE?



WE KNOW THESE TIDAL FRICITION FORCES TEND TO SLOW THE PLANET DOWN. BUT THERE IS ONLY A SLIGHT STABLE EQUILIBRIUM AT 59 DAYS SO THE TIDAL FORCE ACTING LIKE A CONSTANT FORCE PUSHING IT ON PAST THIS EQUILIBRIUM FROM HIGHER FREQUENCIES. HOW IS IT STOPPED FROM GOING OVER THE OTHER SIDE? IT SO HAPPENS WHEN THE PLANET IS MOVING FAST THROUGH ITS PERIGEE THE SUN APPEARS TO IT TO BE GOING BACKWARD IN THE SKY AND THE TIDAL FORCE REVERSES ITS SIGN JUST ENOUGH TO CAPTURE THE PLANET AND START ITS 59 DAY ROTATION.



THIS IS VERY NEAT AND SEEMS TO FIT PRETTY WELL WITH OBSERVATION.



ROTATION OF VENUS

RECALLING AGAIN LAST TIME WE SAID THE PERIOD OF VENUS' YEAR WAS 225 DAYS WHILE IT ROTATED ONCE EVERY 245^{1/2} DAYS. BUT THERE IS SOMETHING VERY STRANGE ABOUT VENUS THAT WE HAVE DISCLOSED WITH OUR RADAR ASTRONOMY. ONE SIDE OF VENUS IS ALWAYS TOWARDS US WHEN THE PLANET IS THE NEAREST TO US - WHICH HAPPENS EVERY 584 DAYS. THERE IS AN "OBSERVABLE" SPOT ON THE PLANET WHICH SCATTERS A LOT OF THE RADAR SIGNALS. THIS SPOT IS ALWAYS IN THE SAME POSITION DURING THIS CLOSE APPROACH.

FIRST WE ARGUE SOMETHING IS THE MATTER WITH THE INSTRUMENT OR THAT SOME EXPLANATION CAN BE FOUND OTHER THAN THE ASTROLOGICAL EXPLANATION THAT EARTH "CONTROLS" THE MOTION OF VENUS. SO THE THEORY THAT SEEMS TO EXPLAIN MERCURY WAS APPLIED TO VENUS. THE POSSIBLE TIDAL FORCE GENERATED BY THE EARTH COULD NOT BE PROPERLY USED TO EXPLAIN THE CAPTURE MECHANISM. ANOTHER FORCE WAS NEEDED, A FRICTIONAL ONE, WHICH WOULD ACT AS A DAMPENING MECHANISM WHICH WOULD BE JUST RIGHT TO DAMPEN THE PREDICT 30,000 YR. PERIOD. THIS FORCE WAS EXPLAINED BY ASSUME VENUS HAS A LIQUID CORE WHICH IMPLIES INTERNAL FRICTION. THIS EXPLANATION HAS NOT MEANT WITH A GREAT DEAL OF SUCCESS AND THE WHOLE ISSUE REMAINS AN OPEN QUESTION YET AS NEW DATA IS COLLECTED ON THE ROTATIONAL MOTION IT SEEMS TO FIT MORE AND MORE.

SO, IN FACT, IT MIGHT TURN OUT YET THAT WE ARE CONTROLLING VENUS AND CAN HAVE WILD ASTROLOGICAL OVERTONES. BUT AT THE SAME TIME THIS DOES NOT EXPLAIN THE RETROGRADE MOTION AT ALL. MAYBE THERE IS SOMETHING PECULIAR IN THE MEASUREMENT WHICH WE HAVE FOUND YET. PERHAPS, THE LITTLE PEOPLE UP THERE ARE PEEPING BACK TO US SOME FORM OF DATA (LIKE OUR PERIOD OF REVOLUTION) IN AN EFFORT TO COMMUNICATE WITH ^{US} THEM AND HERE WE'RE THINKING UP ALL THESE CRAZY THEORIES HOPEFULLY LOOKING FOR A LOGICAL EXPLANATION!

THE SUN

WELL, NOW WE'LL TALK ABOUT THE SUN. AS SOME OF YOU KNOW, IT IS A BIG BALL OF HOT GAS ROTATING ABOUT ITS AXIS AND SPITS OUT ALL SORTS OF CRAP IN THE COURSE OF REVOLUTION. IT SPINS FASTER AT ITS EQUATOR THAN AT ITS POLES. THE ANGULAR VELOCITY IS FITTED TO THE FOLLOWING EQUATION,

$$\omega = 14.38^\circ - 2.77^\circ \sin^2 \theta \quad \frac{\text{degrees}}{\text{day}}$$

WHERE θ IS THE LATITUDE ANGLE.

THE INTERESTING QUESTION TO ASK IS WHY THE BALL ROTATES LIKE THIS IN THE FIRST PLACE. THE EARTH DOES BUT IT'S A SOLID MASS - OR PRESUMABLY SO. BUT A BALL OF LIQUID LIKE A RAIN DROP WOULD SPIN FOREVER AND THE ANGULAR VELOCITY WOULD BE THE SAME ALL OVER THE SPHERE. THE WHOLE BALL OF GAS IS OBVIOUSLY NOT IN EQUILIBRIUM OR ELSE IT WOULD THROW OFF ALL THE STUFF IT DOES. IT DOESN'T WANT ITS LOWEST ENERGY STATE, APPARENTLY. ALL OF THIS IS NOT WELL UNDERSTOOD

STRUCTURE OF THE SURFACE AND ATMOSPHERE

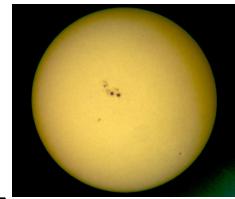
THE EFFECTIVE SURFACE IS TERMED THE PHOTOSPHERE. BELOW THIS LAYER THE OPACITY BEGINS TO INCREASE RAPIDLY AND OBSERVATIONS STOP ABRUPTLY. THIS LAYER IS SEEN AS A HOT EMITTING LAYER WHERE MOST OF THE VISIBLE LIGHT COMES FROM. ABOVE THE PHOTOSPHERE IS THE CHROMOSPHERE WHERE ALL THE JUNK IS SPIT OUT FROM. THERE ARE A LOT OF EMISSION LINES SEEN IN THE SPECTRUM. THESE ARE JUST REVERSALS OF THE FRAUNHOFER ABSORPTION LINES. BEYOND THE CHROMOSPHERE IS THE CORONA WHICH GOES OUT AS FAR AS 2-3 SUN RADII. IT EXHIBITS CONTINUOUS EMISSION LINES FROM THE SCATTERING OF SUNLIGHT BY MANY FREE ELECTRONS. THERE IS A VERY STRONG LINE AT 5303 Å WHICH WAS NOT IDENTIFIED WITH ANY ELEMENT FOR SEVENTY YEARS. IN 1939 IT WAS FINALLY NAMED AS A FORBIDDEN LINE OF IRON WITH THIRTEEN ELECTRONS MISSING. THE IONIZATION TEMPERATURE OF FeIg CORRESPONDS TO A TEMPERATURE OF ONE MILLION DEGREES. BUT THE DENSITY OF THE CORONA IS VERY LOW SO IT IS THEORIZED THAT NOISES ARE GENERATED BY THE SUN WHICH SHAKES THE CORONA TO HELL SUPPLYING THE POWER TO MAINTAIN THIS 10^6 . AS THE DENSITY GETS THINNER AND THINNER, LIKE CRACKING A WHIP

THERE ARE A LOT OF EMPIRICAL THINGS SEEN WHEN OBSERVING THE SUN; THERE ARE SPOTS, SPICULES, GRANULES, SPOTCHES, FILAMENTS, ETC, ETC. IT IS VERY DIFFICULT TO PRODUCE A THEORY FOR REMEMBERING ALL THESE THINGS. I SHALL TRY TO GIVE A GUIDE FOR REMEMBRANCE AS WE GO ALONG.

