# Fitts' Law

speed-accuracy trade-off in human motor control

HCI class, 2020

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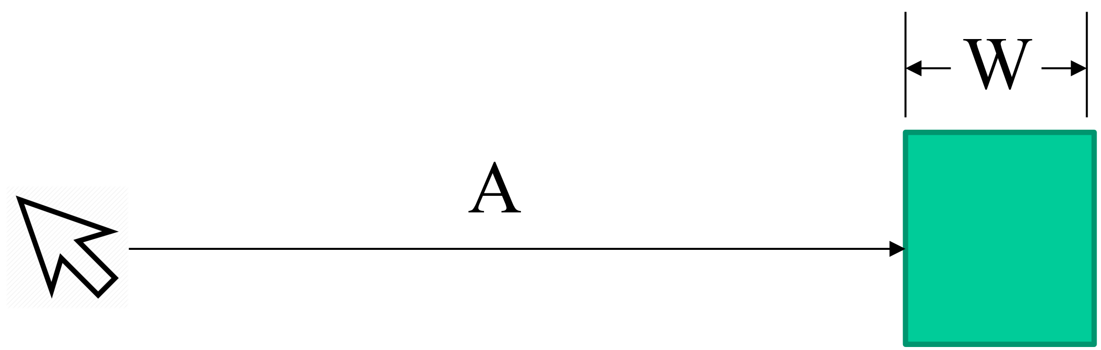
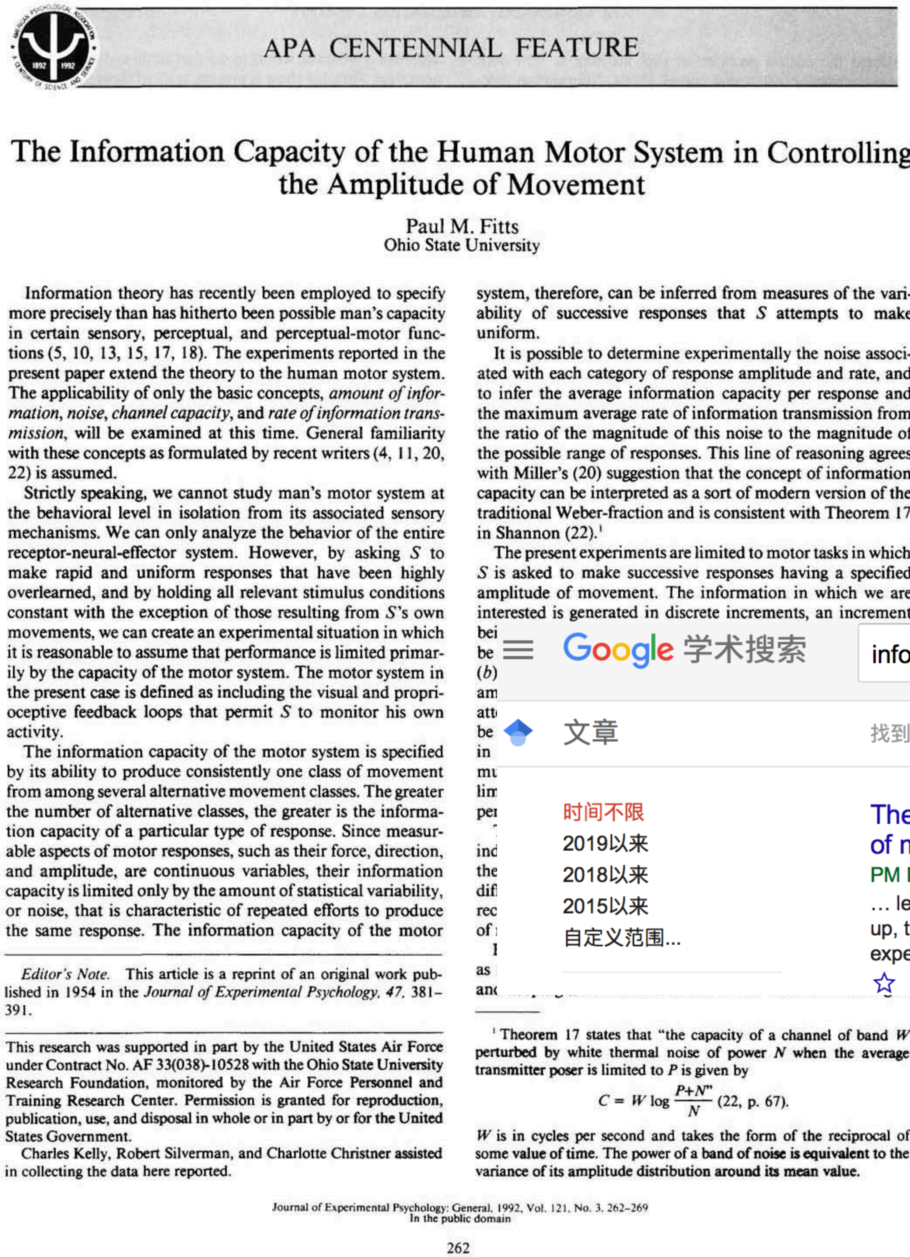
HCI class, 2019



# Fitts' Law (1954, Journal of Experimental Psychology)



APA CENTENNIAL FEATURE

## The Information Capacity of the Human Motor System in Controlling the Amplitude of Movement

Paul M. Fitts
Ohio State University

Information theory has recently been employed to specify more precisely than has hitherto been possible man's capacity in certain sensory, perceptual, and perceptual-motor functions (5, 10, 13, 15, 17, 18). The experiments reported in the present paper extend the theory to the human motor system. The applicability of only the basic concepts, *amount of information, noise, channel capacity*, and *rate of information transmission*, will be examined at this time. General familiarity with these concepts as formulated by recent writers (4, 11, 20, 22) is assumed.

