Homework for Module 5

Quiz, 19 questions

1 point

1.

(Difficulty: $\star$) Select the correct statement among the ones below:

- [ ] In continuous time, there is always a maximum frequency Ω.
- [ ] The total energy of a continuous time signal is always infinite.
- [ ] In continuous time, the only measure of frequency is Hertz (i.e. 1/seconds)
- [x] A signal that is Ω_N-bandlimited is also Ω_M-bandlimited if $\Omega_M \geq \Omega_N$
- [ ] An interpolated discrete-time signal is always bandlimited.

---

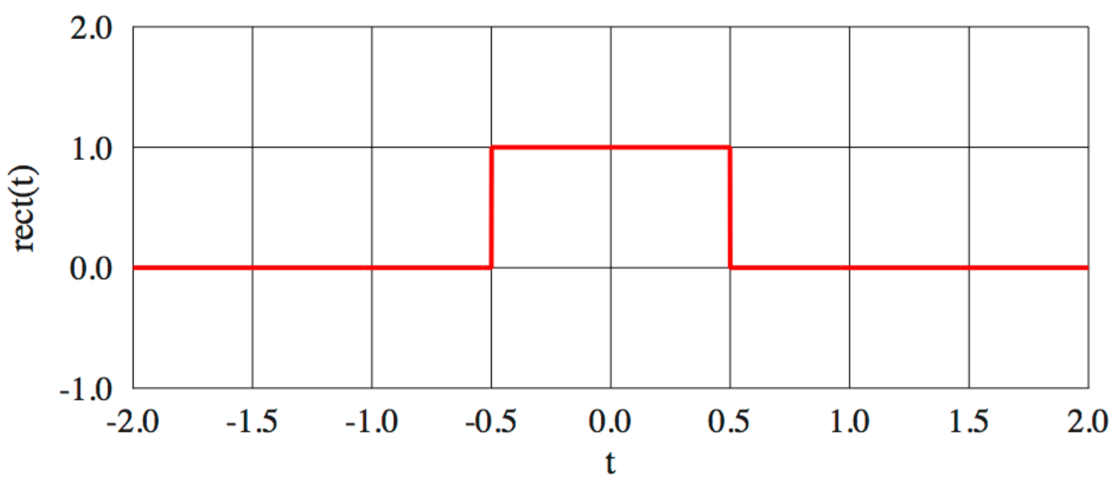
1 point

2.

(Difficulty: $\star$) A piecewise constant function in continuous time can always be expressed as a linear combination of scaled and translated unit step functions. Recall the definition of the unit step is:

$$u(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{otherwise} \end{cases}$$

Consider the function shown in the figure below:



Which is the correct expression for the function in terms of units steps?

- (●) $u(t+\frac{1}{2}) - u(t-\frac{1}{2})$
- ( ) $u(t-\frac{1}{2}) + u(t+\frac{1}{2})$
- ( ) $u(t-1) - u(t+\frac{1}{2})$
- ( ) $\frac{1}{2}\cdot u(t) - \frac{1}{2}\cdot u(t)$
- ( ) $u(t+\frac{1}{2}) - u(t+1)$

1
point

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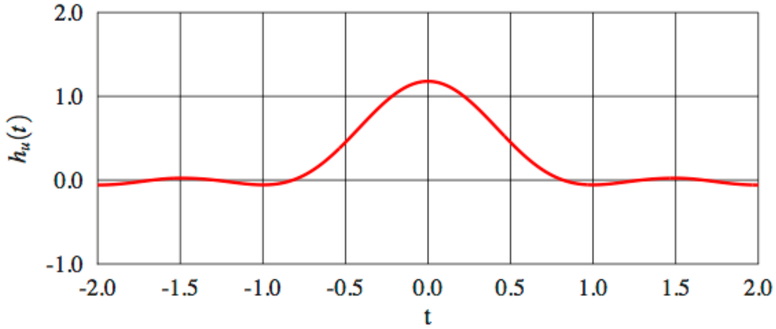
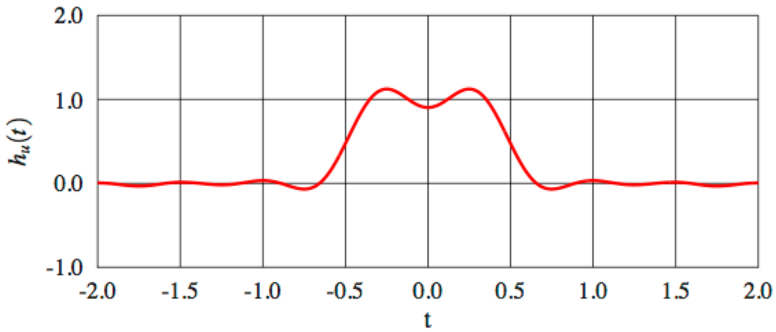
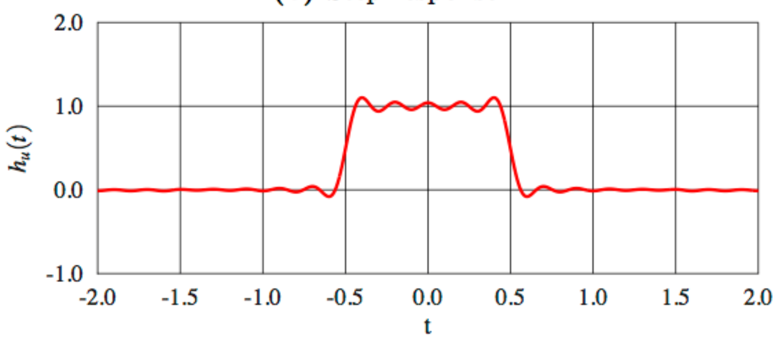
3. (Difficulty: ⋆) The figures below show the result of filtering the signal $x(t) = \mathrm{rect}(t)$ with four ideal lowpass filters with different cutoff frequencies $\Omega_{c_i}, i = 0, 1, 2, 3$.



