博弈论 hw9

RPD1. Given the following matrices for the Prison's Dilemma Game, let β be the discount factor

$$\begin{pmatrix} 2,2 & -1,3 \\ 3,-1 & 0,0 \end{pmatrix}$$

Find

1. Find $\delta \in (0,1)$ so that $\beta > \delta$ implies $\langle PR, PR \rangle$ is a SE
2. Find $\delta \in (0,1)$ so that $\beta > \delta$ implies $\langle TFT, TFT \rangle$ is a SE

Based on Theorem 2, where $\langle PR, PR \rangle$ is a SE if β is large enough $(T > R > P > S, 2R > T + S)$

Supposing that I knows that II is using PR. Payoff to I if I using PR becomes $R + R\beta + R\beta^2 \cdots \to R = (1-\beta)^{-1}$

$$\begin{pmatrix} R,R & S,T \\ T,S & P,P \end{pmatrix}$$

The payoff to Player 1 becomes

$$R(1-\beta^{n-1})(1-\beta)^{-1} + T\beta^{n-1} + P\beta^n(1-\beta)^{-1}$$

For PR to be best response to PR, we need

$$R(1-\beta)^{-1} \geq R(1-\beta^{n-1}) + T\beta^{n-1} + P\beta^n(1-\beta)^{n-1}$$

that means

$$\beta \geq (T-R)/(T-S)$$

For part 1, we get that $\beta > (3-2)/(3-(-1)) = 1/4$, thus $\delta = 1/4$

Based on Theorem 3, where $\langle TFT, TFT \rangle$ is SE if β is large enough $(T > R > P > S, 2R > T + S)$，it suffices to say that when

$$\beta > (T-R)/(R-S)$$

For part 2, we get that $\beta > (3-2)/(2-(-1)) = 1/3$, thus $\delta = 1/3$

RPD2. Given the following playoff matrices for the Prisoner's Dilemma Game, let β be the discount factor

$$\begin{pmatrix} 2,2 & -1,3 \\ 3,1 & 0,0 \end{pmatrix}$$

Let s be a nice strategy (start with Cooperate and never the first one to Defect) such that $< s, s >$ is a SE. Show that there is a constant K independent of s such that $\beta \geq K$

Theorem 4 states that a nice strategy $\langle S, S \rangle$ is a SE. s strategy can be permanent retaliation, where, the player I cooperates until the opponent defects, meaning player I will never be the first one to defect. From the theorem, we can see the above question, that for the strategy $\langle PR, PR \rangle$, the value of $\beta > (3-2)/(3-(-1)) = 1/4$, thus $K = 1/4$. It can also be a TFT strategy. We can use the results from RPD1 to get the value of K

RPD3. Let S be the strategy that it will start with C and continue to do so until the opponent plays D in the previous game. In this case, this strategy will play C with probability 1/3 and D with probability 2/3. Find the transition matrix when Player 1 uses S and Player II uses TFT

Strategy S is a nice strategy. So if we assume the strategy is a TFT. $\langle TFT, TFT\rangle$ is a SE when β is large enough, or more specifically, when

$$\beta > (T-R)/(R-S)$$

If the game is played using these two strategies, the game might look like this,

I C C C ...

II C C C ...

RPD4. Use the Prisoner's Dilemma payoff matrix in Problem RPD1 to show that in a population using $\langle \text{ALL } D, \text{ALL } D\rangle$ and $\langle PR, PR\rangle$ is an ESS when β is sufficiently large.

Based on Theorem 1, $\langle \text{ALL } D, \text{ALL } D\rangle$ is a SE since that if Player I knows II defects all the time, I defects all the time as well (and vice versa). This is an ESS because both players will not change strategies after that. The same concept can be applied to $\langle PR, PR\rangle$, when β becomes sufficiently large, both players will only choose defects, and will no longer switch from that, making it an ESS.

Personal Notes

tragedy of the commons - dilemma arising from the situation in which multiple individuals acting independently and rationally consulting their own self-interest, will ultimately deplete a shared limited resource, even when it is clear that it is not in anyone's long term interest for this to happen.

prisoner's dilemma

- T temptation, R reward, P punishment, S sucker's payoff

how to computer total payoff of a game played infinite number of times?

- discount factor β, x at the n^{th} game is worth $\beta^{n-1}x$,
- All D, defect all times
- PR, Permanent Retaliation, cooperate until , if ever, opponent defects, then defect forever.
- TFT, Tit-for-Tat, cooperate first, then do your opponents previous move
- $AltDC$: alternating defect and cooperate, start with D and then alternatively playing C and D

strategy types

- nice - start cooperating and never first to defect
- retaliatory - it should reliably punish defection by its opponent
- forgiving - having punished defection, it should be willing to try to cooperate again
- clear - it's pattern of play should be consistent and easy to predict