Strictly speaking, we cannot study man's motor system at the behavioral level in isolation from its associated sensory mechanisms. We can only analyze the behavior of the entire receptor-neural-effector system. However, by asking *S* to make rapid and uniform responses that have been highly overlearned, and by holding all relevant stimulus conditions constant with the exception of those resulting from *S*'s own movements, we can create an experimental situation in which it is reasonable to assume that performance is limited primarily by the capacity of the motor system. The motor system in the present case is defined as including the visual and proprioceptive feedback loops that permit *S* to monitor his own activity.

The information capacity of the motor system is specified by its ability to produce consistently one class of movement from among several alternative movement classes. The greater the number of alternative classes, the greater is the information capacity of a particular type of response. Since measurable aspects of motor responses, such as their force, direction, and amplitude, are continuous variables, their information capacity is limited only by the amount of statistical variability, or noise, that is characteristic of repeated efforts to produce the same response. The information capacity of the motor system, therefore, can be inferred from measures of the variability of successive responses that *S* attempts to make uniform.

It is possible to determine experimentally the noise associated with each category of response amplitude and rate, and to infer the average information capacity per response and the maximum average rate of information transmission from the ratio of the magnitude of this noise to the magnitude of the possible range of responses. This line of reasoning agrees with Miller's (20) suggestion that the concept of information capacity can be interpreted as a sort of modern version of the traditional Weber-fraction and is consistent with Theorem 17 in Shannon (22).[1]

The present experiments are limited to motor tasks in which *S* is asked to make successive responses having a specified amplitude of movement. The information in which we are interested is generated in discrete increments, an increment

*Editor's Note.* This article is a reprint of an original work published in 1954 in the *Journal of Experimental Psychology, 47*, 381–391.

This research was supported in part by the United States Air Force under Contract No. AF 33(038)-10528 with the Ohio State University Research Foundation, monitored by the Air Force Personnel and Training Research Center. Permission is granted for reproduction, publication, use, and disposal in whole or in part by or for the United States Government.

Charles Kelly, Robert Silverman, and Charlotte Christner assisted in collecting the data here reported.

[1] Theorem 17 states that "the capacity of a channel of band *W* perturbed by white thermal noise of power *N* when the average transmitter poser is limited to *P* is given by

$$C = W \log \frac{P+N}{N} \text{ (22, p. 67).}$$

*W* is in cycles per second and takes the form of the reciprocal of some value of time. The power of a band of noise is equivalent to the variance of its amplitude distribution around its mean value.

Journal of Experimental Psychology: General, 1992, Vol. 121, No. 3, 262–269
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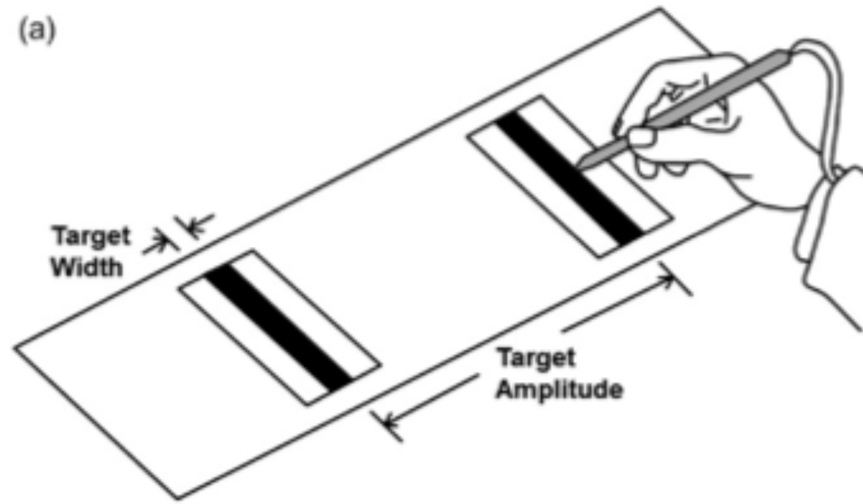
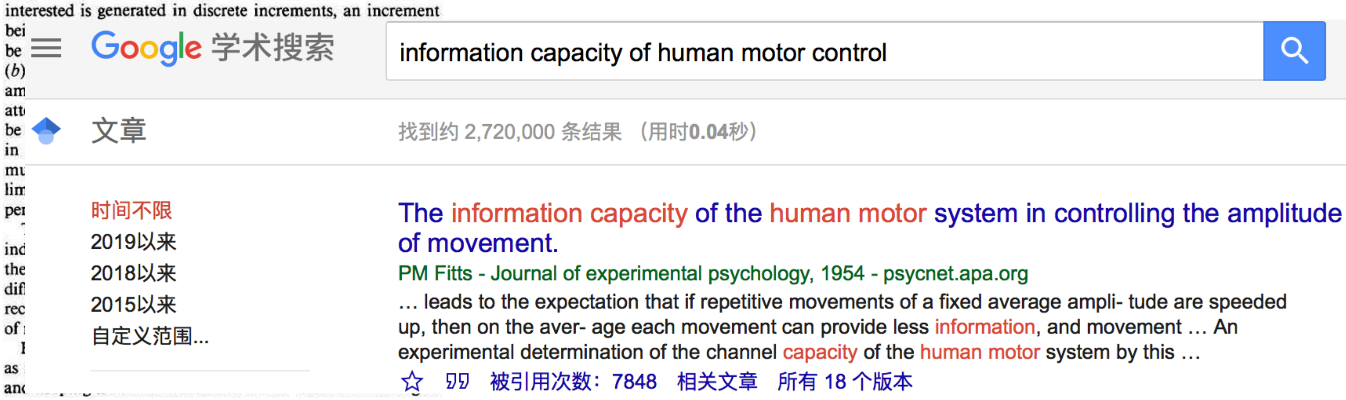


