Problems to find Inverse of square Matrix

① Find the inverse of $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$

change the sign

change the sign

$A^{-1} = \dfrac{\text{adj} A}{|A|}$ adj A = transpose of co-factor matrix

Shortcut:- $\text{adj} A = \begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix}$ $|A| = \begin{vmatrix} 1 & -1 \\ 2 & 4 \end{vmatrix} = 4-(-2)$

$= 6 \neq 0$

$$A^{-1} = \frac{\begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix}}{6}$$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix}$$

Verify

$$A A^{-1} = I$$

$$A A^{-1} = \frac{1}{6}\begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 4+2 & 1-1 \\ 8-8 & 2+4 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

2) Find A^{-1} of $A = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 3 & 1 \\ -4 & 2 \end{vmatrix} = 6 + 4 = 10 \neq 0 \; ; \; A^{-1} \text{ exists}$$

$$\operatorname{adj} A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} \; ; \; A^{-1} = \frac{\operatorname{adj} A}{|A|} = \frac{1}{10}\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$$

$$A \cdot A^{-1} = \frac{1}{10}\begin{bmatrix} 8 & 1 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

(8) Find the inverse of $A = \begin{bmatrix} 2 & -1 & 2 \\ 0 & 1 & 4 \\ 2 & 1 & 7 \end{bmatrix}$

I To find $|A|$:-

$$|A| = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} 2 & -1 & 2 \\ 0 & 1 & 4 \\ 2 & 1 & 7 \end{vmatrix} \end{array} = 2(7-4)+1(0-8)+2(0-2) = -6 \neq 0$$

$\therefore A^{-1}$ exist.

II To find Co-factor matrix

Cofact of 2 $= +3$

Co-fact of $-1 = -(-8) = 8$

Co-fact $2 = +(-2) = -2$

Co-fact $0 = -(-9) = 9$

Co-fact $1 = +10$

Cofact of $4 = -4$

Cofact $2 = +(-6) = -6$

Cofact $1 = -8$

Cofact $7 = +2$

$$\text{Cofact } A = \begin{bmatrix} 3 & 8 & -2 \\ 9 & 10 & -4 \\ -6 & -8 & 2 \end{bmatrix}$$

Step III :- To find adj A

adj A = transpose of cofact A

$$\text{adj } A = \begin{bmatrix} 3 & 9 & -6 \\ 8 & 10 & -8 \\ -2 & -4 & 2 \end{bmatrix}$$

Step IV

To find A^{-1}

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$A^{-1} = \frac{-1}{6} \begin{bmatrix} 3 & 9 & -6 \\ 8 & 10 & -8 \\ -2 & -4 & 2 \end{bmatrix}$$

(3) $A = \begin{bmatrix} 1 & 4 & 2 \\ 0 & 3 & 2 \\ -1 & 4 & 7 \end{bmatrix}$

$|A| = 1(13) - 4(2) + 2(3)$

$|A| = 11 \neq 0$; A^{-1} Exists

Cofact $1 = +13$

Cofact $4 = -2$

Cofact $2 = +3$

Cofact $0 = -20$

Cofact $3 = +9$

Cofact $2 = -8$

Cofact $-1 = +2$

Cofact $4 = -2$

Cofact $7 = +3$

Decomposition of Matrix

Let Consider $a_{11}x + a_{12}y + a_{13}z = b_1$

$$a_{21}x + a_{22}y + a_{23}z = b_2$$

$$a_{31}x + a_{32}y + a_{33}z = b_3$$

$$AX = B$$

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ → matrix of coefficient

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ → matrix of variables

$B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ → matrix of constants

$$AX = B \longrightarrow (1)$$

Decompose A as the product of L & U

$$\boxed{A = LU} \text{ — } (2)$$

$$L\underline{UX} = B \text{ — } (3)$$

Take

$$UX = Y \text{ — } (*)$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$(3) \Rightarrow \boxed{LY = B} \text{ — } (4)$$

Find Y &

Sub in $(*)$

find X