Summary of the Quantum Circuit Born Machine (QCBM) Project

Task 1:

To analyze the performance of the QCBM adhering to conservation laws particularly, particle number distribution.

Quantum Circuit Design:
QCBM circuit with parametric quantum gates corresponding to the various Ising Hamiltonians to include combinations of Rotation Gates (RZ) + Ising Entangling Gates (IsingXY, IsingZZ).

Training Strategy:

- Pre-trained the model using the particle number distribution corresponding the given target distribution.
- Optimized QCBM using Maximum Mean Discrepancy (MMD) Loss Function and Kullback-Leibler (KL) Divergence.
- Used Adam Optimizer from for gradient-based training.

Observations:

- The QCBM circuit with parameterized RZ+IsingXY+IsingZZ gates, performs the best with KL divergence of 1e-3 for small number of qubits.

Task 2:

To further analyze the QCBM circuit chosen at the end of Task 1, to model more complex target distribution involving higher number of qubits.

Quantum Circuit Design:
Constructed QCBM circuit with parametric quantum gates corresponding to Rotation Gates (RZ) + Ising Entangling Gates (IsingXY, IsingZZ).

1. **Training Strategy 1 – Introducing Ancillas:**
    - Pre-trained the model using the particle number distribution corresponding the given target distribution.
    - Introduced ancillas in the circuit.
    - Optimized QCBM using Maximum Mean Discrepancy (MMD) Loss Function and Kullback-Leibler (KL) Divergence.
    - Used Adam Optimizer from for gradient-based training.

    Observations:
    - The QCBM circuit with the pre-training converges with KL divergence of 1e-3.

2. **Training Strategy 2 – Introducing Ancillas with Local Entanglement:**
    - Introduced ancillas locally after every input qubit in the circuit.
    - Rather than Pre-training, I initialized the circuit in a half filled state, where ancillas were all initialized to state |0> and input qubits to state |1>.
    - Optimized QCBM using Maximum Mean Discrepancy (MMD) Loss Function and Kullback-Leibler (KL) Divergence.
    - Used Adam Optimizer from for gradient-based training.

    Observations:
    - The QCBM circuit converges 2x faster than the previous strategy to the same KL divergence of 1e-3

Task 3:

Choosing the above found efficient QCBM Circuit, we further proceed to analyze if one can train a QCBM circuit to efficiently model a Cat – Anticat distribution, simultaneously.

1. **Training Strategy – QCBM Circuit:**
    - Trained the QCBM circuit according to Task 2, to model the Cat (Target) Distribution, by discarding the ancillary qubits at the end of the circuit.

2. **Training Strategy – Encoder Circuit:**
    - Using the trained model in the previous step, the Anti-Cat Distribution is obtained by discarding the input qubits.

3. **Training Strategy – Decoder Circuit:**
    - Analyzed the convergence of a QCBM Circuit equivalently constructed from Task 2, to efficiently model both Cat – Anticat distribution simultaneously at input qubits and ancillary qubits respectively.

    Observations:
    - The Decoder Quantum Circuit converges with KL divergence of 1e-3.

Task 4:

To analyze the convergence of a VQC+QCBM Circuit to model the half filled state.

1. **Training Strategy:**
    - Initialize Cat distribution on the top half of the circuit and particle number distribution of the Anticat in the bottom half of the circuit.
    - The Variational part of the circuit (VQC) is added on the bottom half of the circuit.
    - The adjoint of QCBM is further initialized on the entire circuit.

    Observations:
    - The VQC+QCBM circuit hits a barren plateau at the KL divergence of 1.0

Key Findings:

1. The pre-training of the Ising Hamiltonian based QCBM does not help the model’s convergence any further than the convergence of the fully entangled QCBM (RX+RY+RZ+CNOT).
2. The effective model of Cat – Anticat distribution, was obtained successfully through the Task 3 format.