Exercise

1. For each of the following relations from $A = \{a, b, c, d\}$ to $B = \{1, 2, 3, 4, 5\}$, determine whether or not it is a function from A to B. Whenever your answer is negative, give your reason.

(i). $\{(a, 1), (b, 2), (c, 3)\}$

(ii). $\{(a, 1), (b, 2), (c, 3), (d,4), (d, 5)\}$

(iii). $\{(a, 1), (b, 2), (c, 3), (d, 5)\}$

(iv). $\{(a, 1), (b, 2), (c, 2), (d, 1)\}$

(v). $\{(a, 5), (b, 5), (c, 5), (d, 5)\}$.

2. Identify the domain and range of the three functions in question 1, calling them f, g, h.

3. Can the domain of a function ever be empty? And the range?

4. Draw a diagram to illustrate the non-commutativity of a composite function.

5. Give an example of familiar functions $f, g: N \to N$ such that $f \circ g \neq g \circ f$.

6. Show that we can generalize the definition of composition a little, in the sense that the composition $g \circ f: A \to C$ is a function whenever $f: A \to B$ and $g: B' \to C$ and $B \subseteq B'$. Give an example to show that $g \circ f$ may not be a function when, conversely, $B' \subseteq B$.

7. Let $A = \{1, 2, 3\}$ and $B = \{a, b, c, d\}$. Let $f = \{(1, a), (2, a), (3, b)\}$ and $g = \{(1, a), (2, b), (3, c)\}$.

(i) Explain why neither $f^{-1} = \{(a, 1), (a, 2), (b, 3)\}$ nor $g^{-1} = \{(a, 1), (b, 2), (c, 3)\}$ is a function from B to A.

(ii). Draw a diagram of the example.