# Fitts' Law

**Movement Time = Function?**
**(Movement Amplitude, Target Size)**

- **The law predicts that the time to point at an object using a device is a function of the distance from the target object & the object's size.**
- **The further away & the smaller the object, the longer the time to locate it and point.**

# Three experiments of Fitts

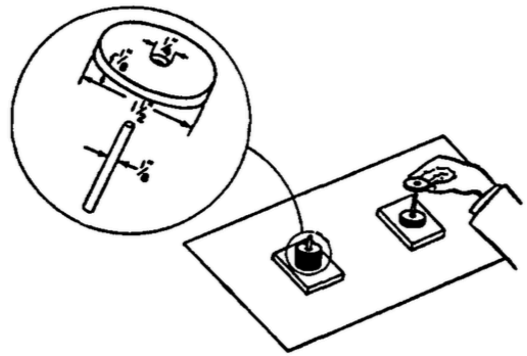


*Figure 2.* Disc transfer apparatus. The task was to transfer eight washers one at a time from the right to the left pin. The inset gives the dimensions for the $W_S$ = ⅛ in. condition.

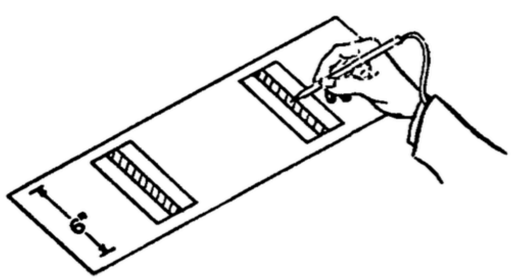


*Figure 1.* Reciprocal tapping apparatus. The task was to hit the center plate in each group alternately without touching either side (error) plate.

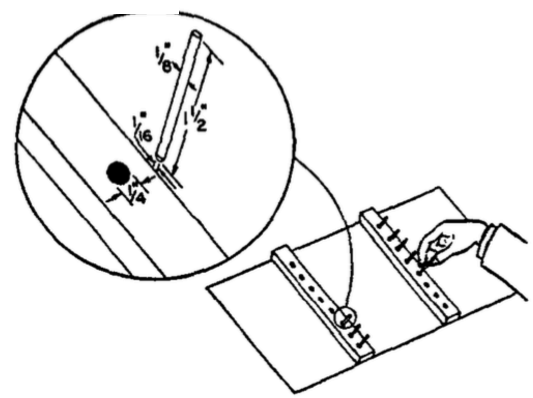


*Figure 3.* Pin transfer apparatus. The task was to transfer eight pins one at a time from one set of holes to the other. The inset gives the dimensions of pins and holes for the $W_S$ = ⅛ in. condition.

# Fitts' Hypothesis

$I_d$: Index of Difficulty

$$I_d = -\log_2 \frac{W_S}{2A} \text{ bits/response,}$$

a difficulty index is needed that will specify the minimum information required on the average for controlling or organizing each movement

$I_p$: Performance of the input

$$I_p = -\frac{1}{t} \log_2 \frac{W_s}{2A} \text{ bits/sec.,} \qquad (2)$$

**Fitts' formula:**

**ID=$\log_2$(2A/W), MT=a+bID**

# Results

TABLE 1

TASK CONDITIONS AND PERFORMANCE DAT
RECIPROCAL TAPPING
($N$ = the same 16 $S$s at ea

| Tolerance and Amplitude Conditions | | | 1-oz. Stylus | | |
|---|---|---|---|---|---|
| $W_s$ | $A$ | $I_d$ | $t$ | Errors (%) | $I_p$ |
| .25 | 2 | 4 | .392 | 3.35 | 10.20 |
| .25 | 4 | 5 | .484 | 3.41 | 10.33 |
| .25 | 8 | 6 | .580 | 2.78 | 10.34 |
| .25 | 16 | 7 | .731 | 3.65 | 9.58 |
| .50 | 2 | 3 | .281 | 1.99 | 10.68 |
| .50 | 4 | 4 | .372 | 2.72 | 10.75 |
| .50 | 8 | 5 | .469 | 2.05 | 10.66 |
| .50 | 16 | 6 | .595 | 2.73 | 10.08 |
| 1.00 | 2 | 2 | .212 | 0.44 | 9.43 |
| 1.00 | 4 | 3 | .260 | 1.09 | 11.54 |
| 1.00 | 8 | 4 | .357 | 2.38 | 11.20 |
| 1.00 | 16 | 5 | .481 | 1.30 | 10.40 |
| 2.00 | 2 | 1 | .180 | 0.00 | 5.56 |
| 2.00 | 4 | 2 | .203 | 0.08 | 9.85 |
| 2.00 | 8 | 3 | .279 | 0.87 | 10.75 |
| 2.00 | 16 | 4 | .388 | 0.65 | 10.31 |

