Apply LU-Decomposition method to solve the Eqns

$2x - 3y + 10z = 3$; $-x + 4y + 2z = 20$; $5x + 2y + z = -12$

$$A = \begin{array}{c} \begin{matrix} x & \;\; y & \;\; z \end{matrix} \\ \begin{bmatrix} 2 & -3 & 10 \\ -1 & 4 & 2 \\ 5 & 2 & 1 \end{bmatrix} \end{array} \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad B = \begin{bmatrix} 3 \\ 20 \\ -12 \end{bmatrix}$$

$$AX = B \quad \text{— } (1)$$

Decompose

$$A = LU \quad \text{— } (2)$$

(2) in (1)

$$L\underline{UX} = B \quad \text{— } (3)$$

$$UX = Y \quad \text{— } (4)$$

$$\boxed{LY = B} \quad \text{— } (*)$$

$$\begin{bmatrix} 2 & -3 & 10 \\ -1 & 4 & 2 \\ 5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 10 \\ -1 & 4 & 2 \\ 5 & 2 & 1 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

$u_{11} = 2$

$u_{12} = -3$

$u_{13} = 10$

$l_{21}u_{11} = -1$

$2l_{21} = -1$

$l_{21} = -1/2$

$l_{31}u_{11} = 5$

$2l_{31} = 5$

$l_{31} = 5/2$

$l_{21}u_{12} + u_{22} = 4$

$\left(-\frac{1}{2}\right)(-3) + u_{22} = 4$

$3/2 + u_{22} = 4$

$u_{22} = 4 - \frac{3}{2} = \frac{5}{2}$

$l_{21}u_{13} + u_{23} = 2$

$\left(\frac{-1}{2}\right)(10) + u_{23} = 2$

$u_{23} = 2 + 5$

$\boxed{u_{23} = 7}$

$$l_{31}u_{12}+l_{32}u_{22}=2$$

$$\left(\frac{5}{2}\right)(-3)+\left(\frac{5}{2}\right)l_{32}=2$$

$$-\frac{15}{2}+\frac{5}{2}l_{32}=2$$

$$\frac{5}{2}l_{32}=2+\frac{15}{2}$$

$$\left(\frac{5}{2}\right)l_{32}=\frac{19}{2}$$

$$l_{32}=\frac{19/\cancel{2}}{5/\cancel{2}}$$

$$\boxed{l_{32}=19/5}$$

$$l_{31}u_{13}+l_{32}u_{23}+u_{33}=1$$

$$\left(\frac{5}{\cancel{2}}\right)(\cancel{10}\,5)+\left(\frac{19}{5}\right)(7)+u_{33}=1$$

$$25+\frac{133}{5}+u_{33}=1$$

$$\frac{258}{5}+u_{33}=1$$

$$u_{33}=1-\frac{258}{5}$$

$$u_{33}=-\frac{253}{5}$$

$$\boxed{LY = B}$$

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$$\begin{array}{ccc} y_1 & y_2 & y_3 \\ \left[\begin{array}{ccc} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 5/2 & 19/5 & 1 \end{array}\right] \end{array} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 20 \\ -12 \end{bmatrix}$$

By forward subs^n, we get

$$\boxed{y_1 = 3} \qquad \boxed{y_2 = 21.5} \qquad \boxed{y_3 = -101.2}$$

$$UX = Y$$

$$\begin{array}{ccc} x & y & z \\ \end{array}$$
$$\begin{bmatrix} 2 & -3 & 10 \\ 0 & 5/2 & 7 \\ 0 & 0 & \frac{-253}{5} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 21.5 \\ -101.2 \end{bmatrix}$$

$$2x - 3y + 10z = 3$$
$$5/2\, y + 7z = 21.5$$
$$-253/5\, z = -101.2$$

$$x = -4$$
$$y = 3$$
$$z = 2$$

II Crout's LU-decomposition

$$A = LU \quad : \quad L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \quad U = \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

III Cholesky's LU-decomposition :-

$$A = LL^T \qquad L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \quad L^T = \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}$$

Eigen Values & Eigen Vectors.-

$|A-\lambda I|=0$ λ_i , x of $[A-\lambda I]X=0$

① Find Eigen values & Eigen vectors for

$$A=\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

Step I :- To find Ch. Eqn of EV

$$|A - \lambda I| = 0$$

$$\times \quad \left| \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\times \quad \left| \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\checkmark \quad \begin{vmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{vmatrix} = 0$$

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

Consider

$$\begin{vmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(3-\lambda) - 8 = 0$$

$$3 - \lambda - 3\lambda + \lambda^2 - 8 = 0$$

$$\lambda^2 - 4\lambda - 5 = 0$$ ch Eq

$$\lambda_1 = 5 \; ; \; \lambda_2 = 1$$ Eigen values

$$\lambda^2 - 4\lambda - 5 = 0$$

$$a\lambda^2 + b\lambda + c = 0$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{4^2 - 4(1)(-5)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{36}}{2}$$

$$\lambda = \frac{4 \pm 6}{2}$$

$$\lambda_1 = \frac{4+6}{2}$$

$$\boxed{\lambda_1 = 5}$$

$$\lambda_2 = \frac{4-6}{2}$$

$$\boxed{\lambda_2 = -1}$$

II Find Eigen vectors

$$\boxed{\lambda_1 = 5}$$

Consider

$$[A - \lambda_1 I]X = 0$$

$$\begin{bmatrix} 1-5 & 4 \\ 2 & 3-5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{bmatrix} -4 & 4 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-4x + 4y = 0$$
$$2x - 2y = 0$$

Consider

$$2x - 2y = 0$$
$$2x = 2y$$

$$\frac{x}{1} = \frac{y}{1}$$

$$X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x = -2y$$

$$\frac{x}{-2} = \frac{y}{1}$$

$$X_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$