For the following matrix game, reduce by domination and solve

1.1

$$\begin{pmatrix} 10 & 3 & 7 & 3 \\ 2 & 6 & 4 & 7 \\ 6 & 2 & 3 & 5 \end{pmatrix}$$

dom dom

$\frac{5}{10}\binom{10}{2} + \frac{5}{10}\binom{3}{6} = \frac{65/10}{40/10} \leq \frac{7}{4}$

$$\begin{array}{c|cc} & q & 1-q \\ \hline p & 10 & 3 \\ 1-p & 2 & 6 \end{array}$$

$10p + 2(1-p) = 3p + 6(1-p)$

$8p + 2 = -3p + 6$

$11p = 4 \quad p = \frac{4}{11}$

$10q + 3(1-q) = 2q + 6(1-q)$

$7q + 3 = -4q + 6$

$11q = 3 \quad q = \frac{3}{11}$

$10\left(\frac{3}{11}\right) + 3\left(\frac{8}{11}\right)$

$\frac{30+24}{11} = \frac{54}{11}$

Player 1 BR $(\frac{4}{11}, \frac{7}{11}, 0)$, Player 2 $(\frac{3}{11}, \frac{8}{11}, 0, 0)$

value of game is $\frac{54}{11}$

$10\left(\frac{4}{11}\right) + 2\left(\frac{7}{11}\right)$

$\frac{40}{11} + \frac{14}{11} = \frac{54}{11}$

1.2

dom dom

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

dom dom

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Diagonal $\frac{1/1}{1/1 + 1/1 + 1/1} = \frac{1}{3} = V$

optimal strategy $\langle (1/V, 1/V, 1/V, 0, 0), (1/V, 0, 1/V, 1/V, 0) \rangle$

1.3

dom dom

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

dom dom

dom

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Diagonal $\frac{1/1}{1/1 + 1/1 + 1/1 + 1/1} = 1/4 = V$

Optimal Strategy $\langle (1/V, 1/V, 0, 1/V, 0), (1/V, 0, 1/V, 0, 1/V) \rangle$

Solve the following symmetric games

2.1

$$\begin{array}{c} p_1 \\ p_2 \\ p_3 \end{array} \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$$

$-p_2 + 2p_3 \geq 0$

$p_1 - 3p_3 \geq 0$

$-2p_1 + 3p_2 \geq 0$

$2p_3 \geq p_2$

$p_1 \geq 3p_3$

$3p_2 \geq 2p_1$

$p_1 + p_2 + p_3 = 1$

$p_1 + \frac{2}{3}p_1 + \frac{1}{3}p_1 = 1$

$p_1 = 3p_3 = \frac{3}{2}p_2$

$2p_1 = 1, p_1 = \frac{1}{2}$

$p_3 = \frac{1}{6}$

$p_2 = \frac{2}{6}$

$p + \frac{1}{2}p + p = \frac{1}{2}$

2.2

$$\begin{matrix} 0 & -2 & 2 \\ 2 & 0 & -3 \\ -2 & 3 & 0 \end{matrix}$$

$2p_2 - 2p_3 \geq 0$

$-2p_1 + 3p_3 \geq 0$

$2p_1 - 3p_2 \geq 0$

$p_2 \geq p_3$

$p_3 \geq \frac{2}{3}p_1$

$p_1 = \frac{3}{2}p_2$

$p_1 = \frac{3}{2}p_2 = \frac{3}{2}p_3$

$p_1 + p_2 + p_3 = 1$

$p_1 + \frac{2}{3}p_1 + \frac{2}{3}p_1 = 1$

$p_1 + \frac{4}{3}p_1 = \frac{7}{3}p_1 = 1$

$p_1 = \frac{3}{7}, \; p_2 = \frac{6}{21}, \; p_3 \; \frac{6}{21}$

3.1 For the following matrix game (0.2, 0.6, 0.2] is safety strategy, of player 1, find The value of game and safety strategy of player 2

$$\begin{array}{cc}
 & \begin{array}{cccccc} 0 & q_1 & q_2 & 0 & q_3 & 0 \end{array} \\
\begin{array}{c} 0.2 \\ 0.6 \\ 0.2 \end{array} & \begin{bmatrix} 12 & -35 & -2 & -2 & 69 & 8 \\ 0 & 6 & -11 & 20 & 0 & -6 \\ -7 & 7 & 25 & -3 & -74 & 10 \end{bmatrix} \\
 & \begin{array}{cccccc} 1 & -2 & -2 & 96 & -2 & 0 \end{array}
\end{array}$$