Note.—$W_s$ is the width in inches of the target plate. $A$ is
two plates. $t$ is the average time in seconds for a movement from one plate to the other. The performance index, $I_p$, is discussed in the text.

TABLE 2

TASK CONDITIONS AND PERFORMANCE DATA FOR 16 VARIATIONS OF A DISC–TRANSFER TASK

($N$ = the same 16 $S$s at each condition)

| Tolerance and Amplitude Conditions | | | Performance | | |
|---|---|---|---|---|---|
| $W_s$ | $A$ | $I_d$ | $t$ | $I_p$ | Rank |
| .0625 | 4 | 7 | .697 | 10.04 | 2.5 |
| .0625 | 8 | 8 | .771 | 10.38 | 1 |
| .0625 | 16 | 9 | .896 | 10.04 | 2.5 |
| .0625 | 32 | 10 | 1.096 | 9.12 | 7 |
| .125 | 4 | 6 | .649 | 9.24 | 6 |
| .125 | 8 | 7 | .734 | 9.54 | 4 |
| .125 | 16 | 8 | .844 | 9.48 | 5 |
| .125 | 32 | 9 | 1.028 | 8.75 | 10 |
| .25 | 4 | 5 | .607 | 8.24 | 12 |
| .25 | 8 | 6 | .672 | 8.93 | 9 |
| .25 | 16 | 7 | .771 | 9.08 | 8 |
| .25 | 32 | 8 | .975 | 8.20 | 13 |
| .5 | 4 | 4 | .535 | 7.47 | 16 |
| .5 | 8 | 5 | .623 | 8.02 | 14 |
| .5 | 16 | 6 | .724 | 8.29 | 11 |
| .5 | 32 | 7 | .902 | 7.76 | 15 |

Note.—$W_s$ is the difference in inches between the diameter of the post and the diameter of the center hole of the disc. $A$ is the center-to-center distance between the two posts. $t$ is the average time in seconds for a movement between two posts.

