Homework for Module 4 Part 1

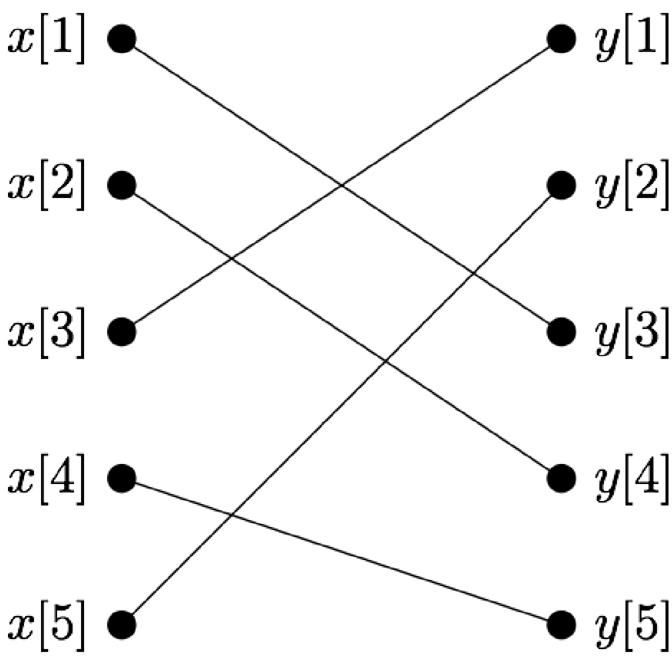
Quiz, 15 questions

1
point

1.
(Difficulty: $\star$) Among the choices below, select all the **linear** systems. (Please note that some of the choices use functions rather than discrete-time signals; the concept of linearity is identical in both cases).

- [x] The DTFT, i.e. transform a sequence $\mathbf{x}$ into $\mathrm{DTFT}\{\mathbf{x}\}$.
- [x] Second derivative, i.e.

  $y(t) = \frac{d^2}{dt^2}x(t)$
- [x] Scrambling, i.e. a permutation to the input sequence, e.g. :



- [x] AM radio modulation, i.e. multiply a signal $x[n]$ by a cosine at the carrier frequency :

  $y[n] = x[n]\cos(2\pi\omega_c n)$
- [x] Time-stretch, i.e. $y(t) = x(\alpha t)$, e.g. if you play an old LP-45 vinyl disc at 33 rpm, the time-stretch coefficient would be $\alpha = 33/45$.
- [ ] Clipping, i.e. enforce a maximum signal amplitude M,e.g.:

  $$y[n] = \begin{cases} x[n] & , x[n] \leq M \\ M & , \text{otherwise} \end{cases}$$
- [ ] Envelope detection (via squaring), i.e. $y[n] = |x[n]|^2 * h[n]$,where $h[n]$ is the impulse response of a *lowpass filter* such as the moving average filter.

1
point

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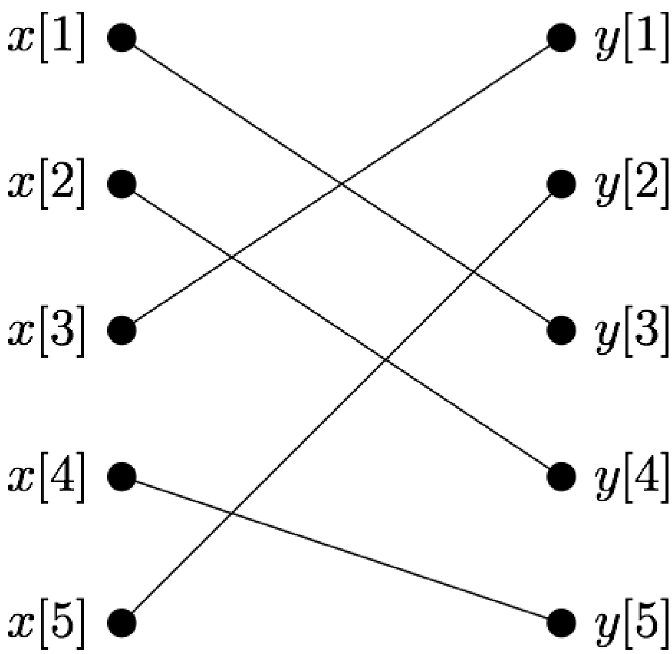
2.

