LU-Decomposition

$$AX = B \quad \text{— } (1)$$

$$A = LU \quad \text{— } (2)$$

(2) in (1)

$$L\underline{UX} = B \quad \text{— } (3)$$

Consider $UX = Y$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Sub. in (3)

$$LY = B \quad \text{— } (4)$$

Find Y then find X

1) Apply LU-Decomposition Method to find the soln of $x+y+z=3$; $2x-y+3z=16$

$3x+y-z=-3$

Step I :-

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 3 & 1 & -1 \end{bmatrix} ; \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} ; \quad B = \begin{bmatrix} 3 \\ 16 \\ -3 \end{bmatrix}$$

$$AX = B \text{ --- } (1)$$

$$\therefore A = LU \sim (2)$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$L \qquad U$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 3 & 1 & -1 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix}$$

$\boxed{u_{11}=1}$ $\boxed{u_{12}=1}$ $\boxed{u_{13}=1}$ $l_{21}u_{11}=2$; $\boxed{l_{21}=2}$

$l_{31}u_{11}=3$; $\boxed{l_{31}=3}$ $l_{21}u_{12}+u_{22}=-1$; $2+u_{22}=-1$ $\boxed{u_{22}=-3}$

$l_{21}u_{13}+u_{23}=3$

$2+u_{23}=3$

$\boxed{u_{23}=1}$

$l_{31}u_{12}+l_{32}u_{22}=1$

$3+(-3)l_{32}=1$

$-3l_{32}=-2$

$\boxed{l_{32}=2/3}$

$$l_{31}u_{13} + l_{32}u_{23} + u_{33} = -1$$

$$3 + \frac{2}{3} + u_{33} = -1$$

$$\frac{11}{3} + u_{33} = -1$$

$$u_{33} = -1 - \frac{11}{3}$$

$$\boxed{u_{33} = -14/3}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2/3 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & 0 & -14/3 \end{bmatrix}$$

$$AX = B$$

$$L\underline{UX} = B$$

$$UX = Y$$

$$\therefore \boxed{LY = B}$$

$$\begin{array}{ccc} y_1 & y_2 & y_3 \\ \end{array}$$
$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2/3 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 16 \\ -3 \end{bmatrix}$$

$$\boxed{y_1 = 3}$$

$$2y_1 + y_2 = 16$$

$$\boxed{y_2 = 10}$$

$$3y_1 + \frac{2}{3}y_2 + y_3 = -3$$

$$9 + \frac{2}{3}(10) + y_3 = 3$$

$$\frac{47}{3} + y_3 = -3$$

$$y_3 = 3 - \frac{47}{3}$$

$$\boxed{y_3 = -\frac{56}{3}}$$

$$Y = \begin{bmatrix} 3 \\ 10 \\ -56/3 \end{bmatrix}$$

$$UX = Y$$

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & 1 \\ 0 & 0 & -14/3 \end{bmatrix} \end{array} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ -56/3 \end{bmatrix}$$

$$-14/3\, z = -56/3$$

$$z = \frac{-56/\cancel{3}}{-14/\cancel{3}}$$

$$\boxed{z = 4}$$

$$-3y + z = 10$$

$$-3y + 4 = 10$$

$$-3y = 6 \quad y = \frac{6}{-3}$$

$$\boxed{y = -2}$$

$$x + y + z = 3$$

$$x + 4 - 2 = 3$$

$$x + 2 = 3$$

$$\boxed{x = 1}$$

Apply LU-Decomposition Method to solve

$10x + y + z = 12$; $2x + 10y + z = 13$; $2x + 2y + 10z = 14$

$AX = B$; $A = LU$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 2 & 2 & 10 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix}$$

$\boxed{u_{11}=10}$

$\boxed{u_{12}=1}$

$\boxed{u_{13}=1}$

$l_{31}u_{11}=2 \; ; \; \boxed{l_{31}=1/5}$

$l_{21}u_{11}=2$

$l_{21}(10)=2$

$l_{21}=2/10$

$\boxed{l_{21}=1/5}$

$l_{21}u_{12}+u_{22}=10$

$(1/5)(1)+u_{22}=10$

$u_{22}=10-\frac{1}{5}$

$\boxed{u_{22}=49/5}$

$l_{21}u_{13}+u_{23}=1$

$(\frac{1}{5}\times 1)+u_{23}=1$

$u_{23}=1-\frac{1}{5}$

$\boxed{u_{23}=4/5}$

$$l_{31}u_{12} + l_{32}u_{22} = 2$$

$$\left(\frac{1}{5}\right)(1) + \left(\frac{49}{5}\right) l_{32} = 2$$

$$\frac{49}{5} l_{32} = 2 - \frac{1}{5}$$

$$\left(\frac{49}{5}\right) l_{32} = \left(\frac{9}{5}\right)$$

$$l_{32} = \frac{9}{\cancel{5}} \times \frac{\cancel{5}}{49} \qquad \boxed{l_{32} = \frac{9}{49}}$$

$$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 10$$

$$\left(\frac{1}{5}\right)(1) + \left(\frac{9}{49} \times \frac{4}{5}\right) + u_{33} = 10$$

$$\frac{36}{245} + u_{33} = 10 - \frac{1}{5}$$

$$u_{33} = \frac{49}{5} - \frac{36}{245}$$

$$\boxed{u_{33} = 9.65}$$

$$LY = B$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/5 & 1 & 0 \\ 1/5 & 9/49 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 13 \\ 14 \end{bmatrix}$$

$\boxed{y_1 = 12}$; $\frac{1}{5} y_1 + y_2 = 13$

$$\frac{12}{5} + y_2 = 13$$

$$y_2 = 13 - \frac{12}{5}$$

$$\boxed{y_2 = \frac{53}{5}}$$

$$\frac{1}{5} y_1 + \frac{9}{49} y_2 + y_3 = 14$$

$$\frac{12}{5} + \left(\frac{9 \times 53}{49 \times 5}\right) + y_3 = 14$$

$$\boxed{y_3 = 9.65}$$

$$U x = Y$$

$$\begin{bmatrix} 10 & 1 & 1 \\ 0 & 49/5 & 4/5 \\ 0 & 0 & 9.6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 53/5 \\ 9.6 \end{bmatrix}$$

$$9.6z = 9.6$$

$$\boxed{z = 1}$$

$$\frac{49}{5}y + \frac{4}{5}z = \frac{53}{5}$$

$$\frac{49}{5}(y) + \frac{4}{5} = \frac{53}{5}$$

$$\frac{49}{5}(y) = \left(\frac{49}{5}\right)$$

$$\boxed{y = 1}$$

$$10x + y + z = 12$$

$$10x + 1 + 1 = 12$$

$$10x = 10$$

$$\boxed{x = 1}$$