FINAL TERM PROJECT REPORT OPERATION RESEARCH (DSA6200)

FACILITY LOCATION PROBLEM

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I.Business problem

In the past decades, location science has attracted many researchers and practitioners from different disciplines and fields. Being a very rich field, location science includes several types of location problems, solution techniques as well as considerable amount of theoretical modeling frameworks and solution techniques. Its connection and interaction with other disciplines such as mathematics, geography, logistics, economics is the main driving force behind its development. We will deal here with facility location, which is a classical optimization problem for determining the sites for factories and warehouses.

An E-Commerce Company in US is planning to install its 8 facilities/warehouses based on 14 demand points in few states. The company has chosen a set of potential facility sites where a facility can be opened, and a set of demand points that must be serviced. Each site has a yearly activation cost i.e., an annual leasing expense that is incurred for using it, independently of the volume it services. This volume is limited to a given maximum amount that may be handled yearly. Additionally, there is transportation cost (in dollars) per unit serviced from each facility to each of the demand point.
The goal is to pick a subset of facilities to open, to minimize the sum of distances from each demand point to its nearest facility i.e. Transportation cost, plus the sum of opening costs of the facilities. A facility location problem is, in which a multiple facility is to be placed, with the only optimization criterion being the minimization of the weighted sum of distances from a given set of point sites.

II.Literature review

The growth in e-commerce has been motivated by several reasons- convenience, ease, pricing and services etc. although e-Commerce is on rise , the challenges has also increased such as logistics [1]. Compared with traditional commerce, e-Commerce has advantages in reducing investment cost and selling cost by using Internet and Web technologies. However, since firms must deliver commodities to customers and orders have the characteristics of numerous multi-variety small batch demands and decentralized locations, the new logistical delivery cost is increasing. Therefore, it is very necessary to reduce the delivery cost in order to improve e-Commerce firms' profits. Looking at the previous work done in the field Ohaswa started with single facility optimization.
As for the most well known basic ones, Ohsawa[2] has focused on a single facility, quadratic Euclidean distance bi-criteria model defined in the continuous space, with convex combination of the minisum and minimax objectives (efficiency and equity). Nickel [3] has extended the classic 2-facility Weber problems to bi-objective ones with regional restrictions. Bhattacharya et al. developed a fuzzy goal programming for their convex multi-facility location problem with minisum (transportation cost) and minimax (distance) objectives with rectilinear distances. Klamroth and Wiecek have developed the median location problem with a line barrier (like rivers, highways, borders, mountain ranges, etc.) which is nonconvex and can have different measures of distance in its minisum objectives. Skriver et al. Ohsawa et al. too, has developed ordered

median problem of two desirable and undesirable facilities (minisum and maxisum objectives) in continuous spaces with Euclidean distances.

Kozanidis [4] has introduced the convex Linear Multiple Choice Knapsack Problem that incorporates a second objective to allocate the available resource among a group of disjoint sets of activities, equitably. The objectives of this problem are in the form of maxisum and minimax functions (profit and equity).

Considering capacities in location problems, there are capacitated and uncapacitated problems in the literature. For instance, Myung et al. have considered an uncapacitated facility location problem with two maxisum objectives (net profit and profitability of the investment) and modeled it as a parametric integer program with fractional and linear objectives. Villegas et al. has modeled a supply network as a bi-objective uncapacitated facility location problem, with minisum and maxisum objectives (cost and coverage). In contrast, Galvão et al. developed an extension of the capacitated model to deal with locating maternity facilities with minisum objective (distance traveled and load imbalance).

In some cases, location problems come to simultaneous allocation issues. For example, Costa et al. has presented a bi-criteria approach to the single allocation hub location problem. The first objective is a minisum form (cost), while the second objective (process time) has two alternative forms: minisum or minimax. Klimberg and Ratick have utilized a different concept in order to formulate and find optimal and efficient facility location/allocation patterns. This concept is called Data Envelopment Analysis (DEA) which defines relative efficiency as the ratio of the sum of weighted output to the sum of weighted input: DEA efficiency = sum of weighted output/sum of weighted input. The more output produced for a given amount of resources, the more efficient (i.e., less wasteful) the process is. Therefore, the purpose in their model is to simultaneously minimize cost and maximize DEA objectives.

One of the most famous categories of location problems is facility location on a network, especially in supply chain context. One examples is Bhaskaran and Turnquist which has studied the relation between transportation cost and coverage objectives in a network multi-facility location in both ordinary and ordered problems. Another example is Blanquero and Carrizosa work in which they dealt with a bi-objective semi-obnoxious location problem with minisum and minimax objectives (cost and negative effect). Fernández et al. presented a bi-objective supply chain design and facility location problem of supermarkets on the plane in which the main objective was to maximize the profit obtained by the chain, and the secondary objective was to minimize the cannibalization suffered by the existing chain-owned facilities. The cannibalization suffered, is the difference between market shares before and after entering a new facility. These are not the only applications of facility location on a network. For example, when you are about to locate a regional rail road or subway network, you will have subtree location problems. George and ReVelle have focused on Median Subtree Location Problems (MSLP) in which they minimized the cost of the subtree as well as minimized travel distance from unconnected nodes as an integer program. There is another famous sub-category for facility location on a network, named reverse logistics. Du and Evans have modeled a reverse logistic network for repair service as a bi-objective optimization problem with minisum objectives (cost and tardiness of cycle time). Fonseca et al. have studied stochastic reverse logistics with several facility echelon, multi-commodities, and technology choices. They model these problems as a two-stage (strategic and tactical) mixed integer program with two minisum objectives (cost and undesirable effect) with Euclidean distances. Some network location problems are combined with routing problems. Current et al. [5] have considered

