MTH 218

Techniques of Integration

If $F(x)$ and $f(x)$ are two functions of x such that

$$\frac{dF(x)}{dx} = f(x)$$

then $F(x)$ is said to be an indefinite integral of $f(x)$ and written as:

$$F(x) = \int f(x)\,dx + c$$

The function $f(x)$ is called the integrand and is said to be integrable if $F(x)$ exists. 'c' is an arbitrary constant which is called a constant of integration that must be included when evaluating an indefinite integral.

Standard Integration Rules:

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$

2. $\int e^x\,dx = e^x + c$

3. $\int a^x dx = a^x \times \log_e a \qquad a > 0$

4. $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| = \int \frac{dx}{x} = \log_e x$

5. $\int \cos ax\, dx = \frac{1}{a} \sin ax$

6. $\int \sin ax\, dx = -\frac{1}{a} \cos ax$

7 $\int \tan ax\, dx = -\frac{1}{a} \log \cos ax = \frac{1}{a} \log$

8. $\int \cosh ax\, dx = \frac{1}{a} \sinh ax$

9. $\int \sinh ax\, dx = \frac{1}{a} \cosh ax$

$$\int \tanh ax\,dx = \frac{1}{a}\cosh ax$$

$$\int \sec x \tan x\,dx = \sec x$$

$$\int \operatorname{cosec}^2 x\,dx = -\cot x$$

$$\int \sec^2 x\,dx = a\tan x \quad \text{etc}$$

Apart from the above standard rules, we have other methods of integration such as

i. Integration by parts
ii. Integration by substitution
iii. Integration by Partial Fraction
iv. Integration using reduction formula
v. Powers of sine and cosine
vi. Integration using Trigonometric identities etc.

INTEGRATION BY PARTS

Integration by Parts (IBP) is one of the important methods of integration. When the product of two functions are given to us then we apply the required formular.

The two functions to be integrated f(x) and g(x) are of the form $\int f(x) \cdot g(x)$. Thus, it can be called a product rule of integration. The first function f(x) is selected such that its derivative formula exists and the second function g(x) is chosen such that an integral of such a function exists.

$$\int f(x) \cdot g(x)\,dx = f(x) \int g(x)\,dx - \int [(f'(x) \int g(x)\,dx) \cdot dx] + C.$$

The Integration of (First function x second function) = (First function) x (integration of secord function) - Integration of (Differentiation of first function x Integration of secord function).

For simplicity, these functions are often represented as 'u' and 'dv' respectively.

The uv integration formular using the notation of 'u' and 'dv' is:

$$\int u\,dv = uv - \int v\,du$$

For choosing the first function u(x), we have to see which of the following function comes first in the following order and then assume it as u.

Logarithmic (L)
Inverse trigonometric (I)
Algebraic (A)
Trigonometric (T)
Exponential (E
This is called the LIATE rule.