

Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with $\mu = 45$ minutes and $\sigma = 8$ minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

- A. 0.3875
- B. 0.2676
- C. 0.5
- D. 0.6987

Ans :- we have a normal distribution with $\mu = 45$ and $\sigma = 8.0$

Let x be the amount of time it takes to complete the repair on a customer's car.

To finish in one hour you must have

$X \leq 50$ So the question is find

$$Pr(x > 50)$$

$$Pr(x > 50)$$

$$= Pr(x \leq 50)$$

$$Z = (x - \mu) / \sigma$$

$$= (x - 45) / 8.0$$

Thus the question can be answered by using the normal table find

$$Pr(x \leq 50) = Pr(z \leq (50 - 45) / 8.0)$$

$$= Pr(z \leq 0.625)$$

$$= 73.4$$

Probability that the service manager will not meet his service manager will not meet his demand will be

$$= 100 - 73.4$$

$$= 26.6\%$$

Or

$$0.2676 \text{ (ANS:- b)}$$

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $\mu = 38$ and Standard deviation $\sigma = 6$. For each statement below, please specify True/False. If false, briefly explain why.

- A. More employees at the processing center are older than 44 than between 38 and 44.
- B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans :- we have a normal distributions with 38 and 6

=let X be the number of employees so according to question.

(A) Ans :- probability of employees greater than age of 44

$$= \Pr(X > 44) = 1 - \Pr(X \leq 44)$$

$$Z = (x - \mu) / \sigma = (X - 38) / 6$$

Thus the question can be answered by using the normal table to find

$$\Pr(X \leq 44)$$

$$= \Pr(Z \leq (44 - 38) / 6)$$

$$= \Pr(Z \leq 1)$$

$$= 84.1345\%$$

Probability that the employees will be greater than age of 44

$$100 - 84.1345$$

$$= 15.8655\%$$

Therefore the statement that “more employees at the processing centre are older than 44 than between 38 and 44” is true.

B . ANS:- PROBABILITY OF EMPLOYEES less than age of 30 = $\Pr(X < 30)$

$$Z = (x - \mu) / \sigma = (30 - 38) / 6$$

Thus the questions can be answered by using the normal table to find

$$\Pr(X \leq 30)$$

$$= \Pr(Z \leq (30 - 38) / 6)$$

$$= \Pr(Z \leq -1.333)$$

$$= 9.12\%$$

So the number of employees with probability 0.0912 of them being under age 30

$$= 0.0912 \times 400 = 36.48$$

(or 36 employees)

Therefore the statement B of the question is also TRUE.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are iid normal random variables, then what is the difference between $2X_1$ and $X_1 + X_2$? Discuss both their distributions and parameters.

ANS :- As we know that if $X \sim N(\mu_1, \sigma^2)$ AND $Y \sim N(\mu_2, \sigma^2)$ are two independence random variable then

$$X + Y \sim N(\mu_1 + \mu_2, \sigma^2 + \sigma^2)$$

Therefore in the questions

$$2X_1 \sim N(2\mu, 4\sigma^2) \text{ and}$$

$$X_1 + X_2 \sim N(\mu + \mu, \sigma^2 + \sigma^2) = N(2\mu, 2\sigma^2)$$

4. Let $X \sim N(100, 20^2)$. Find two values, a and b , symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
- 90.5, 105.9
 - 80.2, 119.8
 - 22, 78
 - 48.5, 151.5
 - 90.1, 109.9

ANS :- D. 48.5, 151.5

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In [1]: 1 from scipy import stats
        2 from scipy.stats import norm
        3 stats.norm.interval(0.99,100,20)
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Out[1]: (48.48341392902199, 151.516586070978)

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $\text{Profit}_1 \sim N(5, 3^2)$ and $\text{Profit}_2 \sim N(7, 4^2)$ respectively. Both the profits are in \$ Million. Answer the following questions about the total profit of the company in Rupees. Assume that \$1 = Rs. 45
- Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
 - Specify the 5th percentile of profit (in Rupees) for the company
 - Which of the two divisions has a larger probability of making a loss in a given year?

ANS :- A): 95% of the **probability lies** between 1.96 **standard deviations** of the **mean**.

Thus **range** is:

$$= (12 - 1.96 \times 5, 12 + 1.96 \times 5)$$

$$= (\$2.2M, \$22.8M)$$

$$= (\text{Rs. } 99M, \text{Rs. } 1026M)$$

B): **Fifth percentile** is calculated as:

$$P\left(Z \leq \frac{p - 12}{5}\right) = 0.05$$

From **p values** of **z score table**, we get:

$$\frac{p - 12}{5} = -1.644$$

$$p = 12 - 8.22 = 3.78$$

Thus at \$3.78M **dollars**, or Rs. 170.1M **amount**, 5th **percentile** of **profit** lies.

Or 5th **percentile** of **profit** is Rs. 170.1 Million.

C): Loss is when profit < 0

Thus: $p < 0$

The first **division** of **company**, thus have **larger probability** of making a loss in a given year.