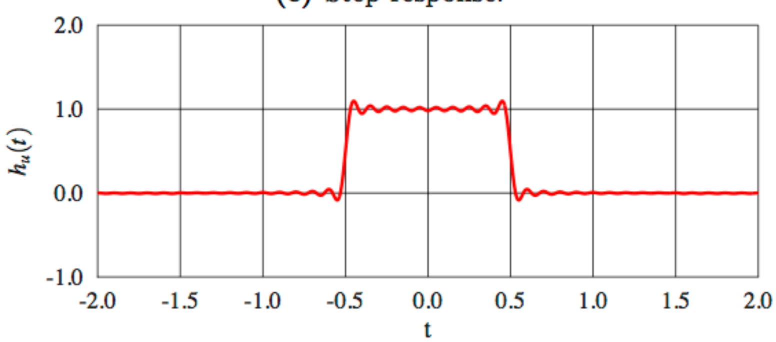
(a) Step response.



(b) Step response.



(c) Step response.



(d) Step response.

Check the correct statements below.

- [ ] There exists a $T \in \mathbb{R}$ such that all the output signals are equal to zero for $|t| > T$.
- [x] The cutoff frequencies Ω_{c_i} are increasing from plot (a) to plot (d).

1 point

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4. Quiz, 19 questions

(Difficulty: ⋆⋆) Consider the interpolator

$$I(t) = \begin{cases} 1 - 2|t| & \text{for } |t| \leq \dfrac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

The interpolator has a triangular shape in the time domain. Select the correct expression for $I(j\Omega)$, the Fourier transform of the interpolator. (Hint: a triangle can be obtained by convolving two rectangles).

- ○ $I(j\Omega) = \frac{1}{2}\mathrm{sinc}^2\left(\frac{\Omega}{2\pi}\right)$
- ○ $I(j\Omega) = \frac{1}{4}\mathrm{sinc}\left(\frac{\Omega}{4\pi}\right)$
- ○ $I(j\Omega) = \frac{1}{2}\mathrm{sinc}\left(\frac{\Omega}{4\pi}\right)$
- ● $I(j\Omega) = \frac{1}{2}\mathrm{sinc}^2\left(\frac{\Omega}{4\pi}\right)$

1 point

5.

(Difficulty: ⋆ ⋆ ⋆) Consider a discrete-time signal $x[n]$ whose spectrum between $-\pi$ and π is

$$X(e^{j\omega}) = \begin{cases} 1 & \text{for } |\omega| \leq \frac{2\pi}{3} \\ 0 & \text{otherwise} \end{cases}$$

with 2π-periodicity over the entire frequency axis.

$x[n]$ is interpolated to a continuous-time signal $x(t) = \sum_{n=-\infty}^{\infty} x[n] I\left(\frac{t-nT_s}{T_s}\right)$ using the interpolator $I(t)$ defined in the previous question:

$$I(t) = \begin{cases} 1 - 2|t| & \text{for } |t| \leq \dfrac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

Which graph represents the resulting spectrum of $x(t)$?

