23rd May 2024 Favour Adewale

MTH 218

TOPIC

Integration is a fundermental concept in caculus that allows us to find the area under curve, calculated accumilated quantity and solva wild range of real life problems. There are several techniques ~~for~~ performing integration, each suited to different types of functions and situations

Power Rule

$$\int x^n \, dx = \frac{1}{n+1} (n+1) + C ; \; n \neq -1$$

Sum Rule.

$$\int (f(x) + g(x)) \, dx = \int f(x) \, dx + g(x) \, dx$$

~~Constant~~ Multiple Rule.

$$\int [(f(x)) \, dx] \, dx = \underbrace{C}_{\text{constant}} \int f(x) \, dx$$

where C is a ~~constant~~ constant

$F(x)$ and $f(x)$ are two functions of x such that $\frac{dF(x)}{dx} = f(x)$ then $F(x)$ is said to be an indefinate integral of small $f(x)$ and this can be written as $F(x) = \int f(x)\,dx + c$

This occures when $f(x)$ exist.

C must be included when evaluating an indeffinite integer.

We have so many integration rules some of them are

1) integration by Part
2) Partial fraction integration techniques.
3) integration by substitution
4) Reduction ~~method~~ formular method.

1 integration by Part:

This is a technique of integrating the product of two functions. It is based by $\int u\,dv = uv - \int v\,du$.

where L – logarithmic-function
I - Inverse trigonometric function
A - Algebric function.
T - Trigonometric function
E - Expodential function.

Steps for using Integration by Parts:

1 Choose 'u' and 'dv': Select parts of the integral to assine as 'u' and 'dv'. you choose u in such a way that its derivative ~~is~~ is simpler than itself.

2) Differentiate and integrate. calculate 'du' which is the derivative of 'u' and 'v' which is the integral of 'dv'.

3) Apply the integration by Part formular.

4) Evaluate the integral

5) simplify and solve

2) Partial fraction integration techniques: This involves breaking down a complex fraction into simplier fractions and then integrating each term seperately.

Steps involved.

1. Factor the denominator.
2. Decompose the fraction.
3. Multiply both sides to eleminate the denominator.
4) Distribute.
5. Combine like terms together.
6. set your Co-efficients.

using Partial fraction method

Example:

$\frac{3x+4}{x^2+5x+6}$ sol

$\frac{3x+4}{x^2+5x+6} = \frac{A}{x+3} + \frac{B}{x+2}$

$= A(x+2) + B(x+3)$

$3x+4 = Ax+2A + Bx+3B$

$3x+4 = Ax + Bx + 2A + 3B$

$5 = A+B$

$4 = 2A+3B$

$3(-2)+4 = A(-2+2)$
$B(-2+3) = -6+4 = B$
$2 = B,\ B = -2$

If $x = -3$

$3(-3)+4 = A(-3+2)+B(-3+3)$

$-9+4 = -A$

$5 = -A \therefore A = 5$

$\frac{5}{x+3} + \left(\frac{-2}{x+2}\right)$

$\frac{5}{x+3} - \frac{2}{x+2}$

2) $\int \frac{2x+3}{(x+1)(x+2)(x-3)}$

sol $\int \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3}$

$2x+3 = A(x+2)(x-3) + B(x+1)(x-3) + C(x+1)(x-2)$

$2x+3 = A(x+2)(x-3)$

$2(-1)+3 = A(-1+2)(-1-3)$

$-2+3 = A(-4)$

$1 = -4A$

$A = -1/4$

when $x = -2$

$-1 = 0 + B(-1)(-5)$

$-1 = 5B \therefore B = -1/5$

when $x = 3$

$6+3 = 0+0+C(4)(5)$

$9 = 0+0+20C$

$9 = 20C \quad C = 9/20$

$$= \int \frac{-1}{4(x+1)} - \frac{1}{5(x+2)} + \frac{9}{20(x-3)}$$

$$= \frac{-1}{4} \ln(x+1) - \frac{1}{5} \ln(x+2)$$

$$= -\frac{1}{4} \ln(x+1) - \frac{1}{5} \ln(x+2)$$

$$+ \frac{9}{20} \ln(x-3)$$

3) Evaluate

$$\int \frac{2x+3}{(x-1)(x-2)(x-3)}$$

~~Solution~~. Solution

$$\int \frac{2x+3}{(x-1)(x-2)(x-3)}$$

4) Reduction formular method.

This is used in integration for working out integrals ~~and~~ of higher order.

Reduction formular

A) for basic exponential expression

(i) $\int x^n e^{mx} dx = \frac{1}{m} x^n e^{mx} - \frac{n}{m} \int x^{n-1} e^{mx} dx.$

(ii) for logarithmic expression.

$$\int \log^n x \, dx = x \log^n x - n \int \log^{n-1} x \, dx$$

$$\int x^n \log^m x \, dx = \frac{x^{n+1}}{n+1} \log^m x$$

$$= \frac{m}{n+1} \int x^n \log^{m-1} x \, dx.$$

C) Algebraic expression

$$\int \frac{x^n}{ax^n+b}dx = \frac{x}{a} - \frac{b}{a}\int \frac{1}{ax^n+b}dx$$

D) Trigonometric functions:

i) $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x \cdot \cos x + \frac{n-1}{n}\int \sin^{(n-2)}x\,dx$

ii) $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x \cdot \sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx$

iii) $\int \sin^n x \cos^m x\,dx = \frac{\sin^{n+1}x \cos^{m-1}x}{n+m} + \frac{m-1}{n+m}\int \sin^n x \cdot \cos^{m-2}x\,dx$

iv) $\int \tan^n x\,dx = \frac{1}{n-1}\tan^{n-1}x - \int \tan^{n-2}x\,dx.$

Example:

