NETWORKS (Repeat)

Ports represent a window through which we provide energy or information or access energy or information.

Each port associated with a physical system having two type of variable. Kinetic variable & potential variable.

Physical system 3 domains

	Electrical Domain	Mechanical	Magnetic
KV	Voltage	Torque, force	MMF = NI
P.V	current	Angular vel., velocity	Flux

In any domain product of pot. variable & Kinetic variable is power variable.

State Variable – variable associated with dynamic element which is responsible for stored energy or information.

I/P and P.V know. To find O/P. This is N/W analysis.
Processing UNIT

I/P and O/P known. This is N/W synthesis.
To find PU

Synthesis – 1) whether system is physically realizable or not.
2) If the system is realizable, one solⁿ to the given prob exists, then ∞ no. of solⁿs are possible.

3) To find which solⁿ or design is the best.
means :- optimal solⁿ. cost ↓ performance ↑

CLASSIFICATION OF SYSTEM / NW.

1) Static / Dynamic

Static – It is the one whose response is totally dependent on present excitation.
eg:- resistor.

Dynamic – Response dependent on present I/P as well as past O/P.
eg:- capacitor or inductor.

2) Continuous / Discrete

Continuous – system whose response is define ~~for~~ w.r.t to independent variable on continuous basis. eg:- R, RC ckt

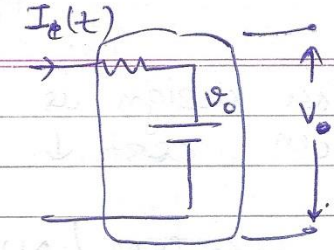
Discrete – Respone is defined whose O/P is discrete w.r.t to independent variable.

3) Linear / Non linear

Homogenity $x_1(t) \to y_1(t)$ $a_1 x_1(t) \to a_1 y_1(t)$
$x_2(t) \to y_2(t)$ $a_2 x_2(t) \to a_2 y_2(t)$

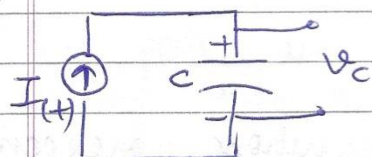
Additivity $a_1 x_1(t) + a_2 x_2(t) \to a_1 y_1(t) + a_2 y_2(t)$

eg:



Non linear even if elements are linear

Q



Initial voltage $\cdot v_c(0^-) = v_0$

$I_1(t) \rightarrow \frac{1}{C}$

$$v_{c_1}(t) = \frac{1}{C}\int_0^t I_{c_1}(t)\,dt + V_0$$

$$V_{c_2}(t) = \frac{1}{C}\int_0^t I_{c_2}(t)\,dt + V_0$$

$$V_{c_1}(t) + V_{c_2}(t) = \frac{1}{C}\int_0^t (I_{c_1}(t) + I_{c_2}(t))\,dt + V_0$$

$\Downarrow$ not $2V_0$

so non linear

→ If mathematical description of a static system is given by linear algebraic eq^n then the system is linear.

If mathematical description of a dynamic system is given by linear differential eq^n or linear difference eq^n then system is stable.

To check linearity: If coeff of differential operator are independent of dependent variable then, diff eq^n is linear diff. eq^n. check coefficients of all.

1) $4\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 3y = 2x + 5\frac{dx}{dt}$ linear (T.I)

2) $y(n+2) + 3y(n+1) + 3y(n) = 6x(n)$ linear (T.I)

3) $4t\frac{dy}{dt} + 2t^2y = 6x + 5\frac{dx}{dt}$ linear (T.V)

\# $\left(\frac{dy}{dx}\right)^2 = 0$ Non linear eqn

but system linear

because $\boxed{\frac{dy}{dx} = 0}$

4) $4\frac{dy}{dt} + 6y + 3\frac{dx}{dt} = \sin t$ Linear

5) $4y\frac{dy}{dt} + 6y = 2x$ Non linear

6) $y^2(n+2) + y(n+1) = 3y(n) + 6x(n)$ Non linear

$\underbrace{y(n+2)\cdot y(n+2)}_{\text{dependent}} + y(n+1) = 3y(n) + 6x(n)$

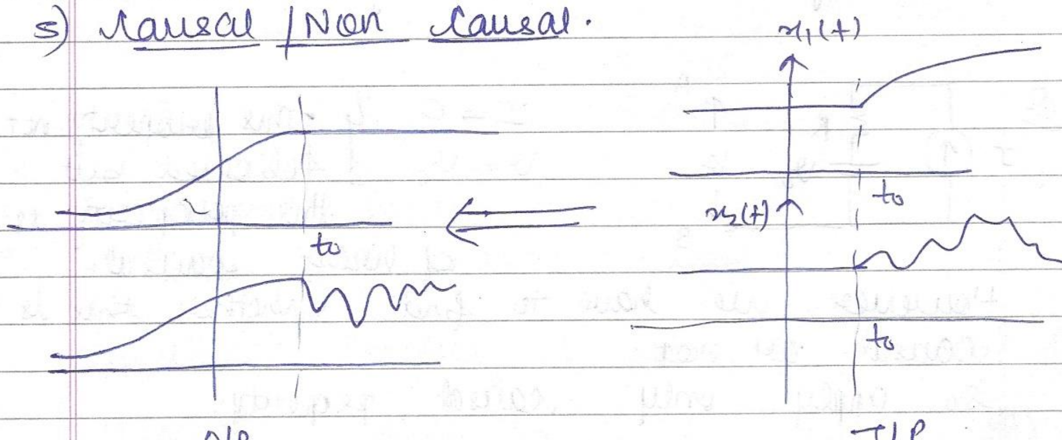
4) Time variant / Time invariant

$x(t) \longrightarrow y(t)$

$x(t-T_0) \longrightarrow y(t-T_0)$

\# If coeff. of ~~dependent variable~~ differential operator is independent of independent variable then it is time invariant.

5) Causal / Non causal.



O/P I/P

$x_1(t) \longrightarrow y_1(t)$

$x_2(t) \longrightarrow y_2(t)$

For causal system if

$x_1(t) = x_2(t)$ for $t \le t_0$

then $y_1(t) = y_2(t)$ $t \le t_0$

Causal system also known as Anticipatory sys. It cannot ancipate what shall be applied to I/P port in future.

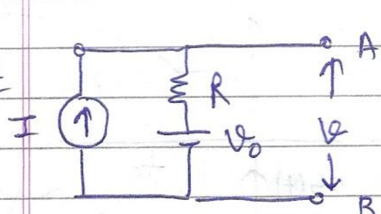
→ Causal signals – These are one which are ~~one~~ zero till $t = 0$

→ Linear causal system –

$$x(t) = x_1(t) - x_2(t) \Longrightarrow y_1(t) - y_2(t)$$

$$\begin{cases} 0 & t \le t_0 \\ x_1(t) - x_2(t) & t > t_0 \end{cases} \qquad y(t) = \begin{cases} 0 & t \le t_0 \\ y_1(t) - y_2(t) & t > t_0 \end{cases}$$

i.e if $x(t) = 0$ then $y(t) = 0$

Q 

$\left.\begin{array}{l} I = 0 \\ v = v_0 \end{array}\right\}$ This property not followed but this property is of linear causal.

However, we have to find whether this is causal or not.

So apply only causal property.

If $I = I_1(t)$ ~~⟹~~ ~~for t≤~~ $v_1(t) = v_2(t)$

$I = I_2(t)$

This is causal system.

causal system with causal signal has response zero till $t = 0$.

6) <u>Lumped / Distributed</u>.

<u>Distributed</u> :— when physical dimension of the system is comparable to wavelength of excitation, then system is said to be of distributed type.

<u>Lumped</u> — when physical dimensions are negligible as compared to wavelength of excitation.

calculate wavelength of ac supply system in India

$$v = \nu \lambda$$

$$\lambda = \frac{v}{\nu} = \frac{3 \times 10^8 \, m/s}{50 \, Hz}$$

$$= 0.6 \times 10^7$$

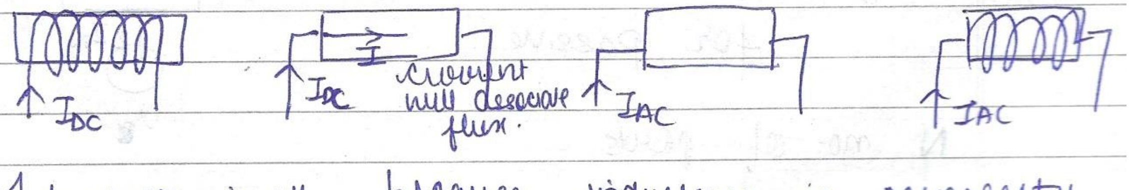
$$= 6 \times 10^6$$

$$\lambda = 6000 \, km$$

→ Distributed — Power tx lines

→ Lumped — n/w used in labs.

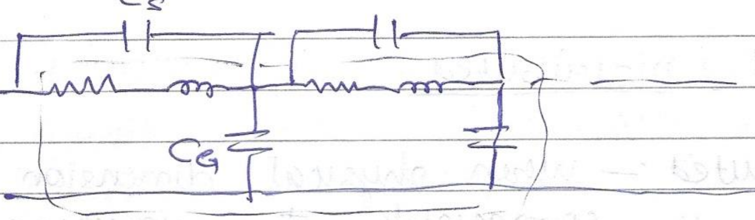
Inductance — property to associate magnetic flux.



Inductance in all. because inductance is property of material.

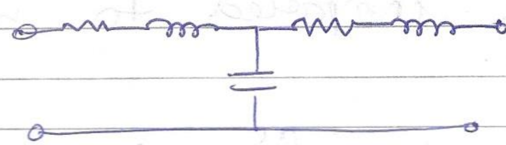
that a medium can withstand

dielectric — max. electric field without polarization
strength



This is distributed.

To convert to lumped — Neglect C_S
Then convert T to Π or Π to T.

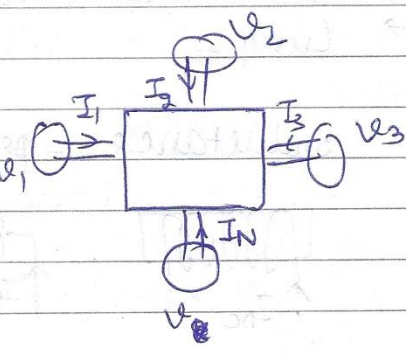


7) finite / Infinite N/W.

8) <u>One dimensional</u> / Multi - dimensional
No. of independent variable 1 — 1 dimensional
" " " " 2 — 2 "

9) <u>Active / Passive</u>
Total energy supplied by excitations to the system is greater than or equal to zero then system is passive

$$\sum_{k=1}^{N} \int_{0}^{t} v_k I_k \, dt \geq 0$$



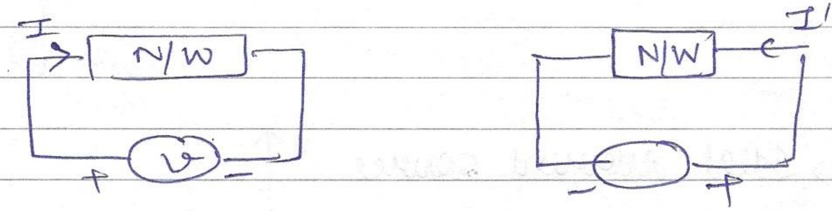
for passive.

N no. of ports

=0 shows internal loss is zero.
Jitna I/P diya exactly utna O/P.

Active – when internal any energy source.

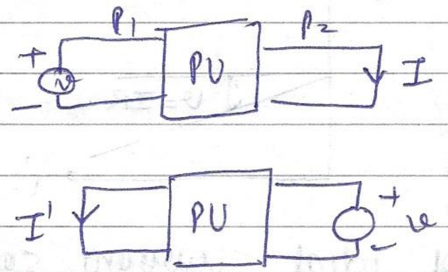
10) <u>Unilateral / Bilateral</u>



I' = I (Bilateral). eg:- to resistor
Unilateral eg:- diode.

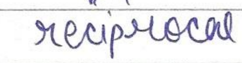
11) <u>Reciprocal / Non Reciprocal N/W</u>

A n/w is reciprocal if interchange of excitation & response doesn't change relationship of excitation & response.

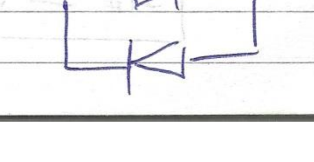


If I' = I then reciprocal N/w.

If the n/w consists of only bilateral elements then the n/w must be reciprocal.
If n/w contains some unilateral elements, then it may be possible that n/w is reciprocal.



Non reciprocal

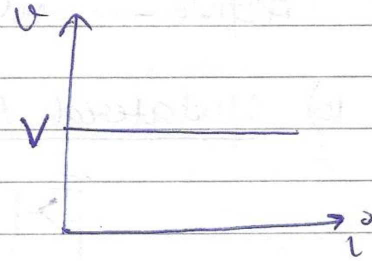


Reciprocal

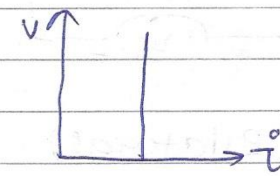
Network Elements

1) Voltage source

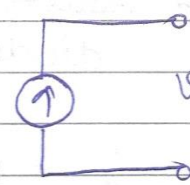
⇒ Ideal voltage source



⇒ Ideal current source

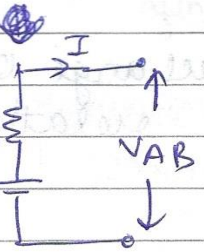


ideal I ~~current~~ source

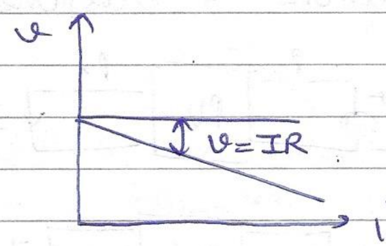


$v \neq 0$ (IMP).

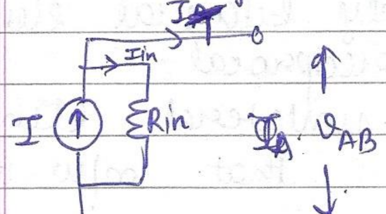
Physical practical system of ideal voltage source



$V_{AB} = v - IR_{AB}$



Practical system of ideal current source



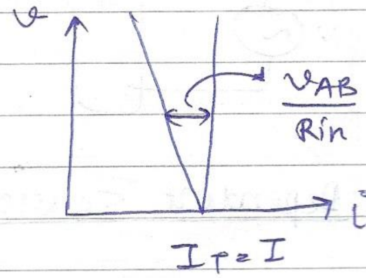
$$I_T = I - \frac{V_{AB}}{R_{in}}$$

$$V_{AB} = (I - I_T)R_{in}$$

$\boxed{I_{in} \simeq 0 \quad \text{not} = 0}$

If in ideal case $R_{in} = \infty$ then I is short ckt i.e O/P voltage $v_{AB} = 0$.
~~so, volt Ho~~ Thus, role of I diminishes as this shows that no energy is supplied
Thus we cant assume voltage across current source as zero.

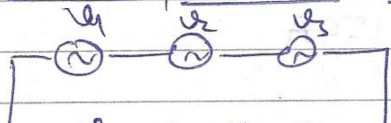
$$I_T = I - \frac{v_{AB}}{R_{in}}$$



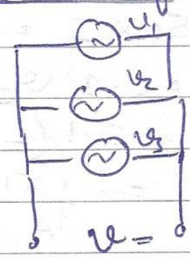
→ ~~It is impossible to achieve~~ . short ckt cond. of ideal voltage source is not a valid representation

→ Open ckt cond. of ideal current source is not valid representation

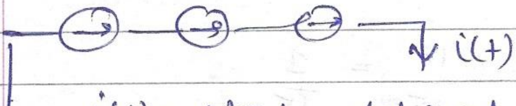
Series / Parallel combination of ideal sources



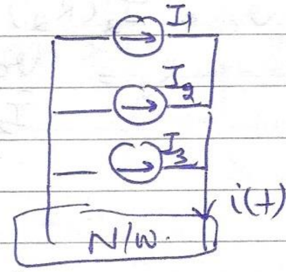
$v = v_1 + v_2 + v_3$



v = not defined

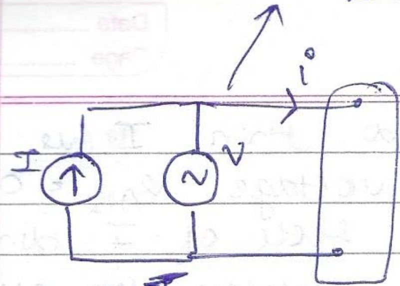


$i(t)$ = Not defined

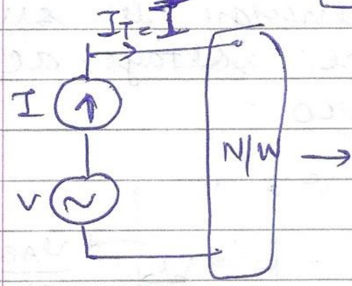


$i(t) = I_1 + I_2 + I_3$

I = some current going to V
some to N/W (i)



voltage is affecting N/W. However i is current to N/W. Here thus I source is redundant.

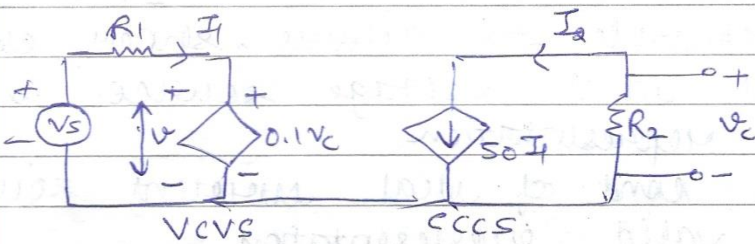


→ Here V source is redundant as I is constant.

Dependent Sources

1/2/2014

Q



$v_c = -10V$ $\quad$ $I_1 = 3A$ $\quad$ $I_2 = ?$ $\quad$ $v = ?$

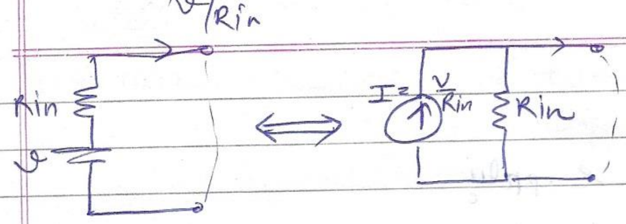
$$I_2 = 50 \times I_1 = 50 \times 3 = 150A$$

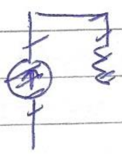
$$v = 0.1\, v_c$$
$$= 0.1(-10)$$
$$= -\frac{1}{10} \times 10 = -1V$$

$$v_c = I_2 (R_2)$$

$$R_2 = \frac{v_c}{I_2} = \frac{+10}{+150} = \frac{1}{15}$$

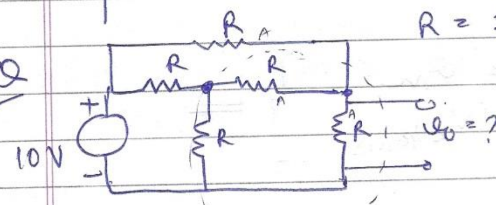
Alexander → 1, 2, 3, 6 | Read

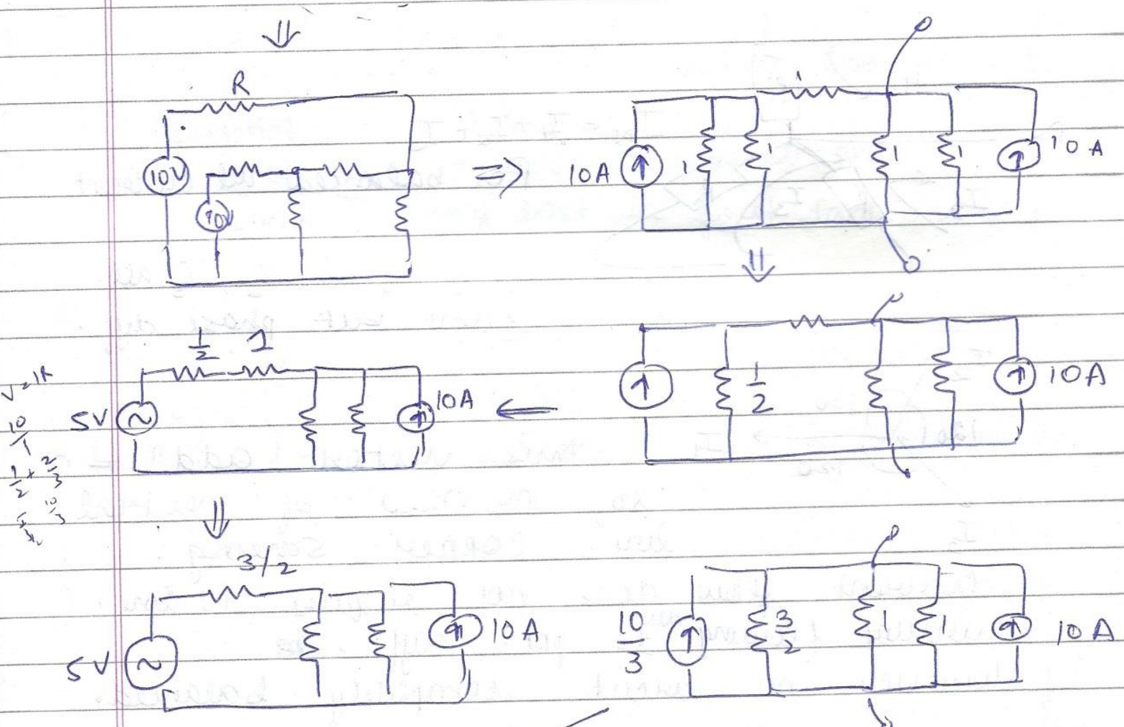




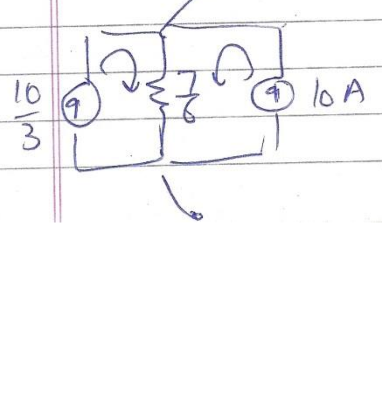
$R = 1\,\Omega$.

Q





$$\frac{2}{3} + \frac{1}{2} = \frac{4+3}{6} = \frac{7}{6}$$



approx.

$\boxed{\underline{\text{Ans}} = 11}$

AC source

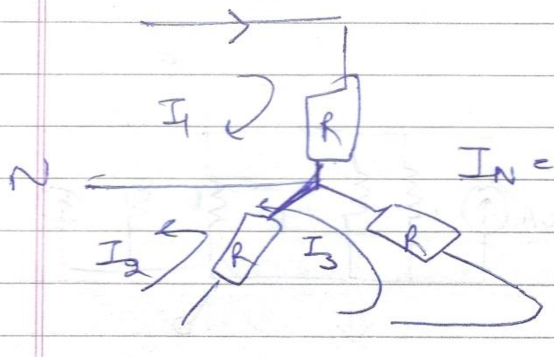
Balanced 3 ϕ supply

$v_R = v_{mR} \sin(\omega t)$

$v_y = v_{my} \sin(\omega t + 120)$

$v_B = v_{mB} \sin(\omega t + 240)$

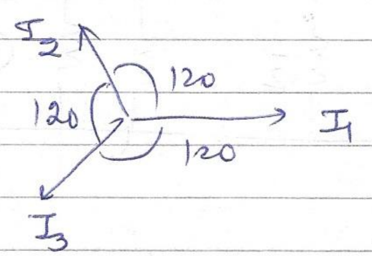
$$\left.\begin{array}{l} v_{mR} = v_{my} = v_{mB} = v_m \\ (v_R)_{RMS} = (v_y)_{RMS} = (v_B)_{RMS} \end{array}\right\} =$$



$I_N = I_1 + I_2 + I_3$

For balanced all current same.

mag. of I_1, I_2, I_3 all same but phase diff.



Thus vector add^n = 0

so, no need of neutral line. Copper saving.

Current flow does not require N line.

Current flowing due to pot. diff.

However, no circuit completely balanced.

Analogous Mechanical system of Capacitor

or volume of tank

H = total capacity of tank to store water
C = " " " capacitor " " charge

H or Volume analogous to C

Charge q represents volume of water stored inside the tank.

I/P in form of K.E	In cap. also, energy
storage " " " P.E	stored in form of P.E

Q

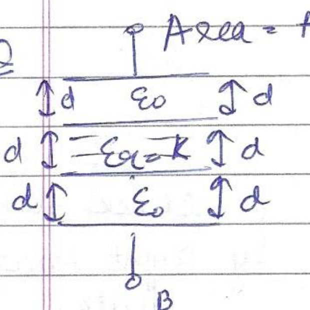
$K_{out} = K_{in} - P_1 - P_{1L} - P_2 - P_{2L}$ → energy lost in 2nd tank

P_1: Potential energy of tank 1

P_{1L}: energy lost in first tank

P_2: Potential energy of second tank

Q



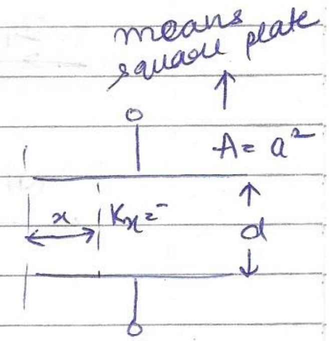
$\varepsilon_0 A/d$

$\varepsilon_0 k A/d$

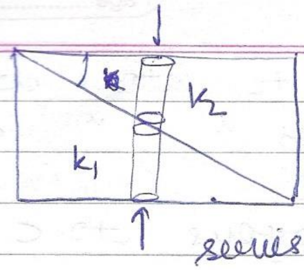
$\varepsilon_0 A/d$

All in series

Q done



Q

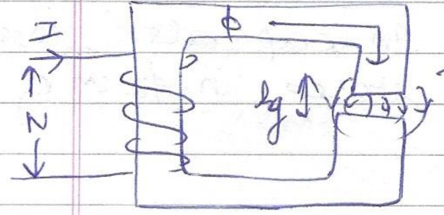


$$\frac{1}{d_c} = \frac{1}{d_{c_1}} + \frac{1}{d_{c_2}}$$

series combination of both

Now after calculating d_c, now parallel combination.

*



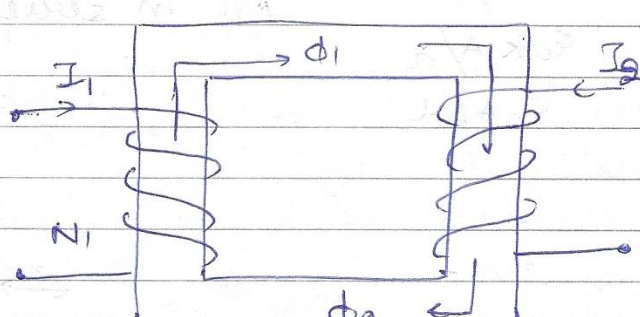
If there is a gap in core we will not take $l_m = l_c + l_g$

If mean path length $= l_C$

then take $l_m = l_C + 2 l_g$ → practically by experiment

R, ϕ all changing.

* Magnetically coupled Ckt



Check flux by Right Hand Rule.

flux same direction = +ve coupling
flux opp " = −ve "

→ L = Ability to ~~generate~~ associate flux with coil due to MMF applied at coil.

M same for both as they both are associated with same core.

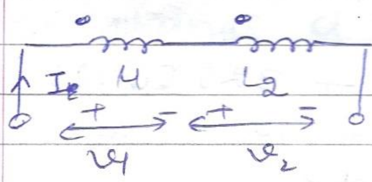
* DOT CONVENTION

→ If current to both the coils are entering to the dotted terminal or leaving from dotted terminal then coupling is +ve.

→ If current through one of coil is entering to the dot and other leaving from dot then coupling −ve.

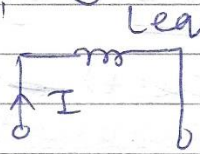
→ Potential of dotted terminals to both the coils is always in phase.

Q



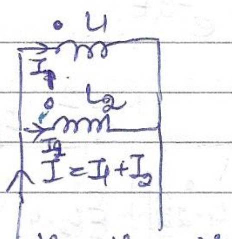
$$v = v_1 + v_2$$
$$= L_1\frac{dI}{dt} + L_2\frac{dI}{dt} + 2M\frac{dI}{dt}$$

⇕



$$I = L_1 + L_2 + 2M$$

Q

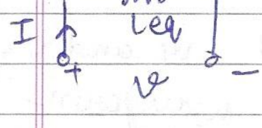


$$v = v_1 = L_1\frac{dI_1}{dt} + \cancel{L_2\frac{dI_2}{dt}} + \cancel{L_2} M\frac{dI_2}{dt}$$

$$v = v_2 = L_2 I_2' + M I_1'$$

$v_1 = v_2 = v$

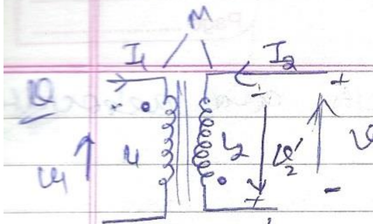
⇕



$$v = L_{eq} I'$$
$$v = L_{eq} I_1' + L_{eq} I_2'$$

Put values of I' and I_2' from above

~~Head of arrow means the~~ head of arrow considered acc. to dotted terminal. Where dotted i.e. +ve.



1Q

$L_1 = 2 \quad L_2 = 3 \quad M = 1$

$I_1 = 325\sin(2\pi 50 t) + 4e^{-2t}$

$I_2 = 325\cos(2\pi 50 t)$

find v_1 and v_2.

$v_1 = L_1 I_1' - M I_2'$

$v_2' = L_2 I_2' - M I_1'$

$v_2 = -(v_2')$

$$v_1 = L_1 \frac{dI_1}{dt} - M\frac{dI_2}{dt}$$

$$= 2\frac{d}{dt}\left(325\sin(\omega t) + 4e^{-2t}\right) - \frac{d}{dt}\left(325\cos(\omega t)\right)$$

$$= 2\left(325\cos(\omega t)(\omega) + 4(-2)e^{-2t}\right)$$

$$+ 325\sin(\omega t)(\omega)$$

Let $\omega = 1$.

$$= 2\left(325\cos\omega t - 8e^{-2t}\right) + 325\sin\omega t$$

$$= 650\cos t - 16e^{-2t} + 325\sin t$$

Mistake

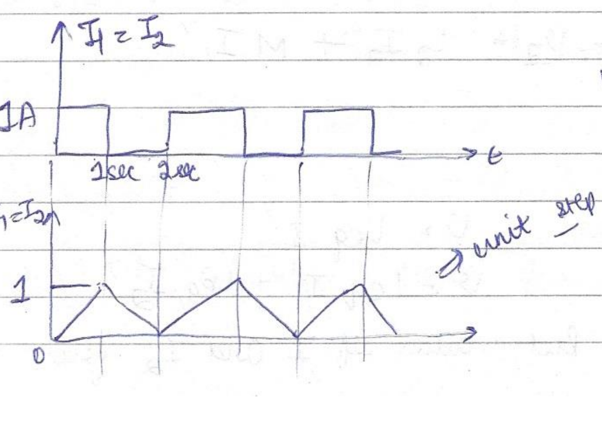
$$v_2 = L_2\frac{dI_2}{dt} - M\frac{dI_1}{dt}$$

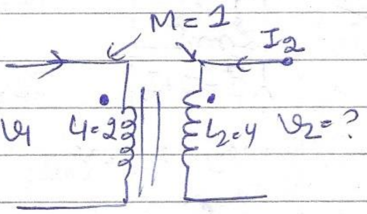
Imp. (Here there will be $-v_2$)

$$= 3(-325\sin t) - 1\left(325\cos(t) - 8e^{-2t}\right)$$

$$v_2 = -975\sin t - 325\cos t + 8e^{-2t}.$$

Q





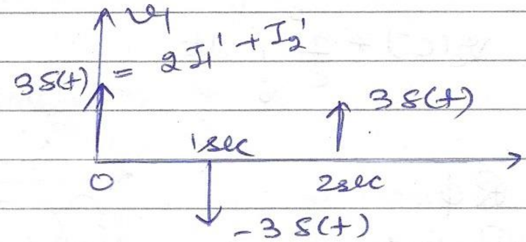
find v_1 and v_2 waveform

Date ....
Page ....

$$v_1 = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt}$$

$$= 2 \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

$$v_2 = L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt} = 4 \frac{dI_2}{dt} + \frac{dI_1}{dt}$$



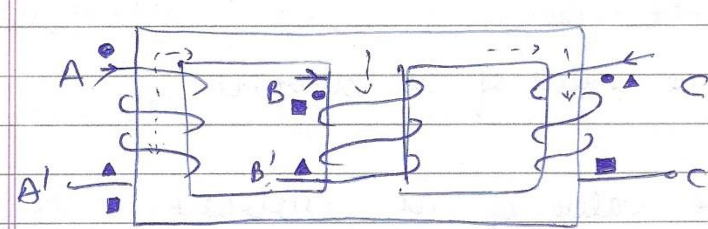
This does not mean that at $t=0$ amplitude 3. Amplitude is ∞. This represents that there is change of 3.

→ $Y = \frac{1}{R}$ = Conductance

→ Reactance $X_L = j\omega L$ (Dynamic elements)
$X_C = \frac{1}{j\omega C}$

Impedance Z = (Used for sinosoidal signals)
Admittance $\frac{1}{Z}$

→



CAPACITOR

say initial charge on cap. is q_0.
then charge at any time t

$$q(t) = q_0 + \int_0^t i_c \, dt$$

$$v_c(t) = \frac{q(t)}{C} = \frac{q_0}{C} + \frac{1}{C}\int_0^t i_c \, dt$$

$$v_c(t) = v_c(0^-) + \frac{1}{C}\int_0^t dt$$

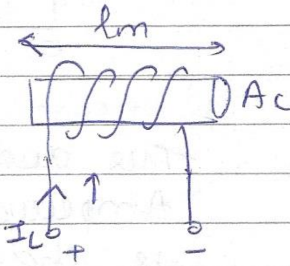
INDUCTOR

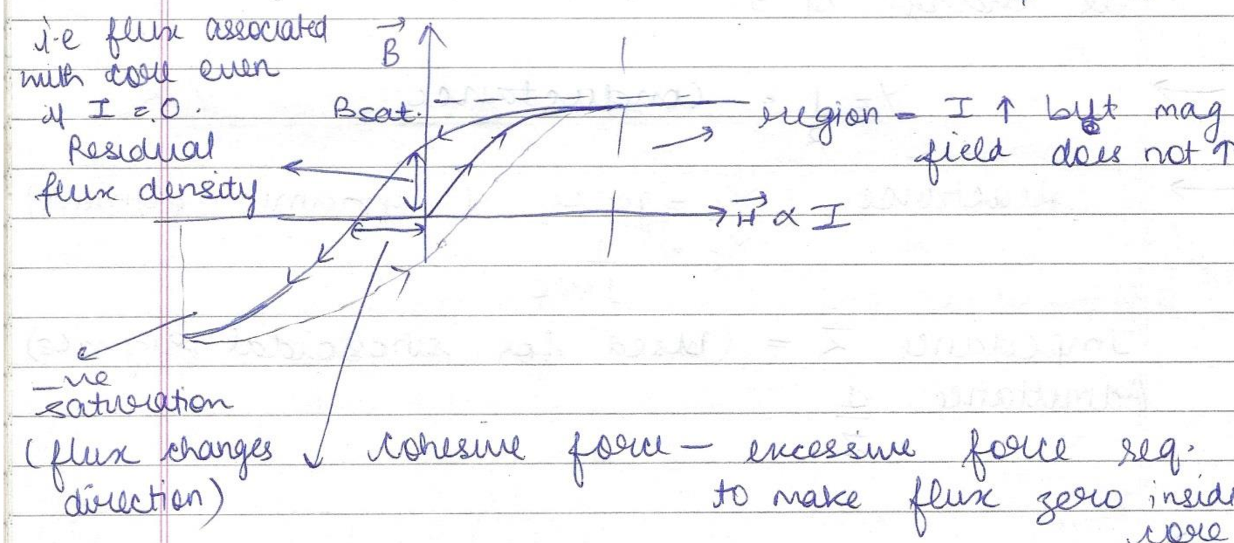
$$NI = R\phi$$

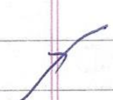
$$NI = R \cdot B \cdot A_c$$

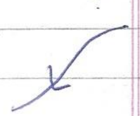
$$= R \cdot \mu_0 \mu_r H A_c$$

$$\boxed{H \propto I}$$





 → for +ve value of ac current

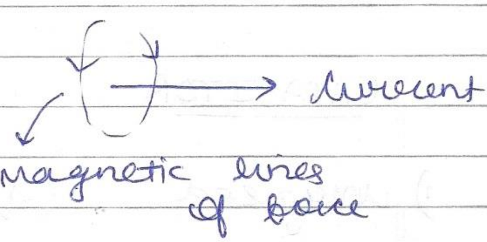
 = for −ve value of ac current.

Area under curve = Hysterisis Loss.

freq. of ac loop = freq. of traversal of hysterisis loss.

More freq of ac current = More loss.

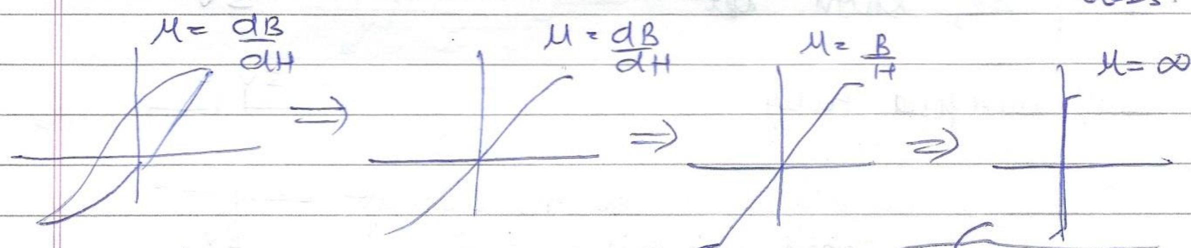
→ Eddy current loss =



Reversal of Ampere's law

current will be induced in core i.e Eddy current.

Itna jada ϕ, utna jada I. ↑ eddy current loss.



Practical core

Ideal core

Hysteris loss is that the electrical energy we give does not totally get converted in magnetic domain

even 0 current sufficient to generate flux.

change in

⇒ Flux is always smooth.

1) Inductor under unergised state. (dc case) $(I_L = 0)$

Initially $\frac{d\phi}{dt}$ very large. Offers ∞ resistance to current. $V = L\frac{d\phi}{dt}$

V = nearly equal to source

so current zero.

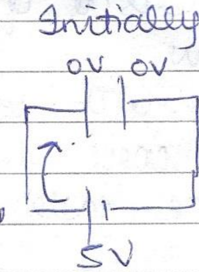
2) Fully energised — $\frac{d\phi}{dt} = 0$. (dc case)

$V = 0$ (i.e. short circuit)

⇒ If ac supply. $\frac{d\phi}{dt}$ will not be zero at any pnt.

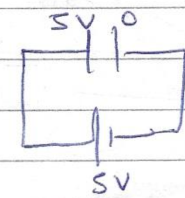
<u>CAPACITOR</u>

1) unergised ($q = 0$)



so flow of current due to Pot. difference
so, short ckt.

2) energised fully

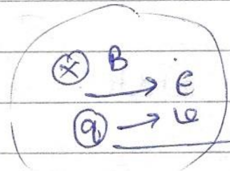


no P.D
so no current

open circuit.