(Difficulty: $\star$) Among the choices below, select all the **time-invariant** systems. (Please note that some of the choices use functions rather than discrete-time signals; the concept of time invariance is identical in both cases).

- [ ] Scrambling : apply a permutation to the input sequence, e.g. :



- [x] Clipping, i.e. enforce a maximum signal amplitude M,e.g.:

$$y[n] = \begin{cases} x[n] & , x[n] \leq M \\ M & , \text{otherwise} \end{cases}$$

- [x] Second derivative, i .e.

$$y(t) = \frac{d^2}{dt^2} x(t)$$

- [ ] The DTFT, i.e. transform a sequence $\mathbf{x}$ into $\mathrm{DTFT}\{\mathbf{x}\}$.

- [ ] Time-stretch, i.e. $y(t) = x(\alpha t)$, e.g. if you play an old LP-45 vinyl disc at 33 rpm, the time-stretch coefficient would be $\alpha = 33/45$

- [x] Envelope detection (via squaring), i.e. $y[n] = |x[n]|^2 * h[n]$,where $h[n]$ is the impulse response of a *lowpass filter* such as the moving average filter.

- [ ] AM radio modulation, i.e. multiply a signal $x[n]$ by a cosine at the carrier frequency :

$$y[n] = x[n]\cos(2\pi\omega_c n)$$

1
point

3.

(Difficulty: ⋆) The impulse response of a room can be recorded by producing a sharp noise (impulsive sound source) in a silent room, thereby capturing the scattering of the sound produced by the walls.

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The impulse response $h[n]$ of Lausanne Cathedral was measured by *Dokmanic et al.* by recording the sound of balloons being popped (hear it!).

The balloon is popped at time $n = 0$ and after a number of samples N, the reverberations die out, i.e. $h[n] = 0$ for $n < 0$ or $n > N$.

The acoustic of this large space can then be artificially recreated by convolving any audio recording with the impulse response, e.g. this cello recording becomes this.

What are the properties of $h[n]$? *(tick all the correct answers)*

- [ ] Anticausal
- [x] FIR
- [x] BIBO stable

1 point

4.

(Difficulty: ⋆) Let

$h[n] = \delta[n] - \delta[n-1]$

$x[n] = \begin{cases} 1 & , n \geq 0, \\ 0 & , \text{else.} \end{cases}$

$y[n] = x[n] * h[n]$.

Compute $y[-1]$, $y[0]$, $y[1]$, $y[2]$ and write the result as space-separated values. E.g.: If you find $y[-1] = -2$, $y[0] = -1$, $y[1] = 0$, $y[2] = 1$, you should enter

```
1   -2 -1 0 1
```

0 1 0 0

1 point

5.

(Difficulty: ⋆⋆) Consider the filter $h[n] = \delta[n] - \delta[n-1]$,

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the signal $x[n] = \begin{cases} n & , n \geq 0, \\ 0 & , \text{else.} \end{cases}$

and the output $y[n] = x[n] * h[n]$.

Compute $y[-1]$, $y[0]$, $y[1]$, $y[2]$ and write the result as space-separated values. E.g.: If you find $y[-1] = -2$, $y[0] = -1$, $y[1] = 0$, $y[2] = 1$, you should enter

```
1   -2 -1 0 1
```

0 0 1 1

---

1 point

6.

(Difficulty: ⋆ ⋆ ⋆) Which of the following filters are BIBO-stable?

Assume $N \in \mathbb{N}$ and $0 < \omega_c < \pi$.

- [ ] The ideal low pass filter with a cutoff frequency ω_c: $H(e^{j\omega}) = \begin{cases} 1 & |\omega| \leq \omega_c \\ 0 & \text{otherwise} \end{cases}$.
- [x] Any filter $h[n]$ with finite support and bounded coefficients.
- [ ] The following smoothing filter: $h[n] = \sum_{k=0}^{\infty} \frac{1}{k+1} \delta[n-k]$.
- [x] The moving average: $h[n] = \frac{\delta[n]+\delta[n-1]}{2}$.