○

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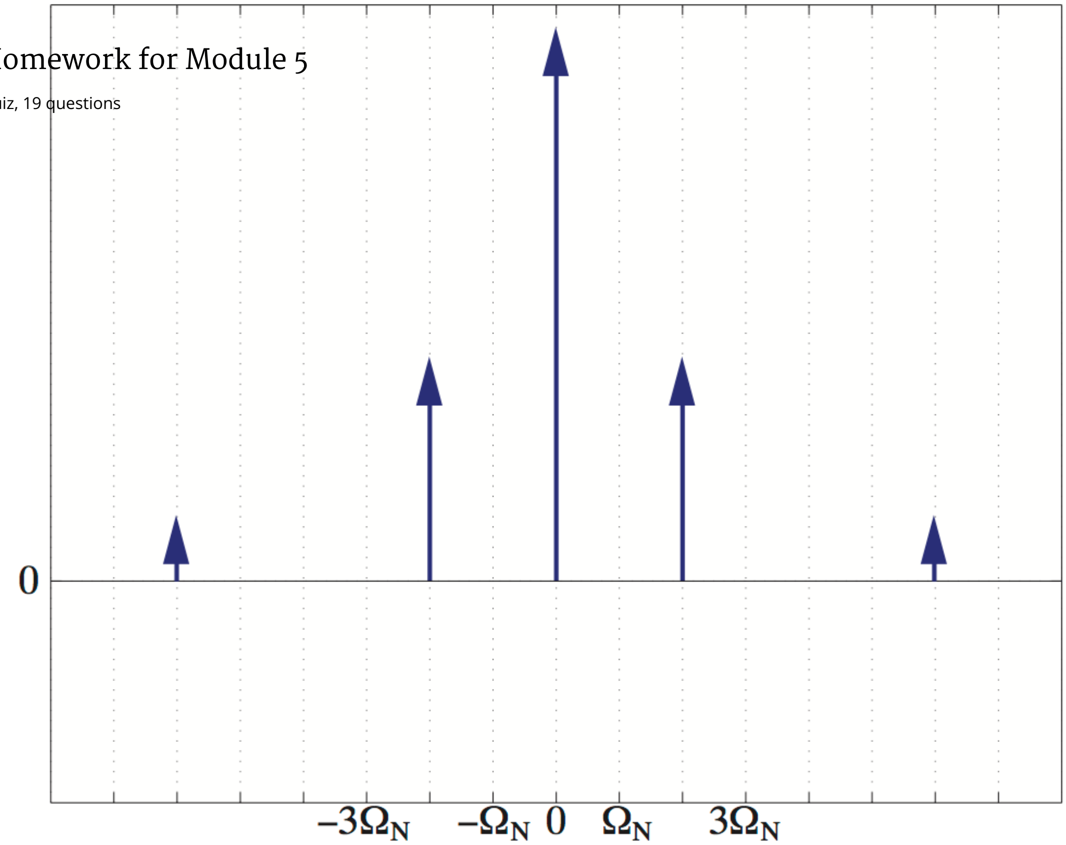
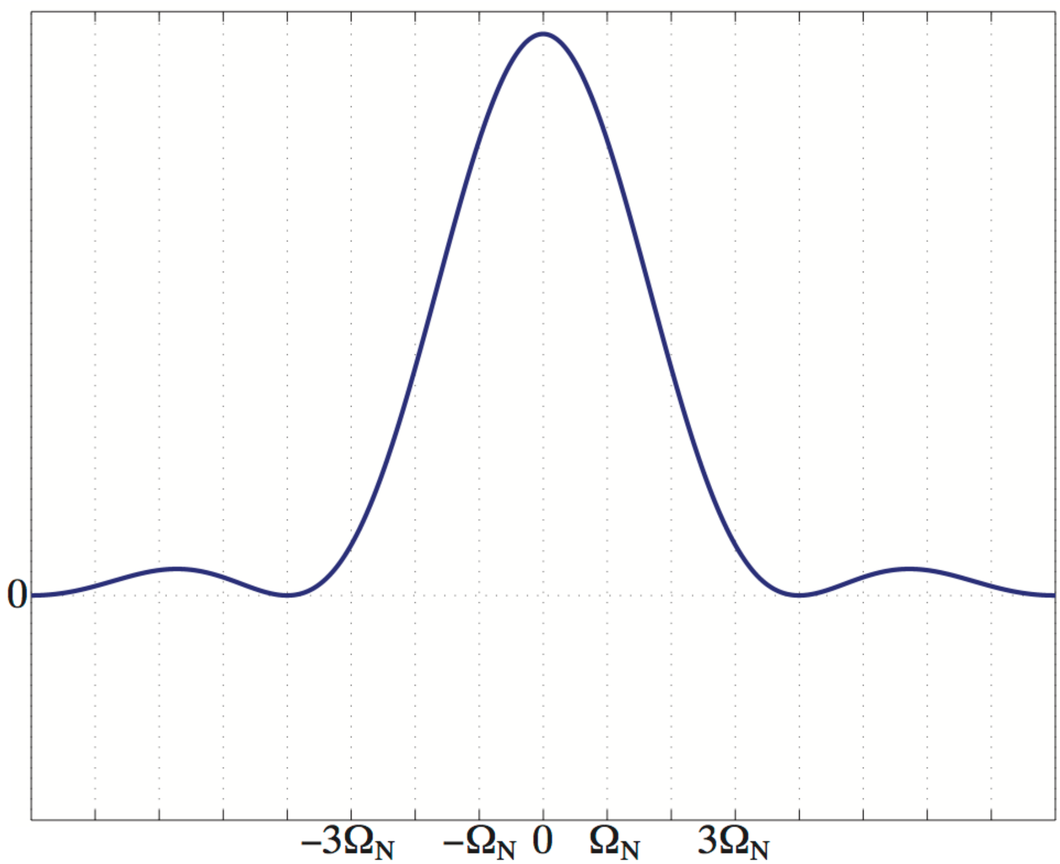
Quiz, 19 questions



