博弈论 hw13

Coal1. Find the characteristic function of the 3-person game in strategic form when the payoff vectors are

If I chooses 1:

		III: 1	III: 2
II:	1	(1, 2, 1)	(3, 0, 1)
	2	(− 1, 6, − 3)	(3, 2, 1)

If I chooses 2

		III: 1	III: 2
II:	1	(− 1, 2, 4)	(1, 0, 3)
	2	(7, 5, 4)	(3, 2, 1)

Homework

$N = \{I, II\}$ $\quad V(\emptyset) = 0, \quad V(N) = Max\{a_{ij} + b_{ij}\}$

$V(I) = Val(A) = $ safety level of I

$V(II) = Val(B^T) = $ Safety level of II

$V(N) =$

$V(I) =$

$$\begin{array}{cc|cccc} & & \multicolumn{4}{c}{II, III} \\ & & 1,1 & 1,2 & 2,1 & 2,2 \\ \hline I & 1 & 1 & \cancel{3} & -1 & \cancel{3} \\ & 2 & -1 & \cancel{1} & 7 & \cancel{3} \end{array}$$

$$\begin{array}{c} p \\ 1-p \end{array} \begin{bmatrix} 1 & -1 \\ -1 & 7 \end{bmatrix}$$

$p - (1-p) = -p + 7(1-p)$

$2p = 8(1-p)$

$10p = 8$

$p = \frac{4}{5}, \frac{1}{5}, \quad V(I) = \frac{3}{5}$

$\frac{4}{5}(1) + \frac{1}{5}(-1)$

$V(I, III) =$

$$\begin{array}{cc|cc} & & \multicolumn{2}{c}{II} \\ & & 1 & 2 \\ \hline I, III & 1,1 & \cancel{2} & \cancel{4} \\ & 1,2 & 4 & 4 \\ & 2,1 & 3 & 11 \\ & 2,2 & \cancel{4} & \cancel{4} \end{array}$$

$V(I, III) = 4$

Coal2. Graph the core for the 3-person game with characteristic function

$v(\emptyset) = 0$, $v(\{1\}) = 0$, $v(\{2\}) = -1$, $v(\{3\}) = 1$, $v(\{1, 2\}) = 3$, $v(\{1, 3\}) = 2$, $v(\{2, 3\}) = 4$, $v(N) = 5$

Graph the core for 3-person game with characteristic function

$V(\emptyset)=0,\ V(1)=0,\ V(2)=-1,\ V(3)=1$

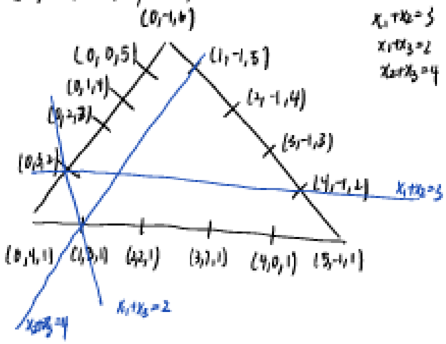
$V(1,2)=3 \quad V(1,3)=2 \quad V(2,3)=4$

$V(1,2,3)=5$

$\{(x_1,x_2,x_3): x_1+x_2+x_3=5,\ x_1\ge 0,\ x_2\ge -1,\ x_3\ge 1\}$

This is a triangle 3-space with vertices

$(0,-1,6),\ (0,4,1),\ (5,-1,1)$



core consists of

$x_1\ge a_1,\ x_2\ge 0,\ x_3\ge 0$

$x_1+x_2\ge a_2 \quad x_1+x_3\ge a_3 \quad x_2+x_3\ge 0$

$x_1+x_2+x_3=a_3$

$x_2=a_3-x_1-x_3\le 0 \quad x_2=0$

$x_1\ge a_2,\ x_1+x_3=a_3$

core is set to $\{(x,0,a_3-x): a_2\le x\le a_3\}$

Coal3. Suppose

$$v(\{1\})=v(\{2\})=v(\{3\})=0,$$
$$v(\{1,2\})=v(\{2,3\})=v(\{1,3\})=a$$
$$v(\{1,2,3\})=3$$

For what values of a is the core non-empty?

Find the core for 3-person games with characteristic function

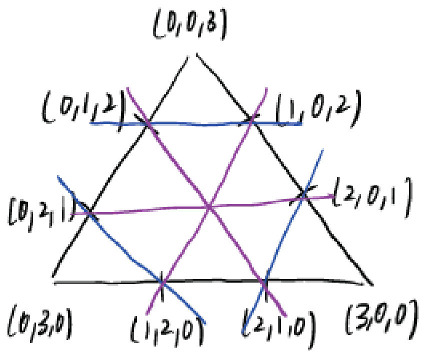
$V(\phi)=0 \quad V(1)=0 \quad V(2)=0 \quad V(3)=0$

$V(1,2)=a \quad V(2,3)=a \quad V(1,3)=a \quad V(1,2,3)=3$

$\{(x_1,x_2,x_3): x_1+x_2+x_3=3 \quad x_1\ge 0,\ x_2\ge 0,\ x_3\ge 0\}$

Triangle 3-space with vertices

$(0,0,3),\ (0,3,0),\ (3,0,0)$



a=2?

a=1?

