

← Homework for Module 4 Part 1

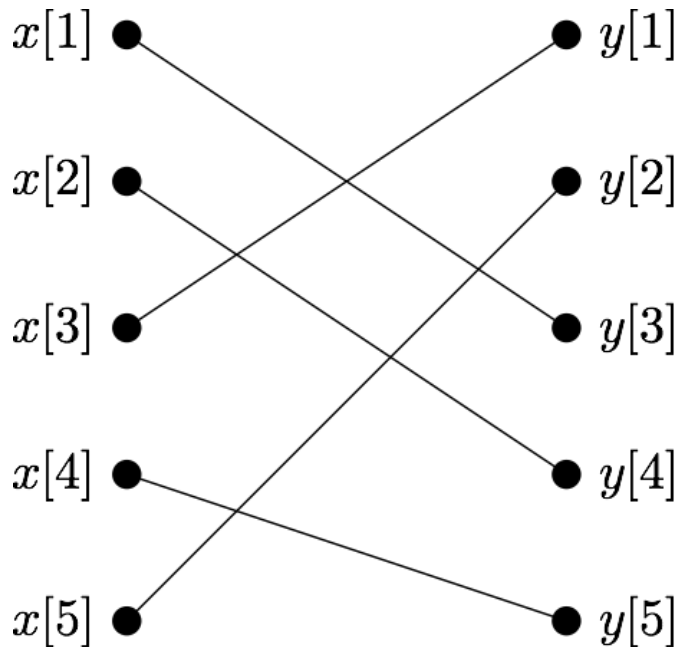
Quiz, 15 questions

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1.

(Difficulty: ★) Among the choices below, select all the **linear** systems. (Please note that some of the choices use functions rather than discrete-time signals; the concept of linearity is identical in both cases).

- ☒ The DTFT, i.e. transform a sequence \mathbf{x} into $\text{DTFT}\{\mathbf{x}\}$.
- ☐ Envelope detection (via squaring), i.e. $y[n] = |x[n]|^2 * h[n]$, where $h[n]$ is the impulse response of a *lowpass filter* such as the moving average filter.
- ☒ AM radio modulation, i.e. multiply a signal $x[n]$ by a cosine at the carrier frequency :
- $$y[n] = x[n] \cos(2\pi\omega_c n)$$
- ☒ Time-stretch, i.e. $y(t) = x(\alpha t)$, e.g. if you play an old LP-45 vinyl disc at 33 rpm, the time-stretch coefficient would be $\alpha = 33/45$.
- ☒ Scrambling, i.e. a permutation to the input sequence, e.g. :



- ☒ Second derivative, i.e.
- $$y(t) = \frac{d^2}{dt^2} x(t)$$
-
- ☐ Clipping, i.e. enforce a maximum signal amplitude M , e.g.:

$$y[n] = \begin{cases} x[n] & , x[n] \leq M \\ M & , \text{otherwise} \end{cases}$$



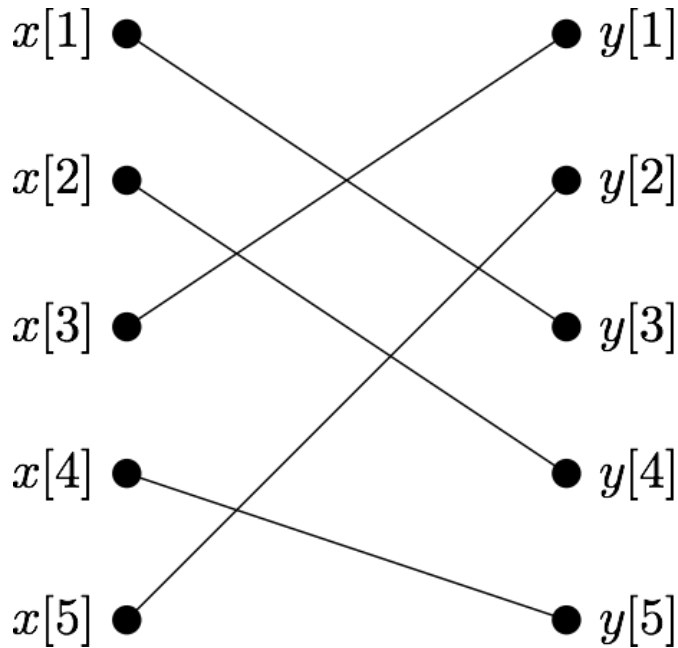
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(Difficulty: ★) Among the choices below, select all the **time-invariant** systems. (Please note that some of the choices use functions rather than discrete-time signals; the concept of time invariance is identical in both cases).