Spectrum



Spectrum

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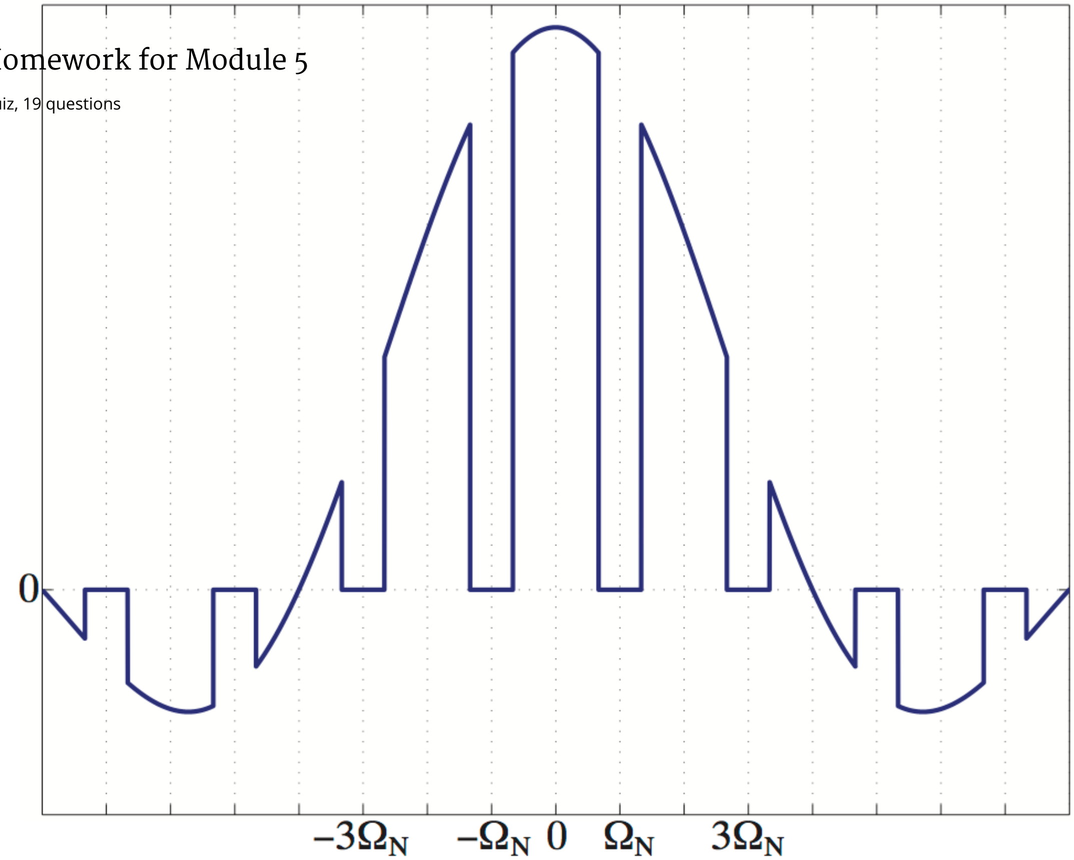
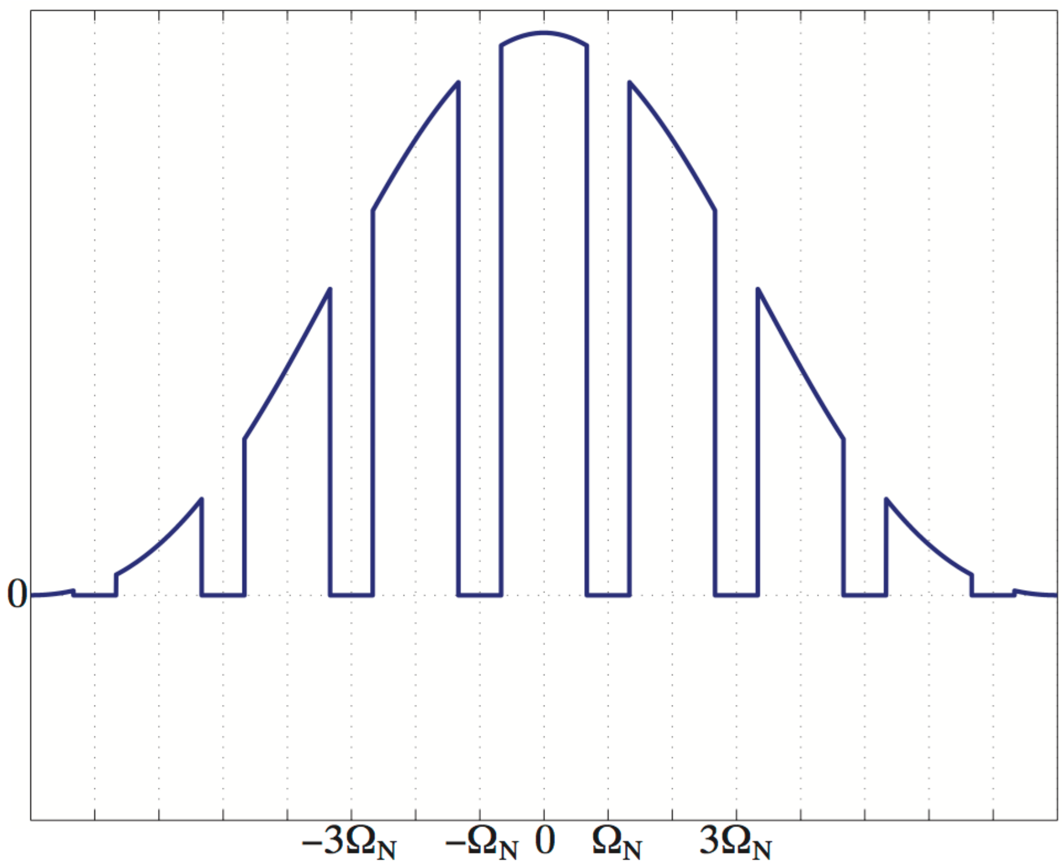
Quiz, 19 questions



