1) Image Histogram (6 points)

L=8

1.1) A 5x5 digital image, $f(x,y)$ for intensity in the range [0, 7] is shown below. Calculate and draw a histogram of this image with bin size = 1

2	3	4	5	5
2	3	4	5	5
2	3	4	4	4
2	3	5	3	3
2	3	5	4	4

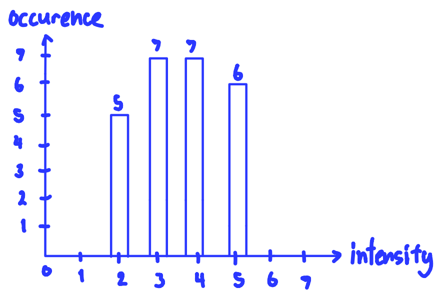
$f(x,y)$

h(2) = 5

h(3) = 7

h(4) = 7

h(5) = 6



1.2) Analyze the characteristics of the image in terms of contrast and brightness

low contrast, avg brightness

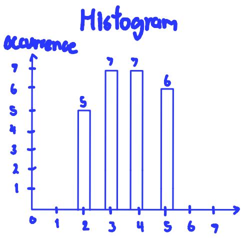
1.3) To perform histogram equalization on an image with intensity range [0, L-1], for each gray level i occurs n_i times, where N is the total number of pixels in the image, each gray level, i can be transformed to

$$\left(\frac{n_0 + n_1 + \cdots + n_i}{N}\right)(L-1)$$

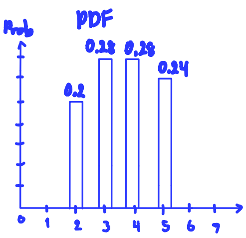
Show how to perform histogram equalization, draw a transformation function, and draw a new image after equalization

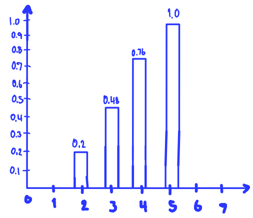
L-1 = 7, N = 25

lets make CDF for easier calculation



÷ N →





gray

0 = 0 : 0

1 = 0 : 0

2 = 0.2×7 : 1

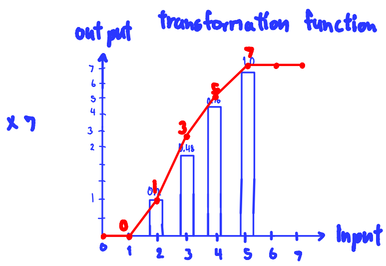
3 = 0.48×7 : 3

4 = 0.76×7 : 5

5 = 1×7 : 7

6 = 1×7 : 7

7 = 1×7 : 7



new image

1	3	5	7	7
1	3	5	7	7
1	3	5	5	5
1	3	7	3	3
1	3	7	5	5

2) (3 points) Spatial Filtering

2.1) Given a small 3x3 image, $f(x,y)$, show how to sharpen an image using 3x3 Laplacian filter (with either positive or negative center) and image boundary pixels are patched with zero. (In this case, intensity can be negative value; positive value can be more than 255)

3	3	3
3	2	2
3	2	1

$f(x,y)$

Laplacian filter : $\begin{matrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{matrix}$

add with original get sharpening : $\begin{matrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{matrix}$

0	0	0	0	0
0	3	3	3	0
0	3	2	2	0
0	3	2	1	0
0	0	0	0	0

0,0 = 0+0+15-3-3 = 9
0,1 = 0-3+15-3-2 = 7
0,2 = 0-3+15+0-2 = 10
1,0 = -3+0+15-2-3 = 7
1,1 = -3-3+10-2-2 = 0
1,2 = -3-2+10+0-1 = 4
2,0 = -3+0+15-2+0 = 10
2,1 = -2-3+10-1+0 = 4
2,2 = -2-2+5+0+0 = 1

The enhanced image

9	7	10
7	0	4
10	4	1

2.2) Describe the difference between the enhanced image and the original image.

Enhanced Images show higher diff of neighboring pixel

3) (3 points) A 3x3 image, $g(x,y)$ is the 4-bit image which is corrupted by salt and pepper noise.

15	15	2	3	3
15	15	2	3	3
1	1	2	0	0
1	1	2	3	3
1	1	2	3	3

$g(x,y)$

3.1) Show how to restore the image from the noise by using 3x3 median filtering with replicate border padding technique.

median = position 5

1 1 2 2 2 15 15 15 15
0 1 2 2 2 3 3 15 15
0 0 2 2 2 3 3 3 3
1 1 1 1 2 2 2 15 15
0 1 1 2 2 2 3 3 15
0 0 2 2 2 3 3 3 3
1 1 1 1 1 1 2 2 2
0 1 1 1 2 2 2 3 3
0 0 2 2 2 3 3 3 3

The restored image ($\hat{f}(x,y)$)

2	2	2
2	2	2
1	2	2

3.2) If you know the original image $f(x,y)$, you can calculate PSNR of the noisy image or the restored image using $f(x,y)$ as the reference image. Compare PSNR of the noisy image and the restored image.

