Assignment on Bimatrix Games:

Bimat1.
The Game of Chicken. Two players speed head-on toward each other and a collision is bound to occur unless one of them chickens out at the last minute. If both chicken out, everything is okay (they both win 1). If one chickens out and the other does not, then it is a great success for the player with iron nerves (payoff = 2) and a great disgrace for the chicken (payoff = −1). If both players have iron nerves, disaster strikes (both lose 2). (a) Set up the bimatrix of this game. (b) What are the safety levels, what are the maximin （safety） strategies, and what is the average payoff if the players use the maximin (safety) strategies? (c) Find all three SE's.

Bimat2.
For the following bimatrix games, find
(i) Security level for each player
(ii) Maximin (safety) strategies for each player

$$\begin{pmatrix} (0,0) & (2,4) \\ (2,4) & (3,3) \end{pmatrix}, \begin{pmatrix} (1,4) & (4,1) \\ (2,2) & (3,3) \end{pmatrix}$$

Bimat3. Find all SE's and the associated payoffs of the following bimatrix games.

$$\begin{pmatrix} (0,0) & (2,4) \\ (2,4) & (3,3) \end{pmatrix}, \begin{pmatrix} (1,4) & (4,1) \\ (2,2) & (3,3) \end{pmatrix}$$

Bimat4.
Suppose that $\langle (p_1,\cdots,p_m)\rangle, \langle (q_1,\cdots,q_n)\rangle$ is a SE in a bimatrix game. If q_j are all positive, then show that $(p_1,\cdots,p_m)$ is an equalizing strategy on the Column player's playoff matrix.

Bimat5. Let [A, B] be a bimatrix game. This game is called a symmetric bimatrix game if A, B are square matrices such that $B = A^T$. Show that for a symmetric bimatrix game if <p, q> is a SE then so is <q, p>.

Bimat6.
A completely mixed strategy is defined to be a mixed strategy assigning positive probability to each of a player's pure strategies. If each player's playoff matrix in a bimatrix game is nonsingular, show that the game can have at most one SE in which both players use completely mixed strategy.
Show that the only SE for the following 3x3 game must be completely mixed. Find this completely mixed SE.

4,5	0,0	5,4
5,4	4,5	0,0
0,0	5,4	4,5

Bimat7.
Consider the 3x3 bimatrix game

$$[A,B]=\begin{pmatrix} 0,0 & 4,0 & 5,3 \\ 4,0 & 0,4 & 5,3 \\ 3,5 & 3,5 & 6,6 \end{pmatrix}$$

Let $< p,q >$ be a Nash equilibrium in [A,B]. Let C(p)be the set of best reply columns with respect to p.

(i) Prove that {Col1, Col2} is not a subset of C(p).
(ii) Prove that C(p) is not equal to {Col2, Col3}.
(iii) Find all Nash equilibria of this game.

Bimat8.
(a) Suppose in the Cournot model that the price function is $\max((a-q),0)$ where q is the total quantity in the market. Suppose the firms have different production costs. Let c_1 and c_2 be the costs of production per unit for firms 1 and 2 respectively, where both c_1 and c_2 are assumed less than a/2. Find the Cournot equilibrium.
(b) What happens, if in addition, each firm has a set up cost? Suppose Player I's cost of producing x is x + 2, and II's cost of producing y is 3y + 1. Suppose also that the price function is p(x, y) = 17 − x − y, where x and y

are the amounts produced by I and II respectively. What is the equilibrium production, and what are the players' equilibrium payoffs?

Bimat9.

Which of the following bimatrix games are potential games? If it is a potential game, find a potential. Otherwise, explain why it is not a potential game.

$$(i)\begin{pmatrix} (3,3) & (0,4) \\ (4,0) & (1,1) \end{pmatrix},\ (ii)\begin{pmatrix} (0,3) & (0,2) \\ (1,0) & (-1,1) \end{pmatrix},\ (iii)\begin{pmatrix} (0,2) & (-1,3) \\ (1,0) & (0,1) \end{pmatrix}$$