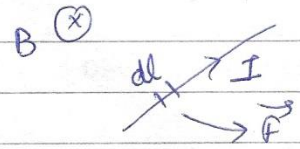
<u>Formulae</u>

1) Lorentz force



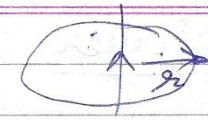
$$\overset{III}{F_L} = q\vec{E} + q(\overset{I}{\vec{v}} \times \overset{II}{\vec{B}})$$

(2)



$$\overset{III}{\vec{dF}} = \overset{I}{i\,\vec{dl}} \times \overset{II}{\vec{B}}$$

③ Biot Savart law.



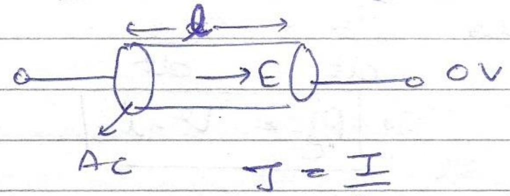
$$\overset{\text{III}}{\vec{dB}} = \frac{\mu_0}{4\pi} \frac{i\,\overset{\text{I}}{\vec{dl}} \times \overset{\text{II}}{\vec{r}}}{|\vec{r}|^3}$$

$\vec{I} \times \vec{II} = \vec{III}$ } all three. Right Hand

finger palm Thumb.

OHM's LAW

$\underline{J \propto \vec{E}}$ (at constant temp.)



$$J = \frac{I}{A_c}$$

$$J = \sigma E$$

$$J = \frac{1}{\rho} E = \frac{1}{\rho} \frac{V}{l}$$

$$\frac{I}{A} = \frac{V}{\rho l}$$

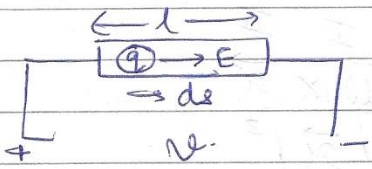
$$V = \left(\frac{\rho l}{A}\right) I$$

↓ Resistance = R

$$R = \frac{dV}{dI} \quad \text{(non linear)}$$

→ Voltage division rule
→ current " rule

⇒ Power



Mechanical Power

$P = \frac{dE}{dt}$

$P = \frac{dw}{dt}$

$F = qE$

$W = F \cdot ds$

$= qE \cdot ds = qE \cdot l \cos(0)$

$W = qE \cdot l$

$W = q \cdot V$

Work done by electrical force $q \cdot E$

$\frac{dw}{dt} = v \frac{dq}{dt}$

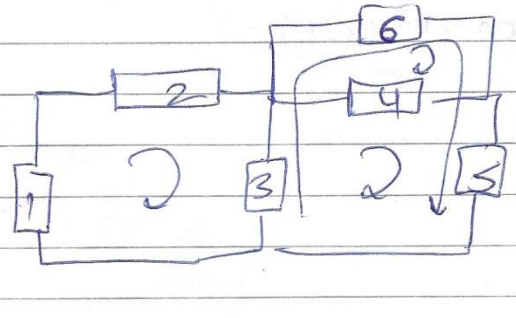
$\boxed{P_e = v \cdot i}$

CRAMER's RULE

KCL / KVL

Loop – closed path.

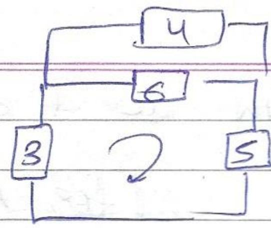
Mesh – fundamental loops are mesh.
i.e smallest closed path present in a n/w which does not include any other closed path.



$e_6, e_4 \rightarrow M$

$e_3\ e_5\ e_6 \rightarrow M$

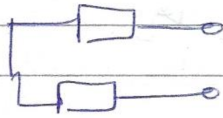
$e_3\ e_6\ e_5 \rightarrow$ also mesh



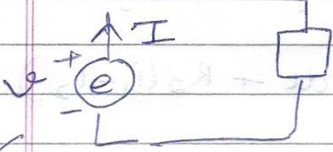
$\Longleftrightarrow$ N/W $e_3\, e_6\, e_5$ also mesh

If asked to count no. of meshes = 4
If no. of mesh eqns = 3 because in that

$\Rightarrow$ Circuit – closed path & single ckt has to be chosen in which no. of meshes = 3

$\Rightarrow$  This is n/w not ckt.

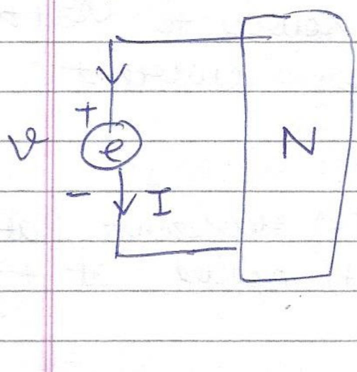
Power Flow



Source delivering power to load as +ve terminal se current bahar ja raha.

Power delivered by element e to rest of n/w is + vI.

Power received by n/w element from N/W – vI



Power delivered by e to rest of N/W = – vI
Power received by e from rest of N/W = vI

KVL applied for both ac & dc.

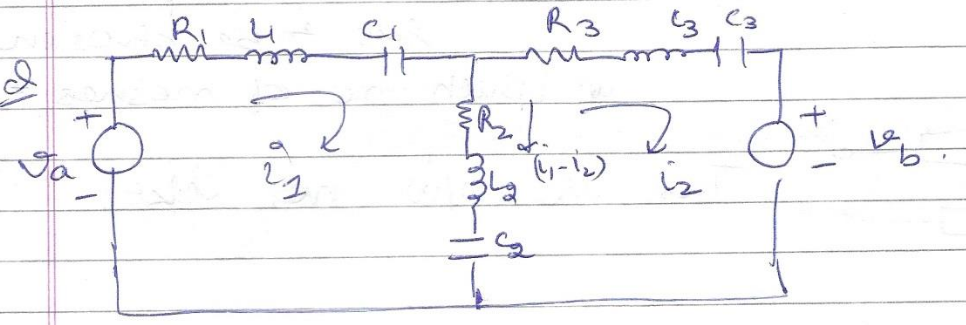
$$\sum_{\forall k} E_k = 0 \quad \text{(for AC)}$$

RMS value with phase

$i_1 = i_2 + x$

Mesh Analysis

Q.



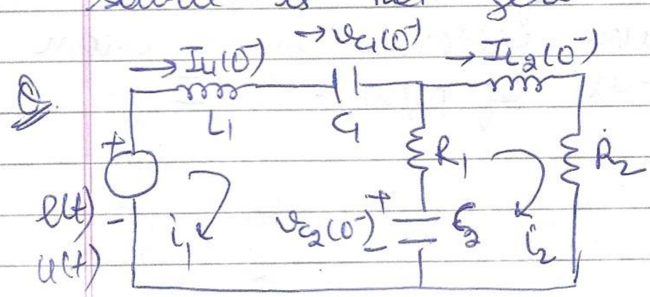
All cap. & inductors initially unenergised.

$$v_a = R_1 i_1 + L_1 \frac{di_1}{dt} + \frac{1}{C_1}\int i_1\,dt + R_2(i_1 - i_2) + L_2 \frac{d(i_1 - i_2)}{dt} + \frac{1}{C_2}\int (i_1 - i_2)\,dt$$

$$R_3 i_2 + L_3 \frac{di_2}{dt} + \frac{1}{C_3}\int i_2\,dt + v_b = R_2(i_1 - i_2) + L_2 \frac{d(i_1 - i_2)}{dt} + \frac{1}{C_2}\int (i_1 - i_2)\,dt$$

In mesh convert, current source to voltage source because voltage across current source is not zero.

Q.



$u(t)$ represents that $e(t)$ applied at $t = 0$

this already includes initial condition

$$e(t)u(t) = L_1 I_{L_1}(0^-) + L_1 \frac{di_1}{dt} + v_{c_1}(0^-) + \frac{1}{C_1}\int_0^t i_1\,dt + (i_1 - i_2)R_1 + v_{c_2}(0^-) + \frac{1}{C_2}\int (i_1 - i_2)\,dt$$

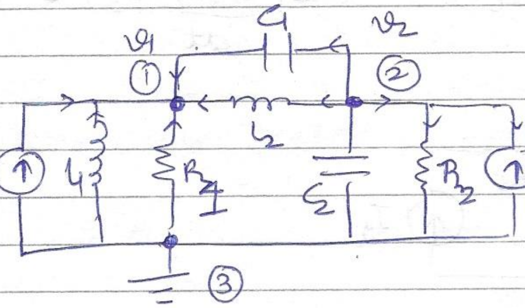
$$\cancel{L_2 I_{L_2}(0^-)} + R_2 i_2 + L\frac{di_2}{dt} = \frac{1}{C_2}\int (i_1 - i_2)\,dt + v_{c_2}(0^+) + R_1(i_1 - i_2)$$

as $v_L(t) = L\frac{dI_L(t)}{dt}$ → at time ($t = t$) which already includes $t = 0$.

Imp. However in capacitor

$$v_c(t) = v_c(0^-) + \underbrace{\frac{1}{C}\int_0^t i\,dt}_{\text{change in voltage.}}$$

Nodal Analysis

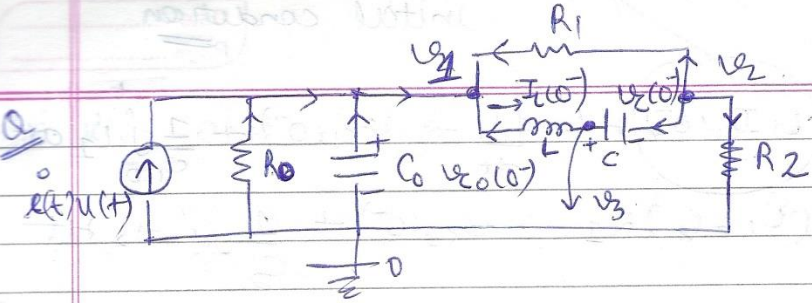


current ↑ pot. to ↓ pot.

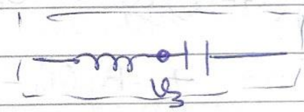
$$I_a + \frac{1}{L_1}\int_0^t (0 - v_1)\,dt + \frac{0 - v_1}{R_1} + C_1\frac{d(v_2 - v_1)}{dt} + \frac{1}{L_2}\int (v_2 - v_1)\,dt = 0$$

$$C_2\frac{dv_2}{dt} + \frac{v_2}{R_2} - I_b + \frac{1}{L_2}\int_0^t (v_2 - v_1) + C_1\frac{d(v_2 - v_1)}{dt} = 0.$$

Q.



Soln



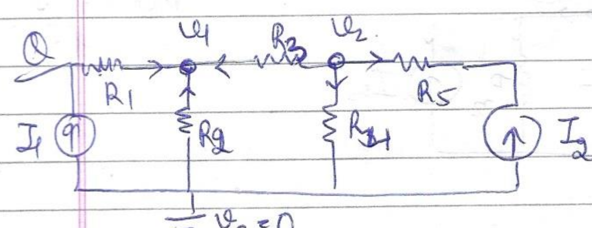
Whenever this situation occurs we have to introduce another node v_3 because we need voltage across inductor & capacitor

$$i(t)u(t) + \frac{(0-v_1)}{R_0} + C_0\frac{d(0-v_1)}{dt} + \frac{(v_2-v_1)}{R_1} + \frac{1}{L}\int_0^t (v_3-v_1)\,dt - I_L(0^-) = 0.$$

$$\frac{v_2}{R_2} + \frac{v_2-v_1}{R_1} + C\frac{d(v_2-v_3)}{dt} = 0.$$

$$\frac{1}{L}\int_0^t (v_1-v_3)\,dt + I_L(0^-) + C_1\frac{d(v_2-v_3)}{dt} = 0$$

Q



$$I_1 + \frac{(0-v_1)}{R_2} + \frac{(v_2-v_1)}{R_2} = 0.$$

$$\frac{v_2-v_1}{R_3} + \frac{v_2}{R_4} = I_2$$

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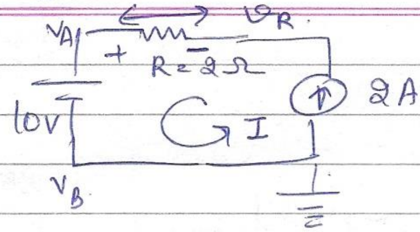
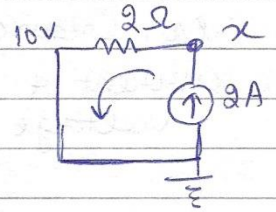
Ideal Source

Q



find v_R and I.

Soln



→ current source delivers constant current

current = 2A

$v_R = 2 \times 2 = 4V$

$\boxed{v_R = -4V}$ Ans

~~(10 − x) = 4~~

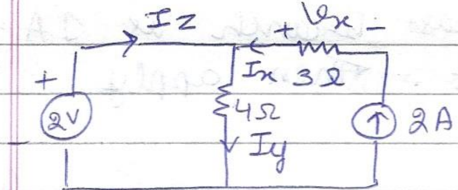
~~10 − (x = 6V)~~

$x - 10 = -4$

$x = -4 + 10$

$= 6$

Q



1) find, I_z, v_x, I_y, I_x
2) Power delivered by current source
3) Power received by voltage source
4) Power dissipated in n/w

Soln

$I_x = 2A$

$V_x = -6V$

$I_y = \frac{1}{2} A$

$I_z + I_x = I_y$

$I_z = \frac{1}{2} - 2 = -\frac{3}{2} A$

Power delivered

Power receive

2) $I_x \cdot V$

$8 \times 2 = 16W$

3) $\frac{2 \times 3}{2} = 3V$

4) $16 - 3$

$= 13W$

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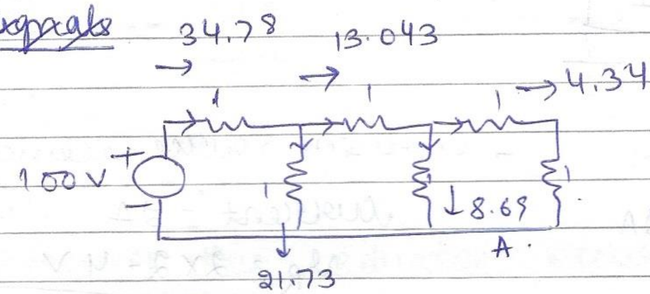
Network Analysis

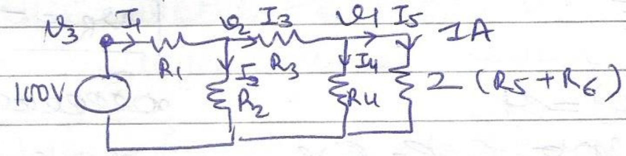
~~Sopagls~~

Q.



Cal. branch current & node voltage.

Sol^n



eq. ~~voltage~~ R = 2

other method

Assume current in last branch is 1A. find voltage for that. Then apply unitary method.

$$v_1 = (R_5 + R_6)$$

$$I_4 = \frac{R_5+R_6}{R_4} \qquad I_3 = I_4 + I_5$$

$$I_3 = I_4 + 1$$

$$v_2 = v_1 + I_3 R_3$$

$$I_2 = \frac{v_2}{R_2} = \frac{v_1 + I_3 R_3}{R_2}$$

$$I_1 = I_2 + I_3$$

$$\boxed{v_3 = I_1 R_1 + v_2}$$

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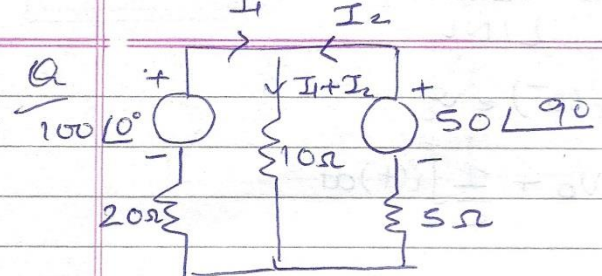
Q find I_1, I_2



$$100\angle 0^\circ = 10(I_1+I_2) + 20 I_1$$
$$100\angle 0^\circ = 30 I_1 + 10 I_2$$

θ alway ... +ve x axis

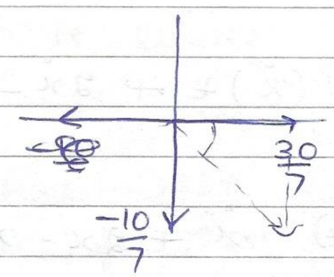
$$50\angle 90^\circ = 10(I_1+I_2) + 5 I_2$$
$$10\ \cancel{50}\angle 90^\circ = \overset{2}{\cancel{10}} I_1 + \overset{3}{\cancel{15}} I_2$$
$$10\angle 90^\circ = 2I_1 + 3I_2$$
$$30\angle 0^\circ = 9I_1 + 3I_2$$

$$10\angle 90 - 30\angle 0^\circ = -7I_1$$
$$7I_1 = 30\angle 0^\circ - 10\angle 90^\circ$$
$$I_1 = \frac{30}{7}\angle 0^\circ - \frac{10}{7}\angle 90^\circ$$

$$\text{Mag} = \sqrt{\left(\frac{30}{7}\right)^2 + \left(-\frac{10}{7}\right)^2}$$

$$\theta = \tan^{-1}\left(\frac{10}{7} \times \frac{7}{30}\right)$$

tan is −ve.



Q L/NL / C/NC / TV/TI

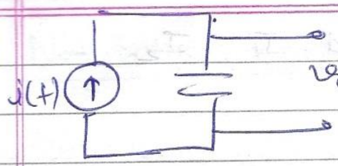
1) $y_n = 6x^2(n-2)$ $\quad y_n = 6x(n-2)\,x(n-2)$ $\quad$ NL, T..., C

2) $n^3 y_n = 6x(n-2)$ $\quad$ L, TV, C

3) $y_n = 3x(n) - 2x(n-1)$ $\quad$ L, TI, C

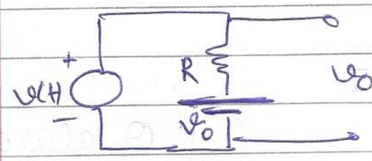
4) $3x(n-1) + x(n+1) = y_n$ $\quad$ L, TI, NC

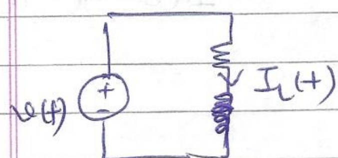
find if these ckts are causal / Non causal
TV / TI , L / NL

6) 

$v_C(0^-) = v_0$

$$v_C(t) = V_0 + \frac{1}{C}\int_0^t i(t)\,dt$$

7) 

8) 

$I_L(0^-) = I_0$

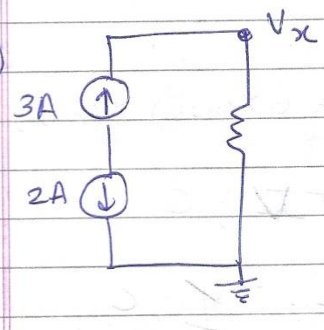
9) $\ddot{x} + 2\dot{x} - 3x = \sin 4t$ L , TI , C

10) $(\ddot{x})^2 + 2\dot{x} - 3x = \sqrt{\sin 4t}$ NL , TI , C

11) $(\ddot{x})t + 2\dot{x} - 3x = \sin 4t$ L , TV , C

12) $\ddot{x} + 2x \cdot x - 3x = 4t$ NL , TI , C

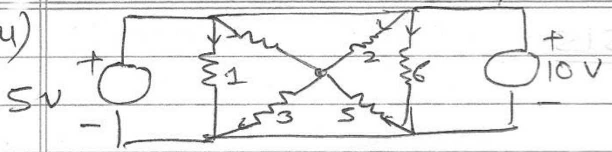
↓ These all are forcing fn. i.e from where energy is obtained.

13) 

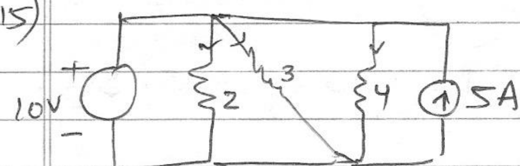
find V_x

Sol Not a valid representation
2 current sources in series
Not possible.

Unrealiable ckt.

14) 

Not valid as 2 voltage source in parallel.

15) 

In this I source redundant.

V across 2, 3, 4 = 10 V

$I_{(2)} = \frac{10}{2} = 5A$

$I_{(3)} = \frac{10}{3}$

$I_{(4)} = \frac{10}{4} = 2.5$

6) Non linear. even though eq^n is linear.

7) Linear

first, for homogenity check that for zero I/P, zero O/P, then further ~~the~~ check if linear.

8) Non linear.

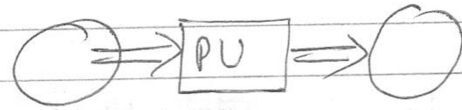
If zero I/P, no zero O/P = Non linear.

$$I_L(t) = I_0 + \frac{1}{L}\int_0^t \left[v(t) - I_L(t).R\right] dt$$

⇒ for linear – diff. eq^n should be linear + for 0 I/P, 0 O/P.

NETWORK ANALYSIS

⇒ Signals



I/P = Energy / Material / Info ⇓ i.e signals

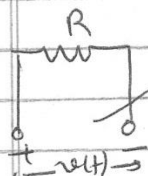
O/P = Energy / Mat. / Info.

eg:- coal given in furnaces.

⇒ Information carriers are called signals

$v(t) = \sin t$ — v(t) not voltage

$v(t) = \sin t \; V$ — v(t) = voltage



$v(t) = \sin t \; V$ ⇒ not strictly signal.

Above all 3, not signals.

Signal → Static = $v(t) = \sin t \; v$ = $v(t) = 5\,V$

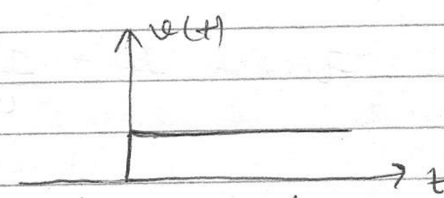
Signal → Dynamic

Strict def: signals carry info which is variable.

Loosely, DC signal also signal

⇒ Signal Representation

1) dc signal.



dc causal signal

Signal 0 for $t < 0$

2) Sinosoidal signal.

$$v(t) = \sqrt{2} A \sin(\omega t + \phi)$$

A = RMS value of $v(t)$

3) Exponential signal

$$v(t) = A e^{st}$$

A, s = complex quantities.

Any continuous time signal can be represented in this form

$\rightarrow$ $s = \sigma + j\omega$

complex, Neper, Radian

$$v(t) = (a+jb) e^{st} + (a-jb) e^{st}$$

$$= 2a e^{st}$$

For any physical system A – real.

$$v(t) = A e^{(\sigma+j\omega)t} + A e^{(\sigma-j\omega)t}$$

$$= A \left(e^{\sigma t} \cdot e^{j\omega t} + e^{\sigma t} \cdot e^{-j\omega t} \right]$$

$$= A e^{\sigma t} \left[e^{j\omega t} + e^{-j\omega t} \right]$$

$$\cancel{\left[e^{(j\omega - j\omega)t} \right]}$$

$$= 2A e^{\sigma t} \cos\omega t$$

$$\boxed{v(t) = 2A e^{\sigma t} \cos\omega t}$$

1) $\cos(2t + 30°)$ $\qquad \sigma = 0$, $\omega = 2$

$s = \pm 2j$

2) $2\cos 3t + 4\sin 3t$

$s = \pm 3j$ $\qquad s = \pm 3j$

$\sigma = 0$
static signal

3) $e^{-2t} + e^{-3t}\cos(4t + \theta)$

$\sigma = -2$

$s = -2 \pm j0$ $\qquad s = -3 \pm 4j$

4) $e^{j2t} + e^{-3t}\cos(4t + \theta)$ $\longrightarrow$ Cos or sin must contain $e^{j\omega t}$ and $e^{-j\omega t}$. That is why $\pm$

only +ve $\quad s = + 2j$ $\qquad s = -3 \pm 4j$

5) $4 + 5e^{2t}\cos(5t + \theta)$

$s = 0$ $\qquad s = 2 \pm 5j$

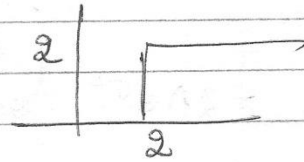
Singularity functions

$\Rightarrow$ used to represent a discontinuous fn OR a fn having discontinuous derivative.

1) Unit step

a) $2u(t-2)$

b) $2u(2-t)$



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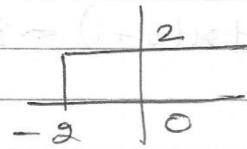
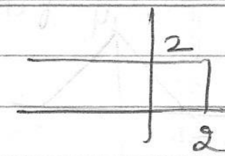
b) $2u(-t+2)$

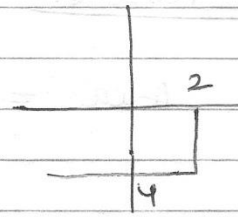


$\Rightarrow$

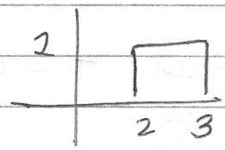


c) $-4\,u(2-t)$

$-4\,u(-t+2)$ =

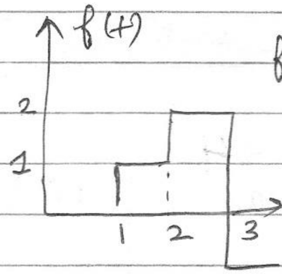


d)



$u(t-2) - u(t-3)$

e)



$f(t) = u(t-1) + u(t-2) - 3u(t-3)$

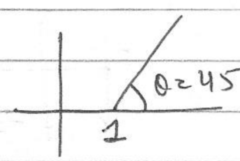
$f(2) = 1 + 1$

$= 2$

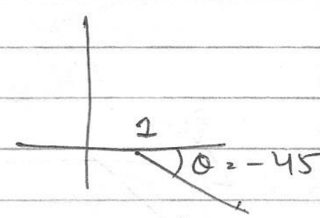
$f(3) = -1$

2) <u>Ramp signal</u>

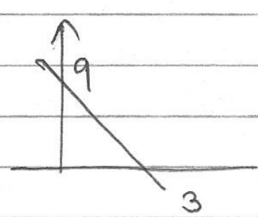
a) $r(t-1)$



b) $-r(t-1)$ $\longrightarrow$



c)

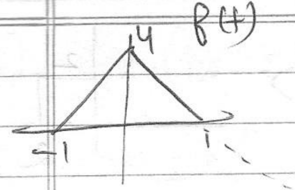


$3r(3-t)$

0,9

3,0

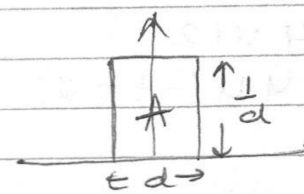
$\frac{9}{3} = 3$

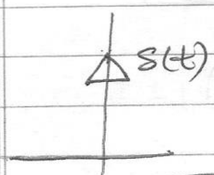
4) 

$4r(t+1) - 8r(t) + 4r(t-1)$

3) Impulse Function

$\lim_{d \to 0}$ Area $= A$



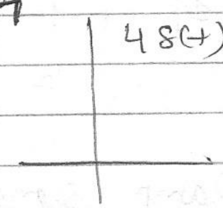


$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \text{not defined or } \infty & t = 0 \end{cases}$$

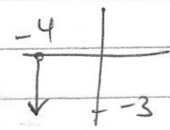
$$\boxed{\int_{0^-}^{0^+} \delta(t)\,dt = 1}$$

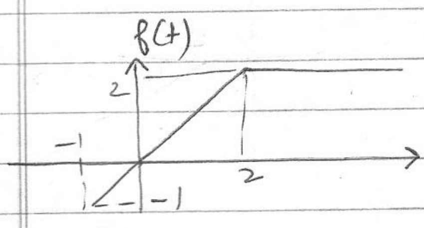
1) 4 represents area under curve = 4
not magnitude

$$\int_{-0}^{0^+} f(t) = 4$$



2) $-3\delta(t+4)$



3) 

Plot $f(t)\,u(t)$, $f(t-1)\cdot u(t)$, $f(t)\cdot u(t-1)$

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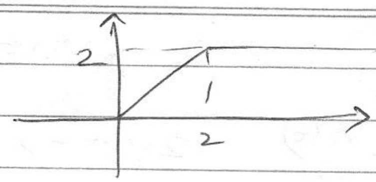
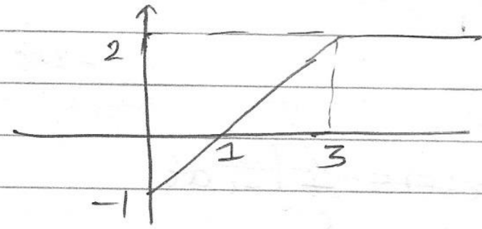
1) $f(t)\, u(t)$

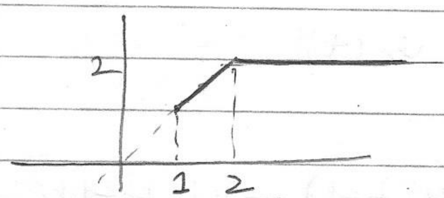


2) $f(t-1) \Rightarrow$



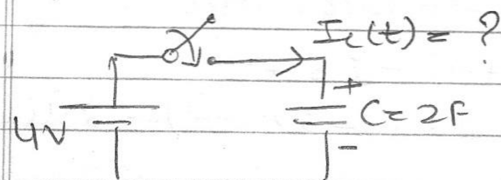
$f(t-1)\, u(t) =$ same

3)



$f(t)\, u(t-1)$

Q



$v_c(0^-) = 0$

find $I_c(t)$

Soln

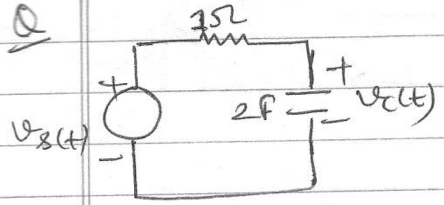
$4u(t) = I/P$ Now close switch

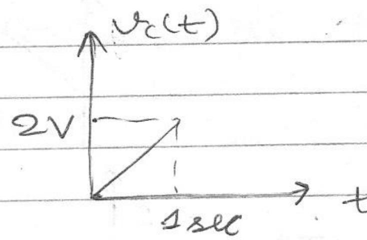
$$I_c = C\frac{dv_c}{dt} = 2\frac{d}{dt}4u(t)$$

$$I_c = 8\,\delta(t)$$

i.e in zero time ∞ current flows and max. charge is attained by capacitor.

Q





Find $v_s(t)$

(1, 2)

(0,0) $\frac{2}{1} = 2$

~~$v_s(t) =$~~

$\cancel{v_c(t)} = \frac{1}{C}\int I_c\, dt$

$$v_s(t) = I_c(t)\, 1 + v_c(t)$$

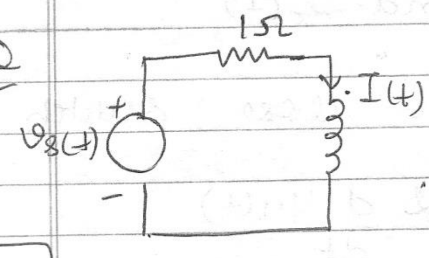
$$= C\frac{d v_c(t)}{dt} + v_c(t)$$

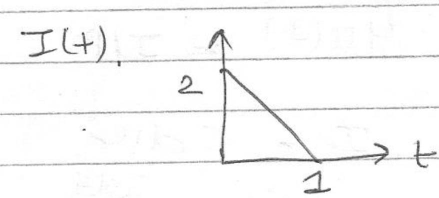
$$= 2\frac{d v_c(t)}{dt} + v_c(t)$$

$$v_c(t) = 2r(t) - 2r(t-1) - 2u(t-1)$$

$$v_s(t) = 2\{2u(t) - 2u(t-1) - 2\delta(t-1)$$
$$+ 2r(t) - 2r(t-1) - 2u(t-1)$$

Q





↙ −ve
↗ +ve

$$v_s(t) = 1\cdot \cancel{I(t)} + L\frac{dI(t)}{dt}$$

(0,2)

(1,0)

$\frac{2}{1}$

$$v_s(t) = I(t) + 1\frac{dI(t)}{dt}$$

$\cancel{2r(t)} = \cancel{2r(t-1)}$ $\cancel{2r(-t-1)}$

Signal = $2u(t) - 2r(t) + 2r(t-1)$

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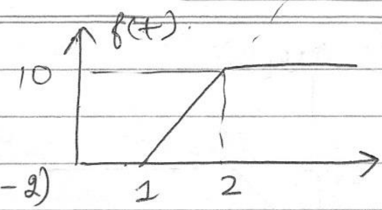
Q. (2,10) (1,0)

$\frac{10}{2-1} = \underline{10}$



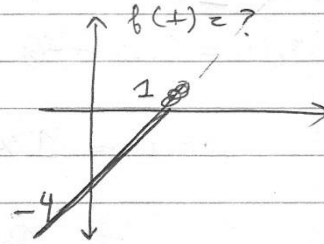
Find f(t) $10\,r(t-1) - 10\,r(t-2)$

~~10 r(t-1) — 10 r(t-2)~~

~~10 u(t-1) + 10 r(t-1) — 10 r(t-2)~~

Q (0,−4) (1,0)

$\frac{-4}{-1} = 4$



$4\,r(t-1)$

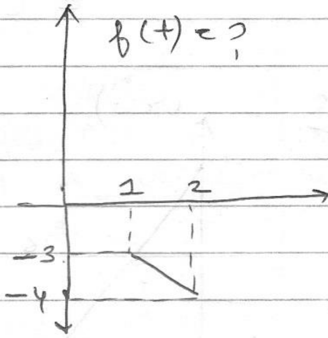
$4\,r(1-t)$

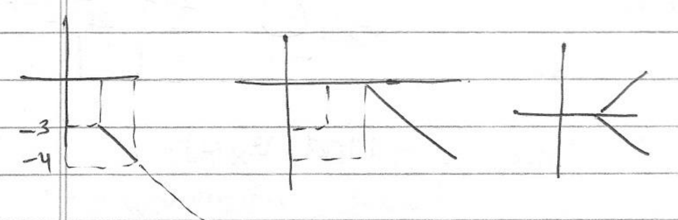
$\boxed{-4\,r(1-t)}$

Q

$-3u(t-1) - r(t-1)$ (1,−3) (2,−4)

$+ 4u(t-2) + r(t-2)$ $\frac{-3+4}{1-2} = \frac{1}{-1}$

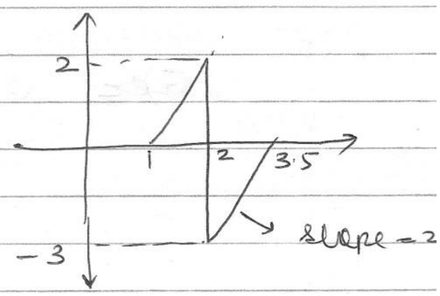




Q

$2\,r(t-1) - 5u(t-2)$

$- 2\,r(t-3.5)$.

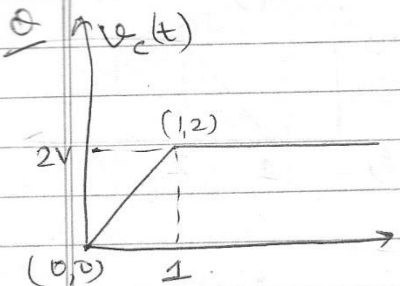


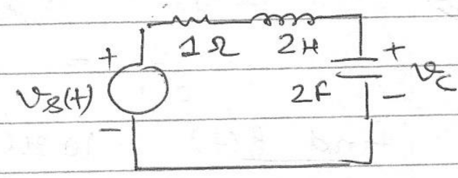
(2, 2) (1,0)

$\frac{2-0}{2-1} = 2$

(0, −3) 0.35

Q





$v_c(t) = 2r(t) - 2r(t-1)$

I through circuit

$$I_c = C\frac{dv_c}{dt}$$

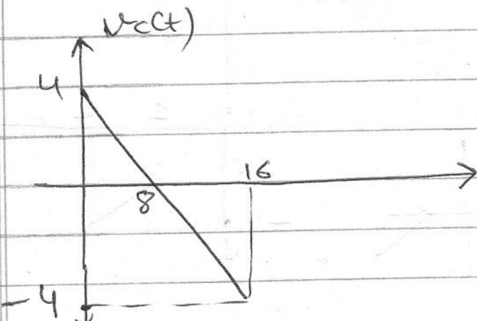
$$v_s(t) = 1 \times I_c + L\frac{dI_c}{dt} + v_c(t)$$

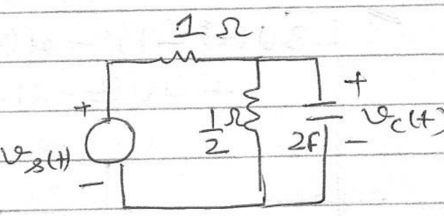
$$= C\frac{dv_c}{dt} + L\frac{d}{dt}\left(C\frac{dv_c}{dt}\right) + v_c(t)$$

$$= C\frac{dv_c}{dt} + LC\frac{d^2v_c}{dt} + v_c(t).$$

Q

(0,4)
(8,0)
$\frac{4-0}{0-8}$
$-\frac{1}{2}$





Find $v_s(t)$.

$$v_s(t) = 1\,i_c(t) + v_c(t)$$

$$v_s(t) = v_c(t) + C\frac{dv_c(t)}{dt} + \frac{v_c(t)}{\frac{1}{2}}$$

$$= v_c(t) + 2v_c(t) + C\frac{dv_c(t)}{dt}$$

$$= 3v_c(t) + 2\frac{dv_c(t)}{dt}.$$

Signal $= 4u(t) - \frac{1}{2}r(t) + 4u(t-16) + \frac{1}{2}r(t-16)$

$$12u(t) - \frac{3}{2} r(t) + 12u(t-16) + \frac{3}{2} r(t-16))$$

$$+ 2 \left(4\delta(t) - \frac{1}{2} u(t) + 4\delta(t-16) + \frac{1}{2} u(t-16)\right)$$

$$12u(t) - \frac{3}{2} r(t) + 12u(t-16) + \frac{3}{2} r(t-16) + 8\delta(t)$$

$$- u(t) + 16\delta(t-16) + u(t-16))$$

$$\Rightarrow 11u(t) - \frac{3}{2} r(t) + 13u(t-16) + \frac{3}{2} r(t-16)$$

$$+ 8\delta(t) + 16\delta(t-16)$$

State of system.

A system is said to be in steady state when its behaviour across all the elements of the system are either a constant or a regular type of periodic fn.

Variation of any state variable during the transition from one steady state to another steady state is smooth. untill unless there is some impulsive type of action happens inside the system. ~~it~~ In presence of impulsive type of action, it may be possible that state variable changes sharply from one state to another.

ith $\frac{d(x)}{dt}$ dt cannot be zero. i.e no impulse.

∞ response means Impulse.

Different Ways of Characterizing LTI system -

1) Differential eq n approach
2) Impulse Response
3) Step Response
4) Black BOX Description.

$\Rightarrow$ In general, total no. of ~~independent~~ dynamic ~~variables~~ =

dynamic elements present in a system = No. of state variables = Order of system = No. of poles = No. of zeroes = No. of natural ν

$\Rightarrow$ No. of poles alway equal to No. of zeroes.

$\Rightarrow$ Natural ν or poles location represents freq. of the system under zero excitation.

i.e freq. of system when it is switching from one ~~transi~~ steady state to another.

$$x(t) = Ae^{st} \longrightarrow \boxed{\text{LTI CTS}} \longrightarrow y(t)$$

$x(t) = Ae^{st}$: eigen signal or characteristic signal for C.T.S

$$y_{P.I} = \frac{g(s)}{f(s)} \cdot Ae^{st}$$

$\frac{g(s)}{f(s)}$: system fn.

