① $y = \underbrace{(1+\sqrt{x^3})}_{u}\underbrace{(x^{-3} - 2\sqrt[3]{x})}_{v}$ 7/1/21

$y = \underbrace{(1+x^{3/2})}_{u}\underbrace{(x^{-3} - 2x^{1/3})}_{v}$

② $y = \dfrac{\sqrt{t} + 2t}{7t - 4t^2}$

$y = \dfrac{t^{1/2} + 2t}{7t - 4t^2} \left(\dfrac{u}{v}\right)$

## Successive differentiation

Successive differentiation is the process of differentiating a given function successively 'n' times and the result of such diffⁿ are called successive differentiation.

**Note:-** ① The higher order differential co-efficients are most important in scientific & Engineering applⁿ

② Let f(x) be a differentiable function & its successive derivatives be denoted by

$f'(x), f''(x), f'''(x), \ldots f^{(n)}(x)$

## Notations:-

1st derivative $\rightarrow \frac{dy}{dx} = y_1 = f'(x) = Dy$

2nd derivative $\rightarrow \frac{d^2y}{dx^2} = y_2 = f''(x) = D^2y$

3rd derivative $\rightarrow \frac{d^3y}{dx^3} = y_3 = f'''(x) = D^3y$

$\vdots$

nth derivative $\rightarrow \frac{d^ny}{dx^n} = y_n = f^{(n)}(x) = D^ny$

$D$ = differential operator $= \frac{d}{dx}$

## problems on successive differentiation

① Find $y_2$ for $y = e^{3x+2}$

Soln

$$y = e^{3x+2}$$

diff. w.r.to $x$, we get

$$y_1 = e^{3x+2} \cdot \frac{d}{dx}(3x+2)$$

$$\boxed{y_1 = 3e^{3x+2}}$$

diff $y_1$ w.r.to $x$

$$y_2 = 3 \cdot e^{3x+2}(3)$$

$$\boxed{y_2 = 9e^{3x+2}}$$

2) find 2nd order derivative of

$y = \log x + a^x$

diff. $y$ w.r.to $x$

$y_1 = \frac{1}{x} + a^x(\log a)$

diff. $y_1$ w.r.to $x$

$y_2 = -\frac{1}{x^2} + (\log a)\, a^x (\log a)$

$$\boxed{y_2 = -\frac{1}{x^2} + (\log a)^2 \cdot a^x}$$

* $\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(x^{-1}) = (-1)\, x^{-1-1}$

$(-1)x^{-2}$

$= -\frac{1}{x^2}$

③ $y = x^3 + 3e^{2x} - 4\sin x$, $y_2 = ?$

$y_1 = 3x^2 + (3\times2)e^{2x} - 4(\cos x)$

$y_1 = 3x^2 + 6e^{2x} - 4\cos x$

$y_2 = 6x + 12e^{2x} + 4\sin x$ //

(4) $y = \underbrace{x^2}_{u}\ \underbrace{\sin(2x)}_{v}$

diff. y. w.r.to x

$y_1 = \underbrace{2x^2}_{u}\ \underbrace{\cos 2x}_{v} + \underbrace{2x}_{u} \cdot \underbrace{\sin 2x}_{v}$

diff $y_1$ w.r.to x

$y_2 = 2x^2(-2\sin 2x) + \cos 2x\,(4x)$
$\quad + 2x(2\cos 2x) + \sin 2x\,(2)$

$= -4x^2\sin 2x + \underline{4x\cos(2x) + 4x\cos(2x)}$
$\quad + 2\sin 2x$

$$\boxed{y_2 = 8x\cos 2x - 4x^2\sin 2x + 2\sin 2x}$$

$* \dfrac{d}{dx}\left(\underbrace{2x^2}_{u} \cdot \underbrace{\cos 2x}_{v}\right)$

$= (2x^2) \cdot (-2\sin 2x)$
$\quad + \cos 2x\,(4x)$

$= -4x^2\sin 2x + 4x\cos 2x$

(5) If $y = 2 + \log x$
then ST $xy_2 + y_1 = 0$

[ $y_1$ ; $y_2$ ; $xy_2$ ; LHS $= 0 =$ RHS ]

Soln $y = 2 + \log x$
diff. w.r.to $x$

$y_1 = 0 + \frac{1}{x}$ ; $\boxed{y_1 = \frac{1}{x}}$

diff $y_1$ w.r.to $x$

$\boxed{y_2 = -\frac{1}{x^2}}$

$xy_2 = \cancel{x}\left(-\frac{1}{x^{\cancel{2}}}\right)$

$\boxed{xy_2 = -\frac{1}{x}}$

LHS $= xy_2 + y_1$

$= -\frac{1}{x} + \frac{1}{x} = 0 =$ RHS

Hence proved $xy_2 + y_1 = 0$

Assignment I :- Find $y_2$

(1) $y = x\sin(2x)$

(2) $y = e^{2x}\cos(3x)$

(3) $y = \frac{x}{1+e^x}$

④ If $y = a\cos mx + b\sin mx$

S.T $y_2 + m^2 y = 0$

⑤ $y = \sin(\log x)$

ST $x^2 y_2 + x y_1 + y = 0$

## Applications of derivatives

① **Position**: It is the location of object and is given as the function of time $x(t)$

② **Velocity**:- It is the derivative of position $\boxed{v = \frac{dx}{dt}}$

* Rate of change of position

③ **Acceleration**:- It is the derivative of velocity. $\boxed{a = \frac{dv}{dt} = \frac{d^2x}{dt^2}}$

* Rate of change of velocity.

**Problem**

① Find velocity at $t = 2$ seconds when displacement is given

Q) $x(t) = t^3 - 2t^2 + 4$.

$x$ is in meters & $t$ is in sec.

Sol $x(t) = t^3 - 2t^2 + 4$

diff. w.r.to $t$

$$\frac{dx}{dt} = \boxed{v = 3t^2 - 4t}$$

put $t = 2$ secs

$$v = 3(2)^2 - 4(2)$$

$$v = 12 - 8$$

$$\boxed{v = 4 \text{ m/sec}}\ *$$