Value of game $= -2$

$$\underset{A}{\begin{bmatrix} -35 & -2 & 69 \\ 6 & -11 & 0 \\ 7 & 23 & -74 \end{bmatrix}} \times \underset{q}{\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}} = \underset{b}{\begin{bmatrix} -2 \\ -2 \\ -2 \end{bmatrix}}$$

$$A \cdot q = b \qquad q_1 = \frac{270}{1123}$$
$$A^{-1}A\, q = A^{-1}b \qquad q_2 \frac{3866}{12353}$$
$$q = A^{-1}b \qquad q_3 = \frac{1339}{12353}$$

3.2 $\left(\frac{52}{143}, \frac{50}{143}, \frac{41}{153}\right]$ opt strat for 1

$$\begin{array}{c} \frac{52}{143} \\ 50/143 \\ 41/153 \end{array} \begin{bmatrix} 0 & 5 & -2 \\ -3 & 0 & 4 \\ 6 & -4 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 5 & -2 \\ -3 & 0 & 4 \\ 6 & -4 & 0 \end{bmatrix} * \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 96/143 \\ 96/143 \\ -96/143 \end{bmatrix}, \qquad \begin{array}{l} q_1 = \frac{4}{13} \\ q_2 = \frac{42}{143} \\ q = \frac{57}{143} \end{array}$$

$$\frac{52}{143} * 0 + \frac{50}{143} * -3 + \frac{41}{153} * 6 = \frac{96}{143} = \text{value of game.}$$

4. Proof: suppose player 2 has optimal strategy which is also equalizing strat, find values in Terms of A.

Equalizing Strategy gives you the same outcome no matter what the opponent does

Optimal Strategy gives you the best option in a give scenario.

Equalizing

$p \in X^*$ when $A(p,q) = A(p,q')$ for any $q, q' \in Y$

- $q \in Y^*$ when $A(p,q) = A(p,q)$ for any $p, p' \in X^*$

BR $A(p^*, q) \geq A(p^*, q^*)$, for all q belong to Y^*

Optimal

player II optimal $= \text{Min}_q \text{Max}_p\, p^T A q$

q' is safety/optimal strategy when

- $\text{Max}_p\, p^T A q' = \text{Min}_q \text{Max}_p\, p^T A q \qquad A($

$$\exists V = p^T A q$$
$$V = p^T A q$$
$$\frac{1}{p^T} \cdot V \cdot \frac{1}{q} = A?$$

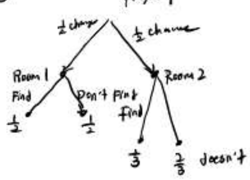
5. Matching Pennies

$$\begin{array}{c|cc} & H & T \\ \hline H & 1 & -1 \\ T & -1 & 1 \end{array} \qquad e_{ij} = \begin{cases} 1, & i=j \\ -1, & \text{otherwise} \end{cases}$$

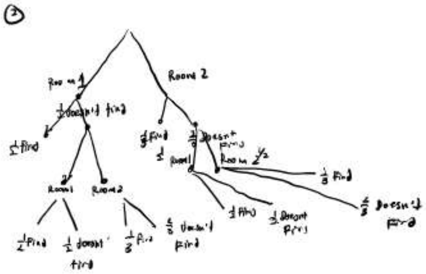
$$-y_H + y_T \leq 1$$
$$y_T - y_H \leq 1$$
$$y_T + y_H = 1$$
$$y_T, y_H \geq 0, \text{ optimal at } y_T = 1/2,\ y_H = 1/2$$

Ext 1

1


player 1
$\frac{1}{2}$ chance
Room 1
Find
$\frac{1}{2}$
Don't Find
$\frac{1}{2}$
Room 2
Find
$\frac{1}{3}$
$\frac{2}{3}$ doesn't

2


Room 1
doesn't find
$\frac{1}{2}$ find
Room 2
Room 1
Room 2
$\frac{1}{2}$ find
$\frac{1}{2}$ doesn't find
$\frac{1}{3}$ find
doesn't find
find
doesn't find
Room 2
$\frac{1}{3}$ find
doesn't find