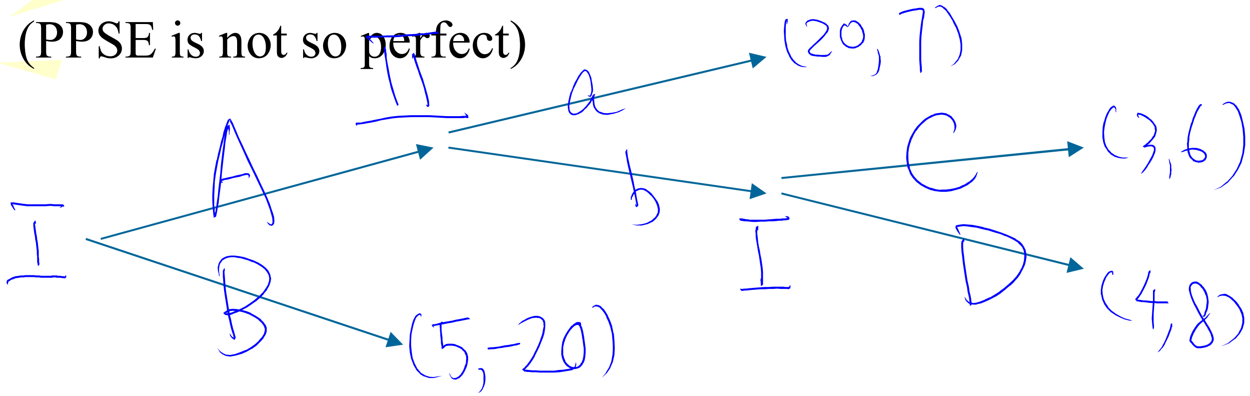
Example: (PPSE is not so perfect)

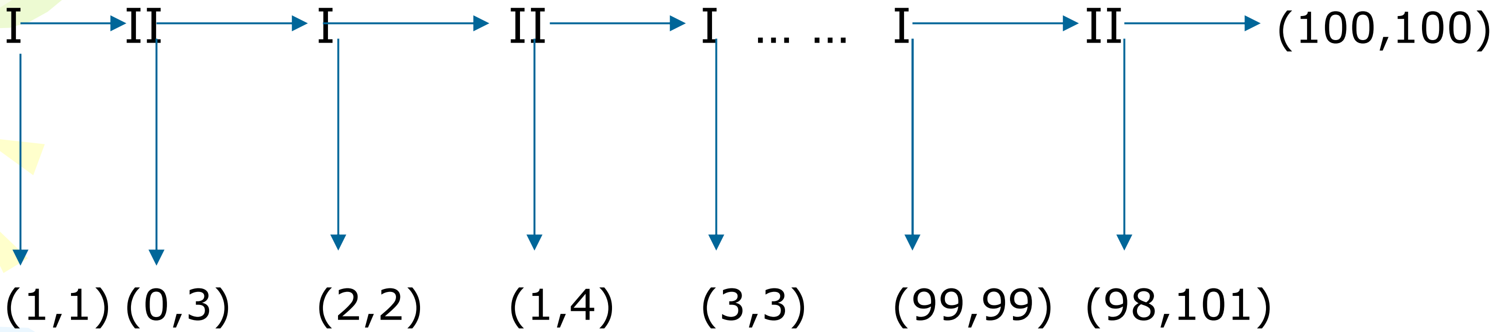


Strategic Form

	a	b
AC	(20,7)	(3,6)
AD	(20,7)	(4,8)
B	(5,-20)	(5,-20)

PPSE: (BD, b)

Example (Rationality can be rather strange): Centipede Game



Player I will go down in the 1st move.

Application of Backward Reasoning:

Sophisticated Voting

- Sincere Voting: Vote for most preferred outcomes.

- Sophisticated Voting: Individual actors vote against their preferences in the earlier votes if they can anticipate the outcome of later votes.

- Example: Assume there are three alternatives, x, y, and z, and three voters, Player 1, 2, and 3, with the following preferences:
- Player 1: $x \geq y \geq z$
- Player 2: $y \geq z \geq x$
- Player 3: $z \geq x \geq y$
- The three alternatives are voted in pairwise comparisons: first x versus y, and then the winner of that vote versus z.

- Player 1: $x \geq y \geq z$
- Player 2: $y \geq z \geq x$
- Player 3: $z \geq x \geq y$

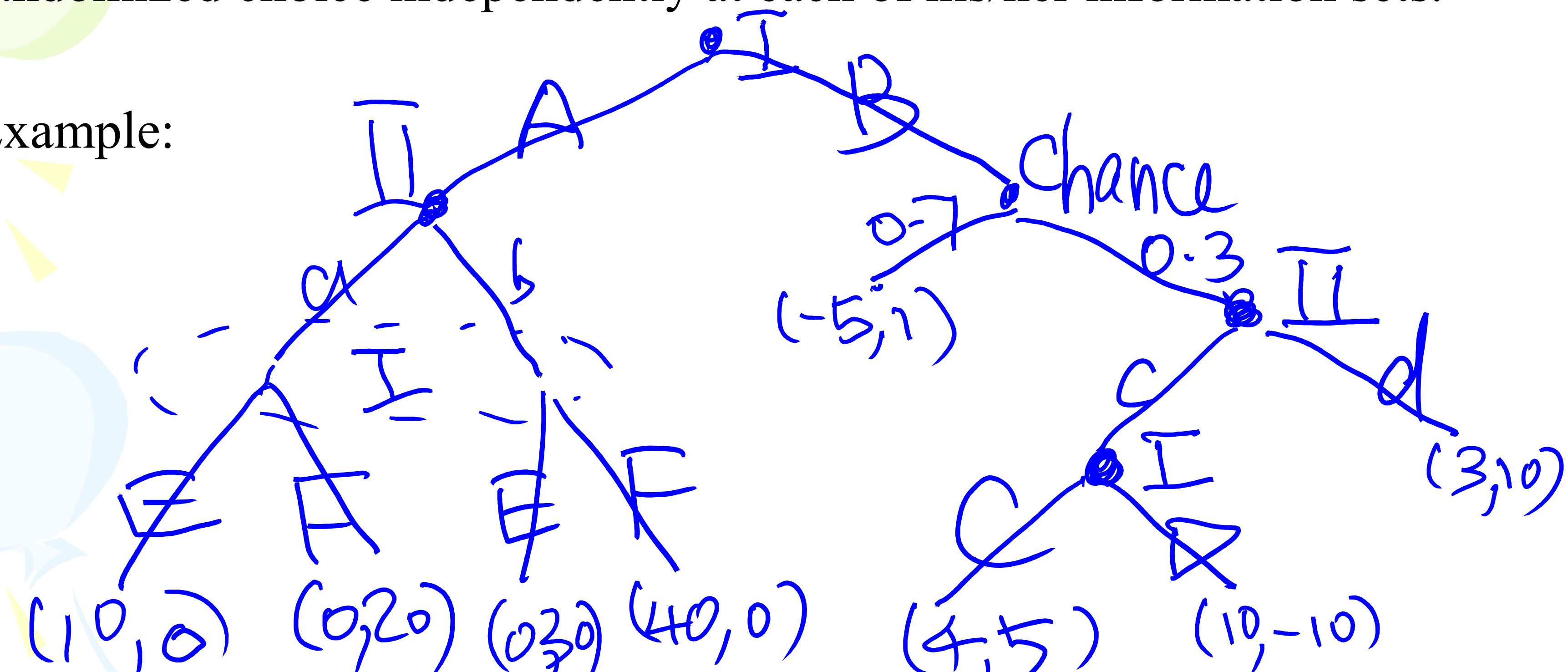
- At the last round of voting, sincere voting is the dominating strategy.
- Then, we can figure out the result of the last round.
- The result of x vs z is z. The result of y vs z is y.
- Player 1 prefers y, Player 2 prefers y and Player 3 prefers z.
- Therefore, Player 1 has a sophisticated voting strategy in round 1 by voting for y. The outcome is then y which Player 1 prefers over z.
- Player 2 will vote for y in round 1 and 2.
- Player 3 does not has the option of sophisticated voting.

The surprise examination paradox: The teacher announces in class: “next week you are going to have an exam, but you will not be able to know on which day of the week the exam is held until that day.” The exam cannot be held on Friday, because otherwise, the night before the students will know that the exam is going to be held the next day. Hence, in the same way, the exam cannot be held on Thursday. In the same way, the exam cannot be held on any of the days of the week.

Is this an application of Backward Induction?

Behavior Strategy: A behavior strategy is obtained if the Player makes a randomized choice independently at each of his/her information sets.

Example:



Player I has 3 information sets. The set of edges coming out from the information sets are P={A,B}, Q={E,F}, R={C,D}.

Player II has two information sets. The set of edges coming out from the information sets are S={a,b}, T={c,d}.

Then the set of behavior strategies for Player I is

$\{(p, q, r): 0 \leq p, q, r \leq 1\}$

Probability p plays A, (1-p) plays B, q plays C, (1-q) plays D, r plays E and (1-r) plays F.

The set of behavior strategies for Player II is

$\{(s, t): 0 \leq s, t \leq 1\}$.

Probability s plays a, (1-s) plays b, t plays c and (1-t) plays d.

A behavior strategy (p, q, r) means

Probability pqr for ACE, p(1-q)r for ADE, pq(1-r) for ACF, p(1-q)(1-r) for ADF. This is a mixed strategy for Player I.

Each behavior strategy induces a probability distribution on the set of terminal vertices (outcomes). Each behavior strategy is a mixed strategy.

Each mixed strategy induces a probability distribution on the set of terminal vertices (outcomes).

Dimension of behavior strategies: Suppose the Player has m information sets such that there are k_i choices at the i^{th} information set. Then, the dimension of behavior strategies for this player is $(k_1 - 1) + (k_2 - 1) + \ldots + (k_m - 1)$. Note that the space of mixed strategies is$(k_1 k_2 \ldots k_m - 1)$.

Example: There are two information sets for Player I. Moves from these are P={S, C}, Q={E, K}.

There is one information set for Player II. Moves from this is R={s, c}.

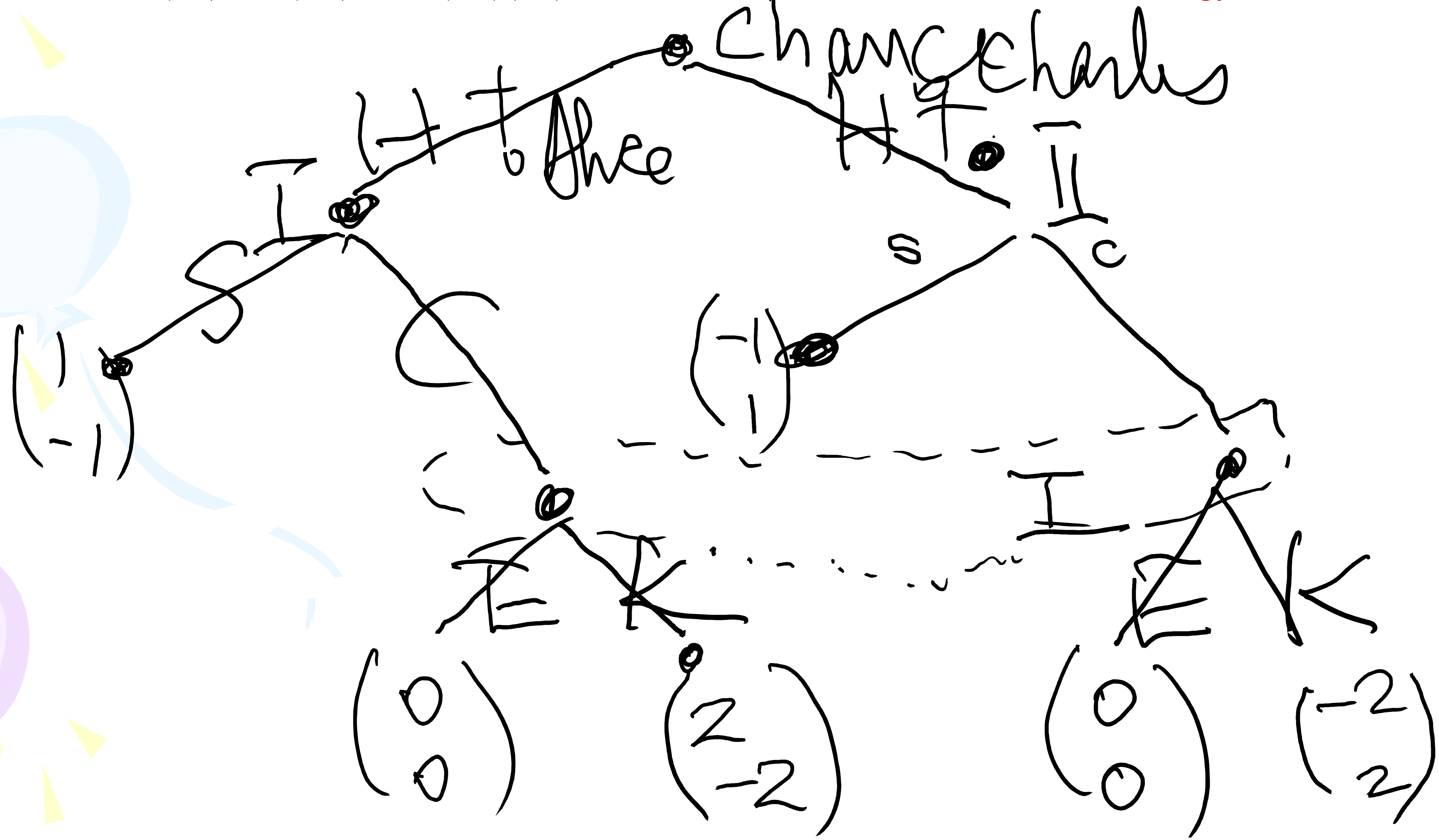
There are 4 pure strategies for Player I: SE, SK, CE, CK, 2 pure strategies for Player II: s, c.

The space of mixed strategies of Player I has dimension=3.

p SE + q SK + r CE + (1-p –q – r) CK, where $0 \leq p, q, r, p+q+r \leq 1$.

The space of behavior strategies for Player I is (sS + (1-s)C)(tE+(1-t)K), $0 \leq s, t \leq 1$.

It is stSE+ s(1-t)SK+(1-s)tCE +(1-s)(1-t)CK. 0.5 SK + 0.5 CE is not a behavior strategy as s=t=0.



Question: Can each mixed strategy be represented by an "equivalent" behavior strategy?

Remark: What do we mean in saying that a mixed strategy is equivalent to a certain behavior strategy?

Answer: They are equivalent if they induce the same probability distribution on the set of vertices (outcomes).

Form the consideration of dimension, the answer should be negative without additional assumptions.

What are the right assumptions to be added?

It is a classical theorem by Harold Kuhn that that answer is positive when the game is of perfect recall.

Theorem (Harold Kuhn); For a game of perfect recall, mixed strategies are equivalent to behavior strategies.

Remark: If we can remember all the past moves and the associated information (perfect recall), then we can randomize as soon as we reach the information set. This is a behavior strategy. However, if we are absentminded, it is best to make plan (what move to choose at each of the information set) before the start of the game. This means choosing a particular pure strategy. Randomization in this case means we use a certain randomization device to choose a particular pure strategy.

Question: If we have perfect recall, do we gain anything if we use behavior strategies?

Kuhn's celebrated theorem says that if we have perfect recall behavior strategies are equivalent to mixed strategies. Hence, it is reasonable for us to use mixed strategies in our discussions.

Bimatrix Games

Two-Person General-Sum Games

General-Sum Strategic Form Games.

The normal or strategic form of a two person game is given by two sets X and Y of pure strategies of the players, and two real-valued functions

$u_1(x, y)$ and $u_2(x, y)$ defined on $X \times Y$, representing the payoffs to the two players.

If Player I chooses $x \in X$ and Player II chooses $y \in Y$, then Player I receives

$u_1(x, y)$ and Player II receives $u_2(x, y)$.

A finite two-person game in strategic form can be represented as a matrix of ordered pairs, sometimes called a bimatrix. The first component of the pair represents Player I's payoff and the second component represents Player II's payoff.

Example :

$$\begin{pmatrix} (1,4) & (2,0) & (-1,1) & (0,0) \\ (3,1) & (5,3) & (3,2) & (4,4) \\ (0,5) & (-2,3) & (4,1) & (2,2) \end{pmatrix}$$

The bimatrix represents the game in which Player I has three pure strategies, the rows, and Player II has four pure strategies, the columns. If Player I chooses row 3 and Player II column 2, then I receives -2

(i.e. he loses 2) and Player II receives 3.

If m and n representing the number of pure strategies of the two players, the game may be represented by two $m \times n$ matrices $\mathbf{A}=(a_{ij})$ and $\mathbf{B} = (b_{ij})$.

The interpretation here is that if Player I chooses row i and Player II chooses column j, then Player I wins a_{ij} and Player II wins b_{ij}, where a_{ij} and b_{ij} are the elements in the i^{th} row, j^{th} column of **A** and **B** respectively.

The game of bimatrix in the above is represented as [**A**,**B**].

Note that the game is zero-sum if and only if the matrix **B** is the negative of the matrix **A**, i.e. $\mathbf{B} = -\mathbf{A}$.

Examples:

1. Prisoner's dilemma (A.W. Tucker)

Two suspected are taken into custody and separated. The district attorney is certain that they are guilty of a specific crime, but he does not have adequate evidence to convict them at a trial.

He points out to each prisoner that each has two alternatives: to confess or not to confess. If they both do not confess, then the district attorney states that he will book them on some minor trumped-up charge such as petty larceny and illegal possession of a weapon, and they will both receive minor punishment,

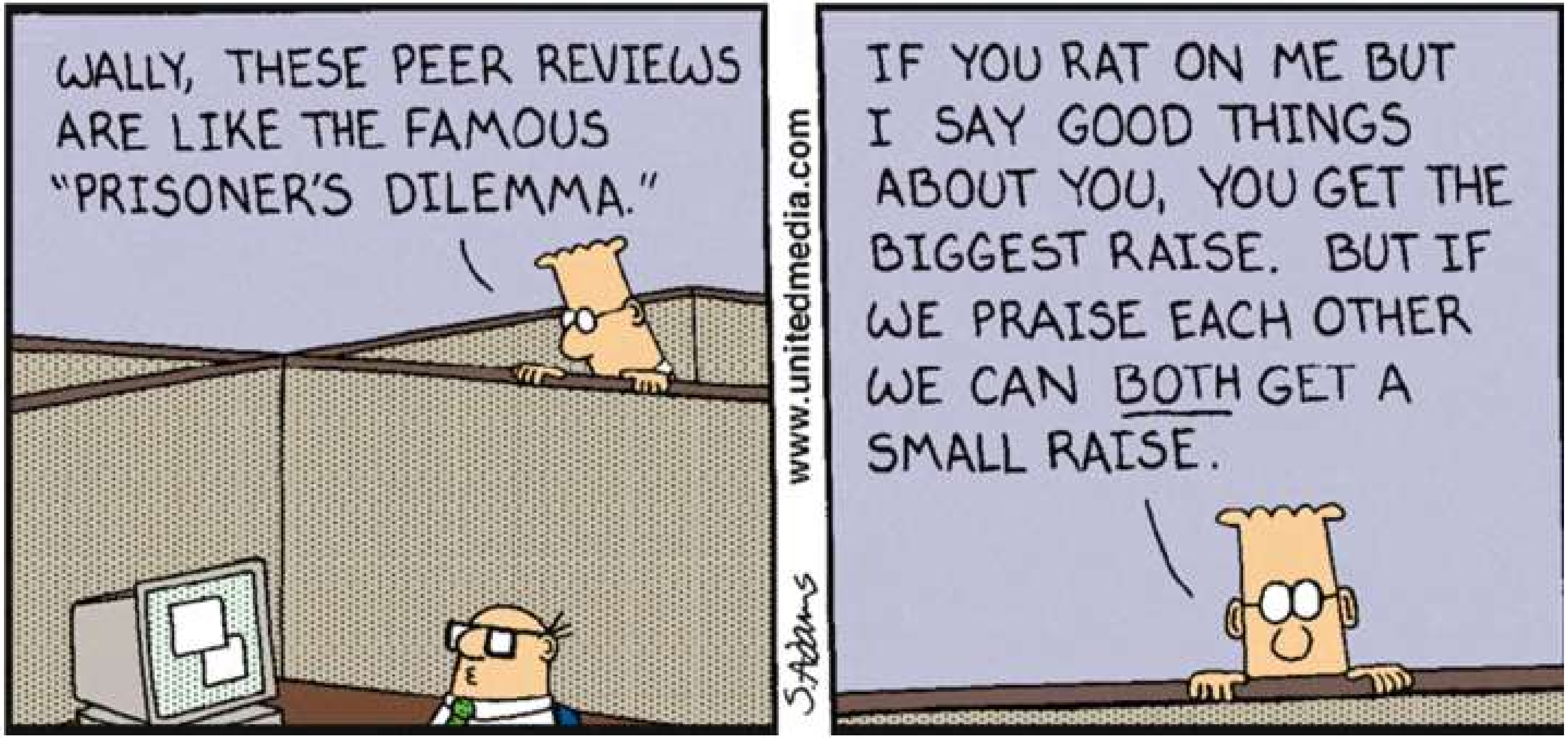
if they both confess they will be prosecuted, but he will recommend less than the most sever sentence; but if one confess and the other does not, then the confessor will receive lenient treatment for turning state's evidence whereas the latter will get "the book" slapped at him.