Spectrum



Spectrum

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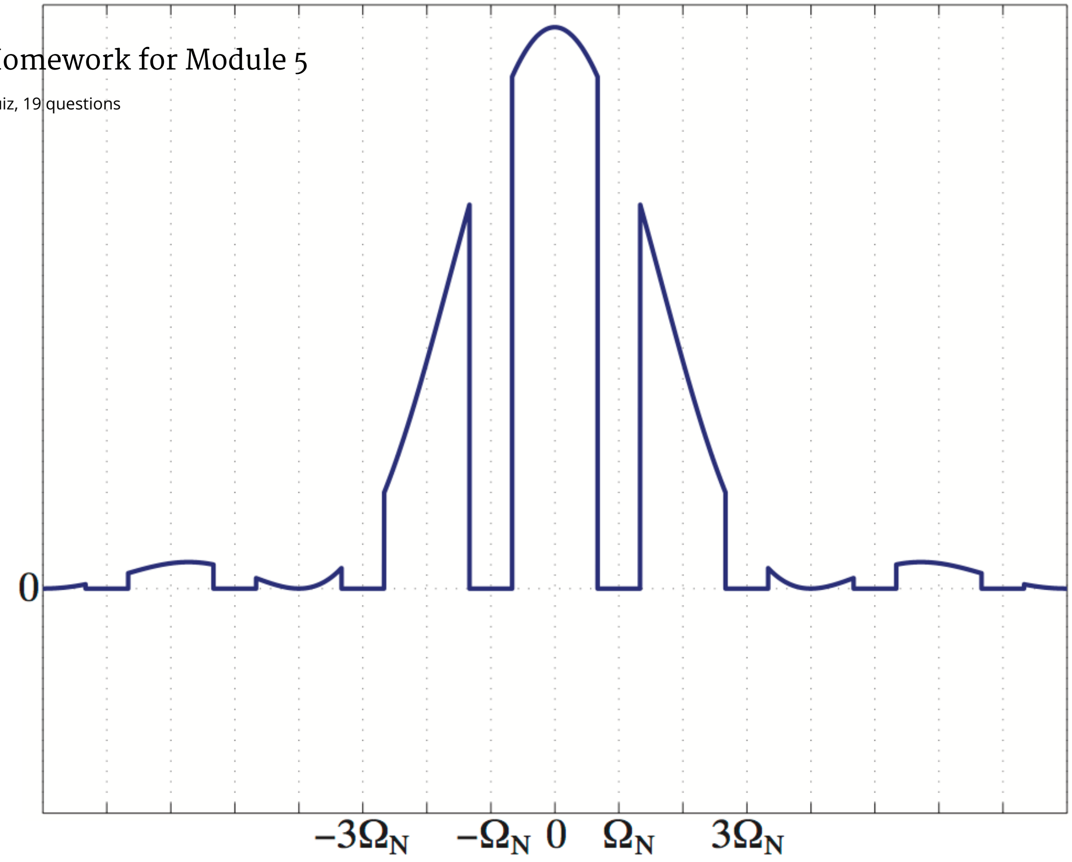


Spectrum

1 point

6.

(Difficulty: ⋆⋆) When using a first-order interpolator (such as the one described in the previous question) to interpolate a **finite-support** sequence, which of the following statements are true?

- [x] The interpolated signal is bandlimited.
- [x] The interpolated signal has finite length in time because of the limited support of the interpolating function $I(t)$.
- [ ] The spectrum between $[-\Omega_N, \Omega_N]$ (the baseband) is distorted by the non-flat response of the interpolating function over the baseband.
- [x] The periodic copies of $X(e^{j\pi\Omega/\Omega_N})$ outside of $[-\Omega_N, \Omega_N]$ are not eliminated by the interpolation filter, since it is not an ideal lowpass.

1 point

7.

(Difficulty: ⋆) Select the correct statement(s).

- [x] Increasing the interpolation interval, T_S results in a wider spectrum of the interpolated signal.
- [x] The sampling of a bandlimited signal $x(t)$ (with maximum frequency F_N) with a sampling frequency $F_S \geq 2 \cdot F_N$ will result in no information loss.
- [x] The sampling theorem implies that the space of bandlimited functions is a Hilbert Space.

1
point

Homework for Module 5

8. Quiz, 19 questions

(Difficulty: ⋆) Consider a real-valued, continuous-time signal $x(t)$. All you know about the signal is that $x(t) = 0$ for $|t| > t_0$. Can you determine a sampling frequency F_S so that when you sample $x(t)$, there is no aliasing?

Select the correct statement(s).

- ○ No, since the exact numerical value for t_0 is not given.
- ○ No. The signal is time-limited, so is not bandlimited. There will always be a certain amount of aliasing in the sampled version.
- ● Yes, since F_S does not depend on the support of $x(t)$, but on the highest frequency contained within it.
- ○ Yes. The signal is time-limited and therefore it is bandlimited. Consequently, there exists a sampling frequency F_S such that no aliasing occurs.

1
point

9.

(Difficulty: ⋆⋆) Listen to the sound of a triangle (the percussive musical instrument):

Below you are given 4 processed version of the original sound. In which one can you hear aliasing artifacts due to downsampling?

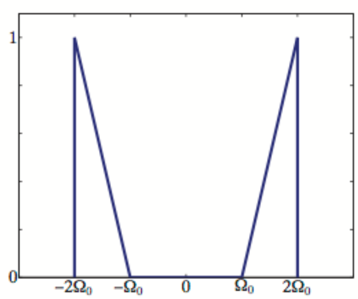
Try to find the answer just by listening. If you are stuck, you may use a numerical package to study the spectrum.

- ○ Sound A
- ○ Sound B
- ○ Sound C
- ● Sound D

1
point

10.

(Difficulty: ⋆) Consider a real-valued, continuous-time signal $x_c(t)$ with the following spectrum:



What is the maximum sampling period T_s that we can use to sample $x_c(t)$ so that the spectral copies caused by sampling do not overlap?

- ○ $T_s = 4\pi/\Omega_0$
- ● $T_s = \pi/\Omega_0$
- ○ $T_s = 2\pi/\Omega_0$

$T_s = \pi/(4\Omega_0)$

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$T_s = \pi/(2\Omega_0)$

Quiz, 19 questions

1 point

11.
(Difficulty: ⋆) Assume $x(t)$ is a continuous-time pure sinusoid at 10 kHz. The signal is raw-sampled at 8 kHz and then interpolated back to a continuous-time signal with an interpolator at 8 kHz. What is the perceived frequency in kHz of the interpolated sinusoid?

