Stat 50, Test 1, Spring 2025

02/25/2025

Instructions. Write your name below, read these instructions but don't start the exam before you are told so. A calculator and a two-page formula sheet are allowed. You need to show your work to get full credit. Simplify your end results (e.g. in the case of fractions) as much as possible and include the units in the end results. Include at least three decimal points in your final answers with rounded decimal expressions unless otherwise stated. If you have questions, raise your hand. No talking, disturbing, collaboration or straying eyes during the exam. No cell phones!

You have 50 minutes to complete 5 problems and a bonus problem at the end.

NAME:

1. The following contingency table includes the number of students in an upper division course (with a total of 80 students enrolled) according to the majors of the students and whether they passed the course:

		Major	
Performance	Math	Physics	Comp. Science
Passed	18	15	21
Didn't pass	7	10	9

A student is randomly selected from this list. Answer the following questions based on the table. Make sure to show the relevant fractions explicitly and use at least three decimal places of rounding.

(a) (2 pts) Find the probability that the student either passed the course or student was not a math major.

(b) (2 pts) Determine if the following events are statistically independent? Justify your answer.

A: The student was a computer science major

B: The student passed the course.

2. The records of a hospital that specializes on lung cancer show that 12% of the patients who were referred to the hospital had lung cancer. The hospital gives each patient a test for lung cancer. Based on their statistics, in 97% of the cases where lung cancer was present, the test result was positive. Moreover, in 95% of the cases in which it was not present, the test was negative. A patient in the hospital is selected at random.

   (a) (2.5 pts) What is the probability that the patient was tested negative for lung cancer?

   (b) (2 pts) Given that the patient was tested negative for cancer, compute the probability that the patient had cancer.

3. (2 pts) In a certain process, the probability of producing a defective component is 0.05. Assume that five independent components were inspected on a specific day. Find the probability that <u>at least one</u> of the items inspected is defective.

4. Let X be the number of hours that a machine is in use in a production facility. Assume that the probability density function (pdf) of X is given by $f(x) = \frac{2}{15}x$, for $1 < x < 4$ (and f is zero otherwise).

   (a) (2 pts) Compute $P(X > 2)$.

   (b) (2 pts) Determine the cumulative distribution function (cdf) of X.

5. A facility has several solar panels but some of the panels are expected to fail within a year due to aging and other reasons. Let Y denote the number of panels that would fail within a year. The probability mass function (pmf), $p(y)$, of Y is estimated as follows:

y :	0	1	2	3	4	5
$p(y)$:	0.15	0.2	0.35	0.2	0.07	0.03

   (a) (1.5 pts) What is the probability that <u>at least 2</u> panels would fail within a year?

(b) (2 pts) Compute the mean value of Y. Include the units.

(c) (2 pts) If the variance of Y is $\sigma_Y^2 = 1.5451$, then compute the z-score of $y = 5$ and briefly explain its meaning in context.

BONUS. (1 pt) Let Y_1 and Y_2 have the same distribution as the r.v. Y as above. Determine the mean of the r.v.$W = \frac{Y_1+Y_2}{2}$.