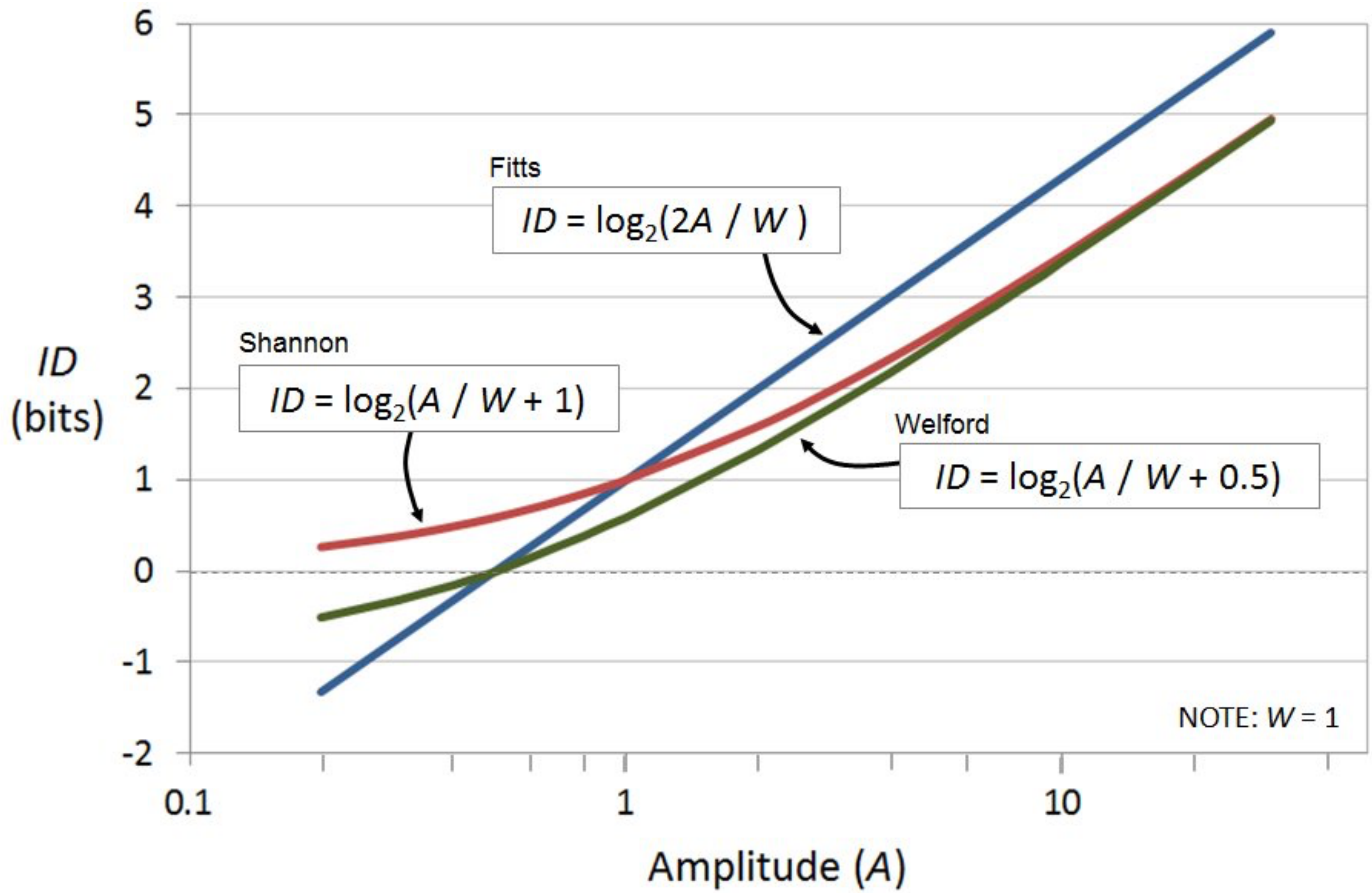
TABLE 3

TIONS AND PERFORMANCE I
0 VARIATIONS OF A PIN–
TRANSFER TASK
same 20 $S$s at each condition)

| Amplitude ons | Performance | | |
|---|---|---|---|
| $I_d$ | $t$ | $I_p$ | R |
| 6 | .673 | 8.92 | |
| 7 | .705 | 9.93 | |
| 8 | .766 | 10.44 | |
| 9 | .825 | 10.91 | |
| 10 | .959 | 10.43 | |
| 5 | .484 | 10.33 | |
| 6 | .518 | 11.58 | |
| 7 | .580 | 12.07 | |
| 8 | .644 | 12.42 | |
| 9 | .768 | 11.72 | |
| 4 | .388 | 10.31 | |
| 5 | .431 | 11.60 | |
| 6 | .496 | 12.10 | |
| 7 | .557 | 12.57 | |
| 8 | .668 | 11.98 | |
| 3 | .326 | 9.20 | |
| 4 | .357 | 11.20 | |
| 5 | .411 | 12.16 | |
| 6 | .486 | 12.34 | |
| 7 | .592 | 11.82 | |

Iote.—$W_s$ is the difference in inches betwee
meter of the pins and the diameter of the hole
which they were inserted. $A$ is the center-to-c
distance between the holes. $t$ is the average ti
seconds for a single movement.

# Versions of Fitts' Law




**Movement of a user's hand to a target**

**---In MacKenzie I S. *A note on the information-theoretic basis for Fitts' law*. Journal of Motor Behavior, 1989, 21(3): 323-330**

# Fitts' Law

- **Fitts' Law describes the time taken to hit a screen target:**

$$T = a + b \log_2(A/W + 1)$$

**a=50, b=150**

**(The constants a and b are determined experimentally )**

**where:** **a and b are empirically determined constants**

**T is movement time**

**A is Distance**

**W is Width of target**


**⇒targets as large as possible**
**distances as small as possible**

# Fitts' Law

- $50+150\log_2(80/50+1)=256\text{ms}$

  $50+150\log_2(80/5+1) \ =663\text{ms}$

$$T = a + b \log_2(A/W +$$

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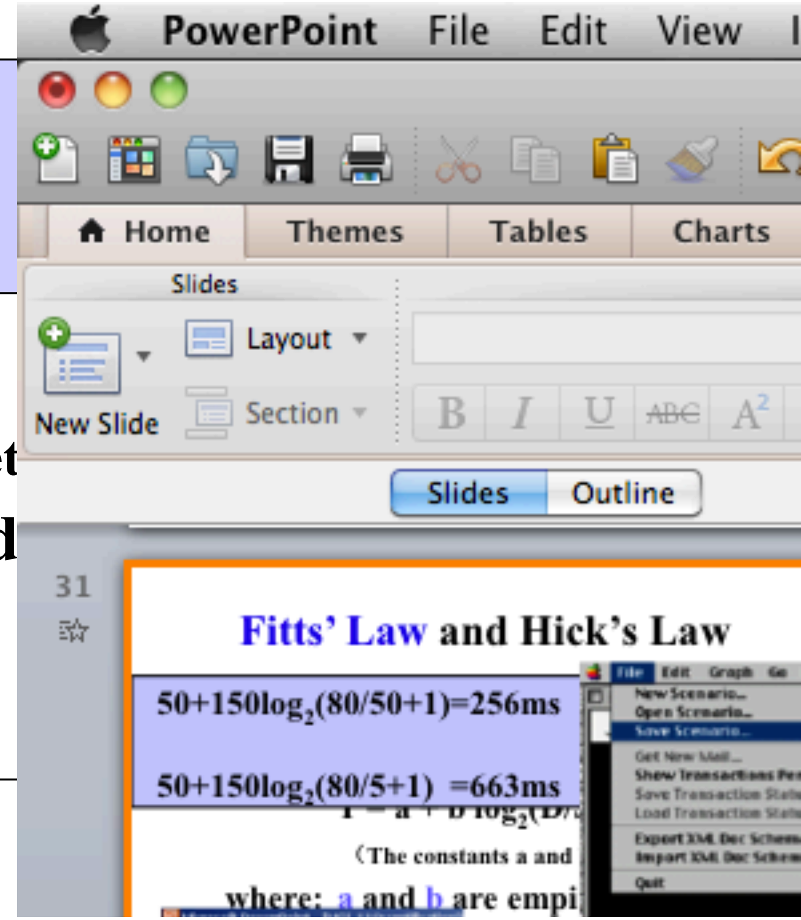



⇒ targets as large as possible
distances as small as possible

# Fitts' Law

- **Fitts' Law demonstration**
  - **Fitts' Law - Vrije Universiteit Amsterdam**
  - **http://simonwallner.at/ext/fitts/**
- **Visit Tog's website and do Tog's quiz, designed to give you Fitts!**
  - **http://www.asktog.com/columns/022DesignedToGiveFitts.html**

# Quiz 1-3

- Microsoft Toolbars offer the user the option of displaying a label below each tool. Name at least one reason why labeled tools can be accessed faster. (Assume, for this, that the user knows the tool and does not need the label just simply to identify the tool.)

- **You have a palette of tools in a graphics application that consists of a matrix of 16x16-pixel icons laid out as a 2x8 array that lies along the left-hand edge of the screen. Without moving the array from the left-hand side of the screen or changing the size of the icons, what steps can you take to decrease the time necessary to access the average tool?**

- A right-handed user is known to be within 10 pixels of the exact center of a large, 1600 X 1200 screen. You will place a single-pixel target on the screen that the user must point to exactly. List the five pixel locations on the screen that the user can access fastest. For extra credit, list them in order from fastest to slowest access.

# Quiz 4-6

- Microsoft offers a Taskbar which can be oriented along the top, side or bottom of the screen, enabling users to get to hidden windows and applications. This Taskbar may either be hidden or constantly displayed. Describe at least two reasons why the method of triggering an auto-hidden Microsoft Taskbar is grossly inefficient.

- **Explain why a Macintosh pull-down menu can be accessed at least five times faster than a typical Windows pull-down menu. For extra credit, suggest at least two reasons why Microsoft made such an apparently stupid decision.**

- What is the bottleneck in hierarchical menus and what techniques could make that bottleneck less of a problem?

# Quiz 7-10

- Name at least one advantage circular popup menus have over standard, linear popup menus.
- What can you do to linear popup menus to better balance access time for all items?
- The industrial designers let loose on the Mac have screwed up most of the keyboards by cutting their function keys in half so the total depth of the keyboard was reduced by half a key. Why was this incredibly stupid?
- What do the primary solutions to all these questions have in common?

# Value of Fitt's Law

- predicting interaction performance
- optimize the interface layout

Figure 1. QWERTY keyboard

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| q | w | e | r | t | y | u | i | o | p |
| a | s | d | f | g | h | j | k | l | |
| | z | x | c | v | b | n | m | | |

K-English

| | | | | | |
|---|---|---|---|---|---|
| z | j | d | g | k | |
| y | l | n | i | c | |
| f | o | a | t | h | w |
| b | u | r | e | s | |
| q | p | m | v | x | |