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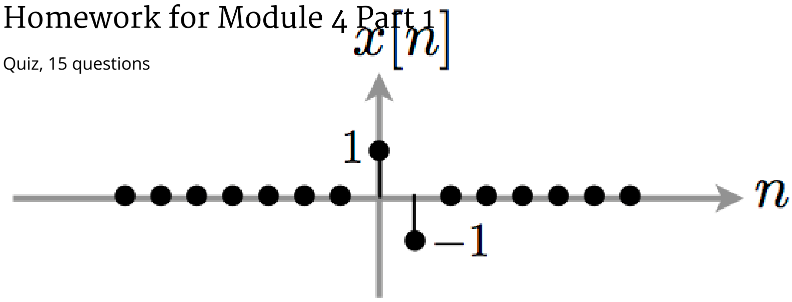
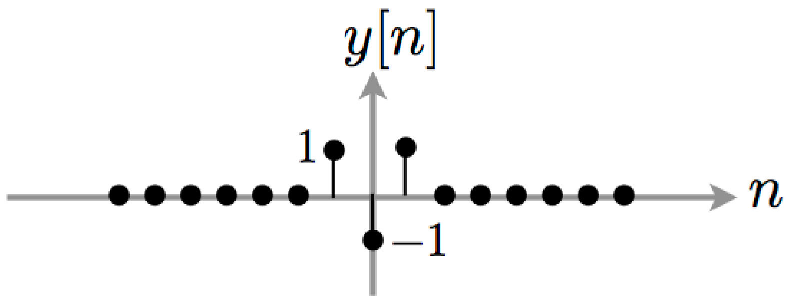
1 point

7.

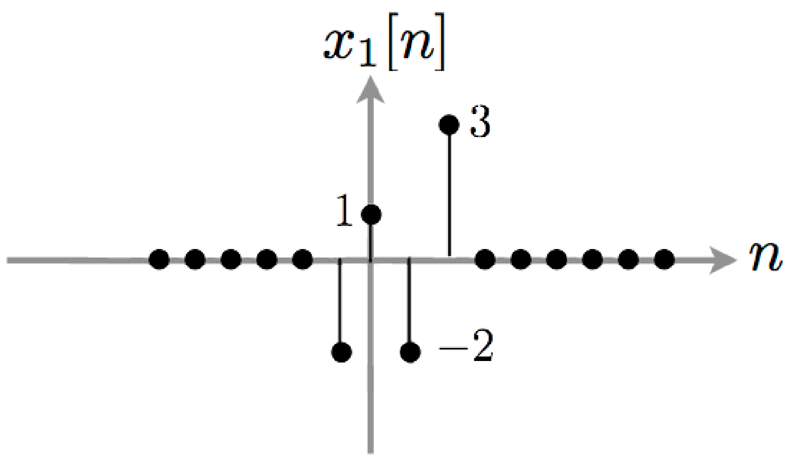
(Difficulty: ⋆⋆) Consider an LTI system $\mathcal{H}$. When the input to $\mathcal{H}$ is the following signal



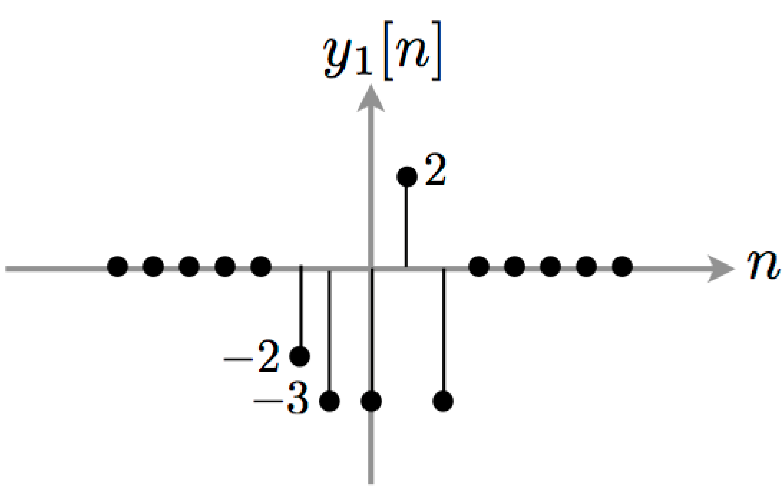
then the output is



Assume now the input to $\mathcal{H}$ is the following signal



Which one of the following signals is the system's output?



