Weekly Challenge 15: Matching in Bipartite Graphs

CS/MATH 113 Discrete Mathematics

Spring 2024

1. n-doku (10 points)

Let us define an n-doku as an $n \times n$ grid which contains all the numbers from 1 to n inclusive in the following manner.

- Each number appears exactly n times in the grid.
- Each number appears exactly once in each row of the grid.
- Each number appears exactly once in each column of the grid.

For example here is a 4-doku.

4	3	1	2
3	4	2	1
2	1	4	3
1	2	3	4

(a) [2 points] Below is a partially completed 5-doku.

1	2	5	3	4
3	5	2	4	1
5	1	4	2	3

Copy and complete the 5-doku.

Solution:

(b) [4 points] Show that filling in the next row of an n-doku is equivalent to finding a matching in some 2n-vertex bipartite graph.

Solution:

(c) [4 points] Prove that a matching must exist in this bipartite graph and, consequently, that an incomplete n-doku can always be completed.

Solution: