Problems on gradient ($\nabla\phi$)

① Find the gradient of $\phi = x+y+z$ at the point $(1,-1,0)$

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k} \quad \text{—— ①}$$

$$\phi = x+y+z$$

$$\frac{\partial\phi}{\partial x} = 1+0+0 = 1$$

$$\frac{\partial \phi}{\partial y} = 0 + 1 + 0 = 1$$

$$\frac{\partial \phi}{\partial z} = 0 + 0 + 1 = 1$$

$$(1) \Rightarrow$$

$$\nabla \phi = 1\hat{i} + 1\hat{j} + 1\hat{k}$$

$$\boxed{\nabla \phi = \hat{i} + \hat{j} + \hat{k}}$$

$$|\nabla \phi| = \sqrt{1^2 + 1^2 + 1^2}$$

$$|\nabla \phi| = \sqrt{3} \text{ units}$$

② find $\nabla\phi$ & $|\nabla\phi|$ at $(0,2,-1)$

where $\phi = x^2 + 2y + 3z^2$

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$\frac{\partial\phi}{\partial x} = 2x + 0 + 0 = 2x$

$\frac{\partial\phi}{\partial y} = 0 + 2 + 0 = 2$

$\frac{\partial\phi}{\partial z} = 0 + 0 + 6z = 6z$

$\frac{\partial\phi}{\partial x}_{(0,2,-1)} = 0$

$\frac{\partial\phi}{\partial y}_{(0,2,-1)} = 2$

$\frac{\partial\phi}{\partial z}_{(0,2,-1)} = -6$

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

&

$$\nabla\phi = 2x\hat{i} + 2\hat{j} + 6z\hat{k}$$

$$\nabla\phi = 0\hat{i} + 2\hat{j} - 6\hat{k}$$

$(0, 2, -1)$

$$|\nabla\phi| = \sqrt{0^2 + 2^2 + 6^2}$$

$(0,2,-1)$

$$= \sqrt{4 + 36}$$

$$|\nabla\phi| = \sqrt{40} \text{ units}$$

(3) find $\nabla\phi$ & $|\nabla\phi|$ at $(1,-1,3)$

where $\phi = 2x^2yz^2$

$$\phi = \underline{2x^2yz^2}$$

$$\frac{\partial\phi}{\partial x} = (2yz^2)\frac{\partial(x^2)}{\partial x} = 4xyz^2$$

$$\frac{\partial\phi}{\partial y} = (2x^2z^2)\frac{\partial(y)}{\partial y} = 2x^2z^2$$

$$\frac{\partial\phi}{\partial z} = (2x^2y)\frac{\partial(z^2)}{\partial z} = 4x^2yz$$

$$\frac{\partial\phi}{\partial x}_{(1,-1,3)} = 4(1)(-1)(3)^2 = -36$$

$$\frac{\partial\phi}{\partial y}_{(1,-1,3)} = 2(1)^2(3)^2 = 18$$

$$\frac{\partial\phi}{\partial z}_{(1,-1,3)} = 4(1)^2(-1)(3) = -12$$

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$\nabla\phi_{(1,-1,3)} = -36\hat{i} + 18\hat{j} - 12\hat{k}$$

$$|\nabla\phi| = \sqrt{36^2 + 18^2 + 12^2}$$

$$= \sqrt{1296 + 324 + 144} = \sqrt{1764} = 42 \text{ units}$$

Properties of gradient

* Unit vector normal to the surface $\hat{n} = \frac{\nabla \phi}{|\nabla \phi|}$

* Normal derivative $= |\nabla \phi|$

* Angle by surfaces ϕ_1 & ϕ_2 $\Rightarrow$ $\cos\theta = \frac{\nabla \phi_1 \cdot \nabla \phi_2}{|\nabla \phi_1| \, |\nabla \phi_2|}$

Problems on grad, div, curl

① Find normal derivative of $x^2+2y^2+3z^2=6$ at a point $(1,1,1)$

Here $\phi = x^2+2y^2+3z^2-6$

$\frac{\partial\phi}{\partial x} = 2x$ | $\frac{\partial\phi}{\partial y} = 4y$ | $\frac{\partial\phi}{\partial z} = 6z$

$\frac{\partial\phi}{\partial x}_{(1,1,1)} = 2$ | $\frac{\partial\phi}{\partial y}_{(1,1,1)} = 4$ | $\frac{\partial\phi}{\partial z}_{(1,1,1)} = 6$

$\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$

$\nabla\phi = 2\hat{i}+4\hat{j}+6\hat{k}$

$|\nabla\phi| = \sqrt{2^2+4^2+6^2}$

$= \sqrt{4+16+36}$

$= \sqrt{56}$

② Find a unit vector normal to the surface

$x^3+y^3+3xyz=3$ at $(1,2,-1)$

$$\frac{-3\hat{i}+9\hat{j}+6\hat{k}}{\sqrt{126}}$$

③ Find the angle btn

$x^2+y^2+z^2=9$; $z=x^2+y^2-3$ at pt

$(2,-1,2)$ ...