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Chapter 4 线性方程组的迭代解法 GS-method

$$\begin{cases} x_1^{(k+1)} = 0.1 x_1^{(k)} + 0.9\times\frac{1}{5}(-2x_2^{(k)} - x_3^{(k)} - 12) \\ x_2^{(k+1)} = 0.1 x_2^{(k)} + 0.9\times\frac{1}{4}(x_1^{(k+1)} - 2x_3^{(k)} + 20) \\ x_3^{(k+1)} = 0.1 x_3^{(k)} + 0.9\times\frac{1}{10}(-2x_1^{(k+1)} + 3x_2^{(k+1)} + 3) \end{cases}$$

$[0,0,0]^T$

$[-2.16, 4.014, 1.743]^T$

$[-4.133, 3.187, 2.049]^T$

$[-4.090, 2.977, 2.015]^T$

$[-4.003, 2.990, 1.999]^T$

$[-3.977, 3.000, 1.999]^T$

6 (1) A 正定 $\Leftrightarrow$ 各顺序主子式 >0

$\Leftrightarrow \begin{vmatrix} 1 & 0 & \alpha \\ 0 & 1 & 0 \\ \alpha & 0 & 1 \end{vmatrix} > 0 \Leftrightarrow 1-\alpha^2 > 0 \Leftrightarrow -1<\alpha<1$

(2) 雅可比迭代解法收敛要求 $2D-A$ 正定

$\begin{vmatrix} 1 & 0 & -\alpha \\ 0 & 1 & 0 \\ -\alpha & 0 & 1 \end{vmatrix} > 0 \Leftrightarrow 1-\alpha^2 > 0$

故 α 取值范围是 $(-1,1)$

(3) 在 $\alpha \in (-1,1)$ 时，A 对称正定，故 G-S 迭代法收敛

10. 对 $\forall \vec{x} = \vec{x}' + \vec{\Delta} \in R^N$

$\varphi(\vec{x}) = \frac{1}{2}(\vec{x}'+\vec{\Delta})^T A(\vec{x}'+\vec{\Delta}) - \vec{b}^T(\vec{x}'+\vec{\Delta})$

$= \frac{1}{2}\vec{x}'^T A\vec{x}' + \frac{1}{2}\vec{x}'^T A\vec{\Delta} + \frac{1}{2}\vec{\Delta}^T A\vec{x}' + \frac{1}{2}\vec{\Delta}^T A\vec{\Delta} - \vec{b}^T\vec{x}' - \vec{b}^T\vec{\Delta}$

$= \frac{1}{2}\vec{x}'^T A\vec{x}' + \frac{1}{2}\vec{\Delta}^T A\vec{\Delta} - \vec{b}^T\vec{x}'$

$= \varphi(\vec{x}') + \frac{1}{2}\vec{\Delta}^T A\vec{\Delta}$ 而 $\vec{\Delta}^T A\vec{\Delta} \geq 0$

故 $\varphi(\vec{x}')$ 为唯一最小值

Chapter5 矩阵特征值计算

(1)

$|\lambda I - A| = \begin{vmatrix} \lambda I - A_{11} & -A_{12} \\ 0 & \lambda I - A_{22} \end{vmatrix} = |\lambda I - A_{11}|\cdot|\lambda I - A_{22}|$

因此 λ_j, λ_k 是 A 的特征值

(2)

$A\vec{x}_j' = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}\begin{bmatrix} \vec{x}_j \\ 0 \end{bmatrix} = \begin{bmatrix} A_{11}\vec{x}_j \\ 0 \end{bmatrix} = \begin{bmatrix} \lambda_j\vec{x}_j \\ 0 \end{bmatrix} = \lambda_j\vec{x}_j'$

2 根据圆盘定理

$D_1: |\lambda-0.3| \leq 1.2$, $D_2: |\lambda+1.2| \leq 1.8$, $D_3: |\lambda-3| \leq 0.6$

D_3 与 D_1, D_2 分解，则仅含一个特征值且为实数

$\therefore \lambda_1, \lambda_2 \in D_1 \cup D_2 \quad \lambda_3 \in [2.4, 3.6]$

对 A^TA 根据圆盘定理

$D_1': |\lambda-1.25| \leq 2$, $D_2': |\lambda-2.16| \leq 2.7$, $D_3': |\lambda-10| \leq 1.7$

同理 $\lambda_1, \lambda_2 \in D_1' \cup D_2' \quad \lambda_3 \in [8.3, 11.7]$

$\rho(A) \in [2.4, 3.6] \quad cond(A)_2 \geq \sqrt{\frac{8.3}{4.86}} = 1.307$

4. $\vec{v}_i = A\vec{u}_{i-1} \quad \lambda_i = \max(\vec{v}_i) \quad \vec{u}_i = \frac{\vec{v}_i}{\lambda_i}$ $[1,1,1]^T$

$[8,6,0]^T$ 8 $[1.075, 0]^T$

$[9.25, 6, -2.75]^T$ 9.25 $[1, 0.649, -0.297]^T$

$[9.541, 5.892, -3.541]^T$ 9.541 $[1, 0.618, -0.371]^T$

$[9.595, 5.841, -3.731]^T$ 9.595 $[1, 0.609, -0.371]^T$

$[9.604, 5.824, -3.775]^T$ 9.604 $[1, 0.606, -0.393]^T$

$[9.603, 5.819, -3.786]^T$ 9.603 $[1, 0.606, -0.394]^T$

$[9.606, 5.87, -3.788]^T$ 9.606 $[1, 0.606, -0.394]^T$

finished