maximum covering/shortest path problems as bi-objective integer programs with cost (minisum) and coverage (maxisum) objectives.
In brief, bi-objective location problems are getting more attention, especially in relation to semi-desirable and undesirable problems, as well as location problems on network. The purpose in most of these problems is minimizing cost, but this objective is in conflict with the other one, which is in most cases, maximizing the distance, or coverage.

III.Model details

In the basic formulation of the facility location problem, the number of possible locations is now finite as we have discrete choices of where to build. The problem has been studied extensively and is classified as capacitated and multi facility location problem. The capacitated facility location problem is the basis for many practical optimization problems, where the total demand that each facility may satisfy is limited. Hence, modeling such problem must take into account both demand satisfaction and capacity constraints.

Let's begin with the problem. The e-commerce company has 8 potential sites for installing its facilities/ warehouses and 14 demand points as in each site (location) j has a yearly activation cost F_j, i.e. , an annual leasing expense that is incurred for using it, independently of the volume it services. This volume is limited to a given maximum amount that may be handled yearly, M_j. Additionally, there is a transportation cost Cji per unit serviced from facility j to demand point i. The following parameters are used in model formulation and a glimpse of data used is shown in Table 1 below.

Demand points *i*	1	2	3	4	………	14		
Annual demand D*i*	220	180	280	230		330		
Facility j	C*ji*						F*j*	M*j*
1	60	95	105	125		100	1000	500
2	55	70	90	65		45	1200	800
3	50	40	65	110		65	900	600
⋮								
8	65	120	50	95		95	1200	600

The problem is formulated as a mathematical optimization model as follow. There are 14 demand points I = 1,2,3,……..,14 and eight sites are available for facilities j = 1,2,3,……, 8. A continuous variable Xij is defined as the amount of serviced from facility j to demand point I and a binary variable yj which equals 1 if a facility is established at location j and if not yj =0.
An integer optimization model for the capacitated facility location problem is specified as follows:-

Parameters :-

i = demand points F[j]= yearly activation cost C[j][i]= transportation cost

j = facilities. D[i]= annual demands M[j]= warehouse capacity

Decision variables

$y[j] \in \{0,1\}$ …. 1 if opening a facility / warehouse otherwise 0.
Int Xji >=0

The model has 246 constraints and 120 variables out of which 8 are binary.

Constraints

$\sum_{j=1}^{8} Xji$ = Di …… for all i $\in$ (1,14)

$\sum_{i=1}^{14} Xji = Mj * yj$ …… for all j $\in$ (1,8)

$Xji <= Di * yj$ …… for all j $\in$ (1,8) & for all i $\in$ (1,14)

$Xji >= 0$ …… for all j $\in$ (1,8) & for all i $\in$ (1,14)

Objective

Minimize $\sum_{j=1}^{8} Fj * yj + \sum_{j=1}^{8} \sum_{i=1}^{14} \quad Cji * Xji$

IV.Model implementation

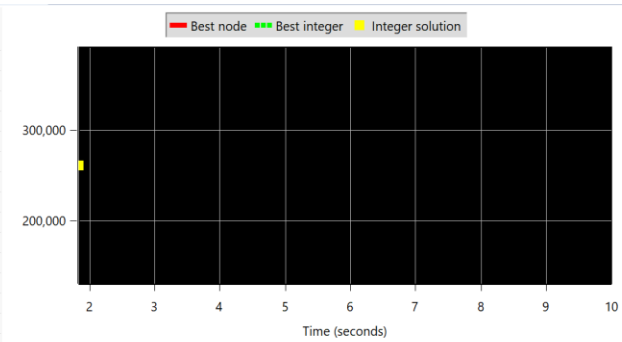
The above model is implemented on self generated dataset using python. The proposed model and solution approaches for determining the facility locations of supplies in response to large-scale demands. The problem was formulated as a minimax problem. The minimax facility location problem, also known as the "P-center" problem. The P-center focuses on a demand point being served by the nearest facility and how all demand points can be covered. The minimax approach involves meeting all the demand requirements to max possible extent and minimizing the cost of transportation from jth facility to ith demand point. The minimax facility location problem is quite different from the minisum facility location problem and the covering problem. The minisum facility location problem considers the locations of general facilities such as distribution centers and inventory, and the covering problem is similar to the minimax facility location problem as it concentrates on optimizing overall system performance within particular distance.
The objective function focuses on both aspects of the problem i.e. minimizing the total cost and covering all the demand points. The constraints are applied so that all the demand points are met

and do not exceed the value for the particular volume demanded by each demand point. The binary variable ‘y’ tell us if the facility is required to be open at the particular site or not. The data consist 14 demand points and 8 facility locations in different areas. The model Is implemented in such a way so that each demand point is easily met by the requirement from nearest facility/ warehouse.

