作业 已知: $f(t) = \sin(t)\cos(2t) + 5\cos(3t)\,4\sin(4t)$

求该数的傅里叶级数

$$\sin(x+y) = \sin x\cos y + \sin y\cos x$$
$$\sin(x-y) = \sin x\cos y - \sin y\cos x$$
$$\sin x\cos y = \frac{1}{2}\left(\sin(x+y) + \sin(x-y)\right)$$

$$\Rightarrow \frac{1}{2}\sin(3t) + \sin(-t) + 5\left(\sin(7t) + \sin(t)\right)$$
$$\frac{1}{2}\left(\sin(3t) + 5\sin(7t) + 4\sin(t)\right)$$

solve for a_0:

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2}\left(\sin(3t) + 5\sin(7t) + 4\sin(t)\right)$$
$$\frac{1}{4\pi}\left(-\frac{1}{3}\cos(3t) - \frac{5}{7}\cos(7t) - 4\cos(t)\right.$$
$$\frac{1}{4\pi}\left(-\frac{1}{3}\cos(3t) - \frac{5}{7}\cos(7t) - 4\cos(t)\right)\Big|_0^{2\pi} = 0$$

solve for a_n

$a_n = 0$ $f(x)$ 为奇函数

Solve for b_n

$$b_n = \frac{2}{2\pi}\int_0^{2\pi} \frac{1}{2}\left(\sin(3t) + 5\sin(7t) + 4\sin(t)\right)\sin(nt)\,dt$$

$$\cos(x+y) = \cos x\cos y - \sin x\sin y$$
$$\cos(x-y) = \cos x\cos y + \sin x\sin y$$
$$\sin x\sin y = \frac{1}{2}\left(\cos(x-y) - \cos(x+y)\right)$$

$$= \frac{1}{4\pi}\int_0^{2\pi}\left[\cos((3-n)t) - \cos((n+3)t) + 5\cos((7-n)t) - 5\cos((1-n)t) + 4\cos((n+1)t)\right]dt$$

$$= \frac{1}{4\pi}\left(\frac{1}{3-n}\sin((3-n)t)\right) - \frac{1}{n+3}\sin((n+3)t) + \frac{5}{7-n}\sin((7-n)t)$$
$$- \frac{5}{n-7}\sin((7-n)t) + \frac{4}{1-n}\sin((1-n)t) - \frac{4}{n-1}\sin((n-1)t)\Big)\Big|_0^{2\pi}$$

$$\lim_{n\to 1} b_n = \frac{1}{4\pi}\left(0-0+0-0+\frac{4}{1-n}\sin((1-n)t) - 0\right)\Big|_0^{2\pi} = \frac{1}{\pi}(2\pi - 0) = 2$$

$$\lim_{n\to 3} b_n = \frac{1}{4\pi}\left(\frac{1}{3-n}\sin(3-n)t - 0+0-0+0-0\right)\Big|_0^{2\pi} = \frac{5}{4\pi}(2\pi - 0) = \frac{1}{2}$$

$$\lim_{n\to 7} b_n = \frac{1}{4\pi}\left(0-0+\frac{5}{7-n}\sin((7-n)t) - 0+0-0\right)\Big|_0^{2\pi} = \frac{5}{4\pi}(2\pi - 0) = \frac{5}{2}$$

$$f(t) = \begin{cases} \sum_{n=1}^{\infty} 2\sin(nt) & n=1 \\ \sum_{n=1}^{\infty} \frac{1}{2}\sin(nt) & n=3 \\ \sum_{n=1}^{\infty} \frac{5}{2}\sin(nt) & n=7 \end{cases}$$ 0, 其他

$$f(t) = 2\sin(t) + \frac{1}{2}\sin(3t) + \frac{5}{2}\sin(7t)$$