Quantum Generative Modelling with Conservation Law based Pretraining

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Motivation

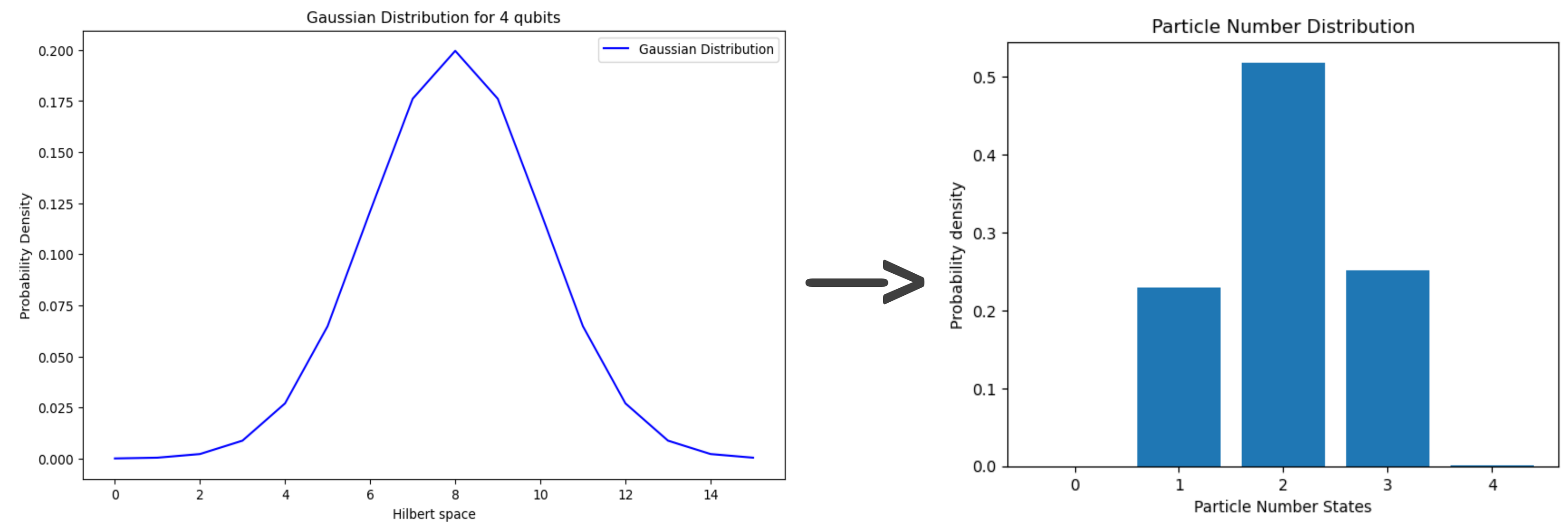
This study explores the integration of conservation laws into quantum generative models specifically by applying particle number conserving system Hamiltonians and/or particle number distribution-based pretraining within the Quantum Circuit Born Machine (QCBM), aiming to model target distributions with reduced effort. Our analysis of such a QCBM focuses on its impact on model convergence and accuracy, using metrics such as Kullback-Leibler (KL) divergence, and Maximum Mean Discrepency (MMD) loss function.

Quantum Circuit Born Machine (QCBM)

Our custom designed QCBM circuits encompasses Ising Hamiltonian-based entangling gates (IsingXY, IsingZZ) combined with rotational gates (RZ) to encode physical constraints effectively [1,2].

Particle Number Distribution

The Particle number distribution corresponding to a given probability distribution of a system refers to the number of particles in specific quantum states.