- ☐ Scrambling : apply a permutation to the input sequence, e.g. :



- ☒ Second derivative, i.e.

$$y(t) = \frac{d^2}{dt^2} x(t)$$

- ☐ The DTFT, i.e. transform a sequence \mathbf{x} into $\text{DTFT}\{\mathbf{x}\}$.

- ☐ AM radio modulation, i.e. multiply a signal $x[n]$ by a cosine at the carrier frequency :

$$y[n] = x[n] \cos(2\pi\omega_c n)$$

- ☒ Clipping, i.e. enforce a maximum signal amplitude M , e.g.:

$$y[n] = \begin{cases} x[n] & , x[n] \leq M \\ M & , \text{otherwise} \end{cases}$$

- ☐ Time-stretch, i.e. $y(t) = x(\alpha t)$, e.g. if you play an old LP-45 vinyl disc at 33 rpm, the time-stretch coefficient would be $\alpha = 33/45$

- ☒ Envelope detection (via squaring), i.e. $y[n] = |x[n]|^2 * h[n]$, where $h[n]$ is the impulse response of a *lowpass filter* such as the moving average filter.

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3.

(Difficulty: ★) The impulse response of a room can be recorded by producing a sharp noise (impulsive sound source) in a silent room, thereby capturing the scattering of the sound produced by the walls.



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The impulse response $h[n]$ of [Lausanne Cathedral](#) was measured by *Dokmanic et al.* by recording the sound of balloons being popped ([hear it!](#)).

The balloon is popped at time $n = 0$ and after a number of samples N , the reverberations die out, i.e. $h[n] = 0$ for $n < 0$ or $n > N$.

The acoustic of this large space can then be artificially recreated by convolving any audio recording with the impulse response, e.g. this [cello recording](#) becomes [this](#).

What are the properties of $h[n]$? (tick all the correct answers)

- ☐ IIR
- ☒ BIBO stable
- ☒ Causal

1
point

4.

(Difficulty: ★) Let

$$h[n] = \delta[n] - \delta[n - 1]$$

$$x[n] = \begin{cases} 1 & , n \geq 0, \\ 0 & , \text{else.} \end{cases}$$

$$y[n] = x[n] * h[n].$$

Compute $y[-1]$, $y[0]$, $y[1]$, $y[2]$ and write the result as space-separated values. E.g.: If you find $y[-1] = -2$, $y[0] = -1$, $y[1] = 0$, $y[2] = 1$, you should enter

1 -2 -1 0 1

0 1 0 0

1
point

5.

(Difficulty: ★★) Consider the filter $h[n] = \delta[n] - \delta[n - 1]$,



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the signal $x[n] = \begin{cases} n, & n = 0, 1, 2, \\ 0, & \text{else.} \end{cases}$
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and the output $y[n] = x[n] * h[n]$.

Compute $y[-1]$, $y[0]$, $y[1]$, $y[2]$ and write the result as space-separated values. E.g.: If you find $y[-1] = -2$, $y[0] = -1$, $y[1] = 0$, $y[2] = 1$, you should enter

1 -2 -1 0 1

0 0 1 1

1
point

6.

(Difficulty: ★ ★ ★) Which of the following filters are BIBO-stable?

Assume $N \in \mathbb{N}$ and $0 < \omega_c < \pi$.

- ☐ The following smoothing filter: $h[n] = \sum_{k=0}^{\infty} \frac{1}{k+1} \delta[n-k]$.
- ☒ Any filter $h[n]$ with finite support and bounded coefficients.
- ☐ The moving average: $h[n] = \frac{\delta[n] + \delta[n-1]}{2}$.
- ☐ The ideal low pass filter with a cutoff frequency ω_c : $H(e^{j\omega}) = \begin{cases} 1 & |\omega| \leq \omega_c \\ 0 & \text{otherwise} \end{cases}$.

