CE1. The payoff matrices are

$$A = \begin{bmatrix} 4 & 1 \\ 6 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & 6 \\ 1 & -3 \end{bmatrix} \qquad A+B = \begin{bmatrix} 8 & 7 \\ 7 & -6 \end{bmatrix}$$

Let the Correlated Equilibrium be

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \qquad p_{11} + p_{12} + p_{21} + p_{22} = 1$$

Then P meets the conditions

$$\begin{cases} (-2) \cdot p_{11} + 4 \cdot p_{12} \geqslant 0 \\ 2 \cdot p_{21} + (-4) \cdot p_{22} \geqslant 0 \\ (-2) \cdot p_{11} + 4 \cdot p_{21} \geqslant 0 \\ 2 \cdot p_{12} + (-4) \cdot p_{22} \geqslant 0 \end{cases} \implies p_{12} + p_{21} \geqslant \max\{p_{11}, 4p_{22}\}$$

So the expected sum of payoff

$$\begin{aligned} & 8 \cdot p_{11} + 7 \cdot p_{12} + 7 \cdot p_{21} + (-6) \cdot p_{22} \\ = {} & 7.5\,(p_{11} + p_{12} + p_{21} + p_{22}) + 0.5\,(p_{11} - p_{12} - p_{21}) - 13.5p_{22} \\ \leqslant {} & 7.5 \cdot 1 + 0.5 \cdot 0 - 13.5 \cdot 0 \\ = {} & 7.5 \end{aligned}$$

The equality holds when

$$P = \begin{bmatrix} 0.5 & 0.25 \\ 0.25 & 0 \end{bmatrix}$$

So the maximum of the expected sum of the two players' payoffs is 7.5.

CE2. (i) *Proof.* The payoff matrices are

$$A = \begin{bmatrix} 6 & 2 \\ 7 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 6 & 7 \\ 2 & 0 \end{bmatrix}$$

Suppose Player II uses the mix strategy $(q, 1-q)$, then the payback of Player I's pure strategies are

$$\begin{bmatrix} 6 & 2 \\ 7 & 0 \end{bmatrix} \cdot \begin{bmatrix} q \\ 1-q \end{bmatrix} = \begin{bmatrix} 4q+2 \\ 7q \end{bmatrix}$$

So the pure best response of Player I is Row 1 if $q \leq 2/3$ and Row 2 if $q \geq 2/3$. Since $B = A^{\mathrm{T}}$, the best response of Player II is similar. Using the Tetraskelion Method, we can find that the three SEs are

$$((1,0),(0,1)) \qquad ((0,1),(1,0)) \qquad ((2/3,1/3),(2/3,1/3))$$

The payoff vector of each SE is

$$(2,7) \qquad (7,2) \qquad (14/3, 14/3)$$ □

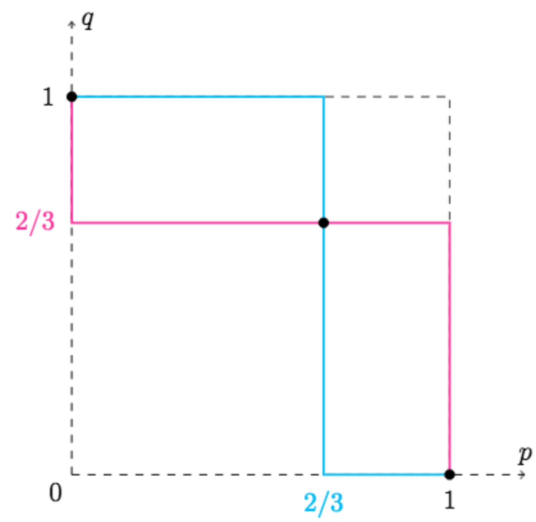

Figure 1: the tetraskelion graph in CE2.(i)

(ii) *Proof.* Let the matrix

$$P = \begin{bmatrix} 1/3 & 1/3 \\ 1/3 & 0 \end{bmatrix}$$

Suppose Player II complies the instructions. If Player I receives the instruction of Row 1, the expected payoff is 4 for compling and 3.5 for deviating. If Player I receives the instruction of Row 2, the expected payoff is 7 for compling and 6 for deviating. Since $B = A^{\mathrm{T}}$, P is a Correlated Equilibrium. As the graph shown, the payoff vector $(5,5)$ is outside the convex hull of the three payoff vectors of SEs. □

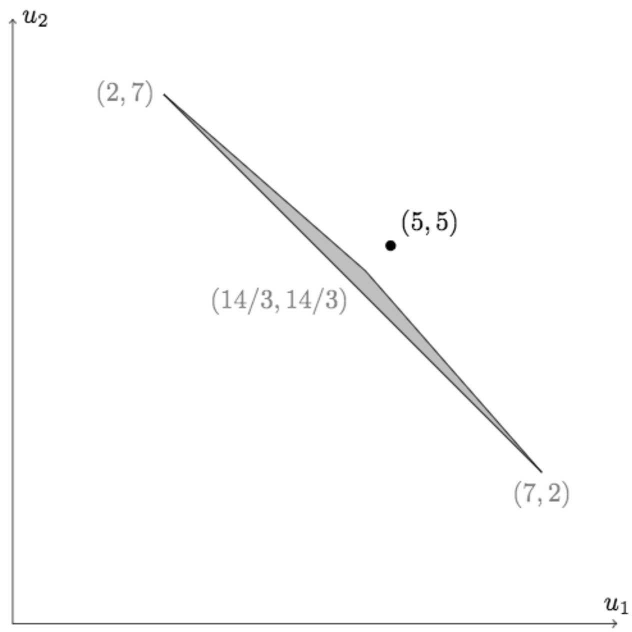

Figure 2: the payoff graph in CE2.(ii)

CE3. For the column chooser, Row 3 is strictly worse than Row 1. After Row 3 is removed the game becomes the same as the one in CE2.. So one of the Correlated Equilibrium is

$$P = \begin{bmatrix} 1/3 & 1/3 & 0 \\ 1/3 & 0 & 0 \end{bmatrix}$$

Since rank $P = 2 > 1$, the CE does not come from a SE.

CE4. *Proof.* Let the matrix $P = (p_{ij})_{n\times n}$. Since A, B, P are all diagonal matrices, for any different integers $i, r = 1, \ldots, n$ we have

$$\sum_{j=1}^{n} p_{ij}\left(a_{ij} - a_{rj}\right) = p_{ii}\left(a_{ii} - a_{ri}\right) = p_{ii}a_{ii} \geqslant 0$$

Similarly, for any different integers $j, s = 1, \ldots, n$ we have

$$\sum_{i=1}^{n} p_{ij}\left(b_{ij} - b_{is}\right) \geqslant 0$$

So P is a Correlated Equilibrium. $\square$