RPD1. In the Prisoner's Dilemma Game

$$T = 3 \qquad R = 2 \qquad P = 0 \qquad S = -1$$

(i) $\langle \mathrm{PR}, \mathrm{PR} \rangle$ is a SE when

$$\beta \geqslant \delta = \frac{T-R}{T-S} = \frac{3-2}{3+1} = \frac{1}{4}$$

(ii) $\langle \mathrm{TFT}, \mathrm{TFT} \rangle$ is a SE when

$$\beta \geqslant \delta = \frac{T-R}{R-S} = \frac{3-2}{2+1} = \frac{1}{3}$$

RPD2. *Proof.* Suppose Player II uses the strategy s. If Player I uses s too, they will always cooperate and the payoff of Player I is

$$\pi(\mathrm{s}, \mathrm{s}) = \sum_{k=0}^{\infty} \beta^k R = \frac{R}{1-\beta} = \frac{2}{1-\beta}$$

If Player I uses the strategy ALLD then Player I always defects while Player II cooperates only in the first game. The payoff of Player I is

$$\pi(\mathrm{ALLD}, \mathrm{s}) = T + \sum_{k=1}^{\infty} \beta^k P = T + \frac{\beta P}{1-\beta} = 3$$

Since $\langle \mathrm{s}, \mathrm{s} \rangle$ is a SE we have that

$$\pi(\mathrm{s}, \mathrm{s}) \geqslant \pi(\mathrm{ALLD}, \mathrm{s}) \implies \frac{2}{1-\beta} \geqslant 3 \implies \frac{1}{3} \leqslant \beta < 1$$

So the constant K can be $\frac{1}{3}$. □

RPD3. The transition matrix is shown in Table 1.

RPD4. *Proof.* Suppose that Player II uses the strategy PR. If Player I uses PR too, they will always cooperate and the payoff of Player I is

$$\pi(\mathrm{PR}, \mathrm{PR}) = \sum_{k=0}^{\infty} \beta^k R = \frac{R}{1-\beta} = \frac{2}{1-\beta}$$

	CC	CD	DC	DD
CC	1	0	0	0
CD	1/3	0	2/3	0
DC	0	1	0	0
DD	0	1/3	0	2/3

Table 1: The transition matrix in RPD3.

If Player II uses ALLD then Player I always defects while Player II cooperates only in the first game. The payoff of Player I is

$$\pi\left(\mathrm{ALLD}, \mathrm{PR}\right) = T + \sum_{k=1}^{\infty} \beta^k P = T + \frac{\beta P}{1-\beta} = 3$$

When $\frac{1}{3} < \beta < 1$ we have that

$$\frac{2}{1-\beta} > 3 \implies \pi\left(\mathrm{PR}, \mathrm{PR}\right) > \pi\left(\mathrm{ALLD}, \mathrm{PR}\right)$$

So PR is an ESS. $\square$