$x_1+x_2=a$

$x_2+x_3=a$

$x_1+x_3=a$

core is non-empty when a=

Coal4. Given the Glove Game of 3 players such that Player I, II, each posses a left glove and Player III a right glove. A pair (left and right) has value 10, one glove or two of the same kind has value 0. Find the core of the game.

Glove Game

$V(\emptyset)=0 \qquad V(1)=V(2)=V(3)=0$

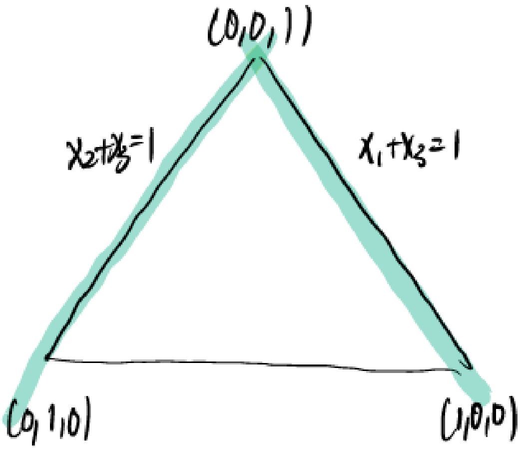
$V(1,2)=0 \quad V(1,3)=1 \quad V(2,3)=1$

$V(1,2,3)=1$

$\{(x_1,x_2,x_3).\ x_1+x_2+x_3=1,\ x_1\ge 0,\ x_2\ge 0,\ x_3\ge 0\}$

Triangle 3-space

$(0,0,1) \quad (0,1,0) \quad (1,0,0)$



$x_1+x_2=0$

$x_1+x_3=1$

$x_2+x_3=1$

Coal5. Suppose (N, v) is a game in coalition form, $N = \{1, 2, 3\}$. Suppose the core is nonempty. Show that the following inequalities are valid.

$$v(1,2,3) \geq v(1) + v(2) + v(3)$$

$$v(1,2,3) \geq v(1,2) + v(3),\ v(1,2,3) \geq v(1,3) + v(2), v(1,2,3) \geq v(2,3) + v(1)$$

$$v(1,2,3) \geq \tfrac{1}{2}v(1,2) + \tfrac{1}{2}v(1,3) + \tfrac{1}{2}v(2,3)$$

Game

$N=\{1,2,3\}$ $\quad v(1,2,3) \geq v(1)+v(2)+v(3)$ (definition)

$v(1,2,3) \geq v(1,2)+v(3)$, $v(1,2,3) \geq v(1,3)+v(2)$, $v(1,2,3) \geq v(2,3)+v(1)$

$v(1,2,3) \geq \frac{1}{2}v(1,2)+\frac{1}{2}v(1,3)+\frac{1}{2}v(2,3)$

$3v(1,2,3) \geq v(1,2)+v(2,3)+v(1,3) + v(1)+v(2)+v(3) \quad (1)$

$2v(1,2,3) \geq v(1,2)+v(2,3)+v(1,3)$

$v(1,2,3) \geq \frac{1}{2}v(1,2)+\frac{1}{2}v(2,3)+\frac{1}{2}v(1,3)$

Coal6.

Let (N, v) be a convex game. For any $T \subseteq N$, we define the T-marginal game v_T on subsets S of N T such that $v_T(S \subset T) - v(T)$. Show that v_T is a characteristic function on N、T and that (N, T, v_T) is a convex game.

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Personal Notes

Many-Person TU Games

in many person cooperative games, there are no restrictions on agreements that may be reached among the players, we assume that all payoffs are measured in the same units, and that there is a transferable utility that allows side payments to be made among the players

characteristic function

coalition, S, defined to be a subset of N (set of players), $S \subseteq N$, and the set of all coalitions is denoted by 2^N

- empty coalition set $\emptyset$
- grand coalition set N
- n = 2, $\{\varnothing, \{1\}, \{2\}, N\}$
- n = 3, $\{\varnothing, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}\}$

coalition form (N, v), where v is a characteristic function

1. $v(\emptyset) = 0$
2. (superadditivity) if S and T are disjoint coalitions (nothing in common), $v(S) + v(T) \leq v(S \cup T) \leq v(S \cup T)$
3. v(S) or worth, power of coalition S when the members act together