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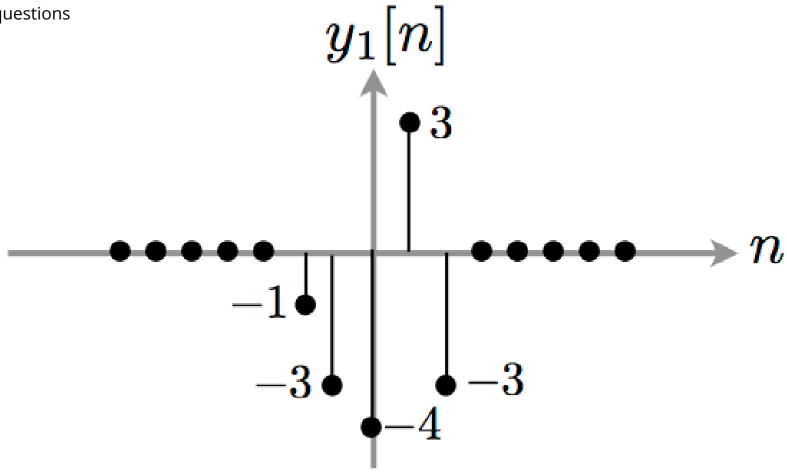

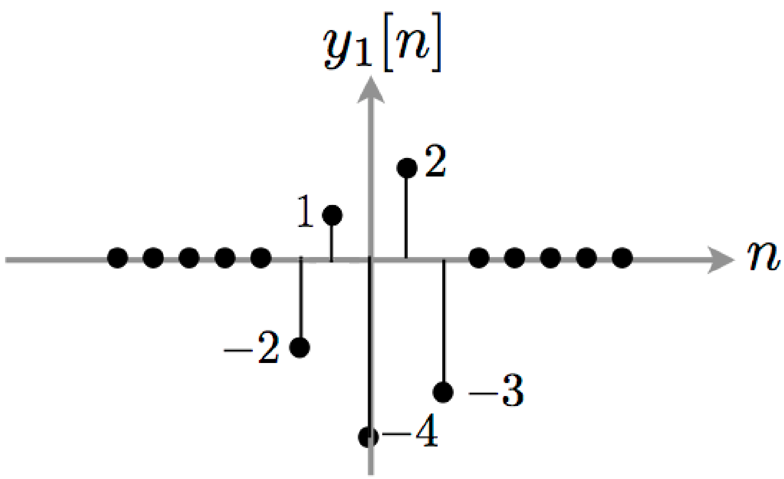

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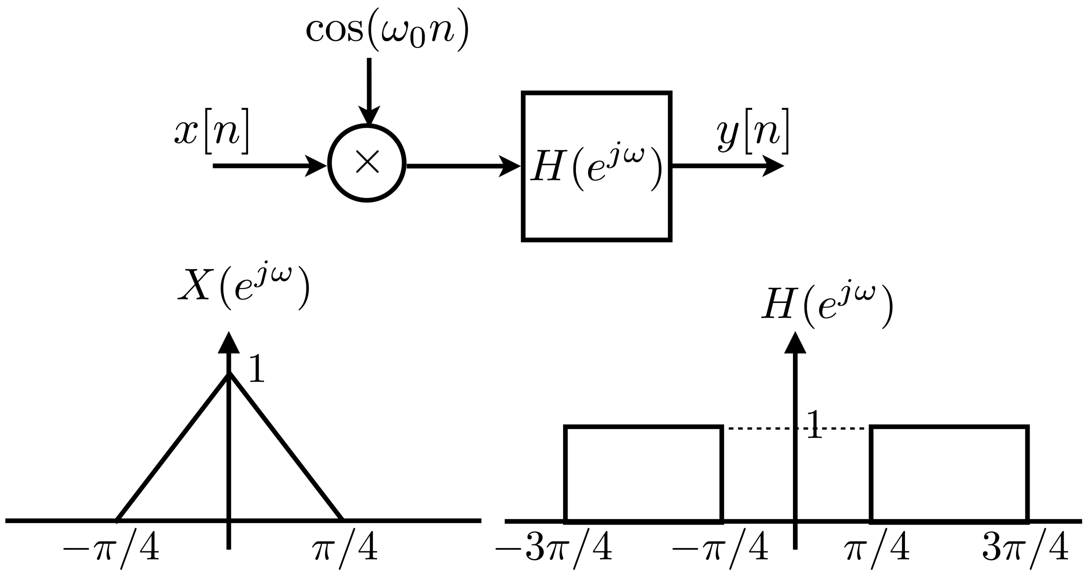
1 point

8.

