Problems definite integrals

18/1/21

① Evaluate:-

$$\int_{-\pi}^{\pi} \cos x \, dx$$

$$I = \left[\sin x\right]_{-\pi}^{\pi} = \left[\sin\pi - \sin(-\pi)\right]$$

$$= \left[\sin\pi + \sin\pi\right] = \left[0+0\right] = 0$$

* $\sin(-\theta) = -\sin\theta$
* $\cos(-\theta) = \cos\theta$
* $\sin\pi = 0$
* $\cos\pi = -1$, $\cos 3\pi = -1$
* $\sin 2\pi = 0$, $\sin n\pi = 0$

$\cos 2\pi = 1$, $\cos 4\pi = 1$

Evaluate:- $\int_0^\infty e^{-x}\,dx$

$$I = \int_0^\infty e^{-x}\,dx = \left[\frac{e^{-x}}{-1}\right]_0^\infty$$

$$= -\left[e^{-x}\right]_0^\infty = -\left[e^{-\infty} - e^{-0}\right]$$

$$= -\left[0-1\right] = 1$$

$$\int e^{ax}\,dx = \frac{e^{ax}}{a}$$

$$e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

$$e^{-0} = \frac{1}{e^{0}} = \frac{1}{1} = 1$$

$$e^{0} = 1$$

Evaluate :- $\int_0^\infty x e^{-x} dx$

$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - \cdots$

$u', u'' \cdots$ derivatives

$v_1, v_2 \cdots$ Integration

$$I = \int_0^\infty \underbrace{x}_{u} \underbrace{e^{-x}}_{v} dx = \left[(x)\left(\frac{e^{-x}}{-1}\right) - (1)\left(\frac{e^{-x}}{(-1)^2}\right)\right]_0^\infty$$

$$= \left[-x e^{-x} - e^{-x}\right]_0^\infty$$

$$= -\left[x e^{-x} + e^{-x}\right]_0^\infty$$

$$= -\left\{\left[0 + 0\right] - \left[0 + e^{-0}\right]\right\}$$

$x = 0$

$$= -\left[-1\right]$$

$$\boxed{I = 1}$$

$e^{-0} = 1$

$$\int u v \, dx = u v_1 - u' v_2 + u'' v_3 - u''' v_4 + \cdots$$

$$\int \underbrace{x}_{u} \, \underbrace{e^{-x}}_{v} \, dx = (x) \cdot \left(\frac{e^{-x}}{-1} \right) - (1) \left(\frac{e^{-x}}{(-1)^2} \right) + 0$$

$$= -x e^{-x} - e^{-x}$$

$$(-1)^2 = 1$$

Vector & Vector Calculus

defn of Scalar : If a physical quantity has only the magnitude then it is called Scalar quantity

Eg:- mass; weight

defn of Vector : If a physical quantity has both magnitude & direction is called Vector quantity

Eg:- Velocity; acceleration

Representation of vectors :-

$$\boxed{\vec{v} = v_1 \hat{i} + v_2 \hat{j} + v_3 \hat{k}} \longrightarrow 3d$$

$v_1, v_2, v_3 \longrightarrow$ functions of x, y, z

$$\boxed{\vec{v} = x \hat{i} + y \hat{j} + z \hat{k}}$$

magnitude $|\vec{v}| = \sqrt{x^2 + y^2 + z^2}$

Dot product of vectors (Scalar product)

Let $\vec{v}_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ & $\vec{v}_2 = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ then

$$\boxed{\vec{v}_1 \cdot \vec{v}_2 = a_1a_2 + b_1b_2 + c_1c_2}$$

$$\hat{i}\cdot\hat{i} = 1 = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k}$$

$$\hat{i}\cdot\hat{j} = 0 = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i}$$

Cross product of vectors (vector product)

Let $\vec{v}_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ & $\vec{v}_2 = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

$$\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$