**Stat 50, Short Answers to Quiz #2, Spring 2025**

1. (1.5 pts) Consider an experiment that consists of rolling a fair die 10 times with independent rolls. Let $X$ denote the random variable for the number of times a six comes up in this experiment. Determine the distribution of $X$ along with its parameters. Then, compute the mean and standard deviation of $X$.

   **Answer**. Each independent roll of the die is a Bernoulli(0.5) trial. So, $X$ has Bin(10,0.5) distribution. Then, its mean is $\mu = np = 10(1/6) = 5/3$ rolls of six and its standard deviation is $\sigma = \sqrt{np(1-p)} = \sqrt{10(1/6)(5/6)} = \frac{5}{6}\sqrt{2} \simeq 1.1785$ rolls.

1. (1.5 pts) The number of cars that enter into a parking garage during morning hours on Mondays has an approximate Poisson distribution with an average of 2.5 cars per minute. Compute the probability that exactly 10 cars enter into the garage during a period of 5 minutes on a Monday morning. Show your work.

   **Answer**. This is a Poisson process with rate parameter $\lambda = 2.5$ and $t = 10$ min so that $\lambda t = 12.5$ cars, and $X_t \sim Poisson(12.5)$ denotes the random variable for the number of cars that enter the garage in that period. So, we compute $P(X_t = 10) = e^{-12.5}\frac{12.5^{10}}{10!} \simeq 0.09564$.

2. (2.5 pts) A shipment of a certain brand of laptop chargers contains 240 items. Assume that five of the chargers in the shipment are defective and consider a random sample of 10 items to be inspected for defects.

   Determine the probability that there is at most one defective charger in the sample (of 10 chargers). Briefly explain which distribution you used for this computation and why.

   **Answer**. This experiment involves sampling without replacement from a finite population, and hence the number of defective items selected has hypergeometric distribution with parameters $N = 240, R = 5$ and $n = 10$. However, it can also be approximated by a binomial distribution since $n/N \leq 0.05$ (check that $n/N = 10/240 = \frac{1}{24} \simeq 0.0417$).

   We can compute $P(X \leq 1)$ using hypergeometric formula:

$$\begin{aligned} P(X &= 0) + P(X=1) = \frac{\binom{5}{0}\binom{235}{10}}{\binom{240}{10}} + \frac{\binom{5}{1}\binom{235}{9}}{\binom{240}{10}} \\ &\simeq 0.98534. \end{aligned}$$

   Similarly, with $p = R/N = 5/240 = \frac{1}{48}$ using $Y \sim Bin(10, 1/48)$ distribution, we get

$$\begin{aligned} P(Y &= 0) + P(Y=1) = (\frac{47}{48})^{10} + \binom{10}{1}\frac{1}{48}(\frac{47}{48})^{9} \\ &\simeq 0.98252. \end{aligned}$$