Stat 50, Quiz #5 (Optional Quiz), Spring 2025 05/05/2025

Instructions. Write your name below, read these instructions before starting the quiz. You have 30 minutes to complete the three questions below. You may use your z and t tables, formulas on the board and a calculator. Show your work and include units to get full credit. Use at least three decimal points when rounding the final probability expressions and proportions. If you have questions, raise your hand. No talking, disturbing, collaboration or straying eyes during the exam. No cell phones!

NAME:

1. (0.75 pts) Let p denote the actual (unknown) proportion of all cars registered in a state, whose emission levels exceed the acceptable state standards. Consider the following hypotheses about p:

   $$H_0 : p \geq 0.25 \text{ versus } H_1 : p < 0.25.$$

   Based on a random sample of cars in the state, about 20% of the cars tested had emission levels above the state standards, and an upper-bound confidence interval for p at 95% confidence level was computed to be $(0, 0.262]$. Using the evidence from the interval above, can you reasonably reject H_0 and conclude that p is smaller than 0.25? Explain.

2. The average investment amount in the bio-tech industry in a state is believed to be at around 20 million dollars. We want to test if this amount is accurate.

   (a) (0.5 pts) Construct the relevant null and alternative hypotheses of a two-tailed test about the mean investment amount, μ.

   (b) (0.5 pts) In a random sample of 16 bio-tech investments in the state, the sample average was 18.15 million dollars with a sample standard deviation of 2.40 million dollars. Moreover, the sample didn't include any outlier. Explain why a z-test doesn't apply to this scenario.

(c) (1 pt) Does a t-test apply to the sample in part (b)? Justify your answers. If you said "yes", then compute also the value of the corresponding t-statistics. You don't need to estimate a P-value in this problem.

3. Data science experts of an engineering company run a large-scale program for a machine learning application to estimate its average run time (in seconds) on a particular computer. In 81 trials of the simulation study, the average run time was 28.0 seconds with a standard deviation of 4.5 seconds.

(a) (1 pt) The experts wonder if it can be concluded that the average run time of the program is below 30 seconds. To this end, the following left-tailed test is designed:

$H_0 : \mu \geq 30$ versus $H_1 : \mu < 30$.

Briefly explain whether a z-test or a t-test can be applied to this problem, and then determine the relevant test statistic (z_0 or t_0).

(b) (1.5 pts) Using a P-value approach, conduct a suitable hypothesis test at 5% level of significance, and state your conclusion in context. Show your work and justify your steps.