$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

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$$\int x\,dx = \frac{x^2}{2}\ ;\ \int x^2\,dx = \frac{x^3}{3}$$

$$\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-2+1}}{-2+1}$$

$$= \frac{x^{-1}}{-1} = -\frac{1}{x}$$

$$\int \frac{1}{\sqrt{x}}\,dx = \int \frac{1}{x^{1/2}}\,dx = \int x^{-1/2}\,dx$$

$$= \frac{x^{-1/2+1}}{-1/2+1} = \frac{x^{1/2}}{1/2} = 2\sqrt{x}$$

① $\int \left(\frac{x^2+4}{x^2}\right)dx$

$$= \int \left(\frac{x^2}{x^2} + \frac{4}{x^2}\right)dx = \int \left(1 + \frac{4}{x^2}\right)dx$$

$$= x + \int 4x^{-2}\,dx$$

$$= x + 4\left[\frac{x^{-2+1}}{-2+1}\right] = x + 4\left(\frac{x^{-1}}{-1}\right)$$

$$\boxed{I = x - \frac{4}{x}}$$

② $\int 5\cos\theta \, d\theta$

$= +5\sin\theta$

③ $\int 9e^{x/4}\,dx$

$= 9 \cdot \frac{e^{x/4}}{(1/4)}$

$= 36e^{x/4}$ //

④ $\int (1+3t)t^2\,dt$

$I = \int (t^2+3t^3)\,dt$

$I = \frac{t^3}{3} + 3 \cdot \frac{t^4}{4}$,

Integration by parts:-

$$\int \underset{\mathrm{I}}{u}\,\underset{\mathrm{II}}{v}\,dx = u\int v\,dx - \int\left[\frac{d(u)}{dx} \cdot \int v\,dx\right]dx$$

I → Inverse

L → Logarithm

A → Algebraic

T → trigonometrical

E → Exponential

① $\int \underset{u}{x} \underset{v}{e^{2x}} dx$

$$\int uv = u\int v\,dx - \int\left(\frac{du}{dx}\cdot\int v\,dx\right)dx$$

$$\int \underset{u}{x}\,\underset{v}{e^{2x}}\,dx = x\int e^{2x}dx - \int\left[\frac{d}{dx}(x)\cdot\int e^{2x}dx\right]dx$$

$$= \frac{xe^{2x}}{2} - \int\left\{1\cdot\left(\frac{e^{2x}}{2}\right)\right\}dx$$

$$= \frac{xe^{2x}}{2} - \int \frac{e^{2x}}{2}dx$$

$$= \frac{xe^{2x}}{2} - \frac{1}{2}\left[\frac{e^{2x}}{2}\right]$$

$$= \frac{e^{2x}}{2}\left[x - \frac{1}{2}\right]$$

② $\int \underset{u}{x}\,\underset{v}{\sin x}\,dx$

$$\int uv\,dx = u\int v\,dx - \int\left[\frac{d}{dx}(u)\cdot\int v\,dx\right]dx$$

$$= x\int \sin x\,dx - \int\left[\frac{d}{dx}(x)\cdot\int \sin x\,dx\right]dx$$

$$= x.(-\cos x) - \int (1)(-\cos x)\,dx$$

$$= -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \sin x$$

$$\boxed{I = \sin x - x\cos x}$$

3 $\int \underset{u}{x^2} \, \underset{v}{\cos x}\,dx$

$$\int u\,v\,dx = u\int v\,dx - \int\left[\frac{d(u)}{dx}\int v\,dx\right]dx$$

$$\int x^2\cos x\,dx = x^2\int \cos x\,dx - \int\left[\frac{d}{dx}(x^2)\cdot\int \cos x\,dx\right]dx$$

$$= x^2(\sin x) - \int 2x.\sin x\,dx$$

$$= x^2\sin x - 2\int \underset{u}{x}\,\underset{v}{\sin x}\,dx$$

$$= x^2\sin x - 2\left[x\int \sin x\,dx - \int\left((1)\int \sin x\,dx\right)dx\right]$$

$$= x^2\sin x - 2\left[x(-\cos x) - \int(-\cos x)\,dx\right]$$

$$= x^2\sin x - 2\left[-x\cos x + \sin x\right]$$

$$= x^2\sin x + 2x\cos x - 2\sin x$$ //

Bernoulli's Rule of Integration by parts

$$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \cdots$$

$u', u'', u''' \ldots \rightarrow$ successive derivatives

$v_1, v_2, v_3 \ldots \rightarrow$ successive integrations.

$$\int \underbrace{x^2}_{u}\,\underbrace{\cos x}_{v}\,dx = uv_1 - u'v_2 + u''v_3 - \cdots$$

$$= (x^2)(\sin x) - (2x)(-\cos x) + (2)(-\sin x) - (0)$$

$$= x^2\sin x + 2x\cos x - 2\sin x$$

$$\int x^3 \sin 2x\,dx$$

$$= (x^3)\left(\frac{-\cos 2x}{2}\right) - (3x^2)\left(\frac{-\sin 2x}{2^2}\right) + (6x)\left(\frac{\cos 2x}{2^3}\right) - (6)\left(\frac{\sin 2x}{2^4}\right)$$

$$= -\frac{x^3\cos 2x}{2} + \frac{3x^2}{4}\sin 2x + \frac{3}{4}x\cos x - \frac{3}{8}\sin 2x$$