第1章练习题 2,4,6,8,11　第2章练习题 2

2. $\sin(x)$ 的求值，数据传递误差，即自变量 x 发生扰动 h 时函数值的误差

(1) $\sin x$ 的绝对误差 $= h\sin' x = h\cos x$

(2) $\sin x$ 的相对误差 $= \frac{h\cos x}{\sin x}$

(3) 条件数 $\text{cond} \approx \left|\frac{x f'(x)}{f(x)}\right| = \left|\frac{x\cos x}{\sin x}\right|$

(4) x 为何值，这个问题高度敏感

$e(\hat{x}) = \hat{x} - x$

$e_r(\hat{x}) = \frac{\hat{x} - x}{x}$

$\frac{f(x+h) - f(x)}{h}$ can be approximated as $f'(x)$,

$\text{cond} = \left|\frac{[f(\hat{x}) - f(x)]/f(x)}{(\hat{x}-x)/x}\right| \approx \left|\frac{x f'(x)}{f(x)}\right|$

$f(x+h) - f(x) = h f'(x)$

$\text{cond} \gg 1$，∴病态

$\left|\frac{x\cos x}{\sin x}\right|_{x=?} = |x\cot x| \gg 1$

当 $\sin x \to 0$，$\text{cond} \to \infty$，$x = k\pi$ 时 $k \neq 0$

4. 设 $Y_0 = 28$，按递推公式

$Y_n = Y_{n-1} - \frac{1}{100}\sqrt{783}$ $(n = 1, 2, 3 \dots)$

Y_{100}，$\sqrt{783} = 27.982$，Y_{100} 有多大误差

Error each time is $\frac{1}{100}\sqrt{783} - \frac{27.982}{100} = \frac{1}{100}\sqrt{783} - .27982$

after n times, error becomes $\frac{n}{100}(\sqrt{783} - .27182)$, $n = 100$

error $= \sqrt{783} - 27182 \approx 1.37 \times 10^{-4}$

6. 序列 $\{y_n\}$ 满足递推关系

$y_n = 10y_{n-1} - 1$

$y_0 = \sqrt{2} \approx 1.41$，计算 y_{10} 误差有多大

Error each time is $10(\sqrt{2} - 1.41)$

n times, error is $10^n(\sqrt{2} - 1.41) = 4.2 \times 10^7$

8. $f(x, y) = x - y$　$f: R^2 \to R$

$|x| + |y|$ 度量二元值 (x, y) 的大小，

并假定 $|x| + |y| \approx 1$

$x - y \approx \varepsilon$　$\varepsilon \ll 1$

考虑 x, y 分别发生扰动的情况下，

证明 f 有条件数 $\text{cond}(f) \approx 1/\varepsilon$，将这个结论与减法的敏感性以及抵消现象联起来说明

$\text{cond} = \left|\frac{[f(\hat{x}) - f(x)]/f(x)}{(\hat{x}-x)/x}\right| \approx \left|\frac{x f'(x)}{f(x)}\right|$

x 发生扰动 h

$$\left|\frac{(f(x+h) - f(x))/f(x,y)}{h/(|x|+|y|)}\right| = \left|\frac{(|x|+|y|)\frac{\partial f}{\partial x}(x,y)}{f(x,y)}\right|$$

$|x| + |y| = 1$，$\frac{\partial f}{\partial x}(x,y) = 1$　∴ $\text{cond}(f) \approx \frac{1}{\varepsilon}$

y 发生扰动 h

$$\left|\frac{(f(y+h) - f(y))/f(x,y)}{h/(|x|+|y|)}\right| = \left|\frac{(x+y)\frac{\partial f}{\partial x}(x,y)}{f(x,y)}\right|$$

$|x| + |y| = 1$　$\frac{\partial f}{\partial x}(x,y) = -1$　∴ $\text{cond}(f) \approx \frac{1}{\varepsilon}$

Since $\varepsilon \ll 1$ 时，$\text{cond}(f) = \frac{1}{\varepsilon} \gg 1$

thus x, y are really sensetive.

Thus, small 扰动 on x, y will be amplified

0.

11. 0.1 对应的 IEE 单精度浮点 to 二进表示

0.1 *	2	0
0.2 *	2	0
0.4 *	2	0
0.8 *	2	1
0.6		

$0.1 = 2^{-4} \times \left(1 + \frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^5} + \frac{1}{2^8} + \cdots\right)$

1 bit　8 bits　23 bits

$x =$ [0　1111100　0001100 1100 ... 1101

(正)　(−4)

符号　指数

截断舍入 $(0.0001100)_2$

最近舍入 $(0.0001101)_2$

2.2

$g(x) = x - \lambda f(x)$

thus $g'(x) = 1 - \lambda f'(x) \in [1-\lambda M, 1-\lambda m] \subseteq (-1, 1)$,

$x \in (-\infty, \infty)$, $g(x) \in (-\infty, \infty)$, so $g(x)$ 全局收敛

thus Iterative function 收敛 towards 不动点，即 $f(x)$ 的根 x^*