←   # Homework for Module 4 Part 1

Quiz, 15 questions

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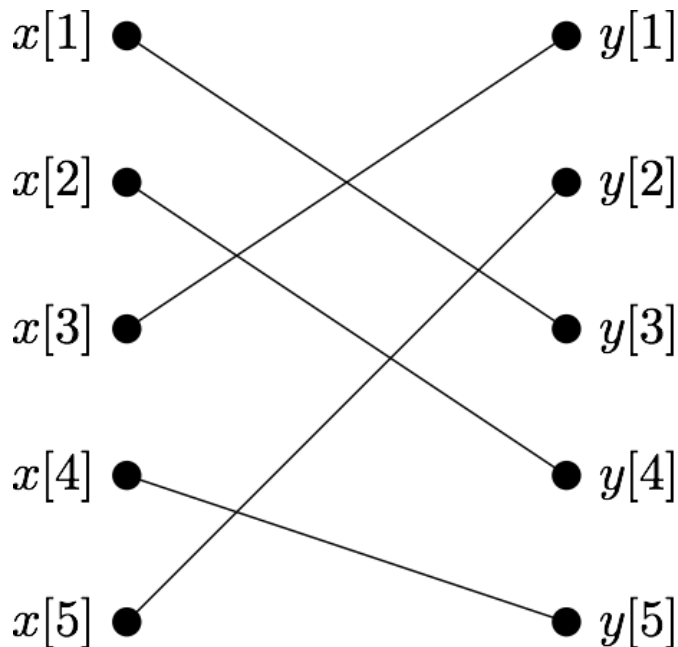
1.

(Difficulty: ★) Among the choices below, select all the **linear** systems. (Please note that some of the choices use functions rather than discrete-time signals; the concept of linearity is identical in both cases).

☐   AM radio modulation, i.e. multiply a signal $x[n]$ by a cosine at the carrier frequency :

$$y[n] = x[n]\cos(2\pi\omega_c n)$$

☐   Scrambling, i.e. a permutation to the input sequence, e.g. :

☐   Clipping, i.e. enforce a maximum signal amplitude M,e.g.:

$$y[n] = \begin{cases} x[n] & , \ x[n] \le M \\ M & , \ \text{otherwise} \end{cases}$$

☐   The DTFT, i.e. transform a sequence **x** into $\mathrm{DTFT}\{\mathbf{x}\}$.

☐   Time-stretch, i.e. $y(t) = x(\alpha t)$, e.g. if you play an old LP-45 vinyl disc at 33 rpm, the time-stretch coefficient would be $\alpha = 33/45$.

☐   Second derivative, i.e.

$$y(t) = \frac{d^2}{dt^2}x(t)$$

☐   Envelope detection (via squaring), i.e. $y[n] = |x[n]|^2 * h[n]$,where $h[n]$ is the impulse response of a *lowpass filter* such as the moving average filter.
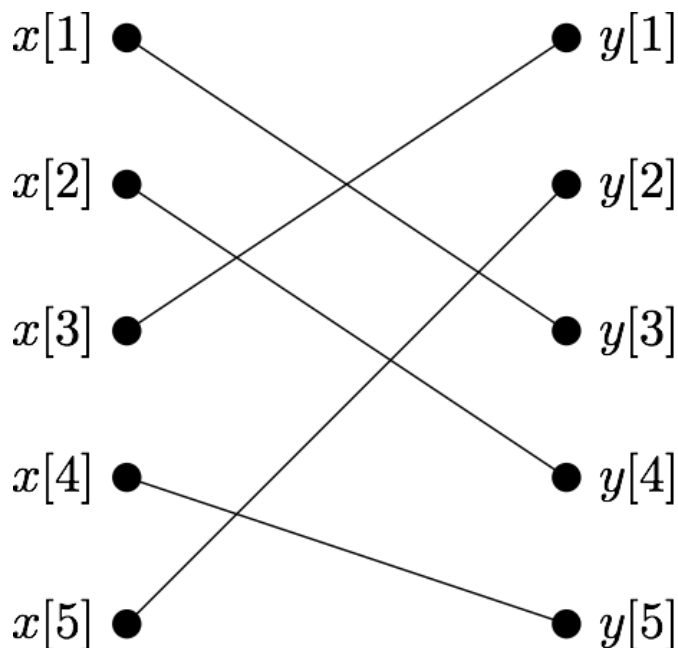
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2.
(Difficulty: ★) Among the choices below, select all the **time-invariant** systems. (Please note that some of the choices use functions rather than discrete-time signals; the concept of time invariance is identical in both cases).

