4) $h[n] = \delta[n] - \delta[n-1]$

$x[n] = \begin{cases} 1, & n \geq 0 \\ 0, & \text{else} \end{cases}$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=0}^{\infty} h[n-k]$$

$$y[-1] = \sum_{k=0}^{\infty} h[-1-k] = \sum_{k=0}^{\infty} (\delta[-1-k] - \delta[-2-k]) = 0$$

$$y[0] = \sum_{k=0}^{\infty} (\delta[-k] - \delta[-1-k]) = \delta[0] - \cancel{\delta[-1]} + \cancel{\delta[-1]} + \cancel{\delta[-2]} + \dots = \delta[0] = 1$$

$$y[1] = \sum_{k=0}^{\infty} (\delta[1-k] - \delta[-k]) = \cancel{\delta[1]} - \delta[0] + \delta[0] - \cancel{\delta[-1]} + \cancel{\delta[-1]} + \cancel{\delta[-2]} + \dots = 0$$

$$y[2] = \sum_{k=0}^{\infty} (\delta[2-k] - \delta[1-k]) = \delta[2] - \delta[1] + \delta[1] - \delta[0] + \delta[0] - \delta[-1] + \dots = 0$$

5) $h(n) = \delta[n] - \delta[n-1]$

$x[n] = \begin{cases} n, & n \geq 0 \\ 0, & \text{else} \end{cases}$

$$y[n] = x[n] * h[n] = \sum_{k=1}^{\infty} k\, h[n-k]$$

$$y[-1] = \sum_{k=1}^{\infty} k(\delta[-1-k] - \delta[-2-k]) = 1(\delta[-2] - \delta[-3]) + 2(\delta[-3] - \delta[-4]) + \dots = 0$$

$$y[0] = \sum_{k=1}^{\infty} k(\delta[-k] - \delta[-1-k]) = 1(\delta[-1] - \delta[-2]) + 2(\delta[-2] - \delta[-3]) + \dots = 0$$

$$y[1] = \sum_{k=1}^{\infty} k(\delta[1-k] - \delta[-k]) = 1(\delta[0] - \delta[-1]) + 2(\delta[-1] - \delta[-2]) + \dots = \delta[0] = 1$$

$$y[2] = \sum_{k=1}^{\infty} k(\delta[2-k] - \delta[1-k]) = 1(\delta[1] - \delta[0]) + 2(\delta[0] - \delta[-1]) + \dots = \delta[0] = 1$$

6) $N \in \mathbb{N}$, $0 < \omega_c < \pi$. BIBO stable?

a) $h[n] = \dfrac{\delta[n] + \delta[n-1]}{2} \Rightarrow \sum_{n=-\infty}^{\infty} |h[n]| = \frac{1}{2} + \frac{1}{2} = 1$ ✓

b) $h[n] = \sum_{k=0}^{N-1} \delta[n-k] \sin(2\pi \frac{k}{N})$ is BIBO since finite support

c) $h[n] = \sum_{k=0}^{\infty} \frac{1}{k+1} \delta[n-k]$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} \left| \sum_{k=0}^{\infty} \frac{1}{k+1} \delta[n-k] \right| \le \sum_{n=-\infty}^{\infty} \sum_{k=0}^{\infty} \left|\frac{1}{k+1}\right| |\delta[n-k]|$$

$$= \sum_{n=-\infty}^{\infty} \left|\frac{1}{n+1}\right| = \infty \quad \text{not BIBO}$$

d) $H(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \omega_c \\ 0, & \text{otherwise} \end{cases} \Rightarrow h[n] = \dfrac{\sin \omega_c n}{\pi n}$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \frac{1}{\pi} \sum_{n=-\infty}^{\infty} \left|\frac{1}{n}\right| |\sin(\omega_c n)| \le \frac{1}{\pi} \sum_{n=-\infty}^{\infty} \left|\frac{1}{n}\right| = \infty \quad \text{not BIBO}$$

7) $x[n] = \delta[n] - \delta[n-1]$

$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$

$y[-1] = \sum_{k=-\infty}^{\infty} x[k] h[-1-k] = \sum_{k=0}^{\infty} x[k] h[-1-k] = 1\, h[-1] - 1 \cdot h[-2] = 1$

$y[0] = -1 = \sum_{k=0}^{\infty} x[k] h[-k] = 1 \cdot h[0] - 1 \cdot h[-1]$

$y[1] = 1 = \sum_{k=0} x[k] h[1-k] = 1 \cdot h[1] - 1 \cdot h[0]$

$y[-2] = 0 = \sum_{k=0}^{\infty} x[k] h[-2-k] = 1 \cdot h[-2] - h[-3] = 0$

$y[2] = 0 = \sum_{k=0}^{\infty} x[k] h[2-k] = 1 \cdot h[2] - 1 \cdot h[1] = 0$

$h[-2] = h[-3] \Rightarrow h[n] = h[n-1]$, $n \le -2$

$h[n] = h[n-1]$, $n \ge 2$

$h[-1] - h[-2] = 1$

$h[0] - h[-1] = -1$

$h[1] - h[0] = 1$

7 cont'd) $\left[\begin{array}{cccc|c} -1 & 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & -1 \\ 0 & 0 & -1 & -1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right]$

