

1. Let X be a continuous r.v. with the probability density function (pdf) given by $f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 1/2, & 1 < x \leq 2 \\ 0, & \text{otherwise} \end{cases}$.

- (a) Write an explicit integral expression for each of the following: $P(\frac{1}{2} < X < \frac{3}{2})$, $\mu_X = E[X]$ and $F(0.7)$ where F is the cdf of X .
- (b) Compute $F(0.7)$.

Short Answers.

$$(a) P(\tfrac{1}{2} < X < \tfrac{3}{2}) = \int_{1/2}^{3/2} f(x)dx = \int_{1/2}^1 xdx + \int_1^{3/2} \tfrac{1}{2}dx \text{ (whose value is } \tfrac{5}{8})$$

$$\mu_X = \int_0^2 xf(x)dx = \int_0^1 x^2dx + \int_1^2 \tfrac{x}{2}dx \text{ (whose value is } \tfrac{13}{12})$$

$$\text{and } F(0.7) = \int_0^{0.7} f(x)dx = \int_0^{0.7} xdx \text{ (since 0.7 is between 0 and 1).}$$

$$(b) F(0.7) = \int_0^{0.7} xdx = 0.245.$$