$\boxed{\text{Laplace}}$

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$\Rightarrow$ 2) If $\sigma = 0$

$s = \pm j\omega$ (Fourier)

$$x(t) = Ae^{\pm j\omega t} \longrightarrow \boxed{\begin{matrix}\text{LTI}\\ \text{CTS}\end{matrix}} \longrightarrow y_{PI} = \frac{g(j\omega)}{f(j\omega)} A e^{\pm j\omega t}$$

3) If excitation available at discrete intervals.

$$x(t) = A e^{sTn} \qquad t = nT.$$

$$\longrightarrow \boxed{\begin{matrix}\text{LTI}\\ \text{(DTS)}\end{matrix}} \longrightarrow y(z) = \frac{g(z)}{f(z)} \cdot x(n)$$

$x(t) = Az^{n}$ $\qquad$ n = variable.

z = complex ν of discrete time system.

$y(z)$ in form of difference equation.

$$a_2\, y_{(n+2)} + a_1\, y_{(n+1)} + a_0\, y_{(n)} = b_2\, x_{(n+2)} + b_1\, x_{(n+1)} + b_0\, x_{(n)}$$

$$(a_2 E^2 + a_1 E + a_0)\, y_{(n)} = [b_2 E^2 + b_1 E + b_0]\, x_n$$

$$y_{(n)} = \left(\frac{b_2 E^2 + b_1 E + b_0}{a_2 E^2 + a_1 E + a_0}\right) x_n$$

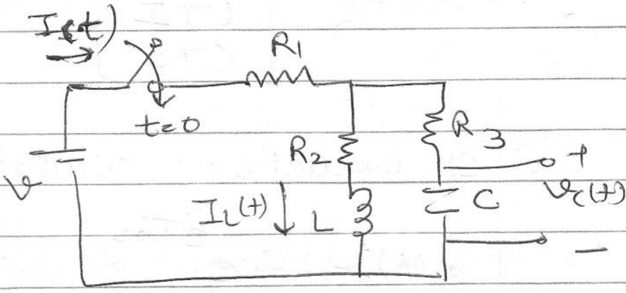
$$= \frac{g(E)}{f(E)}\bigg|\; x(n).$$

$$y_{(PI)} = \frac{g(E)}{f(E)}\bigg|_{E = \text{complex } \nu \text{ i.e } Z}\; x(n)$$

steady state Analysis of Dynamic N/W with DC excitation

1) closed at $t = 0$

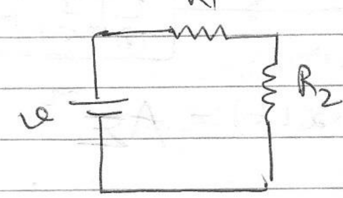
find $i(t)$, $v_C(t)$, $i_L(t)$



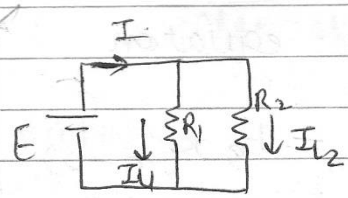
Soln $I_{ss} = \frac{v}{R_1 + R_2}$

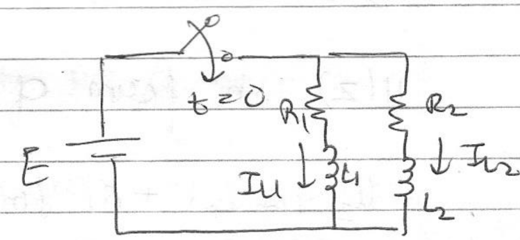
$I_{L_{ss}} = I_{ss}$

$v_{Css} = \frac{R_2}{R_1 + R_2} v$



2)





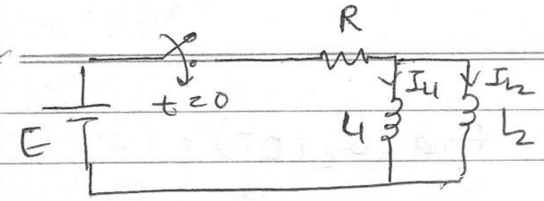
$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

$I = \frac{E(R_1 + R_2)}{R_1 R_2}$

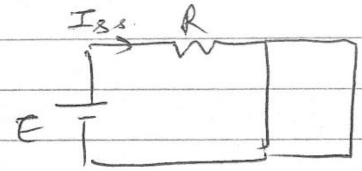
$I_{L1_{ss}} = \frac{R_2}{\cancel{(R_1 + R_2)}} \frac{E\cancel{(R_1 + R_2)}}{R_1 R_2} = \frac{\cancel{R_2} E}{R_1 \cancel{R_2}} = \frac{E}{R_1}$

$I_{L2_{ss}} = \frac{\cancel{R_1} E}{\cancel{R_1} R_2} = \frac{E}{R_2}$

3



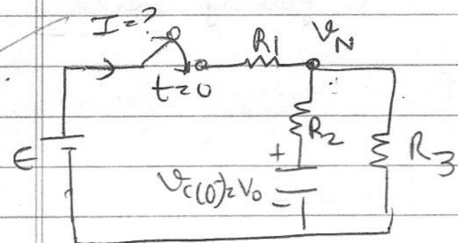
=

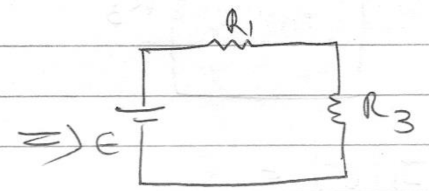


$$I_{ss} = \frac{E}{R}$$

$$I_{L1ss} = I_{L2ss} = \frac{E}{2R}.$$

4



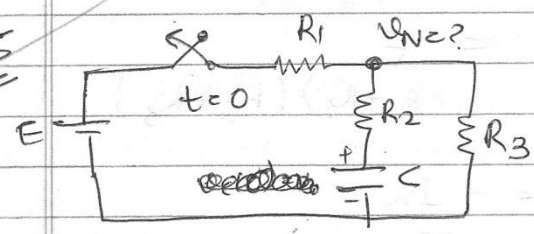


$$I_{ss} = \frac{E}{R_1+R_3}$$

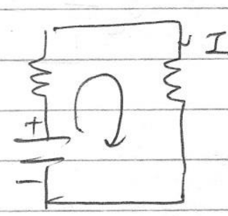
$$v_N = \frac{E}{R_1+R_3} R_3 = V_{0(ss)}$$

v_R ..

5

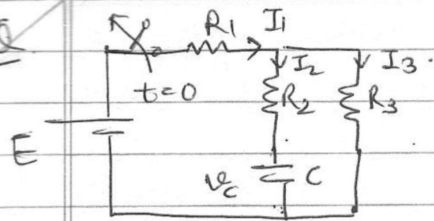


⟹



after ∞ time $I = 0$.

Q

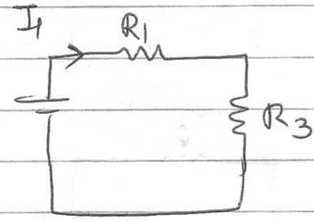


find $v_c(0^-)$.

$$I = \frac{E}{R_1+R_3}$$

$$V = \frac{E}{(R_1+R_3)} R_3 = v_c(0^-) = v_c(0^+).$$

0^-



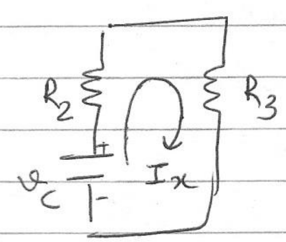
C fully charged

$$I_1(0^-) = \frac{E}{R_1+R_3} \qquad I_2(0^-) = 0$$

$$I_3(0^-) = I_1(0^-).$$

0^+

$$I_1(0^+) = 0.$$



$$I_x = \frac{v_c(0^+)}{R_2+R_3}$$

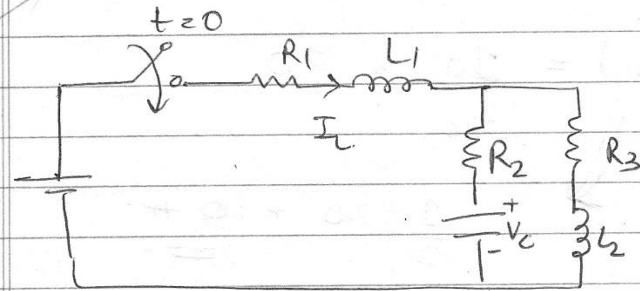
$$= \frac{E}{(R_1+R_3)(R_2+R_3)}$$

$$I_2(0^+) = -I_x$$

$$I_3(0^+) = I_x$$

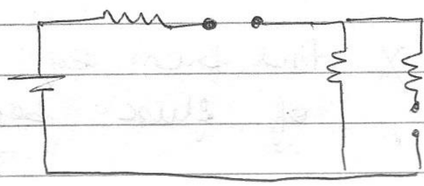
$I_1(\infty) = 0$
$I_2(\infty) = 0$
$I_3(\infty) = 0$
$v_C(\infty) = 0$

Q



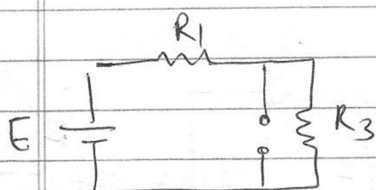
$v_C(0^-) = I_{L1}(0^-)$
$= I_{L2}(0^-) = 0$

(0^+)



$v_C(0^+) = 0$
$I_{L1}(0^+) = 0$
$I_{L2}(0^+) = 0$
$I_C(0^+) = 0$

(∞)



$I_{L1}(\infty) = \dfrac{E}{R_1 + R_3}$ $\qquad I_{L2} = I_{L1}(\infty)$

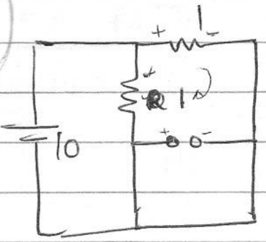
$v_C(\infty) = \dfrac{E R_3}{R_1 + R_3}$

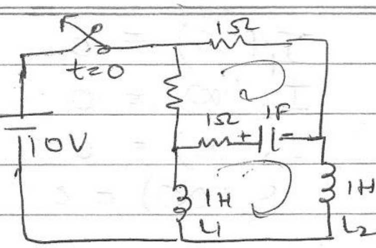
$I_C(\infty) = 0$

$\frac{1\times1}{1+1} = \frac{1}{2}$

Date ....
Page ....

Q (0^-)





$I_{L1}(0^-) = I_{L2}(0^-) = \frac{10}{1/2} = 20$

$= \frac{1}{2} \times 20 = 10\ A.$

$v_C(0^-) = 0\ V$

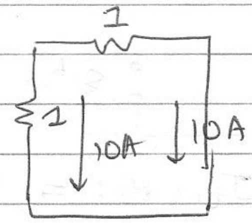
$\left.\begin{array}{l} I_{L2}(0^+) = I_{L2}(0^-) \\ I_{L1}(0^+) = I_{L1}(0^-) \end{array}\right\} \times$ find from eq^n of continuity of flux because

current flow in both loops as there are 3 energy storage devices.

$I_{L1}(\infty) = 0$

$I_{L2}(\infty) = 0$

$v_C(\infty) = 0$



At (0^+)

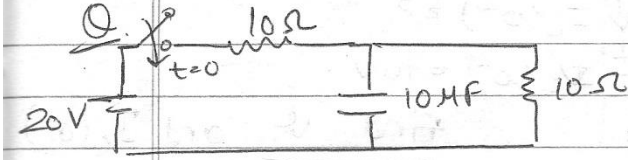
15/2/2014

same current now has to flow through the loop.

i.e impulsive type of action.

current through Inductor is forcibly changed.

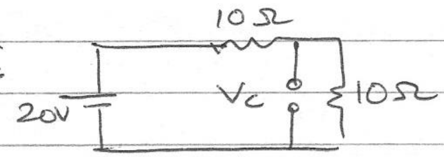
Q.



find $v_c(0^+)$

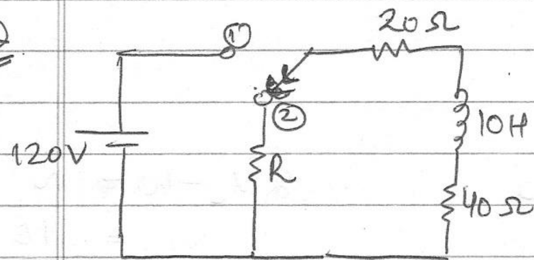
$= 0V$

Soln



$v_c(\infty) = 10V$

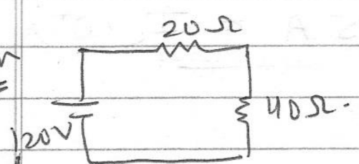
Q



Switch initially at 1 for long time. moved at point 2 at $t = 0$.

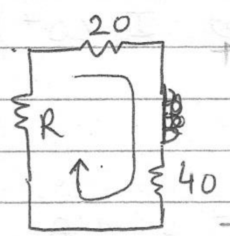
At $t = 0^+$, the voltage across the coil i.e (L with R) is 120 V. The value of resistance R ?

Soln



$I_L(0^-) = \cancel{480}\ \frac{12\cancel{0}}{6\cancel{0}} = 2A$

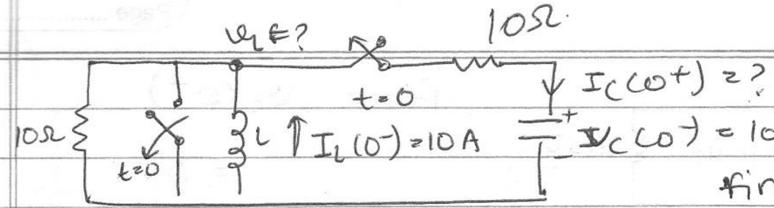
i.e $I_L(0^+) = 2A$



$20 \times 2 + 120 + R(2) = 0$

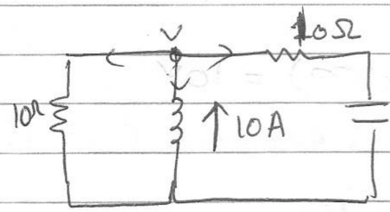
$$\boxed{\frac{120}{20+R} = 40\,\Omega}$$?

Q.



find v_L and $I_C(0^+)$

Soln (0^+)



$$\frac{V_L}{10} + \frac{V_L - 10}{10} = 10$$

$$\frac{2V_L - 10}{10} = 10$$

$$\cancel{3V_L - 10 = 200}$$

$$3V_L = 210$$

$$\cancel{\boxed{V_L = 70}}$$

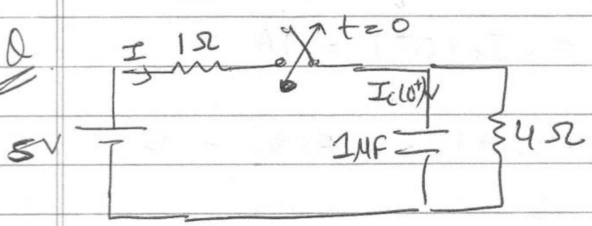
$$2V_L - 10 = 100$$

$$V_L = \frac{110}{2}$$

$$\boxed{V_L = 55}$$

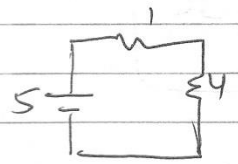
$$\frac{55 - 10}{10} = \frac{45}{10} = \underline{4.5\,A = I_C(0^+)}$$

Q



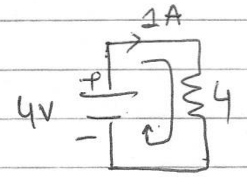
find $I_C(0^+)$.

Soln



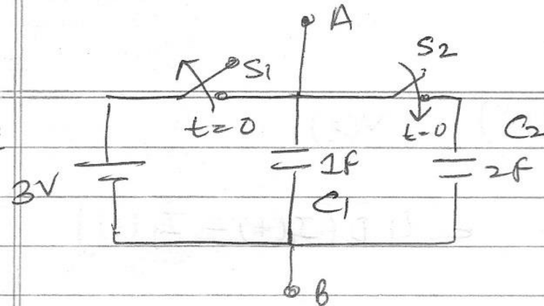
$$I(0^-) = \frac{5}{5} = 1A$$

$$v_C(0^-) = 4 \times 1 = 4V$$



$$\boxed{I_C(0^+) = -1A}$$

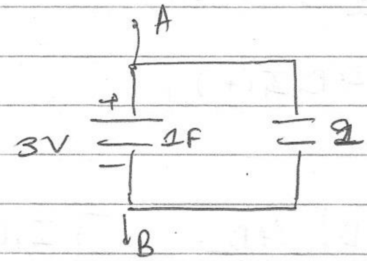
Q



Initially S_1 closed
S_2 open
$v_{C2}(0^-) = 0V$

The voltage across AB $V_{AB}(0^+) = ?$

Soln. $\underline{0^+}$



Impulsive type of action.

In parallel $C_{eq} = C_1 + C_2 = 3F$

$$q = q_1 + q_2$$

$$q = 3 \times 1 = 3C$$

$$\frac{q_1}{1} = \frac{q_2}{2} = \frac{\cancel{3}}{\cancel{3}} = 1$$

$$q_1 = 1$$

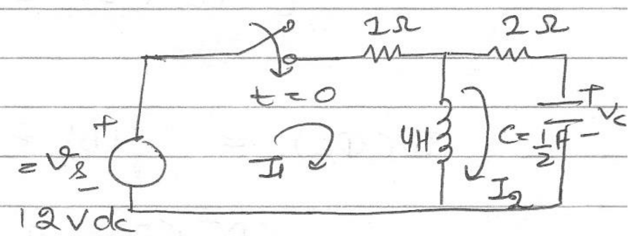
$$q_2 = 2$$

$$\boxed{V_{AB} = \frac{q_1}{C_1} = \frac{1}{1} = 1V}$$

TIME DOMAIN ANALYSIS OF Electrical N/W

Q $I_L(0^-) = 0$
$V_C(0^-) = 0$

cal. $I_1(t)$.



$$I_1(t) + 4\frac{d}{dt}\left(I_1(t) - I_2(t)\right) = V(s).$$

$$2I_2(t) + 2\int I_2(t)\,dt = 4\frac{d}{dt}\left(I_1(t) - I_2(t)\right).$$

$I_1(t) + 4D(I_1(t) - I_2(t)) = V(s)$

$2I_2(t) + 2\frac{1}{D}I_2(t) = 4D(I_1(t) - I_2(t))$

$I_1(t)[1+4D] - 4DI_2(t) = V(s)$

$\left(2+\frac{2}{D}+4D\right) I_2(t) = 4DI_1(t)$

$I_1(t)[1+4D] - 4D\left[\frac{4D}{2+\frac{2}{D}+4D}\right]I_1(t) = V(s)$

$\left[1+4D - 4D\left(\frac{4D}{2+\frac{2}{D}+4D}\right)\right]I_1(t) = V(s)$

$\left[1+4D - \frac{16D^2(D)}{2D+2+4D^2}\right]I_1(t) = V(s)$

$\frac{2D+2+4D^2+8D^2+8D+\cancel{16D^3}-\cancel{16D^3}}{2D+2+4D^2}$

$\left[\frac{12D^2+10D+2}{4D^2+2D+2}\right]I_1(t) = V(s)$

$I_1(t) = \left(\frac{2D^2+D+1}{6D^2+5D+1}\right)V(s)$

$I(cfs) = f(D) = 0$

$6D^2+5D+1 = 0$

$(5)^2 - 4\times 6\times 1$

$25 - 24$

$= 1$

$\frac{-5\pm 1}{2\times 6} = -\frac{6}{2\times 6}$

$= -\frac{1}{2}, -\frac{1}{3}$

$D = -\frac{1}{2}, -\frac{1}{3}$

$k_1 e^{-\frac{1}{2}t} + k_2 e^{-\frac{1}{3}t}$

complex ν present in PI sol^n ~~or~~ of ~~steady state~~ response is same as ν of excitation

$$I_1(PI) = \left.\frac{g(D)}{f(D)}\right|_{D = \text{complex } \nu} \cdot V(s)$$

$= 1 \cdot 12$

$= \underline{12}$

$\therefore \; I_1(t) = 12 + k_1 e^{-\frac{1}{2}t} + k_2 e^{-\frac{1}{3}t}$

At ∞ mag. of this fn $= 0$
but this signal is present

$I_1(0^+) = I_1(0^-) = 0$

$V_C(0^+) = V_C(0^-) = 0$

$I_1(0^+) - I_2(0^+) = I_L(0^+) = 0$

$I_1(0^+) = I_2(0^+)$

$\boxed{I_L'(0^+) = 2\,A/s}$

$I_1(0^+) = 4A = I_2(0^+)$

$v_L(0^+) = 8\,V$

$4 = C\frac{dv_C(0^+)}{dt}$ $\qquad v_C'(0^+) = 8\,V/sec$

$I_1'(0^+), \; I_2'(0^+)$

$V(s) = I_1R_1 + I_2R_2 + \underbrace{2\int I_2\,dt}_{v_C}$ or

$0 = I_1'(R_1) + I_2'R_2 + \cancel{2I_2} \cdot v_C'$

$I_1'(0^+) + I_2'(0^+)\,2 + 8 = 0$

$$I_2 R_2 + v_C = L \frac{d}{dt}(I_1 - I_2)$$

$$2 I_2(0^+) + v_C(0^+) = 4\left[I_1'(0^+) - I_2'(0^+)\right].$$

$$2 \times 4 + 0 = 4\left[I_1'(0^+) - I_2'(0^+)\right]$$

$$4 I_1'(0^+) - 4 I_2'(0^+) = 8$$

$$\cancel{I_1'(0^+) + 2 I_2'(0^+) = -8}$$

$$\cancel{8 I_1'(0^+) - 2 I_2'(0^+) = 16}$$

$$\cancel{9 I_1'(0^+) = 8}$$

$$I_1'(0^+) = -\frac{4}{3} \qquad I_2'(0^+) = -\frac{10}{3}$$

$$I_1(0^+) = 12 + k_1 + k_2$$

$$I_1'(0^+) = 0 + k_1\left(-\frac{1}{2}\right) e^{-\frac{1}{2}t} + k_2\left(-\frac{1}{3}\right) e^{-\frac{1}{3}t}$$

$$I_1(0^+) = 0 - \frac{1}{2} k_1 - \frac{1}{3} k_2$$

$$I_1(t) = \frac{2D^2 + D + 1}{6D^2 + 5D + 1} V(s)$$

roots.

Upper eq^n roots $\quad (1)^2 - 4 \times 2 \times 1$

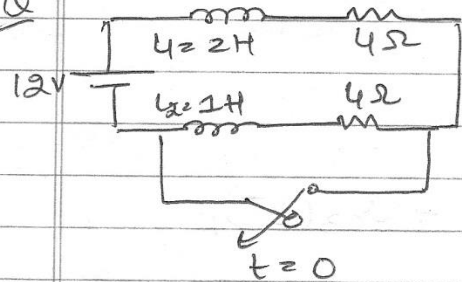
$$1 - 8 = -7$$

$$\frac{-1 \pm \sqrt{7}\, j}{4}$$

$\Rightarrow$ If zeroes = complex ν of excitation then response zero at steady state.

Special case of discontinuity for C and L

Q Case I



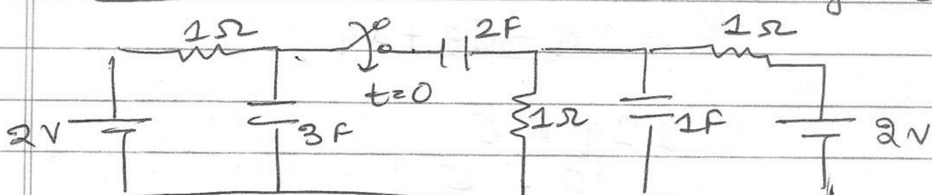
Sol $I_{L_1}(0^-) = 3A$ $I_{L_2}(0^-) = 0$.

Equation of continuity of Flux.

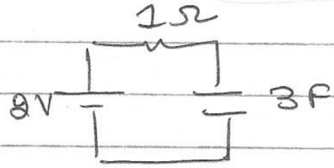
$L_1 I_{L_1}(0^-) + L_2 I_{L_2}(0^-) = L_1 I_{L_1}(0^+) + L_2 I_{L_2}(0^+)$.

tells only magnitude of current.

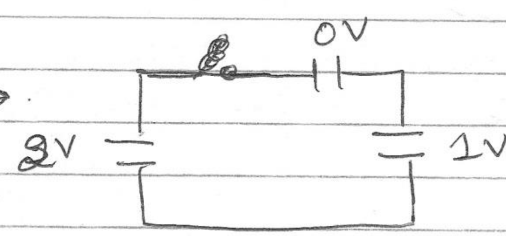
Case II — discontinuous voltage of capacitor



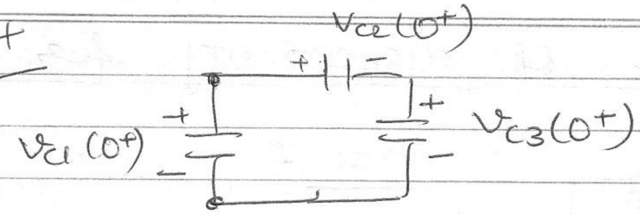
0^-



$0^- \rightarrow$



<u>0^+</u>



Impulsive type of action.

$$v_{c1}(0^+) = v_{c2}(0^+) + v_{c3}(0^+).$$

Let q amount of charge flow through the loop in the interval of $t = 0^-$ to 0^+.

series − q <u>same</u>

$$v_{c1}(0^-) - \frac{q}{C_1} = v_{c2}(0^-) + \frac{q}{C_2} + v_{c3}(0^-) + \frac{q}{C_3}.$$

$$2 - \frac{q}{3} = 0 + \frac{q}{2} + 1 + \frac{q}{1}$$

$$1 = \frac{q}{2} + q + \frac{q}{3}$$

$$6 = 3q + 6q + 2q$$

$$11q = 6$$

$$\boxed{q = \frac{6}{11}}$$

$$i = \frac{dq}{dt} u(t)$$

$$i = \frac{6}{11} \delta(t)$$

Impulse Response

$\delta(t) \rightarrow \boxed{\text{LTI Causal}} \rightarrow \quad h(t) = 0 \;, \; t<0$

$x(t) \rightarrow \boxed{\text{LTI Causal}} \rightarrow \quad y(t) = \int_{0^-}^{t^+} x(\tau)\, h(t-\tau)\, d\tau .$

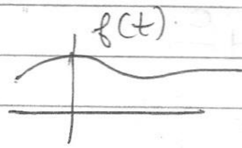
$\downarrow$
causal

Impulse may occur at 0^- to 0^+

If $\tau > t \quad h(t-\tau) = 0$
If impulse occur at t, so integration to be taken till t^+ as it has to be included.

$\Rightarrow$ If convolution integral $= \infty$, i.e system unstable.

Q 1) $f(t) * \delta(t)$.



2) $u(t) * u(t)$

3) $\delta(t) \rightarrow \boxed{\text{LTI} \; \text{I.C}=0} \rightarrow \quad h(t) = (2e^{-2t} - e^{-t})\, u(t)$

Find response if excitation

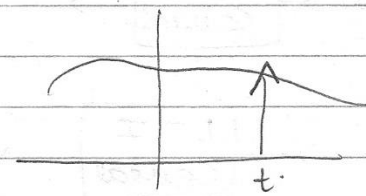
a) $x(t) = u(t)$
b) $x(t) = e^{-3t}\, u(t)$

4) Impulse response of initially relaxed LTI N/W is $4e^{-2t}u(t)$. find response of same N/W to an I/P

a) $u(t)$
b) $e^{-t}\, u(t)$

1) $f(t) * \delta(t)$

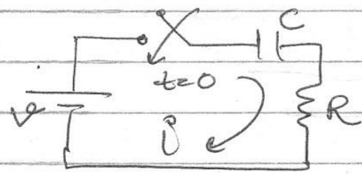
$\int_{-\infty}^{\infty} f(\tau) \cdot \delta(t-\tau)\, d\tau$



5) For a SISO LTI system having system fn $H(s) = \dfrac{2s+1}{s^2+s+1}$. find the complex freq in the natural response of system. Find the forced response to an excitation

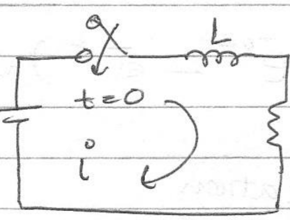
a) e^{-2t}

b) $e^{-\frac{1}{2}t} \cos(\sqrt{3}t + 70°)$

6) 

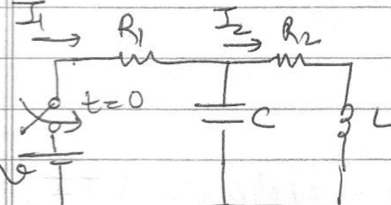
$v_C(0^-) = 0$

cal $i(0^+)$, $i'(0^+)$

7) 

$I_L(0^-) = 0$

find $i(0^+)$ and $i'(0^+)$

8) 

$v_C(0^-) = 0$

$I_L(0^-) = 0$

find $I_1(0^+)$, $I_2(0^+)$

$I_1(\infty)$, $I_2(\infty)$.

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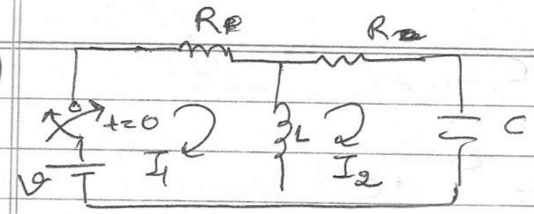
9)

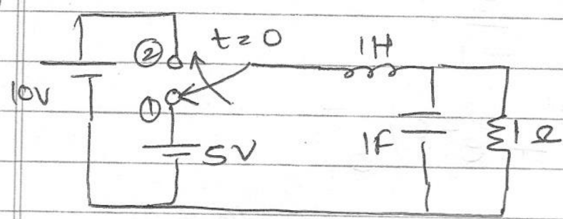


I.C = 0

Cal $I_1(0^+)$, $I_2(0^+)$

$I_1'(0^+)$, $I_2'(0^+)$

10)



① to ② at t = 0

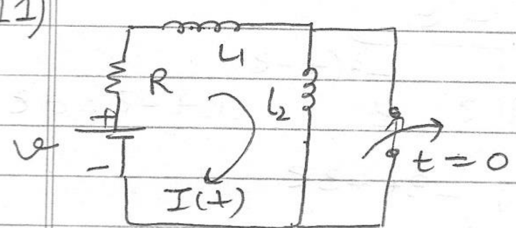
find $I_L(0^-)$, $I_L(0^+)$

$V_C(0^-)$, $I_L'(0^+)$

$V_C(0^+)$, $V_C'(0^+)$

$I_L(\infty)$, $V_C(\infty)$

11)

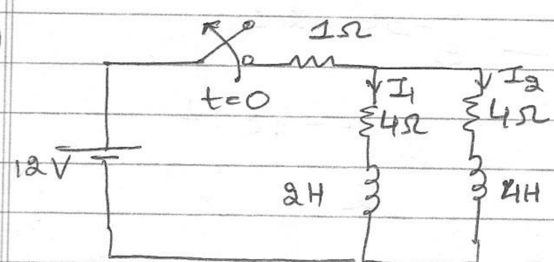


Open at t = 0

Find $I_{L_1}(0^-)$, $I_{L_2}(0^-)$

I(t)

12)



find.

$I_1(0^-)$ & $I_2(0^-)$

$I_1(0^+)$, $I_2(0^+)$

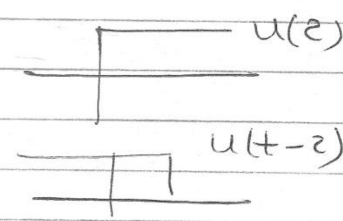
2

$u(t) \circledast u(t)$

$$\int_0^t u(\tau)\, u(t-\tau)\, d\tau$$

$$\int_0^t 1\, d\tau$$

$$= t = r(t)$$



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3. a)

$$\int_0^t \left(2e^{-2\tau} - e^{-\tau}\right) u(\tau)\ u(t-\tau)$$

$$\int_0^t \left(2e^{-2\tau} - e^{-\tau}\right) d\tau$$

$$\frac{2e^{-2\tau}}{-2} + \frac{e^{-\tau}}{+1}$$

$$\left(-e^{-2\tau} + e^{-\tau}\right)_0^t$$

$$-e^{-2t} + e^{-t} + \cancel{e^{0}} - \cancel{e^{0}}$$

$$= \boxed{-e^{-2t} + e^{-t}}$$

b)

$$\int_0^t \left(2e^{-2\tau} - e^{-\tau}\right) u(\tau)\ e^{-3(t-\tau)}\ u(t-\tau)\ d\tau$$

$$\int_0^t \left(2e^{-2\tau} - e^{-\tau}\right) e^{-3t+3\tau}$$

$$\int_0^t 2e^{-3t+\tau} - e^{-3t+2\tau}$$

$$\left| \frac{2e^{-3t+\tau}}{1} - \frac{e^{-3t+2\tau}}{2} \right|_0^t$$

$$2e^{-3t+t} - \frac{e^{-3t+2t}}{2} - 2e^{-3t} + \frac{e^{-3t}}{2}$$

$$2e^{-2t} - \frac{e^{-t}}{2} - 2e^{-3t} + \frac{e^{-3t}}{2}$$

$$= \boxed{2e^{-2t} - \frac{e^{-t}}{2} - \frac{3}{2}e^{-3t}}$$

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4 a) $\int_0^t 4e^{-2\tau} u(\tau)\, u(t-\tau)$

$\int_0^t 4e^{-2\tau}$

$\left| \frac{4e^{-2\tau}}{-2} \right|_0^t = -2e^{-2t} + 2e^{-0}$

$= \boxed{-2e^{-2t} + 2}$

b) $\int_0^t 4e^{-2\tau} u(\tau)\, e^{-(t-\tau)} u(t-\tau)\, d\tau$

$\int_0^t 4e^{-2\tau} e^{-(t-\tau)}\, d\tau$

$= 4e^{-2\tau - t + \tau}\, d\tau$

$= \int_0^t 4e^{-(t+\tau)}\, d\tau$

$= \frac{4e^{-t-\tau}}{-1}$

$= -4e^{-t-t} + 4e^{-t-0}$

$= \boxed{-4e^{-2t} + 4e^{-t}}$

5 $H(s) = \frac{2s+1}{s^2+s+1}$

$s^2 + s + 1 = 0$

$(1)^2 - 4\times1\times1$

$1 - 4 = -3$

$= \frac{-1 \pm \sqrt{3}i}{2}$

forced response for

a) e^{-2t}

$$= \left(\frac{2(-2)+1}{(-2)^2+(-2)+1} \right) e^{-2t} = \frac{-4+1}{4-2+1}$$

$$= \frac{-3}{3} e^{-2t}$$

$$= \underline{\underline{-e^{-2t}}}$$

b) $e^{-1/2t} \cos(\sqrt{3}t + 70)$

complex freq $= -\frac{1}{2} \pm \sqrt{3}j$

$$\frac{2\left(-\frac{1}{2}+\sqrt{3}j\right)+1}{\left(-\frac{1}{2}+\sqrt{3}j\right)^2 + \left(-\frac{1}{2}+\sqrt{3}j\right)+1} \quad \text{I/P}$$

Natural ν i.e pole = complex ν

$$\frac{2s+1}{(s-P_1)(s-P_2)}$$

$(s-P_1) = 0$.

$\therefore \infty$. I/P

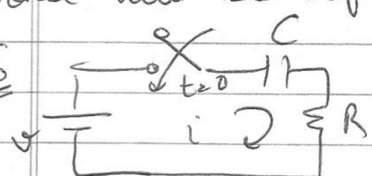
$\cancel{x}_q$ = undefined fn

But steady state response will be defined.

$$\frac{-\cancel{1}+\frac{\sqrt{3}}{2}j+\cancel{1}}{\frac{1}{4}-3-\sqrt{\cancel{3}}j-\frac{1}{2}+\sqrt{\cancel{3}}j+1} = \frac{\sqrt{3}}{2}j \Big/ \frac{1}{4}-2$$

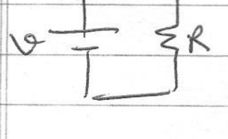
$$X = \frac{\frac{\sqrt{3}}{\cancel{2}}j}{\frac{-7}{\cancel{4}2}} = \underline{\underline{-\frac{2\sqrt{3}j}{7}}} \quad \text{I/P}$$

6



$v_c(0^-) = 0$

find $i(0^+)$, $i'(0^+)$



$$\boxed{i(0^+) = \frac{v}{R}}$$

$v = \frac{1}{C}\int i\,dt + i \times R$

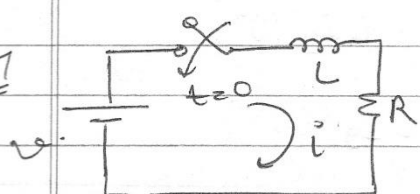
$0 = \frac{1}{C} i + R i'$

$= \frac{1}{C} i(0^+) + R i'(0^+)$

$= i'(0^+) = -\frac{1}{CR} i(0^+)$

$= \frac{-1}{CR}\,\frac{v}{R} = \boxed{-\frac{V}{CR^2}}$

7



$I_L(0^-) = 0$

find $i(0^+)$ & $i'(0^+)$.

$\boxed{i(0^+) = 0}$.

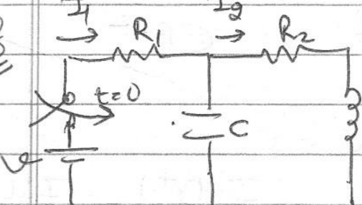
$v = L\frac{di}{dt} + iR$

$v = L i'(0^+) + i(0^+) R$

$v = L i'(0^+) + 0$

$\boxed{i'(0^+) = \frac{v}{L}}$

8

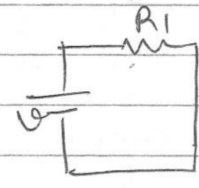


$v_C(0^-) = 0$

$I_L(0^-) = 0$

Find $I_1(0^+)$, $I_1(\infty)$, $I_2(0^+)$, $I_2(\infty$

0^+



i.e $I_2(0^+) = 0$

$I_1(0^+) = \frac{v}{R_1}$

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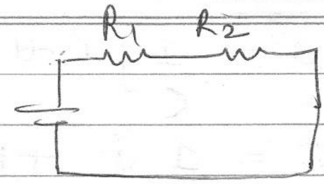
at ∞: $I_1(\infty) = I_2(\infty)$

$$= \frac{v}{R_1 + R_2}.$$



Also find $i_1'(0^+)$, $i_2'(0^+)$, $v_C'(0^+)$, $v_C(0^+)$

$\Rightarrow$ $v_C(0^+) = 0.$

$$v = I_1 R_1 + I_2 R_2 + L\frac{dI_2}{dt}.$$

$$v = I_1(0^+)R_1 + I_2(0^+)R_2 + L\,I_2'(0^+)$$

$$v = \frac{v}{\cancel{R_1}} \times \cancel{R_1} + 0 + L\,I_2'(0^+)$$

$$0 = L\,I_2'(0^+) \qquad \boxed{I_2'(0^+) = 0}$$

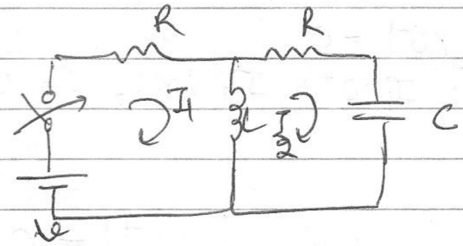
$$v = I_1 R_1 + \frac{1}{C}\int (I_1 - I_2)\,dt.$$

$$0 = I_1'(0^+)R_1 + \frac{1}{C}\left[I_1(0^+) - I_2(0^+)\right].$$

$$0 = I_1'(0^+)R_1 + \frac{1}{C}\left(\frac{v}{R_1} - 0\right)$$

$$I_1'(0^+) = -\frac{1}{C}\frac{v}{R_1 \times R_1} = \frac{-v}{CR_1^2}$$

Q.



