数值分析 lab3

第3章上机题6

编程生成Hilbert矩阵H_n（见例3.4），以及n维向量$b = H_n x$,其中x为所有分量都是1的向量，用Cholesky分解算法求解方程$H_n x = b$,得到近似解$\hat{x}$，计算残差$r = b - H_n \hat{x}$和误差$\Delta x = \hat{x} - x$的$\infty -$范数

1. 设$n = 10$,计算$||r||_\infty, ||\Delta x||_\infty$
2. 在右端顶上施加10^{-7}的扰动然后解方程组，观察残差的变化情况
3. 改变n的值为8和12，求解相应的方程，观察$||r||_\infty, ||\Delta x||_\infty$的变化情况，通过这个是按说明了什么问题？

Cholesky算法实现

In the textbook, the pseudo-code for the Cholesky algorithm is as shown below.

算法 3.10：对称正定矩阵的 Cholesky 分解算法

输入：A，n；输出：A.

For $j=1, 2, \cdots, n$

  For $k=1, 2, \cdots, j-1$

    $a_{jj} := a_{jj} - a_{jk}^2$;

  End

  $a_{jj} := \sqrt{a_{jj}}$; $\left\{ l_{jj} = \sqrt{a_{jj} - \sum_{k=1}^{j-1} l_{jk}^2} \right\}$

  For $i=j+1, j+2, \cdots, n$

    For $k=1, 2, \cdots, j-1$

      $a_{ij} := a_{ij} - a_{ik} a_{jk}$;

    End

    $a_{ij} := a_{ij} / a_{jj}$; $\left\{ l_{ij} = \left(a_{ij} - \sum_{k=1}^{j-1} l_{ik} l_{jk} \right) \Big/ l_{jj} \right\}$

  End

End

Thus, in order to realize the algorithm, we have to realize the code above.

```
function L = cholesky(A)            % input square matrix A
    n = size(A,1);                  % get dimensions of A
    L = zeros(n,n);                 % resulting matrix
    L(1, 1) = sqrt(A(1,1));         % l11 = a11^2, first entry at (1,1)
    for j = 1 : n                   % For j = 1, 2, ..., n
        for k = 1 : j - 1           % For k = 1, 2, ..., j-1
            A(j,j) = A(j,j) - A(j,k) * A(j,k); % ajj := ajj - (ajk)^2
        end
        A(j,j) = sqrt(A(j,j));      % ajj := sqrt(ajj)
        L(j,j) = A(j,j);            % ljj = sqrt(Ajj - sum{(ljk)^2})
        for i = j + 1 : n           % For i = j + 1, j + 2, ..., n
            for k = 1 : j - 1           % For K = 1, 2, ..., j - 1
                A(i,j) = A(i,j) - A(i,k) * A(j,k); % aij := aij - aik * ajk
            end
            A(i,j) = A(i,j) / A(j,j);   % aij = aij/ajj
            L(i,j) = A(i,j);            % lij = aij - si,{lik;kl}/ljj
```

```
        end
    end
end
```

Then we need to calculate $||r||_\infty$ and $||\Delta x||_\infty$

```
n = 10;                  % n = 10
H = hilb(n);             % create Hilbert Matrix of size n
x = ones(n,1);           % ones vector of size n
b = H * x;               % calculate b = Hx to find actual solution
L = cholesky(H);         % calculate Cholesky Method for L
                         % H = LL^T, thus b = LL^Tx, approxixmation
sol = L.' \ (L \ b);     % xbar = (L^T)^-1 * L^-1 * b

% calculate ||r||_\infty and ||\delta x||_infty
r = b - H * sol;
dx = sol - x;
disp("norm(r): " + norm(r,inf));
disp("norm(dx): " + norm(dx,inf));
```

If we were to incorporate a disturbance of 10^-7,

```
bd = b + ones(n,1) * 1e-7;
sold = L.' \ (L \ bd);
rd = bd - H * sold;
dxd = sold - x;
disp("norm(rd): " + norm(rd,inf));
disp("norm(dxd): " + norm(dxd,inf));
```

实验结果

For each of the test cases, $n = 8, 10, 12$

```
% n = 8
>> lab3_6
norm(r): 4.4409e-16
norm(dx): 7.0128e-07
norm(rd): 2.2204e-16
norm(dxd): 0.021622
% n = 10
>> lab3_6
norm(r): 4.4409e-16
norm(dx): 0.00040521
norm(rd): 4.4409e-16
norm(dxd): 0.70073
% n = 12
>>
>> lab3_6
norm(r): 4.4409e-16
norm(dx): 0.055272
norm(rd): 2.2204e-16
norm(dxd): 23.7071
```

实验结果分析

From these results, we can easily see that as n increases, the residual does not change. However, the error, Δx increases significantly, thus showing that is sensitive to disturbance. This proves the fact that Hilbert matrix is ill conditioned.