☐ Scrambling : apply a permutation to the input sequence, e.g. :

$$x[1] \bullet \quad\quad\quad \bullet y[1]$$
$$x[2] \bullet \quad\quad\quad \bullet y[2]$$
$$x[3] \bullet \quad\quad\quad \bullet y[3]$$
$$x[4] \bullet \quad\quad\quad \bullet y[4]$$
$$x[5] \bullet \quad\quad\quad \bullet y[5]$$

☐ Clipping, i.e. enforce a maximum signal amplitude M,e.g.:

$$y[n] = \begin{cases} x[n] & , \ x[n] \leq M \\ M & , \ \text{otherwise} \end{cases}$$

☐ AM radio modulation, i.e. multiply a signal $x[n]$ by a cosine at the carrier frequency :

$$y[n] = x[n] \cos(2\pi\omega_c n)$$

☐ Envelope detection (via squaring), i.e. $y[n] = |x[n]|^2 * h[n]$,where $h[n]$ is the impulse response of a *lowpass filter* such as the moving average filter.

☐ Time-stretch, i.e. $y(t) = x(\alpha t)$, e.g. if you play an old LP-45 vinyl disc at 33 rpm, the time-stretch coefficient would be $\alpha = 33/45$

☐ Second derivative, i .e.

$$y(t) = \frac{d^2}{dt^2} x(t)$$

☐ The DTFT, i.e. transform a sequence $\mathbf{x}$ into $\mathrm{DTFT}\{\mathbf{x}\}$.

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3.

(Difficulty: ★) The impulse response of a room can be recorded by producing a sharp noise (impulsive sound source) in a silent room, thereby capturing the scattering of the sound produced by the walls.

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The impulse response $h[n]$ of <u>Lausanne Cathedral</u> was measured by *Dokmanic et al.* by recording the sound of balloons being popped (<u>hear it!</u>).

The balloon is popped at time $n = 0$ and after a number of samples N, the reverberations die out, i.e. $h[n] = 0$ for $n < 0$ or $n > N$.

The acoustic of this large space can then be artificially recreated by convolving any audio recording with the impulse response, e.g. this <u>cello recording</u> becomes <u>this</u>.

What are the properties of $h[n]$? *(tick all the correct answers)*

☐   BIBO stable

☐   IIR

☐   Causal

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1 point

4.
(Difficulty: ★) Let

$h[n] = \delta[n] - \delta[n-1]$

$x[n] = \begin{cases} 1 & , n \geq 0, \\ 0 & , \text{else.} \end{cases}$ `<br><br>`

$y[n] = x[n] * h[n]$.

Compute $y[-1], y[0], y[1], y[2]$ and write the result as space-separated values. E.g.: If you find $y[-1] = -2, y[0] = -1, y[1] = 0, y[2] = 1$, you should enter

```
1   -2 -1 0 1
```

Enter answer here

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1 point

5.

(Difficulty: ★★) Consider the filter $h[n] = \delta[n] - \delta[n-1]$,

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the signal $x[n] = \begin{cases} n, & 0 \le n \le 6, \\ 0, & \text{else.} \end{cases}$

and the output $y[n] = x[n] * h[n]$.

Compute $y[-1]$, $y[0]$, $y[1]$, $y[2]$ and write the result as space-separated values. E.g.: If you find $y[-1] = -2$, $y[0] = -1$, $y[1] = 0$, $y[2] = 1$, you should enter

```
1   -2 -1 0 1
```

Enter answer here

1
point

6.

(Difficulty: ★★★) Which of the following filters are BIBO-stable?

Assume $N \in \mathbb{N}$ and $0 < \omega_c < \pi$.

☐ The moving average: $h[n] = \frac{\delta[n]+\delta[n-1]}{2}$.

☐ The filter $h[n] = \sum_{k=0}^{N-1} \delta[n-k] \sin\left(2\pi \frac{k}{N}\right)$.