I.C = 0

find $I_1(0^+)$, $I_2(0^+)$, $I_1'(0^+)$, $I_2'(0^+)$

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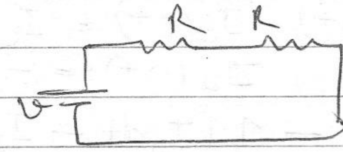
$\underline{0^+}$



$I_1(0^+) = I_2(0^+) = \frac{v}{2R}$

$v = I_1 R + L\frac{d}{dt}(I_1 - I_2)$

$v = I_1(0^+)R + L(I_1'(0^+) - I_2'(0^+))$

$v = \frac{v}{2\cancel{R}}\cancel{R} + L(I_1'(0^+) - I_2'(0^+))$

$\frac{v}{2L} = I_1'(0^+) - I_2'(0^+)$ —— ①

$v = I_1 R + I_2 R + \frac{1}{C}\int I_2\,dt$

$v = I_1'(0^+)R + I_2'(0^+)R + \frac{1}{C} I_2(0^+)$

$v = R[I_1'(0^+) \cancel{R} + I_2'(0^+)] + \frac{1}{C}\frac{v}{2R}$

$\frac{v}{R}\left[1 - \frac{1}{2CR}\right] = I_1'(0^+) + I_2'(0^+)$ —— ②

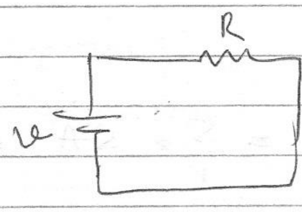
$2I_1'(0^+) = \frac{v}{2L} + \frac{v}{R}\left[1 - \frac{1}{2CR}\right]$

$I_1'(0^+) = \frac{v}{4L} + \frac{v}{2R}\left[1 - \frac{1}{2CR}\right]$

Similarly $\underline{I_2'(0^+)}$

⇒ Another case If switch is open.

$\underline{0^-}$



$I_2(0^-) = 0$

$I_1(0^-) = \frac{v}{R}$

$I_1(0^-) = \frac{v}{R} = \underline{I_L(0^+)}$

$v_c(0^-) = 0 = v_c(0^+)$

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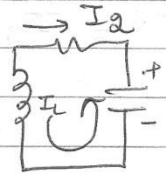
Also

0^+



$$I_1(0^+) - I_2(0^+) = I_L(0^+)$$

$$I_2(0^+) = -I_L(0^+) = -\frac{V}{R}$$

$$L\frac{dI_L}{dt} - \frac{1}{C}\int I_2\,dt - I_2 R = 0$$

$$I_L = I_1 - I_2$$

$$L[I_1' - I_2'] - \underbrace{\frac{1}{C}\int I_2\,dt}\ - I_2 R = 0$$

At 0^+ $v_c(0^+) = 0$

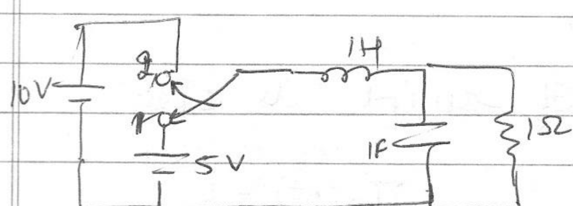
$$L(I_1'(0^+) - I_2'(0^+)) - I_2^{(0^+)} R = 0$$

$$L(I_1'(0^+) - I_2'(0^+)) + \frac{V}{R} \times R = 0$$

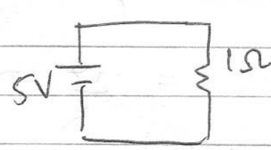
$$I_1'(0^+) - I_2'(0^+) = -\frac{V}{L}$$

$$v_L(0^+) = L\frac{dI_L}{dt} = L\, I_L'(0^+)$$

10



0^-



$$I_L(0^-) = \frac{5}{1} = 5A = I_L(0^+)$$

$$v_c(0^-) = 5V = V_c(0^+)$$

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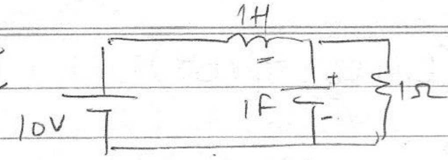
0^+



$$10 = L\frac{dI_L}{dt} + 5$$

$$\frac{10-5}{L} = I_L'(0^+) = \frac{5}{1} = 5A.$$

voltage across $1\Omega = 5$

$$\text{Curie} = \frac{5}{1} = 5A$$

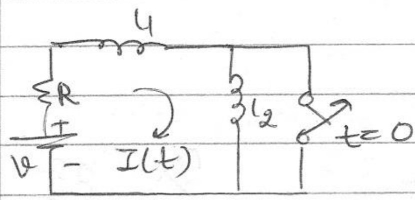
$$C\frac{dv_c}{dt}(0^+) = i_c(0^+).$$

$v_c'(0^+) = 0.$ because current through cap $= 0$ at 0^+.

$i_L(\infty) = 10A$

$v_c(\infty) = 10V$

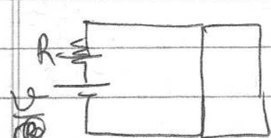
11



find $I_{L1}(0^-)$, $I_{L2}(0^-)$

$I(t)$

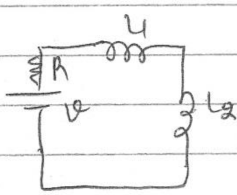
0^-



$$I_{L1}(0^-) = \frac{v}{R}$$

$$I_{L2}(0^-) = \frac{v}{2R}.$$

0^+



$L_1 I_{L1}(0^-) + L_2 I_{L2}(0^-) = L_1 I_L(0^+) + L_2 I_L(0^+)$

because current same

$$L_1 \frac{v}{R} + L_2 \frac{v}{2R} = (L_1 + L_2) I_L(0^+)$$

$$\frac{v}{R}\left(L_1+\frac{L_2}{2}\right) = (L_1+L_2)\, I_L(0^+)$$

$$I_L(0^+) = \frac{v}{R(L_1+L_2)}\left(L_1+\frac{L_2}{2}\right)$$

$$v = L_1\frac{dI_L}{dt} + L_2\frac{dI_L}{dt} + RI_L$$

$$v = (L_1 D + L_2 D)\, I_L + R I_L$$

$$\frac{v}{D(L_1+L_2)+R} = I_L$$

$$D = \frac{-R}{(L_1+L_2)}$$

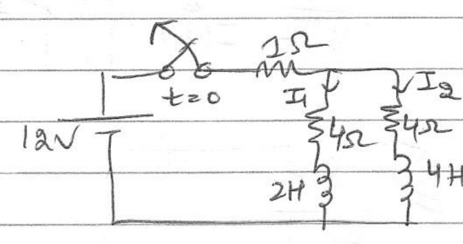
$$I(t) = e^{-\frac{R}{L_1+L_2}t} + \frac{v}{R}$$ forced response

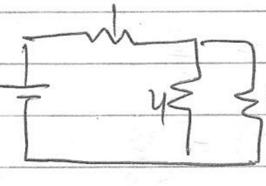
natural response

12.

$I_1(0^-)$, $I_2(0^-)$

$I_1(0^+)$, $I_2(0^+)$





$$\frac{12}{3} = 4A$$

$$I_1(0^-) = 2A = I_2(0^-)$$

Flux eq^n only gives magnitude

$$2\times(2) + 4(2) = (2+4)(x)$$

$$\frac{4+8}{6} = x$$

$$\frac{12}{6} = x$$

$$\underline{\underline{x = 2A}}$$

$$I_1(0^+) = 2A$$
$$I_2(0^+) = \underline{\underline{-2A}}$$

---

* freq. of Impulse Response = CFs freq.
Remaining = PI freq.
i.e excitation freq. (CF)

Suppose impulse response $h(t) = (2e^{-2t} - e^{-t})u(t)$
excitation a) $x(t) = u(t)$
b) $e^{-3t}u(t)$ $\underline{\underline{PI}}$

i) PI = 0 , CFS = −1, −2
PI = −3 , CFS = −2, −1

FF Koo – 1,2,4,5

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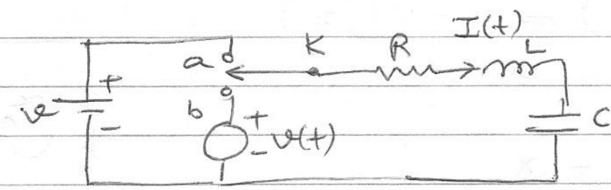
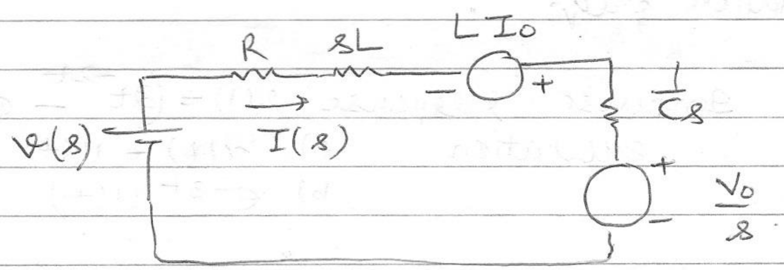
Q



k is held at position a till the time at which current I_0 flows through inductor and cap. charged to voltage V_0.
At that instant switch is thrown to position b. find $I(s)$

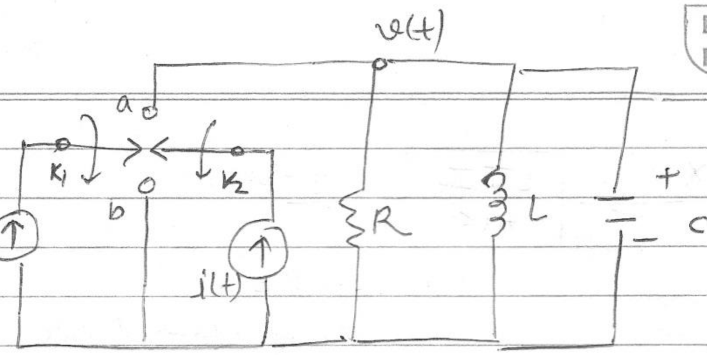
$v_C(0^-) = V_0$
$I_L(0^-) = I_0$



$$V(s) + LI_0 - \frac{V_0}{s} = \left(R + sL + \frac{1}{Cs}\right) I(s)$$

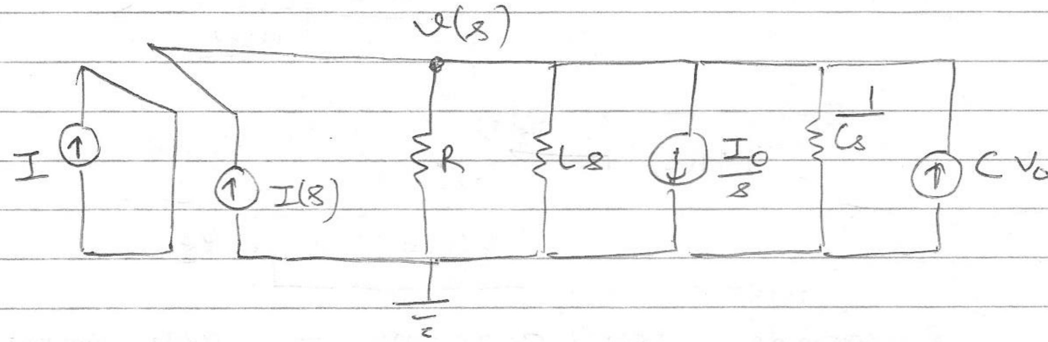
$$\frac{V(s) + LI_0 - \frac{V_0}{s}}{R + sL + \frac{1}{Cs}}$$

Q



Initially switch K_1 held at pos. a and K_2 at b. At an instant when inductor current is I_0 and capacitor is charged to V_0, pos. of K_1 and K_2 interchanged. Find node voltage $v(s)$.

Soln



$$CV_0 + I(s) = \frac{v(s)}{R} + \frac{V(s)}{Ls} + \frac{I_0}{s} + \frac{V(s)}{1/Cs}$$

$$CV_0 + I(s) - \frac{I_0}{s} = v(s)\left[\frac{1}{R} + \frac{1}{Ls} + Cs\right]$$

$$v(s) = \frac{CV_0 + I(s) - I_0/s}{\left(\frac{1}{R} + \frac{1}{Ls} + Cs\right)}$$

or $\text{Voltage} = \dfrac{\text{Current}}{\text{Admittance}} = \text{same}$

$$x(s) \longrightarrow \boxed{\begin{matrix}LTI\\ H(s)\end{matrix}} \longrightarrow y(s).$$

→ LLFPB

Linear Lumped finite passive bilateral

then $H(s)\Big|_{I.C=0} = \text{rational} = \dfrac{g(s)}{f(s)}$

Properties of system function

$$x(s) \longrightarrow \boxed{\begin{matrix}LTI\\ H(s)\end{matrix}} \longrightarrow y(s) = H(s) \cdot Y(s)$$

1) $\delta(t),\ 1 \longrightarrow \boxed{H(s)} \longrightarrow y_\delta(t)$

$Y_\delta(s) = H(s) \cdot 1.$

$\boxed{Y_\delta(s) = H(s)}$

2) Response corresponding to any excitation $x(t)$ in terms of impulse response.

$x(t),\ X(s) \longrightarrow \boxed{H(s)} \longrightarrow y(t)$

$Y(s) = H(s) \cdot X(s)$

3) step response

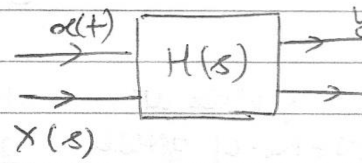
$u(t),\ \frac{1}{s} \longrightarrow \boxed{} \longrightarrow y_u(t) = a(t)$

$Y_u(s) = A(s) = \dfrac{H(s)}{s}$

$\boxed{H(s) = sA(s)}$

Impulse response

4) ~~s~~ Response corresponding to any excitation $x(t)$ in terms of step response.

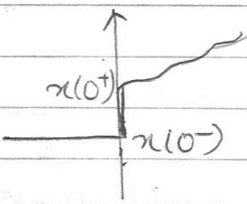


$$Y(s) = H(s) \cdot X(s)$$
$$= s\,A(s).\,X(s)$$
Or $$= [s\,X(s)] \cdot A(s)$$

Multiplication in freq. domain = Convolution in time

$$y(t) = x'(t)\Big|_{t>0^-} * a(t)$$

$$\left[\{x(0^+) - x(0^-)\}\,\delta(t) + x'(t)\Big|_{t>0}\right] * a(t)$$



i.e convolution integral we have to calculate from 0^- to ∞.
whenever there is step response, there are chances that impulse will occur at zero.

5) Concept of poles & zeros of system fn.

$$H(s) = \frac{g(s)}{f(s)} \quad \begin{matrix} \to m \\ \to n \end{matrix} = \text{degree}$$

$g(s) = 0$ known as zeros
$f(s) = 0$ poles

No. of finite zeros not only zeros $= m$
" " " poles $= n$

strictly
* No. of poles $\overset{\text{always}}{=}$ No. of zeros

① If $m > n$	② $m = n$	③ $m < n$
a) Improper system function		Proper Rational fn.
b) $m-n$ = No. of poles will be available at infinity.	No. of poles at ∞ = No. of zeroes at ∞ = 0.	$(n-m)$ = No. of zeroes at infinity.
c) physically unrealizable		Physically realizable

$$H(s) = \frac{(s+1)}{(s+2)(s+3)}.$$

zeroes mean $H(s) = 0$.

Now if $\lim_{s \to a} H(s) = 0$ then it means

$s = a$ is a zero.

Now

$\lim_{s \to \infty} H(s) = 0$ $\therefore$ one location of zero may be ∞.

Poles represent $H(s) = \infty$.

If $H(s) = \frac{(s+1)(s+2)}{(s+3)}$

then $\lim_{s \to \infty} H(s) = \infty$ Thus one location of pole may be ∞.

① $\frac{4s^3+3s^2+s+1}{2s+1}$

Partial frac $K_1 s^2 + K_2 s + ($Proper rational fn$)$

$K_1 \xrightarrow{LI} \delta(t)$.

$K_1(s) \longrightarrow \delta'(t)$

$K_1 s^2 \longrightarrow \delta''(t)$

$\delta(t)$ ka derivative at ∞. $\therefore$ definitely system will blown out. i.e mag. ∞

② $\frac{as+b}{cs+a}$

Partial frac. $K_1 + K_2$ (Proper rat. fn).

one Impulse present.

X * If location of poles = complex " of excitation

eg:- $H(s) = \frac{(s+1)(s+2)}{(s+3)}$

$x(t) = 2e^{-\infty t} \longrightarrow$ means $2e^{-\frac{t}{0}}$

in zero time it decays to $\frac{A}{e}$ i.e Impulse

$X(s) = \frac{2}{(s+\infty)}$

$Y(s) = \frac{(s+1)(s+2)}{(s+3)(s+\infty)}$

$Y^{-1}(s) = \frac{A}{(s+3)} + \frac{B}{(s+\infty)}$

$=$

There will be impulse present in impulse response.

$$Y(s) = H(s) \cdot X(s)$$

Partial fraction expansions of $H(s)X$ will contain 2 parts

terms containing location of poles same as poles of system function.	terms containing poles same as complex ν present in excitation.

Inverse lapace of this ~~to~~ represents natural ν

If $H(s)$ has same pole location as $X(s)$ i.e $s_0(t)$ then both side $\frac{k}{(s-s_0)}$ will come.

In Inverse laplace there will be $k e^{-s_0 t}$

k_1 k_2

Thus $k = k_1 + k_2$

$k_1 e^{-s_0 t}$ for $H(s)$

$k_2 e^{-s_0(t)}$ for $X(s)$

* **forced Response characteristics or steady state Response**

$$x(t)\Big|_{S.S.R/F.R} = H(s)\Big|_{s \to s_0} \cdot x(t)$$

$$= H(s_0) \, A e^{s_0 t}$$

(3) * Freq. Response characteristics i.e excitation purely sinusoidal.

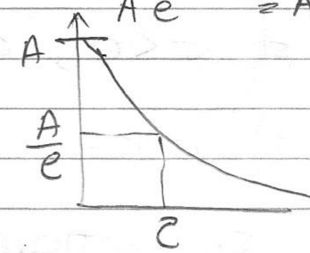
$x(t) = e^{\pm j\omega t}$

$$y(t)\Big|_{SSR} = H(+j\omega)\cos(\omega t).$$

* Free Response or stability

$A e^{-\alpha t} = A e^{-\frac{t}{\alpha}}$

$\tau = \frac{1}{\alpha}$



time where amplitude reduces to $1/e$.

Jitna large τ, utna fast decay

*→ Equilibrium point of the system is defined as the point where the system dynamic becomes zero. i.e $\frac{di_L}{dt}$, $\frac{dv_c}{dt} = 0$.

Instead all dynamics i.e v_c'', i_L'', v_c''' and higher derivatives be zero.

If after disturbance, system going away from operating pnt then unstable.

Marginal stability i.e deviated from operating point but is maintained there

Asymtotic stability — If system going back to $\cong$m point

Exponential stability — If system going back to $\cong$m point at a very fast rate

$S_n = \sigma \pm j\omega$ (natural freq)

1) $\sigma < 0$, $\omega = 0$ Asymtotically stable

2) $\sigma > 0$, $\omega = 0$ Unstable

3) $\sigma = 0$, $\omega = 0$ dc (marginal stability)

4) $\sigma = 0$, $\omega = \pm j\omega$ Marginally stable

5) $\sigma < 0$, $\omega = \pm j\omega$ Asymtotically stable

6) $\sigma > 0$, $\omega = \pm j\omega$ Unstable

Exponentially stable depends on value of σ. All Asymtotically " system are exponentially stable but vice versa not true.

⇒ Initial value theorem

$$f(t) \xrightarrow{LT} F(s)$$

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s).$$

F(s) = ~~System~~ should be proper rational fn for this theorem to be implemented.

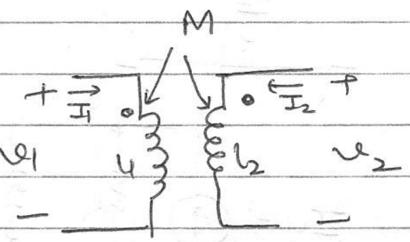
⇒ Final value Theorem

$$f(t) \xrightarrow{LT} F(s)$$

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s).$$

Theorem applicable only when $f(t)$ is non growing signal i.e ~~if $f(t)$ is bounded~~.
i.e if poles on R.H.S, then this not applicable

* Mutually coupled INDUCTORS (Transform Domain Analysis).

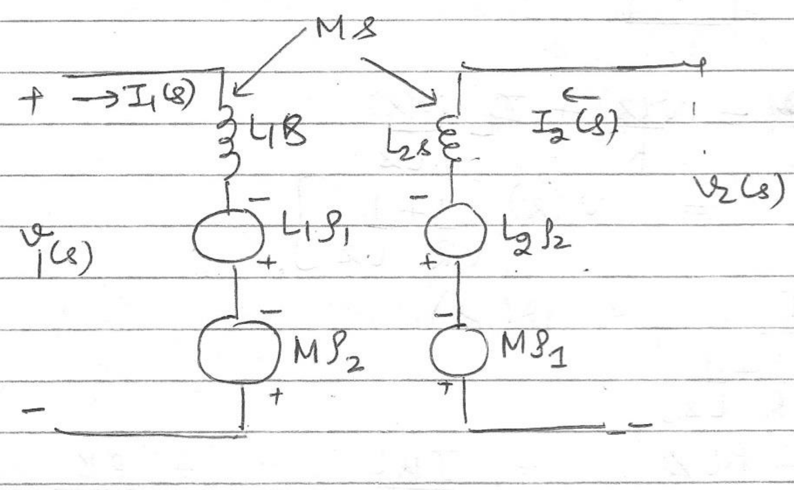


$$I_1(0^-) = \rho_1$$
$$I_2(0^-) = \rho_2$$

$$v_1(t) = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$V_1(s) = L_1\left[sI_1(s) - \rho_1\right] + M\left[sI_2(s) - \rho_2\right]$$

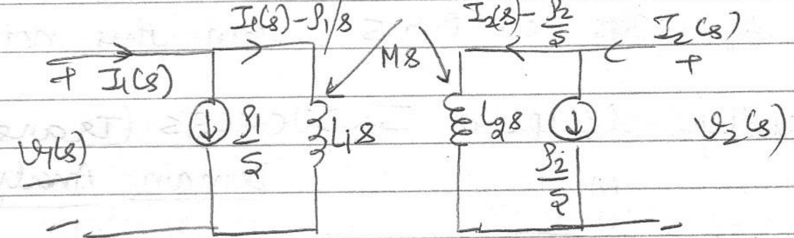
$$V_2(s) = L_2\left[sI_2(s) - \rho_2\right] + M\left[sI_1(s) - \rho_1\right]$$



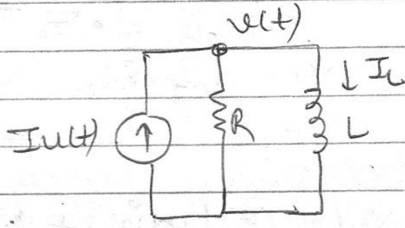
In terms of current source.

$$v_1(s) = L_1 s\left[I_1(s) - \frac{i_1}{s}\right] + Ms\left[I_2(s) - \frac{i_2}{s}\right]$$

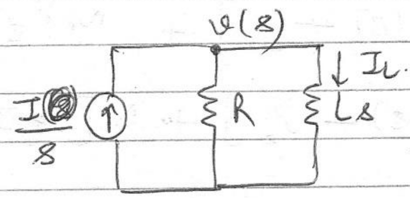
$$v_2(s) = L_2 s\left[I_2 s - \frac{i_2}{s}\right] + Ms\left[I_1 s - \frac{i_1}{s}\right]$$



Q $I_L(0^-) = 0$.
Find $v(t)$
Define ~~defn~~ nature
Of $v(t)$ as $g \to 0$.
i.e conductance



Soln



$$\frac{I}{s} = \frac{v(s)}{R} + \cancel{I_L}\ \frac{v(s)}{Ls}$$

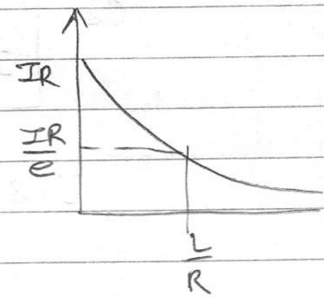
$$\frac{I}{s} = v(s)\left[\frac{1}{R} + \frac{1}{Ls}\right]$$

$$\frac{I}{s\left[\frac{1}{R} + \frac{1}{Ls}\right]} = v(s).$$

$$\frac{I\,RL\cancel{s}}{\cancel{s}(Ls+R)} = \frac{IRL}{s\left[\cancel{L} + \frac{R}{\cancel{s}}\right]} = \frac{R\cancel{L}}{\cancel{L}\left[s + \frac{R}{L}\right]}$$

$$v(t) = IR\, e^{-\frac{R}{L}t}.$$

$$\tau = \frac{L}{R}$$



$R\uparrow$ $\frac{L}{R}\downarrow$ i.e $\tau \downarrow$.

$G \to 0$

means $R \to \infty$.

Mag. goes to ∞. decays in zero time. i.e Impulsive type of action.

$$H(s) = \frac{R}{s+\frac{R}{L}} \quad \text{as } R \to \infty. \qquad (s+\infty)$$

$$e^{-\infty t}$$

$\Downarrow$ Impulse

i.e $v(t) = k\delta(t)$

find value of K

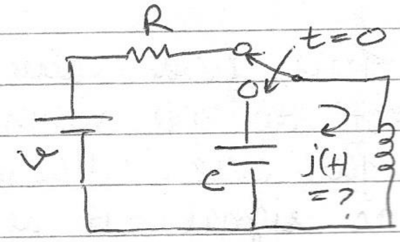
$$\int_0^\infty IR\, e^{-\frac{R}{L}t} = K$$

$$\left[IR \frac{e^{-\frac{R}{L}t}}{-\frac{R}{L}} \right]_0^\infty$$

$$IL\left[-e^{-\frac{R}{L}\infty} + e^{-\frac{R}{L}(0)} \right] = k.$$

$$\boxed{k = I \cdot L}$$

Q. $t=0$ position of switch changed find $i(t)$.



Soln. $I(0^-) = \frac{v}{R}$.

$v(0^-) = 0$.

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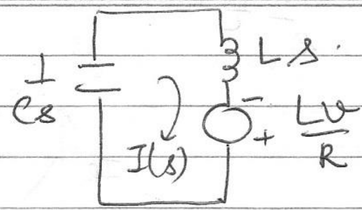

$$I(s) = \frac{\frac{vL}{R}}{\frac{1}{Cs} + Ls} = \frac{\frac{vL}{R}}{\frac{1+LCs^2}{Cs}}$$

$$\frac{s}{s^2+\omega^2} = L[\cos\omega t]$$

$$= \frac{vLCs}{R(1+LCs^2)}$$

$$= \frac{v\cancel{LC}s}{R\cancel{LC}\left(s^2+\frac{1}{LC}\right)}$$

$$= \frac{v}{R}\frac{s}{\left(s^2+\frac{1}{LC}\right)}$$

$$i(t) = \frac{v}{R}\left(\cos\sqrt{\frac{1}{LC}}\,t\right)$$

From here $i(\infty)$ = not defined.

$\Longleftrightarrow$ [From ckt
No. of poles = No. of zeroes = Degree = 2

From final value theorem

$$i(\infty) = \lim_{s\to 0} sF(s) = \frac{v}{R}\frac{s^2}{\left(s^2+\frac{1}{LC}\right)}$$

$$= \frac{\cancel{R}\,vs^2}{R\cancel{s^2}\left[1+\frac{1}{LCs^2}\right]}$$

$$= \frac{v}{R(1+\infty)}$$

$$= \frac{1}{\infty} = \underline{\underline{0}}$$

Thus, final value theorem not to use when poles on $j\omega$ axis. However, for dc signal it is to be used.

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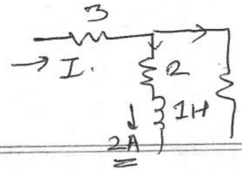
$$\frac{10-3I}{6} = I-2$$

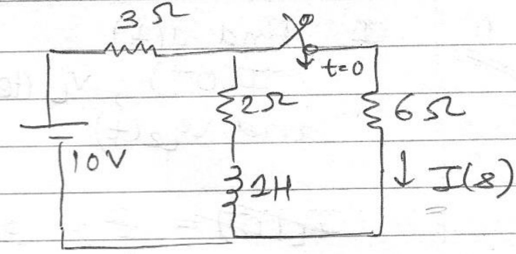
$$I = \frac{22}{9}$$

$$I_{6\Omega} = \frac{4}{9}$$

=



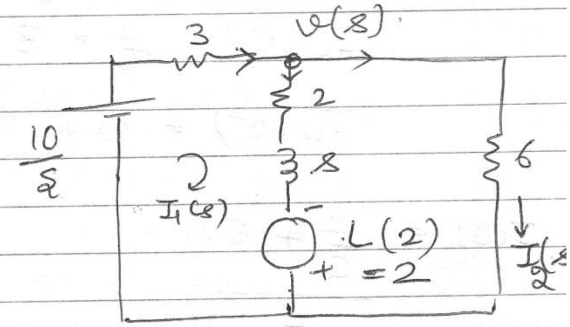
Q. find $I(s)$



$$I_L(0^-) = \frac{10}{5} = 2A$$

~~$\frac{10}{s} + 2L =$~~

$$\frac{v(s) - \frac{10}{s}}{3} = \frac{v(s)}{6}$$



$$\frac{10}{s} = 3I_1(s) + (2+s)(I_1(s) - I_2(s)) - 2$$

$$6I_2(s) + 2 = (2+s)(I_1(s) - I_2(s))$$

$$\frac{10}{s} + 2 = (3+2+s)\,I_1(s) - (2+s)\,I_2(s)$$

$$2 = (2+s)\,I_1(s) - (2+s+6)\,I_2(s)$$

Ans correct

$$I_2(s) = \frac{4s+20}{9s(s+4)}$$ ~~Ans not~~ correct

find $I_2(0^+)$, $I_2(\infty)$.

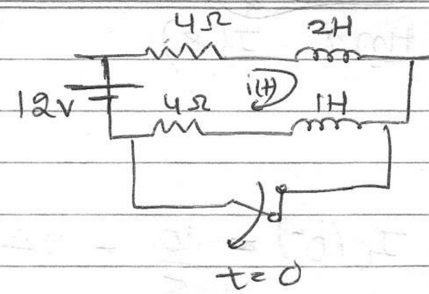
$$\lim_{s\to\infty} \frac{4s+20}{9(s+4)} = \frac{\cancel{s}\left(4+\frac{20}{s}\right)}{9\cancel{s}\left(1+\frac{4}{s}\right)} = \frac{4}{9}$$

$I(0^+) = \frac{5}{9}$

Cross check Ans by finding $I(0^+)$ and $I(\infty)$

From final value theorem $I_2(\infty) = \frac{5}{9}$

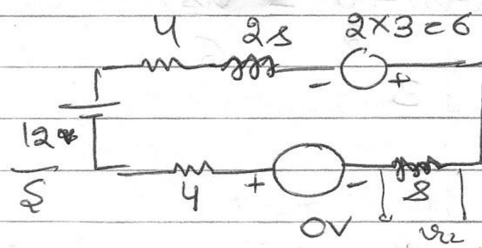
Q. find $i(t)$
$I(0^+)$, $v_{L1}(t)$
and $v_{L2}(t)$



~~$I_L(0^-) = \frac{12}{8} = \frac{3}{2}$~~

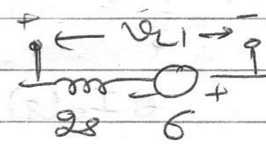
$I_{L1}(0^-) = \frac{12}{4} = 3\ A$

$I_{L2}(0^-) = 0\ A$



$$I(s) = \frac{\frac{12}{s} + 6}{4 + 3s + 4}$$

Voltage across Inductor



$v_{L1}(s) = 2s\, I(s) - 6$

$v_{L2}(s) = s\, I(s)$

$$I(s) = \frac{3/2}{s} + \frac{1/2}{s + 8/3}$$

$$\boxed{i(t) = \frac{3}{2} + \frac{1}{2} e^{-\frac{8}{3}t}}$$

$\boxed{I(0^+) = 2}$

$$v_{L1}(s) = 2s \frac{\left(\frac{12}{s} + 6\right)}{4 + 3s + 4} = \frac{2s(12 + 6s)}{s(4 + 3s + 4)} = \frac{2(12 + 6s)}{4 + 3s + 4} - 6$$

$$v_{L1}(s) = 2s\left[\frac{K_1}{s} + \frac{K_2}{s + \frac{8}{3}}\right]$$

$v_{L1}(s) = \frac{-2(s+4)}{\left(s+\frac{8}{3}\right)}$

$2k_1$ = Constant   Inverse Impulse.

$v_{L1}(s) = -2 - \frac{\frac{8}{3}}{s+\frac{8}{3}}$ $\boxed{v_{L1}(t) = -2\delta(t) - \frac{8}{3}e^{-\frac{8}{3}t}}$

$t \to 0^+$ $\delta(t) \to 0$.

Initial value Thm $v_{L1}(s) = -2s - \frac{\frac{8}{3}s}{s+\frac{8}{3}} = -2s - \frac{\frac{8}{3}}{\left(1+\frac{8}{3s}\right)}$

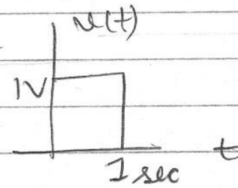
$= -2(\infty) - \frac{8}{3}$

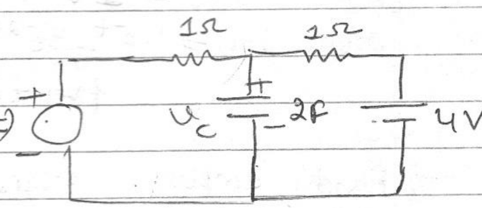
Thus, if there is no proper rational fn, we cant use Initial value theorem

$\boxed{v_{L2}(t) = 2\delta(t) - \frac{4}{3}e^{-\frac{8}{3}t}}$

Natural freq. present in every response through the system is same. Mag. may change till nature of system remains same.

Q. $v(t) =$





find $v_C(t)$

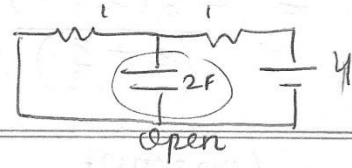
Soln. $v(t) = \cancel{1u(t-1)}\ u(t) - u(t-1)$.

$\frac{1}{s} - \frac{e^{-s}}{s}$ = LT of $u(t) - u(t-1)$

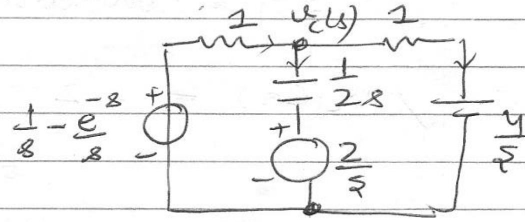
For $t<0$ $v(t) = 0$

At 0^-

$v_C(0^-) = 2V$.



$$\frac{v_C(s) - v(s)}{1} + \frac{v_C(s) - \frac{2}{s}}{\frac{1}{2s}} + \frac{v_C(s) - \frac{4}{s}}{1} = 0.$$



$$v_C(s)\left[1 + 2s + 1\right] - \frac{2}{\cancel{s}} \times 2\cancel{s} - \frac{4}{s} - v(s) = 0$$

$$-4 - \frac{4}{s} - \frac{1}{s} + \frac{e^{-s}}{s} = 0$$

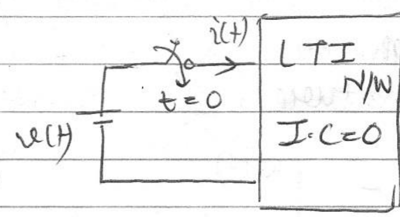
$$\boxed{v_C(s) = \frac{\frac{5}{s} - \frac{e^{-s}}{s} + 4}{2 + 2s}}$$

Ans

$$v_C(t) = 2u(t) + \frac{1}{2}\left[u(t) - e^{-t}u(t)\right] - \frac{1}{2}\left[u(t-1) - e^{-(t-1)}u(t-1)\right]$$

Or find response for $u(t)$, then delay by 1 $u(t-1)$, then add, superposition.

Q. $i(t)$ = Response
$v(t)$ = excitation
When $v(t) = 2V$
then $i(t) = 4e^{-t} - 3e^{-2t}$
for $t > 0$



$v(t) = 1V = u(t)$

1) Find Impulse response
2) Find system fn
3) Response corresponding to excitation $v(t) = e^{-4t}u(t)$
4) Find forced response of system corresponding to excitation $v(t) = \cos 2t$.

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Soln (i) $A(s) = \frac{4}{1+s} - \frac{3}{s+2}$

$H(s) = sA(s)$

$= s\left(\frac{4}{1+s} - \frac{3}{s+2}\right)$

$\boxed{H(s) = \frac{s(s+5)}{(s+1)(s+2)}}$

$\boxed{A(s) = \frac{s+5}{(s+1)(s+2)}}$

(ii) $L^{-1}H(s) =$ System fn.

$\boxed{h(t) = \delta(t) - 4e^{-t} + 6e^{-2t}}$

(iii) $h(t) * x(t)$

$X(s) = \frac{1}{(s+4)}$

$y(t) = -\frac{2}{3}e^{-4t} - \frac{4}{3}e^{-t} + 3e^{-}$

Response $= X(s) \cdot H(s)$.

