① Solve $\dfrac{d^3y}{dx^3} + 2\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = e^{-x} + \sin 2x$

<u>To find CF</u>

$m^3 + 2m^2 + m = 0$ $\quad m(m^2 + 2m + 1) = 0$

$m_2 = -1 \; ; \; m_3 = -1 \; ; \; m_1 = 0$

$CF = C_1 e^{m_1 x} + (C_2 + C_3 x) e^{m_2 x}$

$CF = C_1 e^{0x} + (C_2 + C_3 x) e^{-x}$

$\boxed{CF = C_1 + (C_2 + C_3 x) e^{-x}}$ — ①

$$f(x) = e^{-x} + \sin 2x$$

$$PI = PI_1 + PI_2$$

$$PI_1 = \frac{e^{-x}}{D^3 + 2D^2 + D} \checkmark \quad a = -1$$

$$D = a = -1$$

$$= \frac{e^{-x}}{(-1)^3 + 2(-1)^2 + (-1)} = \frac{e^{-x}}{-1+2-1} = \frac{e^{-x}}{0}$$

x'ly Nr by x, diff Dr

$$PI_1 = \frac{x e^{-x}}{3D^2 + 4D + 1} \checkmark$$

$$D = -1$$

$$PI_1 = \frac{x e^{-x}}{3(-1)^2 + 4(-1) + 1}$$

$$= \frac{x e^{-x}}{0}$$

x'ly Nr by x dif Dr

$$PI_1 = \frac{x^2 e^{-x}}{6D+4}$$

$D = -1$

$$PI_1 = \frac{x^2 e^{-x}}{-6+4}$$

$$\boxed{PI_1 = -\frac{x^2 e^{-x}}{2}} \quad (2)$$

$$PI_2 = \frac{\sin 2x}{D^3+2D^2+D}$$

$$PI_2 = \frac{\sin 2x}{D(D^2)+2D^2+D}$$

* $D^2 = -a^2 = -2^2 = -4$

$$PI_2 = \frac{\sin 2x}{-4D+2(-4)+D}$$

$$= \frac{\sin 2x}{-3D-8}$$

$$= \frac{-\sin 2x}{3D+8}$$

$$= -\frac{\sin 2x\,(3D-8)}{(3D)^2 - 8^2}$$

$$= \frac{-(3D-8)\sin 2x}{9D^2 - 64}$$

$$D^2 = -4$$

$$= -\frac{(3D-8)\sin 2x}{9(-4)-64}$$

$$= \frac{+(3D-8)\sin 2x}{+100}$$

$$= \frac{1}{100}\left[3D(\sin 2x) - 8\sin 2x\right]$$

$$PI_2 = \frac{1}{100}\left[6\cos 2x - 8\sin 2x\right] \text{ --- } (3)$$

$$y = CF + PI_1 + PI_2$$

$$(D^2+4)y = \cos(2x+3)$$

$$CF = C_1 \cos 2x + C_2 \sin 2x$$

$$PI = \frac{\cos(2x+3)}{D^2+4}$$

$$D^2 = -a^2 = -2^2 = -4$$

$$PI = \frac{\cos(2x+3)}{-4+4} = Dr = 0$$

x ly nr by x diffe dr

$$PI = x \cdot \frac{\cos(2x+3)}{2D}$$

$$PI = \frac{x}{2} \cdot \int \cos(2x+3)\,dx$$

$$\frac{1}{D} = \int dx$$

$$PI = \frac{x}{2} \cdot \frac{\sin(2x+3)}{2}$$

$$\boxed{PI = \frac{x \sin(2x+3)}{4}}$$

$$*\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \log 2$$

$$(D^2 - 6D + 9)y = (\log 2)e^{0x}$$

CF

$$m^2 - 6m + 9 = 0$$

$$m = 3, 3$$

$$\boxed{CF = (c_1 + c_2 x)e^{3x}}$$

$$(D^2 - 6D + 9)y = K$$

$$PI = K\left\{\frac{e^{0x}}{D^2 - 6D + 9}\right\}$$

$$e^{0x} = 1$$

$$PI = \frac{\log 2}{D^2 - 6D + 9}$$

$$PI = \log 2\left[\frac{e^{0x}}{D^2 - 6D + 9}\right]$$

Put $D = a = 0$

$$PI = \log 2\left[\frac{e^{0x}}{0 - 6(0) + 9}\right]$$

$$PI = \frac{\log 2}{9}$$

$$y = CF + PI$$

$$y'' + 4y' - 12y = 2^x$$

$$(D^2 + 4D - 12)y = 2^x$$

<u>CF</u> $\Rightarrow$

$$m^2 + 4m - 12 = 0$$

$$m_1 = 2 \; ; \; m_2 = -6$$

$$CF = c_1 e^{2x} + c_2 e^{-6x}$$

$$PI = \frac{e^{\log(2^x)}}{F(D)}$$

$$PI = \frac{e^{(\log 2)x}}{D^2 + 4D - 12}$$

$a = \log 2$

$a = \ln 2$

$$D = \log 2$$

$$PI = \frac{e^{(\log 2)x}}{(\log 2)^2 + 4(\log 2) - 12} = \frac{2^x}{-8.74}$$

Note :-

① $a^x = e^{\log a^x}$

$= e^{x(\log a)}$

$= e^{Ax}$

② $\sinh\theta = \dfrac{e^{\theta} - e^{-\theta}}{2}$

③ $\cosh\theta = \dfrac{e^{\theta} + e^{-\theta}}{2}$

Solve $(D^2+4D+5)y + 2\cosh x = 0$

$$(D^2+4D+5)y = -2\cosh x = -\left(e^{x}+e^{-x}\right)$$

$$-\left[PI_1 + PI_2\right]$$

*CF

$$PI = \frac{-2\cosh x}{F(D)}$$

$$= -2\left[\frac{e^{x}+e^{-x}}{2}\right]$$

$$= -e^{x} - e^{-x}$$

$$(D^2+1)y = \sin(a+x) \quad \text{with} \quad y(0)=0=y'(0)$$

CF $m^2+1=0$; $m=\pm i$

$\therefore CF = e^{0x}\left[C_1\cos x + C_2\sin x\right]$

$$\boxed{CF = C_1\cos x + C_2\sin x}$$

PI : $= \dfrac{\sin(x+a)}{D^2+1}$ $(a=1)$

$D^2 = -a^2 = -1^2 = -1$

$PI = \dfrac{\sin(x+a)}{-1+1} = 0$

$PI = \dfrac{x\sin(x+a)}{2D}$

$PI = \dfrac{x}{2}\int \sin(x+a)\,dx$

$PI = \dfrac{x}{2}\left[-\cos(x+a)\right]$

$$\boxed{y(x) = C_1 \cos x + C_2 \sin x - \frac{x}{2}\cos(x+a)}$$

Condn

$y(0) = 0$

$y(0) = C_1 \cos 0 + \sin 0 - \frac{0}{2}$

$0 = C_1$

$y'(0) = 0$

diff. we get

$y'(x) = -C_1 \sin x + C_2 \cos x - \left\{\frac{x}{2}\cdot[-\sin(x+a)] + \frac{\cos(x+a)}{2}\right\}$

$y'(x) = -C_1 \sin x + C_2 \cos x + \frac{x \sin(x+a)}{2} - \frac{\cos(x+a)}{2}$

$y'(0) = 0 + C_2 + 0 - \frac{\cos a}{2} = 0$