Stat 50, Quiz #3, Spring 2025

04/08/2025

Instructions. Write your name and read these instructions carefully. This quiz consists of two problems and a bonus problem. Some relevant formulas are included below. You may use a calculator and a normal area table. For the end results, include units (if there are any) and use at least three-decimal points of rounding unless otherwise stated. If you have questions, let me know. You have 25 minutes to complete the quiz. Good luck!

NAME:

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Some Useful Formulas.

Let $X_1, X_2, ..., X_n$ be independent random variables with $X_k \sim N(\mu_k, \sigma_k^2)$, for each $k = 1, 2, ..., n$. If $Y = c_1X_1+c_2X_2+...+c_nX_n$, where at least one coefficient (c_k) is non-zero, then Y also has a normal distribution: $Y \sim N(\mu_Y, \sigma_Y^2)$, where $\mu_Y = c_1\mu_1 + c_2\mu_2 + ... + c_n\mu_n$ and $\sigma_Y^2 = c_1^2\sigma_1^2 + c_2^2\sigma_2^2 + ... + c_n^2\sigma_n^2$.

If $X \sim \log N(\mu, \sigma^2)$, then $\mu_X = e^{\mu+\frac{1}{2}\sigma^2}$ and $\sigma_X^2 = e^{2\mu+2\sigma^2} - e^{2\mu+\sigma^2}$. Moreover, $\ln X \sim N(\mu, \sigma^2)$.

1. Let X denote the random variable for the rents (in dollars) of two-bedroom apartmens in a town. If $X \sim \log N(7.8, 0.4^2)$ approximately, answer the following:

   (a) (1 pt) Compute the mean and the standard deviation of the rent amounts for such two-bedroom apartments in the town. You may round the end results to dollar amounts.

   (b) (1.5 pts) Find the probability that a randomly selected two-bedroom apartment in the town is less than 2500 dollars.

2. (2.5 pts) Assume that X_1, X_2 and X_3 are independent normally distributed random variables with the same means, $E[X_1] = 10 = E[X_2] = E[X_3]$, and the following variances: $Var(X_1) = 4 = Var(X_3), Var(X_2) = 5$. Consider the following random variable: $W = X_1 + X_2 - 2X_3$.

   (a) (1pt) Explain why W has a normal distribution with parameters $\mu = 0$ and $\sigma^2 = 25$.

   (b) (1.5 pts) Compute $P(W > 8)$. Show your work.

Bonus. (0.5 pts) Determine the 10th percentile value of W approximately with one decimal place of rounding. Show your work.