RICE GRAINS

THE RICE GRAIN STRUCTURE ARE POLYGONAL CELLS OR SPOTCHES IN THE PHOTOSPHERE. IT IS THEORIZED THAT THEY ARE CONVECTION CELLS IN WHICH JUNK IS COMING UP FROM THEIR CENTER AND SPILLING OUT. THEY ONLY LAST A FEW MINUTES BUT ARE ABOUT 1000 KM ACROSS

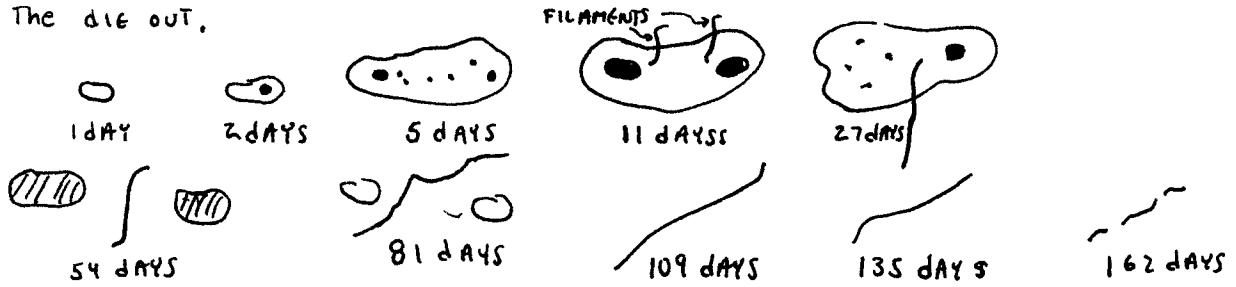
ANOTHER GRAIN STRUCTURE, NOT THE RICE GRAIN, IS CALLED THE SUPER GRANULATION WHERE THEY HAVE A DIAMETER OF 15000 KM AND LAST ABOUT 7000 SECS.



SUNSPOTS

The spots appear black because their temperature is lower than the surrounding photosphere. They come in cycles of about eleven years. They start at a latitude of about 35° and work toward the equator. They tend to dissolve and break up but they do drift with the rotation and some last through a couple of rotations. The spot generate a field of about 1000 Gauss at their center. They show a distinct leading and following edge in which the preceding spot is always lower than the following one. The fields are different for the two spot and the North always leads the South pole, but during the next cycle the polarity is reversed.

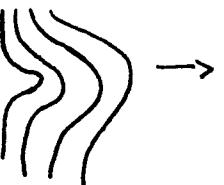
A spectroheliogram is used to observe one particular wavelength and the most pronounced emission line is the K line of calcium. With this K line the sun spots can be "seen" before and after the disk out.



There also develops a dipole moment near the pole which will soon disappear. The strength of the field reaches a maximum after eleven years then reverses polarity.

A theory of the sunspots was put forth by Babcock. He said that inside the sun or right under the surface there are more or less weak magnetic field lines running along the meridians. Due to the difference in rotation speed of the sun and the fact that the lines are tied to the sun they get pulled or bulged out. After going around for several hundred years, the field eventually has so much energy stored in it that it tries to expand and "float" up to the surface. Finally it reaches the surface and the hump bulges out of the surface layer. Thus it appears as a North and South region depending on which way the field lines run. Calculating the area most likely for the bulged to begin from the difference in rotation formula, the lines get the most dense at about 30° .

We still would like to know why the filaments tend to move to the North? Leighton claims that supergranules exist as a convection effect and serves as its own pumping source. Any line coming out of the sun is carried to another place in a random walk manner. If the trailing spot has a South polarity and since it is a little closer to the North pole, it will diffuse there faster and in this way it is possible to predict what the polarity should be.



NORTH POLE
SOUTH POLE

CHAPTER 14

A LECTURE ON PLANETARY ATMOSPHERES

TODAY I WANT TO TALK ABOUT THE RECENT INFORMATION WE HAVE COLLECTED ON THE ATMOSPHERIC COMPOSITION OF THE NEARBY PLANETS - MARS, VENUS, AND JUPITER. THE OLD MATERIAL I HAVEN'T LOOKED AND SUGGEST ANY GOOD ASTRONOMY BOOK IF YOU SO DESIRE.

MARS

FIRST WE LOOK AT MARS. AS YOU PERHAPS KNOW MARS HAS ONLY ONE TENTH THE MASS OF EARTH AND ONE-HALF THE RADIUS, I.E., ABOUT 2000 MILES. IT ROTATES ABOUT ITS AXIS IN ABOUT 25 HOURS MUCH LIKE THE EARTH. MARS IS A REDDISH-ORANGE PLANET AND THAT'S ALL - NO, I'LL TELL YOU MORE.

DURING THE RECENT FLY-BY OF THE MARINER SPACECRAFT WE WERE ABLE TO MEASURE A PHASE SHIFT IN THE RADIO TRANSMISSION AS IT PASSED BY THE EDGE OF THE PLANET. THAT IS, KNOWING ALL THE RELATIVE MOTIONS INVOLVED LIKE THE VEHICLE'S SPEED, MARS' MOTION, THE EARTH'S MOTION, ETC. IT WAS POSSIBLE TO PREDICT THE FREQUENCY OF ~~EMISSION~~ RECEPTION. HOWEVER AS IT PASSED BEHIND THE PLANET, THERE WAS A BRIEF PERIOD IN WHICH THE RADIO TRANSMISSIONS HAD TO PENETRATE THE MARTIAN ATMOSPHERE. THE RESULTING PHASE SHIFT GAVE A RELATIVE MEASURE OF THE MARTIAN ATMOSPHERE.

IT WAS DISCOVERED THAT ABOUT 120 KM FROM THE SURFACE THERE IS AN IONOSPHERE SIMILAR TO THE EARTH'S. IT HAS AN ELECTRON DENSITY OF ABOUT 9×10^4 ELECTRONS/cm³. THE PRESSURE AT THE SURFACE WAS ESTIMATED AT ABOUT 5 MILLIBARS. HOWEVER, THIS FIGURE IS NOT AGREED UPON BY THE ASTRONOMICAL COMMUNITY. OTHER ATTEMPTS USING INFRARED LIGHT HAVE INDICATED HIGHER PRESSURE LIKE 10-20 MB. WHILE THEY ARE ARGUING, WE'LL SAY IT'S ABOUT 10 MB. THE IMPORTANT POINT IS THAT THE PLANET CAN RETAIN AN ATMOSPHERE. FURTHER, THE PHASE SHIFT FITS VERY NICELY AN EXPONENTIALLY DECAYING FUNCTION OF HEIGHT.

THE TEMPERATURE ON THE SURFACE IS RATHER LOW ABOUT 230° K. THERE ARE SPOTS NEAR THE EQUATOR THAT GET WARMER. IT IS INTERESTING TO SEE WHAT HAS BEEN IDENTIFIED IN THE ATMOSPHERE.