Find the integral of $\sin^5 x$ Sol

$\int \sin^5 x = -\frac{1}{5}\sin^{5-1}x \cdot \cos x + \frac{5-1}{5}\int \sin^{(5-2)}x\,dx$

$= -\frac{1}{5}\sin^4 x \cdot \cos x + \frac{4}{5}\left(\int \sin^3 x\,dx\right)$

Recall $\tan x = \sec^2 x$

$\sec^2 x - \tan^2 x = 1.$

$\cos^2 x + \sin^2 x = 1$

$\sin^3 x =$ $\sin^2 x = 1 - \cos^2 x$

$-\frac{1}{3}\sin^2 x \cos x + \frac{2}{3}\int \sin x\,dx$

$= -\frac{1}{3}\sin^2 x \cos x + \frac{2}{3}(-\cos x)\cdot$

$dx + c$

$\therefore \sin^3 x = -\frac{1}{3}\sin^2 x \cos x - \frac{2}{3}\cos x + c.$

$= \int \sin^5 x = -\frac{1}{5}\sin^4 x \cos x + \frac{4}{5}\left(-\frac{1}{3}\sin^2 x \cos x - \frac{2}{3}\cos x + c\right)$

$\int \sin^5 x = -\frac{1}{5}\sin^4 x \cos x - \frac{4}{15}\sin^2 x \cdot \cos x - \frac{8}{15}\cos x + c$

$-\frac{1}{5}(1-\cos^2 x)^2 \cos x - \frac{4}{15}(1-\cos^2 x)\cos x - \frac{8}{15}\cos x + c.$

$= -\frac{1}{5}(1 - 2\cos^2 x + \cos^4 x)\cos x$

$-\frac{4}{15}(\cos x - \cos^3 x) - \frac{8}{15}\cos x + c$

$= \frac{1}{5}\left(\cos x - 2\cos^3 x + \cos^5 x\right) - \frac{4}{15}$

$\cos x + \frac{4}{15}\cos^3 x - \frac{8}{15}\cos x + C$

$= -\frac{1}{5}\cos x + \frac{2}{5}\cos^3 x - \frac{1}{5}\cos^5 x$

$-\frac{4}{15}\cos^3 x - \frac{8}{15}\cos x + C$

$\cancel{-\frac{1}{5}\sin^4 x \cos x - \frac{4}{15}\sin^2 x \cos x}$

$\cancel{-\frac{8}{15}\cos x} \quad C.$

$\int \sin^5 x = -\frac{1}{5}\cos^5 x + \frac{2}{5}\cos^3 x +$

$\frac{4}{15}\cos^3 x - \frac{1}{5}\cos x - \frac{8}{15}\cos x + C$

$= -\frac{1}{5}\cos^5 x + \frac{2}{3}\cos^3 x - 1\cos x + C$

$\therefore \int \sin^5 x = \frac{1}{5}\cos^5 x + \frac{2}{3}\cos^3 x -$

$\cos x + C.$

Example (2)

find

$\cos^5 x$ Sol

$\int \cos^5 x\,dx = \frac{1}{5}\cos^{5-1} x \sin x + \frac{5-1}{5}$

$\int \cos^{5-2} x\,dx.$

$= \frac{1}{5}\cos^4 x \sin x + \frac{4}{5}$

$\left(\int \cos^3 x\,dx\right)$

$\cos^3 x = \frac{1}{3}\cos^2 x \sin x +$

$\frac{2}{3}\int \cos x\,dx$

$= \frac{1}{3}\cos^2 x \sin x + \frac{2}{3}($

$\sin x)\,dx + C$

$\therefore \cos^3 x = \frac{1}{3}\cos^2 x \sin x$

$\frac{2}{3}\sin x + C$

$= \int \sin^5 x = \frac{1}{5}\cos^4 x \sin x$

$+ \frac{4}{5}\left(\frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\right.$

$\left.\sin x + C\right)$

$\cancel{= \frac{1}{5}\cos^4 x \sin x + \frac{4}{15}}$

$\cancel{\sin^2 x + \frac{8}{15}\sin x}$

$\cancel{\sin x + \frac{8}{15}\sin x + C}$

$= \frac{1}{5}($

$= \frac{1}{5}\cos^4 x \sin x + \frac{4}{15}\cos^2 x$

$\sin x + \frac{8}{15}\sin x + C$

$= \frac{1}{5}(1-\sin^2 x)\sin x + \frac{4}{15}$

$(1-\sin^2 x)\sin x + \frac{8}{15}\sin x + C$

$= \frac{1}{5}(1-2\sin^2 x + \sin^4 x)\sin x$

$+ \frac{4}{15}\sin x - \frac{4}{15}\sin^3 x + \frac{8}{15}$

$\sin x + C$

$= \frac{1}{5}\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x$

$+ \frac{4}{15}\sin x - \frac{4}{15}\sin^3 x +$

$\frac{8}{15}\sin x + C$

$\therefore \cos^5 x = \frac{1}{5}\sin^5 x - \frac{2}{3}$

$\sin^3 x + \sin x + C.$