Weekly Challenge 13: Structural Induction

CS/MATH 113 Discrete Mathematics

Spring 2024

1. k-ary tree

Definition 5 in Section 5.3 of our textbook defines a *full binary tree*. We extend this definition to a *full k-ary tree* as follows.

Definition 1 (Full k-ary tree)**.**

Basis Step There is a full k-ary tree consisting only of a single vertex r.

Recursive Step If $T_1, T_2, T_3, \ldots, T_k$ are disjoint full k-ary trees, there is a full k-ary tree, denoted by $T_1 \cdot T_2 \cdot T_3 \cdot \ldots \cdot T_k$, consisting of a root r together with edges connecting the root to each of the roots of $T_1, T_2, T_3, \ldots, T_k$.

We also introduce the following definitions of nodes in a tree.

Definition 2 (Leaf node)**.** A leaf node in a tree is a node that has no children.

Definition 3 (Internal node)**.** An internal node in a tree is a node that is not a leaf node.

Use structural induction to prove the following claim.

Claim 1. *The number of internal nodes in a full k-ary tree with n leaves is $\frac{n-1}{k-1}$.*

Solution: