实验1

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第一章上机题1

实验目标

用MATLAB编程实现例1.4，会出图1-2，体会两种误差队结果不同影响

例 1.4（差商近似 1 阶导数）：对于可微函数 $f: \mathbb{R} \to \mathbb{R}$，考虑 1 阶导数的差商近似[①]

$$f'(x) \approx \frac{f(x+h)-f(x)}{h},$$

其中，h 为步长，试分析按此公式计算 $f'(x)$ 的截断误差与舍入误差，以及它们与 h 值的关系。

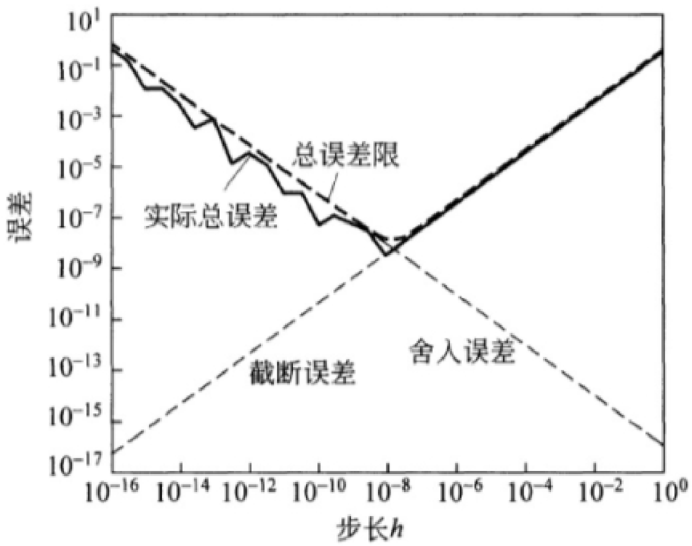


图 1-2　不同步长取值对应的差商近似导数的误差

实验过程

代码

```
h = logspace(-16, 0, 1000);
truncation = h/2;
rounding = 0.0000000000000001 * h .^ -1;
epsilon = truncation + rounding;
error = abs(((sin(1+h)-sin(1)) ./h) - cos(1));
loglog(h,epsilon,'--b', h,truncation, '--k', h, rounding, '--k', h, error,
'r'),axis([0.0000000000000001 1 0.00000000000000001 10]);
```

代码简介

1. $logspace(A, B, N)$ generates a vector of length N, containing points between 10^A and 10^B evenly along a logarithmic axis. We use it to create vector of h values (x-axis)
2. `truncation` represents 截断误差

