Eigen values & Eigen vectors (3×3)

① Find Ev. & Ei vectors of $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$

Step 1 :- $|A - \lambda I| = 0$

$$\begin{vmatrix} -1-\lambda & 3 \\ -2 & 4-\lambda \end{vmatrix} = 0$$

$$(-1-\lambda)(4-\lambda) - (-6) = 0$$

$$-4 + \lambda - 4\lambda + \lambda^2 + 6 = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$ Ch. eqn

~~Step 2 :-~~ To find E.V

$$\lambda^2 - 2\lambda - \lambda + 2 = 0$$

$$\lambda(\lambda - 2) - 1(\lambda - 2) = 0$$

$$(\lambda - 2)(\lambda - 1) = 0$$

$\lambda_1 = 2$ & $\lambda_2 = 1$ → E.Vs

2 → $-2, -1$ → $= -3$

Step III: Eigen vector

$\lambda_1 = 2$

$[A - \lambda_1 I]X = 0$

$$\begin{bmatrix} -1-2 & 3 \\ -2 & 4-2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 3 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-3x + 3y = 0$

$-2x + 2y = 0$

$-3x + 3y = 0$

$+3x = +3y$

$\frac{x}{1} = \frac{y}{1}$

$X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda_2 = 1$

$$\begin{bmatrix} -1-1 & 3 \\ -2 & 4-1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 3 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-2x + 3y = 0$

$+2x = +3y$

$2x = 3y$

$\frac{x}{3} = \frac{y}{2}$

$$X_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 & 2 \end{bmatrix}^T$$

② Find E.V^s & E.vectors for $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

Consider Ch. Eqn $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{vmatrix} = 0$$

$\lambda^3 - 7\lambda^2 + 0\lambda + 36 = 0$

$(1-\lambda)[(5-\lambda)(1-\lambda) - 1] - 1[1-\lambda-3]$

$+3[1 - 3(5-\lambda)] = 0$

...

$(1-\lambda)[5 - \lambda - 5\lambda + \lambda^2 - 1]$

$-1[-\lambda - 2] + 3[1 - 15 + 3\lambda] = 0$

$(1-\lambda)[\lambda^2 - 6\lambda + 4] + \lambda + 2$

$+9\lambda - 42 = 0$

$\lambda^2 - 6\lambda + 4 - \lambda^3 + 6\lambda^2 - 4\lambda + 10\lambda - 40 = 0$

$-\lambda^3 + 7\lambda^2 + 0\lambda - 36 = 0$ ×(-1)

$\lambda^3 - 7\lambda^2 + 0\lambda + 36 = 0$ — (3)

$\lambda_1 = -2 \;;\; \lambda_2 = 6 \;;\; \lambda_3 = 3$

$\lambda^3 - 7\lambda^2 + 36 = 0$

$1 - 7 + 36 \neq 0 \quad x = 1$

$8 - 28 + 36 \neq 0 \quad x = 2$

$x = -2$

$-8 - 28 + 36 = 0$

	x^3	x^2	x	c
$x = -2$	1	−7	0	36
(+)	0	−2	18	−36
	x^2 1	−9	18	0

$(x+2)(x^2 - 9x + 18) = 0$

Find Eigen vectors

$\lambda_1 = -2$ $\quad \frac{x}{-1} = \frac{y}{0} = \frac{z}{1}$

$$\begin{bmatrix} 1-(-2) & 1 & 3 \\ 1 & 5-(-2) & 1 \\ 3 & 1 & 1-(-2) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 3 & 1 & 3 \\ 1 & 7 & 1 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3x + y + 3z = 0$

$x + 7y + z = 0$

$$X = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\frac{x}{\begin{vmatrix} 1 & 3 \\ 7 & 1 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} 3 & 3 \\ 1 & 1 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 3 & 1 \\ 1 & 7 \end{vmatrix}}$$

$$\frac{x}{1-21} = \frac{-y}{3-3} = \frac{z}{21-1}$$

$$\frac{x}{-20} = \frac{-y}{0} \times \frac{z}{20} \times 20$$

$$A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

$|A-\lambda I|=0$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix}=0$$

$(8-\lambda)\left[(7-\lambda)(3-\lambda)-16\right]+6\left[-6(3-\lambda)+8\right]$

$+2\left[24-2(7-\lambda)\right]=0$

$(8-\lambda)\left[21-10\lambda+\lambda^2-16\right]$

$+6\left[6\lambda-10\right]+2\left[2\lambda+10\right]=0$

$40-80\lambda+8\lambda^2-5\lambda+10\lambda^2-\lambda^3$

$+36\lambda-60+4\lambda+20=0$

$-\lambda^3+18\lambda^2-45\lambda+0=0$

$\lambda^3-18\lambda^2+45\lambda-0=0$

$\therefore \lambda(\lambda^2-18\lambda+45)=0$

$0, 3, 15$