Examples of Integration by Part (IBP)

1) Using IBP, solve

$\int xe^x dx$

Soln Step 1: Choose u and dv

$u = x$, $\Rightarrow \frac{du}{dx} = 1$, $\therefore du = dx$

$dv = e^x dx \Rightarrow v = \int e^x dx = e^x$; $\therefore v = e^x$

Step 2 Calculate du and v

$du = dx$

$v = \int dv = e^x$

Step 3: Apply the IBP formula which is $\int u dv = uv - \int v du$

$\int xe^x dx = xe^x - \int e^x dx$

Step 4 Integrate the new integral

$\int e^x dx = e^x$

Step 5: Put all together

$\int xe^x dx = xe^x - e^x + c$

$= e^x(x-1) + c$, where c is the constant of integration.

Example 2

Using IBP method solve $\int x \ln x \, dx$

soln

$u = \ln x$, $\Rightarrow du = \frac{1}{x} dx$

$dv = x dx$, Integrating both sides we have

$v = \frac{x^2}{2}$

$\therefore \int x \ln x \, dx = \ln x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x} dx$

$= \ln x \cdot \frac{x^2}{2} - \int \frac{x}{2} dx$

$\therefore \int x \ln x \, dx = \ln x \cdot \frac{x^2}{2} - \frac{1}{2} \int x dx$

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$\frac{x^2}{2}\ln x - \frac{1}{2}\left(\frac{x^2}{2}\right)+c$

$\Rightarrow \int x \ln x = \frac{x^2}{2}\ln x - \frac{x^2}{4}+c$

Example 3.

$$\int x^2 \cos(x)\,dx$$

step 1: choose u and dv

Let $u = x^2$, $du = 2x\,dx$

Let $dv = \cos(x)dx$, $v = \int dv = \sin(x)$

Step 2: Calculate du and v

$du = 2x\,dx$

$v = \sin(x)$

step 3: Apply the IBP formula

$\int x^2\cos(x)\,dx = x^2\sin(x) - \int 2x\sin(x)dx$

step 4 Integrate the new integral

$\int 2x\sin(x)dx$. This requires another IBP

$\downarrow$

step 1: (for the new integral).

Let $u = 2x$, $\Rightarrow du = 2dx$

Let $dv = \sin(x)dx \Rightarrow v = -\cos(x)$

step 2: (for the new integral).

Calculate du and v

$du = 2dx$

$v = -\cos(x)$

step 3: (for the new integral)

Apply the IBP formula

$\int 2x\sin(x)dx = -2x\cos(x) - \int(-2\cos(x)\,dx)$

step 4. (for the new integral)

Integrate the new integral

$\int(-2\cos(x)\,dx) = 2\int\cos(x)dx$

$\int\cos(x)\,dx = \sin(x)$

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Now, we can continue with the original integral

$$\int x^2 \cos(x)\,dx = x^2 \sin(x) - (-2x\cos(x) - 2\sin(x))$$

Step 5. put all together

$$\int x^2 \cos(x)\,dx = x^2 \sin(x) + 2x\cos(x) - 2\sin(x) + C,$$

where C is the constant of integration.

Integration by Substitution

This is also known as u-substitution. It is a technique used to simplify and solve certain types of integrals. The basic idea is to substitute a new variable (usually denoted as "u") for part of the integrand to make the integral easier to solve.

Steps Involved

Step 1.

Identify a suitable substitution. Look for a part of the integrand that can be simplified by substituting it with a new variable "u". This part should typically be in the form of a function and its derivative appearing elsewhere in the integrand.

Step 2.

Choose "u" and find "du". Choose a suitable expression to substitute for "u" in the integrand. This choice depends on the integral but its often helpful to select the function that appears inside another function or a part of the integrand that can be simplified.

Next, find the derivative of your chosen expression with respect to the variable of integration (usually "x"). This derivative denoted as "du" is crucial for the substitution and helps rewrite the integral in terms of "u".

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step 3

Rewrite the integral in terms of "u". ie replace the chosen expression for "u" and its derivative du in the original integral. This typically involves rewriting the integrand in terms of "u" and replacing "dx" with "du". We then have something like

$$\int f(u)\,du$$

where $f(u)$ represents the simplified expression in terms of u.

step 4

Solve the new integral. Integrate the new expression $\int f(u)\,du$ with respect to u.

step 5

Undo the substitution.

Substitute 'u' back in terms of 'x' using the original substitution you made in step 2.

step 6

Add a constant of integration. Add the constant of integration denoted as 'c' to your final result

step 7.

Check your answer.

Always double-check your solution by differentiating it with respect to "x" to ensure it matches the original integrand.

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Examples using Integration by substitution Method

Example 1:

$$\int 2xe^{x^2}dx$$

Step 1: Identify a suitable substitution.
Here x^2 appears the exponential function, so its a suitable candidate for substitution.

Step 2 - Choose 'u' and find 'du'
Let $u = x^2$, $\Rightarrow du = 2x\,dx$

Step 3 - Rewrite the integral in terms of 'u'
The integral becomes $\int e^u du$

Step 4 - Solve the new integral
$\int e^u du$ is just e^u

Step 5 - Undo the substitution.
Substitute u back in terms of x
e^{x^2}

Step 6 Add a constant of integration
$e^{x^2} + C$

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TRIGONOMETRIC SUBSTITUTION

This is used to simplify and evaluate certain types of integrals that involve radicals and trigonometric functions. It involves substituting a trigonometric identity to transform the integral into a more manageable form.

