Problems on Dot-product

① If $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ & $\vec{b} = 3\hat{i} - 2\hat{k}$ find dot product $\vec{a}$ & $\vec{b}$

Soln

$$\vec{a} \cdot \vec{b} = (\hat{i} - \hat{j} + 2\hat{k}) \cdot (3\hat{i} + 0\hat{j} - 2\hat{k})$$

$$= 3 - 0 - 4$$

$$\boxed{\vec{a} \cdot \vec{b} = -1}$$

(2) Find $\vec{a}\cdot\vec{b}$ if $\vec{a}=\hat{i}$ & $\vec{b}=\hat{k}$

Sol: $\vec{a}\cdot\vec{b} = \hat{i}\cdot\hat{k} = 0$ ($\perp^r$)

(3) ST the vectors $\vec{a} = 6\hat{i} - 2\hat{j} - \hat{k}$ & $\vec{b} = 2\hat{i} + 5\hat{j} + 2\hat{k}$ are $\perp^r$ to each other

Soln Consider $\vec{a}\cdot\vec{b} = (2\hat{i}+5\hat{j}+2\hat{k})\cdot(6\hat{i}-2\hat{j}-\hat{k})$

$$\vec{a}\cdot\vec{b} = 12 - 10 - 2 = 12 - 12$$

$$\boxed{\vec{a}\cdot\vec{b} = 0} \Rightarrow \vec{a} \perp^r \vec{b}$$

(4) Determine the angle b/n $\vec{a} = (3, -4, -1)$ & $\vec{b} = (0, 5, 2)$

Soln

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|} = \frac{(3\hat{i} - 4\hat{j} - \hat{k}) \cdot (0\hat{i} + 5\hat{j} + 2\hat{k})}{\sqrt{3^2 + 4^2 + 1^2} \cdot \sqrt{0^2 + 5^2 + 2^2}}$$

$$\cos\theta = \frac{0 - 20 - 2}{\sqrt{26}\,\sqrt{29}} = \frac{-22}{\sqrt{26 \times 29}} = -0.8011$$

$$\boxed{\theta = \cos^{-1}(0.8011)}$$

(5) Determine the projection of $\vec{a} = (1, 0, -2)$ on $\vec{b} = (2, 1, -1)$

<u>Soln:-</u>

$$\text{Projection} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$= \frac{(\hat{i} + 0\hat{j} - 2\hat{k}) \cdot (2\hat{i} + \hat{i} - \hat{k})}{\sqrt{2^2 + 1^2 + 1^2}} = \frac{2 + 0 + 2}{\sqrt{6}}$$

$$= \frac{4}{\sqrt{6}}$$

(1) projection of $\vec{b} = (2, 1, -1)$ on $\vec{a} = (1, 0, -2)$

Soln projection of $\vec{b}$ on $\vec{a}$ $= \dfrac{(2\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} + 0\hat{j} - 2\hat{k})}{\sqrt{1^2 + 0^2 + 2^2}} = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$

$$= \frac{2 + 0 + 2}{\sqrt{5}} = \frac{4}{\sqrt{5}}$$

Properties of Cross-product.

* $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

* $\vec{a} \times \vec{a} = 0$

* $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$

* Scalar triple product (Box product) $= [\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

* If $\vec{a}, \vec{b}, \vec{c}$ are Co-planar

$$[\vec{a}\ \vec{b}\ \vec{c}] = 0$$

* $\hat{i} \times \hat{i} = 0$; $\hat{j} \times \hat{j} = 0$; $\hat{k} \times \hat{k} = 0$

$\hat{i} \times \hat{j} = \hat{k}$; $\hat{j} \times \hat{k} = \hat{i}$; $\hat{k} \times \hat{i} = \hat{j}$

$\hat{j} \times \hat{i} = -\hat{k}$

Problems on Cross-product

① Find the cross product of $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$
& $\vec{b} = 4\hat{i} + \hat{j} - 3\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \overset{+}{\hat{i}} & \overset{-}{\hat{j}} & \overset{+}{\hat{k}} \\ 2 & -1 & 3 \\ 4 & 1 & -3 \end{vmatrix} = \hat{i}\begin{vmatrix} -1 & 3 \\ 1 & -3 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & 3 \\ 4 & -3 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix}$$

$$= \hat{i}(3-3) - \hat{j}(-6-12) + \hat{k}(2+4)$$

$$= 0\hat{i} + 18\hat{j} + 6\hat{k}$$

(2) $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ & $\vec{b} = \hat{i} - \hat{j} - \hat{k}$

(3) $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ & $\vec{b} = 3\hat{i} - 4\hat{j} + 2\hat{k}$

(4) Find the box product of
$\vec{a} = 2\hat{i} + \hat{k}$; $\vec{b} = 3\hat{i} + \hat{j} + \hat{k}$; $\vec{c} = \hat{i} + \hat{j} + \hat{k}$

Sol $[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} 2^{+} & 0^{-} & 1^{+} \\ 3 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 2(1-1) - 0(\quad) + 1(3-1)$

$= 2$ //

(5) Find λ if $\vec{a}, \vec{b}, \vec{c}$ are co-planar

$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$; $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$; $\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$

$$[\vec{a}\ \vec{b}\ \vec{c}] = 0 \Rightarrow \begin{array}{ccc} + & - & + \end{array}$$
$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

$$\Rightarrow 2(10+3\lambda) + (5+9) + 1(\lambda - 6) = 0$$

$$20 + 6\lambda + 14 + \lambda - 6 = 0$$

$$7\lambda + 28 = 0$$

$$7\lambda = -28$$

$$\lambda = -4$$