Assignment on ESS

ESS1.

Given the symmetric game $[A, A^T]$, where $A = \begin{pmatrix} 5 & 7 & 2 \\ 8 & 6 & 5 \\ 1 & 8 & 4 \end{pmatrix}$. Show that $(1/4, 3/4, 0)$ is an ESS.

ESS2.

Given the symmetric game $[A, A^T]$, where

$$A = \begin{pmatrix} 0 & -1 & 3 & 3 & 3 \\ -1 & 0 & 3 & 3 & 3 \\ 1 & 1 & 0 & -1 & -1 \\ 1 & 1 & -1 & 0 & -1 \\ 1 & 1 & -1 & -1 & 0 \end{pmatrix}.$$

Show that (0.75, 0, 0.25, 0, 0) is an ESS.

ESS3.

For the following symmetric games $[A, A^T]$, find all symmetric equilibrium pairs which use all the pure strategies i.e. find all SE of the form $< p, p >, p_i > 0 \text{ for all } i$. Which of these are ESS?

$$A = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}, A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 2 & 3 & 1 \end{pmatrix}.$$

ESS4.

Given the symmetric game $[A, A^T]$, where $A = \begin{pmatrix} 5 & 7 & 2 \\ 8 & 6 & 5 \\ 1 & 8 & 4 \end{pmatrix}$. Are there any ESS that uses only (i) Row 2 and Row 3, (ii) Row 1 and Row 3?

ESS5.
(Fischer's Fundamental Theorem of Natural Selection says that the rate of increase in fitness of any organism at any time is equal to its genetic variance in fitness at that time.)

Given an nxn symmetric matrix A. Consider the bimatrix game $[A, A^T]$. Suppose $(p_1, \ldots ,p_n)$ satisfies the Replicator Dynamics Equations.

Let $w_i = \sum_j a_{ij} p_j$ and $w=\sum_i p_i w_i$.

Prove the Fundamental Theorem of Natural Selection

that $dw/dt=2 \sum_i p_i (w_i-w)^2$.

(Hint: Write the Replicator Dynamics Equation in terms of w and w_i. Then, differentiate

$w = \sum_{i,j} p_i a_{ij} p_j$)