Matrix Theory

27/1/21

* Matrix, Algebra
* Determinants
* Inverse; 2x2; 3x3
* Eigen values & Eigen vectors
* Decomposition

* Matrix Exp
* Hermitian
* Definiteness
* Quadratic fn

Defn of Matrix :-

A systematic arrangement of elements (real/complex) in the form of Rows [Horizontal] & columns (vertical) within a square brackets, is called matrix.

Eg :- $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}_{2\times 2}$; $B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 4 & 3 \end{bmatrix}_{3\times 2}$; $C = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}_{1\times 3}$

General representation of Matrix :-

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

$m \rightarrow$ no. of Rows

$n \rightarrow$ no. of columns

$a_{11} \rightarrow$ position of Element 1st row & 1st column

$a_{23} \rightarrow$ 2nd row
3rd column

Types of Matrix:-

1) Square

A matrix with no. of rows is Equal no. of columns

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}_{2\times 2}$$

$* \; m = n$

2) Rectangular

A matrix with no. of rows not Equal nor of columns

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 4 & -5 \end{bmatrix}_{2\times 3}$$

$* \; m \neq n$

Identity / Unit Matrix

A diagonal matrix in which all the diagonals is equal to 1 or unity

Eg:- $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$ $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$

Algebra of Matrices :-

① If $A=\begin{bmatrix}1 & -1\\ 4 & 3\end{bmatrix}$ $B=\begin{bmatrix}0 & 1\\ 7 & -5\end{bmatrix}$ find $A+B$ & $3A-4B$

$$A+B=\begin{bmatrix}1 & -1\\ 4 & 3\end{bmatrix}+\begin{bmatrix}0 & 1\\ 7 & -5\end{bmatrix}=\begin{bmatrix}1+0 & -1+1\\ 4+7 & 3+(-5)\end{bmatrix}=\begin{bmatrix}1 & 0\\ 11 & -2\end{bmatrix}$$

$$3A-4B=\begin{bmatrix}3 & -3\\ 12 & 9\end{bmatrix}-\begin{bmatrix}0 & 4\\ 28 & -20\end{bmatrix}=\begin{bmatrix}3-0 & -3-4\\ 12-28 & 9-(-20)\end{bmatrix}=\begin{bmatrix}3 & -7\\ -16 & 29\end{bmatrix}$$

② Find $2A + 3B$ where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & -4 \\ 2 & -1 \\ -2 & 3 \end{bmatrix}$

$$2A + 3B = \begin{bmatrix} 2 & 4 \\ 6 & 8 \\ 2 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -12 \\ 6 & -3 \\ -6 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0 & 4-12 \\ 6+6 & 8-3 \\ 2-6 & 0+9 \end{bmatrix} = \begin{bmatrix} 2 & -8 \\ 12 & 5 \\ -4 & 9 \end{bmatrix}$$

① If $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 6 \end{bmatrix}$ find $A.B$ & $B.A$

verify $A.B \neq B.A$

$$A.B = \underset{2\times 3}{\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \end{bmatrix}} \cdot \underset{3\times 2}{\begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 6 \end{bmatrix}} = \begin{bmatrix} 3+0+3 & 6+8+6 \\ 2+0+6 & 4+16+12 \end{bmatrix} = \underset{2\times 2}{\begin{bmatrix} 6 & 20 \\ 8 & 32 \end{bmatrix}}$$

$\therefore$

$$B \cdot A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 3 & 6 \end{bmatrix}_{3\times 2} \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \end{bmatrix}_{2\times 3}$$

$$= \begin{bmatrix} 3+4 & 2+8 & 1+4 \\ 0+8 & 0+16 & 0+8 \\ 9+12 & 6+24 & 3+12 \end{bmatrix}_{3\times 3} = \begin{bmatrix} 7 & 10 & 5 \\ 8 & 16 & 8 \\ 21 & 30 & 15 \end{bmatrix}_{3\times 3}$$

$$\boxed{A_{m\times p} \cdot B_{p\times n} = AB_{m\times n}}$$

(4) $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -1 & 4 \end{bmatrix}$

find AB & BA

$$A \cdot B = \underset{3\times1}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}} \cdot \begin{bmatrix} 3 & -1 & 4 \end{bmatrix}_{1\times3} = \begin{bmatrix} 3 & -1 & 4 \\ 6 & -2 & 8 \\ 9 & -3 & 12 \end{bmatrix}_{3\times3}$$

$$B \cdot A = \begin{bmatrix} 3 & -1 & 4 \end{bmatrix}_{1\times3} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times1} = \begin{bmatrix} 3 + (-2) + 12 \end{bmatrix} = \begin{bmatrix} 13 \end{bmatrix}_{1\times1}$$

Determinants :-

Every square matrix has a unique value is called its determinant. It is denoted by $|A|$.

Eg :- $A = \begin{bmatrix} 1 & -1 \\ 2 & 7 \end{bmatrix}_{2\times 2}$ $\quad |A| = \begin{vmatrix} 1 & -1 \\ 2 & 7 \end{vmatrix} = 7 - (-2) = 7 + 2 = 9$

$$A = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{bmatrix} 1 & -1 & 2 \\ 5 & 0 & 3 \\ 4 & -1 & 8 \end{bmatrix} \end{array} = 1(0+3) - (-1)(40-12) + 2(-5-0)$$

$$= 3 + 28 - 10 = 21$$

* If $|A| = 0 \Rightarrow A$ is singular matrix

* If $|A| \neq 0 \Rightarrow A$ is non-singular matrix