Steps Involved

Step 1:

Identify the integral that you want to solve and look for expressions that involve square roots or squares of trigonometric functions. Common pattern that suggest trigonometric substitution include:

1. $\int \sqrt{(a^2-x^2)}\, dx$ $\qquad x = a\sin\theta$
2. $\int \sqrt{x^2-a^2}\, dx$
3. $\int \sqrt{(x^2+a^2)}\, dx$

Step 2

Choose the right trigonometric identity

Depending on the form of the integral, you'll choose one of three trigonometric identities to make the substitution.

These identities are

1) If you have $\int \sqrt{(a^2-x^2)}\, dx$, use $x = a\sin(\theta)$
2) If you have $\int \sqrt{(x^2-a^2)}\, dx$, use $x = a\sec(\theta)$
3) If you have $\int \sqrt{(x^2+a^2)}\, dx$, use $x = a\tan(\theta)$

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Step 3

Find dx and Substitute

Next, differentiate the chosen identity to find dx in terms of dθ eg

1) If $x = a\sin(\theta)$, then $dx = a\cos(\theta)d\theta$
2) If $x = a\sec(\theta)$, then $dx = a\sec(\theta)\tan(\theta)d\theta$
3) If $x = a\tan(\theta)$, then $dx = a\sec^2 2(\theta)d\theta$)

step 4

Substitute the expression for x and dx from step3 into the original integral. Also, substitute the trigonometric identity you chose from step2. Your integral should now be in terms of θ

1) for $x = a\sin(\theta)$: $\int\sqrt{(a^2 - a^2\sin^2(\theta))}\,a\cos(\theta)d\theta$
2) for $x = a\sec(\theta)$: $\int\sqrt{(a^2\sec^2(\theta) - a^2)}\,a\sec(\theta)\tan(\theta)d\theta$
3) for $x = a\tan(\theta)$: $\int\sqrt{(a^2\tan^2(\theta) + a^2)}\,a\sec^2(\theta)\,d\theta$

step 5

Simplify the integrand as much as possible, using trigonometric identities to reduce the expression to a more manageable form.

Step 6

Evaluate the integral. Evaluate it using standard integration techniques. (Don't forget to change the limits of integration from x-values to θ-values if necessary.

step 7

Convert back to x once you have found the integral in terms of θ, convert your final answer back to x by using the original trigonometric identity you chose in step 2.

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Examples

Use trigonometric substitution to integrate

(i) $\sin^2(x)$

soln

$$\int \sin^2(x)dx$$

step 1: Use the trigonometric identity

$$\sin^2(x) = \frac{(1-\cos(2x))}{2} = \int \left(\frac{1-\cos(2x)}{2}\right)dx$$

step 2 Distribute the ½ into the integral

$$\left(\frac{1}{2}\right)(1-\cos(2x))dx$$

step 3: Integrate term by term

$$\left(\frac{1}{2}\right)\int (1-\cos(2x))dx = \frac{1}{2}\int 1dx - \left(\frac{1}{2}\right)\int \cos(2x)dx$$

step 4. Integrating each term

$\left(\frac{1}{2}\right)(x) - \left(\frac{1}{4}\right)\sin(2x)+c$, where c is the constant of integration.

(ii) $\int \tan(x)dx$

soln

Rewrite $\tan(x)$ as $\frac{\sin(x)}{\cos(x)}$ we have $\int \left(\frac{\sin(x)}{\cos(x)}\right)dx$

Using u-substitution

Let $u=\cos(x)$, then $du = -\sin(x)dx$

$$\int \left(\frac{-du}{u}\right) = -\int \left(\frac{1}{2}\right)du$$

Integrating we have

$$-\int \left(\frac{1}{u}\right)du = -\ln|u|+c$$

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Substitute back for u in terms of x

$$-\ln|\cos(x)|+c.$$

(iii) $\int \sec^2(x)\,dx$

Soln

Using definition of $\sec^2(x)$ we have

$$\int \sec^2(x)\,dx = \int\left(\frac{1}{\cos^2(x)}\right)dx$$

Using the identity $\cos^2(x) = 1-\sin^2(x)$

$$\int\left(\frac{1}{(1-\sin^2(x))}\right)dx$$

Using the u-substitution

Let $u = \sin(x)$, then $du = \cos(x)dx$

$$\int\left(\frac{1}{(1-u^2)}\right)du$$

Use partial fraction decomposition to simplify more

$$\frac{1}{(1-u^2)} = \frac{1}{(1-u)(1+u)} = \left(\frac{1}{2}\right)\left(\frac{1}{(1-u)} - \frac{1}{(1+u)}\right]$$

$$= \int\left(\frac{1}{2}\right)\left(\frac{1}{(1-u)} - \frac{1}{(1+u)}\right]du$$

Integrating term by term

$$\left(\frac{1}{2}\right)(\ln(1-u) - \ln(1+u|)+c$$

Substituting back for u in terms of x

$$\left(\frac{1}{2}\right)(\ln|1-\sin(x)| - \ln|1+\sin(x)|)+c$$

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