

# Problem Set 15: Graphs and Matchings

CS/MATH 113 Discrete Mathematics

Spring 2024

The problems below make use of concepts and definitions from Sections 10.1 and 10.2 in our textbook. Solving the problems will be tremendously easier if you have gone over the sections and the worked examples included therein, browsed the end-of-section exercises, and consulted their solutions at the back of the book. If you are still stuck at some problem, feel free to consult course staff during their consultation hours as shared on Canvas.

1. Prove that a connected graph with  $n$  vertices has at least 2 vertices of the same degree.

**Solution:**

2. Prove that any cycle graph with an even number of vertices can be 2-colored.

**Solution:**

3. Prove by induction that a graph with  $n$  edges requires no more than  $n + 1$  colors to ensure that no two adjacent edges have the same color.

**Solution:**

4. A *perfect matching* in a graph is a matching that covers every vertex of the graph. Prove that if all the vertices of a bipartite graph have the same degree, then it has a perfect matching.

**Solution:**

5. 100 tourists have arrived at Mohenjodaro. 25 tour guides are available for a one-on-one tour. Each tourist likes at least 10 of the guides. Show that the tours can be arranged such that each tourist tours with a guide that they like and no guide gives more than 10 tours.

Hint: Consider each guide having 10 time slots corresponding to the constraint that each guide conducts at most 10 tours.

**Solution:**