(iv) $H(s)\Big|_{s=j2} \cos 2t$

Ans $\boxed{1.7(\cos 2t + 3.4}$

$\frac{s(s+5)}{(s+1)(s+2)}\Big|_{s=j2} \cos 2t$

$\frac{j\omega(j\omega+5)}{(j\omega+1)(j\omega+2)} \cos 2t$

$\frac{j\omega(j\omega+5)(2-j\omega)(1-j\omega)}{(1-\omega^2)(4-\omega^2)}$

$\frac{(-\omega^2+5j\omega)(2-3j\omega-\omega^2)}{(1+\omega^2)(4+\omega^2)}$

$\frac{(-4+10j)(2-6j-4)}{(1+4)(4+4)} = \frac{(-4+10j)(-6j-2)}{5(8)}$

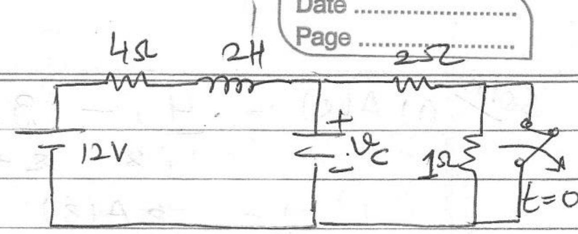
$\frac{17+j}{10} = 1.7+0.1$

Mag $= 1.7$

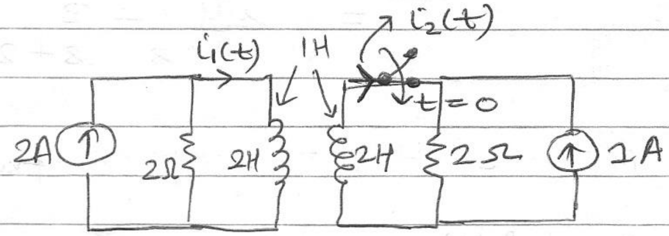
Angle $= \tan^{-1}\frac{0.}{1.}$

$= 3.3^\circ$

Q1 Find v_c', v_c, v_c'', v_c''' at $t = 0^+$

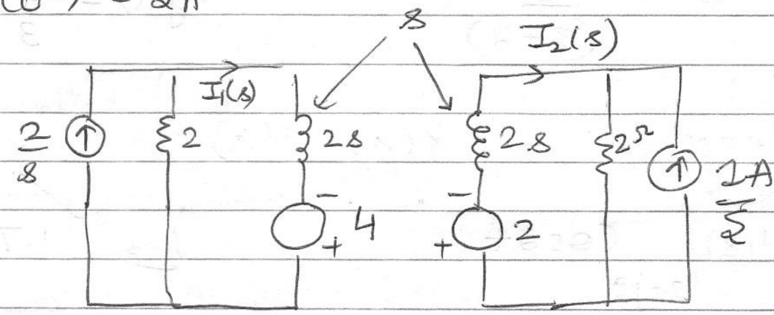


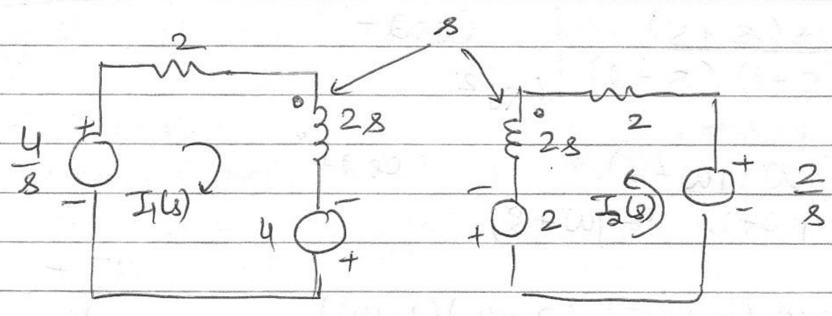
Q2 Find $I_2(t)$



$i_2(0^-) = 0$

$i_1(0^-) = 2A$



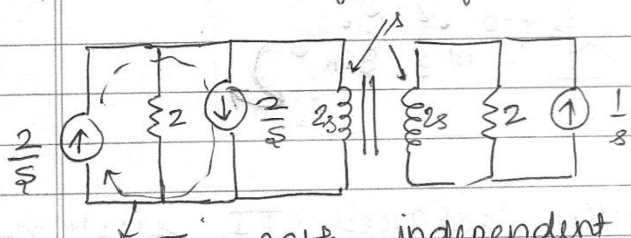


$$\frac{4}{s} = 2I_1(s) + 2s\,I_1(s) - 4 + s\,I_2(s)$$

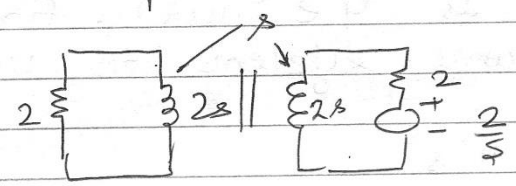
$$\frac{2}{s} = 2I_2(s) + 2s\,I_2(s) - 2 + s\,I_1(s)$$

$I_2(s)$ ko negative karo at last

Also do ques for current source.



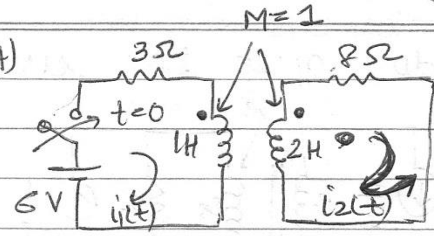
This path independent of whole ckt.



$$\begin{bmatrix} 2s+2 & -s \\ -s & 2s+2 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{2}{s} \end{bmatrix}$$

$$I_2 = \left[-1 + \frac{1}{2} e^{-\frac{2}{3}t} + \frac{1}{2} e^{-2t} \right] u(t)$$

Q3 Find $i_1(t)$ and $i_2(t)$



Q4 Response of initially relaxed LTI system to unit Impulse is $4e^{-t}u(t)$. Find

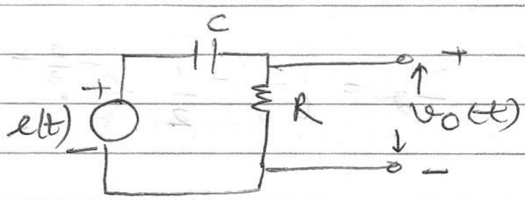
a) response of same system to unit step I/P.

b) Find system fn $H(s)$

c) find response of system to excitation $e^{-t}u(t)$

d) find the steady state response to an excitation $\sin 2t$.

Q.5 Find condition on RC such that the given n/w will act as differentiator.

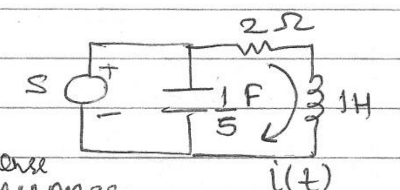


i.e $v_o(t) = k\dfrac{de(t)}{dt}$

Try in both time and laplace domain

Q.6 a) $S = I_s(t)$

b) $S = V_s(t)$



Find the form of ~~free response~~ free response $i(t)$ for both cases.

4. $H(s) = \frac{4}{(s+1)}$

$\frac{4}{\cancel{4}}$

a) $\frac{1}{s} \times H(s) = Y(s)$

$\int$ Impulse response = step respo

$\frac{1}{s} \times \frac{4}{(s+1)} = Y(s)$

$\frac{A}{s} + \frac{B}{s+1} = \frac{4}{s} \cancel{+} \frac{4}{(s+1)}$

$$\boxed{y(t) = 4 - 4e^{-t}}$$

b) $H(s) = \frac{4}{(s+1)}$

c) $y(s) = \frac{4}{(s+1)} \frac{1}{(s+1)} = \frac{4}{(s+1)^2} = 4te^{-t}$.

d) $H(s) = \frac{4}{(s+1)}\Big|_{2j}$ $\sin 2t = \frac{4}{(2j+1)} \sin 2t$.

$= \frac{4}{\sqrt{4+1}} \sin(2t \cancel{+} \tan^{-1}$

$= \frac{4}{\sqrt{5}} \sin(2t - 26.55)$.

5. I across C $\quad C\frac{dv_c}{dt} = C\frac{d}{dt}(e(t) - v_o(t$

$v_o(t) = RC\frac{d}{dt}(e(t) - v_o(t))$

To act as differentiator $\quad v_o(t) \cong 0$.

i.e $R \simeq 0$.

In laplace

$$v_o(s) = \frac{R}{R + \frac{1}{Cs}} e(s) = \frac{RCs}{RCs+1} e(s)$$

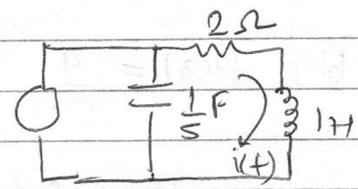
In laplace

$v_o(s)$ should be $k\,s\,e(s)$

i.e $RCs <<< 1$

$$\boxed{s <<< \frac{1}{RC}}$$

6 a) $s = I_s(t)$.



$$I(s) = \frac{\frac{1}{Cs}}{\frac{1}{Cs} + 2 + Ls} I_s(s)$$

$$= \frac{\frac{5}{s}}{\frac{5}{s} + 2 + s} = \frac{5}{5 + 2s + s^2} = \frac{5}{s^2 + 2s + 5}$$

$$= \frac{5}{(s+1-2i)(s+1+2i)}$$

$s = -1 \pm 2j$ represent natural freq. of free response

$I(t) = k e^{-t} \cos(2t + \phi)$

$k_1 e^{-t+2i} + k_2 e^{-t-2i}$

b) When $s = V_s(t)$.

$$2\,I(s) + Ls\,I(s) = V_s(s)$$

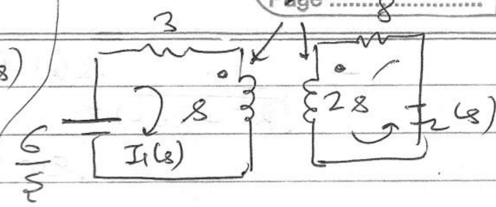
$$(2+s)\,I(s) = V_s(s)$$

$$I(s) = \frac{V(s)}{(s+2)}$$

$$k e^{-2t}$$

3. $\frac{6}{s} = (3+s)\,I_1(s) + s\,I_2(s)$

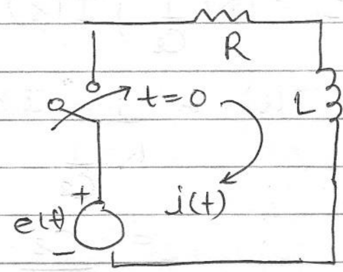
$(8+2s)\,I_2(s) + s\,I_1(s) = 0$



$I_1(t) = k_1 e^{-2t} + k_2 e^{-12t} + 2$

$I_2(t) = k_3 e^{-2t} + k_2 e^{-12t} + 0$

Q. $e(t) = \sin(\omega t + \theta)$
then find $i(t)$



$E(s) = (R + Ls)\,I(s)$

$i(t) = k_1 e^{-\frac{R}{L}t} + k_2 \sin(\omega t + \theta)$

$= k_1 e^{-\frac{R}{L}t} + \frac{1}{R + j\omega L} \sin(\omega t + \theta)$

$$\boxed{i(t) = k_1 e^{-\frac{R}{L}t} + \frac{1}{\sqrt{R^2 + (\omega L)^2}} \sin\left(\omega t + \theta - \tan^{-1}\frac{\omega L}{R}\right)}$$

$i(0) = k_1 + \frac{1}{\sqrt{R^2 + (\omega L)^2}} \sin\left(\theta - \tan^{-1}\frac{\omega L}{R}\right)$

$i(0) = 0$

$k_1 = -\frac{1}{\sqrt{R^2 + (\omega L)^2}} \sin\left(\theta - \tan^{-1}\frac{\omega L}{R}\right)$

find θ in terms of R, L and ω if ~~transient response~~ cf soln is zero.

means $\left(\underbrace{k_1 e^{-\frac{R}{L}t}}_{\text{cf soln}} = 0\right)$. i.e $k_1 = 0$.

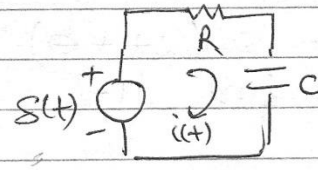
$$-\frac{1}{\sqrt{R^2+(\omega L)^2}} \sin\left(\theta - \tan^{-1}\frac{\omega L}{R}\right) = 0$$

$$\sin\left(\theta - \tan^{-1}\frac{\omega L}{R}\right) = 0$$

$$\theta - \tan^{-1}\frac{\omega L}{R} = n\pi$$

$$\theta = n\pi + \tan^{-1}\frac{\omega L}{R}$$

Q. $1 = \left(R + \frac{1}{Cs}\right) I(s)$



find $i(t)$ for
1) $\delta(t)$
2) $u(t)$
3) $r(t)$

$$I(s) = \frac{1}{R+\frac{1}{Cs}} \cancel{I(s)}\, 1$$

$$= \frac{Cs}{RCs+1} \cancel{I(s)}\, 1$$

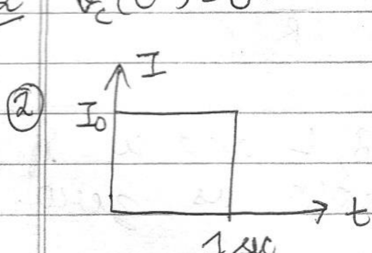
$$= \frac{\cancel{C}s}{R\cancel{C}\left(s+\frac{1}{RC}\right)} = \frac{s}{R\left(s+\frac{1}{RC}\right)}$$

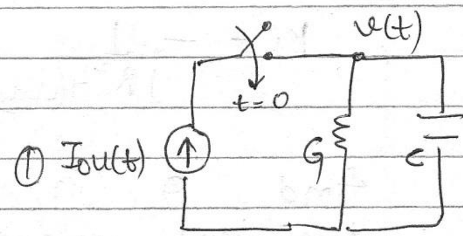
$$= \frac{(RCs+1-1)}{R(RCs+1)}$$

$$= \frac{1}{R}\left[1 - \frac{1}{RCs+1}\right] = \frac{1}{R} - \frac{1}{R(RCs+1)}$$

$$= \frac{1}{R}\delta(t) - \frac{1}{R^2C} e^{-\frac{1}{RC}t}$$

Q $v_c(0^-) = 0$.





(3) $I = I_0 \sin \omega_0 t$

Soln $\frac{I_0(s)}{\cancel{G}} = V(s)\,G + \frac{V(s)}{\frac{1}{Cs}}$

$\frac{I_0(s)}{\cancel{G}} = V(s)\,G + Cs\,V(s)$

$\frac{I_0(s)}{G + Cs} = V(s)$

1) $I_0(s) = \frac{I_0}{s}$

2) $I_0(s) = \frac{I_0}{s} - \frac{I_0}{s} e^{-s}$

3) $I_0 \frac{\omega_0}{s^2 + \omega_0^2}$

$\downarrow$

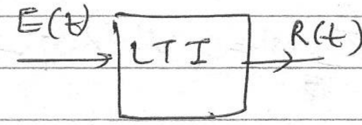
$ke^{-\frac{G}{c}t} +$ put $s = j\omega_0$

$ke^{-\frac{G}{c}t} + \frac{1}{(G + j\omega_0)}$

$ke^{-\frac{G}{c}t} + \frac{I_0}{\sqrt{G^2 + (j\omega_0)^2}}\left(\sin \omega_0 t - \tan^{-1} \frac{c}{G}\right.$

Network function.

N/W fn.



$$H(s) = \left.\frac{L\{R(t)\}}{L\{E(t)\}}\right|_{I.C=0}$$

$$\left.\frac{R(s)}{E(s)}\right|_{I.C=0} = \frac{N(s)}{D(s)}$$

N/W fn

Either Impedance or Admittance

Driving point Immittance fn — when Excitation & Response variable both are associated with same port

Transfer fn. — when excitation & response both are associated with two different ports.

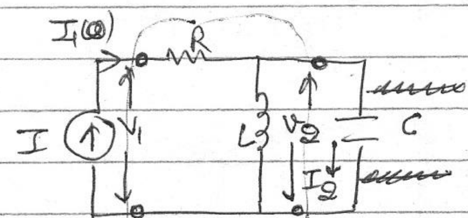
$v_1 \rightarrow V_2$ $\quad H(s) = \frac{V_2}{V_1}$ Ratio fn

$I_1 \rightarrow I_2$ $\quad$ Ratio fn

$v_1 \rightarrow I_2$ $\quad H(s) = \frac{I_2}{V_1}$ Admittance fn

$I_1 \rightarrow V_2$ $\quad$ Impedance fn

Q find D.P Impedance
D.P Admittance
Transfer Impedance
" Admittance
current ratio
voltage ratio



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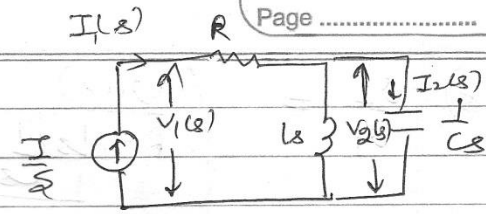


$$= \cancel{\frac{\frac{1}{Cs}+Ls}{\frac{1}{Cs}\times Ls}} + R$$

$$= \frac{1+LCs^2}{\cancel{\frac{L}{C}}} + R$$

$$= \cancel{\frac{C(1+LCs^2)}{L} + R} = \cancel{\frac{C(1+LCs^2)+RL}{L}}$$

$$\cancel{I_1(s) = \frac{\frac{I}{s}}{\frac{C(1+LCs^2)+RL}{L}}}$$

$$\cancel{V_1(s) = I_1(s)}$$

1) $$\frac{V_1(s)}{I_1(s)} = \frac{Ls\cdot\frac{1}{Cs}}{Ls+\frac{1}{Cs}} + R$$

2) $$\frac{I_1(s)}{V_1(s)} = \frac{Ls+\frac{1}{Cs}}{Ls\cdot\frac{1}{Cs} + R\left(Ls+\frac{1}{Cs}\right)}$$

* $\boxed{\text{Driving point Admittance fn} = \frac{1}{\text{D.P Impedance fn}}}$

Not applicable with transfer fn.

3) $$\frac{V_2(s)}{I_1(s)} = Z_{21} = \frac{\left(\frac{Ls}{Ls+\frac{1}{Cs}}\right)\cancel{\frac{I}{s}}\cdot\frac{1}{Cs}}{\cancel{\frac{I}{s}}}$$

4) $$\frac{I_2(s)}{V_1(s)} = \frac{\frac{V_1(s)}{R+\frac{Ls\cdot\frac{1}{Cs}}{Ls+\frac{1}{Cs}}}\left(\frac{Ls}{Ls+\frac{1}{Cs}}\right)}{V_1(s)}$$

5) $$\frac{v_2}{v_1} = \frac{\left(\dfrac{v_1(s)}{R + \dfrac{Ls \cdot \frac{1}{Cs}}{Ls + \frac{1}{Cs}}}\right)\left(\dfrac{Ls}{Ls + \frac{1}{Cs}}\right) \cdot \dfrac{1}{Cs}}{v_1(s)}$$

6) $$\frac{I_2}{I_1} = \frac{\dfrac{Ls}{Ls + \frac{1}{Cs}}(I_1)}{I_1}$$

<u>Soldering type of entry</u> – then ① source maintains architecture of original ckt ② source doesnt

<u>Cutting type of entry</u> – ② source maintains architecture of original ckt. ① source doesnt

⇒ 2 port n/w may be 4 terminal or 3 terminal n/w

2 port means first 2 terminals represent excitation, left " " " response.

However, if n/w 4 terminal, then any 2 terminals as excitation & any 2 as response.

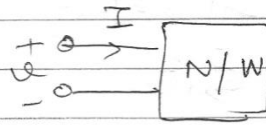
22/3/2014.

Date 22/3/14
Page

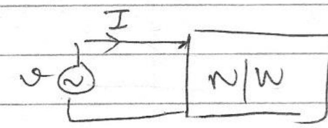
One port n/w

All fns will be driving point fns.

$Z = \frac{V}{I}$, $\frac{I}{V} = Y$

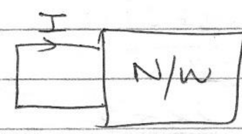


$Y = \frac{I}{V}\Big|_{V=0}$ represents natural ν of system



natural ν, when excitation zero

i.e



$Z = \frac{V}{I}\Big|_{I=0}$ — open ckt impedance parameter

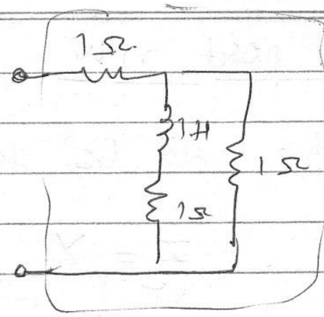
→ ~~response~~ excitation in this is I

$Z(s) = \frac{g(s)}{f(s)}$ $f(s) = 0$ roots of this eqn or poles of $Z(s)$ represents natural ν of n/w under the conditions that ~~both~~ terminals are open.

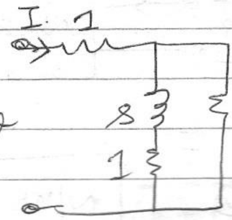
Poles of $Z(s)$ is zeroes of $Y(s)$. Only in case of one port.

$$\boxed{\frac{1}{Y} = Z}$$

Q c ⟶ Define Y(s) and Z(s)



Soln In laplace



$\frac{1(s+1)}{1+s+1} + 1$

$\frac{s+1}{s+2} + 1$

$\frac{s+1+s+2}{s+2}$

$\frac{2s+3}{s+2}$

$Z(s) = \frac{V}{I} = \frac{\cancel{I}\frac{(2s+3)}{(s+2)}}{\cancel{I}}$

$= \frac{2s+3}{s+2}$

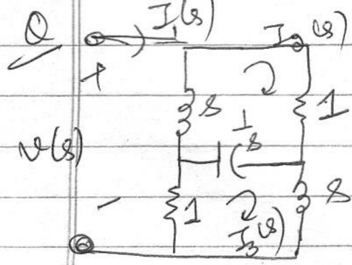
Cross check — (i) Put $s=0$ i.e dc.
i.e Inductor as short ckt.
then eq. resistance $= \frac{3}{2}$

(ii) When s goes to ∞ i.e $X_L = \infty$.
i.e branch of L open ckt.

$\lim_{s \to 0} \frac{2s+3}{s+2} \quad \frac{s\left(2+\frac{3}{s}\right)}{s\left(1+\frac{2}{s}\right)} = 2$

same from ckt.

Q



find $Y(s)$

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Solⁿ $Y(s) = \dfrac{I(s)}{V(s)}$

self Impedance for loop

Mutual Impedance b/w loop 1 a

$$\begin{bmatrix} (s+1) & -s & -1 \\ -s & \left(1+\frac{1}{s}+s\right) & -\frac{1}{s} \\ -1 & -\frac{1}{s} & \left(\frac{1}{s}+s+1\right) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} V_s(s) \\ 0 \\ 0 \end{bmatrix}$$

$$I_1(s) = \frac{\Delta_{11}}{\Delta}\,(V(s))$$

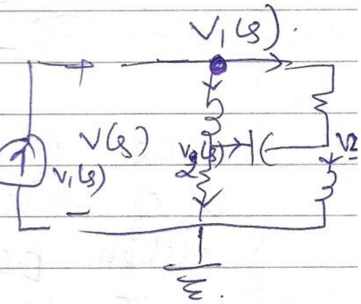
$$\frac{I_1(s)}{V(s)} = \frac{\Delta_{11}}{\Delta}$$

$$\boxed{\text{Ans} = \frac{s^2+s+2}{2s^2+s+1}}$$

This method is more suitable for finding Admittance parameters.

If we have to find impedance parameter us node eqⁿ

$$\underbrace{\begin{bmatrix} 1+\frac{1}{s} & -\frac{1}{s} & -1 \\ -\frac{1}{s} & \left(1+s+\frac{1}{s}\right) & -s \\ -1 & -s & \left(1+s+\frac{1}{s}\right) \end{bmatrix}}_{\text{for outgoing}} \begin{bmatrix} V_1(s) \\ V_2(s) \\ V_3(s) \end{bmatrix} = \underbrace{\begin{bmatrix} I(s) \\ 0 \\ 0 \end{bmatrix}}_{\text{source should be merging towards node}}$$

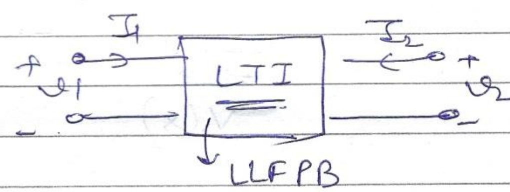


2 Port N/W

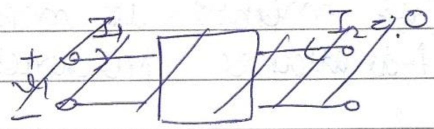
Excitation	Response	Des. fn (Describing fn.)
I_1, I_2	V_1, V_2	Z
V_1, V_2	I_1, I_2	Y
I_1, V_2	V_1, I_2	h
V_1, I_2	I_1, V_2	g
V_1, I_1	$V_2, -I_2$	ABCD or Tx line
V_2, I_2	$V_1, -I_1$	A'B'C'D' or Inve Tx line

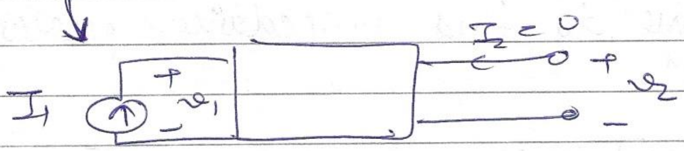
Z parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$



$$Z_{11} = \frac{V_1}{I_1}\Bigg|_{I_2=0}$$





Poles of this n/w fn gives the natural ν of this system under the condition when both ports open.

I_2 is already open. To find natural ν excitation I_1 also zero. i.e Open ckt.

$$Z_{12} = \frac{V_1}{I_2}\Bigg|_{I_1=0}$$

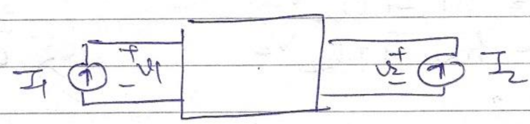
also both port open then natural ν.

Z_{11} = Driving point Impedance fn
Z_{12} = Transfer ~~Admitt~~ Impedance fn
Z_{21} = " "
Z_{22} = " D.P Impedance fn.

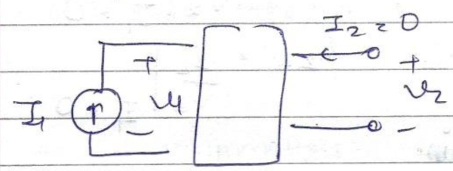
$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} \qquad Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}.$$

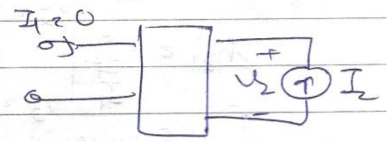
i.e in same state, i.e O.C. the natural ν of n/w is defined. So poles must be same in all. So, lower polynomial in all Z must be same

or chain
use



Apply superposition





$V_1 \Rightarrow \big| I_2 = 0$ $\qquad$ $V_1 \Rightarrow \big| I_1 = 0$
$V_2 \Rightarrow \big| I_2 = 0$ $\qquad$ $V_2 \Rightarrow \big| I_1 = 0$.

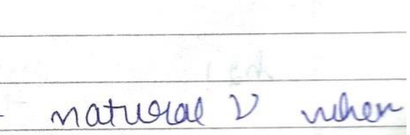
$Z_{11} I_1$, $Z_{21} I_1$ $\qquad$ $Z_{12} I_2$, $Z_{22} I_2$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

[Open ckt parameter

Y parameters

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$



$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}.$$

Poles of this fn represent natural ν when both ports are short ckt.

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} \qquad Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} \qquad Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

Y also called short ckt parameters.

h parameters

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

voltage Backward gain

Gain fn

Ratio fn

$$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0} \qquad h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$$

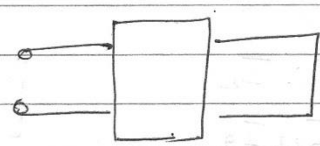
$$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0} \qquad h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$$

Forward current gain

poles of this fn represent natural ν when second port short and first open

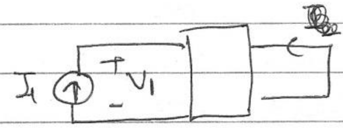
second short first open.

for All —

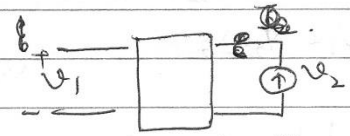


natural ν in this state

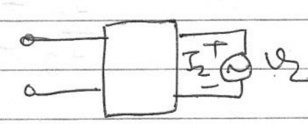
h_{11} →



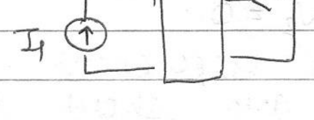
$h_{12} =$



$h_{22} =$



h_{21}



Driving fn – associated with same port
Transfer fn – " " different port

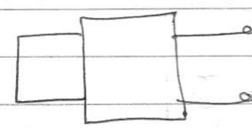
g parameters

$$I_1 = g_{11} V_1 + g_{12} I_2$$
$$V_2 = g_{21} V_1 + g_{22} I_2$$

$g_{11} = \frac{I_1}{V_1}\Big|_{I_2 = 0}$ $g_{12} = \frac{I_1}{I_2}\Big|_{V_1 = 0}$ $g_{21} = \frac{V_2}{V_1}\Big|_{I_2 = 0}$

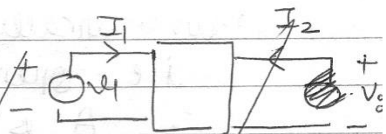
$g_{22} = \frac{V_2}{I_2}\Big|_{V_1 = 0}$

Natural freq →



ABCD parameters

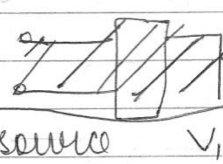
$$V_2 = A V_1 + B I_1$$
$$-I_2 = C V_1 + D I_1$$



$A = \frac{V_2}{V_1}\Big|_{I_1 = 0}$ $B = \frac{V_2}{I_1}\Big|_{V_1 = 0}$ $C = -\frac{I_2}{V_1}\Big|_{I_1 = 0}$

$D = -\frac{I_2}{I_1}\Big|_{V_1 = 0}$

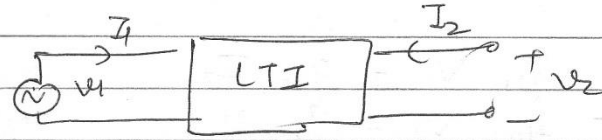
~~Natural freq~~



Power delivered by source $V_1 I_1$
Power received by load $= V_1(-I_2)$

used to calculate power

~~A'B'C'D' parameters~~



In this case excitation is written in terms of response

$$V_1 = A V_2 + B(-I_2)$$
$$I_1 = C V_2 + D(-I_2)$$

~~$V_2 = 0$~~ $A = \left.\dfrac{V_1}{V_2}\right|_{I_2 = 0}$ $\quad B = -\left.\dfrac{V_1}{I_2}\right|_{V_2 = 0}$

$C = \left.\dfrac{I_1}{V_2}\right|_{I_2 = 0}$ $\quad D = -\left.\dfrac{I_1}{I_2}\right|_{V_2 = 0}$

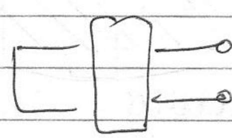
Mathematically if $V_2 = 0$ i.e S.C and $I_2 = 0$ i.e open ckt. contradictory

∴ A, B, C, D are not n/w fn.

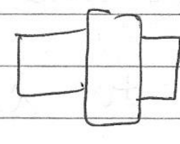
∴ Poles of this fn do not represent natural ν of system.

Inverse of these parameters is n/w fn to define behaviour of n/w because actually v_1 is the excitation

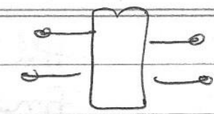
$$\frac{1}{A} = \left.\frac{V_2}{V_1}\right|_{I_2 = 0}$$



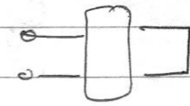
$\dfrac{1}{B} = -\left.\dfrac{I_2}{V_1}\right|_{V_2 = 0}$



$$\frac{1}{C} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0}$$

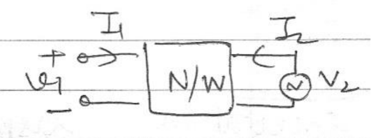


$$\frac{1}{D} = \left.\frac{-I_2}{I_1}\right|_{V_2 = 0}$$



$\underline{A'B'C'D'}$

$$V_2 = A'V_1 + B'\,(-I_1)$$
$$I_2 = C'V_1 + D'\,(-I_1)$$

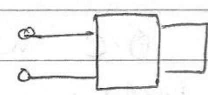


$$A' = \left.\frac{V_2}{V_1}\right|_{I_1 = 0} \qquad B' = \left.\frac{V_2}{-I_1}\right|_{V_1 = 0} \qquad C' = \left.\frac{I_2}{V_1}\right|_{I_1 = 0}$$

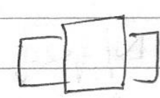
$$D' = \left.\frac{-I_2}{I_1}\right|_{V_1 = 0}$$

Again contradictory

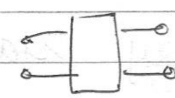
$$\frac{1}{A'} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0} \longrightarrow$$



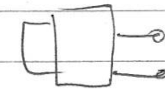
$$\frac{1}{B'} = \left.-\frac{I_1}{V_2}\right|_{V_1 = 0} \longrightarrow$$



$$\frac{1}{C'} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} \longrightarrow$$

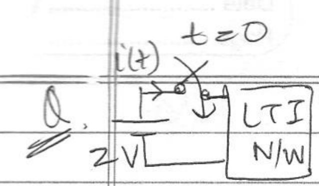


$$\frac{1}{D'} = \left.-\frac{I_1}{I_2}\right|_{V_1 = 0} \longrightarrow$$



i.e poles of these fns represent natural ν of system in the following stage.

Q.



All I.C = 0 $i(t) = 2 - 2e^{-t}$
find steady state response of same n/w with I.C = 0 when exciting fn is Cost.

$$Z(s) = \frac{\frac{2}{s}}{\frac{2}{s} - \frac{2}{(1+s)}} = \frac{\frac{2}{s}}{\frac{2(s+1) - 2s}{s(s+1)}}$$

$$= \frac{\frac{2}{\cancel{s}}}{\frac{\cancel{2s} + 2 - \cancel{2s}}{\cancel{s}(s+1)}} = \frac{\cancel{2}(s+1)}{\cancel{2}} = (s+1)$$

$$Y(s) = \frac{1}{(s+1)}$$

excitation response

$I(s) = Y(s) \cdot V(s)$

$= \frac{1}{(s+1)} V(s)$ $\cancel{Z(s) = (s+1)}$

$= \frac{1}{j+1} V(s)$ $\cancel{= j\omega + 1}$
$\cancel{= j + 1}$

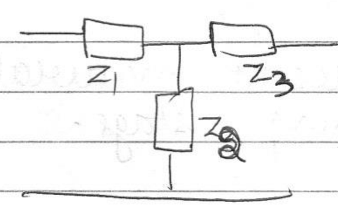
Also find O.C natural ν and short ckt natural freq.

S.C N/freq = -1.   (poles of $Y(s)$).
O.C N freq = ∞.

steady state value = $\frac{1}{j+1}$ Cost

$= \frac{1}{\sqrt{2}} \cos(t - 45^\circ)$

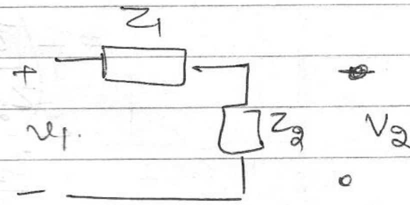
Q



find Z parameter Y, T and h.

$\underline{\underline{Z}}$

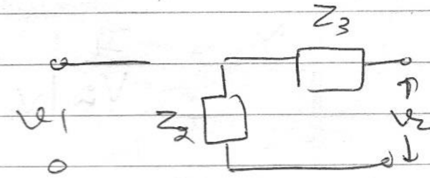
$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0}$$

$$\frac{(Z_1 + Z_2) I_1}{I_1}$$

$$Z_{11} = Z_1 + Z_2.$$



$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0}$$

$$= \frac{Z_2 I_2}{I_2} = Z_2.$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0}.$$

$$= Z_2$$

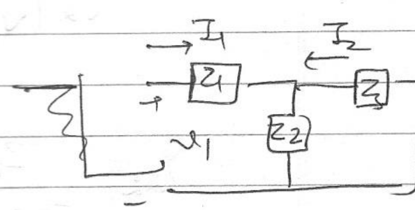
$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0}$$

$$= \frac{I_2 (Z_3 + Z_2)}{I_2}$$

$$= Z_3 + Z_2.$$

$\underline{Y}$ parameters

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$



$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2 = 0}$$

$$= \frac{I_1}{\left(\frac{Z_2 Z_3}{Z_2 + Z_3} + Z_1\right) I_1}$$

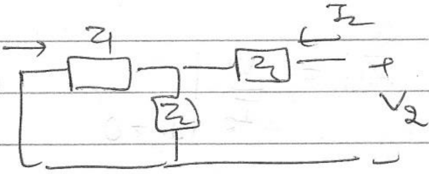
$$Y_{11} = \frac{1}{\frac{Z_2 Z_3}{Z_2 + Z_3} + Z_1} = \frac{Z_2 + Z_3}{Z_2 Z_3 + Z_1 Z_2 + Z_1 Z_3}$$

$$Y_{21} = \frac{I_2}{V_1}$$

$$= -\frac{\left(\frac{Z_2}{Z_2+Z_3} I_1\right)}{\left(\frac{Z_2 Z_3}{Z_2+Z_3} + Z_1\right) I_1} = -\frac{Z_2}{\cancel{Z_2+Z_3}} \frac{(\cancel{Z_2+Z_3})}{(Z_2 Z_3 + Z_1 Z_2 + Z_1 Z_3)}$$

$$= \boxed{\frac{-Z_2}{Z_2 Z_3 + Z_1 Z_2 + Z_1 Z_3}}$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1 = 0}$$



$$Y_{12} = -\left[\frac{\frac{Z_2}{Z_1+Z_2}(\cancel{I_2})}{\left(\frac{Z_1 Z_2}{Z_1+Z_2} + Z_3\right)\cancel{I_2}}\right]$$

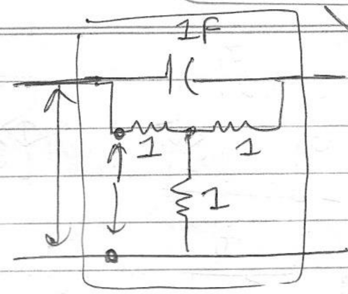
$$= \frac{-Z_2}{Z_1 Z_2 + Z_3 Z_1 + Z_3 Z_2}$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1 = 0} = \frac{\cancel{I_2}}{\left(\frac{Z_1 Z_2}{Z_1+Z_2} + Z_3\right)(\cancel{I_2})}$$

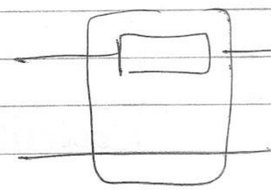
$$Y_{22} = \frac{Z_1+Z_2}{Z_1 Z_2 + Z_3 Z_1 + Z_3 Z_2} = \cancel{\frac{Z_1 Z_3 + Z_2 Z_3}{Z_1 Z_2 + Z}}$$

Q5 Find Z & Y.

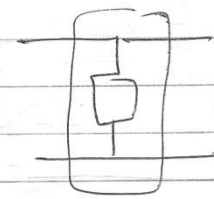
Parallel connection ←



Q6 Find Z, Y, h, T

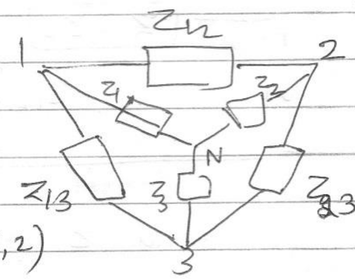


Q7 Find Z, Y, h, T



$\Pi \Leftrightarrow \Delta\ \nabla$

$\star \Leftrightarrow Y \Leftrightarrow T$



$$Z_{12} = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}$$ (different from 1,2)

$$Z_{13} = \frac{\Delta}{Z_2}$$ $Y \rightarrow \Delta$

$$Z_{23} = \frac{\Delta}{Z_1}$$

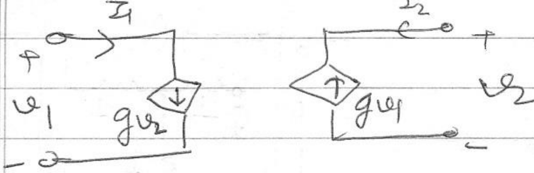
$$Z_1 = \frac{Z_{12}\cdot Z_{13}}{\Sigma Z}$$ $\Delta \rightarrow Y$

$$Z_2 = \frac{Z_{21}Z_{23}}{\Sigma Z}$$ → those connected to node 2 $\Sigma Z = Z_{12} + Z_{13} + Z_{23}$.

$$Z_3 = \frac{Z_{13}\cdot Z_{23}}{\Sigma Z}$$

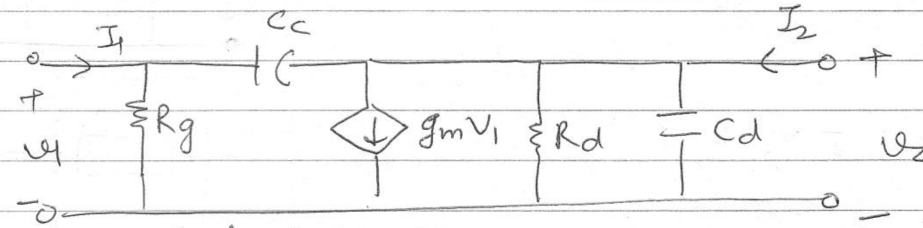
Date ....
Page ....

