May 13th 2024

163. MTH212 - Real Analysis

Limits

DEF: Limit of function f(x) as x tends or approaches c is denoted as

$$\lim_{x \to c} f(x) = L$$

It means that x gets arbitruarily close to c. the valve of f(x) gets arbitruarily close to L.

Properties of Limit

1. A limit exhist if and only if, the right hand limit and the left hand limit exhist and are equal.
2. If a limit exist, it is unique.
3. Limit Laws (Rules)

(a) Sum or Differences ±

$$\lim_{x \to c} [f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to x} g(x)$$

(b) Product Law:

$$\lim_{x \to c} [f(x) \cdot g(x)] = \left[\lim_{x \to c} f(x)\right] \cdot \left[\lim_{x \to c} g(x)\right]$$

(c) Quotient Law:

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} \left(\text{if } \lim_{x \to c} g(x) \neq 0\right)$$

(d) Scalar Multiple Law:

$$\lim_{x \to c} [b(f(x))] = b\left[\lim_{x \to c} f(x)\right]$$

(e) Power Law:

$$\lim_{x \to c} [f(x)]^n = \left[\lim_{x \to c} f(a)\right]$$

1. Simply substitute C into the function if it does not result in an undefined epression.
2. Factorizing and Cancelly: Factorize and simplify to eliminate common terms.

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3. Multiply by Conjugate to remove radicals from the denominator
4. Special Trigonometric Limits: these utilize the trigonometric identities to simplify ~~the~~ expressions involving trigonometric functions.

Types of Limit
1. Finite Limit: The limit exist and is in real numbers
2. Infinite Limit: The limit tends to positive or negative infinity
3. Infinity Limit (Limit at Infinity): The behaviour of the function [f(x)] as x approaches infinity.
4. One-Sided Limit: The one sided limit are differentiated as the right hand limit (when the limit approaches from the right) and the left hand limit (when the limit approaches from the left). Where as the other limit are sometimes referred to as two-sided limit.

Left-hand Limit:

$$\lim_{x \to c^-} f(x) = L$$

Right-hand Limit:

$$\lim_{x \to c^+} f(x) = L$$