FOR MANY YEARS CO₂ HAS BEEN OBSERVED BY ABSORPTION LINES IN THE INFRARED REGION OF THE SPECTRUM. MAYBE THERE IS NOTHING ELSE. BUT PEOPLE HAVE CONTRIVED VARIOUS MODEL ATMOSPHERES TO EXPLAIN ALL THE OBSERVED PHENOMENA. A STANDARD ATMOSPHERE NOWADAYS CONSISTS OF 40% CO₂ AND 60% NITROGEN. BUT THE NITROGEN IS NOT EASILY SEEN AND, IN FACT, THERE MAY NOT BE ANY AT ALL. MAYBE IT'S ARGON; MAYBE WE DON'T KNOW FOR SURE.

SMALL AMOUNTS OF WATER VAPOR HAVE BEEN OBSERVED NEAR THE POLES-
BY SOME PEOPLE. BUT IT IS NOW GENERALLY BELIEVED TO REALLY EXIST.
THE OBSERVATIONS OF THE POLAR CAPS DISAPPEARING IN THE MARTIAN SPRING
IS BELIEVED TO BE H₂O VAPOR. THE CONCENTRATION OF H₂O IS SO SMALL
THAT IT COULD NEVER PRECIPITATE. THE DEEPMEST AMOUNT IS GIVEN AS MUCH LESS
THAN 14 MICRONS OR THE "DEPTH" OF THE WATER IF IT CONDENSED ALL OVER
THE SURFACE. IN OTHER WORDS, THAT'S HOW DEEP THEIR OCEAN WOULD BE.
IT AIN'T MUCH WATER!

IN ADDITION CO HAS NOW BEEN IDENTIFIED USING A BETTER INTERFEROMETER
WHICH IS EVEN CAPABLE OF DETECT ISOTOPES OF THE VARIOUS ELEMENTS. ONCE
METHANE WAS OBSERVED BUT NEVER FOUND AGAIN. THIS IS VERY STRANGE BECAUSE
METHANE COULD NOT EXIST IN EQUILIBRIUM WITH CO₂ AND H₂O BECAUSE THE
CH MOLECULE WOULD BE EATEN RIGHT UP BY OXYGEN ATOMS. SO MAYBE THERE
ARE LITTLE CREATURES ARE MANUFACTURING IT

THERE HAVE BEEN PROPOSED VARIOUS THEORIES ABOUT THE MARTIAN
WEATHER. CO₂ IS A GOOD ABSORBER AND RADIATOR OF IR LIGHT WHICH SOMEHOW
EFFECTS THE TIME DURATION OF SURFACE STORMS WHICH TENDS TO MAKE THEM
VERY LONG. SO THEY MIGHT LAST A MONTH OR SO AT A TIME.

WHAT DO WE SEE ON THE SURFACE? WELL, THERE ARE THE POLAR CAPS WHICH
GROW AND DIMINISH IN A SEASONAL MANNER. BASICALLY THE PLANET IS
REDDISH-ORANGE WITH DARK AREAS COVERING ABOUT ONE-FOURTH OF ITS SURFACE.
THESE DARK AREAS UNDERGO CHANGES IN SIZE AND COLOR DURING THE MARTIAN
YEAR. DARKS "CANALS" WERE ONCE OBSERVED BY P. LOWELL WHO SUGGESTED
THEY WERE THE DOINGS OF INTELLIGENT BEINGS. HE WAS A VERY MISGUIDED INDIVIDUAL
BUT AT THE TIME HE HAD THE BEST TELESCOPE AND BEST OBSERVATION CONDITION
(IN ARIZONA) SO THAT ANYONE WHO DIDN'T SEE THEM JUST DIDN'T HAVE
THE RIGHT CONDITIONS. IT IS THE SAME P. LOWELL WHO "SAW" VAST ARMIES
OF ANTS MOVING ACROSS THE CRATER OF THE MOON.

HOWEVER, THIS ANTIQUATED IDEA IS GIVING WAY TO A MORE SOPHISTICATED
INTERPRETATION OF WHAT WE SEE. NOW WE CLAIM WE DON'T KNOW WHAT IT
IS WE SEE.

THE MARINER ALSO TOOK PICTURES OF CRATERS ON THE SURFACE WHICH APPEAR
LIKE THE ONES WE SEE ON THE MOON. THIS IS NOT A STARTLING DISCOVERY
BECAUSE THE MARTIAN AIR IS THIN ENOUGH THAT METEORS COULD BOMBARD IT
WITHOUT BURNING UP. THEY APPEAR TO HAVE FLATTER BOTTOMS, ARE SMALLER THAN
THE MOON'S AND LESS FREQUENT. THERE SEEMS TO BE FEWER PER SQUARE
FOOT THAN THE MOON. THE REASON FOR THIS IS BELIEVED TO BE DUE TO
THE EROSION OF THE CRATER EDGES WHICH FILLS UP THE BOTTOMS. SINCE MARS
IS CLOSER TO THE ASTEROID BELT WE WOULD EXPECT MORE SO WE NEED
A MECHANISM OF OBLITERATION TO EXPLAIN WHY THERE ARE FEWER
THAN THE MOON. BUT KNOW THERE ARE TOO MANY UNCERTAINTIES TO
GIVE AN ACCURATE THEORY RIGHT NOW.

IN ONE OF MARINER'S PICTURE OF THE MARTIAN HORIZON THERE APPEARED A SMALL HALO OF SCATTERED LIGHT JUST ABOVE THE SURFACE. AT FIRST IT WAS THOUGHT TO BE A SPOT ON THE CAMERA LENS BUT IT WAS LATER DECIDED THAT IT WAS A CLOUD. BELIEVED TO BE A CO₂ CLOUD OF DRY ICE SUBLIMING NOT ICE CLOUDS AS MORE COMMONLY BELIEVED.

THE MARKINGS ON THE SURFACE SOMETIMES DISAPPEAR. EVEN THE POLAR CAPS DO THIS. MAYBE THERE IS A BIG DUST STORM PLANET WIDE WHICH BLOWS OVER THE SURFACE LEAVING BEHIND A LAYER OF DIRT ALL OVER INCLUDING THE ICE. LATER THE ICE COULD REDEPOSIT OVER THE DIRT. MORE IMPORTANT IS THE SUDDEN CHANGE IN CONTRAST OF THE VARIOUS REGIONS. THEY SUDDENLY APPEAR AND GRADUALLY FADE OUT. THE DARKER AREAS MAY BE HIGHER AND THE DUST BLOWS OFF INTO THE VALLEYS BETWEEN RIDGES. MARINER COULD DISCERN THE CHANGE IN BRIGHTNESS BUT IT DIDN'T TELL US THE QUALITY OF THE SURFACE.

THE FEYNMAN THEORY IS: BIG FIELDS OF DARK BROWN POPPIES GET COVERED WITH DUST AND CRAP AND THEN ARE BLOWN CLEAN BY THE GENTLE MARTIAN WINDS.

WHEN THE POLAR CAPS MELT, WHICH IS REALLY SUBLIMATION OF CO₂, THE SURFACE FEATURES DARKEN IN A GENERAL DIRECTION DOWNWARD FROM THE POLE. SOME PEOPLE LIKE BOB LEIGHTON DON'T THINK THIS IS A REAL PHENOMENA - THIS WAVE OF DARKENING. WHILE THEY CLAIM TO SEE COLORS SUCH AS BLUE AND GREEN, I THINK THEY'RE WRONG HERE. COLOR JUDGMENT IS NOT AN OBJECTIVE PROCESS BUT DEPENDS ON THE BACKGROUND AND HOW ONE INTERPRETS THE BACKGROUND.