Compare orignal PSNR must greater than noisy and restore
restore PSNR must greater than noisy

4. (6 points) Given an 8-bit 3x3 image, $f(x,y)$ below, $F(u,v)$ is the Fourier transform of $f(x,y)$.

4.1) Show how to calculate the Fourier transform at 1 and. 2 in the image below.

9	0	0
0	18	0
0	0	9

$f(x,y)$

0,0	0,+1	0,-1
1 a	-0.5-0.87 j b	-0.5+0.87 j c
-0.5-0.87 j d (+1,0)	-0.5+0.87 j e (+1,+1)	4 f (+1,-1)
2 g (-1,0)	4 h (-1,+1)	-0.5-0.87 j i (-1,-1)

$F(u,v)$

-1,-1 i	-1,0 g	h -1,+1
0,-1 c	a	b 0,+1
f +1,-1	d +1,0	e +1,+1

① $\frac{1}{9}\sum_{x=0}^{2}\sum_{y=0}^{2} f(x,y)\, \cancel{e^{-j2\pi(\frac{ux}{3}+\frac{vy}{3})}} = \frac{36}{9} = 4$

4.2) What is the Fourier Spectrum at 1 and 2 ?

② $\frac{1}{9}\sum_{x=0}^{2}\sum_{y=0}^{2} f(x,y)\, e^{-j2\pi(\frac{2x}{3})}$

$u=2, v=0$

4.3) Determine the shifted Fourier transform

$\sin -\theta = -\sin\theta$

$\cos -\theta = \cos\theta$

x,y

0,0 $9 \times 1 = 9$

1,1 $18 \times e^{-j\frac{4\pi}{3}} = 18(\cos(-\frac{4\pi}{3}) + j\sin(-\frac{4\pi}{3}))$

$= 18(\cos\frac{4\pi}{3} - j\sin\frac{4\pi}{3})$

$= 18(-\frac{1}{2} + \frac{\sqrt{3}}{2}j)$

2,2 $9 \times e^{-j\frac{8\pi}{3}} = 9(\cos\frac{8\pi}{3} - j\sin\frac{8\pi}{3})$

$= 9(-\frac{1}{2} - \frac{\sqrt{3}}{2}j)$

$= 9 - 13.5 + 4.5\sqrt{3}j = \frac{-4.5 + 4.5\sqrt{3}j}{9}$

$= -0.5 + 0.5\sqrt{3}j$

a b c / d e f / g h i → g h i / a b c / d e f → i g h / c a b / f d e

-0.5-0.87j	-0.5+0.87j	4
-0.5+0.87j	4	-0.5-0.87j
4	-0.5-0.87j	-0.5+0.87j

4.4) Design a 3x3 ideal high pass filter with cutoff frequency (D_0) = 1 pixel. Due to the small size of the filter, use the condition below.

$$H(u,v) = \begin{cases} 0 & if\ D(u,v) < D_0 \\ 1 & if\ D(u,v) \boxed{\geq} D_0 \end{cases}$$

1 1 = $\sqrt{2}$

0 1 = 1

0 0 = 0

where $H(u,v)$ is the ideal high-pass filter, $D(u,v)$ is distance of (u,v) from image center.

Ideal high-pass Filter

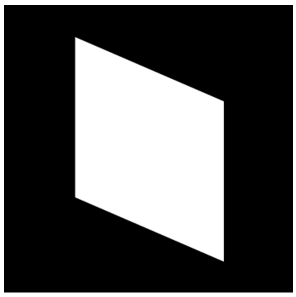
1	1	1
1	0	1
1	1	1

4.5) From Fourier transform of an image in 4.1), show how to compute the result of high-pass filtering in frequency domain. just multiply it, only center become zero

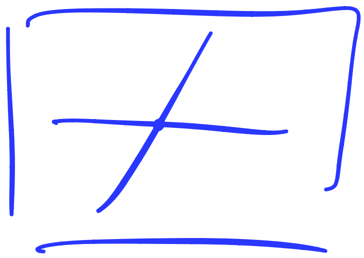
4.6) After passing the high pass filter, then calculate the inverse Fourier Transform of $\hat{F}(u,v)$ to get the filtered image in spatial domain, $\hat{f}(x,y)$. Analyze the characteristics of $\hat{f}(x,y)$. (Do not need to calculate iDFT of $\hat{F}(u,v)$) highlight all edge