6

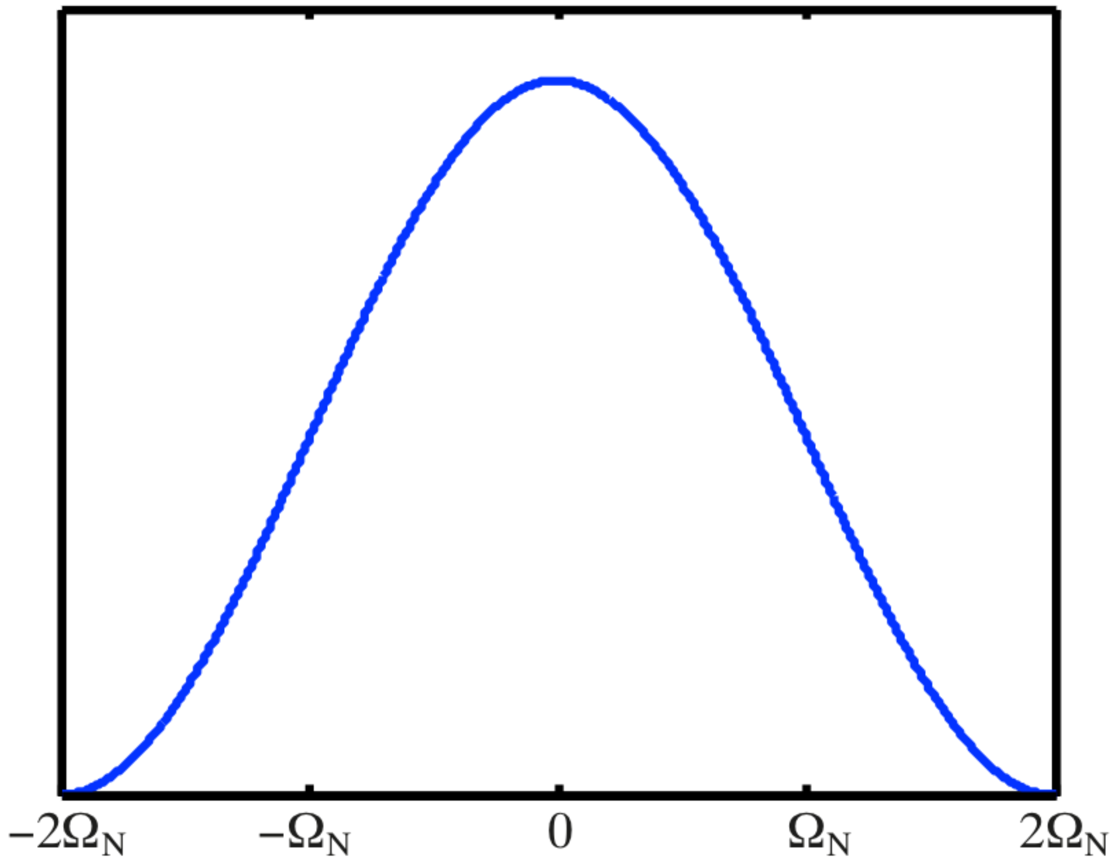
1 point

12.

(Difficulty: ⋆⋆⋆) Consider a real-valued continuous-time signal $x_c(t)$ whose bandlimited Fourier transform is

$$X_c(j\Omega) = \begin{cases} \cos^2(\frac{\pi}{4}\frac{\Omega}{\Omega_N}) & \text{for } |\Omega| < 2\Omega_N \\ 0 & \text{otherwise} \end{cases}$$

$X_c(j\Omega)$ is shown here between $-2\Omega_N$ and $2\Omega_N$:



The signal $x(t)$ is now sampled with $T_s = \pi/\Omega_N$; this defines the periodized spectrum

$\tilde{X}_c(j\Omega) = \sum_k X_c(j(\Omega - 2k\Omega_N))$

Which of the following pictures depicts $\tilde{X}_c(j\Omega)$?

