① $y = \dfrac{6x^2}{2-x}$ $\begin{matrix} = u \\ = v \end{matrix}$ 31/12/2020

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$

$$\frac{dy}{dx} = \frac{(2-x)\,d(6x^2) - 6x^2\,d(2-x)}{(2-x)^2}$$

$$= \frac{(2-x)(12x) - 6x^2(-1)}{(2-x)^2}$$

$$= \frac{24x - 12x^2 + 6x^2}{(2-x)^2}$$

$$= \frac{24x - 6x^2}{(2-x)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{6x(4-x)}{(2-x)^2}}$$

② $y = \dfrac{\sqrt{t} + 2t}{7t - 4t^2}$ find $\dfrac{dy}{dt}$

diff. w.r.to t

$$\frac{dy}{dt} = \frac{(7t - 4t^2)\,d(t^{1/2} + 2t) - (\sqrt{t} + 2t)\,d(7t - 4t^2)}{(7t - 4t^2)^2}$$

$$= \frac{(7t - 4t^2)\left(\frac{1}{2\sqrt{t}} + 2\right) - (\sqrt{t} + 2t)(7 - 8t)}{(7t - 4t^2)^2}$$

Simplify

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(3) $y = \dfrac{x+\tan x}{1+\operatorname{cosec} x}$

diff. w.r.t x

$$\frac{dy}{dx} = \frac{(1+\operatorname{cosec} x)(1+\sec^2 x) - (x+\tan x)(-\operatorname{cosec} x \cot x)}{(1+\operatorname{cosec} x)^2}$$

$$= \frac{(1+\operatorname{cosec} x)(1+\sec^2 x) + (x+\tan x)(\operatorname{cosec} x \cot x)}{(1+\operatorname{cosec} x)^2}$$ //

(4) $y = 2\cos x - 6\sec x + 3$

$\frac{dy}{dx} = -2\sin x - 6\sec x \tan x$ //

(5) $y = 10\tan x - 2\cot x$

$\frac{dy}{dx} = 10\sec^2 x + 2\operatorname{cosec}^2 x$

(6) $y = \underset{u}{\tan x}\ \underset{v}{\sec x}$

$\frac{dy}{dx} = \tan x \frac{d}{dx}(\sec x) + \sec x \frac{d}{dx}(\tan x)$

$= \tan x\,(\sec x \tan x) + \sec x\,(\sec^2 x)$

$$= \sec x \tan^2 x + \sec^3 x$$

$$= \sec x (\tan^2 x + \sec^2 x)$$

$$y = 6 + 4\sqrt{x}\, \operatorname{cosec} x$$

$$\frac{dy}{dx} = 0 + 4\sqrt{x}\cdot(-\operatorname{cosec} x \cot x) + \operatorname{cosec} x \cdot \left(\frac{\cancel{4}^{2}}{\cancel{2}_{1}\sqrt{x}}\right)$$

$$= \frac{2}{\sqrt{x}} \operatorname{cosec} x - 4\sqrt{x}\, \operatorname{cosec} x \cot x$$

$$\boxed{\frac{dy}{dx} = 2\operatorname{cosec} x\left(\frac{1}{\sqrt{x}} - 2\cot x\right)}\,//$$

$$y = 2e^x - 8^x$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}; \quad \frac{d}{dx}(a^x) = a^x \log a$$

$$\frac{dy}{dx} = 2e^x - 8^x \log 8 \,//$$

$$y = 4\log x - e^x \cdot x^5$$

$$\frac{dy}{dx} = \frac{4}{x} - \left[e^x x^5 + 5x^4 e^x\right]$$

$$\frac{dy}{dx} = \frac{4}{x} - e^x x^4 (x+5)$$

$y = \dfrac{1+5x}{\log x}$

diff. w.r.t. x using quotient rule

$$\frac{dy}{dx} = \frac{\log x \, d(1+5x) - (1+5x)\, d(\log x)}{(\log x)^2}$$

$$= \frac{5\log x - \left(\frac{1+5x}{x}\right)}{(\log x)^2}$$

$$= \frac{5x\log x - (1+5x)}{x(\log x)^2}$$

find tangent of $f(x) = 7^x + 4e^x$ at $x=0$

$$f(x) = 7^x + 4e^x$$

$$\text{tangent} = \frac{dy}{dx} = 7^x \log 7 + 4e^x$$

at $x=0$,

$$\boxed{\left(\frac{dy}{dx}\right)_{x=0} = \log 7 + 4}$$

* $\log 7 = \ln(7)$ — base e

① $y = 2\cos x + 6\cos^{-1} x$

(2) $y = \operatorname{cosec}^{-1} x - 4\cot^{-1} x$

(3) $y = 5x^6 - \sec x \tan x$

(4) $y = \sin t + t^2 \tan^{-1} t$

(5) $y = \dfrac{\sin^{-1} x}{1+x}$

Problems on chain-Rule

$$\frac{d}{dx} f[g(x)] = f'(g(x)) \cdot \frac{d}{dx} g(x)$$

(1) $y = \sin(x^2)$

diff. w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin(x^2)\right]$$

$$= \cos(x^2) \cdot \frac{d}{dx}(x^2)$$

$$dy/dx = 2x\cos(x^2)$$

(2) $y = \cos\left[\log(2x)\right]$

diff y w.r.to x

$$\frac{dy}{dx} = -\sin\left[\log(2x)\right] \cdot \frac{d}{dx} \log(2x)$$

$$= -\sin[\log(2x)] \cdot \frac{1}{2x} \cdot \frac{d}{dx}(2x)$$

$$= -\sin[\log(2x)] \cdot \frac{\cancel{2}}{\cancel{2}x}$$

$$= -\frac{\sin(\log(2x))}{x}$$

③ $y = \sin^{-1}(x^2)$ $\quad z$

$$\frac{d}{dx}(\sin^{-1} z) = \frac{1}{\sqrt{1-z^2}}$$

$$\frac{d}{dx}\sin^{-1}(x^2) = \frac{1}{\sqrt{1-(x^2)^2}} \cdot \frac{d}{dx}(x^2)$$

$$= \frac{1}{\sqrt{1-x^4}} \cdot 2x$$

$y = \tan^{-1}\left(\frac{2}{x}\right)$

diff. w.r.to x

$$\frac{d(\tan^{-1}x)}{dx} = \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{1}{1+(2/x)^2} \cdot \frac{d}{dx}\left(\frac{2}{x}\right)$$

$$= \frac{1}{1+\left(\frac{4}{x^2}\right)} \cdot 2 \cdot \left(-1x^{-1-1}\right)$$

$$= \frac{-2x^{-2}}{1+4/x^2}$$

$$\boxed{\frac{dy}{dx} = \frac{-2}{x^2\left[1+\frac{4}{x^2}\right]}}$$

$$= \frac{-2}{\cancel{x^2}\left[\frac{x^2+4}{\cancel{x^2}}\right]} = \frac{-2}{x^2+4}$$

$$y = e^{1-\cos x} = e^{z}$$

diff. w.r.t. x

$$\frac{dy}{dx} = e^{1-\cos x} \cdot \frac{d}{dx}(1-\cos x)$$

$$\boxed{\frac{dy}{dx} = \sin x \, e^{1-\cos x}}$$

① $y = (6x^2+7x)^4$

② $y = (4t^2-3t+2)^{-2}$

③ $y = \tan^{-1}(3x-1)$