☐ The following smoothing filter: $h[n] = \sum_{k=0}^{\infty} \frac{1}{k+1}\delta[n-k]$.

☐ The ideal low pass filter with a cutoff frequency ω_c: $H(e^{j\omega}) = \begin{cases} 1 & |\omega| \le \omega_c \\ 0 & \text{otherwise} \end{cases}$.
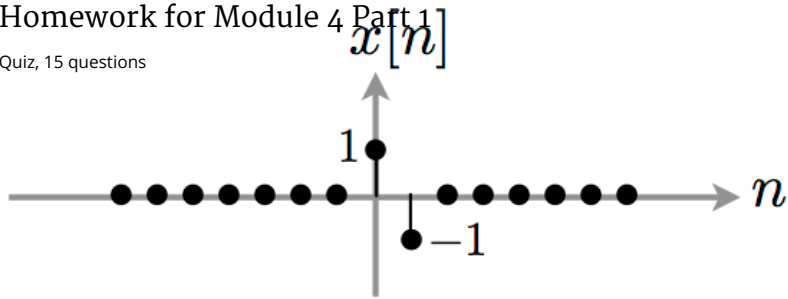

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7.

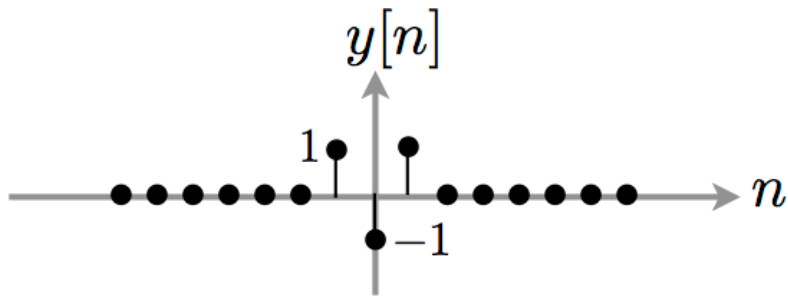(Difficulty: ★★) Consider an LTI system $\mathcal{H}$. When the input to $\mathcal{H}$ is the following signal
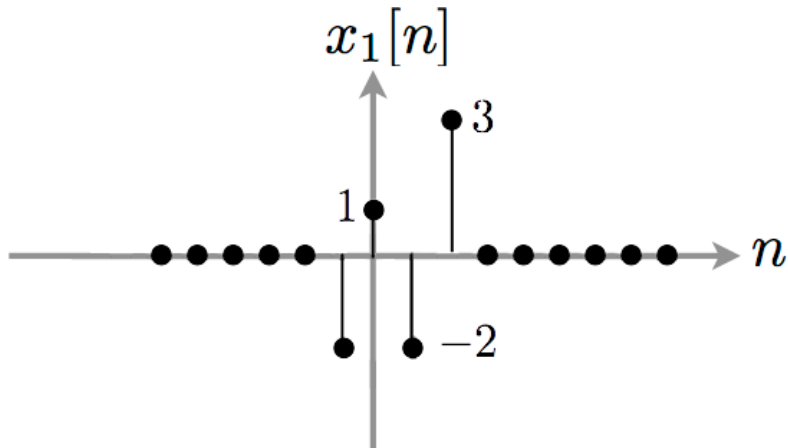
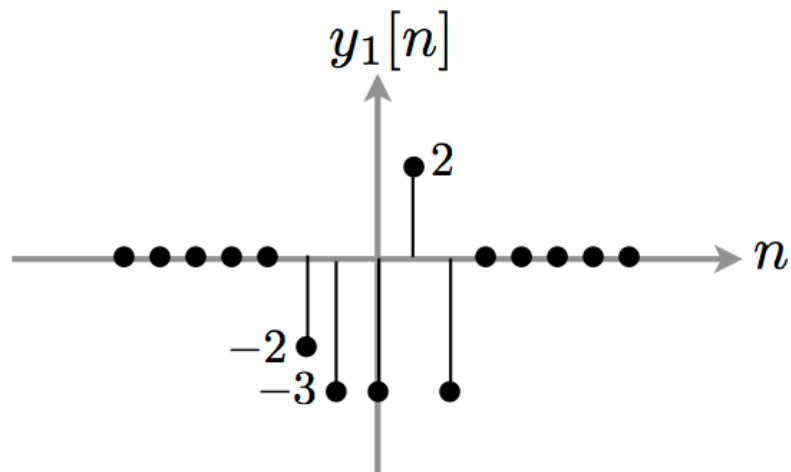← **Homework for Module 4 Part 1**