$$\text{截断误差} = \frac{Mh}{2}, M = 1$$

3. `rounding` represents 舍入误差

$$\text{舍入误差} = \frac{2\epsilon}{h}, \epsilon = 10^-16$$

4. total error 总误差 is represented by `epsilon = rounding + truncation`

   since we know that

$$f(x) = \sin(x), f'(x) \approx \frac{f(x+h) - f(x)}{h},$$

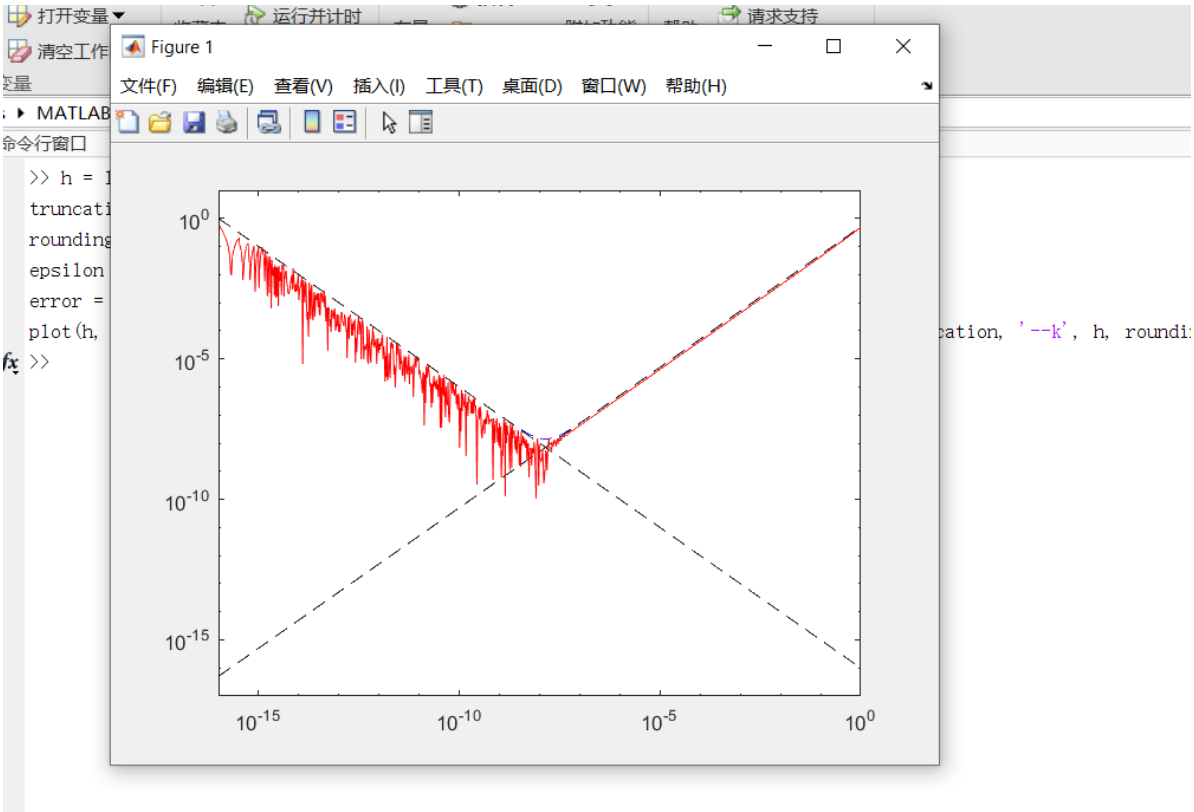
   error $x = 1$ with h is equal to approximation - actual

$$\left| \frac{\sin(1+h) - \sin(1)}{h} - \cos(1) \right|$$

   this represents 实际总误差

5. then we plot the values using loglog which plots *x*- and *y*-coordinates using a base 10 logarithmic scale on the *x*-axis and the *y*-axis, it is useful when dealing with logarithmic scales

实验结果



第一章上机题3

实验目的

编程观察无穷级数

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

的求和计算

1. 采用IEEE单精度浮点数，观察当n位何值时，求和结果不再变化，将它于理论分析的结论进行比较（注：在MATLAB中可用single命令将变量转成单精度浮点数）
2. 用IEEE双精度浮点计算（1）中前n项的和，评估IEEE单精度浮点数计算结果的误差。
3. 如果采用IEEE双精度浮点数，估计当n为何值时求和结果不在变化，这在当前做实验的计算机上大概需要多场的计算时间

实验过程

code is located in lab1_3.m

`sum = single(1)` converts the matrix 1 into a single precision

1. Use a while loop to record the sum value, stay in loop until the difference between sum and last approximately equals 0.

```
sum = single(1);
last = single(0);
j = 1;

while sum - last ~= 0
    last = sum;
    j = j + 1;
    sum = sum + 1/j;
end
disp(j);
```

2. same as part 1, with the removal of single function

```
single_sum = single(1);
single_last = single(0);

j = 1;
while single_sum - single_last ~= 0
    single_last = single_sum;
    j = j + 1;
    single_sum = single_sum + 1 / j;
end

double_sum = 1;
i = 1;
while i ~= j
    i = i + 1;
    double_sum = double_sum + 1/i;
end

disp(single_sum);
disp(double_sum);
disp((single_sum - double_sum) / double_sum);
```

实验结果

1.
```
2097152
```

2.
```
15.4037
15.1333
0.0179
```

3. Double precision numbers have at most 16 significant digits, so when $1/n = 5 \times 10^{-16}$, the sum no longer changes,where $n = 2 \times 10^{15}$. The frequency of the computer used in the experiment is 2.8GHz, so it takes will take about 2×10^{6} seconds, or about 23 days.