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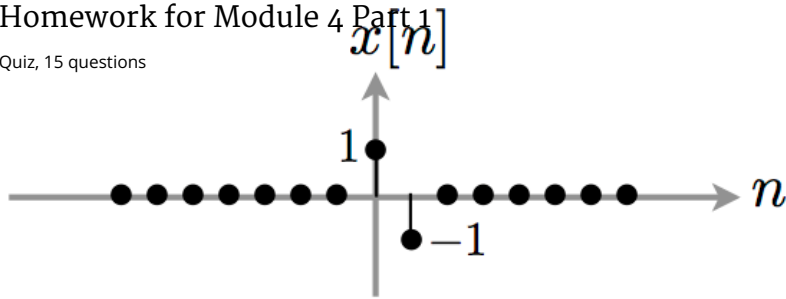
7.

(Difficulty: ★★) Consider an LTI system \mathcal{H} . When the input to \mathcal{H} is the following signal

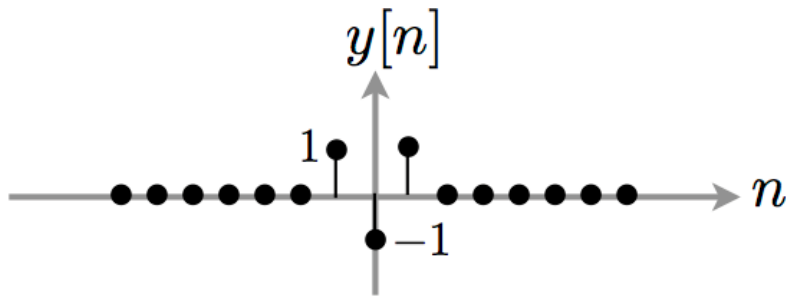


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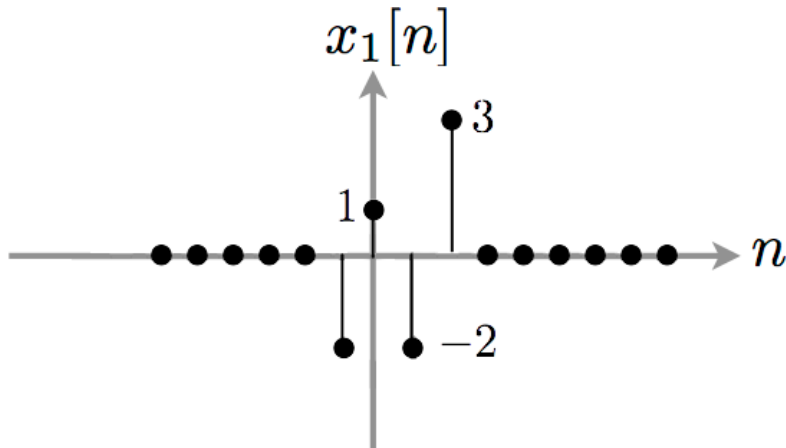
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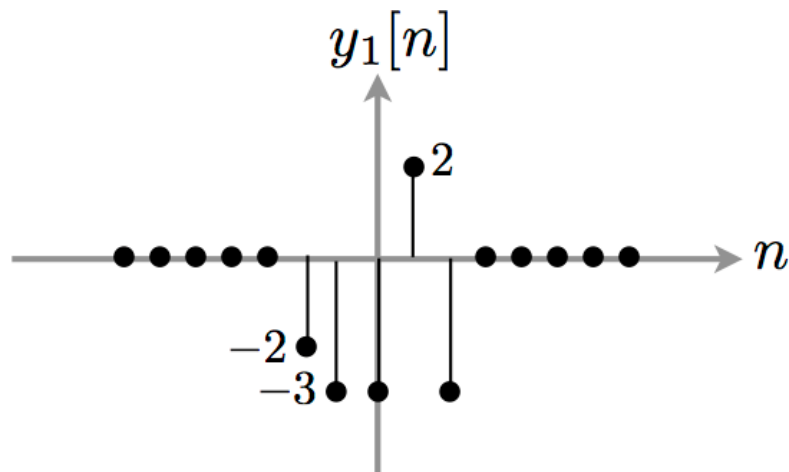
then the output is