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$$x[n]$$

then the output is

$$y[n]$$

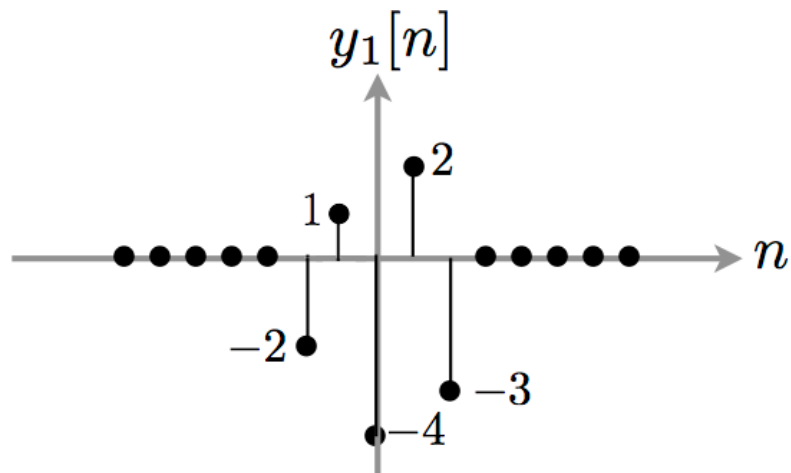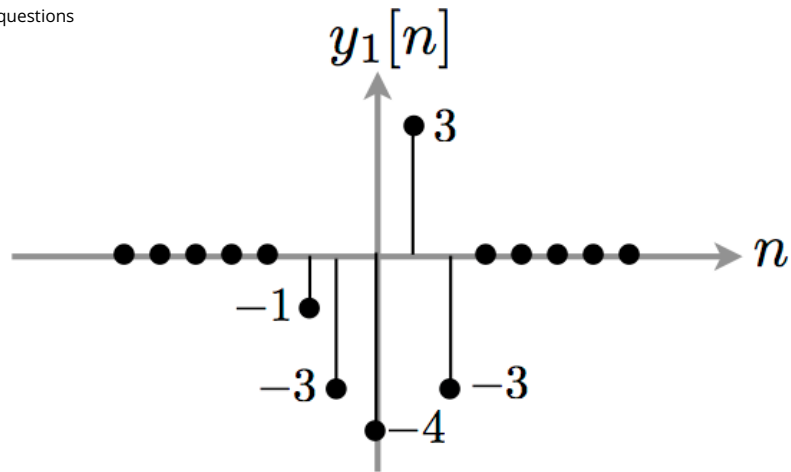Assume now the input to $\mathcal{H}$ is the following signal

$$x_1[n]$$

Which one of the following signals is the system's output?

○

$$y_1[n]$$

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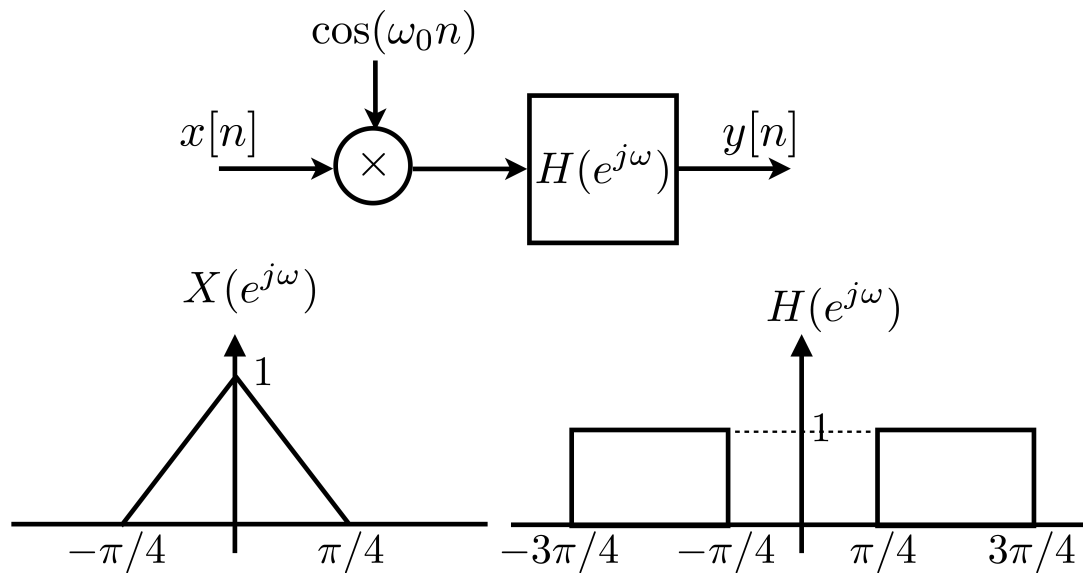
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8.

