STA 211 (MTH 219)

Probability Distribution Theory. I

1. Random Variable
2. The mean & variance of a random variable.
3. Kinds of probability distribution in both discrete and continuous variable.
4. Moments and moment generating function

1. A random variable is a numerical outcome of a random process. It's also a variable that takes on different numerical values according to

chance mechanism. It is usually denoted by capital letters such as x and y.

Example.

Suppose that a coin is tossed twice, so that the sample space : $S = \{HH, HT, TH, TT\}$. Let X represent the number of S that can come up. With each point, we can associate a number for X as shown below. Thus for example in the case of HH let X = 2, while in the case of HT and TH, let X = 1. It follows that X is a random variable.

Table 1

Sample Points	HH	HT	TH	TT
X	2	1	1	0

A random variable that takes on a finite or countably infinite ~~variable~~ num of valu is called a discrete random variable. While one which takes on a non-countable infinite number of values is called a continuous random variable.

2. Expected value (Mean) and Variance of A Random Variable: The expected value of a random variable is the mean or average value of a random

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variable over the population on which the random variable is defined. For a random variable X, its expected value is usually denoted by $E(x)$ or μ_x or simply μ. In the case of discrete random variable, the mean is given as $\mu = E(X) = \sum_{i=1}^{n} (x_i\, p(x_i))$ . . . *

where $p(x) = Pr(X=x)$ which is the probability or the probability mass function of the random variable while continuous random variable is determined by computing the expression

$$\mu = E(x) = \int_{-\infty}^{\infty} x f(x)\, dx \quad \cdots \quad (**)$$

lower bound of x

Where $f(x) = P(X=x)$ where x is any number. Which is the probability density function of the random variable. Essentially the expected value is a weighted average in which all possible values of the random variable is weighted by its probability. Mathematically $x \cdot P(x)$. The probability function $p(X)$ or $f(X)$ must satisfy the following conditions

(1) $P(x_i) \geq 0 \quad \forall i$

(2) $\sum_{i=1}^{\infty} P(x_i) = 1$

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(3) $0 \leq P(x_i) \leq 1$

The variance of the discrete random variable ~~X~~ is denoted by

$$\sigma_x^2 = \sum_{i=1}^{n} (x_i - \mu_x)^2 P(X = x) \quad \dots \text{(xx)}$$

While for continuous random variable is given as

$$\sigma_x^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 f(x)$$

The variance of x is also denoted by $\Sigma(x)$ $V(x)$ or simply σ^2. The standard deviation is the square root of the variance σ_x

Example: Find the mean, the variance and the standard deviation of x when x denotes the number that shows up when a fair die is rolled.

Solution.

x is a random variable that has the numerical outcome of the fair die rolled.

$X = \{1, 2, 3, 4, 5, 6\}$

The probability function of x and the necessary calculations are summarized in table 2 below.

Table.

x	P(x)	x P(x)	$x^2 P(x)$
1	1/6	1/6	1/6

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2	1/6	2/6	4/6
3	1/6	3/6	9/6
4	1/6	4/6	16/6
5	1/6	5/6	25/6
6	1/6	6/6	36/6

$$\mu = \sum_{i=1}^{N} x_i p(x_i) = \frac{21}{6} = 3.5$$

$$\sigma^2 = \sum x^2 p(x) - u^2$$

$$= \frac{91}{6} - \left(\frac{21}{6}\right)^2$$

$$= 2.92$$

$$Sd = \sqrt{\sigma^2} = \sqrt{2.92} = 1.71$$

Hence Mean = 3.5, Variance = 2.92 and Standard deviation = 1.71

Given $f(x) = 1.5x^2$ for $-1 < x < 1$.

Find the mean & variance of x.

Solution

$$E(x) = \int_{-1}^{1} x(1.5x^2)\,dx = \int_{-1}^{1} 1.5x^3\,dx$$

$$= \left.\frac{1.5x^4}{4}\right|_{-1}^{1} = 0$$

$$V(x) = \int_{-1}^{1} 1.5x^2(x-0)^2\,dx$$

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$$= 1.5 \int_{-1}^{1} x^4 \, dx = 1.5 \left. \frac{x^5}{5} \right|_{-1}^{1}$$

$$V(x) = 0.6$$

Probability Distribution

A probability distribution is a model which describes a specific kind of random process.