5. (4 points) Transformation Patterns

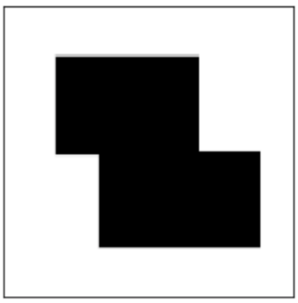
5.1) Given the image of parallelogram below, when apply Fourier transform on this image, draw the output spectrum (shifted version) that is likely to occur from the Fourier transform. (Ignore the color shade, only draw patterns that you think it should have values in Fourier spectrum)



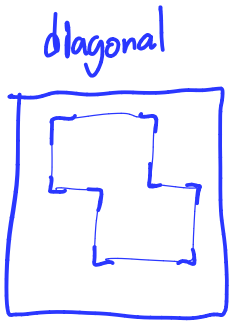
Parallelogram image

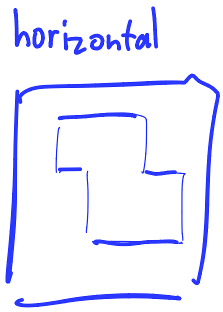


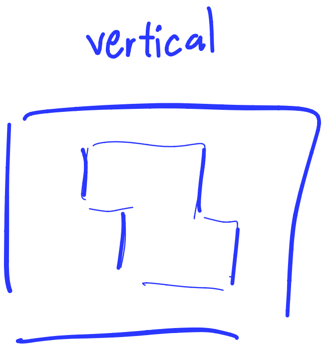
5.2) Given the image of "z" letter below, when apply wavelet transform (Haar) on this input image, draw the patterns of wavelet coefficients that are likely to occur in diagonal, horizontal and vertical directions. (Ignore the color shade, only draw patterns that you think it should have values in terms of wavelet coefficients)



"z" image





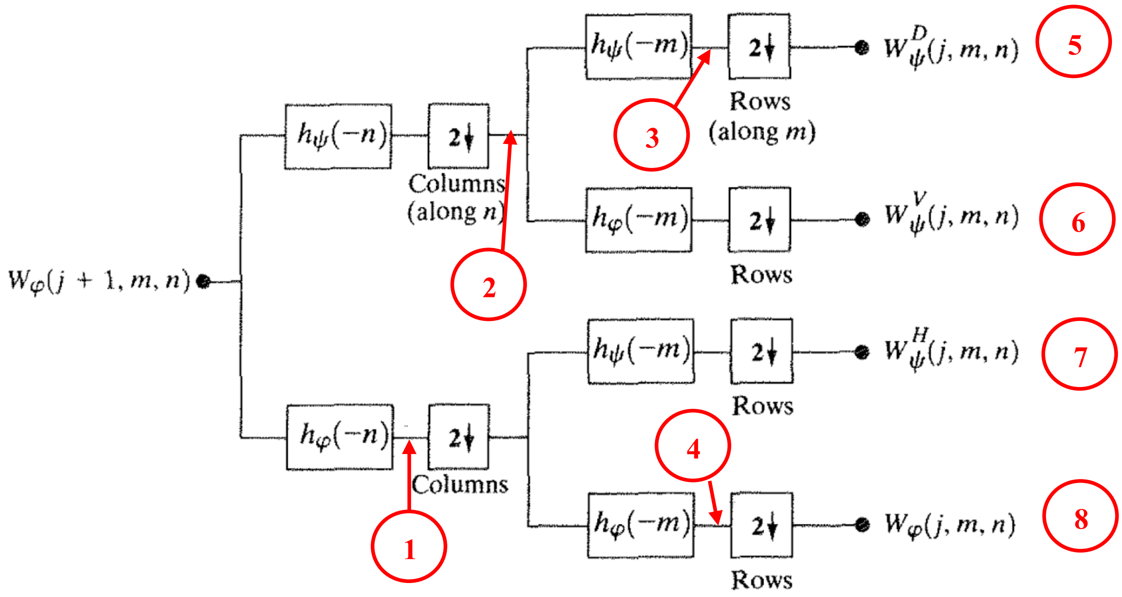


6. (6 points) Apply 2-D Wavelet transform using Haar Wavelet on a 2x2 image below. Given Haar scaling vector $h_\varphi(n) = \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ and Haar wavelet vector $h_\psi(n) = \left[\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right]$

3	-1
1	-3

A 2x2 image, $f(x,y)$

Considering the analysis filter bank below, answer the following questions.



Analysis filter bank

6.1) How many and what are the levels of image pyramid are used in this case?

2 level, cuz j=0 It only got one pixel, and j=1

6.2) Show how to calculate the results at 1 - 4 and four wavelet coefficients (approximation, horizontal, vertical and diagonal wavelet coefficients) 5 - 8

6.3) Any zero in wavelet coefficients? If so, explain why it is zero?

