MTH212 31st May 2020 Friday:

6 Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n!}$ converges

solution.

Apply ratio test let $a_n = \frac{1}{n!}$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{1/(n+1)!}{1/n!}$$

$$= \lim_{n\to\infty} \frac{n!}{(n+1)n!} = \lim_{n\to} \frac{1}{n+1} = 0$$

Since the limit is less than 1, the series $\sum_{n=1}^{\infty} \frac{1}{n!}$ converges by the ratio test.

7. Determine the convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

solution.

This is an alternating series $\sum (-1)^{n+1} b_n$, IF $b_n \geq 0$, $b_{n+1} \leq b_n$ (monotonically decreasing) and $\lim_{n\to 0} b_n = 0$ then the series converges.

Here $b_n = \frac{1}{n}$

$b \geq 0$ is true

$b_{n+1} = \frac{1}{n+1} \leq \frac{1}{n} = b_n$ is decreasing

$$\lim_{n\to\infty} \frac{1}{n} = 0$$

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All the condition of alternating serie test are met, so the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ Converges.

8. Determine the convergence of the series $\sum_{n=1}^{\infty} \frac{3^n}{n!}$

Solution.

Use the ratio test let $a_n = \frac{3^n}{n!}$

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \frac{3^{n+1}/(n+1)!}{3^n/n!} = \frac{3 \cdot 3^n/(n+1) \cdot n!}{3^n/n!}$$

$$= \lim_{n \to \infty} \frac{3}{n+1} = 0$$

Since the limit is less than 1, the series $\sum_{n=1}^{\infty} \frac{3^n}{n!}$ converge by the ratio test.

9. Determine if the series converges absolutely, conditionally or diverges $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

Solution

(i) Absolute Convergence test $\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{n} \right| = \sum_{n=1}^{\infty} \frac{1}{n}$

The series $\sum \frac{1}{n}$ is the harmonic series, which diverges. Therefore the series does not converge absolutely.

(ii) Conditional Convergence Test.

We apply the alternating series test (leibniz's test)

$a_n = \frac{1}{n}$ which is positive, decreasing and

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$$\lim_{n\to\infty} \frac{1}{n} = 0.$$

Therefore, by the alternating series test

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges.

Since the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges but not

absolutely, it converges conditionally.

10. (9) Test the series for absolute and conditional convergence $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$

Solution.

Absolute convergence test $\sum_{n=1}^{\infty} \left| \frac{(-1)^n n}{n^2+1} \right| = \sum_{n=1}^{\infty} \frac{n}{n^2+1}$

We compare the series to $\sum \frac{1}{n}$

$$\frac{n}{n^2+1} - \frac{n}{n^2} = \frac{1}{n}$$

Since $\sum \frac{1}{n}$ diverges (harmonic series), we suspect $\sum \frac{n}{n^2+1}$ diverges as well. To confirm we use the limit comparison test with $b_n = \frac{1}{n}$

$$\lim_{n\to\infty} \frac{n/n^2+1}{1/n} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$$

Since the limit is a positive finite number and

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$\sum \frac{1}{n}$, diverges $\sum \frac{n}{n^2+1}$ also diverges.

- Conditional Convergence Test.
- We apply the alternating series test (Leibniz's test)

$a_n = \frac{n}{n^2+1}$, we check if a_n is decreasing and

$$\lim_{n \to \infty} a_n = 0$$

- $a_n = \frac{n}{n^2+1} = \frac{n}{n^2} = \frac{1}{n}$

- a_n is decreasing and $\lim_{n \to \infty} a_n = 0$

Since the series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$ converges by the alternating series test but not absolutely, it converges conditionally.

11. Determine if the series converges absolutely, conditionally or diverges.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p} \quad \text{for } p > 0$$

Solution.

1. Absolute convergence test $\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{n^p} \right| = \sum_{n=1}^{\infty} \frac{1}{n^p}$

This is a p-series. The p-series $\sum \frac{1}{n^p}$ converges if:

- $p > 1$, the series converge absolutely.

1. If ~~p~~ $0 < p \leq 1$ the series diverges

2. Conditional Convergence test

If $0 < p \leq 1$, we apply the alternating series test

$a_n = \frac{1}{n^p}$, which is positive, decreasing and

$$\lim_{n \to \infty} \frac{1}{n^p} = 0$$

Therefore, if $0 < p \leq 1$, the series converges conditionally by the alternating series test

i.e For $p > 1$: the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converges

- For $0 < p \leq 1$: the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converges conditionally.

(2) Examine the series for conditional convergence or absolute convergence

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$a_n = \frac{1}{n^p}$, which is positive, decreasing and $\lim_{n\to\infty} \frac{1}{n^p} = 0$

Therefore, If $0 < p \leq 1$, the series Converges Conditionally by the alternating series test.

i.e For $p > 1$: the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ Converges absolutely.

- For $0 < p \leq 1$: the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ Converges Conditionally.

(2) Examine the series for Conditional Convergence or absolute Convergence.

① $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)^2}$ ② $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$

Solution

1st Consider the absolute Convergence

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