Stat 50, Short Answers to Quiz #3, Spring 2025

1. Let X denote the random variable for the rents (in dollars) of two-bedroom apartmens in a town. If $X \sim \log N(7.8, 0.4^2)$ approximately, answer the following:

   (a) (1 pt) Compute the mean and the standard deviation of the rent amounts for such two-bedroom apartments in the town. You may round the end results to dollar amounts.
   Answer. Mean: $\mu_X = e^{\mu+\frac{1}{2}\sigma^2} = e^{7.8+(0.5)(0.4)^2} = 2643.9 \simeq 2644$ dollars
   Variance: $\sigma_X^2 = e^{2\mu+2\sigma^2} - e^{2\mu+\sigma^2} = e^{2(7.8)+2(0.4)^2} - e^{2(7.8)+(0.4)^2} = 1.2129 \times 10^6$
   Std. dev.: $\sigma_X = \sqrt{1.2129 \times 10^6} = 1101.3 \simeq 1101$ dollars.

   (b) (1.5 pts) Find the probability that a randomly selected two-bedroom apartment in the town is less than 2500 dollars.
   Answer. Since $\ln X \sim N(7.8, 0.4^2)$ and $Z = \frac{\ln X - 7.8}{0.4} \sim N(0,1)$,

$$\begin{aligned} P(X &< 2500) = P(\ln X < \ln 2500) \\ &= P(Z < \frac{\ln 2500 - 7.8}{0.4}) \\ &\simeq P(Z < 0.06) \simeq 0.524. \end{aligned}$$

2. (2.5 pts) Assume that X_1, X_2 and X_3 are independent normally distributed random variables with the same means, $E[X_1] = 10 = E[X_2] = E[X_3]$, and the following variances: $Var(X_1) = 4 = Var(X_3), Var(X_2) = 5$. Consider the following random variable: $W = X_1 + X_2 - 2X_3$.

   (a) (1pt) Explain why W has a normal distribution with parameters $\mu = 0$ and $\sigma^2 = 25$.
   Answer. Since W is a linear combination of three independent normally distributed random variables, W also has a normal distribution. Moreover, its mean is

$$\begin{aligned} E[W] &= E[X_1] + E[X_2] - 2[EX_3] \\ &= 10 + 10 - 20 = 0 \end{aligned}$$

   and its variance is

$$\begin{aligned} Var(W) &= Var(X_1) + Var(X_2) + 4Var(X_3) \\ &= 4 + 5 + 4(4) = 25. \end{aligned}$$

   (b) (1.5 pts) Compute $P(W > 8)$. Show your work.
   Answer. Let $Z = \frac{W}{5}$ which has $N(0,1)$ distribution. Then, using the area table and the complement rule,

$$\begin{aligned} P(W &> 8) = P(Z > 1.6) \\ &= 1 - 0.9452 = 0.0548. \end{aligned}$$

Bonus. (0.5 pts) Determine the 10th percentile value of W approximately with one decimal place of rounding. Show your work.
Answer. First, find an approximate z-score from the table: $z \simeq -1.28$ (a more accurate result is -1.282). Then, $x = \mu + z\sigma = 0 - 1.28(5) = -6.4$.