博弈论 hw7

CE1 Given the following bimatrix game

$(4,4)$	$(1,6)$
$(6,1)$	$(-3,-3)$

Find the correlated equilibrium that maximizes the expected sum of the two player's payoffs.

$$A=\begin{pmatrix}4 & 1\\ 6 & -3\end{pmatrix},\ B=\begin{pmatrix}4 & 6\\ 1 & -3\end{pmatrix}$$

CE1 Bimatrix Game

$$\begin{array}{ccc} & 1 & 2 \\ 1 & \multicolumn{2}{c}{\left[\begin{array}{cc}(4,4) & (1,6)\\ (6,1) & (-3,-3)\end{array}\right]} \\ 2 & & \end{array}$$

Inequalities

$p_{11}(a_{11}-a_{21})+p_{12}(a_{12}-a_{22})\geq 0$

$p_{21}(a_{21}-a_{11})+p_{22}(a_{22}-a_{12})\geq 0$

$p_{11}(b_{11}-b_{12})+p_{21}(b_{21}-b_{22})\geq 0$

$p_{12}(b_{12}-b_{11})+p_{22}(b_{22}-b_{21})\geq 0$

$p_{11}+p_{12}+p_{21}+p_{22}=1 \quad p_{ij}\geq 0$

Thus the inequalities for this game are

$p_{11}(4-6)+p_{12}(1+3)\geq 0$

$p_{21}(6-4)+p_{22}(-3-1)\geq 0$

$p_{11}(4-6)+p_{21}(1+3)\geq 0$

$p_{12}(6-4)+p_{22}(-3-1)\geq 0$

$-2p_{11}+4p_{12}\geq 0$

$2p_{21}-4p_{22}\geq 0$

$-2p_{11}+4p_{21}\geq 0$

$2p_{12}-4p_{22}\geq 0$

$2p_{12}\geq p_{11}$

$p_{21}\geq 2p_{22}$

$2p_{21}\geq p_{11}$

$p_{12}\geq 2p_{22}$

$p_{11}+p_{21}+p_{12}+p_{22}=1$

$p_{12}=\max\left(\tfrac{1}{2}p_{11},\ 2p_{22}\right)$

$p_{21}=\max\left(\tfrac{1}{2}p_{11},\ 2p_{22}\right)$

Many Answers!

sum(4,4)

$8p_{11}+7p_{12}+7p_{21}-6p_{22}$ subject to

$p_{12}+p_{21}=1-p_{11}-p_{22}$, $p_1=p_2=0$

$p_{12}+p_{21}=1$

$8p_{11}+7(p_{12}+p_{21})-6p_{22}$

expected sum $=7$

CE2 Given the following bimatrix game

$(6,6)$	$(2,7)$
$(7,2)$	$(0,0)$

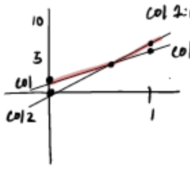
i. Find all SE using Tetraskelion method. Show that the payoff vectors of the SEs are $(2,7),(7,2),(14/3,14/3)$

ii. Show that there exists a correlated equilibrium such that its payoff vector is outside the convex hull of the payoff vectors of the three SEs in (i)

CE2 Bimatrix Game

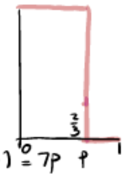
$$\begin{bmatrix} (6,6) & (2,7) \\ (7,2) & (0,0) \end{bmatrix} \quad A = \begin{bmatrix} 6 & 2 \\ 7 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 6 & 7 \\ 2 & 0 \end{bmatrix}$$

BR of B to A $(p, 1-p)$



col 1: $6p + 2(1-p)$

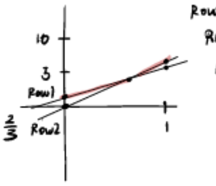
col 2: $7p$



$6p + 2(1-p) = 7p$ p

$2 - 2p = p$, $3p = 2$, $p = \frac{2}{3}$

BR of A to B $(q, 1-q)$

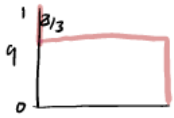


Row 1 $6q + 2(1-q)$

Row 2 $7q$

$6q + 2(1-q) = 7q$

$q = 2/3$



$0 \le p < \frac{2}{3}$ BR is col 1, $q = 1$

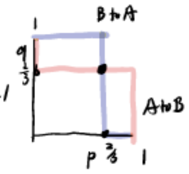
$p = \frac{2}{3}$ BR is everywhere $0 \le q \le 1$