Methodology

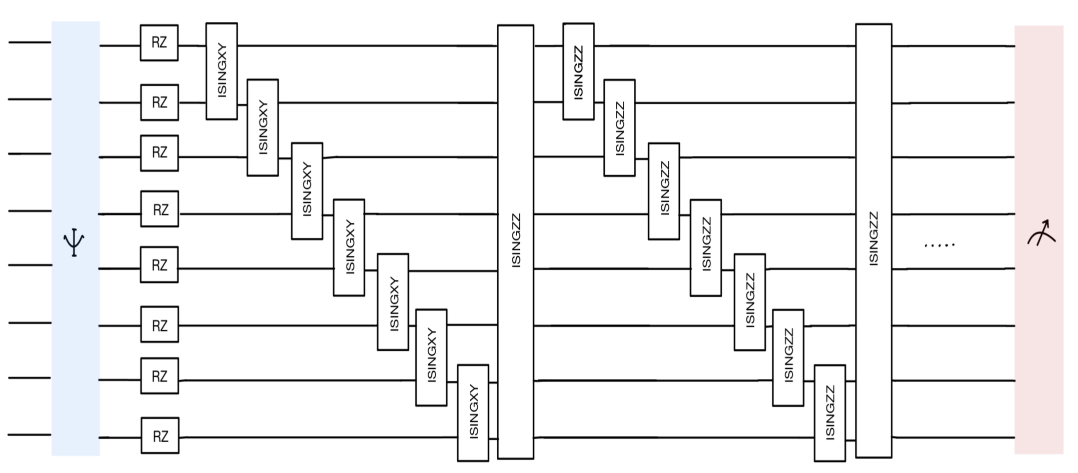
Our approach focuses on leveraging conservation laws for quantum generative modeling using QCBMs. We structured our methodology as follows:

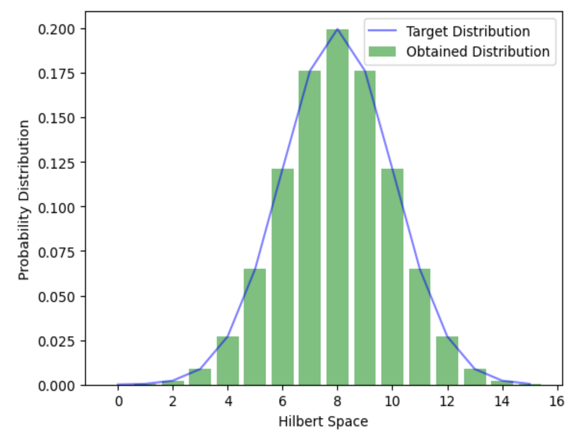
Task 1: Standard QCBM Training

Purpose: To check if the model converges to map the given target distribution.

Task 2: Pretrained QCBM

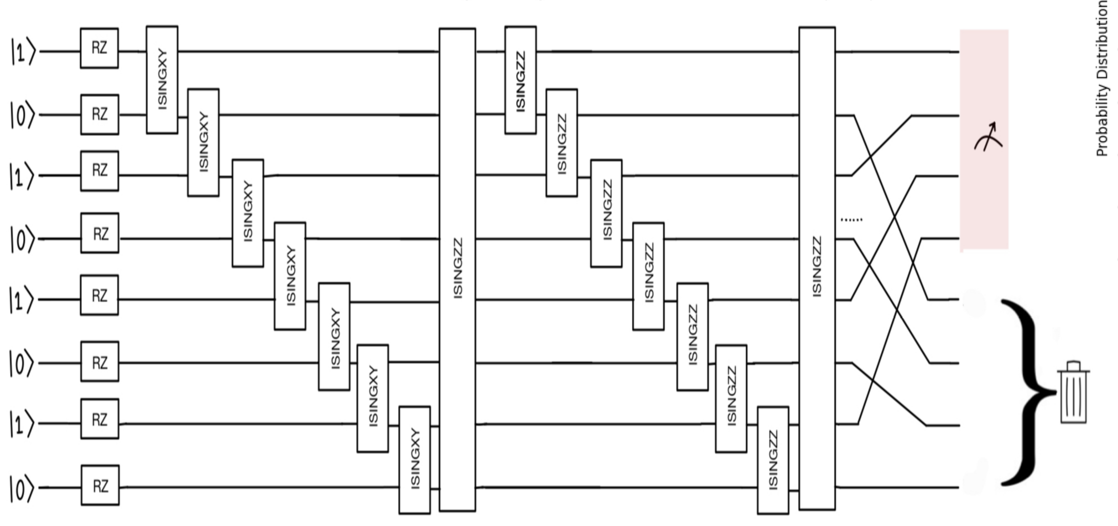
Hypothesis: Pretraining should accelerate the convergence of the model.

