Temple Run

Problem Explanation

We are given N cities and M roads between cities u and v where *1* <= *u,v* <= *N*. Due to Corona the City was closed and because of that a lot of animals damaged all the roads and the temples - we need to reconstruct either the road or temple both of which have an associated cost. The cost associated with the road is denoted as ***b*** and the same for a temple in the city is denoted as ***a***. Since the people living in cities are religious we have to find the minimum cost required such that each city either has a temple or there exists a path to some city that has a temple in it.

Prerequisites

Connected components in a Graph

Resources

Refresher on theory and algorithms on Connected components in Graphs
http://web.stanford.edu/class/archive/cs/cs161/cs161.1176/Lectures/CS161Lecture10.pdf

Approach

There are 2 important observations :

1. If the cost of construction of a temple is less than or equal to the cost of repairing the road i.e., ***a<=b*** then simply construct a temple in every city and hence the cost will be **a X N**, where N is the total number of cities. This is because if we connect a city with another city which has a temple then the cost incurred is ***b*** which is more than the cost of constructing the temple.

2. If the cost of construction of a temple is greater than the cost of repairing the road i.e., ***a>b*** then for each connected component, find the total number of cities in that connected component.

We will then construct one temple for each connected component and connect the other cities in a tree like structure.(i.e. If there are N nodes then the number of edges required to connect all of them is N-1).

i.e. cost incurred will be $\sum$ (a + b X (c_i - 1)) where 1<=i<=Total Number of Connected Components and c_i is the total number of cities in i^{th} component.

Time Complexity: O (N)
Space Complexity: O(N)

Where *N* is the total number of cities.