Assume now the input to \mathcal{H} is the following signal



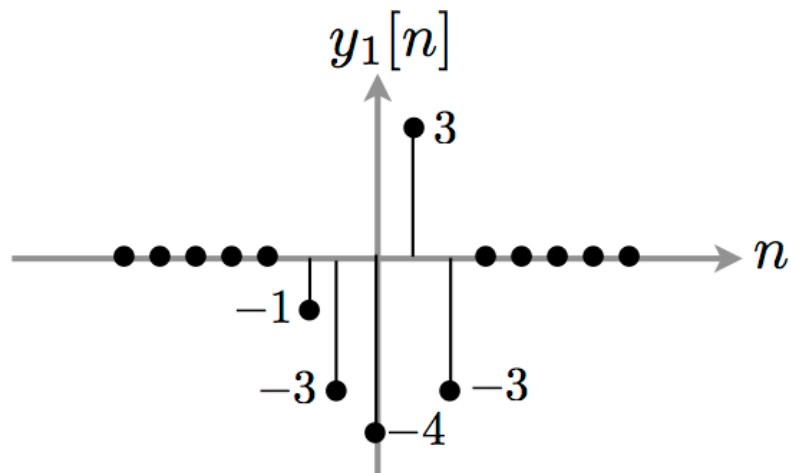
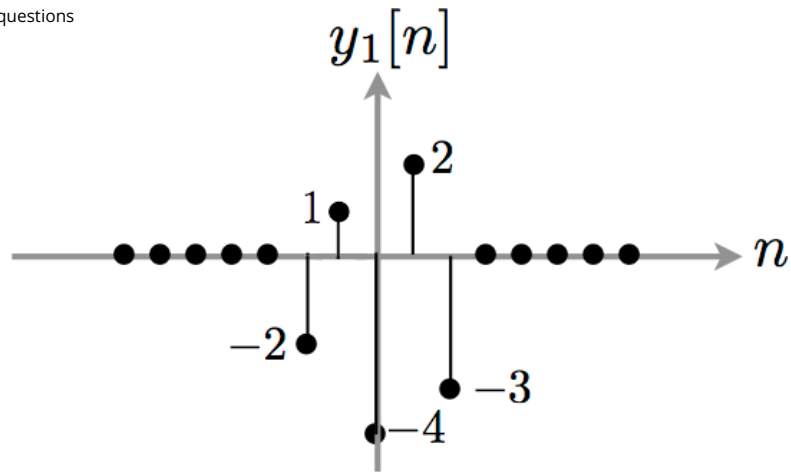
Which one of the following signals is the system's output?





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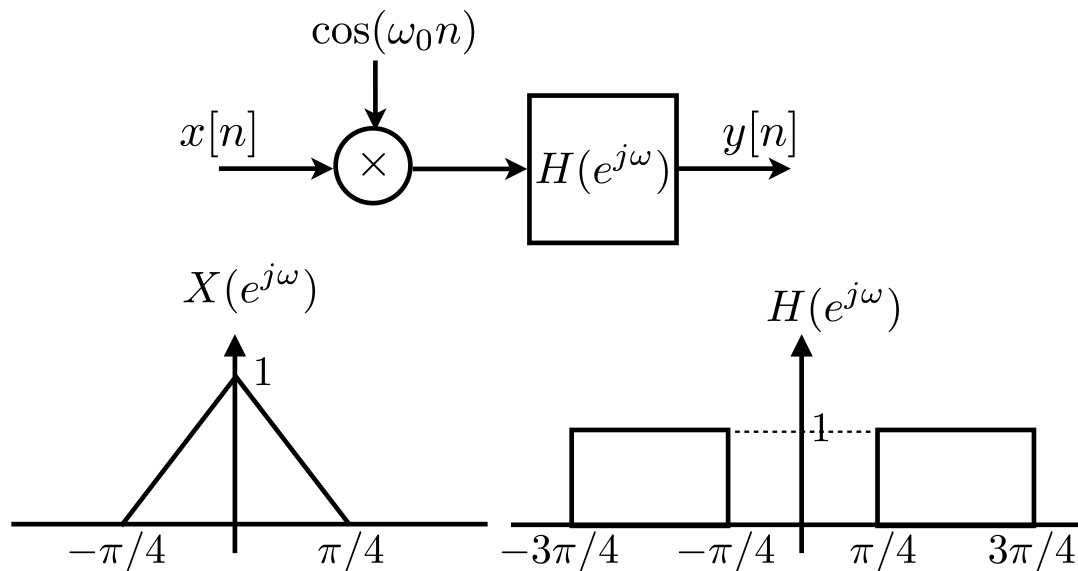
8.