In terms of years in prison, the strategic problem might reduce to:

		Prisoner 2	
		Not Confess	Confess
Prisoner 1	Not Confess	1 yr each	10 yrs for 1, 3 months for 2
	Confess	3 months for 1, 10 yrs for 2	8 yrs each

To put payoff matrix in terms of utility, we may get the following.

		Prisoner 2	
		Not Confess	Confess
Prisoner 1	NotConfess	(5,5)	(-1,6)
	Confess	(6,-1)	(0,0)


WALLY, THESE PEER REVIEWS ARE LIKE THE FAMOUS "PRISONER'S DILEMMA."
www.unitedmedia.com
S.Adams
IF YOU RAT ON ME BUT I SAY GOOD THINGS ABOUT YOU, YOU GET THE BIGGEST RAISE. BUT IF WE PRAISE EACH OTHER WE CAN BOTH GET A SMALL RAISE.

2. Chicken

In the film "Rebel Without a Cause" James Dean plays a teenager spoiled by prosperity who is challenged to a deadly car race. The two players will drive their cars at maximum speed towards a cliff. The loser (declared chicken) is the first to jump out of his moving car.

The following game matrix is for two cars driving head on. The one who swerve to avoid collision is called the "Chicken"

		Player 2	
		Don't swerve	Swerve
Player 1	Don't swerve	(-3,-3)	(2,0)
	Swerve	(0,2)	(1,1)

3. Battle of sexes

A woman, Player 1, and a man, Player 2, each has two choices for evening's entertainment. Each can go to watch ballet or to watch football. Following the usual cultural stereotype, the woman much prefers the ballet and the man the football, however, to both it is more important that they go out together than that each sees the preferred entertainment.

		Player 2: Ballet	Player 2: Football
Player 1	Ballet	(2,1)	(0,0)
	Football	(0,0)	(1,2)

4. Hawk and Dove

Two birds take the roles of Player 1 and Player 2 in the game. Each has two behaviors, Hawk and Dove. Two hawkish birds fight and hence risk injury. Two dovelike birds share whatever resource is in dispute. A hawkish bird chases off a dovelike bird.

		Player 2: Hawk	Player 2: Dove
Player 1	Hawk	(-25,-25)	(50,0)
	Dove	(0,50)	(15,15)

Overview: The analysis of two-person games is necessarily more complex for general-sum games than for zero-sum games. When the sum of the payoffs is no longer zero (or constant), maximizing one's own payoff is no longer equivalent to minimizing the opponent's payoff.

The Maximin Principle does not apply to bimatrix games!

One can no longer expect to play “optimally” by simply looking at one’s own payoff matrix and guarding against the worst case.

Clearly, one must take into account the opponent’s payoff matrix and the reasonable strategy options of the opponent. In doing so, we must remember that the opponent is doing the same. The general-sum case requires other more subtle concepts of solution.

The theory is generally divided into two branches, the noncooperative theory and the cooperative theory.

In the noncooperative theory, either the players are unable to communicate before decisions are made, or if such communication is allowed, the players are forbidden or are otherwise unable to make a binding agreement on a joint choice of strategy. The main noncooperative solution concept is the *strategic equilibrium* (SE).

In the cooperative theory, it is assumed that the players are allowed to communicate before the decisions are made. They may make threats and counter-threats, proposals and counter-proposals, and hopefully come to some compromise. They may jointly agree to use certain strategies, and it is assumed that such an agreement can be made binding.

