Solution of Non-homogeneous diff. eqn ($RHS \neq 0$)

If $f(D)y = f(x)$ ($RHS \pm 0$)

Then $\overset{\text{soln}}{\;}$ $y = CF + PI$

* CF = Complementary fun
* PI = Perticular Integral

I PI for $f(x) = e^{ax}$

Consider

$$PI = \frac{f(x)}{F(D)}$$ provided $F(a) \neq 0$ $(D = a)$ $(\underline{Dx} \neq 0)$

$$PI = x\frac{f(x)}{F'(D)}$$ provided $F'(a) \neq 0$

$$PI = x^2\frac{f(x)}{F''(D)}$$ $F''(a) \neq 0$ --- cont.

① Solve:-

$$(D^2 - 5D + 6)y = e^{2x}$$

Step I :- To find C.F

AE, $m^2 - 5m + 6 = 0$

$m_1 = 3$ (or) $m_2 = 2$

$m_1 \neq m_2$ [real & distinct]

$CF = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

$$\boxed{CF = c_1 e^{3x} + c_2 e^{2x}} \quad \text{--- ①}$$

Step II :- To find P.I

$f(x) = e^{2x}$; $a = 2$*

$$PI = \frac{f(x)}{f(D)} \quad , f(a) \neq 0$$

$$PI = \frac{e^{2x}}{D^2 - 5D + 6} \quad \text{put } D = a = 2$$

$$= \frac{e^{2x}}{2^2 - 5(2) + 6} = \frac{e^{2x}}{0}$$

$\underline{Dr} = 0$

$$PI = \frac{x\, e^{2x}}{2D - 5}$$

x^{ly} nr by x

diff dr

$$= \frac{x\, e^{2x}}{2(2) - 5}$$

$D = 2 = 9$

$$= \frac{x\, e^{2x}}{4 - 5} = \frac{x e^{2x}}{-1}$$

$$\boxed{PI = -x e^{2x}} \quad (2)$$

$y = CF + PI$

$$\boxed{y = c_1 e^{3x} + c_2 e^{2x} - x e^{2x}}$$

② Solve

$(D^2 - 6D + 9)y = e^{3x}$

$m^2 - 6m + 9 = 0$

$m_1 = 3$ & $m_2 = 3$
(Real & Equal)

$CF = (c_1 + c_2 x)e^{m_1 x}$

$$\boxed{CF = (c_1 + c_2 x)e^{3x}} \quad \text{—} ①$$

P.I :-

$$PI = \frac{f(x)}{F(D)}$$

$$PI = \frac{e^{3x}}{D^2-6D+9}$$

$D=3$

$$PI = \frac{e^{3x}}{3^2-6(3)+9} = \frac{e^{3x}}{0}$$

$[Dr = 0]$

xly nr by x & diff. the dr

$$PI = \frac{xe^{3x}}{2D-6}$$

$D=3$

$$PI = \frac{xe^{3x}}{2(3)-6}$$

$$= \frac{xe^{3x}}{0}$$

$[dr = 0]$

$x'y$ nr sy x

diff dn

$$\boxed{PI = \frac{x^2 e^{3x}}{2}} \quad ②$$

$\therefore$ Soln

$$y = CF + PI$$

Case II :- $f(x) = \sin ax / \cos ax / \sin(ax+b) / \cos(ax+b)$

$$PI = \frac{\sin ax}{f(D^2)} \; ; D^2 = -a^2$$

$$f(-a^2) \neq 0$$

$$PI = \frac{x \sin ax}{f'(D^2)} \; ; D^2 = -a^2$$

$$f'(-a^2) \neq 0$$

① Solve: $(D^2-5D+6)y = \sin 2x$

I CF:

$m^2 - 5m + 6 = 0$

$m_1 = 3$ & $m_2 = 2$

(real & distinct)

$CF = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

$$\boxed{CF = c_1 e^{3x} + c_2 e^{2x}} \quad \text{—①}$$

II To find P.I

$$PI = \frac{f(x)}{F(D)} \quad X$$

$$PI = \frac{\sin 2x}{D^2-5D+6} \quad (a=2)$$

$* \boxed{D^2 = -a^2}$ $\quad \boxed{D^2 = -2^2 = -4}$

$$PI = \frac{\sin 2x}{-4-5D+6} = \frac{\sin 2x}{2-5D}$$

$$PI = \frac{\sin 2x}{2-5D} \times \frac{2+5D}{2+5D}$$

$$(a+b)(a-b) = a^2 - b^2$$

$$= \frac{(2+5D)\sin 2x}{2^2 - 5^2 D^2}$$

$$= \frac{(2+5D)\sin 2x}{4-25D^2}$$

$$* \; D^2 = -4$$

$$= \frac{(2+5D)\sin 2x}{4-25(-4)}$$

$$= \frac{(2+5D)\sin 2x}{104}$$

$$= \frac{2\sin 2x + 5D(\sin 2x)}{104}$$

$$PI = \frac{2\sin 2x + 5(2\cos 2x)}{104}$$

$$PI = \frac{2\sin 2x + 10\cos 2x}{104} \quad \text{—— (2)}$$

$$y = CF + PI$$

① Solve.

$$(D^2+4)y = \cos(2x+3)$$

$m^2+4=0$; $m^2+0m+4=0$

$m = 0 \pm 2i$ (Im & distinct)

$$CF = e^{\alpha x}[c_1\cos\beta x + c_2\sin\beta x]$$

$$CF = e^{0x}[c_1\cos 2x + c_2\sin 2x]$$

$$\boxed{CF = c_1\cos 2x + c_2\sin 2x} \text{ — } (1)$$

To find PI

$$PI = \frac{\cos(2x+3)}{D^2+4}$$

put $D^2 = -a^2 = -2^2 = -4$

$$PI = \frac{\cos(2x+3)}{-4+4} = \frac{\cos(2x+3)}{0}$$

($Dr = 0$ $\therefore$ diff Dr & $\cancel{x}$ ly nr by x)

$$PI = \frac{x\cos(2x+8)}{2D}$$

[x^{ly} & $\div$ by D
· Other product]

$$PI = \frac{1}{2}\left[\frac{x\cos(2x+3)}{D}\right]$$

$$PI = \frac{1}{2}\int x\cos(2x+3)\,dx$$

(Int)