$\begin{bmatrix} A & H \\ V & D \end{bmatrix}$ A : 0 cuz avg = 0 > No DC component

D : different in diagonal is cancelling

3	-1
1	-3

A 2x2 image, $f(x, y)$

6.2

① $* \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] = \begin{bmatrix} \frac{3}{\sqrt{2}}, & \boxed{\frac{2}{\sqrt{2}}}, & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}, & \boxed{\frac{-2}{\sqrt{2}}}, & \frac{-3}{\sqrt{2}} \end{bmatrix}$ ⑦

เพราะมันเป็น -n

② $* \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] = \begin{matrix} -\frac{3}{\sqrt{2}}, & \boxed{\frac{4}{\sqrt{2}}}, & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}, & \boxed{\frac{4}{\sqrt{2}}}, & -\frac{3}{\sqrt{2}} \end{matrix} = \begin{bmatrix} \frac{4}{\sqrt{2}} \\ \frac{4}{\sqrt{2}} \end{bmatrix}$

convo ต้อง flip

down sampling

③ $* \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] = -\frac{4}{2}, 0, \frac{4}{2} = \left[-2, \boxed{0}, 2\right]$ ⑤

④ $* \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] = \frac{2}{2}, 0, \frac{-2}{2} = \left[1, \boxed{0}, -1\right]$ ⑧

⑥ $* \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] = \frac{4}{2}, \frac{8}{2}, \frac{4}{2} = 2, \boxed{4}, 2$ ⑥

⑨ $* \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] = -\frac{2}{2}, \frac{4}{2}, \frac{-2}{2} = -1, \boxed{2}, -1$ ⑨

7) A gray-level 8-bit image of 4 x 7 pixels is shown below

0	100	200	250	250	250	100
50	100	150	250	250	250	100
50	100	150	250	250	250	100
50	100	150	250	250	250	100

7.1) The image intensity levels are equally quantized from 8-bit to have 4 intensity levels (0, 1, 2 and 3). Write the quantized image below.

0	1	3	3	3	3	1
0	1	2	3	3	3	1
0	1	2	3	3	3	1
0	1	2	3	3	3	1

8bit = 0-255

$2^{8-2} = 2^6 = 64$

0-63 = 0

64-127 = 1

128-191 = 2

192-255 = 3

7.2) What is the entropy of the quantized image compared with the original image?

$\hookrightarrow H(z) = -\sum p(z_k) \log_2 p(z_k)$

Quantized

$p(0) = 4/28 = 1/7$

$p(1) = 8/28 = 2/7$

$p(2) = 3/28$

$p(3) = 13/28$

$H_Q = 0.401 + 0.516 + 0.345 + 0.514$

$= 1.776$

Original

$p(0) = 1/28$

$p(50) = 3/28$

100 = 8/28 = 2/7

150 = 3/28

200 = 1/28

250 = 12/28

$H_0 = 0.172 + 0.345 + 0.516$
$+ 0.345 + 0.172 + 0.524$
$= 2.094$

7.3) Represent these 4 levels with minimum fixed bits encoded in "**Code #1**" in the table below. And find the probability of each intensity level r_k at kth level.

Intensity level (r_k)	Code #1	Probability of r_k occurs $P(r_k)$
0	00	1/7
1	01	2/7
2	10	3/28
3	11	13/28

7.4) Implement Huffman Coding of the 4 intensity levels in 7.1) and encode in terms of **"Code #2"** in the table below.

Intensity level (r_k)	Code #2
0	010
1	00
2	011
3	1

$p(3) = 13/28$ 1

$p(1) = 8/28$ 00

$p(0) = 4/28$ 010

$p(2) = 3/28$ 011

$7/28$ 01

$15/28$ 0

$15/28$ 0

7.5) Determine the average number of bits (L_{avg}) required to code Code #1 and Code #2 using

$$L_{avg} = \sum_{k=0}^{L-1} l(r_k)P_k(r_k)$$

where $l(r_k)$ is bit-length of intensity level, r_k and $P_k(r_k)$ is the probability of r_k occurs in the image.

Code #1 $L_{avg} = 2$

Code #2 $L_{avg} = 4/28 \times 3 + 8/28 \times 2 + 3/28 \times 3 + 13/28 \times 1$

$= \frac{12}{28} + \frac{16}{28} + \frac{9}{28} + \frac{13}{28} = 1\frac{22}{28} = 1.785$ #

8) The resized images A and B shown below are the results of enlarging an image to 1000 times its original size using bilinear interpolation and nearest-neighbor interpolation. Which of the following images was resized using **bilinear interpolation**, and which was resized using **nearest-neighbor interpolation**? Why are the outputs different? Please explain.

Resized image A

blocky, jagged edge
: NN

Resized image B

smooth, blurry transition
: bilinear