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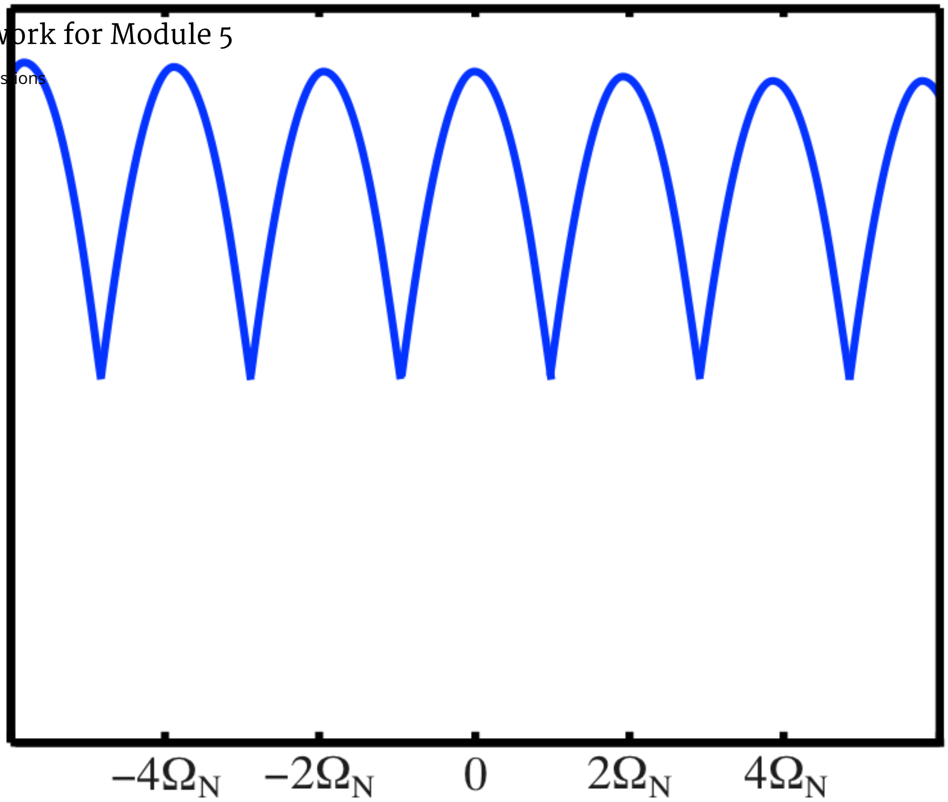
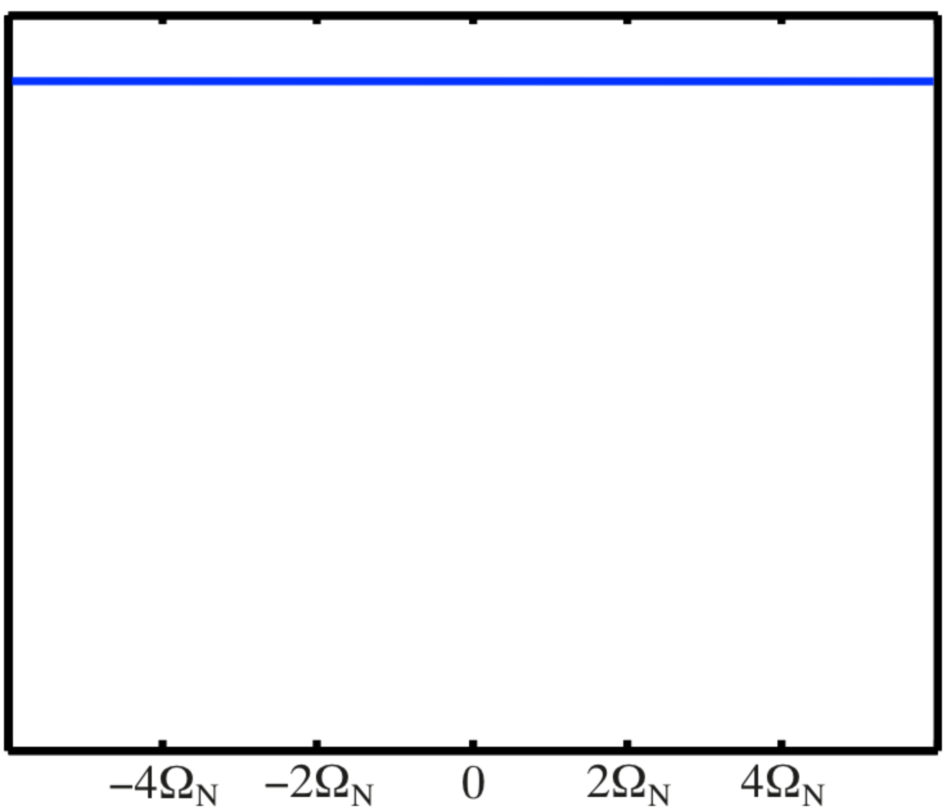
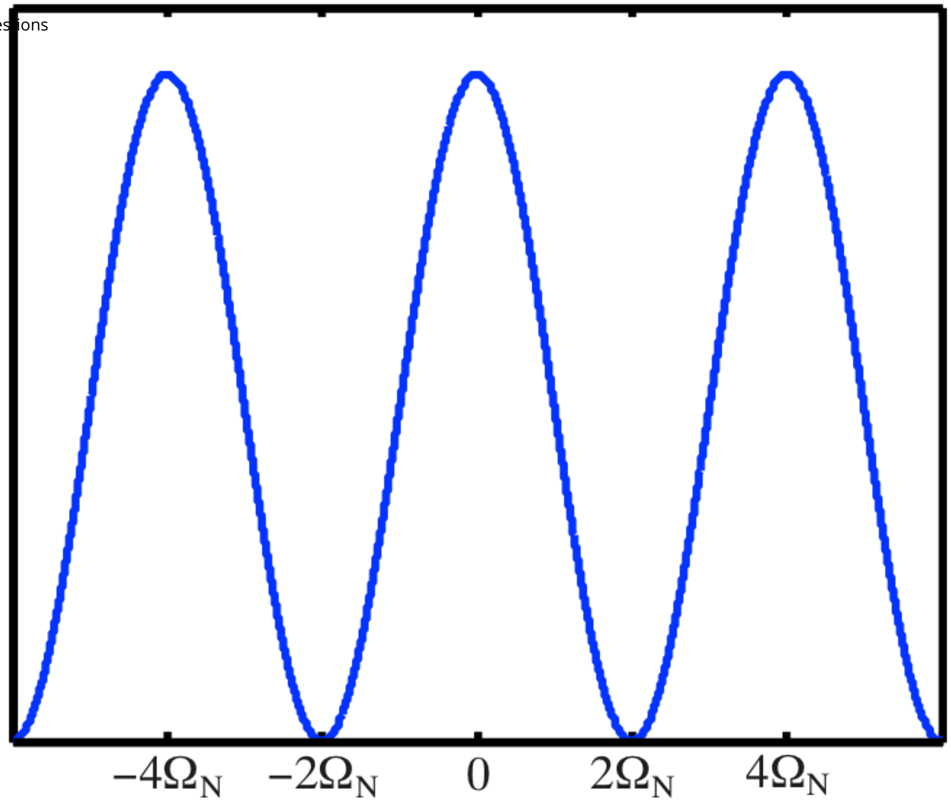
Quiz, 19 questions



Spectrum



Spectrum



Spectrum

1 point

13.