$$\Rightarrow h[-1] = h[1]$$
$$h[-2] = h[0]$$
$$h[0] = h[1] - 1$$

Set $h[-2] = 0 \Rightarrow h[0] = 0 \Rightarrow h[1] = 1 \Rightarrow h[-1] = 1$

$$h[n] = \begin{cases} 0, & n \leq 2 \\ 1, & n = -1 \\ 0, & n = 0 \\ 1, & n \geq 1 \end{cases}$$

for $x_1[n] = \begin{cases} -2, & x = -1 \\ 1, & x = 0 \\ -2, & x = 1 \\ 3, & x = 2 \\ 0, & \text{otherwise} \end{cases}$

$$y_1[n] = x_1[n] * h[n] = \sum_{k=-\infty}^{\infty} x_1[k]\, h[n-k] = \sum_{k=-1}^{2} x_1[k]\, h[n-k]$$

$$y_1[-2] = -2h[-1] + h[-2] - 2h[-3] + 3h[-4] = -2$$
$$y_1[-1] = -2h[0] + h[-1] - 2h[-2] + 3h[-3] = 1$$
$$y_1[0] = -2h[1] + h[0] - 2h[-1] + 3h[-2] = -2-2 = -4$$
$$y_1[1] = -2h[2] + h[1] - 2h[0] + 3h[-1] = -2+1+0+3 = 2$$
$$y_1[2] = -2h[3] + h[2] - 2h[1] + 3h[0] = -2+1-2 = -3$$
$$y_1[3] = -2h[4] + h[3] - 2h[2] + 3h[1] = -2+1-2+3 = 0$$

9) $x[n] = \cos\frac{\pi}{3}n + \sin\frac{\pi}{4}n + \frac{1}{2}\cos\frac{3\pi}{4}n$

$$X(e^{j\omega}) = \frac{1}{2}\left[\tilde{\delta}(\omega - \tfrac{\pi}{3}) + \tilde{\delta}(\omega + \tfrac{\pi}{3})\right] + \frac{j}{2}\left[\tilde{\delta}(\omega - \tfrac{\pi}{4}) - \tilde{\delta}(\omega + \tfrac{\pi}{4})\right]$$
$$+ \frac{1}{2}\cdot\frac{1}{2}\left[\tilde{\delta}(\omega - \tfrac{3\pi}{4}) + \delta(\omega + \tfrac{3\pi}{4})\right]$$

$$Y(e^{j\omega}) = X(e^{j\omega}) H(e^{j\omega})$$

$$H(e^{j\omega}) = \begin{cases} 1 + \frac{2\omega}{\pi}, & \omega \in [-\frac{\pi}{2}, 0] \\ 1 - \frac{2\omega}{\pi}, & \omega \in [0, \frac{\pi}{2}] \\ 0, & \text{otherwise} \end{cases}$$

$$y[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} Y(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\cdot\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\delta(\omega-\tfrac{\pi}{5})+\delta(\omega+\tfrac{\pi}{5}) - j\left\{\delta(\omega-\tfrac{\pi}{4}) - \delta(\omega+\tfrac{\pi}{4})\right\} + \tfrac{1}{2}\left[\delta(\omega-\tfrac{3\pi}{4})+\delta(\omega+\tfrac{3\pi}{4})\right]\right] H(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\cdot\frac{1}{2}\left[H(e^{j\frac{\pi}{5}})e^{j\frac{\pi}{5}n} + H(e^{-j\frac{\pi}{5}})e^{-j\frac{\pi}{5}n} + \frac{1}{j}\left\{H(e^{j\frac{\pi}{4}})e^{j\frac{\pi}{4}n} - H(e^{-j\frac{\pi}{4}})e^{-j\frac{\pi}{4}n}\right\}\right]$$

$$= \frac{1}{2\pi}\cdot\frac{1}{2}\left[\frac{3}{5}\left(e^{j\frac{\pi}{5}n} + e^{-j\frac{\pi}{5}n}\right) + \frac{1}{j}\left\{\frac{1}{2}\left(e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}\right)\right\}\right]$$

$$= \frac{1}{2\pi}\left[\frac{3}{5}\cos\left(\frac{\pi}{5}n\right) + \frac{1}{2}\sin\left(\frac{\pi}{4}n\right)\right]$$

10) $h[n] * h[-n] \xleftrightarrow{DTFT} H(e^{j\omega})H(e^{-j\omega})$

$$G(e^{j\omega}) = H(e^{j\omega})H(e^{-j\omega}) \quad \ni \angle G(e^{j\omega}) = 0$$

11) $x[n] = \cos(\frac{\pi}{2}n)$, $h[n] = \frac{1}{5}\,\mathrm{sinc}\left(\frac{n}{5}\right)$, $\mathrm{sinc}(x) = \begin{cases} \frac{\sin(\pi x)}{\pi x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

