



Fig. 1.8: Venn diagram of subjects

Step 2: From the Venn diagram above, we have:

No. of students who had taken only math = 24,

No. of students who had taken only chemistry = 60,

No. of students who had taken only physics = 22.

Step 3: Total no. of students who had taken only one course: = $24 + 60 + 22 = 106$.

Hence, the total number of students who had taken only one course is 106.

Exercises

Use \in or \notin to indicate whether the given object is an element of the given set in the following problems.

12 $\{1, 2, 3, 4, \dots\}$

5 $\{x: x \text{ is a natural number greater than } 5\}$

6 $\{x: x \text{ is a natural number less than } 6\}$

3 \emptyset

Problems 5–8, write the following sets a second way.

$\{x: x \text{ is a natural number less than } 8\}$

$\{x: x \text{ is a natural number greater than } 6, \text{ less than } 10\}$

$\{3, 4, 5, 6, 7\}$ 8. $\{7, 8, 9, 10, \dots\}$

In Problems 9 and 10, which of \emptyset , A, and B are subsets of B?

9. $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$

10. $A = \{a, b, c, d\}$ and $B = \{c, d, a, b\}$?

11. Is $A \subseteq B$ if $A = \{a, b, c, d\}$ and $B = \{a, b, d\}$?

12. Is $A \subseteq B$ if $A = \{6, 8, 10, 12\}$ and $B = \{6, 8, 10, 14, 18\}$?

Use \subseteq notation to indicate which set is a subset of the other in Problems 13–16.

13. $C = \{a, b, 1, 2, 3\}$ and $D = \{a, b, 1\}$

14. $E = \{x, y, a, b\}$, $F = \{x, 1, a, y, b, 2\}$

15. $A = \{6, 8, 7, 4\}$, $B = \{8, 7, 6, 4\}$

16. $D = \{a, e, 1, 3, c\}$, $F = \{e, a, c, 1, 3\}$

In Problems 17–20, indicate whether the following pairs of sets are equal.

17. $A = \{a, b, \pi, \sqrt{3}\}$, $B = \{a, \pi, \sqrt{3}, b\}$

18. $A = \{x, g, a, b\}$, $D = \{x, a, b, y\}$

19. $D = \{x: x \text{ is a natural number less than } 4\}$, $E = \{1, 2, 3, 4\}$

20. $F = \{x: x \text{ is a natural number greater than } 6\}$, $G = \{7, 8, 9, \dots\}$

21. From the following list of sets, indicate which pairs of sets are disjoint.

$A = \{1, 2, 3, 4\}$

$= \{x: x \text{ is a natural number greater than } 4\}$

$= \{4, 5, 6, \dots\}$

$D = \{1, 2, 3\}$

22. If A and B are disjoint sets, what does $A \cap B$ equal?

In Problems 23–26, find $A \cap B$, the intersection of sets A and B .

23. $A = \{2, 3, 4, 5, 6\}$ and $B = \{4, 6, 8, 10, 12\}$

24. $A = \{a, b, c, d, e\}$ and $B = \{a, d, e, f, g, h\}$

25. $A = \emptyset$ and $B = \{x, y, a, b\}$

26. $A = \{x: x \text{ is a natural number less than } 4\}$ and $B = \{3, 4, 5, 6\}$

In Problems 27–30, find $A \cup B$, the union of sets A and B .

27. $A = \{1, 2, 4, 5\}$ and $B = \{2, 3, 4, 5\}$

28. $A = \{a, e, i, o, u\}$ and $B = \{a, b, c, d\}$

29. $A = \emptyset$ and $B = \{1, 2, 3, 4\}$

30. $A = \{x: x \text{ is a natural number greater than } 5\}$ and $B = \{x: x \text{ is a natural number less than } 5\}$

In Problems 31–42, assume that $A = \{1, 5, 8, 7, 2\}$, $B = \{4, 3, 8, 10\}$, $C = \{2, 4, 6, 10\}$ and that U is the universal set of natural numbers less than 11. Find the following.

31. A^c

32. B^c

33. $A \cap B^c$

34. $A^c \cap B^c$

35. $(A \cup B)^c$

36. $(A \cap B)^c$

37. $A^c \cup B^c$

38. $(A^c \cup B)^c$

39. $(A \cap B') \cup C'$

40. $A \cap (B' \cup C')$

41. $(A \cap B')' \cap C$

42. $A \cap (B \cup C)$

The difference of two sets, $A - B$, is defined as the set containing all elements of A except those in B . That is, $A - B = A \cap B^c$.