VENUS

WE KNOW VERY LITTLE ABOUT THIS PLANET EVEN THOUGH IT IS THE CLOSEST ONE TO US. WE CAN'T SEE ANY MARKINGS DUE TO THE THICK CLOUDS COVERING THE WHOLE PLANET. WE SEE LAYERS OF CLOUDS AT A TEMPERATURE OF 240°K WHICH WE BELIEVE IS HIGH ICE. THAT IS, IT IS ALMOST CERTAINLY IDENTIFIED AS ICE BECAUSE IT HAS BEEN FITTED WITH CURVES ON EARTH TAKEN FROM DATA ON OUR ATMOSPHERE. CO₂ HAS ALSO BEEN IDENTIFIED IN THE ATMOSPHERE.

IF WE OBSERVE VENUS AT MICROWAVE FREQUENCIES, 3-30 CM, WE CAN MEASURE THE TEMPERATURE OF THE SURFACE DUE TO RADIATION. THE DATA INDICATED A SURFACE TEMPERATURE OF AROUND 700°K. THAT'S A DAMN HOT SURFACE! WHAT WE SEE WHEN WE USE .4 TO 1 CM IS 285°K OR THE TEMPERATURE AT HIGHER LEVELS SINCE THE SHORTER WAVELENGTH DON'T PENETRATE AS FAR. WE ARE THEN LOOKING AT THE TOP OF THE CLOUD LAYER. WHY IS IT SO HOT? HOW CAN THE ATMOSPHERE BE ADIABATICALLY STABLE WITH A TEMPERATURE GRADIENT OF NEARLY 500°K?

A THEORY SUGGESTS THE ATMOSPHERE IS VERY DEEP, PERHAPS A HUNDRED TIMES THE THICKNESS OF OURS. IF IT IS THICK ENOUGH, THEN IT COULD BE ADIABATICALLY STABLE AS YOU GO HIGHER UP. SOME ~~EXCE~~ SUSPECT SOME KIND OF SURFACE NOISE IS ADDING TO THE RADIATION AND THUS THE SURFACE IS ACTUALLY COOLER THAN WE THINK. MAYBE THERE ARE A LOT OF RADIO AND T.V. STATIONS TRANSMITTING ALL THIS ENERGY! BUT MOST PEOPLE NOW ACCEPT THIS HIGH TEMPERATURE AND SEEK TO EXPLAIN IT.

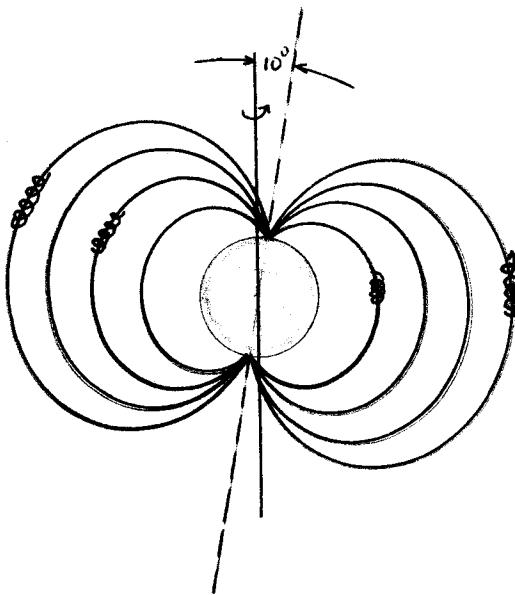
AS A THERMAL SOURCE, IT EMITS A PRETTY NICE BLACK BODY SPECTRUM WITH THE INTENSITY GOING AS $\propto T^2/d^2$. FORTUNATELY THE SURFACE DOES NOT COOL OFF DURING ITS NIGHT (123 DAYS) OR ELSE WE WOULD BE IN REAL TROUBLE TRYING TO EXPLAIN THE MECHANISM RESPONSIBLE. AS IT STANDS NOW, THE THICK ATMOSPHERE MODEL SEEMS TO BE A PRETTY GOOD ONE.

JUPITER

NOW HERE COMES A VERY INTERESTING STORY AND THERE ARE MANY THINGS TO SAY. THE RADIO EMISSIONS CAN BE SEPARATED INTO TWO REGIONS: THE DECA AND DECIMETER REGIONS. THE DECAMETER REGION IS BELOW MC/SEC AND THE DECIMETER IS ABOVE 200 MC/SEC.

THE DECAMETER REGION IS CHARACTERIZED BY SPORADIC EMISSIONS, ITS ELLIPTICALLY (NOT LINEARLY OR CIRCULARLY) POLARIZED, IS EXTREMELY INTENSE (AT TIMES) AND THE SOURCE IS SMALL COMPARED TO THE DIAMETER OF THE PLANET (ABOUT 5-15 SEC OF ARC WHILE THE PLANET SUBTENDS A MINUTE). THE DECIMETER REGION IS CHARACTERIZED BY A STEADY EMISSION, LINEARLY POLARIZED, AND COVERS AN AREA SEVERAL TIMES THE PLANET'S SIZE.

IT HAS BEEN SUGGESTED THAT JUPITER HAS AN INTRINSIC MAGNETIC POLE WHICH IS NOT AXIAL WITH THE SPIN AXIS BUT LIES SOME 10° OFF TO ONE SIDE. AS A RESULT SYNCHROTRON RADIATION OCCURS IN THE OUTER REGIONS WHEN ELECTRONS GET TRAPPED IN THE FIELD LINES. THE AXIS OF THE FIELD IS PERIODICALLY TIPPING BACK AND FORTH WITH A HIGHLY REGULAR PERIOD OF $9 \text{ hrs } 55 \text{ min } 29.37 \text{ sec}$. THERE IS A SLIGHT VARIATION IN THE INTENSITY OF THE FIELD BEING MORE INTENSE IN THE PLANE OF THE EQUATOR.



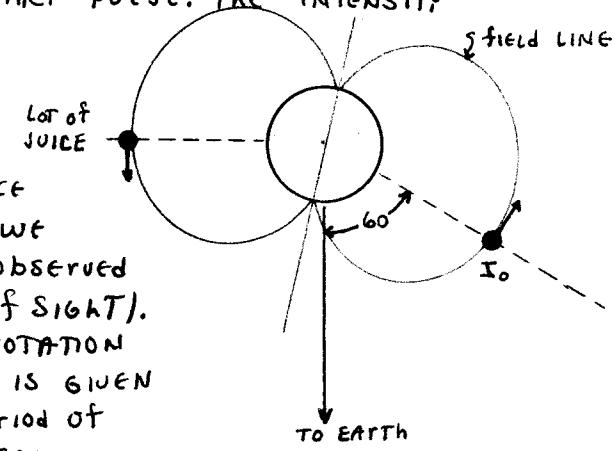
However, The USUAL ENERGY SPECTRUM FROM SYNCHROTRON GOES AS dE/E^S where S IS 2.7 or 2.8, FOR JUPITER $S=1$. SO IT IS NOT AS RAPIDLY DECAYING WHICH SUGGEST IT MIGHT BE OCCURRING IN SOME CHAOTIC CLOUDS.

AT ANY RATE, SYNCHROTRON RADIATION IS BELIEVED TO BE THE SOURCE OF THE DECI-METER EMISSION AND THEREFORE, PRETTY WELL UNDERSTOOD.

THE DECA-METER EMISSIONS.