The cooperative theory itself breaks down into two branches, depending on whether or not the players have comparable units of utility and are allowed to make monetary *side payments* in units of utility as an incentive to induce certain strategy choices. The corresponding solution concept is called:

- TU cooperative value if side payments are allowed.
- NTU cooperative value if side payments are forbidden or otherwise unattainable.

The initials TU and NTU stand for "transferable utility" and "non-transferable utility" respectively.

Safety Levels.

In a bimatrix game with m×n matrices **A** and **B**, Player I can guarantee winning on the average at least

$v_I = \max_{\mathbf{p}} \min_j (p_1 a_{1j}+\ldots+p_m a_{mj})= \mathrm{Val}(\mathbf{A})$.

This is called the **safety level of Player I**.

Player I can achieve this payoff without considering the payoff matrix of

Player II. A strategy, **p**, that achieves the maximum is called a **maxmin strategy for Player I**.

Similarly, the **safety level of Player II** is

$v_{II}=\max_{\mathbf{q}} \min_i(b_{ij}q_j+\ldots+b_{in}q_n)= \mathrm{Val}(\mathbf{B}^T)$,

since Player II can guarantee winning this amount on the average. Any strategy **q**, that achieves the maximum is a **maxmin strategy for Player II**.

Example: For the following bimatrix game, find the maximin strategies and safety levels for each player.

$$\begin{pmatrix} (2,0) & (1,3) \\ (0,1) & (3,2) \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 3 \\ 1 & 2 \end{pmatrix}$$

$v(A) = 3/2, \quad v(B^T) = 2$

Safety Level of I = $3/2$, Maximin strategy for I : $(3/4, 1/4)$

Safety Level of II = 2, Maximin strategy for II : $(1/2, 1/2)$

Noncooperative Games

Strategic Equilibria

A finite n-person game in strategic form is given by n nonempty finite sets,

$X_1, X_2, \ldots, X_n$, and n real-valued functions $u_1, u_2, \ldots, u_n$, defined on

$X_1 \times X_2 \times \cdot \cdot \cdot \times X_n$.

The set X_i represents the pure strategy set of player i and $u_i(x_1, x_2, \ldots, x_n)$ represents the payoff to player i when the pure strategy choices of the players are

$x_1, x_2, \ldots, x_n$, with $x_j \in X_j$

for $j = 1, 2, \ldots, n$.

It is useful to extend this definition to allow the players to use mixed strategies.

We denote the set of probabilities over k points by P_k:

$P_k = \{\mathbf{p} = (p_1, \ldots, p_k) : p_i \geq 0$ for $i = 1, \ldots, k$, and $p_i + \ldots + p_n = 1\}$.

Let m_i denote the number of pure strategy choices of player i, so that the set X_i has m_i elements. Then the set of mixed strategies of player i is just P_{m_i}

It is denoted by X^*_i where $X^*_i = P_{m_i}$

We denote the set of elements of X_i by the first m_i integers, $X_i = \{1, 2, \ldots, m_i\}$.

Suppose that for i = 1, 2, . . . , n, Player i uses $\mathbf{p}_i = (p_{i1}, p_{i2}, \ldots, p_{imi}) \in X^*_i$. Then the average payoff to player j is

$$u_j(\mathbf{p}_1,\ldots,\mathbf{p}_n) = \sum_{i_1} \cdots \sum_{i_n} p_{1i_1} \cdots p_{ni_n} u_j(i_1,\ldots,i_n)$$

Definition. A vector of mixed strategy choices (also called a strategy profile) $(\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n)$ with $\mathbf{p}_i \in X^*_i$ for $i = 1, \ldots, n$ is said to be a strategic equilibrium, or SE for short, if for all $i = 1, 2, \ldots, n$, and for all $\mathbf{p} \in X^*_i$,

$$u_i(\mathbf{p}_1, \ldots, \mathbf{p}_{i-1}, \mathbf{p}_i, \mathbf{p}_{i+1}, \ldots, \mathbf{p}_n) \geq u_i(\mathbf{p}_1, \ldots, \mathbf{p}_{i-1}, \mathbf{p}, \mathbf{p}_{i+1}, \ldots, \mathbf{p}_n). (*)$$

Any mixed strategy $\mathbf{p}_i$ that satisfies (*) for all $\mathbf{p} \in X^*_i$ is a **best response** of player i to the mixed strategies of the other players.

Thus, a particular selection of mixed strategy choices of the players forms an SE if and only if each player is using a best response to the strategy choices of the other players. No player can gain by unilaterally changing strategy.

Note that a PSE is a special case of an SE when the strategies are all pure strategies.

Remark: If players in a noncooperative game are allowed to communicate and do reach some informal agreement, it may be expected to be a strategic equilibrium. Since no binding agreements may be made, the only agreements that may be expected to occur are those that are

self-enforcing, in which no player can gain by unilaterally violating the agreement.

Each player is maximizing his return against the strategy the other player announced he will use.

It is not what you get.
It is what you do!

正其誼不謀其利
明其道不計其功

The first question that arises is “Do there always exist strategic equilibria?”.