Q 8



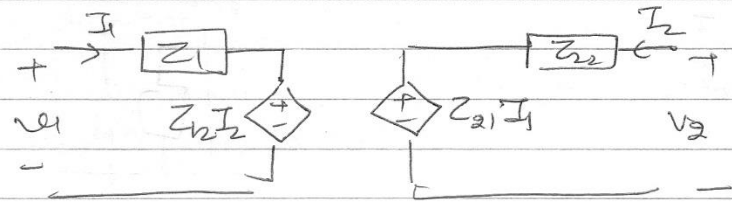
calculate Y parameter.

Q 9



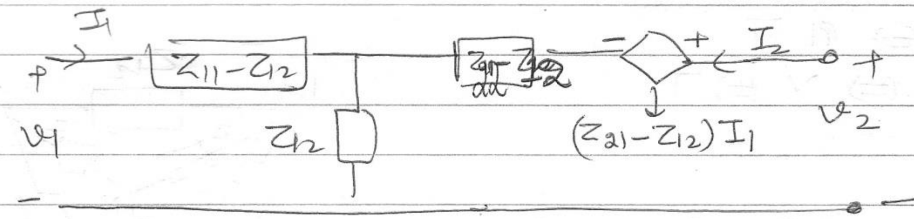
calculate Y.

Q 10



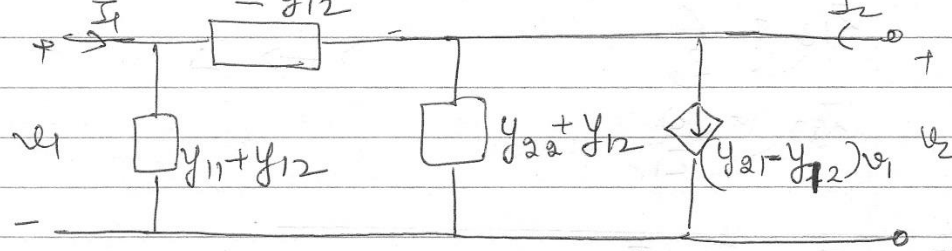
calculate Z parameter

Q 11



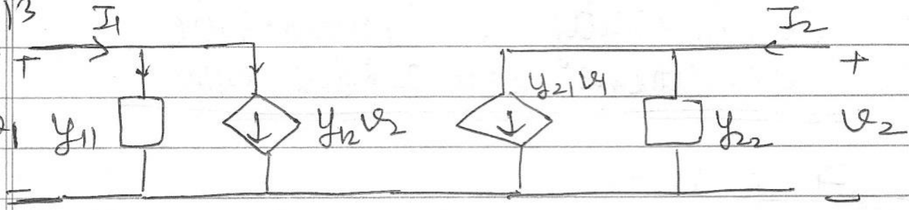
find Z parameters.

Q 12



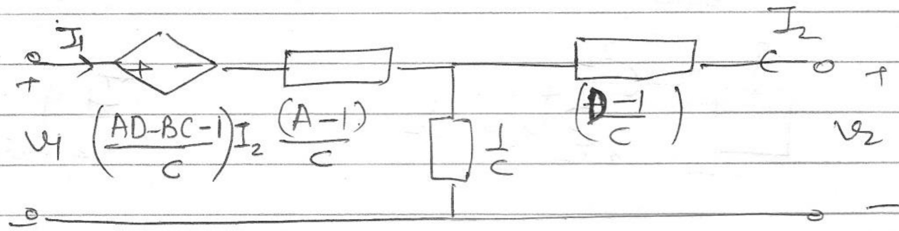
find Y parameter

Q 13



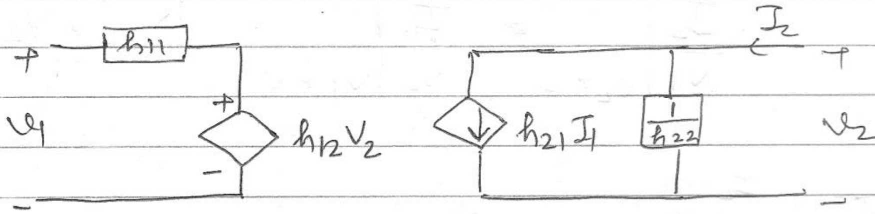
Calculate Y parameter.

Q 14



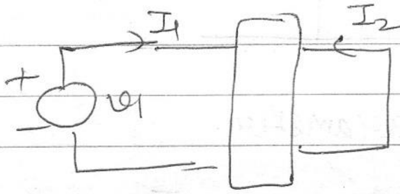
Calculate ABCD parameters.

Q 15



Calculate h parameter

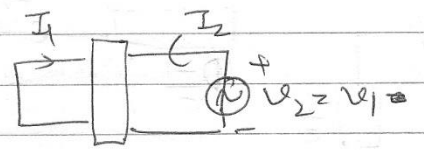
Reciprocity & Symmetric Conditions for 2 port N/W



$v_1 = E$

$I_2 = R.$

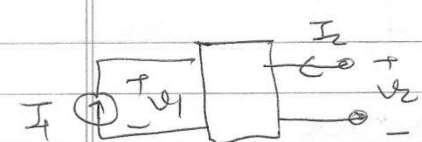
$$\left.\frac{I_2}{V_1}\right|_{V_2=0}$$



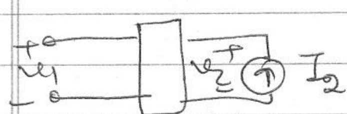
$$\left.\frac{I_1}{V_2}\right|_{V_1=0}$$

Now if these two are equal then N/W reciprocal.

$$\left.\frac{I_2}{V_1}\right|_{V_2=0} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \qquad \boxed{Y_{12} = Y_{21}}$$



$$\left.\frac{V_2}{I_1}\right|_{I_2=0} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

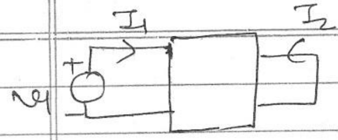


for reciprocal N/W.

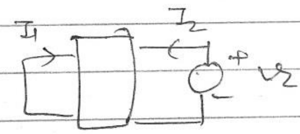
$\therefore$ for Reciprocal N/W $\boxed{Z_{12} = Z_{21}}$

In reciprocity, E and R at diff. ports.

In symmetry, E & R at same port

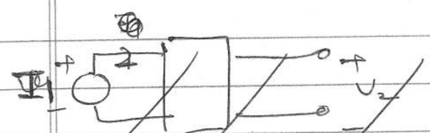


$\cancel{\left.\frac{V_1}{I_1}\right|_{V_2=0}}$ $\qquad$ $\left.\frac{I_1}{V_1}\right|_{V_2=0}$

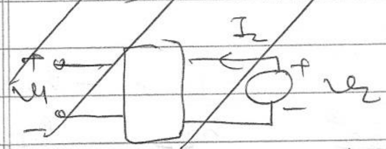


$\cancel{\left.\frac{V_2}{I_2}\right|_{V_1=0}}$ $\qquad$ $\left.\frac{I_2}{V_2}\right|_{V_1=0}$

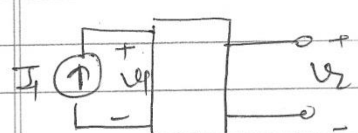
$\boxed{Y_{11}=Y_{22}}$ ~~Voltteer~~ for symmetric N/W



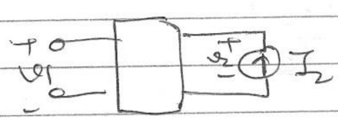
$\cancel{\left.\frac{I_1}{V_1}\right|_{I_2=0}}$



$\cancel{\left.\frac{I_2}{V_2}\right|_{I_1=0}}$



$\left.\frac{V_1}{I_1}\right|_{I_2=0}$



$\left.\frac{V_2}{I_2}\right|_{I_1=0}$

$$\boxed{Z_{11}=Z_{22}}$$

h parameter

$V_1 = h_{11} I_1 + h_{12} V_2$
$I_2 = h_{21} I_1 + h_{22} V_2$

condition of reciprocity is

$$\left.\frac{V_1}{I_2}\right|_{I_1=0} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad Z_{12}=Z_{21}$$

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$$0 = I_1 h_{21} + V_2 h_{22}$$

$$\left.\frac{V_2}{I_1}\right|_{I_2=0} = -\frac{h_{21}}{h_{22}} = Z_{21}$$

$$I_2 = V_2 h_{22}$$

$$I_2 = \frac{V_1}{h_{12}} h_{22}$$

$$\boxed{\left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{h_{12}}{h_{22}} = Z_{12}}$$

$$\cancel{\frac{h_{12}}{h_{22}} = \frac{h_{21}}{h_{22}}} \qquad \frac{h_{12}}{h_{22}} = -\frac{h_{21}}{h_{22}}$$

$$\boxed{h_{12}h_{22} + h_{21}h_{22} = 0}$$

for ABCD Reciprocity

$$V_1 = AV_2 - B(I_2)$$
$$I_1 = CV_2 - D I_2$$

$$\left.\frac{V_1}{I_2}\right|_{I_1=0} = \qquad \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$\Rightarrow \quad CV_2 = D I_2$

$\Rightarrow \quad V_1 = AV_2 \,, \quad I_1 = CV_2$

$$\frac{V_2}{I_1} = \frac{1}{C}$$

$$\left(\frac{V_1 + BI_2}{A}\right) C = D I_2$$

$$C(V_1 + BI_2) = AD I_2$$

$$CV_1 = (AD - BC) I_2$$

$$\left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{AD-BC}{C}$$

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$$\frac{AD-BC}{\cancel{C}} = \frac{1}{\cancel{C}}$$ $\boxed{AD-BC=1}$

h parameter symmetry

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$0 = h_{21}I_1 + h_{22}V_2$$

$$0 = h_{21}I_1 + h_{22}\left(\frac{V_1 - h_{11}I_1}{h_{12}}\right)$$

$$0 = \left(h_{21} - \frac{h_{11}h_{22}}{h_{12}}\right)I_1 - \frac{h_{22}}{h_{12}}V_1$$

$$\left.\frac{V_2}{I_2}\right|_{I_1=0}$$ $$I_2 = h_{22}V_2$$

$$\frac{V_2}{I_2} = h_{22}$$

$$\frac{V_1}{I_1} = \frac{h_{21} - \frac{h_{11}h_{22}}{h_{12}}}{\frac{h_{22}}{h_{12}}} = \frac{h_{21}h_{12} - h_{11}h_{22}}{h_{22}}$$

$$\frac{h_{21}h_{12} - h_{11}h_{22}}{h_{22}} = \frac{1}{h_{22}}$$

$$\boxed{h_{21}h_{12} - h_{11}h_{22} = 1}$$

g parameter.

$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$

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$$\left.\frac{V_1}{I_2}\right|_{I_1=0} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$g_{11}V_1 + g_{12}I_2 = 0$$

$$\frac{V_1}{I_2} = -\frac{g_{12}}{g_{11}}$$

$$\frac{V_2}{I_1} = \frac{g_{21}}{g_{11}}$$

$$-\frac{g_{12}}{g_{11}} = \frac{g_{21}}{g_{11}}$$

$$\boxed{g_{21}+g_{12}=0}$$

Symmetry

$$\frac{V_1}{I_1} = \frac{1}{g_{11}}$$

$$g_{11}V_1 + g_{12}I_2 = 0$$

$$g_{11}\left(\frac{V_2 - g_{22}I_2}{g_{21}}\right) + g_{12}I_2 = 0$$

$$V_2\left(\frac{g_{11}}{g_{21}}\right) - \left(\frac{g_{22}g_{11}+g_{12}}{g_{21}}\right)I_2 = 0$$

$$= \frac{\dfrac{g_{22}g_{11}+g_{12}g_{21}}{\cancel{g_{21}}}}{\dfrac{g_{11}}{\cancel{g_{21}}}}$$

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$$\frac{g_{22}g_{11} + g_{12}g_{21}}{\cancel{g_{11}}} = \frac{1}{\cancel{g_{11}}}$$

$$\boxed{g_{22}g_{11} + g_{12}g_{21} = 1}$$

ABCD (Symmetry)

$\cancel{V_1}$
$\cancel{I_1}$

$$V_1 = AV_2$$
$$I_1 = CV_2$$
$$\frac{V_1}{I_1} = \frac{A}{C}$$

$$CV_2 = DI_2$$
$$\frac{V_2}{I_2} = \frac{D}{C}$$
$$\frac{D}{C} = \frac{A}{C} \qquad \boxed{A = D}$$

Inverse Tx line (Reciprocity)

$$V_2 = A'V_1 - B'I_1$$
$$I_2 = C'V_1 - D'I_1$$

$$V_2 = AV_1$$
$$I_2 = CV_1 \qquad \frac{V_1}{I_2} = \frac{1}{C}$$

$$CV_1 = DI_1$$
$$C\left[\frac{V_2 + BI_1}{A}\right] = DI_1 \qquad \boxed{D'A' - B'C' = 1}$$
$$\frac{C}{A}V_2 = \left(\frac{D - BC}{A}\right)I_1$$
$$\frac{V_2}{I_1} = \frac{DA - BC}{C}$$

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Symmetry

$$\frac{V_1}{I_1} = \frac{D}{C}$$

$$V_2 = AV_1$$
$$I_2 = CV_1$$
$$\frac{A}{C}$$

$$\frac{D}{C} = \frac{A'}{C}$$ $\boxed{A' = D'}$

* Inter Relationship b/w different types of Parameters

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$V = [Z][I]$$
$$I = [Y][V]$$

$$V = [Z][Y][V]$$

$$[Z][Y] = 1$$

$$\boxed{[Z] = [Y]^{-1}}$$ This doesnt mean $Y_{11} = \frac{1}{Z_{11}}$

h & T

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$V_1 = A V_2 - B I_2$$
$$I_1 = C V_2 - D I_2 .$$

$$V_1 = h_{11}\left(\frac{I_2 - h_{22} V_2}{h_{21}}\right) + h_{12} V_2 .$$

$$V_1 = \frac{h_{11}}{h_{21}} I_2 - \left(\frac{h_{22}}{h_{21}} - h_{12}\right) V_2$$

$$\boxed{A = -\left(\frac{h_{22}}{h_{21}} - h_{12}\right)}$$

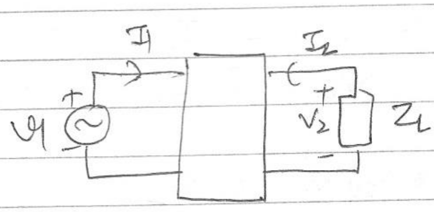
$$\boxed{-B = \frac{h_{11}}{h_{21}}}$$

$$I_1 = \frac{I_2}{h_{21}} - \frac{h_{22} V_2}{h_{21}}$$

$$\boxed{C = -\frac{h_{22}}{h_{21}}} \qquad \boxed{D = -\frac{1}{h_{21}}}$$

2 port N/W terminated at a load

$$\boxed{v_2 = v_L = -Z_L I_2}$$



$$\cancel{z_{11} = \frac{v_1 - z_{12} I_2}{I_1}}$$

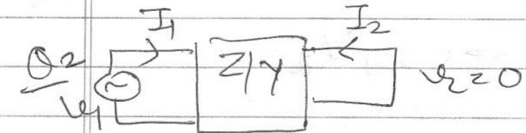
$$\cancel{= v_1 - z_{12}}$$

Interconnection of Multiple 2 port N/W

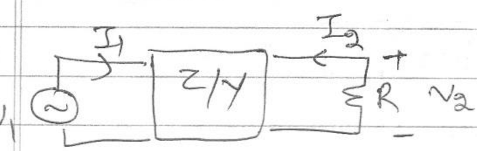
1) Parallel Interconnection (already done).
2) series
3) Cascade.

Q1

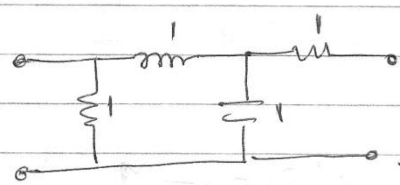

calculate $\frac{V_2}{V_1}$ in terms of Z & Y.

Q2


find $\frac{I_2}{I_1}$

Q3


calculate $\frac{I_2}{V_1}$, $\frac{V_2}{I_1}$ and $\frac{V_2}{V_1}$

Q4


Or all in series
find Z, h, Y, T parameters.
→ one T connected in series and

1.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$V_1 = Z_{11} I_1$$
$$V_2 = Z_{21} I_1$$

$$\boxed{\frac{V_2}{V_1} = \frac{Z_{21}}{Z_{11}}}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$Y_{21} V_1 = -Y_{22} V_2$$

$$\boxed{\frac{V_2}{V_1} = -\frac{Y_{21}}{Y_{22}}}$$

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2. $V_1 = Z_{11} I_1 + Z_{12} I_2$

$0 = Z_{21} I_1 + Z_{22} I_2$

$$\boxed{\frac{I_2}{I_1} = -\frac{Z_{21}}{Z_{22}}}$$

$I_1 = Y_{11} V_1$

$I_2 = Y_{21} V_1$

$$\boxed{\frac{I_2}{I_1} = \frac{Y_{21}}{Y_{11}}}$$

3. $V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$V_2 = -R I_2$

$-R I_2 = Z_{21} I_1 + Z_{22} I_2$

$-(R + Z_{22}) I_2 = Z_{21} I_1$

$$V_1 = Z_{11}\left(-\frac{(R+Z_{22})}{Z_{21}} I_2\right) + Z_{12} I_2$$

$$V_1 = \left[-\frac{Z_{11}(R+Z_{22})}{Z_{21}} + Z_{12}\right] I_2$$

$$\frac{I_2}{V_1} = \frac{1}{-\frac{Z_{11}(R+Z_{22})}{Z_{21}} + Z_{12}}$$

$$\boxed{\frac{I_2}{V_1} = \frac{Z_{21}}{-Z_{11}R - Z_{11}Z_{22} + Z_{12}Z_{21}}}$$

$$V_2 = -R\left(-\frac{Z_{21}}{R+Z_{22}}\right) I_1$$

$$\boxed{\frac{V_2}{I_1} = \frac{RZ_{21}}{R+Z_{22}}}$$

$\cancel{V_2 = -R\left(\frac{V_1 - Z_{11} I_1}{Z_{12}}\right)} \Rightarrow V_2 = \cancel{-R}\,(V_1$

$\cancel{-R\,(V_1 + Z_{11}(R+Z_{22}}$

$$\frac{V_2}{V_1} = \frac{Z_{11}I_1 + Z_{12}I_2}{Z_{21}I_1 + Z_{22}I_2}$$

$$= \frac{-\dfrac{Z_{11}(R+Z_{22})}{Z_{21}}I_2 + Z_{12}I_2}{-Z_{21}\left(\dfrac{R+Z_{22}}{Z_{21}}\right)I_2 + Z_{22}I_2}$$

$$= \frac{-\dfrac{Z_{11}}{Z_{21}}(R+Z_{22}) + Z_{12}}{-\dfrac{Z_{21}}{Z_{21}}(R+Z_{22}) + Z_{22}}$$

$$= \frac{-Z_{11}(R+Z_{22}) + Z_{12}Z_{21}}{Z_{21}\left[-(R+Z_{22}) + Z_{22}\right]}$$

8. $I_1 = g v_2$ $\qquad$ $I_1 = Y_{11}V_1 + Y_{12}V_2$

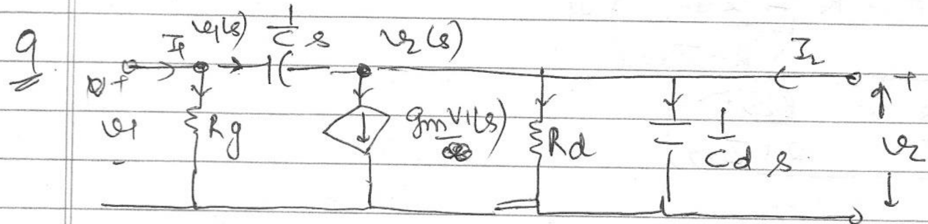
$I_2 = -g v_1$ $\qquad$ $I_2 = Y_{21}V_1 + Y_{22}V_2$

$\dfrac{I_1}{V_2} = g$ $\qquad$ $\dfrac{I_2}{v_1} = -g$

$\boxed{Y_{12} = g}$ $\qquad$ $\boxed{Y_{21} = -g}$.

$\boxed{Y_{11} = 0}$ $\qquad$ $\boxed{Y_{22} = 0}$

9.



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$$\frac{v_1(s)}{Rg} + \cancel{\frac{v_1(s) - v_2(s)}{\frac{1}{Cs}}} = 0$$

$$I_1(s) = \frac{v_1(s)}{Rg} + \frac{v_1(s) - v_2(s)}{\frac{1}{Cs}}$$

$$I_1(s) = v_1(s)\left[\frac{1}{Rg} + Cs\right] - v_2(s)(Cs)$$

$$I_2(s) = v_2(s)\left[Cds + \frac{1}{Rd}\right] + gmV(s) + \frac{v_2(s) - v_1(s)}{\frac{1}{Cs}}$$

$$I_2(s) = v_2(s)\left[Cds + \frac{1}{Rd} + Cs\right] + (gm - Cs)V_1(s)$$

$$Y_{11} = \frac{1}{Rg} + Cs \qquad Y_{12} = -Cs$$

$$Y_{21} = gm - Cs \qquad Y_{22} = Cds + Cs + \frac{1}{Rd}.$$

10 $v_1 = I_1 Z_1 + Z_{12} I_2$ $\qquad$ $v_1 = I_1 Z_{11} + Z_{12} I_2$

$v_2 = I_2 Z_{22} + Z_{21} I_1$ $\qquad$ $v_2 = Z_{21} I_1 + Z_{22} I_2$

Thus, this is eq. ckt of Z parameters.

11 $v_1 = I_1(Z_{11} - Z_{12}) + Z_{12}(I_1 + I_2)$

$v_2 = I_2(Z_{22} - Z_{12}) + (Z_{21} - Z_{12})I_1 + Z_{12}(I_1 + I_2)$

$v_1 = \cancel{I_1(Z_{11} + Z_{12})}.\ (Z_{11} - \cancel{Z_{12}} + \cancel{Z_{12}})I_1 + Z_{12}I_2$

$v_1 = Z_{11}I_1 + Z_{12}I_2$

$v_2 = I_2(Z_{22} - \cancel{Z_{12}} + \cancel{Z_{12}}) + (Z_{21} - \cancel{Z_{12}} + \cancel{Z_{12}})I_1$

$v_2 = I_2 Z_{22} + Z_{21} I_1$

$v_1 = Z_{11} I_1 + Z_{12} I_2$ // eq. to Z n/w.

$\boxed{\begin{matrix} V = IR \\ I = VY \end{matrix}}$

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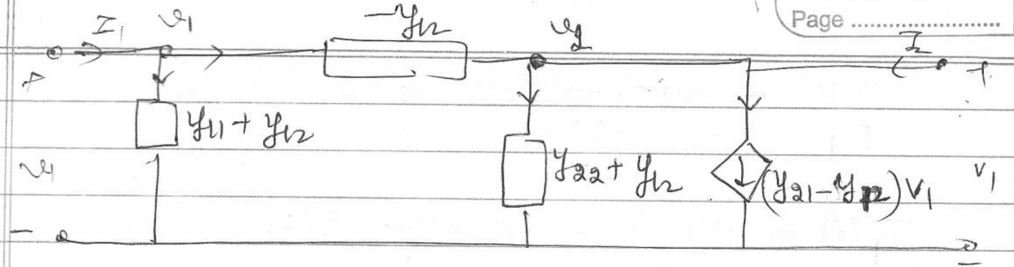

$I_1 = v_1(y_{11}+y_{12}) + (v_1 - v_2)(-y_{12})$

$I_1 = v_1(y_{11} - \cancel{y_{12}} + \cancel{y_{12}}) + v_2 y_{12}$

$I_2 = (y_{21} - y_{12})V_1 + V_2(y_{22}+y_{12}) + \underline{v_2 - v_1}(-y_{12})$

~~$I_2 = V_1(y_{21} + y_{12})$~~

$I_2 = (y_{21} - \cancel{y_{12}} + y_{12})V_1 + (y_{22} + \cancel{y_{12}} - \cancel{y_{12}})V_2$

$Y_{11} = y_{11} - \cancel{y_{12}} + \cancel{y_{12}}$ $\quad Y_{12} = y_{12}$

$Y_{21} = y_{21} - \cancel{y_{12}} + y_{12}$ $\quad Y_{22} = y_{22}$. $\quad$ eq. $\underline{\underline{Y\ n/w}}$

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$I_1 = V_1(y_{11}) + y_{12}v_2$
$I_2 = V_2 y_{22} + y_{21}v_1$

$Y_{11} = y_{11}$ $\quad Y_{22} = y_{22}$
$Y_{12} = y_{12}$ $\quad Y_{21} = y_{21}$

eq. ckt of Y parameters.

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$$v_1 = \left(\frac{AD-BC-1}{C}\right) I_2 + I_1\left(\frac{A-1}{C}\right) + \frac{1}{C}(I_1+I_2)$$

$$v_2 = \left(\frac{D-1}{C}\right) I_2 + \frac{1}{C}(I_1+I_2)$$

$$v_1 = \left(\frac{AD-BC-1}{C} + \frac{1}{C}\right) I_2 + \left(\frac{A-1}{C} + \frac{1}{C}\right) I_1$$

$$v_1 = \left(\frac{AD-BC-\cancel{1}+\cancel{1}}{C}\right) I_2 + \left(\frac{A-\cancel{1}+\cancel{1}}{C}\right) I_1$$

$$\rightarrow v_1 = \left(\frac{AD-BC}{C}\right) I_2 + \left(\frac{A}{C}\right) I_1$$

$$v_2 = \left(\frac{D-1}{C} + \frac{1}{C}\right) I_2 + \frac{1}{C} I_1$$

$$\rightarrow v_2 = \frac{D}{C} I_2 + \frac{1}{C} I_1$$

$$v_1 = \left(\frac{AD-BC}{C}\right) I_2 + \left(\frac{A}{C}\right)\left[\left(v_2 - \frac{D}{C} I_2\right) C\right]$$

$$v_1 = \left(\frac{AD-BC}{C}\right) I_2 + A v_2 - \left(\frac{AD}{C}\right) I_2$$

$$v_1 = \left(\frac{AD-BC}{C} - \frac{AD}{C}\right) I_2 + A v_2$$

$$v_1 = A v_2 \cancel{\left(\frac{A^2}{C} - \frac{AD-BC}{C}\right) I_2} - B I_2$$

$$I_1 = C\left(v_2 - \frac{D}{C} I_2\right)$$

$$I_1 = C v_2 - D I_2$$

$A = A$ $\quad C = C$ $\quad$ eq. ckt of ABCD parameters

$B = B$ $\quad D = D$

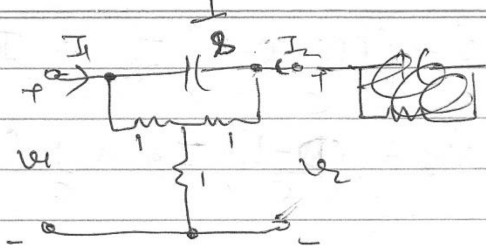
15

$v_1 = h_{11} I_1 + h_{12} V_2$ $\quad$ eq. ckt

$I_2 = v_2 h_{22} + h_{21} I_1$. $\quad$ of h parameter

Q.

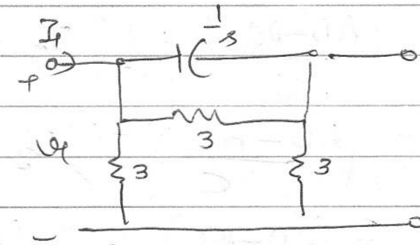
$Z_{12} = \dfrac{1+1+1}{1}$



$Z_{12} = 3$

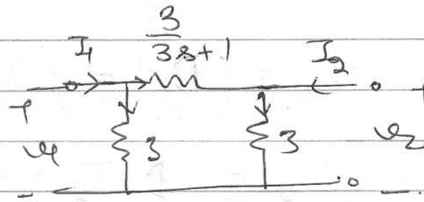
$Z_{21} = \dfrac{3}{1} = 3$

$Z_{13} = 3$



$\dfrac{\frac{1}{s} \times 3}{\frac{1}{s} + 3} = \dfrac{\frac{3}{s}}{\frac{1+3s}{s}}$

$= \dfrac{3}{3s+1}$



$I_1 = \dfrac{v_1(s)}{3} + \dfrac{v_1(s) - v_2(s)}{\frac{3}{3s+1}}$

$I_1(s) = v_1(s)\left[\dfrac{1}{3} + \dfrac{3s+1}{3}\right) - \dfrac{3s+1}{3} v_2(s).$

$I_2(s) = \dfrac{v_2(s)}{3} - \left(\dfrac{v_1(s) - v_2(s)}{\frac{3}{3s+1}}\right).$

$I_2(s) = v_2(s)\left[\dfrac{1}{3} + \dfrac{3s+1}{3}\right] - \dfrac{3s+1}{3} v_1(s))$

$Y_{11} = \dfrac{2+3s}{3}$

$Y_{12} = -\dfrac{3s+1}{3}$

$Y_{21} = -\dfrac{3s+1}{3}$

$Y_{22} = \dfrac{2+3s}{3}$

Q. $V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$\cancel{I_1 = \dfrac{V_1}{Z_{11}} - \dfrac{Z_{12}}{Z_{11}} I_2}$

$\cancel{I_1 = \dfrac{V_1}{Z_{11}} - \dfrac{Z_{12}}{Z_{11}}\left[\dfrac{V_2}{Z_{22}} - \dfrac{Z_{21}}{Z_{22}} I_1\right)}$

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(c) ~~ollbee~~

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$V_1 = \frac{I_1}{Y_{11}} - \frac{Y_{12}}{Y_{11}}V_2$$

$$V_1 = \frac{I_1}{Y_{11}} - \frac{Y_{12}}{Y_{11}}\left(\frac{I_2 - Y_{21}V_1}{Y_{22}}\right)$$

$$V_1 = \frac{I_1}{Y_{11}} - \frac{Y_{12}}{Y_{11}}\frac{I_2}{Y_{22}} + \frac{Y_{12}Y_{21}V_1}{Y_{11}Y_{22}}$$

$$V_1\left(1 - \frac{Y_{12}Y_{21}}{Y_{11}Y_{22}}\right) = \frac{I_1}{Y_{11}} - \frac{Y_{12}}{Y_{11}Y_{22}}I_2$$

$$V_1\left(\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{\cancel{Y_{11}}Y_{22}}\right) = \frac{I_1}{\cancel{Y_{11}}} - \frac{Y_{12}}{\cancel{Y_{11}}Y_{22}}I_2$$

$$V_1 = \frac{Y_{22}I_1}{Y_{11}Y_{22} - Y_{12}Y_{21}} - \frac{Y_{12}}{Y_{11}Y_{22} - Y_{12}Y_{21}}I_2$$

$$Z_{11} = \frac{Y_{22}}{|Y|} = \frac{\frac{2+3s}{3}}{\left(\frac{2+3s}{3}\right)^2 - \left(\frac{3s+1}{3}\right)^2}$$

$$Z_{12} = \frac{-Y_{12}}{|Y|} = \frac{\frac{2s+3}{3}}{\frac{1}{9}\left[4+\cancel{9s^2}+12s-\cancel{9s^2}-1-6s\right]}$$

$$= \frac{\frac{2s+3}{3}}{\frac{1}{9}\left[6s+3\right]}$$

$$= \frac{2s+3}{\cancel{3}} \times \frac{\cancel{9}}{\cancel{3}(2s+1)}$$

$$= \boxed{\frac{2s+3}{2s+1}}$$

$$Z_{12} = \frac{\frac{3s+1}{3}}{\frac{1}{9}\left[6s+3\right]} = \boxed{\frac{3s+1}{2s+1}}$$

$$Z_{21} = \frac{3s+1}{2s+1}$$

$$Z_{22} = \frac{2s+3}{2s+1}$$

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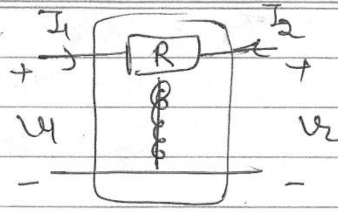
Soln



$I_1 = -I_2$

$v_1 = I_1 R + v_2$

$I_1 R = v_1 - v_2$

$I_1 = \frac{1}{R} v_1 - \frac{1}{R} v_2$ $\qquad Y_{11} = \frac{1}{R} \qquad Y_{12} = -\frac{1}{R}$

$I_2 = -\frac{1}{R} v_1 + \frac{1}{R} v_2$ $\qquad Y_{21} = -\frac{1}{R} \qquad Y_{22} = \frac{1}{R}$

$Z_{11} = \frac{Y_{22}}{\Delta Y} = \frac{1/R}{\frac{1}{R^2} - \frac{1}{R^2}} = \infty$

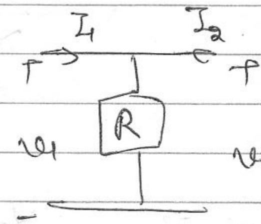
$Z_{12} = Z_{21} = Z_{22} = \infty$

$h_{11} = R$ $\qquad h_{21} = \frac{-\frac{1}{R}}{\frac{1}{R}} = -1$

$h_{12} = \frac{+\frac{1}{R}}{\frac{1}{R}} = 1$ $\qquad h_{22} = 0$

$A = \frac{-\frac{1}{R}}{-\frac{1}{R}} = 1 \qquad B = R \qquad C = 0 \qquad D = 1$

Soln



$v_1 = v_2$

$v_1 = (I_1 + I_2) R$

$v_1 = I_1 R + I_2 R$

$v_2 = I_1 R + I_2 R$

$Z_{11} = R$

$Z_{12} = R$

$Z_{21} = R$

$Z_{22} = R$

$Y_{11} = \frac{Z_{22}}{|Z|} = \frac{R}{R^2 - R^2} = \infty$

$Y_{12} = \frac{-Z_{11}}{\Delta Z} = \infty$

$Y_{21} = \frac{-Z_{21}}{\Delta Z} = \infty \qquad Y_{22} = \frac{Z_{11}}{\Delta Z} = \infty$

$h_{11} = \frac{\Delta Z}{Z_{22}} = 0$ $\quad$ $h_{21} = -\frac{Z_{21}}{Z_{22}} = -1$

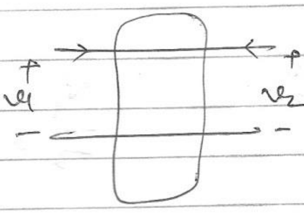
$h_{12} = \frac{Z_{12}}{Z_{22}} = 1$ $\quad$ $h_{22} = \frac{1}{Z_{22}} = \frac{1}{R}$

$T =$ $\quad A = \frac{Z_{22}}{Z_{12}} = 1$.

$B = \frac{\Delta Z}{Z_{12}} = 0$

$C = \frac{1}{Z_{12}} = \frac{1}{R}$ $\quad$ $D = \frac{Z_{11}}{Z_{12}} = 1$.

Q $T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$



Z, Y, h, g not defined

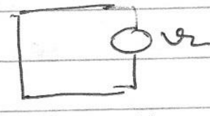
$v_1 = v_2$ $\quad$ $A = 1$ $\quad$ $B = 0$

$I_1 = -I_2$ $\quad$ $D = 1$ $\quad$ $C = 0$

$g_{11} = \frac{I_1}{V_1}\Big|_{I_2 = 0}$ $\quad$ $g_{12} = \frac{I_1}{I_2}\Big|_{V_1 = 0}$

$g_{21} = \frac{v_2}{v_1}\Big|_{I_2 = 0}$ $\quad$ $g_{22} = \frac{V_2}{I_2}\Big|_{V_1 = 0}$.

a) $g_{11} = 0$ $\quad$ b)

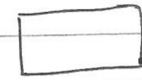


not a valid representation as both arms of V_2 are short circuit.

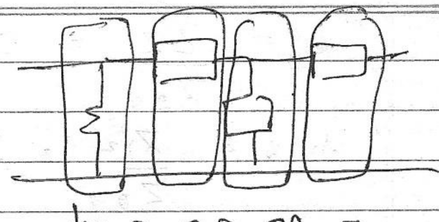
c) $g_{21} = 1$

d) $g_{22} =$ undefined (same as g_{12}).

This is no valid circuit ←

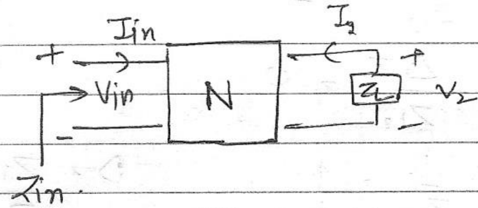


Q all in cascade



$[T_1]\,[T_2]\,[T_3]\,[T_4]$

Q



$$Z_{in} = \frac{V_1}{I_1} = -Z_L$$

find h parameters

if $\frac{V_1}{V_2} = k$

Soln

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

$$-Z_L I_2 = V_2$$

$$\frac{V_2}{I_2} = -Z_L \qquad \frac{V_2}{I_2} = \frac{V_1}{I_1}$$

$$\boxed{\frac{V_2}{V_1} = \frac{I_2}{I_1} = \frac{1}{k}}.$$

$$\frac{V_1}{V_2} = \frac{I_1}{I_2} = k$$

$$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0} = 0$$

$$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0} = k.$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0} = \frac{1}{k}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0} = 0$$

$$\begin{bmatrix} 0 & k \\ \frac{1}{k} & 0 \end{bmatrix}$$

⇒ check whether n/w is active or passive.

