Problems on differentiation

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① $y = 6x^5 - 4x^2 + 9x + 10$

$\frac{d}{dx}(x^n) = nx^{n-1}$

Soln: diff w.r.to x, we get

$$\frac{dy}{dx} = 6\frac{d}{dx}(x^5) - 4\frac{d}{dx}(x^2) + 9\frac{d}{dx}(x) + \frac{d}{dx}(10)$$

$$= 6(5x^4) - 4(2x) + 9(1) + 0$$

$$\boxed{\frac{dy}{dx} = 30x^4 - 8x + 9}$$

$\frac{d}{dx}(k) = 0$; $\frac{d}{dx}(x) = 1$; $\frac{d}{dx}(x^2) = 2x$; $\frac{d}{dx}(x^3) = 3x^2$

② $y = \sqrt{x} + 8\sqrt[3]{x} - 2\sqrt[4]{x}$

* $\sqrt{x} = x^{1/2}$; $\sqrt[3]{x} = x^{1/3}$; $\sqrt[4]{x} = x^{1/4}$ *

Soln

$$\frac{dy}{dx} = \frac{d}{dx}(x^{1/2}) + 8\frac{d}{dx}(x^{1/3}) - 2\frac{d}{dx}(x^{1/4})$$

$$= \frac{1}{2}x^{\frac{1}{2}-1} + 8\cdot\frac{1}{3}x^{\frac{1}{3}-1} - \cancel{2}\cdot\frac{1}{\cancel{4}_2}x^{\frac{1}{4}-1}$$

$$= \frac{1}{2}x^{-\frac{1}{2}} + \frac{8}{3}x^{-\frac{2}{3}} - \frac{1}{2}x^{-3/4}$$

$$= \frac{1}{2x^{\frac{1}{2}}} + \frac{8}{3x^{2/3}} - \frac{1}{2x^{3/4}}$$

$$= \frac{1}{2\sqrt{x}} + \frac{8}{3\sqrt[3]{x^2}} - \frac{1}{2\sqrt[4]{x^3}}$$

(3) $f(t) = \frac{4}{t} - \frac{1}{6t^3} + \frac{8}{t^5}$

$f(t) = 4.t^{-1} - \frac{1}{6}t^{-3} + 8t^{-5}$

diff. w.r.to 't', we get

$\frac{d f(t)}{dt} = 4(-1t^{-1-1}) - \frac{1}{6}(-3t^{-3-1})$

$+8(-5t^{-5-1})$

$= -4t^{-2} + \frac{\cancel{3}^{1}}{\cancel{6}_{2}}t^{-4} - 40t^{-6}$

$= -\frac{4}{t^2} + \frac{1}{2t^4} - \frac{40}{t^6}$ //

(4) $y = x(3x^2 - 9)$

$y = 3x^3 - 9x$

diff. w.r.to x

$\frac{dy}{dx} = 3(3x^2) - 9(1) = 9x^2 - 9$

$= 9(x^2 - 1)$ //

(5) $y = \dfrac{4x^3 - 7x + 3}{x^3}$

$y = \dfrac{4\cancel{x^3}}{\cancel{x^3}} - \dfrac{7x^1}{x^{\cancel{3}2}} + \dfrac{3}{x^3}$

$y = 4 - \dfrac{7}{x^2} + \dfrac{3}{x^3}$

$y = 4 - 7x^{-2} + 3x^{-3}$

diff. w.r. to x

$\dfrac{dy}{dx} = 0 - 7(-2x^{-2-1}) + 3(-3x^{-3-1})$

$= 14x^{-3} - 9x^{-4}$

$= \dfrac{14}{x^3} - \dfrac{9}{x^4}$ //

(6) $R(z) = \dfrac{6}{\sqrt{z^3}} + \dfrac{1}{8z^4} - \dfrac{1}{3z^{10}}$

$R = 6z^{-3/2} + \dfrac{1}{8}z^{-4} - \dfrac{1}{3}z^{-10}$

diff. w.r. to z.

$\dfrac{dR}{dz} = {}^{3}\cancel{6}\left(-\dfrac{3}{\cancel{2}}z^{-\frac{3}{2}-1}\right) + \dfrac{1}{\cancel{8}_{2}}\left(-\cancel{4}^{1}z^{-4-1}\right)$

$- \dfrac{1}{3}(-10z^{-10-1})$

$$\frac{dR}{dz} = -9z^{-5/2} - \frac{1}{2}z^{-5} + \frac{10}{3}z^{-11}$$

$$= \frac{-9}{\sqrt{z^5}} - \frac{1}{2z^5} + \frac{10}{3z^{11}}$$

7) Determine tangent $\left(\frac{dy}{dx}\right)$ line to $f(x) = 7x^4 + 8x^{-6} + 2x$ at $x = -1$

$$y = f(x) = 7x^4 + 8x^{-6} + 2x$$

diff. w.r.to x.

$$\frac{dy}{dx} = 7(4x^3) + 8(-6x^{-6-1}) + 2(1)$$

$$= 28x^3 - 48x^{-7} + 2$$

$$\boxed{\frac{dy}{dx} = 28x^3 - \frac{48}{x^7}} + 2$$

$$\left(\frac{dy}{dx}\right)_{x=-1} = 28(-1)^3 - \frac{48}{(-1)^7} + 2$$

$$= -28 + 48 + 2$$

$$\boxed{\left(\frac{dy}{dx}\right)_{x=-1} = 22}$$

Practice $g(x) = \frac{16}{x} - 4\sqrt{x}$ at $x = 4$

Problems on Quotient & Product Rule

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} \quad \text{(product)}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$

① $f(t) = \underbrace{(4t^2 - t)}_{u}\underbrace{(t^3 - 8t^2 + 12)}_{v}$

diff. w.r.t. t

$$f'(t) = (4t^2 - t)\frac{d}{dt}(t^3 - 8t^2 + 12) + (t^3 - 8t^2 + 12)\frac{d}{dt}(4t^2 - t)$$

$$= (4t^2 - t)[3t^2 - 16t + 0] + (t^3 - 8t^2 + 12)(8t - 1)$$

$$= 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - t^3 - 64t^3 + 8t^2 + 96t - 12$$

$$= 20t^4 - 132t^3 + 24t^2 + 96t - 12$$

② $y = (1 + \sqrt{x^3})(x^{-3} - 2\sqrt[3]{x})$

diff. w.r.t. x

$$y = \left(1 + \sqrt{x^3}\right)$$