Leture Notes

Elementary Algebra and Analysis.

Part 1: Set theory, Cartesian Products, Mappings.
Peano's axioms. Construction of integers and rational numbers. Dedekind cuts and real numbers.

Part 2: Enumerable, Non-Inum-erable Sets. Cardinal Numbers.

Part 3: Division Algorithm. Primes. Fundamental Theorem of Arithmetic. G.C.D and L.C.M

Books Recommended

1. A Summary of Modern Algebra by G. Birkhoff and S. Maclane (Macmillan) Chapters I, II, IV, XII, XIII
2. Algebra Vol I by Cohn, P.M. (Wiley) Chapters 1, 2
3. Topics in Algebra by Herstein. I.N. (Wiley) Chapters 1

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Venue: 500 LT 'W2'

DR. I. C. ALUFOHAI (Room 14)

Maths Department.

4. Principles of Mathematical Analysis by Rudin, W. (McGraw-Hill) Chapters 1, 2

5. A Course of pure Mathematics by G. H. (Cambridge Univ. Press) Chapter 1

6. Finite Mathematics - Theory and Problems of - by S. Lipschute (Shaum) Chapters 5, 6, 7, 8

7. A Second Course in Mathematical Analysis by J. C. Burkill and H. B- -urkill (Cambridge Univ. Press) Chapter 1.

8. Algebra by F. Ayres (Schaum)

9. Theory of Numbers by L. Dickson

10. Mathematical Analysis: A modern approach to Advanced Calculus - by T. M. Apostol (Addison - -Wesley) Chapters 1, 2.

Holds: Tuesdays 8am-9am. and Fridays 10am-12am.

Class Begins

Def 1.1: 1. By a set, we mean a well-defined collection of objects.

Note: By well-defined, we mean that if A is a set then given any object x, either x belongs to A or x does not belong to A.

Notation: If A is a set, then we shall write $x \in A$ if x belongs to A, and $x \notin A$ if x does not belong to A.

2. If A, B are sets, and if $x \in A$ implies $x \in B$, then A is said to be a subset of B while B is said to be a superset of A.

We write this in symbols as $A \subset B$ (or, $B \supset A$). In this case,

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We shall also say that B contains A, or that A is contained in B.

3. If X is a fixed set and we are considering its subsets, then X is called the Universal set.

4. The object in a set A are called the points, or elements, or members, of the set A.

5. A set which has no members is called an empty, denoted by the symbol $\emptyset$ (read 'phi') a Greek (small) letter.

6. If X is a Universal set, then the empty set of X (i.e the set which does not contain any point of X) is denoted by $\emptyset_X$ (read as phi subscript x)

7. Two sets A and B are said to be equal if $A \subset B$ and also $B \subset A$ (written in symbols as $A = B$)

i.e A is equal to B or A equals B

Note: (i) An empty set is also called a null set.

(ii) If A is a subset of B, then every point of A belongs to B.

Notation: If there is a point x of A which does not belong to B, then A is not a subset of B. We write this as $A \not\subset B$, and say that A is not contained in B.

8. If A, B are sets, and if $A \subset B$ but $B \not\subset A$, we say that A is a proper set of B.

Notation: Given a set X, we shall use the notation $A = \{a \in X : P(a)\}$ to mean that A is the set of all points x of X for which the property

$P(.)$ holds, that is, A is the set {all points a of X such that $P(a)$ holds}. For instance, the set of all integers x, such that $3 < x < 5$} is the set whose only member is. We write this set as {4}
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9. A set which has only one element is called a singleton set

Some Examples: (i) {all integers x: $\frac{1}{x} = 2$} is the empty set $\emptyset$ of the set $\mathbb{Z}$ of all integers $0, \pm 1, \pm 2, \ldots$

(ii) $\{a, b, c\} = \{a, b, c\}$ and $\{c, a, b\} = \{b, c, a\}$.

(iii) $\{a, b\} \subset \{a, b, c\}$ but $\{a, b\} \neq \{a, b, c\}$.