Find $A - B$ for each pair of sets in Problem 43–46 if $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

43. $A = \{1, 3, 7, 9\}$ and $B = \{3, 5, 8, 9\}$

44. $A = \{1, 2, 3, 6, 9\}$ and $B = \{1, 4, 5, 6, 7\}$

45. $A = \{2, 1, 5\}$ and $B = \{1, 2, 3, 4, 5, 6\}$

46. $A = \{1, 2, 3, 4, 5\}$ and $B = \{7, 8, 9\}$

47. Out of 120 students in a school, 5% can play Cricket, Chess and Carroms. If it happens that the number of players who

- can play any and only two games are 30. The number of students who can play Cricket alone is 40. What is the total number of those who can play Chess alone or Carroms alone?
8. Draw the diagram that best represents the relationship among the given classes: Animal, Tiger, Vehicle, Car
9. At an overpriced department store, there are 112 customers. If 43 have purchased shirts, 57 have purchased pants, and 38 have purchased neither, how many purchased both shirts and pants?
10. In a group, 25 people like tea or coffee; of these, 15 like tea and 6 like coffee and tea. How many like coffee?
11. An advertising agency finds that, of its 170 clients, 115 use Television, 110 use Radio and 130 use Magazines. Also 85 use Television and Magazines, 75 use Television and Radio, 95 use Radio and Magazines, 70 use all the three. Draw Venn diagram to represent these data. Find: (i) how many use only Radio? (ii) how many use only Television? (iii) how many use Television and Magazine but not radio?
12. In a town 85% of the people speak Tamil, 40% speak English and 20% speak Hindi. Also 32% speak Tamil and English, 13% speak Tamil and Hindi and 10% speak English and Hindi, find the percentage of people who can speak all the three languages.
- In a group of students, 65 play football, 45 play hockey, 42 play cricket, 20 play football and hockey, 25 play football and cricket, 15 play hockey and cricket and 8 play all the three games. Find the total number of students in the group (Assume that each student in the group plays at least one game).
- A group of 62 students were surveyed, and it was found that each of the students surveyed liked at least one of the following three fruits: apricots, bananas, and cantaloupes.
- 34 liked apricots.
30 liked bananas.
33 liked cantaloupes.
11 liked apricots and bananas.
15 liked bananas and cantaloupes.
17 liked apricots and cantaloupes.
19 liked exactly two of the following fruits: apricots, bananas, and cantaloupes.
- a. How many students liked apricots, but not bananas or cantaloupes?
b. How many students liked cantaloupes, but not bananas or apricots?
c. How many students liked all of the following three fruits: apricots, bananas, and cantaloupes?
d. How many students liked apricots and cantaloupes, but not bananas?
55. 150 college freshmen were interviewed. 85 were registered for a Math class, 70 were registered for an English class, 50 were registered for both Math and English.
- (a) How many signed up only for a Math Class?
(b) How many signed up only for an English Class?
(c) How many signed up for Math or English?
(d) How many signed up neither for Math nor English?
56. 100 students were interviewed. 28 took PE, 31 took BIO, 42 took ENG, 9 took PE and BIO, 10 took PE and ENG, 6 took BIO and ENG, 4 took all three subjects.
- (a) How many students took none of the three subjects?
(b) How many students took PE but not BIO or ENG?
(c) How many students took BIO and but not ENG?
57. Given $U = \{1, 2, 3, 4, 5, 6, 7, 8, 10\}$, $X = \{1, 2, 6, 7\}$ and $Y = \{1, 3, 4, 5, 8\}$.

Find $X \cup Y$ and draw a Venn diagram to illustrate $X \cup Y$.

58. Given $U = \{1, 2, 3, 4, 5, 6, 7, 8, 10\}$, $X = \{1, 6, 9\}$ and $Y = \{1, 3, 5, 6, 8, 9\}$

Find $X \cup Y$ and draw a Venn diagram to illustrate $X \cup Y$.

59. Given: $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $X = \{1, 2, 6, 7\}$ and $Y = \{1, 3, 4, 5, 8\}$

(a) Draw a Venn diagram to illustrate $(X \cup Y)^c$. (b) Find $(X \cup Y)^c$.

60. If $D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ with subsets A, B and C where $A = \{4, 6, 8\}$ and $B = \{6, 7, 8, 9\}$ and $C = \{1, 2, 3, 4\}$, find the following: (a). $A \cap B$ (b). $B \cap C$ (c). $A \cup B$ (d). $B \cup C$ (e). $(A \cup B \cup C)^c$.

61. Given two sets: $A = \{1, 4, 8, 9\}$, and $B = \{3, 4, 9\}$. Write down the intersection of the sets.

Given that the sets A and B are $A = \{3, 6, 9, 10\}$, and $B = \{1, 3, 5, 8, 10\}$. Prove that the commutative property of intersection holds for them.

Prove that the associativity property of intersection holds for the following sets: $A = \{1, 5, 8\}$, $B = \{2, 5, 8, 9\}$, $C = \{1, 8, 9\}$.

64. Let A and B be two finite sets such that $n(A) = 20$, $n(B) = 28$ and $n(A \cup B) = 30$. Find $n(A \cap B)$.

65. If $n(A - B) = 18$, $n(A \cup B) = 70$ and $n(A \cap B) = 25$, then find $n(B)$.

66. In a group of 60 people, 27 like cold drinks and 42 like hot drinks and each person likes at least one of the two drinks. How many like both coffee and tea?

67. There are 35 students in art class and 57 students in dance class. Find the number of students who are either in art class or in dance class.

(i). When two classes meet at different hours and 12 students are enrolled in both activities.

(ii). When two classes meet at the same hour.

68. In a group of 100 persons, 72 people can speak English and 43 can speak French. How many can speak English only? How many can speak French only and how many can speak both English and French?