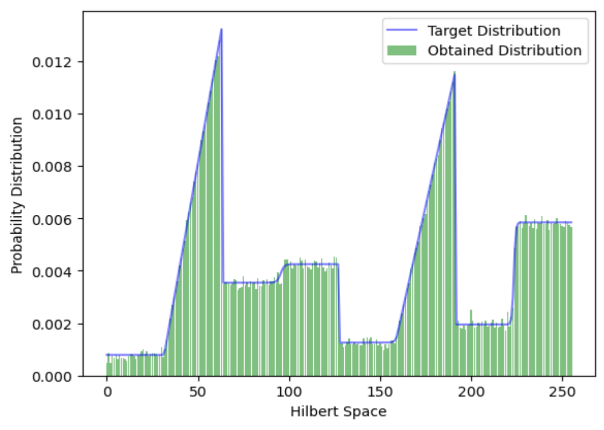




Task 3: Ancilla - Assisted QCBM

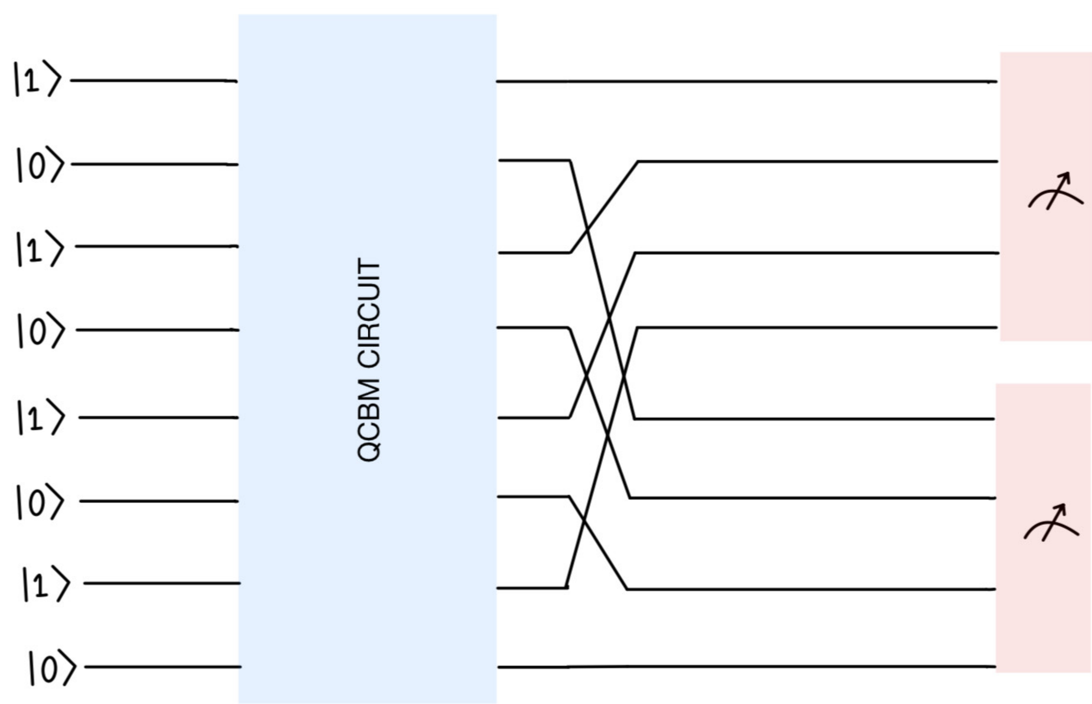
Hypothesis: Ancilla-based strategies should improve the convergence speed and model more complex distribution involving higher number of qubits.

