

Summary

The time complexity of the solution is equal to the number of balanced parentheses combinations possible, which is equivalent to the n th Catalan Number.

Hence the complexity is $O((1 / (n + 1)) * (2n C n))$ which is approximated to

$O((4^n) / \sqrt{n})$.

Read more about Catalan Numbers and its applications in combinatorial problems at

<https://cp-algorithms.com/combinatorics/catalan-numbers.html>

The space complexity is the same as the time complexity.

Problem Pattern

1. Given the number of pairs of parentheses, we have to generate all possible combinations of well-formed parentheses.
2. For eg, if $n = 2$, out output should be
 - a. $()()$
 - b. $(())$
3. We can create a function which generates all the $2^{(2 * n)}$ possibilities, and checks whether each possibility is correct or not.
 - The complexity of such a solution will be $O(2^{(2 * n)})$ for generation, and then checking each possibility shall take an extra $O(n)$, hence the total complexity will be $O((2^{(2 * n)}) * n)$.
 - Can we do better than this?

Problem Approach

1. We can create a backtracking algorithm which generates all of the correct possibilities, and never backtracks for unbalanced possibilities.
2. Let's say the function is `backtrack(S, left, right)`, in which `S` is the currently generated string, `left` is the number of opening parenthesis we have already added, and `right` is the same for the closing parenthesis.
 - Now we shall only iterate for cases where the bracketing will always be balanced.
 - The way we do this is by checking the following conditions:
 - a. if the current number of open parenthesis is less than n , then we can recurse for `S + '('`.
 - b. if the current number of closing parenthesis is less than the number of opening parenthesis, then we can recurse for `S + ')'`.

Problem Pseudocode

```
generateParenthesis(N):
```

```
    ans = []
```

```
    backtrack('', 0, 0, N, ans)
```

```
    return ans
```

```
backtrack(S, left, right, N, ans):
```

```
    if len(S) == 2 * N:
```

```
        ans.append(S)
```

```
        return
```

```
    if left < N:
```

```
        backtrack(S+'(', left+1, right, N, ans)
```

```
    if right < left:
```

```
        backtrack(S+')', left, right+1, N, ans)
```