1) $y[n] + 2y[n-1] = 3x[n] + 2.5x[n-1]$

$Y(z)[1 + 2z^{-1}] = X(z)(3 + 2.5z^{-1})$

$H(z) = \frac{Y(z)}{X(z)} = \frac{3 + 2.5z^{-1}}{1 + 2z^{-1}} \Rightarrow z = -2$ (pole)

not stable

$x[n] = \delta[n] - \delta[n+1] \Rightarrow X(z) = 1 - z$

$\Rightarrow Y(z) = H(z)X(z) = (1-z)\left(\frac{3 + 2.5z^{-1}}{1 + 2z^{-1}}\right) = \frac{3 - 3z + \frac{5}{2}z^{-1} - \frac{5}{2}}{1 + 2z^{-1}}$

$= \frac{-3z + \frac{1}{2} + \frac{5}{2}z^{-1}}{1 + 2z^{-1}}$

2) ROC for $x[n]$ is $r_2 < |z| < r_1$

$Z[x^*(-n)] = \sum_{n=-\infty}^{\infty} x^*[-n] z^{-n} = \sum_{n=-\infty}^{\infty} x^*[n] z^{n}$

$= \overline{\sum_{n=-\infty}^{\infty} x[n] z^{-n}} = \overline{\sum_{n=-\infty}^{-1} x[n] z^{-n} + \sum_{n=0}^{\infty} x[n] z^{-n}}$

$= \overline{\sum_{n=1}^{\infty} x(n) z^{n}} + \overline{\sum_{n=0}^{M} x[n] z^{-n}}$

$= \overline{X_a(z)} + \overline{X_c(z)}$

$\Rightarrow$ ROC is $\frac{1}{r_1} < |z| < \frac{1}{r_2}$ due to the reversal by conjugation

4) $y[n] - \frac{1}{2}y[n-1] = 2x[n] - 5x[n-1] - x[n-2]$

$Y(z)\left(1 - \frac{1}{2}z^{-1}\right) = X(z)\left[2 - 5z^{-1} - z^{-2}\right]$

$H(z) = \frac{Y(z)}{X(z)} = \frac{2 - 5z^{-1} - z^{-2}}{1 - \frac{1}{2}z^{-1}}$

$H(e^{j\pi}) = H(-1) = \frac{2 + 5 - 1}{1 + \frac{1}{2}} = \frac{6}{\frac{3}{2}} = 4$

7) $y[n] = \frac{3}{8}y[n-2] + \frac{5}{4}y[n-1] + \frac{1}{4}x[n-1] + x[n]$

$Y(z)\left[1 - \frac{3}{8}z^{-2} - \frac{5}{4}z^{-1}\right] = X(z)\left[1 + \frac{1}{4}z^{-1}\right]$

$$\frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{4}z^{-1}}{1 - \frac{5}{4}z^{-1} - \frac{3}{8}z^{-2}} = \frac{z^2 + \frac{1}{4}z}{z^2 - \frac{5}{4}z - \frac{3}{8}} = \frac{z(z+\frac{1}{4})}{z^2 - \frac{5}{4}z - \frac{3}{8}}$$

Poles: $z = \frac{\frac{5}{4} \pm \sqrt{\frac{25}{16} + 4(\frac{3}{8})}}{2} = \frac{\frac{5}{4} \pm \sqrt{\frac{49}{16}}}{2} = \frac{\frac{5}{4} \pm \frac{7}{4}}{2}$

$\Rightarrow z = \frac{12}{8} = \frac{3}{2}, \; z = -\frac{1}{4}$

8) $y[n] = x[n] + f$

$g = -f$

$f = -x[n] + \frac{1}{2}y[n] + g$

$y[0] = x[0]$

$f = -x[0] + \frac{1}{2}y[0]$

$y[1] = x[1] - x[0] + \frac{1}{2}y[0] = x[1] - \frac{1}{2}y[0]$

$f = -x[1] + \frac{1}{2}y[1] + x[0] - \frac{1}{2}y[0] = -x[1] + \frac{1}{2}x[1] - \frac{1}{4}y[0] + \frac{1}{2}y[0]$

$y[2] = x[2] - x[1] + x[0] + \frac{1}{2}y[1] - \frac{1}{2}y[0]$

$f = -x[2] + \frac{1}{2}y[2] + x[1] - \frac{1}{2}y[1] - x[0] + \frac{1}{2}y[0]$

9) $y[n] = x[n] - 0.95^2 x[n-2] + x[n-2]$

$= x[n] + (1 - 0.95^2)x[n-2]$

$Y(z) = X(z)\left[1 + (1 - 0.95^2)z^{-2}\right] \Rightarrow \frac{Y(z)}{X(z)} = 1 + (1 - 0.95^2)z^{-2}$

5) $Y(z)\left[1 + 2z^{-1}\right] = X(z)\left[z - \frac{1}{2}\right]$

$H(z) = \frac{z - \frac{1}{2}}{1 + 2z^{-1}}$ pole at $z = -2$ unstable