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MTH212

Example1: Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges

Solution

This is a p-series with $p=2$. According to the p-series test: $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p>1$

Since $2>1$, the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

Example2: Test the series $\sum_{n=1}^{\infty} \frac{1}{n}$ for convergence

Solution

This is a harmonic series. The harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ is known to diverge.

Alternatively, apply the p-series test for p-series $\sum_{n=1}^{\infty} \frac{1}{n}$. the series converges. If $p>1$, since $p=1$, the series diverges.

Example3: Determine whether the series $\sum_{n=1}^{\infty} \frac{n!}{2^n}$ converges.

Solution

We apply the ratio test let $a_n = \frac{n!}{2^n}$

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Then $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \frac{(n+1)!/2^{n+1}}{n!/2^n}$; $\Rightarrow \lim_{n \to \infty} \frac{(n+1)! \cdot 2^n}{n! \cdot 2^{n+1}}$

$\Rightarrow \lim_{n \to \infty} \frac{(n+1) n! \, 2^n}{n! \, 2^n \cdot 2}$; $\Rightarrow \lim_{n \to \infty} \frac{(n+1)}{2}$

As n approaches infinity, $\frac{(n+1)}{2} \to \infty$

Since the limit is greater than 1, the series

$\sum_{n=1}^{\infty} \frac{n!}{2^n}$ diverges by ratio test.

Class Work 6 : 27th May 2024 Monday

1. Determine if the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ converges.

2. Determine if the series $\sum_{n=1}^{\infty} \frac{n^n}{(n!)^2}$ converges