Problems on differentiation 6/1/2021

① $y = e^{2x}$

$\frac{dy}{dx} = 2e^{2x}$; $y = e^{ax}$

$\frac{dy}{dx} = ae^{ax}$

② $y = x^3$; $\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{dy}{dx} = 3x^2$

③ $y = \sin 2x$

$\frac{dy}{dx} = 2\cos 2x$

④ $y = \log(3x)$

$\frac{dy}{dx} = \frac{1}{3x} \times 3 = \frac{1}{x}$

⑤ $y = e^{x^2}$

$\frac{dy}{dx} = e^{(x^2)}(2x)$

$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

⑥ $y = \underset{u}{x}\,\underset{v}{e^{2x}}$

$\frac{dy}{dx} = x \cdot 2e^{2x} + e^{2x} \cdot (1)$

$$y = \frac{x^2}{1+x} = \frac{u}{v}$$

$$\times \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$

$$\frac{dy}{dx} = \frac{(1+x)(2x) - x^2(1)}{(1+x)^2}$$

$$= \frac{2x+2x^2-x^2}{(1+x)^2} = \frac{2x+x^2}{(1+x)^2}$$

$$= \frac{x(2+x)}{(1+x)^2}$$

Logarithmic differentiation

$$y = f(x)^{g(x)}$$

Consider Log on b.s

$$\log y = \log\left[f(x)^{g(x)}\right]$$

$$\log y = g(x)\log\left[f(x)\right]$$

diff. w.r.to x

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{d}{dx}\left[g(x)\cdot\log(f(x))\right]$$

① find $\frac{dy}{dx}$ where $y = x^{\sin x}$

Solⁿ

$$y = x^{\sin x} \quad \text{——— ①}$$

Consider $\log_e$ on b.s

$$\log_e y = \log\left(x^{\sin x}\right)$$

$$\boxed{\log_e y = \underbrace{\sin x}_{u} \cdot \underbrace{\log_e x}_{v}}$$

diff. w.r.to x

$$\frac{1}{y}\frac{dy}{dx} = \sin x \frac{d}{dx}\left(\log_e x\right) + \log x \frac{d}{dx}(\sin x)$$

$$\frac{1}{y}\frac{dy}{dx} = \sin x\left(\frac{1}{x}\right) + \log x\,(\cos x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\sin x}{x} + \cos x \log x$$

$$\frac{dy}{dx} = y\left[\frac{\sin x}{x} + \cos x \log x\right]$$

$$dy/dx = x^{\sin x}\left[\frac{\sin x}{x} + \cos x \log x\right]$$

② $y=(\sin x)^x$

$$\log y = \log[(\sin x)^x]$$

$$\log y = \underset{u}{x}\ \underset{v}{\log(\sin x)}$$

diff. w.r.to x

$$\frac{1}{y}\frac{dy}{dx} = x\left[\frac{1}{\sin x}\cdot\cos x\right] + \log(\sin x)$$

$$\frac{1}{y}\frac{dy}{dx} = x\cot x + \log(\sin x)$$

$$\frac{dy}{dx} = y[\quad]$$

$$\frac{dy}{dx} = (\sin x)^x\left[x\cot x + \log(\sin x)\right]$$

③ $y=(\tan x)^{(x^2)}$

$$\log y = \underline{x^2}\ \underline{\log(\tan x)}$$

$$\frac{1}{y}\frac{dy}{dx} = x^2\cdot\left(\frac{1}{\tan x}\cdot\sec^2 x\right) + 2x\log(\tan x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{x^2\sec^2 x}{\tan x} + 2x\log(\tan x)$$

$$\frac{dy}{dx} = (\tan x)^{x^2}\left[\frac{x^2\sec^2 x}{\tan x} + 2x\log(\tan x)\right]$$

Practice

① $(x^3)^{[\cos(x^2)]}$

differentiation of parametric function

① Find $\frac{dy}{dx}$ where

$x = \log t \quad ; \quad y = \sin t$

$x = f(t) \quad ; \quad y = g(t)$

diff. w.r.t. t

$$\frac{dx}{dt} = \frac{1}{t} \quad ; \quad \frac{dy}{dt} = \cos t$$

$$\boxed{\frac{dy}{dx} = \frac{dy/dt}{dx/dt}} = \frac{\cos t}{(1/t)}$$

$$\boxed{\frac{dy}{dx} = t\cos t}$$

② $x = t^2$ & $y = \cos t$

diff. x & y w.r.t. to t we get

$$\frac{dx}{dt} = 2t \quad ; \quad \frac{dy}{dt} = -\sin t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-\sin t}{2t}$$

$$\boxed{\frac{dy}{dx} = -\frac{\sin t}{2t}}$$

③ $x = a\cos^3\theta \quad ; \quad y = a\sin^3\theta$

diff. w.r.to θ

$$\frac{dx}{d\theta} = a \cdot 3\cos^2\theta \cdot (-\sin\theta)$$

$$\frac{dy}{d\theta} = a\,3\sin^2\theta \cdot (\cos\theta)$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cancel{3a}\,\sin^{\cancel{2}}\theta\,\cancel{\cos\theta}}{-\cancel{3a}\,\cos^{\cancel{2}}\theta\,\cancel{\sin\theta}}$$

$$\frac{dy}{dx} = -\frac{\sin\theta}{\cos\theta}$$

$$\boxed{\frac{dy}{dx} = -\tan\theta}$$

practice

① $x = ct$; $y = -\frac{c}{t}$

② $x = a(\theta - \sin\theta)$
$y = a(1 - \cos\theta)$

*Successive differentiation :-

$y = f(x)$

$\frac{dy}{dx} \rightarrow$ 1st derivative

$\frac{d^2y}{dx^2} \rightarrow$ 2nd derivative

$\frac{d^3y}{dx^3} \rightarrow$ 3rd derivative

$3x^2$; $6x$; 6 ; 0