(Difficulty: ⋆) Consider the system shown below, consisting of a cosine modulator at frequency ω_0 followed by an ideal bandpass filter $h[n]$ whose frequency response is also shown in the figure; assume that the input to the system is the signal $x[n]$, whose spectrum is shown below.

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$$\cos(\omega_0 n)$$

$$x[n] \xrightarrow{\qquad} \otimes \xrightarrow{\qquad} \boxed{H(e^{j\omega})} \xrightarrow{y[n]}$$

$X(e^{j\omega})$

$H(e^{j\omega})$

Determine the value of $\omega_0 \in [0, 2\pi]$ that maximizes the energy of the output $y[n]$ when the input is $x[n]$.

Remember that π must be entered in the answer box as pi.
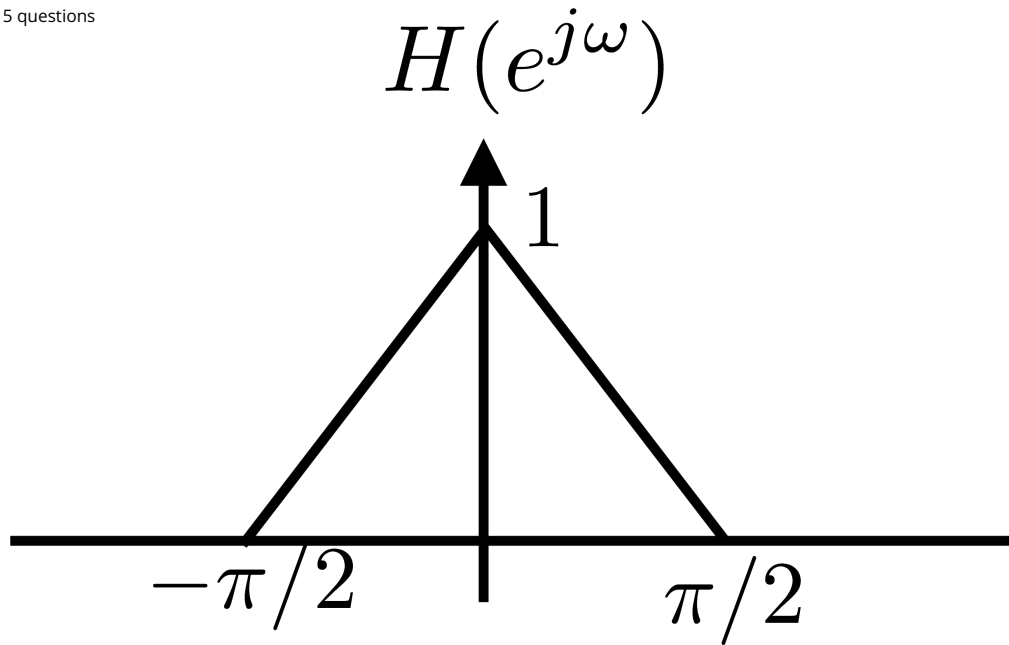
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9.

(Difficulty: ★) Consider a lowpass filter with the following frequency response.