V.Model and run characteristics

The model has show exceptionally good performance. The model provided the optimal solution of $ 261500 by minimizing all the cost factors as the mean objective value is $339525 which way greater than the optimal solution value. Mean objective value is calculated by taking the mean of objective values returned by the model considering all the possible solution for problem.

Statistic	Value
Cplex	solution (optimal) with objective 261500
Constraints	246
Variables	120
Binary	8
Other	112
Non-zero coefficients	568
MIP	
Objective	261,500
Incumbent	261,500
Nodes	0
Remaining nodes	0
Iterations	23
Solution pool	
Count	2
Mean objective	339,525



The below fig shows the run characteristics of the model. The y values shows the facility locations to be opened in future and X values shows the transfer of volume from j th facility to i th demand point which are more clearly represented in the result section

```
// solution (optimal) with objective 261500
// Quality Incumbent solution:
// MILP objective                              2.6150000000e+05
// MILP solution norm |x| (Total, Max)         4.80800e+03  5.50000e+02
// MILP solution error (Ax=b) (Total, Max)     0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)             0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)       0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)         0.00000e+00  0.00000e+00
//

y = [1
         1 1 1 1 1 1 1];
X = [[0 0 0 0 0 30 0 0 470 0 0 0 0 0]
             [0 0 0 230 0 0 0 0 0 0 0 300 0 270]
             [220 180 0 0 0 0 200 0 0 0 0 0 0 0]
             [0 0 0 0 0 0 0 370 0 0 0 0 0 0]
             [0 0 250 0 0 0 0 0 0 0 250 0 0 0]
             [0 0 0 0 0 60 200 0 0 310 0 0 0 60]
             [0 0 0 0 550 250 0 0 0 0 0 0 0 0]
             [0 0 30 0 0 310 0 0 0 0 0 0 260 0]];
```

Scenario Analysis

What if in future the demand for some demand points increases?

- Then the possible solution is to open a new facility location to met the demand point requirement or storage capacity of existing facility can be expanded. The optimal solution will depend upon the after comparing cost for opening a new facility or upgradation, operation but for second we have to optimize the model to include the upgradtion cost.

What if in future there is shortage of demand by demand points ?

- If company face such problem in future then it has to update the data and pass it to model to find the new optimal facility locations as needed. As tested on artificial data by eliminating 3 demand points the no of facility locations to be opened decreased by one.

VI.Results insights

This section provides the result and performance of the model including a bit of analysis.

What facilities or warehouse to be established in future?

F1	F2	F3	F4	F5	F6	F7	F8
1	1	1	1	1	1	1	1

As seen in the model results all the facilities are necessary to be established to provide smooth operation of the project and the demands fulfilled by each facility location of each demand point is shown blow in the Table.

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14
F1	0	0	0	0	0	30	0	0	470	0	0	0	0	0
F2	0	0	0	230	0	0	0	0	0	0	0	300	0	270
F3	220	180	0	0	0	0	200	0	0	0	0	0	0	0
F4	0	0	0	0	0	0	0	370	0	0	0	0	0	0
F5	0	0	250	0	0	0	0	0	0	0	250	0	0	0
F6	0	0	0	0	0	60	200	0	0	310	0	0	0	0
F7	0	0	0	0	550	250	0	0	0	0	0	0	0	0
F8	0	0	30	0	0	310	0	0	0	0	0	0	260	0

The optimal solution obtained suggest all facility locations are required to be opened in future and the solution incurs the minimal total cost of $265100.

VII.Conclusion

The solutions derived from the model can be used to support the decisions regarding distribution center location i.e. facility locations for B2C firm.

First, the proposed model considers the characteristics of firm and variables are employed in the model to represent the delivery cost, market supply and customer demand.. In addition, a more reasonable setup cost function of the distribution center is presented in the model, and it overcomes the current literature limitations which neglecting to optimize the capacity of distribution centers. So the model is suitable for the location of facility centers for e-commerce company.

Secondly , there is no violation of any constraint in the model. All the requirements are fully met. It can work well in spite of the lack of historical data as data is self generated and model has shown an extraordinary performance in curbing the company from huge expenses by minimizing the cost factor.

In terms of future research, two directions have been identified. First, the proposed model can be extended to multiple objectives, such as maximizing the total service level, minimizing total delivery time and so on. In this case, due to the limited resources and different goals of decision makers, the extended model can be modified or adjusted by adding objectives or constraints before it is applied to solve real problems. Second, we intend to develop a multi-objective algorithm with respect to the extended model, which can obtain a optimal solution set for decision makers instead of only one optimal solution. Therefore, the extended model and algorithm will become more universal and more convenient to use for decision makers.

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