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Question.............................

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Soln

1. Absolute convergence test

$$\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{n^p}\right| = \sum_{n=1}^{\infty} \frac{1}{n^p}$$

This is a p-series. The p-series $\sum \frac{1}{n^p}$ converges if and only if $p>1$.

- If $p>1$, the series converged absolutely
- If $0<p\le 1$ the series diverges

2. Conditional Convergence Test

- If $0<p\le1$, we apply the alternating Series Test:

$a_n = \frac{1}{n^p}$, which is positive, decreasing

and $\lim_{n\to\infty} \frac{1}{n^p} = 0$

Therefore, if $0<p\le 1$, the series converges conditionally by the Alternating series Test.

1d. For $p>1$: the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^p}$ converges absolutely.

- For $0<p\le1$: the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^p}$ converges conditionally.

(2) examine the series for conditional convergence or absolute convergence

(1) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)^2}$ (2) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$

$p \ge 0$

Soln

1st consider the absolute convergence series

1st $\sum_{n=1}^{\infty} \left|\frac{(-1)^{n+1}}{(n+1)^2}\right| = \sum_{n=1}^{\infty}\frac{1}{(n+1)^2}$

now determine whether this series converges, NB that this is essentially a p-series with $p=2$

$$\sum_{n=1}^{\infty}\frac{1}{(n+1)^2}$$

the p-series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges if $p>1$

Here, $p=2$ which is greater than 1

$\therefore$ the series $\sum_{n=1}^{\infty}\frac{1}{(n+1)^2}$ converges

Since the series converges absolutely.

(2) Soln

also, 1st consider the absolute convergence of the series

1st $\sum_{n=1}^{\infty} \left|\frac{(-1)^{n+1}}{n\sqrt{n}}\right| = \sum_{n=1}^{\infty}\frac{1}{n\sqrt{n}}$

rewrite $\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$

This is a p-series with $p=3/2$. The p-series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges if $p>1$

Here, $p=3/2$ which is greater than 1.

$\therefore$ the series $\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}$ converges absolutely

$p \ge 0$

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(3) $\sum_{n=2}^{\infty} \frac{(-1)^n n}{n^3-1}$

Soln

1st consider the absolute convergence test

$\therefore \sum_{n=2}^{\infty} \left|\frac{(-1)^n n}{n^3-1}\right| = \sum_{n=2}^{\infty} \frac{n}{n^3-1}$

For large n, the term n^3-1 is approximately n^3. $\therefore$ we can approximate the term as

$\frac{n}{n^3-1} \approx \frac{n}{n^3} = \frac{1}{n^2}$,

the series $\sum_{n=2}^{\infty} \frac{1}{n^2}$ is a p-series with $p=2$. A p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if $p>1$

Since $p=2>1$, the series $\sum_{n=2}^{\infty} \frac{1}{n^2}$ converges

We can also use the comparison test with $b_n = \frac{1}{n^2}$ (limit comparison)

$a_n = \frac{n}{n^3-1}$, $b = \frac{1}{n^2}$

we compute the limit

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n}{n^3-1}}{\frac{1}{n^2}}$

p.t.o

$= \lim_{n\to\infty} \frac{n^3}{n^3-1} = \lim_{n\to\infty} \frac{n^3}{n^3(1-\frac{1}{n^3})} =$

$\lim_{n\to\infty} \frac{1}{1-\frac{1}{n^3}} = 1$

Since the limit is a positive finite number and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, the series $\sum_{n=2}^{\infty} \frac{n}{n^3-1}$ also converges

Therefore, the series $\sum_{n=2}^{\infty} \left|\frac{(-1)^n n}{n^3-1}\right|$ converges. the original series converges absolutely.

(4) Determine if the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ is conditionally convergent

Soln ~~check for convergence~~
check using the Alternating Series Test

the series is $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$

• Alternating form: the series is ~~in~~ the form $\sum (-1)^{n+1} a_n$ where $a_n = \frac{1}{n}$,

• monotonicity: $a_n = \frac{1}{n}$ is a decreasing sequence since $\frac{1}{n+1} < \frac{1}{n}$ for all n.

• limit: $\lim_{n\to\infty} \frac{1}{n} = 0$

Since all conditions of alternating series Test are met, the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ converges

p.t.o

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(ii) Check for absolute convergence

Consider the series

$$\sum_{n=1}^{\infty} \left|(-1)^{n+1}\frac{1}{n}\right| = \sum_{n=1}^{\infty}\frac{1}{n},$$

this is harmonic series which is known to diverge

Since the $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges the original series does not converge absolutely.

$\therefore$ the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$ is conditionally convergent.

(5) Determine if the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ is conditionally convergent

Soln

using the alternating series test

the series $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\sqrt{n}}$

Alternating Form: the series is in the form $\sum(-1)^n a_n$ where

$a_n = \frac{1}{\sqrt{n}}$

monotonicity: $a_n = \frac{1}{\sqrt{n}}$ is a decreasing sequence since

$\frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}}$ converges

(ii) Check for absolute convergence

Consider the series

$$\sum_{n=1}^{\infty}\left|(-1)^n\frac{1}{\sqrt{n}}\right| = \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$$

use the integral test to determine if this series converges

$$\int_1^{\infty}\frac{1}{\sqrt{x}}dx = 2\sqrt{x}\Big|_1^{\infty} = 2\sqrt{\infty}$$

The integral diverges, indicating that the series $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$

we conclude that the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ is conditionally convergent.

(6) Determine if the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}$ is conditionally convergent

Soln

using Alternating Series test: At the form $\sum(-1)^{n+1}a_n$ where $a_n = \frac{1}{n^2}$

monotonicity: $a_n = \frac{1}{n^2}$ is a decreasing sequence since

$\frac{1}{(n+1)^2} < \frac{1}{n^2}$ for all n.

limit: $\lim_{n\to\infty}\frac{1}{n^2} = 0$

Since the condition of alternating series are met, the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^2}$ converges

(ii) check for absolute convergence

Consider

$$\sum_{n=1}^{\infty}\left|(-1)^{n+1}\frac{1}{n^2}\right| = \sum_{n=1}^{\infty}\frac{1}{n^2}$$

PTO

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: this is p-series with $p=2$ which converges because $p>1$

in conclusion $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ converges absolutely and therefore

it is not conditionally convergent.