NOW THIS IS CRAZY; ABSOLUTELY CRAZY. THERE ARE LOTS OF BURSTS AND NOISES AND OTHER THINGS GOING ON BUT MORE STIRRING IS A VERY PERIODIC MAXIMUM IN THE FLUCTUATIONS. IT SEEMS THAT THE MOON OF JUPITER Io HAS A MAJOR ROLE IN THE OBSERVING OF THIS MAXIMUM BECAUSE WHENEVER IT IS 90° TO THE LINE OF SIGHT WE GET A MAXIMUM. ALSO WHEN IT GOES BETWEEN US AND IS 60° PAST, WE GET ANOTHER PULSE. THE INTENSITY FADES AS IT GOES AWAY.

Io IS CLOSE ENOUGH TO JUPITER TO LIE IN ITS FIELD LINES AND DISRUPT THE FLOW OF ELECTRONS ALONG A PARTICULAR LINE. AT THE SURFACE THE MOTION IS TANGENTIAL TO THE SURFACE AND AS THE FIELD ROCKS BACK AND FORTH WE GET STRONG PULSES WHEN THAT PLANE IS OBSERVED NORMALLY (I.E., AT RIGHT ANGLE TO OUR LINE OF SIGHT). THE FLUCTUATION IS PECULIAR IN THAT THE ROTATION OF JUPITER AS MEASURED BY ITS RED SPOT IS GIVEN AS $9\text{ hr } 55\text{ min } 37\text{ sec}$ WHICH IS NOT THE PERIOD OF OSCILLATION OF THE FIELD AXIS BY ABOUT 7 SECs.

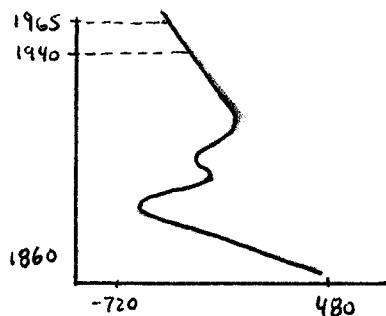


JUPITER'S ROTATION

LAST TIME WE WERE TALKING ABOUT JUPITER AND THE APPARENT UNEQUAL PERIODS OF ROTATION DUE TO THE RED SPOT AND MAGNETIC POLE OSCILLATIONS. RECALL A JUPITER DAY IS 9^{hr} 55^{min} 29^{sec} AND THE MAGNETIC POLE PERIOD IS 9^{hr} 55^{min} 37^{sec} WHICH IS DEFINITELY NOT EQUAL TO THE FORMER.

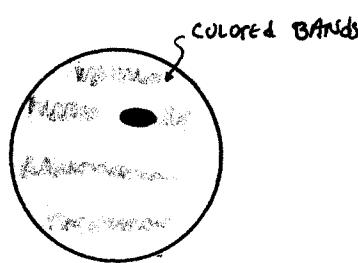
The Red Spot is an observed phenomena of Jupiter which is ALWAYS there; AT LEAST for the last hundred years or so of OBSERVATIONS. IT CHANGES ITS SHAPE AND COLOR A LITTLE BUT REMAINS fixed, more or less, at the SAME LATITUDE. If we pick ANY period whatever different from a Jupiter day and PLOT IT VERSUS THE RED SPOT RATE, IT WILL APPEAR TO BE CONSTANTLY DRIFTING TO ONE SIDE. The seemingly drifting motion is just a result of BEING off the correct ROTATIONAL RATE.

If we plot the MAGNETIC POLE PERIOD AS A FUNCTION of ROTATIONAL ANGLE VS TIME for the LAST HUNDRED YEARS IT LOOKS SOMETHING SCREWY LIKE



SO THERE ISN'T A PERPETUAL DRIFT AND OVER THE LAST 25 YEARS THE RATE HAS BEEN RATHER STEADY. If we PICKED A DIFFERENT PERIOD THE CURVE WOULD REMAIN MORE OR LESS THE SAME SHAPE BUT WOULD BE SHIFTED IN SLOPE A LITTLE. SEVERAL THEORIES HAVE BEEN PROPOSED TO ACCOUNT FOR THIS PHENOMENA. ONE PLACES THE BLAME ON ATMOSPHERIC DISTURBANCES OF SOME KIND; LIKE OVER A VOLCANO OR MOUNTAIN. OR MAYBE THE MOMENT OF INERTIA IS CHANGED SOMEHOW. WELL, THIS IS ANOTHER MYSTERY WE DON'T UNDERSTAND.

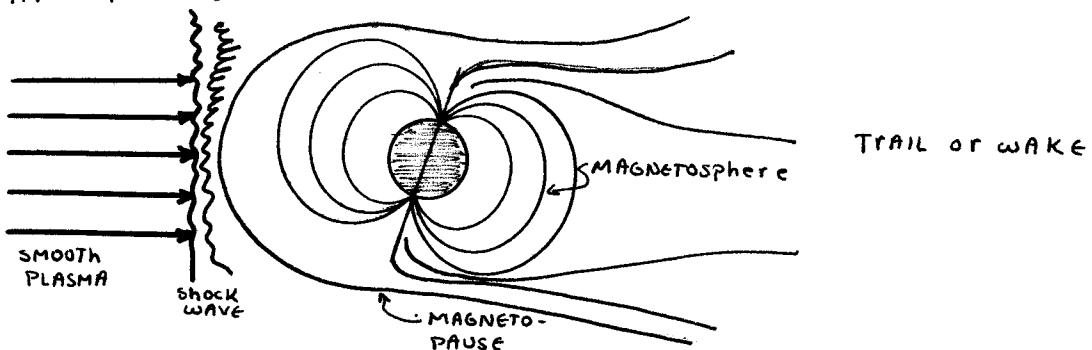
AND THAT'S ALL I CARE TO SAY ABOUT THE PLANET. There IS A LOT I DIDN'T SAY BUT YOU CAN FIND THAT IN A GOOD ASTRONOMY BOOK.



The Sun AND The Solar Winds

NOW RETURNING TO THE SUN, I WANT TO TALK ABOUT THE SOLAR WINDS AND OTHER RELATED PHENOMENA ON EARTH DUE TO IT. IN TRYING TO GATHER MATERIAL, PRESENTABLE MATERIAL, I GOT INTO A REAL HORNET'S NEST. IT SEEMS NO ONE KNOWS A DAMN THING & FOR REAL. THERE ARE THEORIES AND SOME EXPERIMENTS WHICH SORT OF CONFIRM THE THEORIES - IF YOU KNOW THE THEORIES.

THE GENERAL IDEA IS THAT THE SOLAR WINDS COME FROM THE CORONA AS AN OUTSTREAM OF PLASMA AND THE EARTH HAPPENS TO BE IN THE WAY. IT DOES NOT BLOW PERFECTLY REGULARLY. THE EARTH'S MAGNETIC FIELD IS RATHER STRONG AND DEFLECTS THE PARTICLES. Thus IT SERVES AS A SHIELD AND THIS IS CALLED THE MAGNETOSPHERE. THE OUTER MOST SHELL IS CALLED THE MAGNETOPAUSE. BETWEEN THE MAGNETO PAUSE AND THE SMOOTH STREAM OF INCOMING PLASMA THERE IS A RESULTING SHOCK WAVES AND CONFUSED MESS. DURING SOLAR BURSTS THE SHOCK WAVE PUSHES HARDER ON THE MAGNETO PAUSE WHICH IN TURN CAUSES A DISTORTION OF THE EARTH'S MAGNETIC FIELD. THE RESULT IS MAGNETIC STORMS OR DISTURBANCES IN OUR ATMOSPHERE.