(Difficulty: ★) Consider the system shown below, consisting of a cosine modulator at frequency ω_0 followed by an ideal bandpass filter $h[n]$ whose frequency response is also shown in the figure; assume that the input to the system is the signal $x[n]$, whose spectrum is shown below.



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Determine the value of $\omega_0 \in [0, 2\pi]$ that maximizes the energy of the output $y[n]$ when the input is $x[n]$.

Remember that π must be entered in the answer box as pi.

Preview

$$\frac{\pi}{2}$$

pi/2

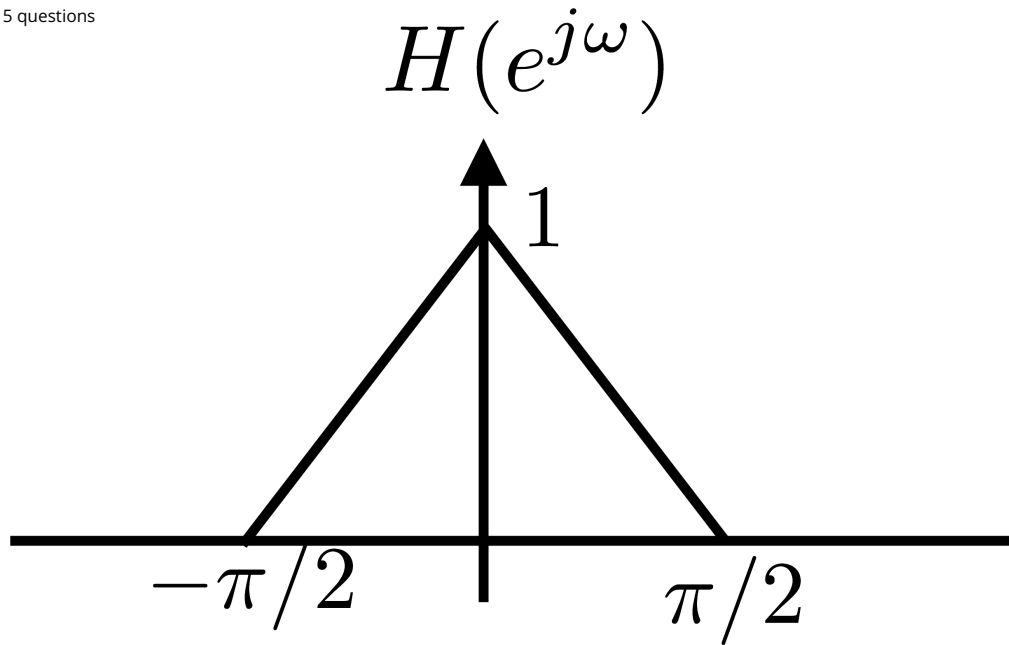
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9.

(Difficulty: ★) Consider a lowpass filter with the following frequency response.

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What is the output $y[n]$ when the input to this filter is $x[n] = \cos(\frac{\pi}{5}n) + \sin(\frac{\pi}{4}n) + 0.5\cos(\frac{3\pi}{4}n)$?

Preview

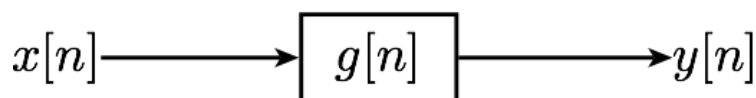
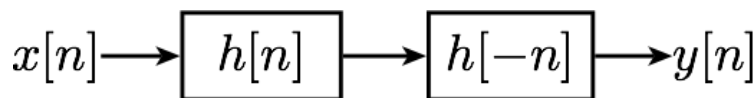
$$\frac{1}{2} \sin\left(\frac{\pi n}{4}\right) + \frac{3}{5} \cos\left(\frac{\pi n}{5}\right)$$

$$3/5 * \cos(\pi/5 * n) + 1/2 * \sin(\pi/4 * n)$$

1
point

10.