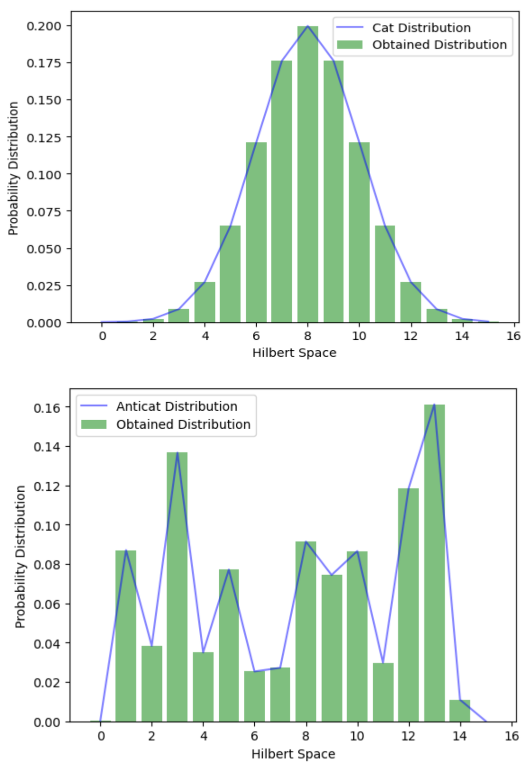




Task 4: Encoder (QCBM) - Decoder (Generative) Circuit

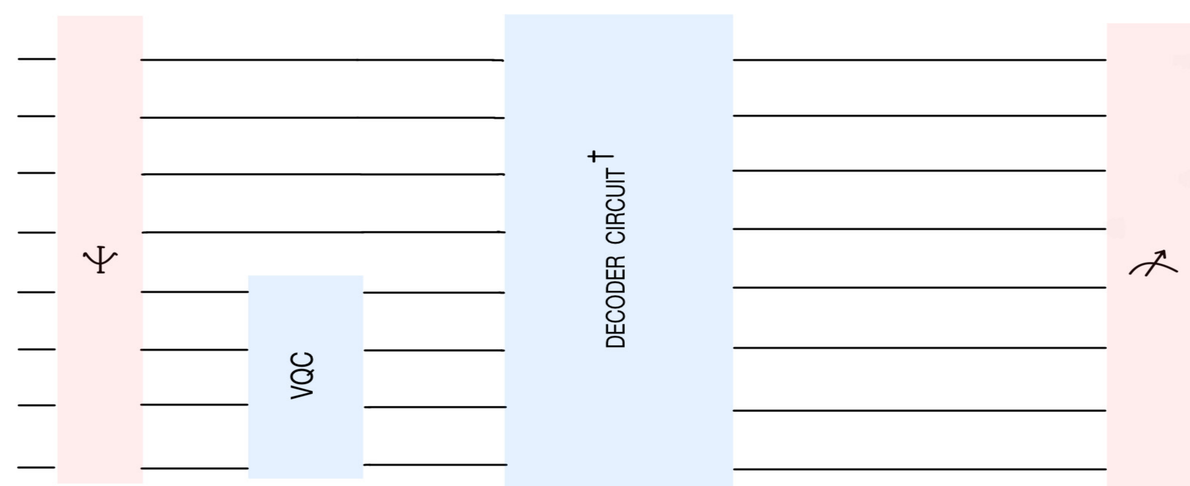
Purpose: To check the convergence of the Decoder circuit.





Task 5: Hybrid VQC + QCBM

Hypothesis: The model should converge to superposition of all half filled particle states.



Results

Model	KL Divergence	Observations
Standard QCBM	∞	Model doesn't converge
Pretrained QCBM	~1e-3	Model successfully converges
Ancilla-based QCBM (Ancillas + pretraining)	~1e-3	Successfully models more complex distributions
Ancilla-based QCBM (Ancillas with local entanglement)	~1e-3 (2x faster)	Models complex distributions faster
Decoder QCBM	~1e-3	Model successfully converges
Hybrid VQC + QCBM	~1.0	Struggles with barren plateau

Key Takeaways

- ✅ Pretraining improves convergence, but fully entangled QCBM (RX+RY+RZ+CNOT) performs similarly.
- ✅ Ancilla-based pretraining enhances convergence speed significantly.
- ✅ Successfully trained QCBM model for Cat–Anticat distribution.
- ❌ Hybrid VQC+QCBM struggled with barren plateaus, limiting its usefulness [3].

Future Work

1. Investigate the ability of a hybrid VQC + QCBM model to successfully capture the Anticat Distribution for a given Cat Distribution.
2. Train the model on various datasets to ensure model's generalization.

[1] : M. Benedetti et al Quantum Sci. Technol. 4 043001 (2019)
[2] : J. Liu, L. Wang Phys. Rev. A 98, 062324 (2018)
[3] : K. Sharma, M. Cerezo, et al. Phys. Rev. Letters 128.18 (2022): 180505