(Difficulty: $\star$) Consider the system shown below, consisting of a cosine modulator at frequency ω_0 followed by an ideal bandpass filter $h[n]$ whose frequency response is also shown in the figure; assume that the input to the system is the signal $x[n]$, whose spectrum is shown below.



Determine the value of $\omega_0 \in [0, 2\pi]$ that maximizes the energy of the output $y[n]$ when the input is $x[n]$.

Remember that π must be entered in the answer box as pi.

Preview

$\frac{\pi}{2}$

pi/2

1
point

9.

(Difficulty: ⋆) Consider a lowpass filter with the following frequency response.

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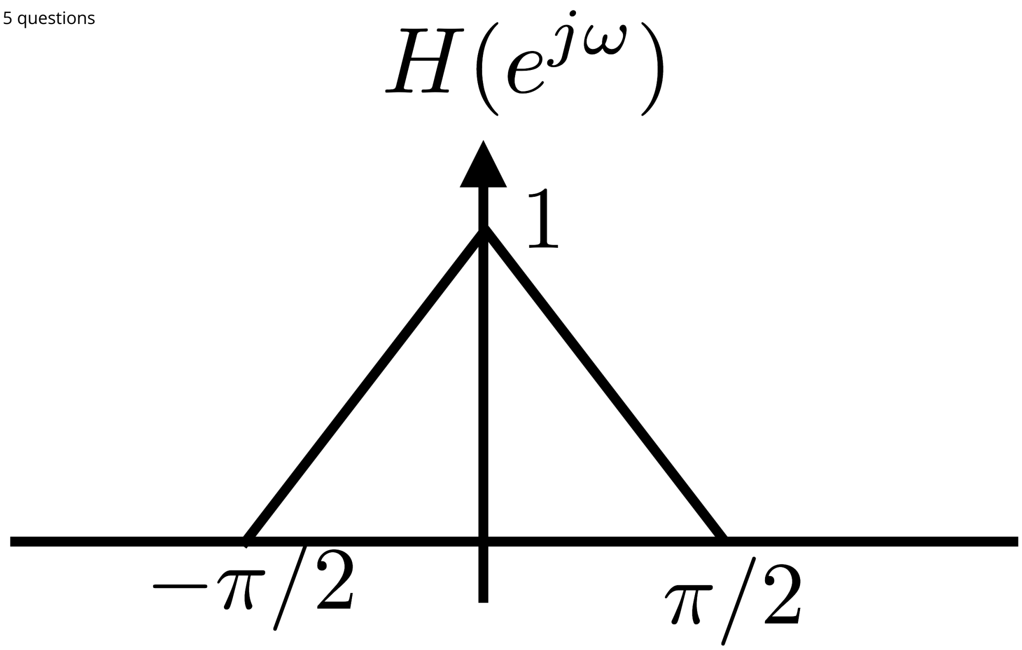


What is the output $y[n]$ when the input to this filter is $x[n] = \cos(\frac{\pi}{5}n) + \sin(\frac{\pi}{4}n) + 0.5\cos(\frac{3\pi}{4}n)$?

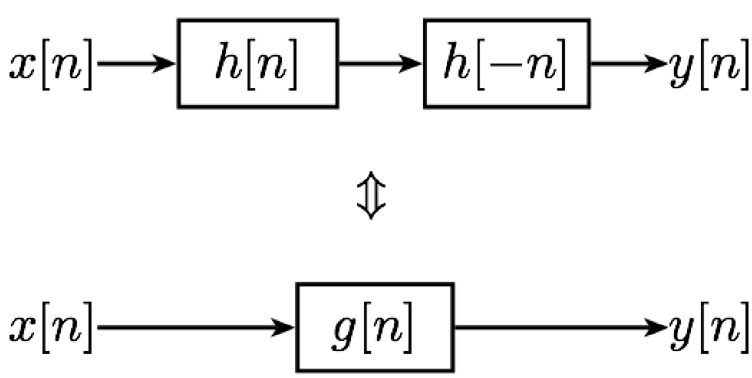
Preview

$\frac{1}{2}\sin\left(\frac{\pi n}{4}\right) + \frac{3}{5}\cos\left(\frac{\pi n}{5}\right)$