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$$H(e^{j\omega})$$

What is the output $y[n]$ when the input to this filter is $x[n] = \cos(\frac{\pi}{5}n) + \sin(\frac{\pi}{4}n) + 0.5\cos(\frac{3\pi}{4}n)$?
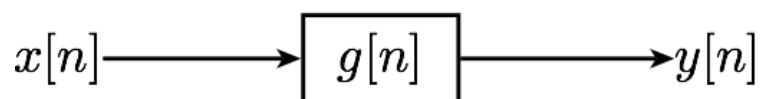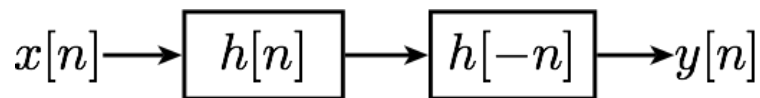
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10.
(Difficulty: ★) Consider a filter with real-valued impulse response $h[n]$. The filter is cascaded with another filter whose impulse response is $h'[n] = h[-n]$, i.e. whose impulse response is the time-reversed version of $h[n]$:

The cascade system can be seen as a single filter with impulse response g[n].

What is the phase of G(ejω)?

Preview

← Homework for Module 4 Part 1

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11.
(Difficulty: ★) Let $x[n] = \cos(\frac{\pi}{2}n)$ and $h[n] = \frac{1}{5}\text{sinc}(\frac{n}{5})$. Compute the convolution $y[n] = x[n] * h[n]$, and write the value of $y[5]$.
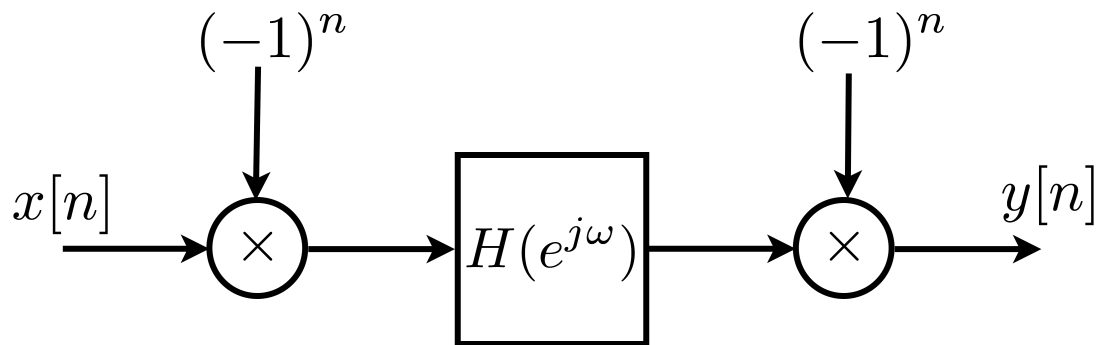
Hint: First find the convolution result in the frequency domain.

Enter answer here

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12.
(Difficulty: ★★) Consider the system below, where $H(e^{j\omega})$ is an ideal lowpass filter with cutoff frequency $\omega_c = \pi/4$:

Consider two input signals to the system:

- $x_1[n]$ is bandlimited to $[-\pi/4, \pi/4]$
- $x_2[n]$ is band-limited to $[-\pi, -3\pi/4] \cup [3\pi/4, \pi]$.

Which of the following statements is correct?

☐ Both $x_1[n]$ and $x_2[n]$ are eliminated by the system.

☐ $x_2[n]$ is not modified by the system while $x_1[n]$ is eliminated.

☐ $x_1[n]$ is not modified by the system while $x_2[n]$ is eliminated.

☐ Both $x_1[n]$ and $x_2[n]$ are not modified by the system.

1
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13.

(Difficulty: ⋆) $x[n]$ and $y[n]$ are two square-summable signals in $\ell_2(\mathbb{Z})$; $X(e^{j\omega})$ and $Y(e^{j\omega})$ are their corresponding DTFTs.

