Assignment on Games in Extensive Form

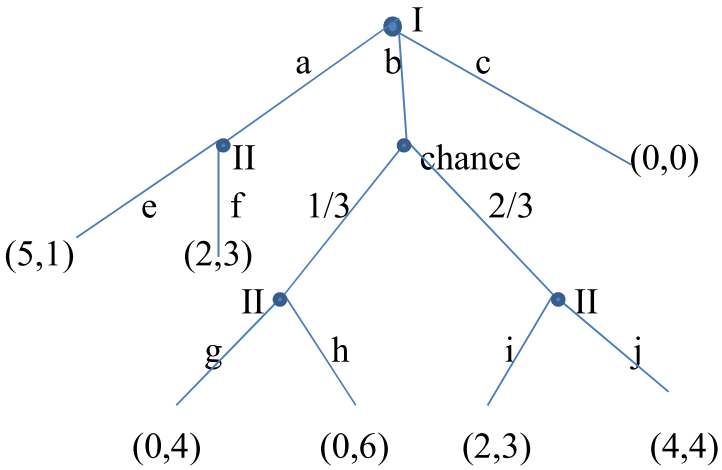
Ext1.
The Silver Dollar. Player II chooses one of two rooms in which to hide a silver dollar. Then, Player I, not knowing which room contains the dollar, selects one of the rooms to search. However, the search is not always successful. In fact, if the dollar is in room #1 and I searches there, then (by a chance move) he has only probability 1/2 of finding it, and if the dollar is in room #2 and I searches there, then he has only probability 1/3 of finding it. Of course, if he searches the wrong room, he certainly won't find it. If he does find the coin, he keeps it; otherwise the dollar is returned to Player II. (i) Draw the game tree. (ii) Suppose Player II is given one more chance to search if he/she does not see the coin. Draw the game tree.

Ext2.
Coin A has probability 1/2 of heads and 1/2 of tails. Coin B has probability 1/3 of heads and 2/3 of tails. Player I must predict "heads" or "tails". If he predicts heads, coin A is tossed. If he predicts tails, coin B is tossed. Player II is informed as to whether I's prediction was right or wrong (but she is not informed of the prediction or the coin that was used), and then must guess whether coin A or coin B was used. If II guesses correctly she wins 1 dollar from I. If II guesses incorrectly and I's prediction was right, I wins 2 dollars from II. If both are wrong there is no payoff.
(a) Draw the game tree.
(b) Find the equivalent strategic form of the game.

Ext3.
Player I has two coins. One is fair (probability 1/2 of heads and 1/2 of tails) and the other is biased with probability 1/3 of heads and 2/3 of tails. Player I knows which coin is fair and which is biased. He selects one of the coins and tosses it. The outcome of the toss is announced to II. Then II must guess whether I chose the fair or biased coin. If II is correct there is no payoff. If II is incorrect, she loses 1. Draw the game tree. Find the equivalent strategic form and solve.

Ext4.

Given the following game tree, find the PPSE using the method of Backward Induction.



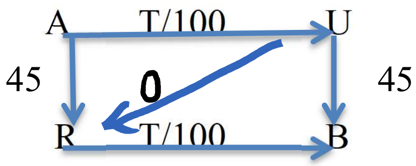

Ext5.
Find all PSEs of the game described in Ext1(i).

Ext6.
Find all PSE's of the following games in strategic form.

(a) $$\begin{pmatrix} (-3,-4) & (2,-1) & (0,6) & (1,1) \\ (2,0) & (2,2) & (-3,0) & (1,-2) \\ (2,-3) & (-5,1) & (-1,-1) & (1,-3) \\ (-4,3) & (2,-5) & (1,2) & (-3,1) \end{pmatrix}.$$

(b) $$\begin{pmatrix} (0,0) & (1,-1) & (1,1) & (-1,0) \\ (-1,1) & (0,1) & (1,0) & (0,0) \\ (1,0) & (-1,-1) & (0,1) & (-1,1) \\ (1,-1) & (-1,0) & (1,-1) & (0,0) \\ (1,1) & (0,0) & (-1,-1) & (0,0) \end{pmatrix}.$$

Ext7.
4000 drivers drive from A to B. The following diagram shows the connections between different locations and the travel time.




where T is the number of drivers using this road segment.

This is a 4000-person game such that each player has three strategies, A-U-B A-R-B and A-U-R-B.
Show that 4,000 choosing A-U-R-B is a PSE.

Ext8.
Consider the game of 3-person Scissor-Rock-Paper such that each loser will pay \$1 and the winner(s) will share equally on the payoff from the loser(s). Show that the strategy for each player to play the mixed strategy (1/3, 1/3, 1/3) is a Strategic Equilibrium for the game.

Ext9.
Is the following game the strategic form of an extensive form game with perfect information? Draw the game tree if your answer is positive. Justify your answer if your answer is negative.

$$\begin{pmatrix} (0,-4) & (0.5,-0.5) & (1,4) \\ (-1,5) & (-2,1) & (2,3) \end{pmatrix}$$

Ext10.
Suppose Alice and Bob form a team that we call Player I. Player II consists of Charles alone. The game is for Alice, Bob and Charles to call out "Heads" or "Tails" simultaneously. If the calls are the same, Player I wins. Otherwise, Player II wins. (i) Draw the Kuhn tree. (ii) Show that Player I has a mixed strategy that wins with probability 0.5 but no behavior strategy that wins with probability greater than 0.25.