# Hick's Law

- **When you have to choose to take 1 among n**

$$Rt = a + b \log_2(n + 1)$$

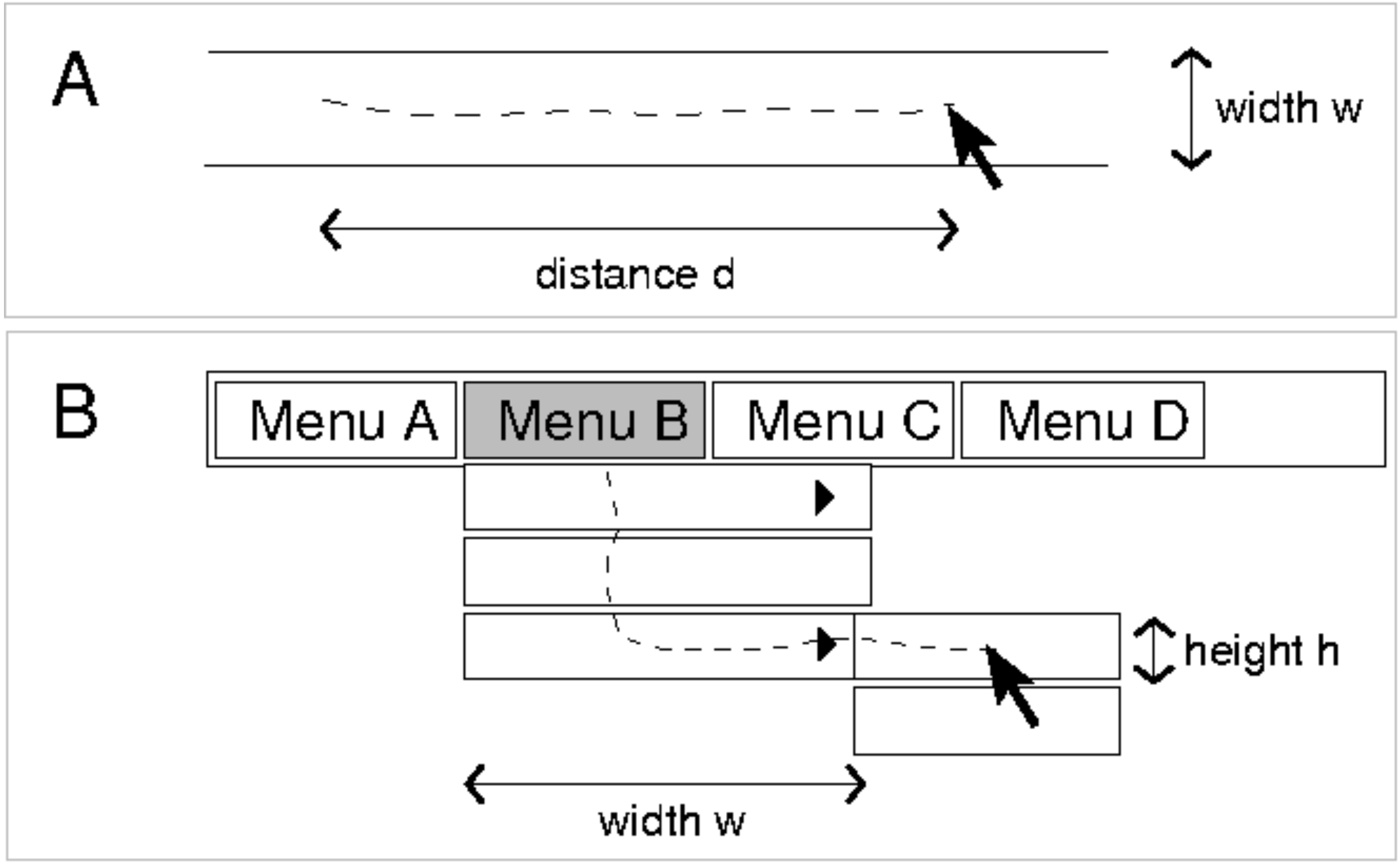
**If i**[th] **choice is** $p(i)$, $H=\sum p(i)\log_2(1/p(i)+1)$

⇒ **Giving a user many choices simultaneously is usually faster than is organizing the same choices into hierarchical groups**

$$a+b\log_2(6+1) < a+3b$$
$$2(a+b\log_2(3+1)) = 2a+4b$$

**— in Hick, W. E. (1 March 1952). On the rate of gain of information. Quarterly Journal of Experimental Psychology, 4(1):11-26**

# Steering Law



# Steering Law Equation

$T_{msec} = a + b\ (d/s)$

a, b = empirically-derived constants
d = distance, s = width of tunnel
ID (Index of Difficulty) = (d/s)

Index of Difficulty now *linear*, not logarithmic
(i.e. steering is more difficult then pointing)


Adapted from Robert Miller

# Extends Fitts' Law

# Fitts' Law (cont')

- Fitts' Law（error rate = 4%）

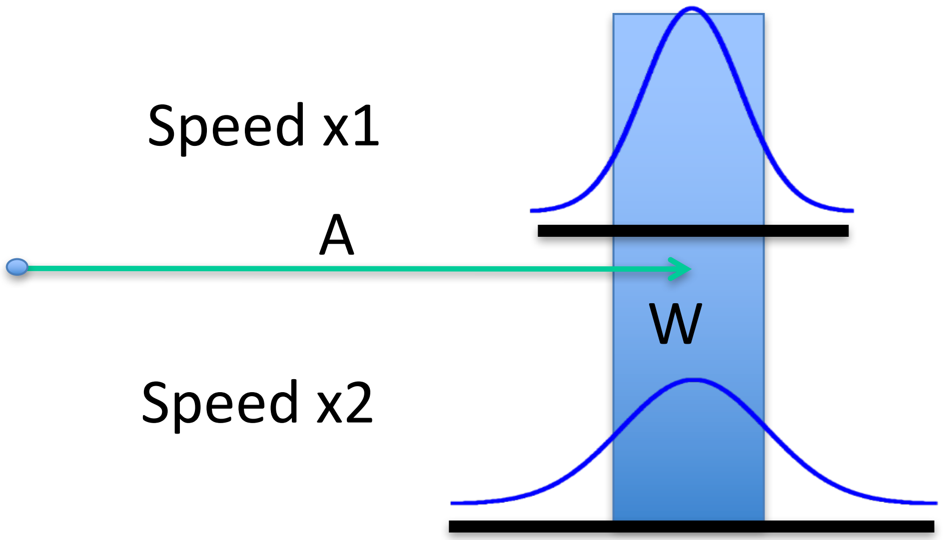
$$MT = a + b \times \log\left(\frac{A}{W} + 1\right)$$