(iv) $\{a\}$, $\{b\}$, $\emptyset$, $\{c, a\}$ are all proper subsets of $\{a, b, c\}$

Note: The empty set $\emptyset$ is a subset of every set, since the condition $x \in A \Rightarrow x \in B$ is satisfied for $A = \emptyset$ and B any set. The symbol '$\Rightarrow$' means implies.

(ii) $x \in A \Rightarrow x \in B$ means whenever $x \in A$ then x must be in B.

Definitions 1.2:

1. Given a set J, we say that J serves as an index set for the family $= \{A_\alpha\}_{\alpha \in J}$ of sets A_α's, or that the sets A_α's are indexed by J, if for every $\alpha \in J$ there exist a set A_α in the family $\mathcal{A}$ and also that J exhausts all the members of the family $\mathcal{A}$:
read as script a.

2. By the union of the sets A_α's indexed by a set J,

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written in symbols as $\bigcup_{\alpha \in J} A_\alpha$,

we mean the set

$\{x : x \in A_\alpha$ for at least one α in $J\}$.

Thus, $x \in \bigcup_{\alpha \in J} A_\alpha$ if, and only if, (i.e. when, and only when) $x \in A_{\alpha_p}$ for some $\alpha_p \in J$.

8. By the intersection of the sets A_α's indexed by a set J, written in symbols as $\bigcap_{\alpha \in J} A_\alpha$, we mean the set

Hence ':' means 'such that'

$\{x : x \in A_\alpha$ for every α in $J\}$.

Thus, $x \in \bigcap_{\alpha \in J} A_\alpha$ when, and only when, $x \in A_\alpha \ \forall \alpha \in J$.

The symbol '$\forall$' means 'for every' or 'for each'

e.g:

(i) $\bigcup_{n=1}^{\infty} A_n$ = the whose set R' of all rational numbers, where $A_n = \{x \in R' : -n < x < n\}$ for $n = 1, 2, 3, \ldots$

(ii) $\bigcap_{n=1}^{\infty} A_n = \{0\}$, where $A_n = \{x \in R' : -\frac{1}{n} < x < \frac{1}{n}\}$ for $n = 1, 2, 3, \ldots$

(iii) $\bigcap_{n=1}^{\infty} A_n$ = the empty set, where $A_n = \{x \in R' : x \geq n\}$ for $n = 1, 2, 3, \ldots$

(iv) $\bigcap_{n=1}^{\infty} A_n = (-1, 1)$, where $A_n = \{x \in R' : -n < x < n\}$ for $n = 1, 2, 3, \ldots$ and $(-1, 1)$ means the open interval with end-points -1 and 1 i.e. $(-1, 1) = \{x \in R' : -1 < x < 1\}$

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4. Two sets A and B are said to be <u>disjoint</u> if their intersection $A \cap B$ is the empty set $\emptyset$. Here the index set has only two points we can call them 1 and 2 (or α_1 and α_2). So A_{α_1} can be taken as A while A_{α_2} is B in our definition 1.2 part (3).

<u>Note</u>: The union $A \cup B$ of two sets A and B is the set $\{x : x$ belongs to at least one of the sets $A, B\}$ with $J = \{\alpha_1, \alpha_2\}$, $A_{\alpha_1} := A$; $A_{\alpha_2} = B$ in part(2) of Def1.2

5. Given two sets A and B, the difference set, denoted $A \setminus B$, is defined as the set $\{x \in A : x \notin B\}$ while the difference set $B \setminus A$ is the set $\{x \in B : x \notin A\}$.

6. Given two sets A and B, their symmetric difference, denoted $A \Delta B$, is defined as the set: $(A \setminus B) \cup (B \setminus A)$. It is the set of all <u>those</u> points which belong to one or the other of the two sets A and B but not to both.

7. The complement A^c of a set A is defined as the set $\{x : x \notin A\}$. So, if X is the Universal set, then $A^c = X \setminus A$.