$$\vec{F} = \underbrace{x^2}_{f_1}\hat{i} + \underbrace{y^2}_{f_2}\hat{j} + \underbrace{z^2}_{f_3}\hat{k} \quad ; \quad \phi = x^2 + y^2 + z^2$$

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$\boxed{\nabla\phi = 2x\,\hat{i} + 2y\,\hat{j} + 2z\,\hat{k}}$$

$$\operatorname{div}\vec{F} = \nabla\cdot\vec{F} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

$$\boxed{\operatorname{div}.\vec{F} = 2x + 2y + 2z}$$

$$\text{Curl } \vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & y^2 & z^2 \end{vmatrix}$$

$$= \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(0-0) = \vec{0}$$

$\phi = x^2 + y^2 + z^2$

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$$

$$= 2 + 2 + 2 = 6 \,//$$

*

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Laplacian operator

Vector Algebra

* $\vec{a} = (a_1 \; a_2 \; a_3) = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$
* $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = (a_1 \; a_2 \; a_3)^T$
* $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

* In general vectors in n-dimensional space is $x = (x_1 \; x_2 \; x_3 \; \cdots \; x_n)^T$

* The norm of a vector is (length)

$$\|x\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$$

* Addition & Subtraction of vectors

$$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} \quad \vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \quad \vec{u} \pm \vec{v} = \begin{pmatrix} u_1 \pm v_1 \\ u_2 \pm v_2 \\ \vdots \\ u_n \pm v_n \end{pmatrix}$$

* The dot product of $\vec{u}$ & $\vec{v}$

$$\vec{u} \cdot \vec{v} = u_1 v_1 + v_2 u_2 + \dots + u_n v_n = \sum_{i=1}^{n} u_i v_i$$

(inner product)

Norms :- p-norm & L_p-norm

$$\|x\|_p = \sqrt[p]{|x_1|^p + |x_2|^p + |x_3|^p + \cdots + |x_n|^p}$$

$$\|x\|_p = \left[|x_1|^p + |x_2|^p + |x_3|^p + \cdots + |x_n|^p\right]^{1/p}$$

3-most widely used norms are

① L_1-norm:- $L_1 = \|x\|_1 = [|x_1| + |x_2| + |x_3| + \cdots |x_n|]$

$\rightarrow$ 1-norm

② L_2-norm:-

$\|x\|_2 = \sqrt{|x_1|^2 + |x_2|^2 + |x_3|^2 + \cdots + |x_n|^2}$

$\longrightarrow$ Euclidean Norm

③ L_∞-norm :-

$$\|x\|_\infty = \left[|x_1|^\infty + |x_2|^\infty + \dots |x_n|^\infty\right]^{1/\infty}$$

or

$$\|x\|_\infty = \max\{|x_1|, |x_2|, \dots |x_n|\} = x_{max}$$

$$\|x\|_\infty = \lim_{p \to \infty} \left(\sum |x_i|^p\right)^{1/p}$$

① $u = \begin{bmatrix} \underset{u_1}{1} & \underset{u_2}{2} & \underset{u_3}{3} \end{bmatrix}^T$ & $v = \begin{bmatrix} \underset{v_1}{1} & \underset{v_2}{-2} & \underset{v_3}{-1} \end{bmatrix}^T$

<u>L_1-norm</u>

$$\|u\|_1 = [\,|u_1| + |u_2| + |u_3|\,] = [1 + 2 + 3] = 6$$

$$\|v\|_1 = [\,|v_1| + |v_2| + |v_3|\,] = [\,|1| + |-2| + |-1|\,]$$
$$= [1 + 2 + 1] = 4$$

L_2-norm :-

$$\|u\|_2 = \sqrt{|u_1|^2 + |u_2|^2 + |u_3|^2} = \sqrt{|1|^2 + |2|^2 + |3|^2} = \sqrt{14}$$

$$\|v\|_2 = \sqrt{|v_1|^2 + |v_2|^2 + |v_3|^2} = \sqrt{|1|^2 + |-2|^2 + |-1|^2} = \sqrt{6}$$

L_∞ norm

$$\|u\|_\infty = \max\{|u_1|, |u_2|, |u_3|\} = \{|1|, |2|, |3|\}$$

$$= 3$$

$$\|v\|_\infty = \max\{|v_1|, |v_2|, |v_3|\} = \{|1|, |-2|, |-1|\}$$

$$= 2$$

$$u+v=\begin{bmatrix}1 & 2 & 3\end{bmatrix}^T+\begin{bmatrix}1 & -2 & -1\end{bmatrix}^T$$

$$=\begin{bmatrix}1+1 & 2+(-2) & 3+(-1)\end{bmatrix}^T$$

$$=\begin{bmatrix}2 & 0 & 2\end{bmatrix}^T=w$$

$$\|w\|_1=2+0+2$$

$$=4 \,//$$

$$\|w\|_2=\sqrt{2^2+0^2+2^2}$$

$$=\sqrt{8} \,//$$

$$\|w\|_\infty=\max(|2|,|0|,|2|)$$

$$=2 \,//.$$

Find L_1, L_2, L_∞-norms for

$u+v$ & $u-v$

$$u = [1\ \ -2\ \ 4\ \ -5]^T$$

$$v = [0\ \ -1\ \ 1\ \ 5]^T$$

$$u+v = [1\ \ -3\ \ 5\ \ 0]^T$$

$$u+v = X$$

L_1-norm

$$\|X\|_1 = \{|1|+|-3|+|5|+|0|\} = 9$$

L_2-norm

$$\|X\|_2 = \sqrt{|1|^2+|-3|^2+|5|^2+0^2} = \sqrt{35}$$

$$u-v = [1\ \ -1\ \ 3\ \ -10]^T$$

$$u-v = Y$$

L_1-norm

$$\|Y\|_1 = \{|1|+|-1|+|3| + |-10|\} = 15$$

L_2-norm

$$\|Y\|_2 = \sqrt{|1|^2+|-1|^2+|3|^2 + |-10|^2} = \sqrt{111}$$

$$\|X\|_\infty = 5$$

$$\|Y\|_\infty = 10$$