$\frac{2}{3} < p \le 1$ BR is col 2, $q = 0$

$0 \le q < 2/3$ BR is Row 1 $p = 1$

$q = 2/3$ BR is everywhere $0 \le p \le 1$

$\frac{2}{3} < q \le 1$ BR is Row 2 $p = 0$



The 3 SE are $\frac{2}{3}(6) + \frac{1}{3}(2)$

i) $\langle$Row 1, Col 2$\rangle$, $\langle (\frac{2}{3}, \frac{1}{3}), (\frac{2}{3}, \frac{1}{3}) \rangle$, $\langle$Row 2, Col 1$\rangle$

and the payoffs are $\langle 2,7 \rangle$ $\langle 14/3, 14/3 \rangle$ $\langle 7,2 \rangle$

ii) Inequalities

$p_{11}(6-7) + p_{12}(2-0) \ge 0$ $2p_{12} \ge p_{11}$

$p_{21}(7-6) + p_{22}(0-2) \ge 0$ $p_{21} \ge 2p_{22}$

$p_{11}(6-7) + p_{21}(2-0) \ge 0$ $2p_{21} \ge p_{11}$

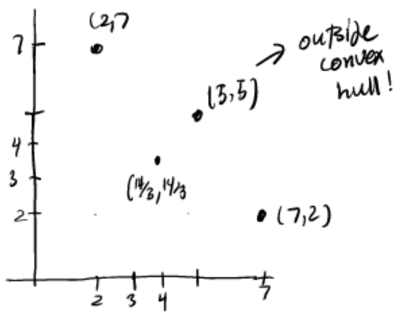
$p_{12}(7-6) + p_{22}(0-2) \ge 0$ $p_{12} \ge 2p_{22}$

$p_{11} + p_{12} + p_{21} + p_{22} = 0$

$p_{11} = p_{12} = p_{21} = \frac{1}{3}$, $p_{22} = 0$

payoff $6 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} + 7 \cdot \frac{1}{3} = \frac{15}{3} = 5$

payoff = $\langle 5,5 \rangle$



CE3 For the following 2×3 game, find a CE which does not come from a SE

(6,6)	(2,7)	(3,5)
(7,2)	(0,0)	(8,1)

CE3 Matrix

$$\begin{bmatrix} (6,6) & \boxed{(2,7)} & (3,5) \\ \boxed{(7,2)} & (0,0) & (8,1) \end{bmatrix}$$

Nash Equilibrium / SE

2 PURE SE $\langle$Row 1, Col 2$\rangle$, $\langle (1,0), (0,1,0) \rangle$

$\langle$Row 2, Col 1$\rangle$ $\langle (0,1), (1,0,0) \rangle$

1 MIXED SE $\langle (2/3, 1/3), (2/3, 1/3, 0) \rangle$

Inequalities as described

$m = 2$

$n = 3$

$\Sigma_j p_{ij} (a_{ij} - a_{rj}) \geq 0$ for $i, r = 1, \dots m$

$\Sigma_i p_{ij} (b_{ij} - b_{is}) \geq 0$ for $j, s = 1, \dots n$

$1 \leq i, r \leq m$

$1 \leq j, s \leq n$

But we know we can use theory of domination.
Thus, we can exclude the last column of the game, and we can use the results from previous question to show that there is in fact a CE that is not SE!

CE4

Let $[A, B]$ be a bimatrix game such that both A and B are diagonal matrices with nonnegative diagonal entries. Show that any diagonal matrix (p_{ij}) such that $(p_{ij}) \geq \Sigma_{i,j} p_{ij} = 1$, is a CE

If the elements off the diagonal are zero then there are no profitable deviations when the other player follows the correlated equilibrium:

Suppose player 2 follows the correlated equilibrium. Then you must prove that no other strategy will improve your payoff in expected value. Showing this is very simple in your case: suppose you get the ith signal. Player 2 will play his ith action and your best response is also to play your ith action since the payoff matrix is diagonal. Use this argument for each signal to see that the correlated equilibrium maximizes your payoff for each signal ii, so it must maximize your expected payoff as well。

Personal Notes

Correlated Equilibrium - generalizes Nash Equilibrium