(Difficulty: ⋆) A chirp is a sinusoidal signal whose frequency increases over time. Consider the linear chirp defined as

$x(t) = \cos(2\pi f_0 t + \alpha\pi t^2)$.

By setting $\alpha = \frac{f_1 - f_0}{t_1}$~~, then f_0 is the inital frequency of the chirp at time $t = 0$, and $\frac{f_0+f_1}{2}$ is the instantaneous frequency at time~~ $t = t_1$ and the frequency increases linearly with time.

Write a program in your favorite programming language that computes a discrete-time version of the chirp at a sampling frequency $F_s = 8000\mathrm{Hz}$ for $0 \leq t \leq 2$ seconds. Set $f_0 = 0\mathrm{Hz}$ and $f_1 = 10\mathrm{Hz}$ and $t_1 = 2$ seconds.

Use the program to count how many times the signal crosses the abscissa. How many zero-crossings can you count in the generated chirp?

Enter the number of zero-crossings as an integer.

800

1 point

14.

(Difficulty: ⋆⋆) Consider the same chirp signal as in the previous question. Which of the following statements are true for the signal?

Homework for Module 5

Quiz, 19 questions

Tick all the correct statements.

- [ ] A linear chirp needs the same time to go from 100Hz to 200Hz as to go from 1000Hz to 2000Hz.
- [ ] If the runtime for the chirp generation is not limited, we cannot set a sampling frequency such that there will be no aliasing.
- [ ] If we sample the signal with a very low sampling frequency, the resulting signal will have an instantaneous frequency that varies between 0 Hz and 2π
- [ ] The instantaneous frequency of the chirp is linearly increasing with time.

---

1 point

15.
(Difficulty: ⋆) Consider the stochastic process defined as

$$Y[n] = X[n] + \beta X[n-1]$$

where $\beta \in \mathbb{R}$ and $X[n]$ is a zero-mean wide-sense stationary process with autocorrelation

$$R_X[k] = \sigma^2 \alpha^{|k|}, \qquad |\alpha| \leq 1$$

$Y[n]$ can also be expressed as filtered version of $X[n]$ where the filter's impulse response $h[n]$ is:

- ( ) $h[n] = \delta[n] - \beta\delta[n+1]$
- ( ) $h[n] = \delta[n] + \beta\delta[n+1]$
- ( ) $h[n] = \delta[n] + \beta\delta[n-1]$
- (●) $h[n] = \delta[n] - \beta\delta[n-1]$
- ( ) $h[n] = \delta[n+1] + \beta\delta[n]$

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1 point

16.
(Difficulty: ⋆ ⋆ ⋆) Using the same setup as in the previous question, select the correct expression for the power spectral density $P_Y(e^{j\omega})$.

- (●) $P_Y(e^{j\omega}) = \sigma^2 \frac{1+\alpha^2+2\alpha cos(\omega)}{1+\beta^2-2\beta cos(\omega)}$
- ( ) $P_Y(e^{j\omega}) = \alpha(1-\sigma)\frac{1-\beta^2+2\beta cos(\omega)}{1+\alpha^2-2\alpha cos(\omega)}$
- ( ) $P_Y(e^{j\omega}) = \alpha^2 \frac{\beta^2+2\beta cos(\omega)}{\alpha^2-2\alpha cos(\omega)}$
- ( ) $P_Y(e^{j\omega}) = \sigma(1-\alpha)\frac{1+\beta+2cos(\omega)}{1+\alpha-2cos(\omega)}$
- ( ) $P_Y(e^{j\omega}) = \frac{1+\beta^2+2\beta cos(\omega)}{1+\alpha^2-2\alpha cos(\omega)}$
- ( ) $P_Y(e^{j\omega}) = \sigma^2(1-\alpha^2)\frac{1+\beta^2+2\beta cos(\omega)}{1+\alpha^2-2\alpha cos(\omega)}$

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1 point

17.

(Difficulty: ⋆⋆) Using the same setup as in the previous question, assume that the output $Y[n]$ turns out to be a white noise sequence. Which of the following statements are necessarily true ?

- [ ] The power spectrum $P_Y(e^{j\omega})$ must be constant.
- [ ] $\beta = -\alpha$
- [ ] The samples of $Y[n]$ must be uncorrelated.
- [ ] $\beta = 2\alpha$
- [x] The samples of $Y[n]$ must be correlated.
- [ ] $\beta = \alpha$

1 point

18.

(Difficulty: ⋆) What is the minimum number of bits per sample you must use in order to sample and uniformly-quantize an analog signal with at least 80 dB of SNR?

Hint : you can use the "rule of thumb" from the lecture notes.

480

1 point

19.

(Difficulty: ⋆ ⋆ ⋆) A uniformly-distributed, zero mean stochastic signal with power spectral density $P_x(e^{j\omega}) = \sigma_x^2$ is quantized by means of a uniform linear quantizer with input range from $-2\sqrt{3}\sigma_x$ to $+2\sqrt{3}\sigma_x$ and resolution of R bits per sample.

What is the SNR at the output of the quantizer?

- ( ) $\frac{2^R}{2}$
- (•) $6R$
- ( ) $\frac{2^{2R}}{6}$
- ( ) $\frac{\sigma_x^2}{2^{2R}}$
- ( ) $\frac{2^{2R}}{4}$

- [x] I, **Mark R. Lytell**, understand that submitting work that isn't my own may result in permanent failure of this course or deactivation of my Coursera account.

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