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作业1：推导卷积的微分公式

$$\frac{d}{dt}\left[f_1(t)*f_2(t)\right]\overset{②}{=}f_1(t)*\left[\frac{d}{dt}f_2(t)\right]\overset{①}{=}\left[\frac{df_1(t)}{dt}\right]*f_2(t)$$

① $f_1(t)*f_2(t)=\int_{-\infty}^{+\infty}f_1(t-\tau)f_2(\tau)d\tau$

$$=\int_{-\infty}^{+\infty}\frac{d}{dt}\left[f_1(t-\tau)f_2(\tau)\right]d\tau$$

$$=\int_{-\infty}^{+\infty}\left[f_1'(t-\tau)\,f_2(\tau)+f_2'(\tau)f_1(t-\tau)\right]d\tau$$

$$=\int_{-\infty}^{+\infty}f'(t-\tau)f_2(\tau)\,dt$$

$$=\frac{d}{dt}f_1(t)*f_2(t)\quad\square$$

② 由交换律，$f_1(t)*f_2(t)=\int_{-\infty}^{\infty}f_2(t-\tau)f_1(\tau)d\tau$

同理 $\frac{d}{dt}[f_1(t)*f_2(t)]=\frac{d}{dt}f_2(t)*f_1(t)$

作业2：推导卷积的积分公式

设 $g(\lambda)=\int_{-\infty}^{t}f_1(\lambda)d\lambda$

$g(\lambda-\tau)=\int_{-\infty}^{t}f_1(\lambda-\tau)d\lambda$

$$\int_{-\infty}^{t}(f_1*f_2)(\lambda)d\lambda=f_1(t)*\int_{-\infty}^{t}f_2(\lambda)\overset{①}{=}\left(\int_{\infty}^{t}f_1(\lambda)d\lambda\right)*f_2(t)$$

$$\int_{-\infty}^{t}(f_1*f_2)(\lambda)d\lambda=\int_{-\infty}^{t}\int_{-\infty}^{\infty}f_1(\lambda-\tau)f_2(\tau)\,d\lambda\,d$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{t}f(\lambda-\tau)\,d\lambda\;f(\tau)\,d\tau$$

$$=\int_{-\infty}^{\infty}g(\lambda-\tau)\,f_2(\tau)\,d\tau$$

$$=g(\lambda)*f_2(t)$$

$$=\int_{-\infty}^{t}f_1(\lambda)d\lambda*f_2(t)$$

② 由交换律，$f_1(t)*f_2(t)=\int_{-\infty}^{+\infty}f_2(t-\tau)f_1(\tau)d\tau$

同理 $\int_{-\infty}^{t}(f_1*f_2)(\lambda)d\lambda=f_1(t)*\int_{-\infty}^{t}f_2(\lambda)d\lambda$

作业3：推导一个函数与单位阶跃函数的卷积卷积等于该函数的积分，

$$f(t)*u(t)=\int_{-\infty}^{t}f(t)\,dt$$

设 $u(t)=\begin{cases}1, & t>0\\ 0, & t<0\end{cases}$

由卷积的定义

$$f(t)*u(t)=\int_{-\infty}^{\infty}u(t-\tau)\,f(\tau)\,d\tau$$

$$u(t-\tau)=\begin{cases}1, & t>\tau\\ 0, & t<\tau\end{cases}$$

① $\int_{-\infty}^{t}1\cdot f(\tau)\,d\tau+\int 0\cdot f(\tau)\,d\tau=\int_{-\infty}^{t}f(t-\tau)$

因为 $(f_1*f_2)^{n}(t)-f_1^{(m)}(t)*f_2^{(n-m)}(t)$

$$f(t)*u(t)=(f*u)^{(0)}(t)$$

$$=f^{(-1)}(t)*u^{(1)}(t)$$

$u^{(1)}(t)=0\quad(t\neq0)$

且 $\int_{-\infty}^{\infty}u^{(1)}(t)\,dt=\int_{-\infty}^{+\infty}\frac{du}{dt}dt$

$=\int_{-\infty}^{\infty}du=u(t)\Big|_{-\infty}^{\infty}=1,\quad u^{(1)}(t)=\delta(t)$

即 $f^{(-1)}(t)*u^{(1)}(t)=f^{(-1)}(t)*\delta(t)$

$$=f^{(-1)}(t)=\int_{-\infty}^{t}f(\tau)\,d\tau\quad\square$$