① $m^3 - 6m^2 + 11m - 6 = 0$

$m_1 = 1 \quad m_2 = 2 \quad m_3 = 3$

② $m^2 + 5m + 6 = 0$

$m_1 = -2 \quad m_2 = -3$

③ $m^2 + 6m + 9 = 0$

$m_1 = -3 \quad m_2 = -3$

$$1\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$$

* Higher order linear diff equation with const. coff

$$\frac{dy}{dx} = m \quad \frac{d^2y}{dx^2} = m^2 \quad \frac{d^3y}{dx^3} = m^3$$

$$\boxed{m^3 - 6m^2 + 11m - 6 = 0}$$

$m_1 = 1 \;;\; m_2 = 2 \;;\; m_3 = 3$ — Real & distinct

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x} = c_1 e^{x} + c_2 e^{2x} + c_3 e^{3x}$$

$$m^2+6m+9=0 \qquad \frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=0$$

$$m_1=-3 \; : \; m_2=-3$$

real & repeated roots

$$y=(c_1+c_2x)e^{m_1x}=(c_1+c_2x)e^{-3x}$$

$$m_1=2,\; m_2=2,\; m_3=2$$

$$y=(c_1+c_2x+c_3x^2)e^{m_1x}=(c_1+c_2x+c_3x^2)e^{2x}$$

general form

$$\frac{d^n y}{dx^n} + k_1 \frac{d^{n-1} y}{dx^{n-1}} + k_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots + k_n y = 0$$

$K_1, k_2, k_3 \cdots k_n$ are constant coefficients

$RHS = 0 \implies y = C.F$

Nature of roots	Roots	$y = C.F$
1) Real & distinct (different)	$m_1 \neq m_2$	$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$
	$m_1 \neq m_2 \neq m_3$	$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$
2) Real & Equal (repeated)	$m_1 = m_2$	$y = (c_1 + c_2 x) e^{m x}$
	$m_1 = m_2 = m_3$	$y = (c_1 + c_2 x + c_3 x^2) e^{m_1 x}$

Imaginary & distinct Q	$m = \alpha \pm i\beta$	$y = e^{\alpha x}\left[c_1 \cos\beta x + c_2 \sin\beta x\right]$
Bi Q	$m = \alpha_1 \pm i\beta_1$ $m = \alpha_2 \pm i\beta_2$	$y = e^{\alpha_1 x}\left[c_1 \cos\beta_1 x + c_2 \sin\beta_1 x\right]$ $+ e^{\alpha_2 x}\left[c_3 \cos\beta_2 x + c_4 \sin\beta_2 x\right]$
Cubic	$m_1 = a$ (Real) $m_2 = \alpha \pm i\beta$ (Im) ⋮	$y = c_1 e^{ax}$ $+ e^{\alpha x}\left[c_2 \cos\beta x + c_3 \sin\beta x\right]$

Imaginary & repeated	$m_1 = \alpha \pm i\beta = m_2$	$y = e^{\alpha x}\left[(C_1 + C_2 x)\cos\beta x + (C_3 + C_4 x)\sin\beta x\right]$
	$m_1 = m_2$ $m_3 = \alpha \pm i\beta = m_4$	$y = (C_1 + C_2 x)e^{m_1 x} + e^{\alpha x}\left[(C_3 + C_4 x)\cos\beta x + (C_5 + C_6 x)\sin\beta x\right]$

$m_1 \neq m_2$

$m_3 = \alpha \pm i\beta$

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + e^{\alpha x}\left[c_3 \cos\beta x + c_4 \sin\beta x\right]$$

$m_1 = m_2$

$m_3 = \alpha \pm i\beta$

$$y = (c_1 + c_2 x) e^{m_1 x} + \text{——''——}$$

Solve :-

① $$\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=0$$

Step I :- $D^2y+6Dy+9y=0$

$[D^2+6D+9]y=0$

Step II :- $Dy=m$: $D^2y=m^2$

$$\boxed{m^2+6m+9=0}$$ A.E

Step III :- $m_1=-3$; $m_2=-3$

Real & Equal

* $\frac{d}{dx}=D$
* $D=m \rightarrow$ Auxillary Eqn
* Solve A.E
* Identify the nature root
* $=y=C.F$

Step IV :- $y = (c_1 + c_2 x) e^{m_1 x}$

$$\boxed{y = (c_1 + c_2 x) e^{-3x}}$$

② $[D^3 - 6D^2 + 11D - 6]y = 0$

To find A.E $D = m$.

$$m^3 - 6m^2 + 11m - 6 = 0$$

$m_1 = 1$; $m_2 = 2$; $m_3 = 3$

$y = CF = C_1 e^{x} + C_2 e^{2x} + C_3 e^{3x}$

③ $(D^3 + 1) y = 0$

$m^3 + 1 = 0$

$m^3 + 0m^2 + 0m + 1 = 0$

$S \Longleftrightarrow D$

$\alpha + \beta i$

$m_1 = -1 \; ; \; m_2 = \frac{1}{2} + \frac{\sqrt{3}}{2} i \quad \left(\frac{1 \pm i\sqrt{3}}{2} \right)$

$m_3 = \frac{1}{2} - \frac{\sqrt{3}}{2} i$

$\alpha - \beta i$

$$y = CF = c_1 e^{-x} + e^{\frac{1}{2}x}\left[c_2 \cos\left(\frac{\sqrt{3}}{2}x\right) + c_3 \sin\left(\frac{\sqrt{3}}{2}\right)x\right]$$

(4) $(D^3 - 3D^2 + 3D - 1)y = 0$

$m^3 - 3m^2 + 3m - 1 = 0$

$m_1 = 1,\ m_2 = 1,\ m_3 = 1$

$$y = (c_1 + c_2 x + c_3 x^2)e^{x}$$

(5) $(D^3 - 3D + 2)y = 0$

(1) $(D^2 + 4)y = 0$