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**Algorithm 1** Karatsuba Algorithm

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**Input:**  $a, b$  - two numbers represent as strings

**Input:**  $n$  - length of the two strings

**Output:** *product* - the product of two numbers

```
1: procedure Karatsuba( $a, b$ )
2:   PadZeroes( $a, b$ )                                ▷ make sure strings are of equal length
3:   if  $a$  and  $b$  are single digit numbers then          ▷ base case
4:      $\text{return } a * b$ 
5:   end if
6:    $al \leftarrow a[0, n/2)$                                 ▷ left of  $a$ 
7:    $ar \leftarrow a[n/2, n)$                                 ▷ right of  $a$ 
8:    $bl \leftarrow b[0, n/2)$                                 ▷ left of  $b$ 
9:    $br \leftarrow b[n/2, n)$                                 ▷ right of  $b$ 
10:   $p1 \leftarrow \text{Karatsuba}(al, bl)$ 
11:   $p2 \leftarrow \text{Karatsuba}(al + ar, bl + br)$ 
12:   $p3 \leftarrow \text{Karatsuba}(ar, br)$ 
13:   $\text{return } (10^n * p1) + 10^{n/2} * (p2 - p1 - p3) + p3$ 
14: end procedure
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**Algorithm 2** Graphics

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**Input:**  $x, y$  - arrays representing points on x and y axis

**Input:**  $n$  - the number of  $x$  and  $y$  points given

**Input:**  $p$  - point given

**Output:** how many lines OP intersects

```
1: procedure Graphics( $x, y$ )
2:   QuickSort( $x$ )
3:   QuickSort( $y$ )
4:   for  $i = 0$  to  $n - 1$  do
5:     if  $(0 - x[i]) * (p.y - 0) - (y[i] - 0) * (p.x - x[i]) > 0$  then          ▷ToLeft algorithm
6:        $\text{return } i$ 
7:     end if
8:   end for
9:    $\text{return } n - 1;$                                 ▷ points intersect all numbers
10: end procedure
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**Algorithm 3** Long Multiplication

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**Input:**  $a, b$  - two numbers represented as strings

**Input:**  $m, n$  - the length of two strings

**Output:** the product of the two numbers

```
1: procedure LongMult( $a, b$ )
2:    $\text{Prod} \leftarrow \text{array}[m + n]$                                 ▷ result is at most size  $m + n$ 
3:    $\text{Carry} \leftarrow 0$                                            ▷ keep track of carry
4:   if  $n < m$  then Swap( $a, b$ )                                     ▷ smaller number on bottom
5:   end if
6:   for  $i = m - 1$  to  $0$  do                                       ▷ multiply starting from end of smaller number
7:     for  $j = n - 1$  to  $0$  do                                       ▷ multiply digit by digit of larger number
8:        $\text{TempProd} \leftarrow \text{Prod}[i + j + 1] + (a[i] * a[j])$       ▷ store ones digit
9:        $\text{Prod}[i + j + 1] \leftarrow \text{TempSum} \bmod 10;$ 
10:       $\text{Prod}[i + j] \leftarrow \text{TempSum} / 10$                     ▷ store sum carry in previous element
11:    end for
12:  end for
13:  return RemoveLeadingZero(Sum)                                ▷ Remove leading zero
14: end procedure
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