MTH 210 (201) Elementary Algebra and Analysis. 14th May 2024

Prop 1.3: For any set B, we have that 1.3.1: $A = (A \cap B) \cup (A \setminus B)$ holds for each set A.

Note $A \setminus B$ is called the complement of B in A.

Proof (of prop 1.3): Note if $x \in A \cap B$ then, by definition, x belongs to both A and B, hence $A \cap B$ is a subset of A. By def, if $x \in A \setminus B$ then x belongs to A but $x \notin B$, hence $A \setminus B$ is a subset of A. Thus $(A \cap B) \cup (A \setminus B)$ is a subset of A. So, the inclusion 1.3.2 $(A \cap B) \cup (A \setminus B) \subset A$ holds. It remains to prove that the reverse inclusion holds in 1.3.2. If $x \in A$ then either $x \in B$, (as B is a set). If $x \in B$ then x belongs to $A \cap B$, (since x belongs to A). On the other hand, if $x \notin B$ then $x \in A \setminus B$, (as $x \in A$). Thus, x belongs to the union of the two sets $A \cap B$ and $A \setminus B$. Hence A is a subset of $(A \cap B) \cup (A \setminus B)$. So, A equals $(A \cap B) \cup (A \setminus B)$.

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The proof of prop1.3 is now complete.
Note that the sets $A \cap B$ and $A \setminus B$ are disjoint: that is, their intersection is the empty set.

Exercise 1: Prove the De Morgan's Laws that for all sets A, B

(i) $(A \cup B)^c = A^c \cap B^c$

(ii) $(A \cap B)^c = A^c \cup B^c$

2. Prove that:

(i) for any set A, $(A^c)^c = A$

(ii) for any collection (or family) $\mathcal{A} = \{A_\alpha\}_{\alpha \in J}$ of sets A_α's indexed by a set J, the following equalities hold:

$$\left(\bigcup_{\alpha \in J} A_\alpha\right)^c = \bigcap_{\alpha \in J} A_\alpha^c$$

and

$$\left(\bigcap_{\alpha \in J} A_\alpha\right)^c = \bigcup_{\alpha \in J} A_\alpha^c$$

Def 1.4: For any sets A and B, the Cartesian product of A with B, denoted $A \times B$ (read as A cross B), is defined as the set of all

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ordered pairs (a, b) where $a \in A$ and $b \in B$ with equality of pairs defined by 1.4.1 : $(a, b) = (a', b')$ when and only when $a = a'$ and $b = b'$ both hold.

Remark: The Cartesian product of n number of sets $A_1, \ldots, A_n$ in that order (i.e. A_1 with A_2 with A_{n-1} with A_n), denoted by the symbol $A_1 \times A_2 \times \cdots \times A_{n-1} \times A_n$ is defined as the set of all ordered n-tuples $(x_1, x_2, \cdots, x_{n-1}, x_n)$ where $x_1 \in A_1$, $x_2 \in A_2$, $\ldots$, $x_{n-1} \in A_{n-1}$, and $x_n \in A_n$ with equality of n-tuples defined by 1.4.2 : $(x_1, x_2, \cdots x_{n-1}, x_n) = (x_1', x_2', \ldots, x_{n-1}', x_n')$ when, and only when, $x_j = x_j'$ $\forall j$, $(j = 1, 2, \ldots, n)$.
Finally, if the A_j's are all equal, say to A, then we shall have their Cartesian product as A^n

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