(Difficulty: ★) Consider a filter with real-valued impulse response $h[n]$. The filter is cascaded with another filter whose impulse response is $h'[n] = h[-n]$, i.e. whose impulse response is the time-reversed version of $h[n]$:



The cascade system can be seen as a single filter with impulse response $g[n]$.

What is the phase of $G(e^{j\omega})$?

Preview

0



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11.

(Difficulty: ★) Let $x[n] = \cos(\frac{\pi}{2}n)$ and $h[n] = \frac{1}{5}\text{sinc}(\frac{n}{5})$. Compute the convolution $y[n] = x[n] * h[n]$, and write the value of $y[5]$.

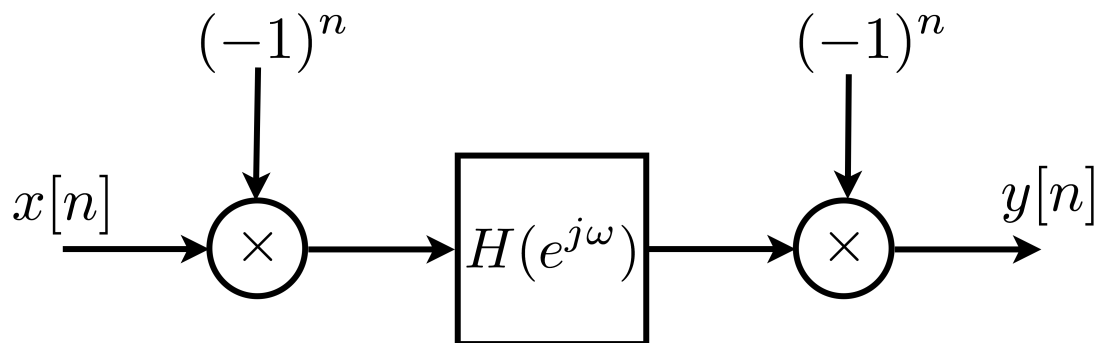
Hint: First find the convolution result in the frequency domain.

0

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12.

(Difficulty: ★★) Consider the system below, where $H(e^{j\omega})$ is an ideal lowpass filter with cutoff frequency $\omega_c = \pi/4$:



Consider two input signals to the system:

- $x_1[n]$ is bandlimited to $[-\pi/4, \pi/4]$
- $x_2[n]$ is band-limited to $[-\pi, -3\pi/4] \cup [3\pi/4, \pi]$.

Which of the following statements is correct?

- ☒ $x_2[n]$ is not modified by the system while $x_1[n]$ is eliminated.
- ☐ Both $x_1[n]$ and $x_2[n]$ are not modified by the system.
- ☐ $x_1[n]$ is not modified by the system while $x_2[n]$ is eliminated.
- ☐ Both $x_1[n]$ and $x_2[n]$ are eliminated by the system.

1
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13.

(Difficulty: ★) $x[n]$ and $y[n]$ are two square-summable signals in $\ell_2(\mathbb{Z})$; $X(e^{j\omega})$ and $Y(e^{j\omega})$ are their corresponding DTFTs.



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We want to compute the value.

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$$\sum_{n=-\infty}^{\infty} x[n]y^*[n].$$

in terms of $X(e^{j\omega})$ and $Y(e^{j\omega})$. Select the correct expression among the choices below.