$y[n] = x[n] * h[n]$ $\quad \left[\frac{1}{5} = \frac{\omega_c}{\pi} \Rightarrow \omega_c = \frac{\pi}{5}\right]$

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \frac{1}{2}\left[\delta(\omega-\tfrac{\pi}{2})+\delta(\omega+\tfrac{\pi}{2})\right]\mathrm{rect}\left(\frac{5\omega}{2\pi}\right)$$

$$y[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{2}\left[\delta(\omega-\tfrac{\pi}{2})+\delta(\omega+\tfrac{\pi}{2})\right]\mathrm{rect}\left(\frac{5\omega}{2\pi}\right) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\cdot\frac{1}{2}\int_{-\frac{\pi}{5}}^{\frac{\pi}{5}} \left[\delta(\omega-\tfrac{\pi}{2})+\delta(\omega+\tfrac{\pi}{2})\right] e^{j\omega n} d\omega = 0$$

12)

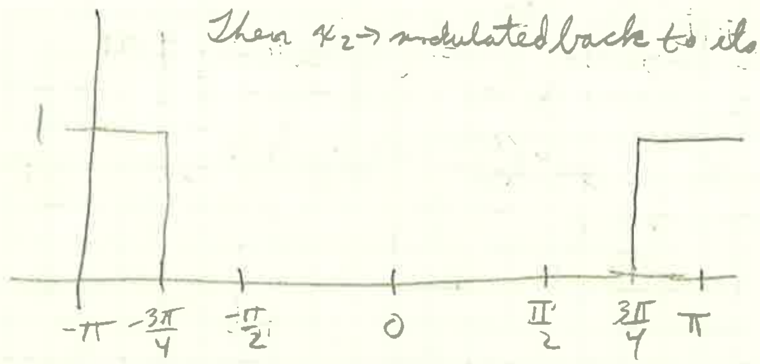
$$x[n] \longrightarrow \underset{(-1)^n}{\otimes} \longrightarrow \boxed{H(e^{j\omega})} \longrightarrow \underset{(-1)^n}{\otimes} \longrightarrow y[n]$$

$H(e^{j\omega})$ - ideal low pass filter, $\omega_c = \frac{\pi}{4}$

$$DTFT\{x(n) e^{-j\pi n}\} = X(e^{j(\omega+\pi)})$$

$X(e^{j(\omega+\pi)})H(e^{j\omega}) \Rightarrow$ x_1 - modulated to $\omega_0 = \pi$, thus filtered
x_2 - modulated to $\omega_0 = 0$, thus unmodified

Then $x_2 \to$ modulated back to its original location



14) $h[n]$ ideal lowpass, $\omega_c < \frac{\pi}{2}$

$$\cos^2 x = \frac{1}{2}(1+\cos 2x)$$

$$y(n) = x(n) * \{\cos^2(\tfrac{\pi}{2}n)h(n)\} - x(n) * \{\cos^2(\tfrac{\pi}{2}n)h[n]\} * h[n]$$

$$= \frac{1}{2}\left[x[n] * \{(1+\cos \pi n)h[n]\} - x[n] * \{(1+\cos(\pi n))h[n]\} * h[n]\right]$$

$$Y(e^{j\omega}) = \frac{1}{2}\left[\cancel{X(e^{j\omega})H(e^{j\omega})} + X(e^{j\omega})\frac{1}{2}\left(H(e^{j(\omega-\pi)}) + H(e^{j(\omega+\pi)})\right) - \cancel{X(e^{j\omega})H^2(e^{j\omega})}\right.$$

$$\left. - X(e^{j\omega})\frac{1}{2}\left(H(e^{j(\omega-\pi)}) + H(e^{j(\omega+\pi)})\right)\cancel{H(e^{j\omega})}\right]$$

$$Y(e^{j\omega}) = \frac{1}{2}\left[X(e^{j\omega})\frac{1}{2}\left(H(e^{j(\omega-\pi)}) + H(e^{j(\omega+\pi)})\right)\right]$$ Highpass filter with cutoff frequency $\pi - \omega_c$

15) $y[n] = (e^{j\pi n}+1)w[n] = (e^{j\pi n}+1)(x(n) * h(n))$, $x[n] = \delta[n]$

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) * DTFT\{1 + e^{j\pi n}\}$$

$$= H(e^{j\omega}) * (\tilde{\delta}(\omega) + \tilde{\delta}(\omega - \pi))$$

$$= \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} H(e^{j(\omega-\sigma)})\delta(\sigma)\, d\sigma + \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} H(e^{j(\omega-\sigma)})\delta(\sigma-\pi)\, d\sigma$$

$$= \frac{1}{2\pi} + \frac{1}{2\pi} = \frac{1}{\pi}$$

$$y[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{\pi} e^{j\omega n}\, d\omega = \frac{1}{2\pi^2} \cdot \frac{1}{jn} e^{j\omega n}\Big|_{-\pi}^{\pi} =$$