DURING VIOLENT SOLAR FLARES OR SOLAR PROMINENCES A PRONOUNCED COSMIC RAY INCREASE IS RECORDED AT EARTH. WHEN WE SEE A FLARE LASTING A COUPLE OF MINUTES, A FEW MINUTES LATER WE GET AN INCREASE IN COSMIC RAY FLUX. THIS IS BECAUSE THE PARTICLES ARE NOT COMPLETELY RELATIVISTIC. AFTER THE INITIAL RISE OR SUDDEN COMMENCEMENT, THERE FOLLOWS IN A DAY OR TWO ANOTHER FLUX INCREASE DUE TO THE PLASMA SHOCK INDUCING A MAGNETIC STORM. OUR RECENT SATELLITES HAVE CONFIRMED THAT THIS PLASMA EXISTS UNIFORMLY AND IS CONTINUOUS.

THE VELOCITY OF THE INCOMING PARTICLES AT THE EARTH IS ABOUT $400 \frac{\text{km}}{\text{sec}}$ BUT VARIES OVER THE RANGE $320-700 \frac{\text{km}}{\text{sec}}$. THE FLUX DENSITY IS GIVEN AS $4 \times 10^8 \frac{\text{PROTONS}}{\text{cm}^2 \cdot \text{sec}}$ OR ABOUT $10 \frac{\text{PROTONS}}{\text{cc}}$.

THE TEMPERATURE OF THE GAS IS REALLY NOT KNOWN BUT NOMINALLY GIVEN AS $10^5-10^{5.5} \text{ }^\circ\text{K}$.

The MAGNETIC FIELD OUTSIDE THE EARTH'S MAGNETOSPHERE IS GIVEN AS
 $4 - 5 \gamma$
 where $\gamma = 10^{-5}$ GAUSS.

THE FLOW VELOCITY IS MUCH HIGHER THAN THE THERMAL VELOCITY AND
 THE EXTRA ENERGY COMES FROM THE MAGNETIC FIELD. THE THERMAL
 ENERGY AND MAGNETIC FIELD ENERGY ARE ABOUT THE SAME
 $NKT \sim \frac{B^2}{8\pi}$

THE HIGHER THE VELOCITIES THE BIGGER THE MAGNETIC FIELD VIOLENCE (OR
 NERVOUSNESS K_p) ON EARTH.

THE WINDS ARE BASICALLY GROUPED INTO THREE TYPES:

- (1) GENERAL OUTFLOW (FROM CORONA)
- (2) JETS FROM STUFF ON THE SURFACE
- (3) SHARP PULSE FLARE

Theory of General Emission

WE CONSIDER THE CORONA EXPANDING HYDRODYNAMICALLY INTO THE VACUUM AROUND IT. THE CURIOUS THING IS HOW THE CORONA MAINTAINS A TEMPERATURE OF 10^6 °K WHILE THE LOWER SURFACE TEMPERATURE IS AROUND 4000 °K (THE CHROMOSPHERE). THE CORONA IS EMITTING AND KICKING OUT PARTICLES AT SUPERSONIC VELOCITIES. Thus WHAT IS LEFT BEHIND IS NOT AFFECTED BY WHAT LEAVES. THE MEAN FREE PATH OF THESE PARTICLES IS LARGER THAN SEVERAL SUN RADII.

IT THEN SEEMS CONTRADICTORY TO USE THE IDEA OF AN INTERACTING GAS WHEN, IN FACT, WE ARE DESCRIBING A COLLISIONLESS SYSTEM. HOWEVER, EVEN IF THE PARTICLES DO NOT INTERACT PHYSICALLY, THEY CAN BE COUPLED ELECTRO-MAGNETICALLY AND, THEREFORE, NOT COMPLETELY INDEPENDENT OF ONE ANOTHER. BUT THIS IS NOT WORKED OUT SATISFACTORILY. SO I SORTA OF AVOID THE QUESTION OF HOW WE CAN TREAT THIS GAS HYDRODYNAMICALLY.

AT ANY RATE THE GAS CAN BE DESCRIBED BY A PRESSURE, TEMPERATURE, AND VELOCITY. TO WRITE THE EQUATION OF THE GAS WE HAVE

$$\text{CONSERVATION OF MATTER} \quad \rho \pi r^2 4\pi = Q \quad \text{CONTINUOUS FLOW}$$

TO BALANCE THE ACCELERATING FORCES

$$-\frac{dP}{dr} - \frac{GM\rho p}{R^2} = \frac{d\mathbf{v}}{dT} = \frac{\partial \mathbf{v}}{\partial t} - (\mathbf{v} \cdot \nabla) \mathbf{v}$$

$$-\frac{dP}{dr} - \frac{GM\rho p}{R^2} = \rho \frac{d\mathbf{v}}{dt} \cdot \mathbf{v}$$

where $\frac{\partial \mathbf{v}}{\partial t} = \text{RATE OF CHANGE OF ONE POINT} = 0$

CONSIDERING ON STEADY FLOW

$$-\frac{dp}{\rho} - \frac{GM}{R^2} dR = v dv$$

WE CAN INTRODUCE THE SPEED OF SOUND SINCE IT IS RELATED TO THE RATE OF CHANGE OF PRESSURE WITH DENSITY

$$C_s^2 = \frac{dp}{d\rho}$$

OR,

$$dp = C_s^2 d\rho$$

Then

$$-C_s^2 \frac{dp}{\rho} - \frac{GM}{R^2} dR = v dv$$

RECALLING

$$4\pi \rho v R^2 = Q \quad \text{WE HAVE}$$

$$\frac{4\pi v R^2 dp + 4\pi \rho R^2 dv + 4\pi \rho v R dR}{4\pi \rho v R^2} = \frac{0}{Q}$$

$$\frac{dp}{\rho} + \frac{dv}{v} + 2 \frac{dR}{R} = 0$$

$$\frac{dp}{d\rho} = - \frac{dv}{v} - 2 \frac{dR}{R}$$

ON SUBSTITUTING

$$-C_s^2 \left[-\frac{dv}{v} - 2 \frac{dR}{R} \right] - \frac{GM}{R^2} dR = v dv$$

$$\cancel{\frac{dp}{d\rho}} \left[\frac{C_s^2 dv}{v} - v dv \right] = -2 C_s^2 \frac{dR}{R} + \frac{GM}{R^2} dR$$

$$\left[\frac{v^2}{C_s^2} - 1 \right] \frac{dv}{v} = \left(2 - \frac{GM}{C_s^2 R} \right) \frac{dR}{R}$$

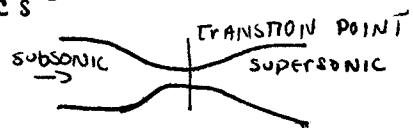
WE SEE IF $v < C_s$ (I.E., THE SUBSONIC RANGE), THE PARTICLE CAN INCREASE ITS VELOCITY AS THE RADIUS INCREASES UNTIL IT BECOMES SUPERSONIC. THE CRITICAL POINT WHERE IT BECOMES SUPERSONIC IS

$$2 - \frac{GM}{R_{\text{crit}} C_s^2} \geq 1$$

$$\text{or} \quad 2 = \frac{GM}{R_{\text{crit}} C_s^2} \quad \text{or} \quad \frac{R_{\text{crit}}}{R_{\text{sun}}} = \frac{GM}{R_{\text{sun}} 2 C_s^2}$$

$$\text{SINCE } \frac{1}{2} v_{\text{escape}}^2 = \frac{GM}{R_e} \quad \frac{R_{\text{crit}}}{R_{\text{sun}}} = \frac{1}{4} \frac{(v_c)^2}{C_s^2}$$