3/5*cos(pi/5*n)+1/2*sin(pi/4*n)

1
point

10.
(Difficulty: ⋆) Consider a filter with real-valued impulse response $h[n]$. The filter is cascaded with another filter whose impulse response is $h'[n] = h[-n]$, i.e. whose impulse response is the time-reversed version of $h[n]$:



The cascade system can be seen as a single filter with impulse response g[n].

What is the phase of G(ejω)?

Preview

0

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1 point

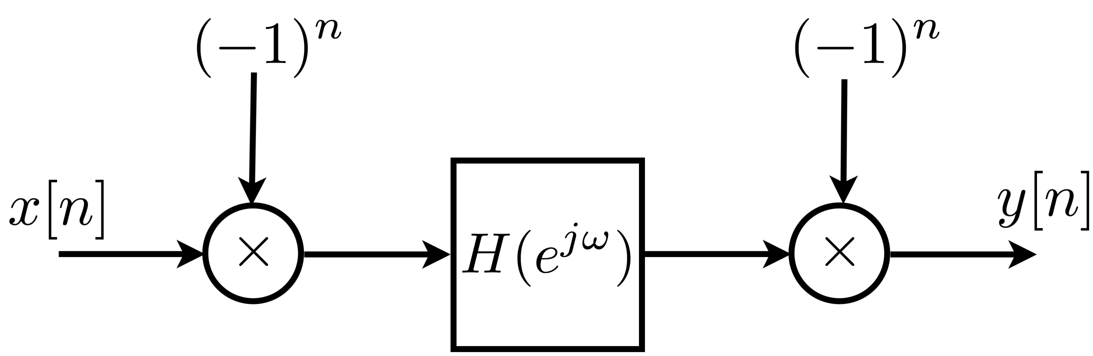
11.
(Difficulty: ⋆) Let $x[n] = \cos(\frac{\pi}{2}n)$ and $h[n] = \frac{1}{5}\mathrm{sinc}(\frac{n}{5})$. Compute the convolution $y[n] = x[n] * h[n]$, and write the value of $y[5]$.

Hint: First find the convolution result in the frequency domain.

0

1 point

12.
(Difficulty: ⋆⋆) Consider the system below, where $H(e^{j\omega})$ is an ideal lowpass filter with cutoff frequency $\omega_c = \pi/4$:



Consider two input signals to the system:

- $x_1[n]$ is bandlimited to $[-\pi/4, \pi/4]$
- $x_2[n]$ is band-limited to $[-\pi, -3\pi/4] \cup [3\pi/4, \pi]$.

Which of the following statements is correct?

- [ ] Both $x_1[n]$ and $x_2[n]$ are eliminated by the system.
- [ ] Both $x_1[n]$ and $x_2[n]$ are not modified by the system.
- [ ] $x_1[n]$ is not modified by the system while $x_2[n]$ is eliminated.
- [x] $x_2[n]$ is not modified by the system while $x_1[n]$ is eliminated.

1 point

13.

(Difficulty: $\star$) $x[n]$ and $y[n]$ are two square-summable signals in $\ell_2(\mathbb{Z})$; $X(e^{j\omega})$ and $Y(e^{j\omega})$ are their corresponding DTFTs.

We want to compute the value.

$\sum_{n=-\infty}^{\infty} x[n]y^*[n]$.