- "Effective width" model（error rate ≠ 4%）

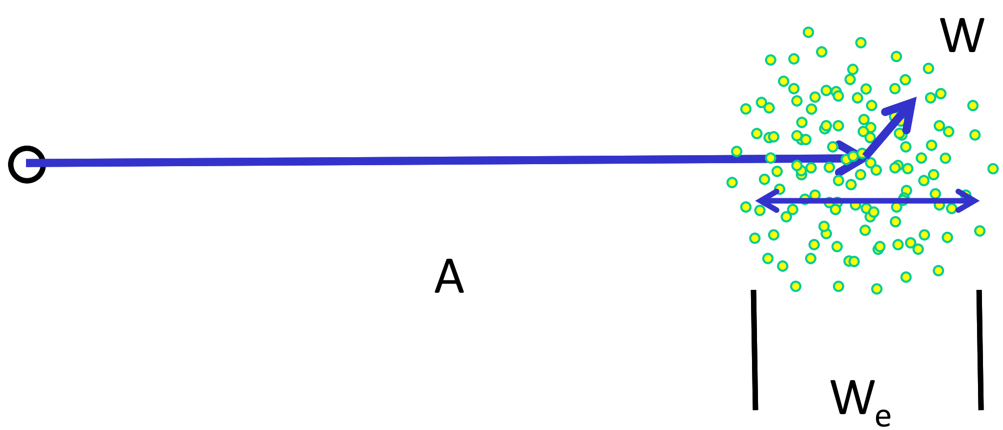
$$MT = a + b \times \log\left(\frac{A}{\sqrt{2\pi e}\sigma} + 1\right)$$

$\sigma$ denotes the standard deviation of endpoint coordinates

# "Effective Width" Model




**Moving faster means more inaccurate**



# Fitts' Law (cont')

- Fitts' Law（error rate = 4%）

$$MT = a + b \times \log\left(\frac{A}{W} + 1\right)$$

- "Effective width" model（error rate ≠ 4%）

$$MT = a + b \times \log\left(\frac{A}{\sqrt{2\pi e}\sigma} + 1\right)$$

$\sigma$ denotes the standard deviation of endpoint coordinates

# Fitts' Law for finger touch

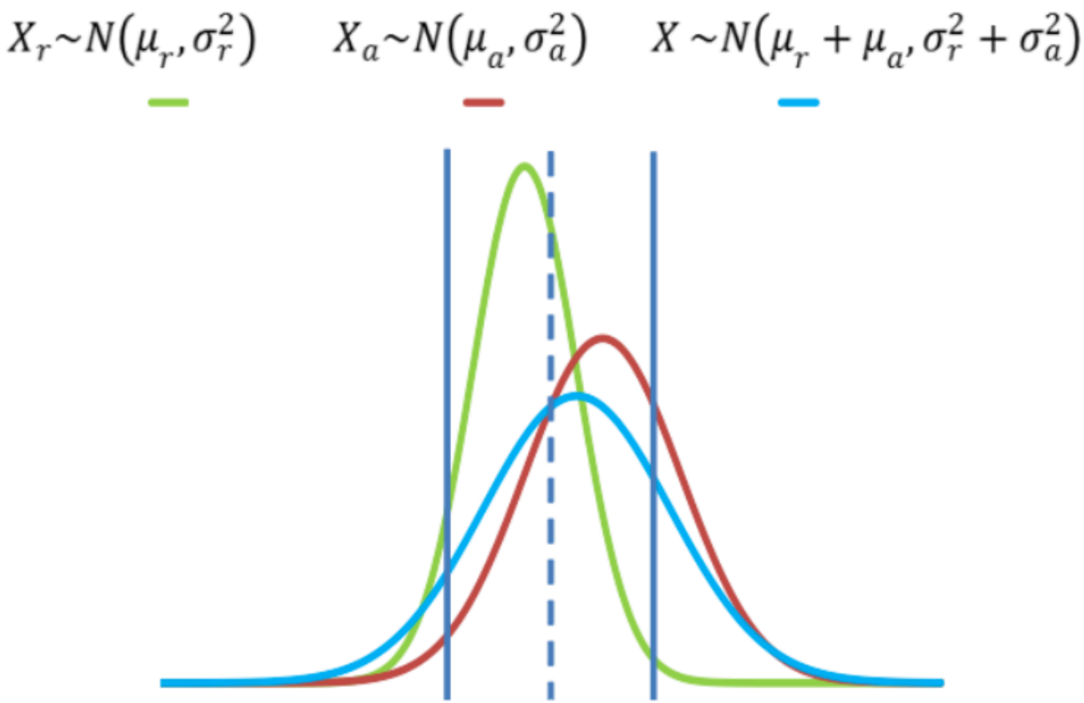



**Figure 1. Dual distribution hypothesis in 1D Fitts' tasks. The two solid vertical lines represent the target, and the dashed line is the target center. The green, red and light blue curves show distributions of $X_r$, $X_a$, and $X$.**

Xr: controlled by the speed accuracy tradeoff of the performer

$Xa$: reflects the absolute precision of a motor system that includes the implement

# Fitts' Law for finger touch

$$T = a + b\log_2\left(\frac{A}{\sqrt{2\pi e(\sigma^2 - \sigma_a^2)}} + 1\right)$$

$\boldsymbol{\sigma_a}$ : 0.94 mm for 1D task; 1.5 mm for 2D task

Bi, Xiaojun, Yang Li, and Shumin Zhai. "FFitts law: modeling finger touch with fitts' law." *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems*. ACM, 2013.