☐ $X(e^{j\omega}) * Y(e^{-j\omega})$

☐ $\frac{1}{2\pi} X(e^{j\omega}) Y(e^{-j\omega})$

☐ $\frac{1}{2\pi} X(e^{j\omega}) Y^*(e^{j\omega})$

☒ $\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) Y^*(e^{j\omega}) d\omega$

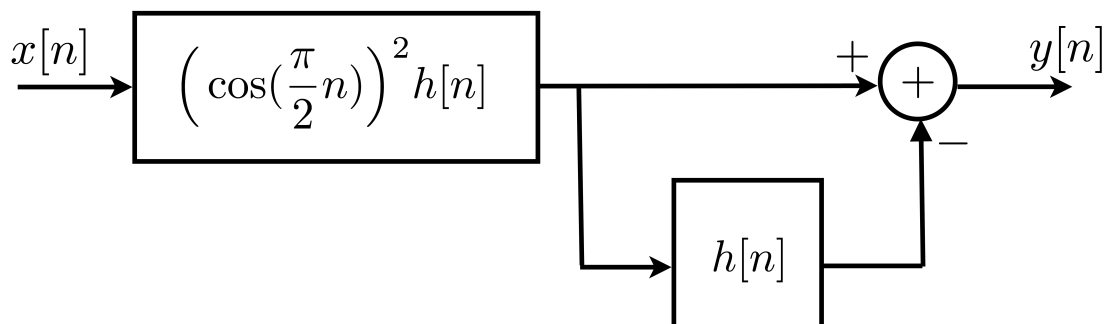
☐ $\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) Y(e^{-j\omega}) d\omega$

☐ $X(e^{j\omega}) Y(e^{-j\omega})$

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point

14.

(Difficulty: ★ ★ ★) $h[n]$ is the impulse response of an ideal lowpass filter with cutoff frequency $\omega_c < \frac{\pi}{2}$. Select the correct description for the system represented in the following figure?



Hint: Use the trigonometric identity $\cos(x)^2 = \frac{1}{2}(1 + \cos(2x))$.

☐ A lowpass filter with gain 1 and cutoff frequency ω_c .

☐ A lowpass filter with gain $\frac{1}{2}$ and cutoff frequency $2\omega_c$.

☒ A highpass filter with gain $\frac{1}{2}$ and cutoff frequency $\pi - \omega_c$.

☐ A highpass filter with gain $\frac{1}{4}$ and cutoff frequency ω_c .

☐ A highpass filter with gain 1 and pass band $[\omega_c, \pi - \omega_c]$.

☐ A lowpass filter with gain 1 and cutoff frequency $\omega_c/2$.

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point

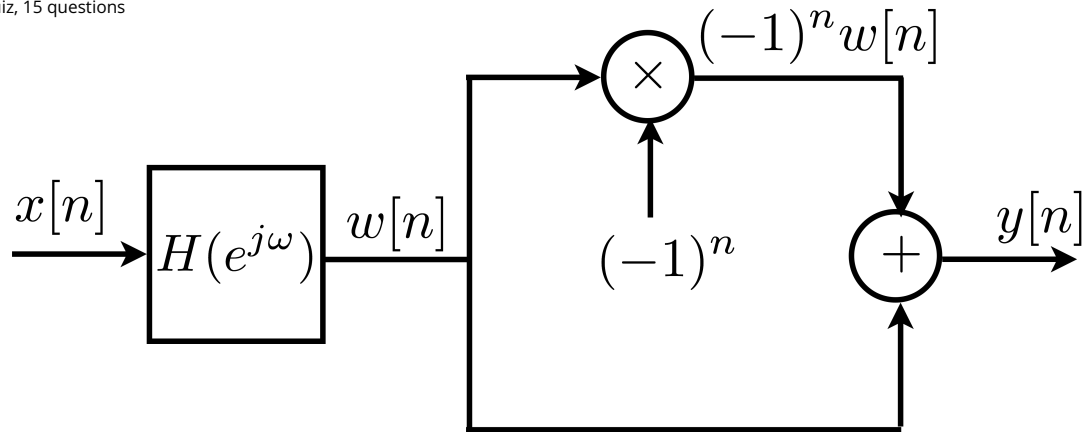
15.

(Difficulty: ★ ★ ★) Consider the following system, where $H(e^{j\omega})$ is a half-band filter, i.e. an ideal lowpass with cutoff frequency $\omega_c = \pi/2$:



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Assume the input to the system is $x[n] = \delta[n]$. Compute

$$\sum_{n=-\infty}^{\infty} y[n]$$

- Hint: Perform the derivations in the frequency domain.



I, **Mark R. Lytell**, understand that submitting work that isn't my own may result in permanent failure of this course or deactivation of my Coursera account.

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