1) a) $\frac{2\pi}{N} = \frac{\pi}{6} \Rightarrow N = 12$

b) $\frac{2\pi}{N} = \frac{2\pi}{3} \Rightarrow N = 3$

2) a) $DTFT\{x[n]\cos\frac{5\pi}{8}n\} = \frac{1}{2}\left[X(e^{j(\omega-\frac{5\pi}{8})}) + X(e^{j(\omega+\frac{5\pi}{8})})\right]$

b) $s[n] := IDTFT\{X(e^{j(\omega-\frac{5\pi}{8})})\} \Rightarrow DTFT\{s[n]\} = X(e^{j(\omega-\frac{5\pi}{8})})$

c) $s[n] := \sin(\frac{21\pi}{8}n)\cdot x[n] \in \mathbb{R}$

$$DTFT\{\sin(\frac{21\pi}{8}n)\cdot x[n]\} = DTFT\{-\frac{1}{2}j\left[e^{j\frac{21\pi}{8}n}x[n] - e^{-j\frac{21\pi}{8}n}x[n]\right]\}$$

$$= -\frac{1}{2}j\left\{\sum_{n=-\infty}^{\infty} x[n]e^{j\frac{21\pi}{8}n}e^{-j\omega n} - \sum_{n=-\infty}^{\infty} x[n]e^{-j\frac{21\pi}{8}n}e^{-j\omega n}\right\}$$

$$= -\frac{1}{2}j\left\{\sum_{n=-\infty}^{\infty} x[n]e^{-j(\omega-\frac{21\pi}{8})n} - \sum_{n=-\infty}^{\infty} x[n]e^{-j(\omega+\frac{21\pi}{8})n}\right\}$$

$$= -\frac{1}{2}j\left[X(e^{j(\omega-\frac{21\pi}{8})}) - X(e^{j(\omega+\frac{21\pi}{8})})\right]$$ in the frequency support

d) $s[n] = e^{j\frac{5\pi}{8}n}x[n] \notin \mathbb{R}$

$$DTFT\{s[n]\} = \sum_{n=-\infty}^{\infty} x[n]e^{j\frac{5\pi}{8}n}e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x[n]e^{-j(\omega-\frac{5\pi}{8})n}$$

$$= X(e^{j(\omega-\frac{5\pi}{8})})$$

So $s[n] \notin \mathbb{R}$ for b either

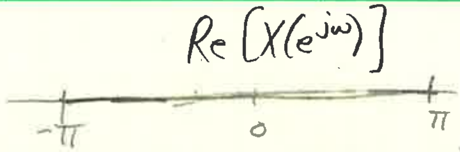
3) $x[n] = \begin{cases} 1, & 0 \le n \le M-1 \\ 0, & M \le n \le L-1 \end{cases}$

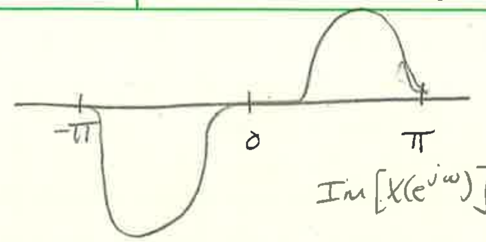
$$DFT\{x[n]\} = \sum_{n=0}^{L-1} x[n]e^{-j\frac{2\pi}{L}nk} = \sum_{n=0}^{M-1} e^{-j\frac{2\pi}{L}nk}$$

$$= \frac{1-e^{-j\frac{2\pi}{L}Mk}}{1-e^{-j\frac{2\pi}{L}k}} = \frac{e^{-j\frac{\pi}{L}Mk}(e^{j\frac{\pi}{L}Mk} - e^{-j\frac{\pi}{L}Mk})}{e^{-j\frac{\pi}{L}k}(e^{j\frac{\pi}{L}k} - e^{-j\frac{\pi}{L}k})}$$

$$= \frac{\sin(\frac{\pi}{L}Mk)}{\sin(\frac{\pi}{L}k)}e^{-j\frac{\pi}{L}(M-1)k}$$

4)





- $x[n]$ has 0 mean since $X(e^{jw})[0]=0$
- $x[n] \in \mathbb{R}$ since the imaginary part is anti-symmetric
  - ↳ $x[n]$ is real-antisymmetric
- $x[n]$ is Hermitian-antisymmetric since $X(e^{jw})$ is pure imaginary

5) $x[n]$ with $X(e^{jw})$

$$\text{IDTFT}\left\{ j\frac{d}{dw}X(e^{jw})\right\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} j\frac{d}{dw}X(e^{jw})\, e^{jwn}\,dw$$

$$= \frac{j}{2\pi}\left[\underbrace{e^{jwn}\cancel{X(e^{jw})}\Big|_{-\pi}^{\pi}}_{0 \text{ periodicity}} - \int_{-\pi}^{\pi} X(e^{jw})\, jn e^{jwn}\,dw\right]$$

$$= \frac{j}{2\pi}\left[-jn\int_{-\pi}^{\pi} X(e^{jw})\, e^{jwn}\,dw\right]$$

$$= n\,x[n]$$

6) Compute $\text{IDTFT}\left\{ 4X(e^{jw})/\pi - 2\right\}$ using linearity of $x[n]$ - known

7) $\tilde{x}[n] = x[n \bmod N]$ and DFS $\tilde{X}[k]$

$\Rightarrow \tilde{X}[k] = X[k]$

12) $(-\frac{\pi}{10}, \frac{\pi}{10})$ has length $\frac{\pi}{5}$

a) $\frac{\pi}{3} - \frac{\pi}{10} = \frac{10\pi - 3\pi}{30} = \frac{7\pi}{30} > \frac{\pi}{10}$ ✓

b) $\frac{9\pi}{10} - \frac{\pi}{10} = \frac{8\pi}{10} > \frac{\pi}{10}$, $\frac{9\pi}{10} + \frac{\pi}{10} = \pi$ ✓

c) $\frac{11\pi}{12} - \frac{\pi}{10} = \frac{55\pi - 6\pi}{60} = \frac{49\pi}{60}$, $\frac{11\pi}{12} + \frac{\pi}{10} = \frac{61\pi}{60}$ ✓