$I_1 = I_2 k$

$V_1 = V_2 k$

$V_1 \times I_1 + V_2 I_2$

$k V_2 \cdot k I_2 + V_2 I_2$

$V_2 I_2 \left(k^2 + 1\right)$

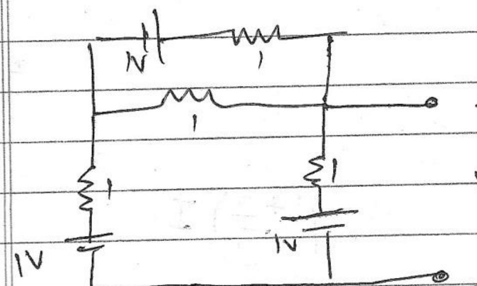
$= (1+k^2)(-Z_L) I_2^2$

$= -(1+k^2)(Z_L) I_2^2$

Result ≤ 0 so passive n/w.
i.e O/P

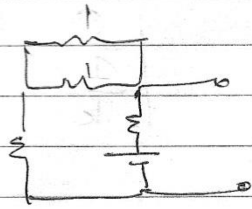
N/W Theorems

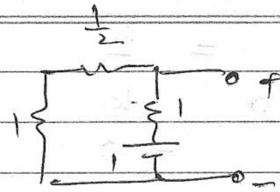
Q



$V_{AB} = ?$

This is non linear N/W





$$1 = \left(1 + \frac{1}{2} + 1\right) I$$

$$1 = \left(\frac{2+1+2}{2}\right)$$

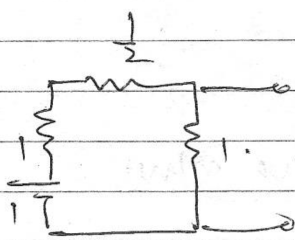
$$2 = 5I \qquad I = \frac{2}{5}$$

$\underline{\underline{V_{AB}}} =$

$$-1 + 1 \times \frac{2}{5} + V_{AB} = 0$$

$$V_{AB} = +1 - \frac{2}{5}$$

$$= \frac{3}{5}\ V$$

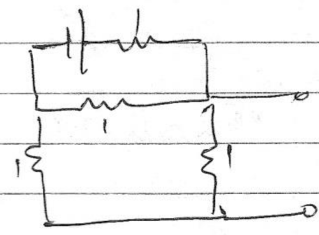


$$1 = \frac{5I}{2}$$

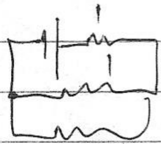
$$I = \frac{2}{5}$$

$$V_{AB} = \frac{1 \times 2}{5} = \frac{2}{5}\ V.$$

1V



2



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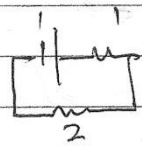


$\frac{2}{3}$

$$1 = \left(1 + \frac{2}{3}\right) I$$

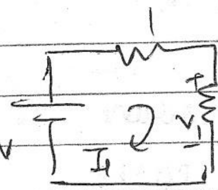
$$1 = \frac{5}{3} I \qquad I = \frac{3}{5}$$

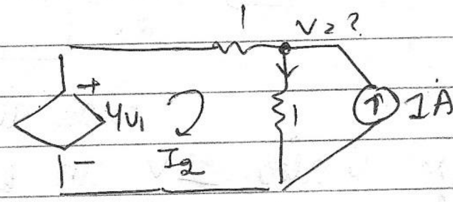
$\frac{1}{\cancel{3}} \times \frac{\cancel{3}}{5}$ $\qquad$ $I = \frac{1}{5}$

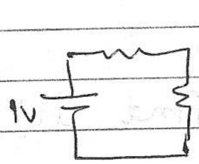
$v = \frac{1}{5}$

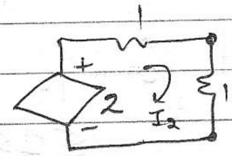
Total $V_{AB} = \frac{3}{5} + \frac{2}{5} + \frac{1}{5} = \frac{6}{5}$ V

Q









$1 = I_1 + I_1$

$1 = 2I_1$

$I_1 = \frac{1}{2}$

$V_1 = \frac{1}{2} \times 1 = \frac{1}{2}$

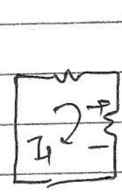
$2I_2 - 2 = 0$

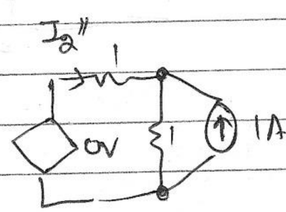
$2I_2 = 2$

$I_2 = 1$

$1 \times I_2$

$\boxed{V = 1}$





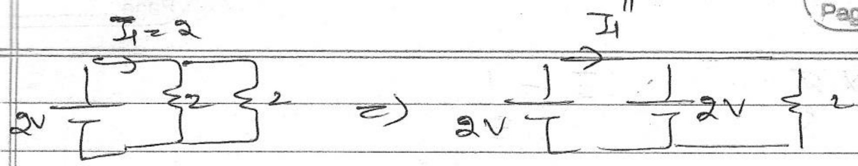
$I = \frac{1}{2} \times 1$

$V = \frac{1}{2}$ V

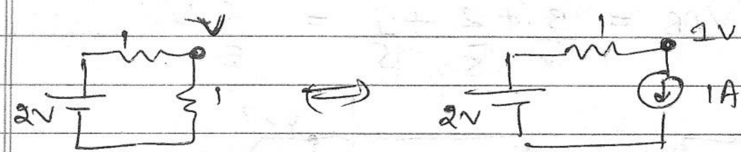
$I_2'' = -\frac{1}{2}$ A

$V = 1 + \frac{1}{2} = \frac{3}{2}$ V

Substitution



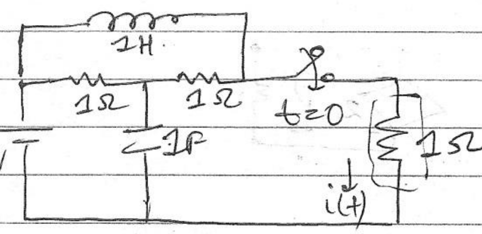
Cant be used as initially I_1 was defined to be 2. However I_1'' is not defined



Now also current is 1A

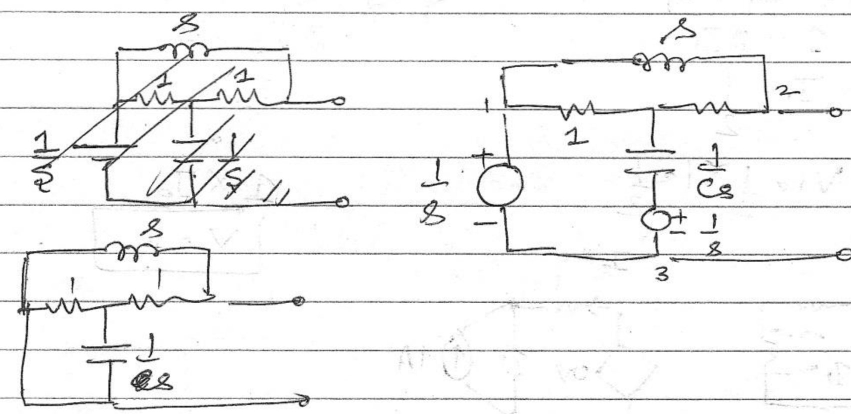
Voltage V also 1V in both cases.

Q



Find $i(t)$

Soln



$$1 + \frac{1}{s} + \frac{1}{s} = 1 + \frac{2}{s} = \frac{s+2}{s}$$

$$Z_{12} = \frac{\frac{s+2}{\cancel{s}}}{\frac{1}{\cancel{s}}} = s+2$$

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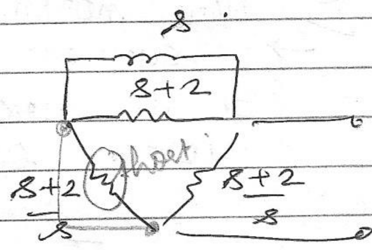
$$Z_{13} = \frac{\frac{s+2}{s}}{1} = \frac{s+2}{s}$$

$$Z_{23} = \frac{\frac{s+2}{s}}{1} = \frac{s+2}{s}$$



$$\frac{s(s+2)}{s+s+2} = \frac{s(s+2)}{2s+2} + \frac{s+2}{s}$$

$$= \frac{s^2(s+2) + (s+2)2(s+1)}{2(s+1)s}$$

so

$$\frac{s(s+2)}{2s+2} \,\|\, \frac{s+2}{s} = \frac{s^2(s+2) + 2(s+2)(s+1)}{2(s+1)(s)}$$

$$= \frac{\frac{(s+2)\left[s^2+2(s+1)\right]}{2(s+1)(s)}\left(\frac{s+2}{s}\right)}{\frac{(s+2)\left[s^2+2(s+1)\right]}{2(s+1)(s)} + \frac{s+2}{s}}$$

Ans

correct Ans

$$\boxed{Z_0(s) = \frac{s(s+2)}{s^2+2s+2}}$$

$$\frac{\frac{\cancel{(s+2)}\,(s^2+2(s+1))}{\cancel{2(s+1)(s)}}\;\frac{(s+2)}{s}}{\frac{\cancel{(s+2)}\,(s^2+2(s+1)+2(s+1)(\cancel{s+2})}{\cancel{2(s+1)(s)}}}$$

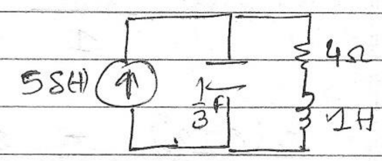
$$= \frac{\frac{(s^2+2(s+1))(s+2)}{s}}{s^2+2s+2+2s+2}$$

$$= \frac{(s^2+2s+2)\,\cancel{(s+2)}}{s(s+2)^{\cancel{2}}}$$

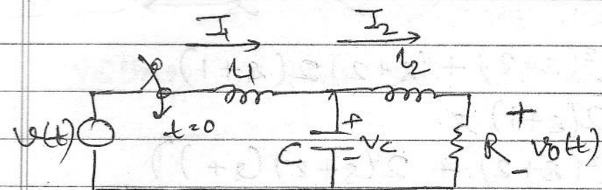
$v_{oc} = \frac{1}{s}$.

$I(t) = 1 - \frac{1}{2} e^{-t} + t e^{-t}$

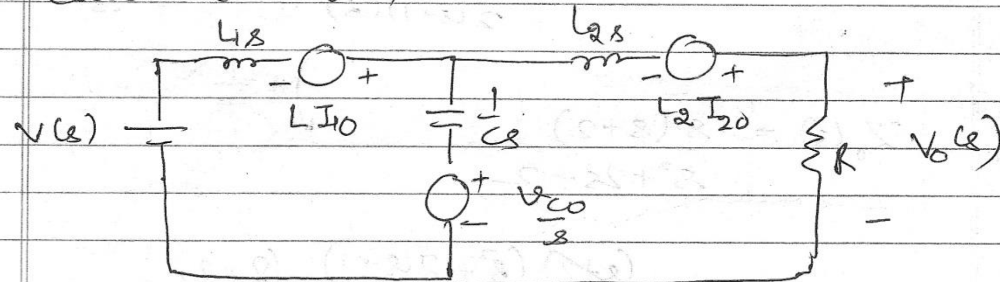
Q Here norton
→ current = 5
It is already in
Norton eq. circuit

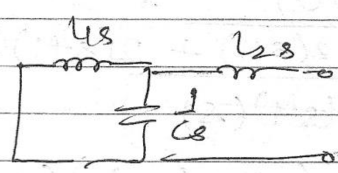


Q



$I_1(0^-)$ · are all non $= I_{10}$
$I_2(0^-)$ zero values $= I_{20}$
$v_c(0^-)$ $= V_{c0}$
Calculate $v_0(s)$





$$\frac{L_1 \cancel{s} \times \frac{1}{C\cancel{s}}}{L_1 s + \frac{1}{Cs}} = \frac{\frac{L_1}{C}}{\frac{CL_1 s^2 + 1}{Cs}} = \frac{L_1 s}{L_1 C s^2 + 1} + L_2 s$$

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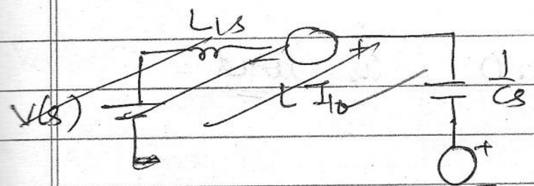
$$R_{eq} = \frac{L_1 s + L_2 s L_1 C s^2 + L_2 s}{L_1 C s^2 + 1}$$



$$I(s) = \left[\frac{V(s) + L_1 I_{10} - \frac{V_{C0}}{s}}{L_1 s + \frac{1}{Cs}} \right]$$

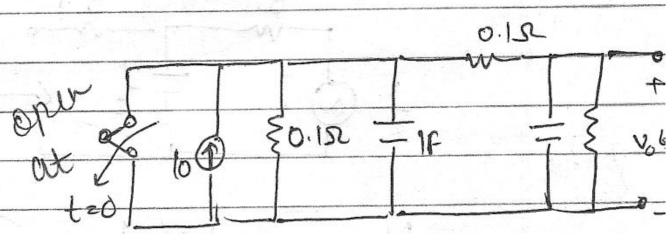
$$I(s) = \left[\frac{V(s) - \frac{V_{C0}}{s} + L_1 I_{10}}{\frac{L_1 C s^2 + 1}{Cs}} \right]$$

$$V_{oc}(s) = V(s) - I_1(s) L_1(s) + L_1 I_{10} + L_2 I_{20}$$

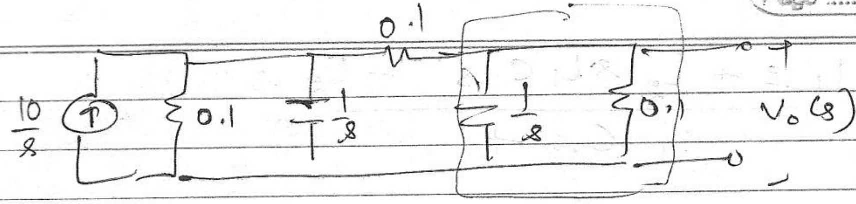
because of presence of $L_2 I_{20}$

$I_2 = 0$ $\qquad$ $L_2 s = 0$

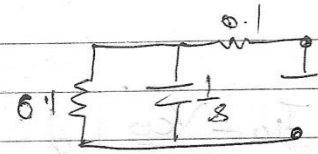
Q.



We found all IC = 0



Assume both $\frac{1}{s}$ and 0.1 as load

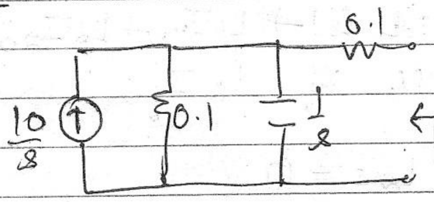


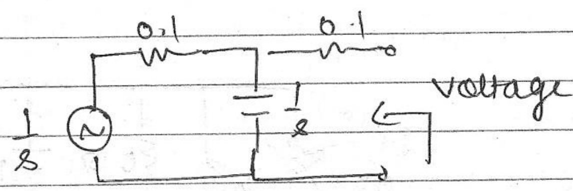
$$\frac{0.1 \times \frac{1}{s}}{0.1 + \frac{1}{s}} + 0.1 = \frac{\frac{0.1}{s}}{\frac{0.1s+1}{s}}$$

$$\frac{0.1}{0.1s+1} + 0.1$$

$$\frac{0.1 + 0.01s + 0.1}{0.1s+1}$$

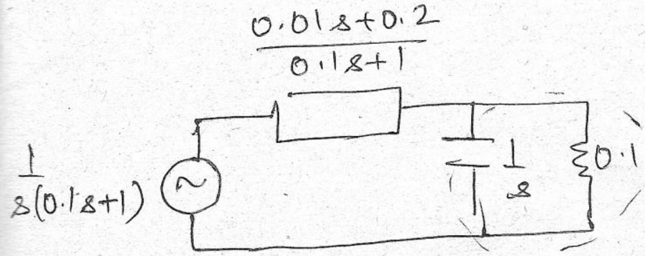
$$Z_{eq} = \frac{0.01s + 0.2}{0.1s+1}$$





$$V_{th} = \frac{\frac{1}{s} \times \frac{1}{s}}{\frac{1}{s} + 0.1} = \frac{\frac{1}{s}}{\frac{1+0.1s}{s}} \times \frac{1}{s}$$

$$V_{th} = \left(\frac{1}{0.1s+1}\right)\frac{1}{s}$$



$$\frac{\frac{1}{s}\times 0.1}{\frac{1}{s}+0.1} = \frac{\frac{0.1}{\cancel{s}}}{\frac{1+0.1s}{\cancel{s}}}$$

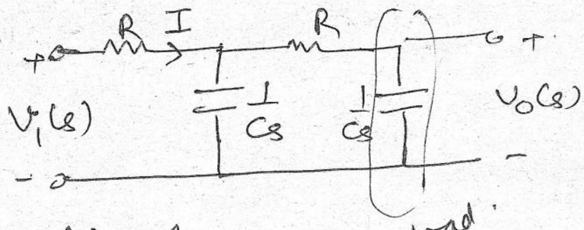
$$\frac{0.1}{1+0.1s}$$

$$V_0(s) = \frac{\frac{0.1}{1+0.1s}}{\frac{0.1}{1+0.1s}+\frac{0.01s+0.2}{0.1s+1}}\left(\frac{1}{s(0.1s+1)}\right)$$

$$= \frac{0.1}{0.1+0.01s+0.2}\left(\frac{1}{s(0.1s+1)}\right)$$

$$V_0(s) = \left(\frac{0.1}{0.01s+0.3}\right)\left(\frac{1}{s(0.1s+1)}\right)$$

Q



All cap. Initially uncharged.

Find $\frac{V_0(s)}{V_i(s)}$

Angle of $\frac{V_0(s)}{V_i(s)} = \pi$ at which ν.

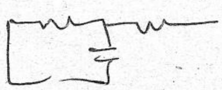
Find mag. of $F(s)$ at this freq.

Let $V_0(s) = 1$

$\cancel{I \times \frac{1}{Cs} = 1}$

$\cancel{I = Cs}$

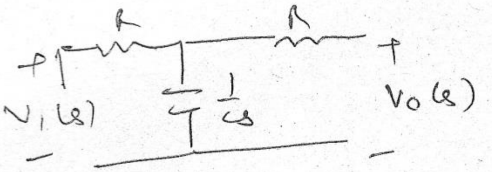
$$\cancel{\frac{\frac{1}{Cs}}{\frac{1}{Cs}+R+\frac{1}{Cs}}} = Cs$$



$$\frac{R\times\frac{1}{Cs}}{R+\frac{1}{Cs}} = \frac{\frac{R}{\cancel{Cs}}}{\frac{RCs+1}{\cancel{Cs}}}$$

$$\frac{R}{RCs+1} + R$$

$$\frac{R + R^2Cs + R}{RCs+1}$$



$$V_{Th} = \frac{\frac{1}{Cs}}{\frac{1}{Cs}+R} V_i(s) = \underline{\underline{\frac{1}{1+RCs} V_i(s)}}$$

$$V_o(s) = \frac{\frac{1}{Cs}}{\frac{1}{Cs}+R_{TH}} V_o(c)$$

$$V_o(s) = \frac{\frac{1}{Cs}}{\frac{1}{Cs}+\frac{2R+R^2Cs}{RCs+1}} \cancel{V_o(s)} \cdot \frac{1}{1+Rs} V_i(s)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{\cancel{Cs}}}{\frac{RCs+1+Cs(2R+R^2Cs)}{(RCs+1)(\cancel{Cs})}}$$

$$= \frac{1}{RCs+1+Cs(2R+R^2Cs)} \left[\frac{1}{1+Rs}\right]$$

Now put $s = j\omega$

then $\frac{V_o(s)}{V_i(s)} = a+jb$.

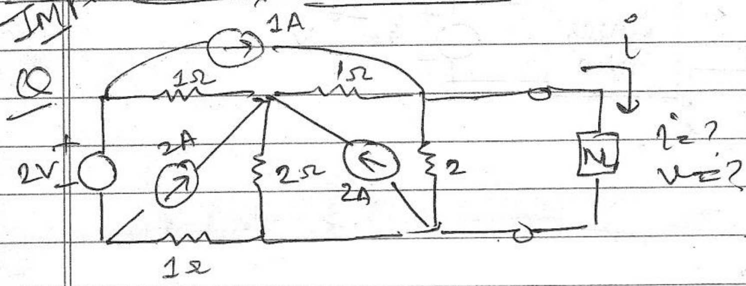
$$\tan^{-1}\frac{b}{a} = \pi$$

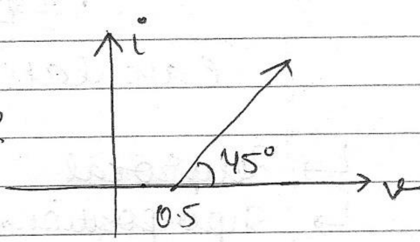
$$\omega = ?$$

* Dont use theorems in Time domain especially in Transient Analysis.

IMP

Q

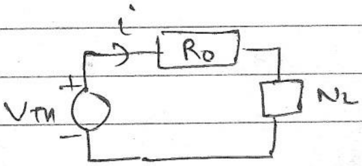




find i and v

$$i = \begin{cases} 0 & v \leq 0.5 \\ v - 0.5 & v > 0.5 \end{cases}$$

$v_{TH} - iR_0 = v$

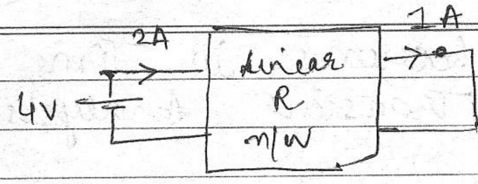


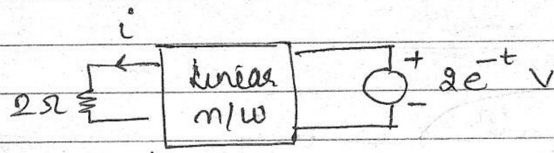
for $v \leq 0.5$

$v_{TH} = v$

If $v > 0.5$
then no soln in this reg.

Q





Calculate i

- ↳ Reciprocal
- ↳ Superposition
- ↳ Subtitution

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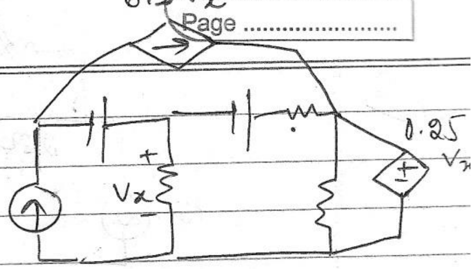
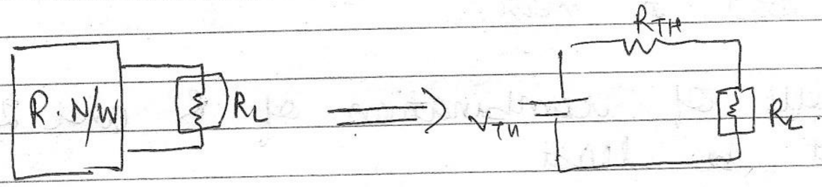


Maximum Power Transfer Theorem.



$$P_L = \left(\frac{V_{TH}}{R_{TH}+R_L}\right)^2 R_L = \frac{V_{TH}^2}{(R_{TH}+R_L)^2} R_L.$$

$$\frac{dP}{dR_{TH}} = 0$$

$$\frac{(R_{TH}+R_L)^2 V_{TH}^2 - V_{TH}^2 R_L \, 2(R_{TH}+R_L)[1}{(R_{TH}+R_L)^4}$$

$$\boxed{\cancel{R_{TH} = R_L}}$$

Internal parameters of n/w delivering power to fixed load R_L is maximum.

$\boxed{R_{TH} = R_L}$ for $\frac{dP}{dR_L}$.

condition on well known fixed N/W to variable load.

When $R_{TH} = 0$ P_L maximum } when R_{TH} varies
$R_{TH} = \infty$ P_L minimum.

$R_L = R_{TH}$ P_L max. } when R_L varies.
$R_L = 0, \infty$ P_L min.

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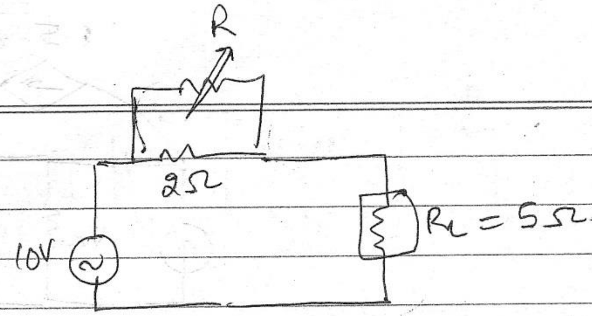
Q



find value of R so that power delivered to load to max.

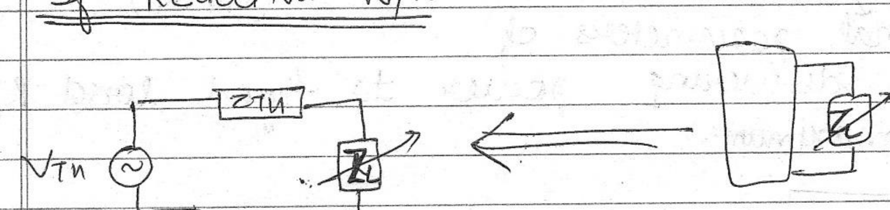
sol Value of combination of R and 2 must be least

$$Req = \frac{2R}{R+2} \quad \therefore \boxed{R = 0}$$

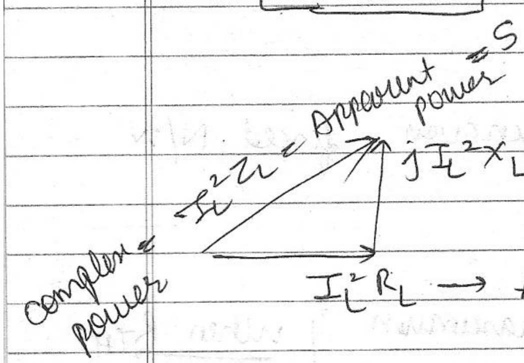
$Req = 0$

then power Transfer max.

If Reactive N/W



$Z_L = R_L + jX_L$



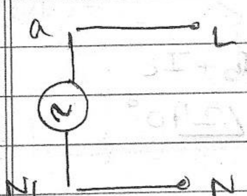
$jI_L^2 X_L \longrightarrow$ Reactive power represents stored energy in reactive element

$I_L^2 R_L \longrightarrow$ ~~Apparent~~ power
Active $= V_L I_L \cos\phi$

We have to maximise active power.

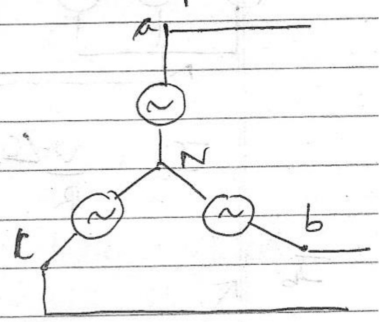
$$P_L = V_L I_L \cos\phi \longrightarrow I_L^2 R \cos\phi$$
$$= \left(\frac{V_{TH}}{Z_{TH}+Z_L}\right)^2 Z_L \cos\phi$$
$$\frac{dP_L}{dZ_L} = 0$$
we get $\boxed{Z_L = Z_{TH}^*}$

Three Phase Circuit

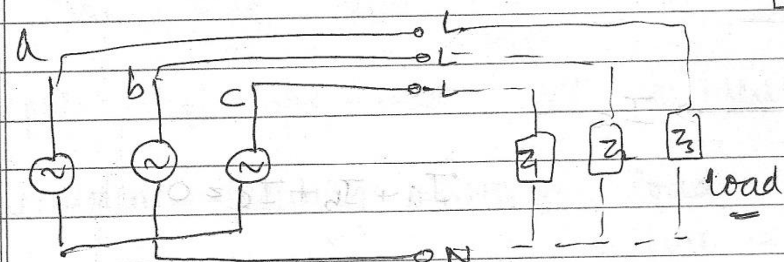


single ϕ generator

practically available



3ϕ generator



3ϕ four wire system.

$\Rightarrow$ **Balanced 3ϕ Supply**

voltage generated by generator 1

$$V_{RMS}(a) = V_{RMS}(b) = V_{RMS}(c).$$

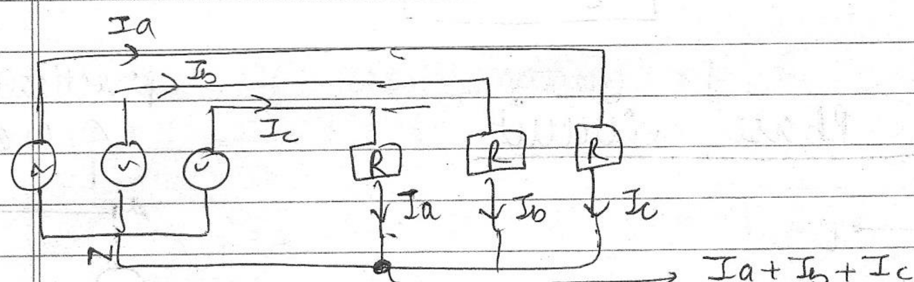
Phase diff. b/w voltage signal generated by each phase is $120°$.

$$V_a \angle 0°, \quad V_b \angle 120°, \quad V_c \angle 240°$$

Balanced load

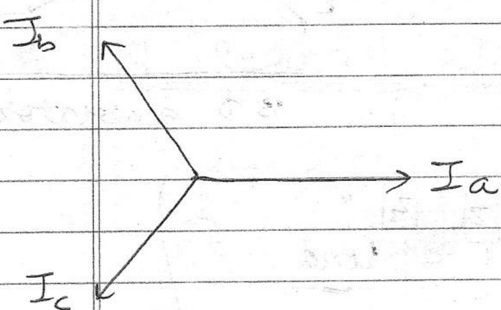
when $Z_1 = Z_2 = Z_3$

means $Z_1 = R_1 + jX_1$ $Z_2 = R_1 - jX_1$ are not equal because there is no modulus.



$I_a = \frac{V\angle 0^\circ}{R}$ $I_b = \frac{V\angle 120^\circ}{R}$ $I_c = \frac{V\angle 240^\circ}{R}$

Mag. of all same



$I_a + I_b + I_c = 0$

If exactly balanced type of supply & load are present, then fourth wire N can be removed.
Thus balanced 3ϕ, 3 wire supply.

If load not equal, then current will flow back, distorting whole N/W making even source unbalanced.

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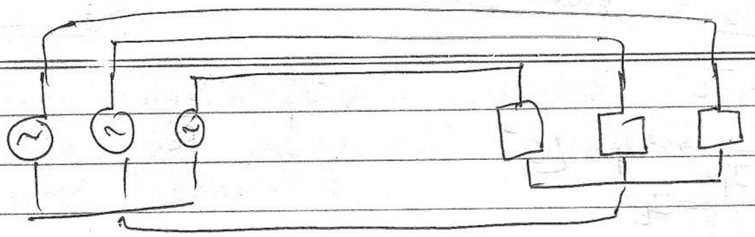
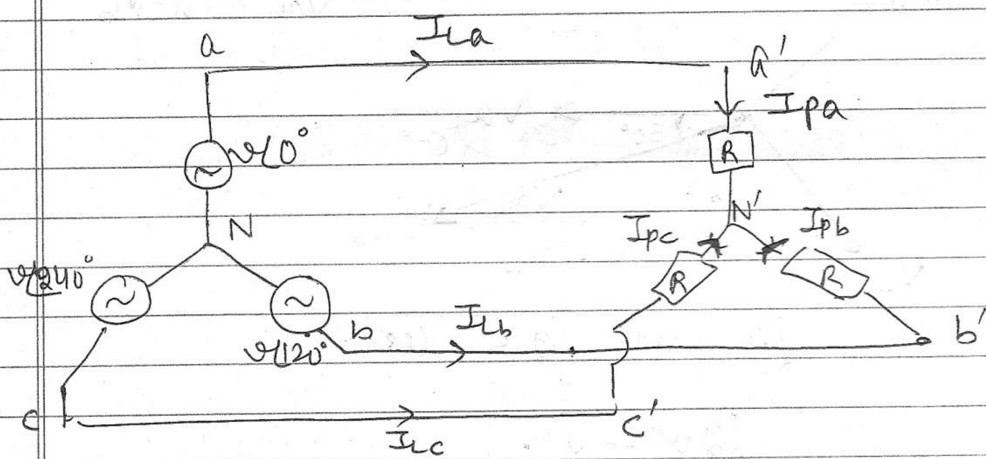


Star connection (either 3 or 4 wire).



pot. diff b/w two lines — line voltage
V_{ab}, V_{ac}, V_{bc}

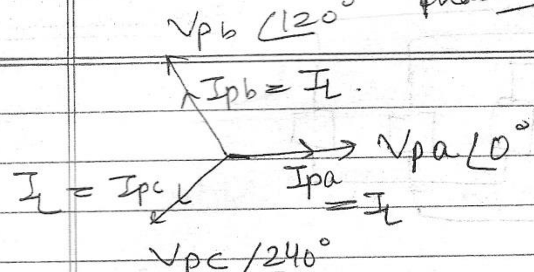
pot. diff b/w ~~two~~ line to Neutral - phase voltage

current flowing through load - phase current
" " " line - line "

In this case
$I_L = I_p$.

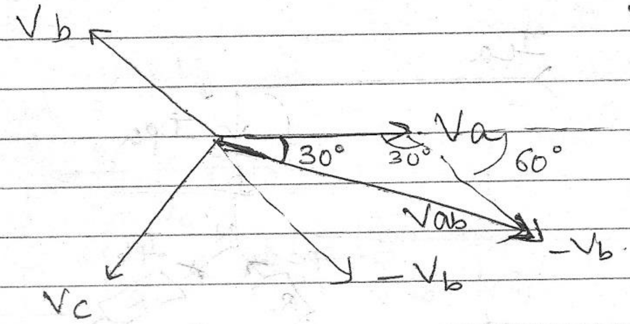
If balanced load, even if it is reactive
phase b/w currents remains same.
ie $I_{pa}\angle 0°$, $I_{pb}\angle 120°$, $I_{pc}\angle 240°$

phase voltage



$I_p = \frac{V_p}{R}$

$V_{ab} = V_a - V_b$



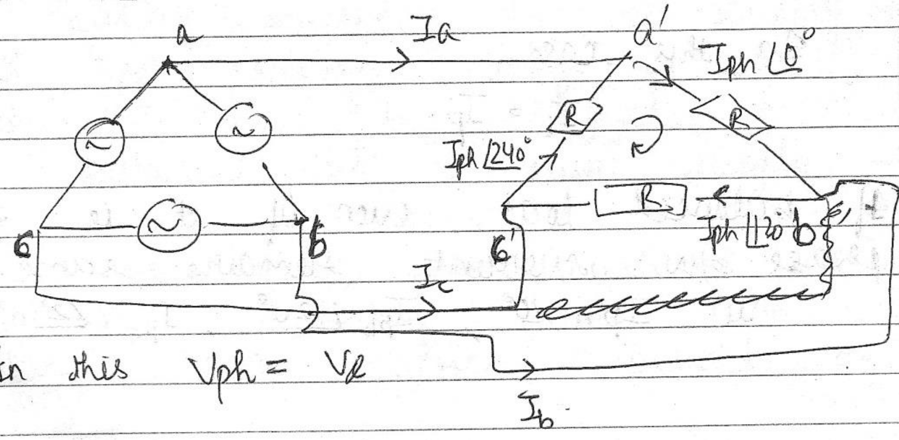
$\sqrt{v^2 + v^2 + 2v \cdot v \cos 30}$

$\sqrt{2v^2 + v^2}$

$\boxed{V_L = \sqrt{3} v_{ph}}$

$\boxed{I_L = I_{ph}}$

* Delta N/W



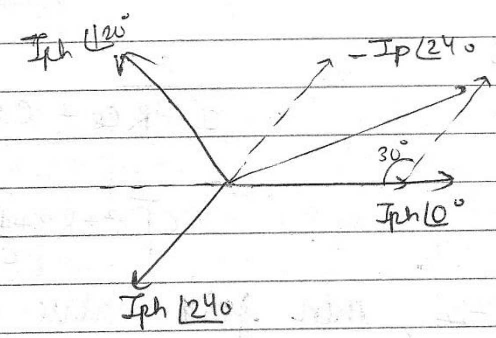
In this $V_{ph} = V_L$

Even if distortion created in this ~~loop~~. ckt, then the remaining current circulates in load loop.

$$\boxed{V_l = V_{ph}}$$

$$I_a + I_{ph} \angle 240^\circ = I_{ph} \angle 0^\circ$$

$$I_a = I_{ph} \angle 0^\circ - I_{ph} \angle 240^\circ$$



$$\boxed{I_L = \sqrt{3}\, I_{ph}}$$

Star Ckt | Delta ckt

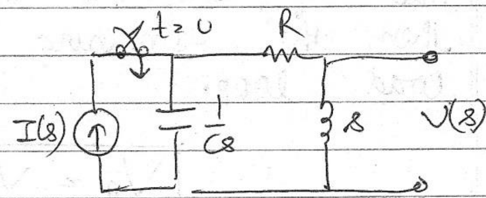
Star Ckt	Delta ckt
Power/phase = $V_{ph} I_{ph}$	$V_{ph} I_{ph}$
Total power = $3 V_{ph} I_{ph}$	$3 V_{ph} I_{ph}$
$= \cancel{3} \frac{V_L}{\cancel{\sqrt{3}}} I_L$	$3 V_L \frac{I_L}{\sqrt{3}}$
$= \sqrt{3} V_L I_L$	$\sqrt{3} V_L I_L$

Total active power = $\sqrt{3} I_L V_L \cos\phi$

$\boxed{6/4/2014}$

Network function

Q. find $\dfrac{V(s)}{I(s)}$



$$V(s) = \left[\frac{\frac{1}{Cs}}{\frac{1}{Cs}+R+s}\right] I(s) \cdot s$$

$$\frac{V(s)}{I(s)} = \frac{\frac{1}{\cancel{Cs}} \times s}{\frac{1+RCs+Cs^2}{\cancel{Cs}}} = \frac{s}{1+RCs+Cs^2}$$

$$= \frac{s}{C\left[s^2+Rs+\frac{1}{C}\right]}$$

Q. If poles are $-2, -1$, then find value of R, C

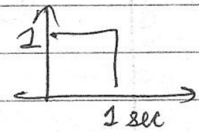
$$(s+2)(s+1) = 1+RCs+Cs^2$$

$$s^2+3s+2 = 1+RCs+Cs^2$$

$\cancel{\boxed{C=1}}$ $\cancel{RC=3}$ $\cancel{\boxed{R=3}}$. $s^2+3s+2 = s^2+Rs+\frac{1}{C}$

$\boxed{R=3}$

$\frac{1}{C}=2$ $\boxed{C=\frac{1}{2}}$

Q. If $i(t) =$  then find $v(t)$

$$i(t) = u(t) - u(t-1)$$
$$= \frac{1}{s} - \frac{e^{-s}}{s}$$

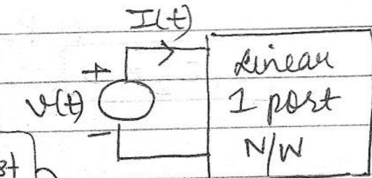
$$Z(s) = \frac{2s}{s^2+3s+2}$$

$$v(s) = \frac{2s}{s^2+3s+2}\left[\frac{1}{s} - \frac{e^{-s}}{s}\right].$$

Result $v(t) = 10\left[e^{-t} - e^{-2t}\right]u(t) - 10\left[e^{-(t-1)} - e^{-2(t-1)}\right]u(t)$

Q

$v(t)$	$i(t)$
$6V(dc)$	0
$\sin(t)$	$0.6\sin t + 0.8\cos t$
$\sin 2t$	?



Here system was assumed to be initially relaxed

$\to k\cos(t-\theta)$ · or $\sin(t+\theta)$

$$\frac{I(s)}{V(s)} = \frac{0.6\sin t + 0.8\cos t}{\sin(t)}$$

$$= \frac{0.6\frac{1}{s^2+1} + 0.8\frac{s}{s^2+1}}{\frac{1}{s^2+1}}$$

$$H(s) = \frac{0.6 + 0.8s}{1}$$

$$I(s) = H(s) \cdot V(s)$$
$$= (0.6 + 0.8s)\left[\frac{2}{s^2+4}\right]$$
$$= \frac{1.2}{s^2+4} + \frac{1.6s}{s^2+4}$$
$$= 0.6\sin 2t + 0.8\cos 2t$$

Q. If poles of $Y(s) = s = -1 \pm j1$ and 2 zeroes are present. find response corresponding to $\sin 2t$.

$$\frac{I(s)}{V(s)} = Y(s) = \frac{(s-Z_1)(s-Z_2)}{(s+1+j1)(s+1-j1)}$$

(i) $S_0 = 0$ [steady state]. for first case

$$0 = \frac{Z_1 Z_2}{P_1 P_2} \Big| .6V$$

$$\boxed{Z_1 Z_2 = 0}$$

for this ques.

$v(t)$	$i(t)$ (steady state)
6 V	0
$\sin t$	$0.6 \sin t + 0.8 \cos t$
$\sin 2t$	?

