Assignment on Games in Coalition Form

Coal1.

Find the characteristic function of the 3-person game in strategic form when the payoff vectors are:

If I chooses 1:

	III: 1	III: 2
II: 1	(1, 2, 1)	(3, 0, 1)
II: 2	(− 1, 6, − 3)	(3, 2, 1)

If I chooses 2

	III: 1	III: 2
II: 1	(− 1, 2, 4)	(1, 0, 3)
II: 2	(7, 5, 4)	(3, 2, 1)

Coal2.

Graph the core for the 3-person game with characteristic function: v(∅) = 0, v({1}) = 0, v({2}) = −1, v({3}) = 1, v({1,2}) = 3, v({1,3}) = 2, v({2,3}) = 4, and v(N) = 5.

Coal3.

Suppose $v(\{1\}) = v(\{2\}) = v(\{3\}) = 0$

$$v(\{1, 2\}) = v(\{2, 3\}) = v(\{1, 3\}) = a$$

$$v(\{1, 2, 3\}) = 3$$

For what values of a is the core non-empty?

Coal4.

Given the Glove Game of 3 players such that Player I, II each possesses a left glove and Player III a right glove. A pair (left and right) has value 10, one glove or two of the same kind has value 0. Find the core of this game.

Coal5.

Suppose (N, v) is a game in coalition form. N={1, 2, 3}. Suppose the core is nonempty. Show that the following inequalities are valid.

$v(1,2,3) \geqq v(1) + v(2) + v(3)$

$v(1,2,3) \geqq v(1,2) + v(3)$, $v(1,2,3) \geqq v(1,3) + v(2)$, $v(1,2,3) \geqq v(2,3) + v(1)$,

$v(1,2,3) \geqq \frac{1}{2} v(1,2) + \frac{1}{2} v(1, 3) + \frac{1}{2} v(2,3)$.

Coal6.

Let (N, v) be a convex game. For any $T \subseteq N$, we define the T-marginal game v_T on subsets S of $N \setminus T$ such that $v_T(S) = v(S \cup T) - v(T)$.

Show that v_T is a characteristic function on $N \setminus T$ and that $(N \setminus T, v_T)$ is a convex game.

(In fact, this condition is also sufficient.)