2) $\vec{V}^{(0)} = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}, \quad \vec{V}^{(1)} = \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{bmatrix}, \quad \vec{V}^{(2)} = \begin{bmatrix} 1/2 \\ -1/2 \\ 1/2 \\ -1/2 \end{bmatrix}$

Find $\vec{V}^{(3)}$, row reduce matrix and find the standard basis columns. Then, apply Gram-Schmidt

$$A = \begin{bmatrix} 1/2 & 1/2 & 1/2 & 1 & 0 & 0 & 0 \\ 1/2 & 1/2 & -1/2 & 0 & 1 & 0 & 0 \\ 1/2 & -1/2 & 1/2 & 0 & 0 & 1 & 0 \\ 1/2 & -1/2 & -1/2 & 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & -1 & -1 & 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & -1 & -1 & -1 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & -1 & -1 & -1 & 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 1 & 1 & 0 & 0 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

The basic vectors are $A_{*0}, A_{*1}, A_{*2}, A_{*4}$

$$\left\{ \vec{V}^{(0)}, \vec{V}^{(1)}, \vec{V}^{(2)}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

Gram-Schmidt

$$\vec{\psi}^{(3)} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} - \left\langle \vec{V}^{(0)}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\rangle \vec{V}^{(0)} - \left\langle \vec{V}^{(1)}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\rangle \vec{V}^{(1)} - \left\langle \vec{V}^{(2)}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\rangle \vec{V}^{(2)}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} - 1/2 \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix} + 1/2 \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{bmatrix} - 1/2 \begin{bmatrix} 1/2 \\ -1/2 \\ 1/2 \\ -1/2 \end{bmatrix} = \begin{bmatrix} -1/4 \\ 1/4 \\ 1/4 \\ -1/4 \end{bmatrix}$$

$$\Rightarrow \vec{V}^{(3)} = \begin{bmatrix} -1/2 \\ 1/2 \\ 1/2 \\ -1/2 \end{bmatrix}$$

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