Stat 50, Quiz #1, Section 6, Spring 2022

02/10/2025

Instructions. Write your name, read these instructions but don't start the quiz before you are told so. This quiz consists of three problems and is worth 5 points. You may utilize the formulas below and a simple or scientific calculator. Include units for the end results (if there are any), and use at least two-decimal points when rounding your final answers unless otherwise stated. If you have questions, let me know. You have 25 minutes to complete the quiz. Good luck!

NAME: Short Answers

Some Useful Formulas.

Defining and computing formulas for the sample variance: $s^2 = \frac{\sum (x_k-\bar{x})^2}{n-1} = \frac{\sum x_k^2 - \frac{\left(\sum x_k\right)^2}{n}}{n-1}$

Interquartile range (IQR): $IQR = Q_3 - Q_1$

Lower and upper limits: $LL = Q_1 - (1.5)IQR$ and $UL = Q_3 + (1.5)IQR$

Z-score for sample data: $z = \frac{x-\bar{x}}{s}$

Total probability formula: $P(A) = P(A \cap B) + P(A \cap B^c)$

Inclusion-exclusion formula: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Consider the following scenario and data set for the problems 1-2 below:

A study has been conducted at a burned site of a national park in California. Researchers have collected data about the diameter, species and status of trees in the site. The following diameter measurements are recorded in a sample of 17 trees at a particular location of the site:

7, 10, 12, 16, 18, 19, 21, 23, 25, 26, 30, 34, 37, 45, 52, 72, 80 (in inches).

1. The five-number summary of the sample data consists of 7, 17, 25, 41 and 80 inches. Utilizing this information, answer parts (a) and (b) below:

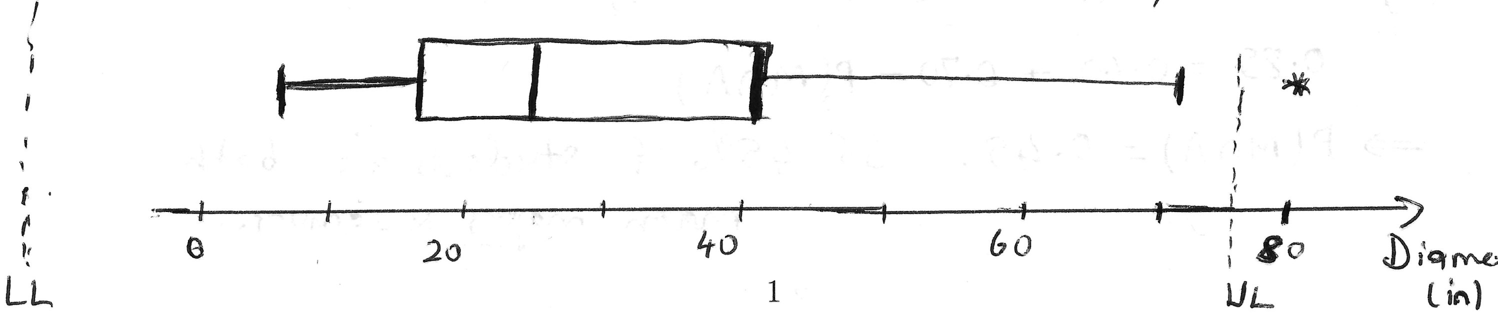
   (a) (1 pt) Determine the IQR as well as the lower and upper limits. Then, identify any potential outlier(s) for the sample data. Show work.

   $IQR = Q_3 - Q_1 = 41 - 17 = 24$ in.
   $LL = -19$ in
   $UL = 77$ in

   The only outlier is the largest measurement, $x = 80$ in

   (b) (1 pt) Draw a horizontal boxplot of the data set by also showing any outlier(s) explicitly. Make sure to use a reasonable scale of diameter values.

   (right-skewed dist.)



2. You are given the following sums for this sample of 17 tree diameter measurements:

$\sum_{i=1}^{17} x_k = 527$ and $\sum_{i=1}^{17} x_k^2 = 23,243$. Moreover, the sample mean is $\bar{x} = \frac{527}{17} = 31$ in.

(a) (0.75 pts) Determine the standard deviation of the diameter measurements in the sample.

Applying the computing formula, we get

$$S = \sqrt{\frac{23,243 - \frac{527^2}{17}}{16}} \simeq 20.776 \text{ in (or } 20.78 \text{ in)}$$

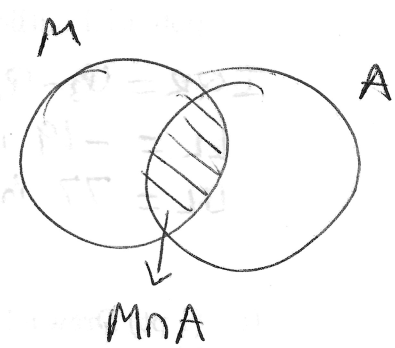
(b) (1.25 pts) Using one-decimal point of rounding, compute the z-score of a specific diameter measurement, $x = 45$ inches, and briefly explain its meaning in context.

$$z = \frac{x - \bar{x}}{S} = \frac{45 - 31}{20.776} \simeq 0.67 \simeq 0.7.$$

So, this measurement is about 0.7 std. dev. larger than the average measurement in the sample, and is a usual value.

3. (1 pt) Among the students who are enrolled in a large math course, about 85% of them are either math majors or seniors (or both). If about 60% of the students are math majors and 70% of them are seniors, determine the percent of students who are both math majors and seniors. Show your work.

Hint: You may use probability rules and/or a Venn diagram.



Let M denote math majors and A denote seniors.

Then, we are given $P(M \cup A) = 0.85$, $P(M) = 0.60$ and $P(A) = 0.70$.

By the addition rule for unions,

$$0.85 = 0.60 + 0.70 - P(M \cap A)$$

$\Rightarrow P(M \cap A) = 0.45$. So 45% of students are both math majors & seniors.