This question was resolved in 1951 by John Nash in the following theorem which generalizes von Neumann’s minimax theorem. In honor of this achievement, strategic equilibria are also called **Nash Equilibria (NE)**.

Theorem. *Every finite n-person game in strategic form has at least one strategic equilibrium.*

EQUILIBRIUM POINTS IN N-PERSON GAMES

By John F. Nash, Jr.*

Princeton University

Communicated by S. Lefschetz, November 16, 1949

One may define a concept of an n-person game in which each player has a finite set of pure strategies and in which a definite set of payments to the n players corresponds to each n-tuple of pure strategies, one strategy being taken for each player. For mixed strategies, which are probability distributions over the pure strategies, the pay-off functions are the expectations of the players, thus becoming polylinear forms in the probabilities with which the various players play their various pure strategies.

Any n-tuple of strategies, one for each player, may be regarded as a point in the product space obtained by multiplying the n strategy spaces of the players. One such n-tuple counters another if the strategy of each player in the countering n-tuple yields the highest obtainable expectation for its player against the $n - 1$ strategies of the other players in the countered n-tuple. A self-countering n-tuple is called an equilibrium point.

The correspondence of each n-tuple with its set of countering n-tuples gives a one-to-many mapping of the product space into itself. From the definition of countering we see that the set of countering points of a point is convex. By using the continuity of the pay-off functions we see that the graph of the mapping is closed. The closedness is equivalent to saying: if $P_1, P_2, \ldots$ and $Q_1, Q_2, \ldots, Q_n, \ldots$ are sequences of points in the product space where $Q_n \rightarrow Q$, $P_n \rightarrow P$ and Q_n counters P_n then Q counters P.

Since the graph is closed and since the image of each point under the mapping is convex, we infer from Kakutani's theorem[1] that the mapping has a fixed point (i.e., point contained in its image). Hence there is an equilibrium point.

In the two-person zero-sum case the "main theorem"[2] and the existence of an equilibrium point are equivalent. In this case any two equilibrium points lead to the same expectations for the players, but this need not occur in general.

* The author is indebted to Dr. David Gale for suggesting the use of Kakutani's theorem to simplify the proof and to the A. E. C. for financial support.

[1] Kakutani, S., *Duke Math. J.*, 8, 457–459 (1941).

“An n-tuple of strategies, one for each player, may be regarded as a point in the product space obtained by multiplying the n strategy spaces of the players. One such n-tuple counters another if the strategy of each player in the countering n-tuple yields the highest obtainable expectation for its player against the n-1 strategies of the other players in the countered n-tuple. A self-countering n-tuple is called an equilibrium point.”

“The correspondence of each n-tuple with its set of countering n-tuples gives a one-to-many mapping of the product space into itself”

“From the definition of countering we see that the set of countering points of a point is convex.”

“By using the continuity of the pay-off functions we see that the graph of the mapping is closed.”

“The closedness is equivalent to saying: if $P_1, P_2, \ldots$ and $Q_1, Q_2, \ldots$ are sequences of points in the product space where $Q_n \to Q$, $P_n \to P$ and Q_n counters P_n, then Q counters P.”

“Since the graph is closed and since the image of each point under the mapping is convex, we infer from Kakutani’s theorem that the mapping has a fixed point (i.e., point contained in its image). Hence there is an equilibrium pair.”

Question: How good are SE's, or equivalently does SE gives players reasonable payoffs?

Theorem: Let [A, B] be a bimatrix game.

Let <p, q> be a SE.

Then, $p^T A q \geq v_I$, $p^T B q \geq v_{II}$.

Remark: This theorem says that using SE both players can get payoff better than their safety levels.

Proof: Let p_0 be the safety strategy for Player I.

Then, $p^T A q \geq p_0{}^T A q$ because p is a BR to q.

As p_0 is a safety strategy, $p_0{}^T A q \geq v_I$.

Hence, $p^T A q \geq v_I$.

Similarly, we can prove $p^T B q = q^T B^T p \geq v_{II}$.

Remark: One of the difficulties of the noncooperative theory is that there are usually many equilibria giving different payoff vectors as we shall see in the following example.

$$\begin{pmatrix} (20,7) & (3,6) \\ (20,7) & (4,8) \\ (5-20) & (5,-20) \end{pmatrix}$$

Another difficulty is that even if there is a unique strategic equilibrium, it may not be considered as a reasonable solution or a predicted outcome.

As in the case for 2-person 0-sum games, we can prove easily the following characterization for SE's.

Proposition:

(i) p is a BR to q whenever $p_i>0$ implies Row i is a BR to q.

(ii) q is a BR to p whenever $q_j>0$ implies Column j is a BR to p.

Theorem: <p, q> is a SE whenever the following two conditions are valid.

(i) $p_i>0$ implies Row i is a BR to q.

(i) $q_j>0$implies Column j is a BR to p.

Labeling algorithm to check for SE's

Let [A,B] be a bimatrix game.

Given a mixed strategy $p=(p_1,\ldots p_m)$ for Player I, and $q=(q_1,\ldots,q_n)$ a mixed strategy for Player II, there is a simple labeling algorithm to determine whether $\langle p, q \rangle$ is a SE.

