Bernoulli's Rule

(1) $\int x^2 \cos(4x)\, dx$

$\int uv\, dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \cdots$

$$\int \underset{u}{x^2}\, \underset{v}{\cos(4x)}\, dx = (x^2)\left(\frac{\sin 4x}{4}\right) - (2x)\left(-\frac{\cos 4x}{4^2}\right) + (2)\left(-\frac{\sin 4x}{4^3}\right)$$

$$= \frac{x^2 \sin 4x}{4} + \frac{\cancel{2}x\cos 4x}{\cancel{16}\,8} - \frac{\cancel{2}\sin 4x}{\cancel{64}\,32}$$

$$= \frac{x^2 \sin 4x}{4} + \frac{x\cos 4x}{8} - \frac{\sin 4x}{32}$$

Definite Integral

$$\int_a^b f(x)\,dx = \Big[g(x)\Big]_{x=a}^{b} = g(b) - g(a)$$

Eg:-

$$\int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_{x=0}^{1}$$

$$= \left[\frac{1^3}{3} - \frac{0^3}{3}\right]$$

$$= \frac{1}{3}$$

$$\because \int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$I = \int_0^2 e^{2x}\,dx = \left[\frac{e^{2x}}{2}\right]_{x=0}^{2}$$

$$= \frac{1}{2}\left[e^4 - e^0\right]$$

$$= \frac{1}{2}\left[e^4 - 1\right]$$

Problems on Definite integrals

① $$\int_1^4 (5x^2 - 8x + 5)\,dx$$

$$I = \left\{ \frac{5x^3}{3} - \frac{\overset{4}{\cancel{8}}\,x^2}{\cancel{2}} + 5x \right\}_{x=1}^{4}$$

$$= \left\{ \left[\frac{5x^3}{3}\right]_1^4 - \left[4x^2\right]_1^4 + \left[5x\right]_1^4 \right\}$$

$$= \left\{ \left[\frac{5\times 64}{3} - \frac{5}{3}\right] - \left[4\times 16 - 4\right] + \left[5\times 4 - 5\right] \right\}$$

② $$\cancel{\int_1^5 \left(\frac{5}{x^3}\right) dx}$$

$$= \left\{ \left[\frac{320}{3} - \frac{5}{3}\right] - \left[64 - 4\right] + \left[20 - 5\right] \right\}$$

$$= \left\{ \frac{\overset{105}{\cancel{315}}}{\cancel{3}} - 60 + 15 \right\}$$

$$= 60$$

② $\int_1^5 \left(5/x^3\right) dx$

$$I = \int_1^5 5(x^{-3})\,dx$$

$$= 5\left(\frac{x^{-3+1}}{-3+1}\right)_1^5$$

$$= 5\left[\frac{x^{-2}}{-2}\right]_1^5$$

$$= \frac{-5}{2}\left[\frac{1}{x^2}\right]_1^5$$

$$= \frac{-5}{2}\left[\frac{1}{5^2} - \frac{1}{1^2}\right]$$

$$= -\frac{5}{2}\left[\frac{1}{25} - \frac{1}{1}\right]$$

$$= -\frac{5}{2}\left[\frac{1-25}{25}\right] = -\frac{\cancel{5}^{1}}{\cancel{2}_{1}}\left(\frac{-\cancel{24}^{12}}{\cancel{25}_{5}}\right) = \frac{12}{5}$$

③ Evaluate $\int_0^{\pi/2} (2\sin\theta - 5\cos\theta)\,d\theta$

$* \sin \pi/_2 = 1$

$* \cos \pi/_2 = 0$

$* \sin 0 = 0$

$* \cos 0 = 1$

$$I = \Big[2(-\cos\theta) - 5(\sin\theta) \Big]_{\theta=0}^{\pi/2}$$

$$= -\Big[2\cos\theta + 5\sin\theta \Big]_0^{\pi/2}$$

$$= -\Big\{ \Big[2\cos(\pi/_2) + 5\sin(\pi/_2) \Big] - \Big[2\cos 0 - 5\sin 0 \Big] \Big\}$$

$$= -\Big\{ [0+5] - [2-0] \Big\}$$

$$= -\{ 5-2 \}$$

$$= -3 \quad //$$

(9) Evaluate $\int_0^{\pi} x \sin x \, dx$

$$I = \left\{ (x)\left(-\cos x\right) - (1)\left(-\sin x\right) \right\}_0^{\pi}$$

$$= \left\{ \sin x - x\cos x \right\}_{x=0}^{\pi}$$

$$= \left\{ \left[\sin\pi - \pi\cos\pi\right] - \left[\sin 0 - 0\cos 0\right] \right\}$$

$$= \left\{ \left[0 - \pi(-1)\right] - \left[0 - 0\right] \right\} = \pi$$

* $\sin\pi = 0$
* $\sin 0 = 0$
* $\cos\pi = -1$
* $\cos 0 = 1$