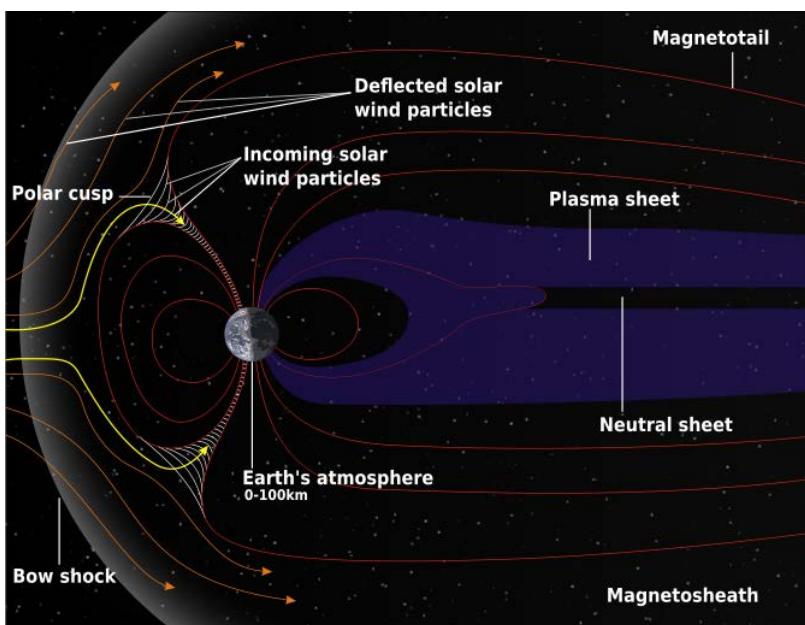
The Phenomena is like a supersonic nozzle



WE HAVE ONE IMPORTANT THING TO BRING IN AND THAT IS THE CONSERVATION OF ENERGY. IN THE CORONA THE THERMAL ENERGY CORRESPONDS TO ONLY 0.1 KILOVOLTS WHILE ON THE EARTH WE GET ABOUT 3.1 KILOVOLTS. SO WHY THE DIFFERENCE? ACTUALLY IN THE CORONA IT IS 0.2 KV CONSIDERING BOTH ELECTRONS AND PROTONS.

IF WE USE A $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$ WE CAN'T GET MORE THAN 0.3 KV IN THE CORONA. THE EARTH SEES PARTICLES GOING AT $450 \frac{\text{km}}{\text{sec}}$ OR AT 1.1 KV. BUT WE FORGOT ABOUT THE GRAVITATIONAL ENERGY NEEDED TO LIFT THE PROTONS AWAY FROM THE SUN. THIS CORRESPONDS TO A GRAVITATIONAL ENERGY OF 2 KV OR TO GIVE A TOTAL OF 3.1 KV.

BUT THE PLASMA IS CONTINUALLY BEING HEATED LIKE A NOZZLE WITH AN AFTERBURNER. THERE MUST BE SOME MECHANISM FOR GENERATING NOISE OUT BEYOND THE CORONA BECAUSE THERE IS A HELLUVA LOT OF ENERGY OUT IN THAT SUPERSONIC RANGE. THE EMISSION FROM THE SUN OBVIOUSLY MUST NOT BE PORT AND REGULAR.



Theory of The Corona

LAST TIME WE WERE DISCUSSING THE MECHANISM OF CORONAL EFFECTS ON COSMIC RAYS. ONE MAJOR PROBLEM WAS HOW TO ACCOUNT FOR THE LARGE AMOUNT OF THERMAL ENERGY PER CUBIC FOOT AT DISTANCES OF 3 SUN RADII. AT THAT POINT THE SEPARATION BETWEEN SUB AND SUPERSONIC VELOCITIES OCCURS. BEYOND THIS POINT AND OUT AS FAR AS $20 R_\odot$ THERE IS PROPOSED A CONSTANT THERMAL BLANKET OF NEARLY 1.5×10^6 °K. WHY IS THIS REGION SO ISOTHERMAL?

There MUST BE A BIG THERMAL CONDUCTIVITY IN THIS REGION such that there is a very rapid heat flux from the surface on out to $20 R_\odot$. Now to understand why the conductivity could be so high we must analyze the plasma streaming out. It consists of high energy protons and electrons. So we have a two component gas in which the protons are moving about 160 Km/sec through the subsonic zone. But the electrons are 2000 times lighter and would thus be accelerated nearly 40 ($\sim \sqrt{2000}$) times faster than the speed of sound. So they ~~ever~~ literally leave the protons standing still and travel ahead of the main plasma stream. They then serve as a communication carrying the hot news from the bottom of the corona to the far out regions. So this isothermal proposition is no longer impossible to believe. There this is the unexplained mechanism of heat generation; whether or not it is some whip like phenomena resulting from noise at the lower level is not well established.

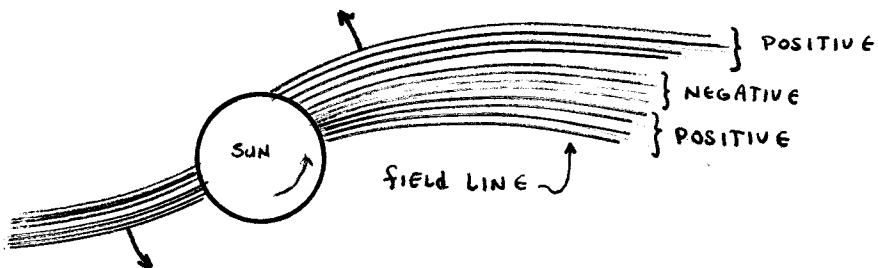
To determine the equilibrium condition in the corona we must study the degree of ionization or more explicitly the probability of recombination. In the low density of the corona the rate of radiative capture of one electron by an ion is given by



But a more probable reaction is where the first electron excites the ion to a higher level permitting the capture of two electrons. This reaction is much more important for it gives a correction to the expected temperature in the corona.

The CHARACTER of The MAGNETIC FIELD IN The PLASMA

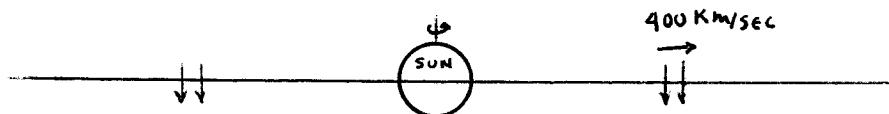
IT IS SAFE TO SAY THE SUN ROTATES AND AS IT GOES ROUND IT TURNS THE CORONA WITH IT; AS FAR OUT AS 10 R_S. THIS CORONAL JUNK ACTS AS A PERFECT CONDUCTOR AND THUS TRAPS THE MAGNETIC FIELD IN THE MATTER. THIS MEANS THE FIELD IS IN TURN ROTATING WITH THE WHOLE SYSTEM. AS THESE LINES ARE PULLED OUT, THEY FORM A RADIAL FIELD BUT ARE ALTERNATELY POSITIVE AND NEGATIVE - LIKE PULLED OUT TAFFY.