- Put the label BR to the best response rows to q
- Put the label + at the i^{th} row where p_i is positive
- Put the label BR to the best response columns to p
- Put the label + at the j^{th} column where q_j is positive
- <p, q> is a SE whenever a + label is accompanied by a BR label

Labelling Criterion to check whether <p, q> is a SE

Example: Is <(0,3/8, 5/8), (½, ½)> a SE for the following bimatrix game?

$$\begin{bmatrix} 2,7 & 0,3 \\ 3,-1 & 5,4 \\ 4,4 & 1,1 \end{bmatrix}$$

$$
\begin{array}{c|cc|}
 & + & + \\
\hline
0 & 2,7 & 0,3 \\
3/8 & 3,-1 & 5,4 \\
5/8 & 4,4 & 1,1 \\
\hline
 & \frac{17}{8} & \frac{17}{8} \\
 & \text{BR} & \text{BR}
\end{array}
\qquad
\begin{array}{c|cc|cc}
 & \frac{1}{2} & \frac{1}{2} & & \\
\hline
 & 2,7 & 0,3 & 1 & \\
+ & 3,-1 & 5,4 & 4 & \text{BR} \\
+ & 4,4 & 1,1 & \frac{5}{2} & \\
\hline
\end{array}
$$

$\langle (0, 3/8, 5/8), (\frac{1}{2}, \frac{1}{2}) \rangle$ is not a SE.

We reformulate the Proposition of characterizing BR strategies.

Let $\mathbf{1}_m \in \mathbb{R}^m$ such that all its entries equal to 1.

Let $\mathbf{1}_n \in \mathbb{R}^n$ such that all its entries equal to 1.

Proposition: p is a BR to q whenever

p, α satisfy the following optimization problem.

Max $(p^T A q - \alpha)$

Subject to the following constraints

$Aq - \alpha \mathbf{1}_m \leq 0$

$p^T \mathbf{1}_m - 1 = 0$

$0 \leq p$

Remark: To maximize (p^TAq - α), we like to let p^TAq as large as possible, and α as small as possible.

The constraint $Aq - \alpha\, \mathbf{1}_m \leq 0$ says that α is larger than the payoff when Player I uses any of his/her pure strategy.

Therefore, the maximum of (p^TAq - α) achieves at a BR to q.

Proposition: q is a BR to p whenever

q, β satisfy the following optimization problem.

Max $(p^T B q - \beta)$

Subject to the following constraints

$p^T B - \beta \, \mathbf{1}_n^{\,T} \leq 0$

$q^T \, \mathbf{1}_n - 1 = 0$

$0 \leq q.$

We can then formulate finding SE as a quadratic programming problem.

Theorem: <p, q> is a SE iff p,q, α, β satisfy the following optimization problem.

$$\text{Max } (p^T(A+B)q - \alpha - \beta)$$

Subject to the following constraints

$$Aq - \alpha \, \mathbf{1}_m \leq 0$$

$$p^T \mathbf{1}_m - 1 = 0$$

$$p^T B - \beta \, \mathbf{1}_n{}^T \leq 0$$

$$q^T \mathbf{1}_n - 1 = 0$$

$$0 \leq q, \; 0 \leq p$$

Remark: The optimal value is 0. Note that the objective function is quadratic in its variables.

For 0-sum games, i.e. A=-B, and hence $-\alpha = \beta$, the quadratic programming problem becomes finding p,q, α

satisfying the following constraints

$Aq - \alpha \mathbf{1}_m \leq 0$

$p^T \mathbf{1}_m - 1 = 0$

$-p^T A + \alpha \mathbf{1}_n \leq 0$

$q^T \mathbf{1}_n - 1 = 0$

$0 \leq q,\ 0 \leq p$

Remark: For Linear Programming problems, we have polynomial time algorithms. In fact, the Simplex Algorithm performs very well (polynomial time) “generically”.

For bimatrix game, finding SE can be formulated as a quadratic programming problem. What is its complexity?

Computational complexity of finding SE

Christos Papadimitriou:

But the most interesting aspect of the Nash equilibrium concept to our community is that *it is a most fundamental computational problem whose complexity is wide open.* Suppose that $n = 2$ and S_1, S_2 are finite sets. *Is there a polynomial algorithm for computing a (mixed) Nash equilibrium in such a game?* Because of the guaranteed existence of a solution, the problem is unlikely to be NP-hard; in fact, it belongs to a class of problems “between” P and NP, characterized by reliance on *the parity argument* for the existence proof [24]. In a different direction, as we have already pointed out, this problem is a generalization of linear programming; in fact, there is an algorithm for it that is a combinatorial generalization of the simplex algorithm [2] (as a corollary, the solution is always a vector of rational numbers, something that is not true in general for $n \geq 3$ players).

Together with factoring, *the complexity of finding a Nash equilibrium is in my opinion the most important concrete open question on the boundary of P today.*

At the 2018 International International Congress of Mathematicians, Contantinos Daskalakis of MIT was arwaded the Nevanlinna Prize for his contributions on the computational complexity of Nash Equilibrium.