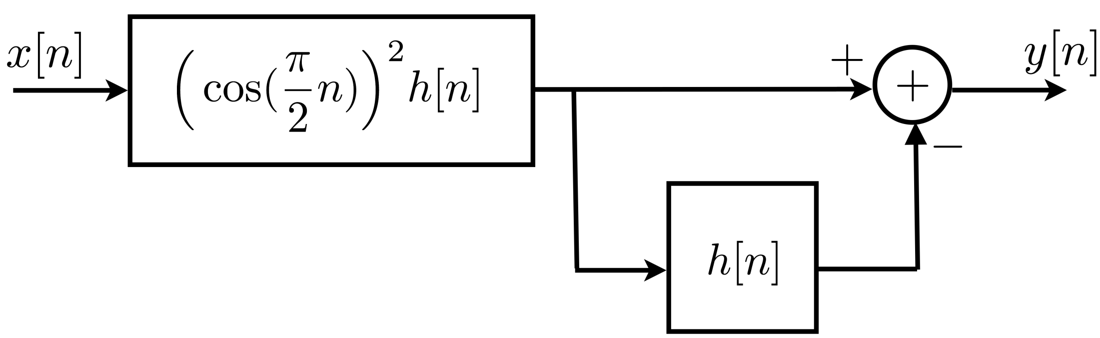
in terms of $X(e^{j\omega})$ and $Y(e^{j\omega})$. Select the correct expression among the choices below.

- $\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega})d\omega$
- $\frac{1}{2\pi} X(e^{j\omega})Y^*(e^{j\omega})$
- $\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y(e^{-j\omega})d\omega$
- $X(e^{j\omega}) * Y(e^{-j\omega})$
- $\frac{1}{2\pi} X(e^{j\omega})Y(e^{-j\omega})$
- $X(e^{j\omega})Y(e^{-j\omega})$

1 point

14.

(Difficulty: $\star\star\star$) $h[n]$ is the impulse response of an ideal lowpass filter with cutoff frequency $\omega_c < \frac{\pi}{2}$. Select the correct description for the system represented in the following figure?



Hint: Use the trigonometric identity $\cos(x)^2 = \frac{1}{2}(1+\cos(2x))$.

- A lowpass filter with gain $\frac{1}{2}$ and cutoff frequency $2\omega_c$.
- A lowpass filter with gain 1 and cutoff frequency $\omega_c/2$.
- A lowpass filter with gain 1 and cutoff frequency ω_c.
- A highpass filter with gain $\frac{1}{2}$ and cutoff frequency $\pi - \omega_c$.
- A highpass filter with gain $\frac{1}{4}$ and cutoff frequency ω_c.
- A highpass filter with gain 1 and pass band $[\omega_c, \pi - \omega_c]$.

1 point

15.

(Difficulty: ⋆⋆⋆) Consider the following system, where $H(e^{j\omega})$ is a half-band filter, i.e. an ideal lowpass with cutoff frequency $\omega_c = \pi/2$:

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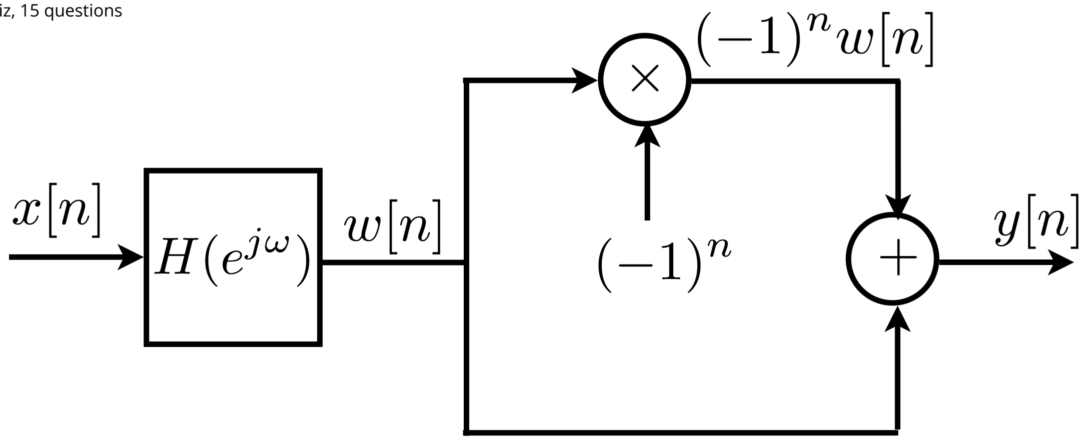

Assume the input to the system is $x[n] = \delta[n]$. Compute

$\sum_{n=-\infty}^{\infty} y[n]$

- Hint: Perform the derivations in the frequency domain.

1

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