THE ALTERNATING FIELD LINE BUSINESS IS NOT PLEASING TO ME. THIS MEANS OUTGOING AND INCOMING LINES WITH OPPOSITE CHARGE ARE COMING VERY CLOSE TO EACH OTHER - WE'LL COME BACK TO THIS. WE REALIZE NOW THAT MATERIAL SQUIRT OUT FROM THE SURFACE FOLLOWS THESE LINES. IT LOOKS LIKE ONE OF THE LAWN SPRINKLED THAT SPINS AROUND AS THE WATER COMES OUT. THE WATER COMES OUT IN A NICE LOGARITHMIC SPIRAL

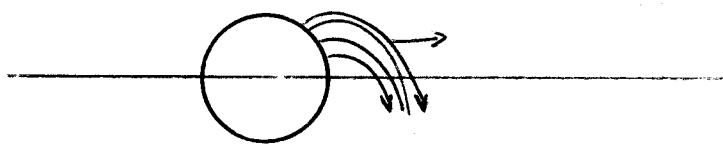
WE DO, IN FACT, SEE THIS GARDEN SPRINKLER EFFECT AS VERIFIED BY THE IMP AND MARINER SPACE PROBES. SO THIS IS A GOOD THEORY TO GUIDE US IN UNDERSTANDING THE DATA. BUT IT HAS NOT BEEN CHECKED TOO CAREFULLY AND WHEN THEY WERE TRYING TO MEASURE A FEW GAMMAS OF SOLAR FIELD, THEY GOT CONFUSED BY INDUCED FIELD WITHIN THE SPACECRAFT ITSELF. THEY HAD A REAL PROBLEM IS COMPENSATING FOR THESE ERRORS.

MARINER HAS SHOWN THAT THERE EXISTS A MAGNETIC FIELD COMPONENT IN A PLANE PARALLEL TO THE ECLIPTIC AND IT ALWAYS HAS THE SAME SIGNS. THAT IS, THE NORMAL COMPONENT WHICH SHOULD BE ZERO ISN'T; IT'S ABOUT 1 G IN STRENGTH WITH THE SAME SIGN,



THESE "DOWN" POINTING LINES MOVE OUTWARD IN A RADIAL FASHION WITH GOOD VELOCITIES AND REPRESENT A HEILUVA LOT OF FLUX - MUCH MORE THAN IS IN THE DIPOLE OF THE SUN. THIS IS A REAL DIRTY WORLD WE LIVE IN; EVERYBODIES CRAZY!

WE CERTAINLY NEED SOME SORT OF THEORY TO UNDERSTAND WHY 70-80% OF THE FIELD LINE ARE OBSERVED GOING DOWN. AS IT TURNS OUT, THOUGH, WE WOULD EXPECT MOST OF THE FIELD LINES TO BE DIRECTED DOWNWARDLY BECAUSE WHEN WE DISCUSSED THE FIELDS FROM THE SUN SPOTS WE SAW HOW ONE SPOT FORMED A LITTLE BEHIND AND NORTHLY FROM THE LEADING SPOT. AS THE SPOTS DIFFUSE AND SPREAD THEIR POLARIZATION THE FIELD OF THE SUN IS NOT A REAL DIPOLE. IN FACT, THE AVERAGING EFFECT OF THE NORTHERN AND SOUTHERN HEMISPHERES MIGHT STILL LEAVE A NET POSITIVE CHARGE OVER MORE THAN TO HALF THE SUN AS THESE LINES ARE SQUIRTED OUT THEY CROSS THE ELLIPTIC IN A DOWNWARD DIRECTION.

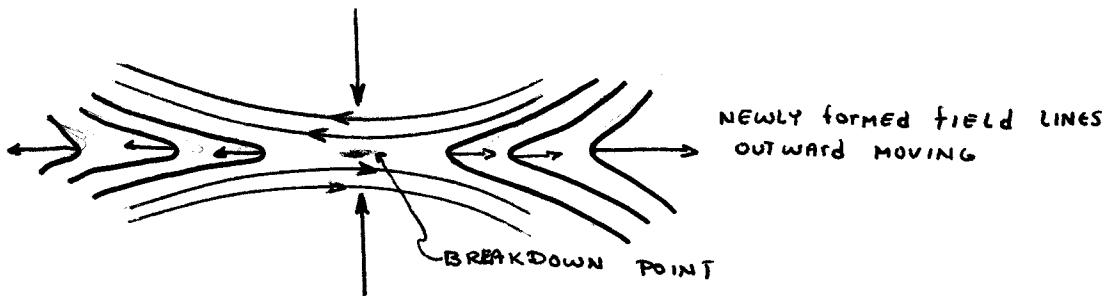


BY NO MEANS IS THIS THEORY COMPLETE JUST AS IT IS INCOMPLETE TO JUDGE AN INTEGRAL BY ONE POINT.

THEORY OF PETSCHER

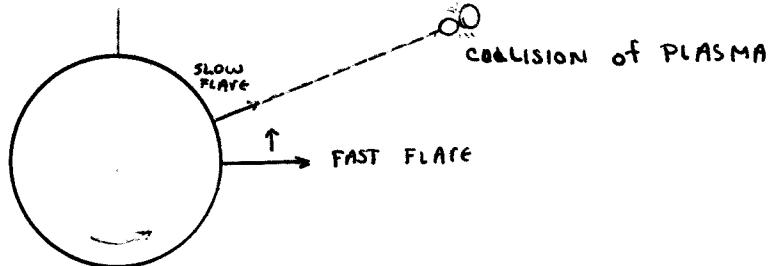
(THE FOLLOWING IS A POOR REPRODUCTION OF FEYNMAN'S TALK)

RETURNING TO THIS PROBLEM OF OPPOSING FIELD LINES, I WANT TO DISCUSS A VERY INTERESTING THEORY DEVELOPED BY A FELLOW NAMED PETSCHER. IF WE HAVE TWO OPPOSING FIELD LINES RUNNING CLOSE TO EACH OTHER, AN UNSTABLE SITUATION WILL RESULT IF THE LINES ARE SQUEEZED TOGETHER. PETSCHER PROPOSES THAT THE SQUEEZING REACHES A POINT OF BREAKDOWN WHERE A "HOLE" IS FORMED WHICH ACTS AS A FIELD SINK PULLING IN MORE AND MORE LINES GENERATING TREMENDOUS JOULE HEATING AT THE POINT OF BREAKDOWN.



THE PETSCHER PROCESS IS ACTUALLY AN ANNIHILATION-RECONSTRUCTION ONE WHERE CLOSELY PACKED FIELD LINES BREAK OFF AND FORM NEW LINES. I THINK HE HAS JUST DISCOVERED SOMETHING VERY IMPORTANT. IT COULD VERY WELL EXPLAIN SOLAR FLARES AND EFFECT SOLAR WINDS. BUT MORE THAN THAT IT COULD CONTAIN SOMETHING IMPORTANT TO OUR UNDERSTANDING OF MAGNETIC FIELDS IN GENERAL.

There IS A LITTLE MORE I WANTED TO SAY About SOLAR WINDS.
If we look at SOLAR FLARE ACTIVITIES we realize that they throw
out JUNK AT DIFFERENT RATES. SOME KICK HARDER THAN OTHERS.
Owing to the SOLAR ROTATION, IT IS HIGH PROBABILITY THAT TWO
CLOSERLY OCCURRING SOLAR FLARES COULD OVERLAP AND GENERATE A
SHOCK WAVE.



WHEN THE SECOND LUMP OF PLASMA OVERTAKES THE FIRST, THERE IS A
DEFINITE SHOCK RESULTING WHICH WE HAVE NOT EXPLAINED TO ANY LENGTH.

THAT, MORE OR LESS, WRAPS THINGS UP FOR THIS YEAR'S LECTURE.
FRANKLY, I'M GLAD TO SEE THE END COME; THIS MATERIAL HAS COMPLETELY
SWAMPED ME AND I CAN'T GIVE COMPREHENSIBLE AND INTERESTING
TALKS ANYMORE