Module I - Mathematical foundation

* The essence of an algorithm
* Big-O Notations
* Differentiation & Integration
* Vector & vector calculus
* Matrices & Matrix decompositions

Differentiation & Integration

24/12/20

$f(x)$	$\frac{d}{dx}$
K (constant)	0
x^n	nx^{n-1}
e^{ax}	ae^{ax}
$\ln x$ $[\log_e x]$	$\frac{1}{x}$
a^x	$a^x \log_e a$
$\sin x$	$\cos x$
$\cos x$ *	$-\sin x$
$\tan x$	$\sec^2 x$
$\operatorname{cosec} x$ *	$-\operatorname{cosec} x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$ *	$-\operatorname{cosec}^2 x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

$$\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$$

$$\frac{d}{dx}(\underset{\mathrm{I}}{u}.\underset{\mathrm{II}}{v}) = u\frac{dv}{dx} + v\frac{du}{dx}$$ product rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$ quotient rule

$$\frac{d}{dx}[K f(x)] = K\frac{d}{dx}f(x)$$

Simple problems on differentiation

① If $y = x^2 + 2x$ find $\frac{dy}{dx}$

Soln $y = x^2 + 2x$

differentiate w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}(2x^1)$$

$$= 2x^{2-1} + 2(1 \cdot x^{1-1})$$

$$\boxed{\frac{dy}{dx} = 2x + 2}$$

$* \frac{d(x^n)}{dx} = nx^{n-1}$

$x^0 = 1$

$* \frac{d}{dx}(1) = 0$

$* \frac{d}{dx}(x) = 1$

$* \frac{d}{dx}(x^2) = 2x$

$* \frac{d}{dx}(x^3) = 3x^2$

$* \frac{d}{dx}(x^4) = 4x^3$

$\frac{d(x^n)}{dx} = nx^{n-1}$

② differentiate $y = e^{3x} + x^5 - 2^x$

Soln $y = e^{3x} + x^5 - 2^x$

differentiate w.r.to x

$$\frac{dy}{dx} = \frac{d}{dx}(e^{3x}) + \frac{d}{dx}(x^5) - \frac{d}{dx}(2^x)$$

$$\boxed{\frac{dy}{dx} = 3e^{3x} + 5x^4 - 2^x \log 2}$$

③ $y = 5e^x - \log x - 3\sqrt{x}$

differentiate w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx}(5e^x) - \frac{d}{dx}(\log x) - \frac{d}{dx}(3\sqrt{x})$$

$$= 5 \cdot e^x - \frac{1}{x} - 3\frac{d}{dx}(x^{1/2})$$

$$= 5e^x - \frac{1}{x} - 3\left[\frac{1}{2}x^{1/2 - 1}\right]$$

$$= 5e^x - \frac{1}{x} - \frac{3}{2} \cdot x^{-\frac{1}{2}}$$

$$\boxed{\frac{dy}{dx} = 5e^x - \frac{1}{x} - \frac{3}{2\sqrt{x}}}$$

$$\frac{d}{dx}[\sqrt{x}] = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}(1/x) = \frac{d}{dx}(x^{-1}) = -1x^{-1-1}$$

$$= -x^{-2}$$

$$= -\frac{1}{x^2}$$

(9) $y = \frac{1}{x^{4/3}} - \frac{3}{x^{3/2}}$

diff. w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}\left[x^{-4/3}\right] - 3\frac{d}{dx}\left[x^{-3/2}\right]$$

$$* = \frac{-4}{3} \cdot x^{-4/3-1} - 3\left(\frac{-3}{2}\right) \cdot x^{-3/2-1}$$

$*\frac{d}{dx}(x^n) = nx^{n-1}$

$$= \frac{-4}{3} x^{-\frac{7}{3}} + \frac{9}{2} x^{-5/2}$$

$$= -\frac{4}{3} \cdot \frac{1}{x^{7/3}} + \frac{9}{2} \cdot \frac{1}{x^{5/2}}$$

Note:- $-\frac{4}{3} - \frac{1 \times 3}{3}$

$$= \frac{-4}{3} - \frac{3}{3}$$

$$= \frac{-4-3}{3} = \frac{-7}{3}$$