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We want to compute the value.
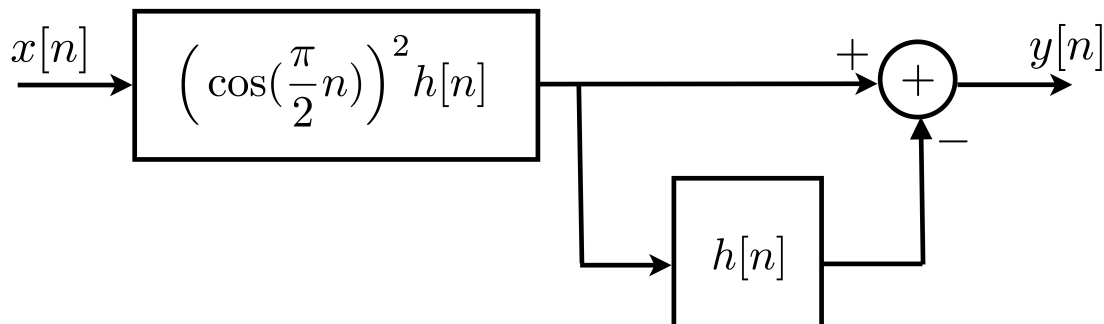Quiz, 15 questions

$$\sum_{n=-\infty}^{\infty} x[n]y^*[n].$$

in terms of $X(e^{j\omega})$ and $Y(e^{j\omega})$. Select the correct expression among the choices below.

○ ~~$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y(e^{-j\omega})d\omega$~~

○ ~~$\frac{1}{2\pi}X(e^{j\omega})Y^*(e^{j\omega})$~~

○ $X(e^{j\omega})Y(e^{-j\omega})$

○ ~~$\frac{1}{2\pi}X(e^{j\omega})Y(e^{-j\omega})$~~

○ $X(e^{j\omega}) * Y(e^{-j\omega})$

○ ~~$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega})d\omega$~~

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1 point

14.

(Difficulty: ⋆ ⋆ ⋆) $h[n]$ is the impulse response of an ideal lowpass filter with cutoff frequency $\omega_c < \frac{\pi}{2}$. Select the correct description for the system represented in the following figure?

Hint: Use the trigonometric identity $\cos(x)^2 = \frac{1}{2}(1 + \cos(2x))$.

○ A highpass filter with gain 1 and pass band $[\omega_c, \pi - \omega_c]$.

○ A lowpass filter with gain 1 and cutoff frequency $\omega_c/2$.

○ A highpass filter with gain $\frac{1}{4}$ and cutoff frequency ω_c.

○ A lowpass filter with gain 1 and cutoff frequency ω_c.

○ A highpass filter with gain $\frac{1}{2}$ and cutoff frequency $\pi - \omega_c$.

○ A lowpass filter with gain $\frac{1}{2}$ and cutoff frequency $2\omega_c$.
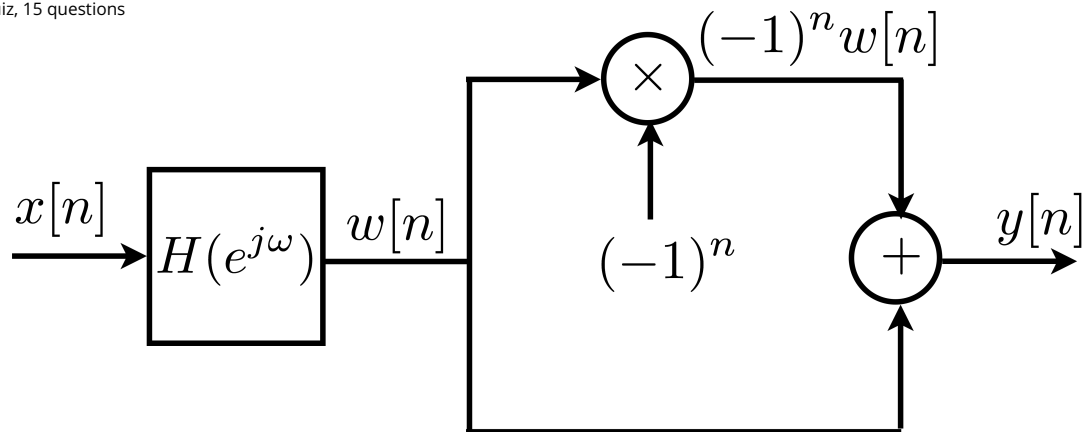
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15.

(Difficulty: ★ ★ ★) Consider the following system, where $H(e^{j\omega})$ is a half-band filter, i.e. an ideal lowpass with cutoff frequency $\omega_c = \pi/2$:

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Assume the input to the system is $x[n] = \delta[n]$. Compute

$$\sum_{n=-\infty}^{\infty} y[n]$$

- Hint: Perform the derivations in the frequency domain.

Enter answer here

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