(ii) Complex $\nu = \pm j1$.

$$\sin(t+\theta) = \frac{(j-Z_1)(j-Z_2)}{(j+1+j)(\cancel{j}+1-\cancel{j})} \Big| \sin t$$

$$\frac{(j-Z_1)(j-Z_2)}{(2j+1)}$$

$a+jb$, $a-jb$ } only when zeroes are complex

$$\left[\frac{(j-(a+jb))(j-(a-jb))}{(2j+1)} \right.$$

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$$\frac{(j(1-b)-a)\ \cancel{(j(1+b)-a)}}{(2j+1)} \quad \tan^{-1}$$

$$\frac{\sqrt{(1-b)^2+a^2}\ \cancel{\sqrt{(1+b)^2+a^2}}}{\sqrt{4+1}}$$

$$\frac{\sqrt{1+b^2-2b+a^2}\ \cancel{\sqrt{1+b^2+2b+a^2}}}{\sqrt{5}}$$

$$\cancel{z_1 z_2 = 0}$$
$$\cancel{a^2+b^2 = 0}$$
$$\cancel{\text{i.e.}\quad a=b=0}$$
$$\cancel{\text{so}\quad z_1 = z_2 = 0}$$

$$Y(s) = \frac{s^2}{(s-p_1)(s-p_2)}$$

$$\frac{-1-jz_1-jz_2+z_1z_2}{2j+1} \qquad z_1 z_2 = 0$$

$$\frac{-1-j(z_1+z_2)}{2j+1} \quad \Big|\ \sin t$$

Now let $z_1 = a_1 + jb_1$
$z_2 = a_2 + jb_2$

$$\frac{-1-j(a_1+jb_1+a_2+jb_2)}{2j+1}$$

$$\frac{-1-j(a_1+a_2)+b_1+b_2}{(2j+1)} = \frac{\sqrt{(b_1+b_2-1)^2+(a_1+a_2)^2}}{\sqrt{5}}$$

| ans zeroes are 0 & -2 |

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$$\tan^{-1} - \frac{(a_1+a_2)}{(b_1+b_2-1)} - \tan^{-1} 2$$

$Z_1 Z_2 = 0 \longrightarrow$ means $Z_1 = 0$

$(a_1+jb_1)(a_2+jb_2) = 0$

$a_1 a_2 + j a_1 b_2 + j b_1 a_2 - b_1 b_2 = 0$

$(a_1 a_2 - b_1 b_2) + j(a_1 b_2 + a_2 b_1) = 0$

$$\frac{j(j-Z_2)}{1+2j} = \frac{-1-jZ_2}{1+2j}$$

$$= -\frac{(1+jZ_2)}{1+2j} = 0.6\sin t + 0.8\cos t$$

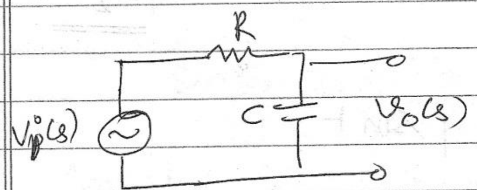
$\downarrow$

convert in complex

$Z_2 = a+jb$

then compare.

Q



$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{Cs}}{\frac{1}{Cs}+R} = \frac{1}{1+sCR}$$

$$H(j\omega) = \frac{1}{1+j\omega CR}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega CR)^2}}$$

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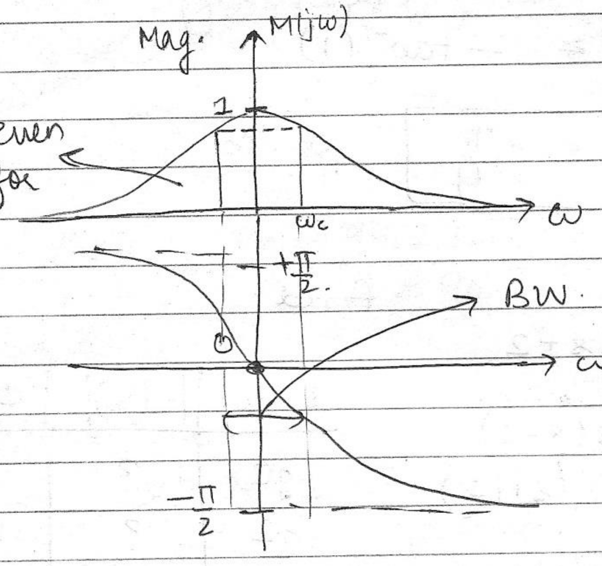
$\angle H(j\omega) = -\tan^{-1}\omega CR$

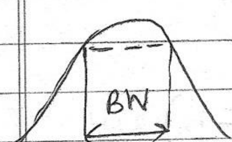


Mag. even fn so for −ve ω, −ve of +ω

Behaving as LPF

BW. Phase plot almost linear.

cutoff ν — ν at which mag. reduces by $\frac{1}{\sqrt{2}}$



* If I/P contains ν from $-\omega_c$ to ω_c approx. then shape of waveform will be unaltered. If it goes beyond the linear reg. of phase curve, then the shape will be distorted by significant amount.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+(\omega CR)^2}}$$

Phase at ω_c.

$$\frac{1}{2} = \frac{1}{1+\omega^2 C^2 R^2}$$

$$1+\omega_c^2 C^2 R^2 = 2$$

$$\omega_c = \frac{1}{RC}$$

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$$= -\tan^{-1}\left(\frac{1}{RC} \times RC\right)$$

$$\phi = -\tan^{-1}(1)$$

$$\boxed{\phi = -\frac{\pi}{4}}$$

Q $H(s) = \dfrac{4s}{s^2+2s+2}$ find

$s(j\omega)$	$\lvert H(j\omega)\rvert$	$\phi(H(j\omega))$
$j0$	?	?
$j2$	?	?
$j\infty$	?	?

Soln $H(j\omega) = \dfrac{4\,\cancel{j\omega}(s-0)}{(s+1+j)(s+1-j)}$

$$H(s) = \frac{4(j0-0)}{(j0+1+j)(j0+1-j)}$$

Mag $= 0$

$$\tan^{-1}\frac{0}{0} - \tan^{-1}\frac{1}{1} - \tan^{-1}\left(\frac{-1}{1}\right)$$

$$90 - \cancel{45} + \cancel{45}$$

$$90^\circ$$

(ii) $\dfrac{4(j2-0)}{(j2+1+j)(j2+1-j)}$

$$90 - \tan^{-1}3 - 45$$

$$45 - \tan^{-1}3$$

$$= -26.8$$

Mag

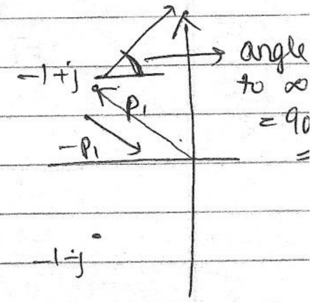
$$\frac{2\times 4}{\sqrt{10}\;\sqrt{2}}$$

$$\frac{8}{\sqrt{20}}$$

$$= 1.78$$

(iii) $j(\infty)$ $\quad \dfrac{4(j\infty - 0)}{(j\infty+1+j)\,(j\infty+1-j)}$ $\quad \lim \dfrac{\infty}{\infty^2}$

$$\lim \frac{4 \cdot \infty}{\infty \cdot \infty} = 0$$



Phase $\quad 90 - 90 - 90$
$= \underline{90}$

$$\Rightarrow H(s) = \frac{4s}{s^2+2s+2}$$

$$\frac{4j(0)}{\underbrace{0+j0}+2} = \frac{4j0}{2} = 2j(0)$$

neglected bcoz it has real part

Mag = 0
Phase 90°

$$H(s) = \frac{4j(\infty)}{-\infty+2j\infty} \qquad \tan^{-1}\frac{\infty}{\infty} \quad \text{underterminate form.}$$

$\therefore$ Do it graphically

Q $\quad H(s) = \dfrac{4s}{s^2+2s+2}$

$$H(j\omega) = \frac{4j\omega}{(j\omega)^2+2(j\omega)+2} = \frac{4j\omega}{-\omega^2+2j\omega+2}$$

$$M(j\omega) = \frac{4\omega}{\sqrt{(2-\omega^2)^2+(2\omega)^2}} = \frac{4}{\sqrt{4+\left(\frac{2}{\omega}-\omega\right)^2}}$$

$$\phi(j\omega) = \frac{\pi}{2} - \tan^{-1}\left(\frac{2\omega}{2-\omega^2}\right)$$

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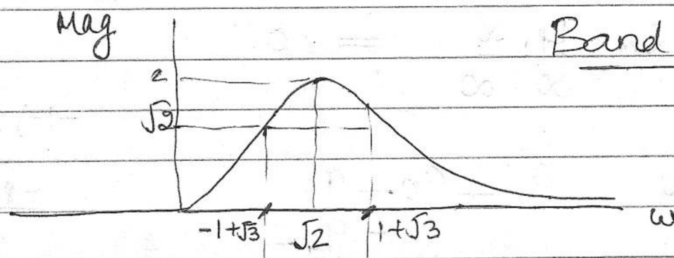
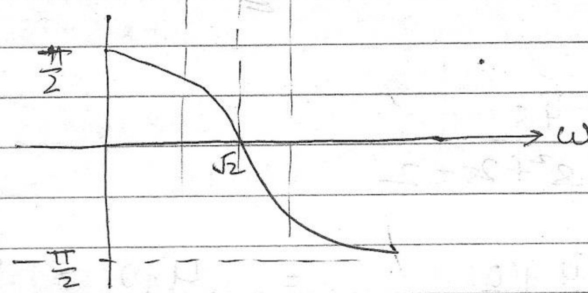
$$\phi(j\omega) = -\tan^{-1}\left(\frac{\omega^2-2}{2\omega}\right)$$

Band pass





$M_{max} = 2$

$$\frac{2}{\sqrt{2}} = \sqrt{2}$$

At which ω mag $\sqrt{2}$

$$\sqrt{2} = \frac{4}{\sqrt{4+\left(\frac{2}{\omega}-\omega\right)^2}}$$

$$2 = \frac{16}{4+\left(\frac{2}{\omega}-\omega\right)^2}$$

If $\Rightarrow \frac{4}{\sqrt{4+\left(\omega-\frac{2}{\omega}\right)^2}}$

$\Downarrow$

then roots on +ve side $= 1 \pm \sqrt{3}$

$$4+\left(\frac{2}{\omega}-\omega\right)^2 = 8$$

$$\left(\frac{2}{\omega}-\omega\right)^2 = 4$$

$$\frac{2}{\omega} - \omega = 2$$

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$2 - \omega^2 = 2\omega$

$2\omega + \omega^2 - 2 = 0$

$\omega^2 + 2\omega - 2 = 0$

$(2)^2 - 4 \times 1 \times (-2)$

$4 + 8 = 12 = 2\sqrt{3}$

$$\begin{array}{r|l} 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array}$$

$$\frac{-2 \pm 2\sqrt{3}}{2}$$

$$\boxed{-1 \pm \sqrt{3}}$$

find phase at cutoff freq.

$$\tan^{-1}\left(\frac{(1+\sqrt{3})^2 - 2}{2(1+\sqrt{3})}\right)$$

$$-\tan^{-1}\frac{1+3+2\sqrt{3}-2}{2+2\sqrt{3}}$$

$$-\tan^{-1}\left(\frac{2+2\sqrt{3}}{2+2\sqrt{3}}\right)$$

$$-\tan^{-1}(1) = -\frac{\pi}{4}$$

$$\text{for } -1+\sqrt{3} = \frac{\pi}{4}$$

$$H(s) = (s^2 + \omega_0^2)\, F_1(s).$$

$$H(s) = \underbrace{(s + j\omega_0)(s - j\omega_0)}_{\text{Add}^n \text{ of zeroes.}}\, F_1(s) \nearrow \begin{array}{c}\text{Assum} \\ k\angle\theta\end{array}$$

$$(j\omega + j\omega_0)(j\omega - j\omega_0)$$

$$k\angle\theta \; (\omega + \omega_0)\angle 90^\circ \quad (\omega - \omega_0)\angle -90^\circ$$

$$k(\omega + \omega_0)(\omega - \omega_0) \angle\theta$$

Total mag. & phase $\boxed{k(\omega^2 - \omega_0^2)\angle\theta}$

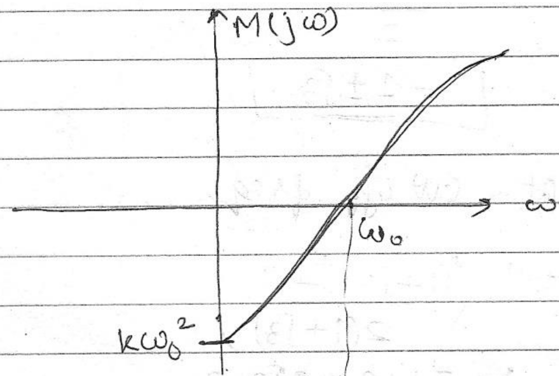
This is applicable only when $0 < \omega < \omega_0$

When $\omega > \omega_0$

$$F_1(s) \quad (s - j\omega_0) \quad (s + j\omega_0)$$

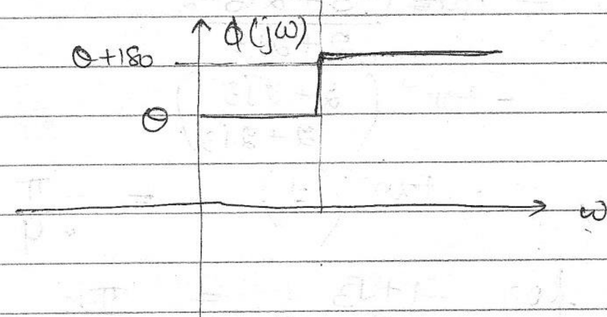
$$k \angle \theta \quad (\omega - \omega_0) \angle 90 \quad (\omega + \omega_0) \quad \angle 90$$

$$k(\omega^2 - \omega_0^2) \quad \angle \theta + 180$$



+180° phase shift

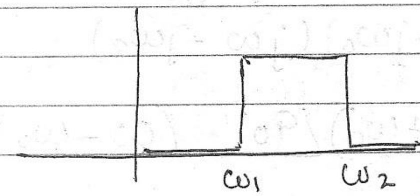
thus there is change from (−) to (+) in mag.



A $\omega = \omega_0$ indeterminate phase otherwise upper value

Q $\dfrac{s^2 + \omega_1^2}{s^2 + \omega_2^2} = H(s)$ Mag. & phase plot

$\omega_1 < \omega_2$



Phase

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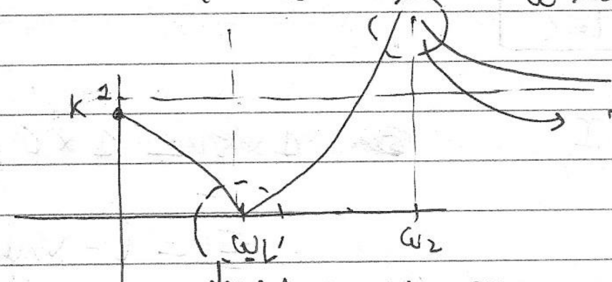
$$M(j\omega) = \frac{\omega_1^2 - \omega^2}{\omega_2^2 - \omega^2}$$

$$\frac{(j\omega)^2 + \omega_1^2}{(j\omega)^2 + \omega_2^2} \qquad \left.\frac{\omega_1^2 - \omega^2}{\omega_2^2 - \omega^2}\right\}$$ This is now real for i.e angle 0 but this is wrong.

$$H(j\omega) = \frac{(j\omega - j\omega_1)\,(j\omega - (-j\omega_1))}{(j\omega - j\omega_2)\,[j\omega - (-j\omega_2)]}$$

$$\angle H(j\omega) = \begin{cases} 0^\circ & 0 < \omega < \omega_1 \\ 0^\circ + 180^\circ & \omega_1 < \omega < \omega_2 \\ 0^\circ & \omega > \omega_2 \end{cases}$$



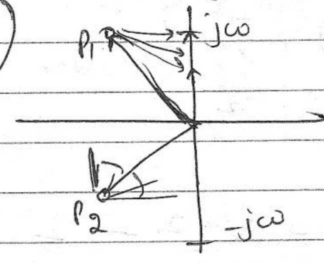
when plot mag., consider prod.
Pole location exactly on $j\omega$ axis, means max. magnitude, most selectivity, least B.W.
This is practically not realizable.

$$\frac{1}{(j\omega - p_1)(j\omega - p_2)}$$
$\angle\theta_1 \downarrow$ $\quad \angle\theta_2 \uparrow$

for P_2
resultan 90

$\frac{1}{\uparrow} = \downarrow$

i.e phase ↓.



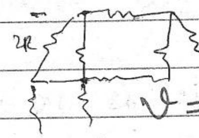
resultant approaching towards ω i.e at last angle 0

~~20/4/2014~~ 30/8/2014

Steady State Analysis of dc circuit

~~Single Tuned circuit~~

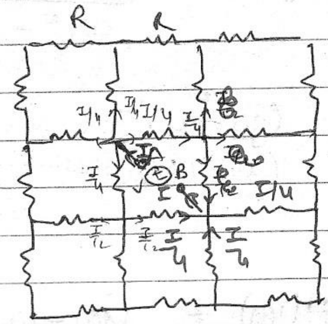
Q 1.



$$V = \frac{I}{4}R + \frac{I}{4}R$$

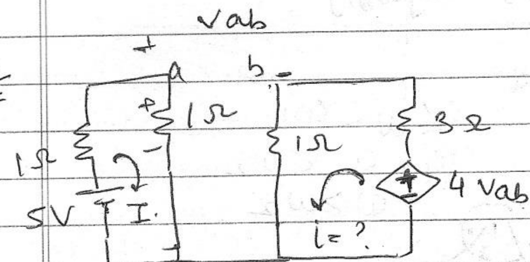
$\frac{I}{4}$ = 3x+·

$$\boxed{\frac{V}{I} = \frac{R}{2}}$$



Identical nodes.

2.



$$5 = (1+1)I$$

$$\frac{5}{2} = I$$

$$1 \times \frac{5}{2} - 1 \times i - V_{ab} = 0$$

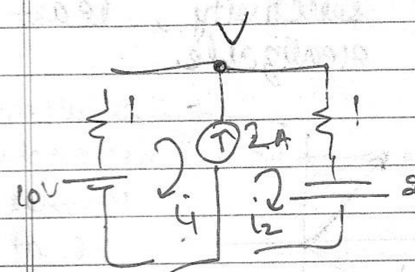
$$\frac{5}{2} - i - V_{ab} = 0$$

$$\frac{5}{2} = 2i$$

$$\boxed{\frac{5}{4} = i}$$

$\cancel{4}V_{ab} \pm \cancel{4}i$

3.



$$\frac{V-10}{1} + \frac{V+2}{1} = 2$$

$$2V - 8 = 2$$

$$2V = 10$$

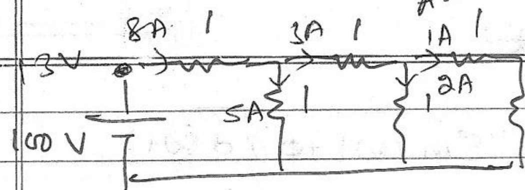
$$V = 5$$

$$i_1 = \frac{10-5}{1} = 5A$$

$$i_2 = \frac{5+2}{1} = 7A.$$

Assume 1A.

Q



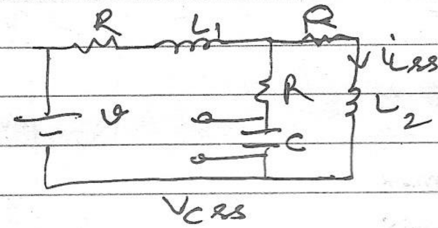
Current through each branch

13V — 1A.

$100V — \frac{1}{13} \times 100 = \_\_ A.$

Principle of homogenity.

Q



find

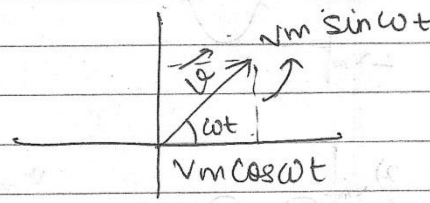
$i_{ss} = \frac{v}{2R}$

$v_{Css} = \frac{v}{2}$

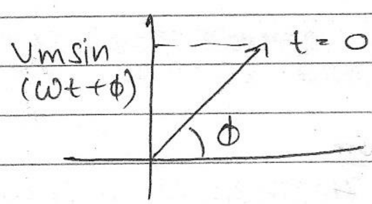
Steady state Analysis of AC ckt.

$\vec{v}(t) = V_m \sin\omega t$

rotating with speed ω.



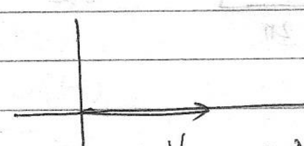
$v_1(t) = V_m \sin(\omega t + \phi)$



$v(t) = V_m \sin\omega t$

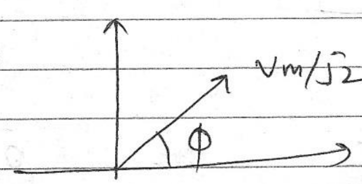
Phasor notation

i.e in terms of RMS



$V_{rms} = \frac{V_m}{\sqrt{2}}$

$V_m \sin(\omega t + \phi)$



→ RMS value $= \sqrt{\dfrac{1}{T}\int_0^T v^2\,dt}$

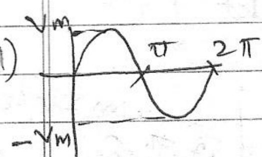
$= \sqrt{\dfrac{1}{2\pi}\int_0^{2\pi} v_m^2 \sin(\omega t + \phi)\, d(\omega t)}$

→ Average value $= \dfrac{\int_0^T v\,dt}{T}$

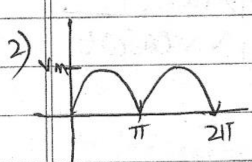
→ Form factor $= \dfrac{V_{RMS}}{V_{average}}$

→ Peak factor $= \dfrac{V_{max}}{V_{RMS}}$

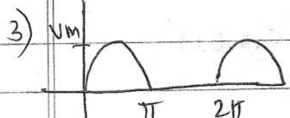
Q v(t) v_{RMS} v_{AV} FF PF

1) 

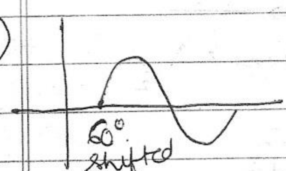
v_m 0

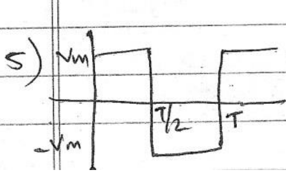
2) 

v_m v_m

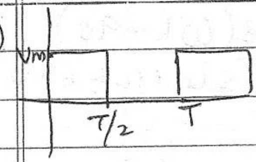
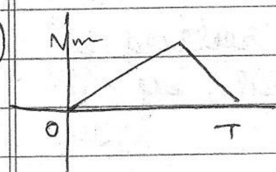
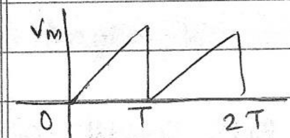
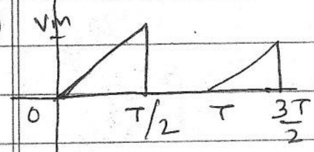
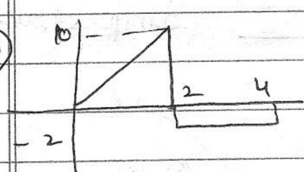
3) 

$\dfrac{v_m}{\sqrt{2}}$ $\dfrac{v_m}{2}$

4) 

5) 

v_m 0

$v(t)$	v_{RMS}	v_{AV}	FF	PF
6) 	$\frac{v_m}{\sqrt{2}}$	$\frac{v_m}{2}$		
7) 				
8) 	$y = \frac{V_m}{T} t$ $\sqrt{\frac{1}{T}\int_0^T \left(\frac{V_m t}{T}\right)^2}$ $= \sqrt{\frac{1}{T}\frac{V_m^2}{T^2}\frac{t^3}{3}}$ $= \sqrt{\frac{1}{T^3} V_m^2 \frac{T^3}{3}} = \frac{V_m}{\sqrt{3}}$			
9) 				
10) 				

→  source always expressed as RMS value.

$v_1(t) = V_m \sin\omega t$ $\quad V_2(t) = V_m \cos\omega t$

$v_1(t) = -j\, v_2(t)$

$v_2(t) = +j\, v_1(t)$

$v_1'(t) = v_m\omega \cos\omega t$

$= +\omega j\, v_1(t) = +j\omega\, v_m \sin\omega t$

$\sin\omega t = \frac{e^{j\omega t} - e^{j\omega t}}{2j}$

$\left[j\sin\omega t = j\left(\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right) \right. \qquad \frac{e^{j(\omega t + 90)} - e^{-j(\omega t + 9}}{2j}$

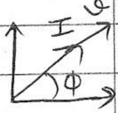
$$\frac{\cos(\omega t+90) + i\sin(\omega t+90) - \cos(\omega t+90) - i\sin(\omega t+90)}{2i}$$

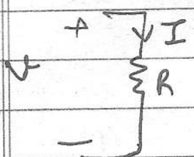
→ $\boxed{v_1' = j\omega v_1}$

→ $\boxed{v_2' = j\omega v_2}$

i.e under steady stat derivative replaced by $j\omega$ and integral by $\frac{1}{j\omega}$

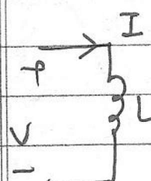
→ $\int_0^t v_1(t)\,dt = \frac{1}{j\omega} v_1(t)$





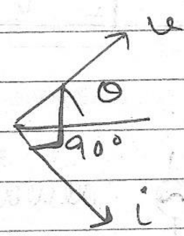
$v = IR$. v = RMS value of sinosoidal source

No phase change b/w v & I

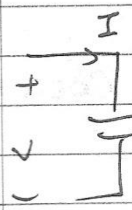


$v = L\frac{di}{dt} = \underline{j\omega L I}$. jX_L Impedance

$i = \frac{v}{j\omega L}$.

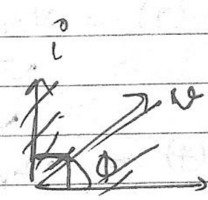


lagging type of N/W

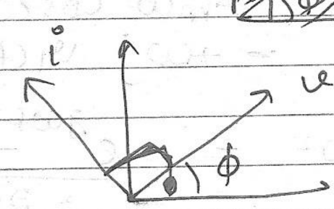


$I = C\frac{dv_c}{dt}$

$I = j\omega C v_c$



leading type of n/w



$jX_C = \frac{1}{j\omega C} = -jX_C$

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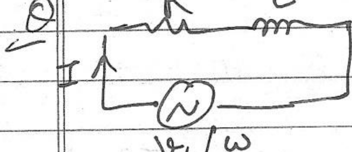
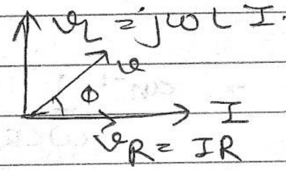
Take I as reference

Q) Calculate voltage across R & L.



$v_R = IR$

$v_L = j\omega L I$



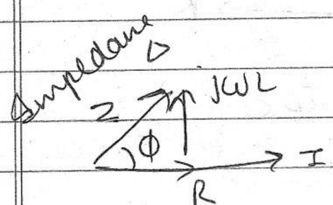
$v = v_R + v_L$

$\rightarrow \phi = \tan^{-1} \frac{\omega L}{R}$

Angle b/w v and I

$\cos\phi$ = power factor $\quad 0 < \cos\phi < 1$

$Z = \frac{V}{I} = R + j\omega L$



$\phi = \tan^{-1} \frac{\omega L}{R}$

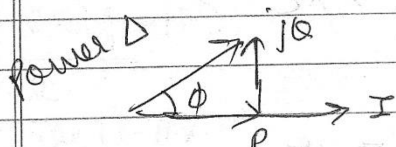
Total power $\quad vI = v_R I + v_L I$

$S = I^2R + j\omega L I^2$

complex power = P_{res} + reactive power of inductor

Active power due to resistor

$\boxed{S = P + jQ}$



$\phi = \tan^{-1} \frac{\omega L}{R}$

$\rightarrow \boxed{P = |S| \cos\theta}$

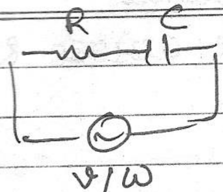
$\rightarrow \boxed{Q = |S| \sin\theta}$

S = Only Magnitude

Current

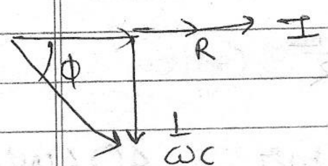
Reference voltage

Q



$$v = \left(R + \frac{1}{j\omega C}\right) I.$$

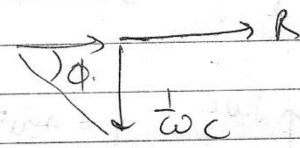
$$v_C = \frac{-j}{\omega C} I \qquad v_R = IR.$$



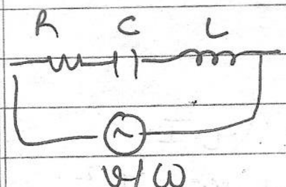
$$\tan^{-1} \frac{\frac{1}{\omega C}}{R} = \tan^{-1} \frac{1}{\omega C R}$$

leading N/W.

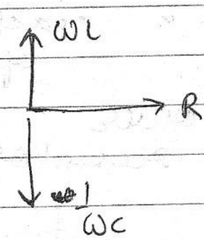
$$Z = R - \frac{j}{\omega C}$$



Q



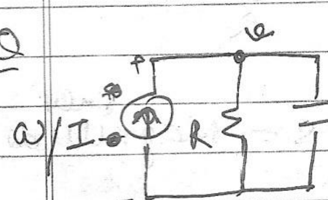
$$v_R = IR$$
$$v_L = j\omega L I$$
$$v_C = \frac{-j}{\omega C} I$$



$$\tan^{-1}\left[\frac{\omega L - \left(\frac{1}{\omega C}\right)}{R}\right]$$

Assume $X_L > X_C$

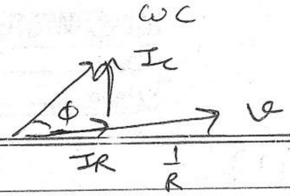
Q



$$v_R = v_C = v$$

$$I = I_R + I_C$$
$$I = \frac{v}{R} + \frac{v\omega C}{j}$$

$j\omega C v$



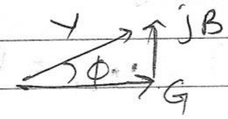
$\boxed{\phi = \tan^{-1}\omega CR}$

Current Δ

In parallel ~~ckt~~ Z cannot be added directly so Y.

$Y = G + jB$ $\qquad G = \frac{1}{R}$ $\qquad B = \omega C$

Angle b/w conductance & Admittance



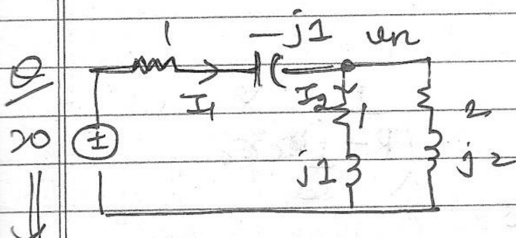
Admittance ϕ

Y and G = ϕ

$S = vI$ $\qquad P = v I_R$ $\qquad Q = v I_C$

$\boxed{S = P + jQ}$

Q



Calculate I_1, I_2 & v_n

$20\sqrt{2}\sin\omega t$

$$\frac{v_n - 20}{1 - j1} + \frac{v_n}{1 + j1} + \frac{v_n}{2 + j2} = 0$$

$$\cancel{\frac{(v_n - 20)(1 - j1)}{2}} + \cancel{\frac{v_n(1 - j1)}{2}} + \cancel{\frac{v_n(2 - j2)}{8}}$$

$$\cancel{\frac{v_n}{(1-j1)(1+j1)(2+j2)}} = \cancel{2 \times 2(1+j)}$$

$$\cancel{4(1+j)}$$

$$4(v_n - 20)(1 + j1) + 4v_n(1 - j1) + 2v_n(1 - j1)$$

$$\cancel{2v_n(1-j1)} \; 6v_n(1 - j1)$$

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$4(v_n - 20)(1+j1) + 6v_n(1-j1)$

$4(v_n + jv_n - 20 - 20j) + 6v_n - 6v_nj$

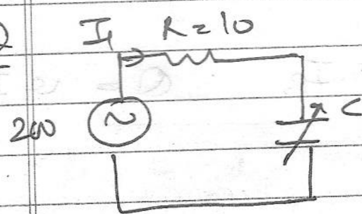
$10v_n - 2v_nj - 80 - 80j = 0$

$v_n(10-2j) = 80(1+j)$

$v_n = \frac{80(1+j)}{(10-2j)} = \frac{80(1+j)}{2(5-j)}$

$v_n = \frac{40(1+j)}{(5-j)}$

Q

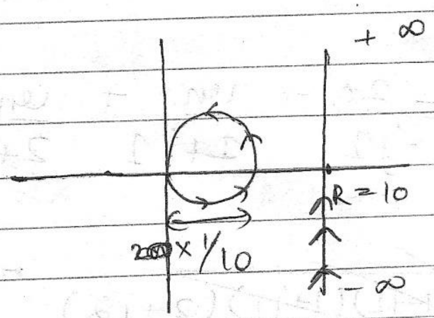


$X_C \rightarrow (0 \rightarrow \infty)$

Plot locus of I_1

$V = IZ$

$I = \frac{V}{Z} = \left(\frac{1}{Z}\right)V$ $\quad \frac{1}{(R - jX_C)}V$

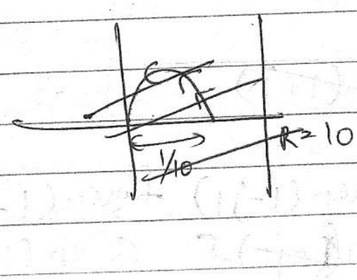


$R - jX_C$

$R - \infty$ $\quad \omega\ \infty$

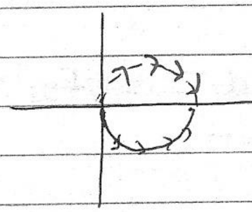
R

~~0 to ∞~~

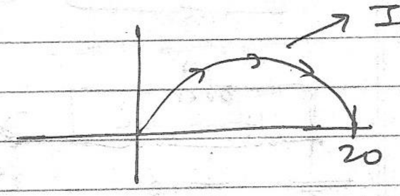


$R - jX_C \rightarrow 0 \rightarrow -\infty$

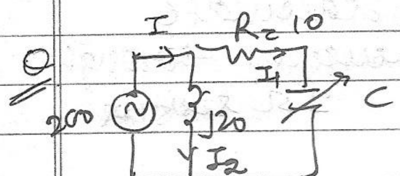
movement from lower value to higher value



Ans



Q



$X_C\ (0 \rightarrow \infty)$

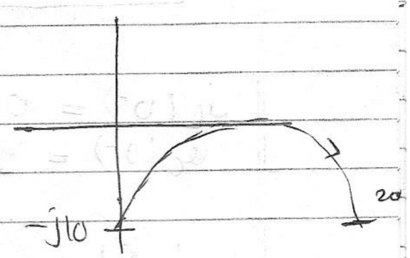
locus of I_2

locus of I

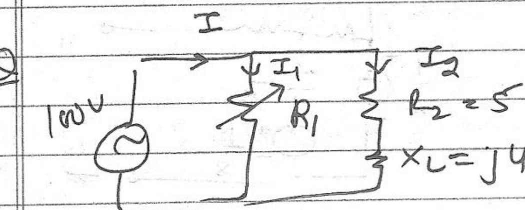
$I_2 = \frac{200}{j20} = -j10$

$I = I_1 + I_2$

$= -j10 + \frac{200}{10 + jX_C}$



Q



$R_1 \rightarrow (0 - \infty)$

find locus of I_1 & I

$I = I_1 + I_2$

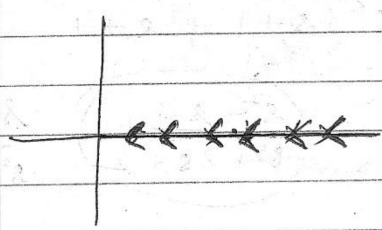
$I_2 = \frac{100}{5 + j4}$

$I_1 = \frac{100}{R_1}$

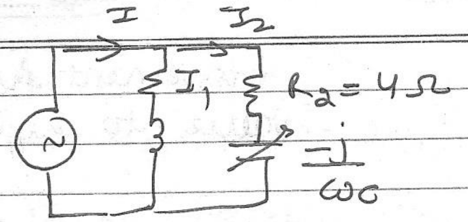
$I_1 = \left(\frac{1}{R_1}\right) 100$

$R_1 \rightarrow 0 \rightarrow \infty$

$\frac{1}{R_1} \quad \infty \rightarrow 0$



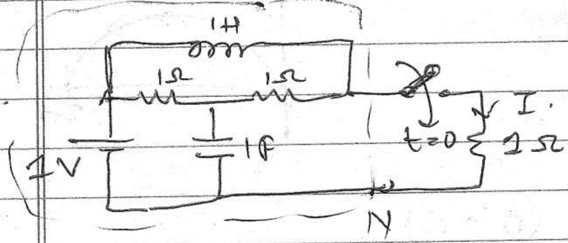
Q



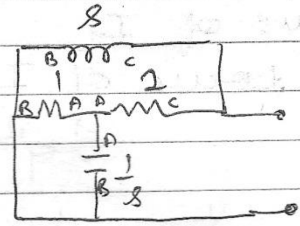
$\omega \rightarrow -\infty$ to ∞

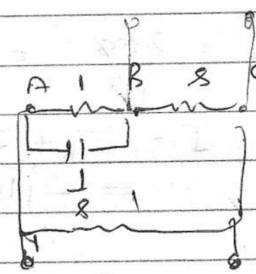
find I & I_2

Q.



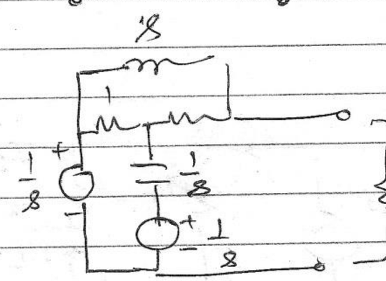
calculate current through 1Ω resistor





$i_L(0^-) = 0$

$v_C(0^-) = 1$



$$\frac{\frac{1}{s}\times 1}{\frac{1}{s}+1} = \frac{\frac{1}{s}}{\frac{1+s}{s}}$$

$$= \frac{1}{1+s} + s$$

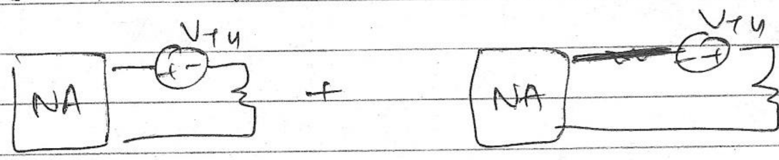
$$\frac{1+s(1+s)}{(1+s)} = \frac{1+s+s^2}{1+s}$$

$$\frac{\frac{(s^2+s+1)}{(s+1)}}{\frac{s^2+s+1}{(s+1)}+1} = \frac{s^2+s+1}{s^2+s+1+s+1}$$

$$= \frac{s^2+s+1}{s^2+2s+2} \qquad \frac{s(s+2)}{s^2+2s+2}$$

$V_{TH} = \frac{1}{s}$

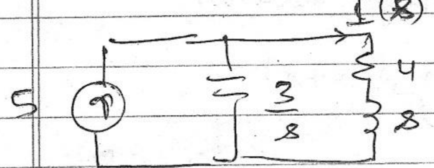
This N/W equivalent to

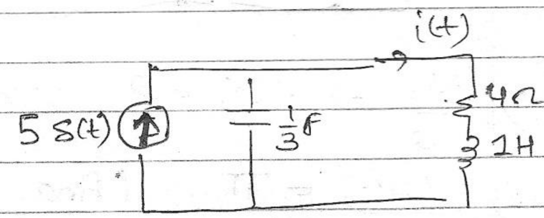


i.e total voltage across switch = 0 when switch open

$$v_{sw} = v_{oc} = V_{TH}$$

Q Find $i(t)$.





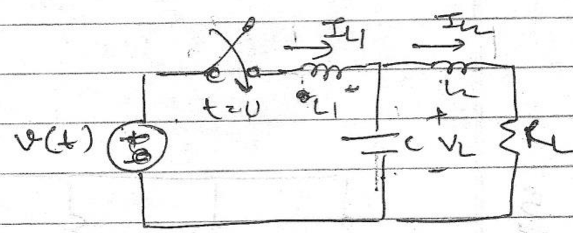
$$I(s) = \frac{\frac{3}{s}}{4 + s + \frac{3}{s}} \times 5$$

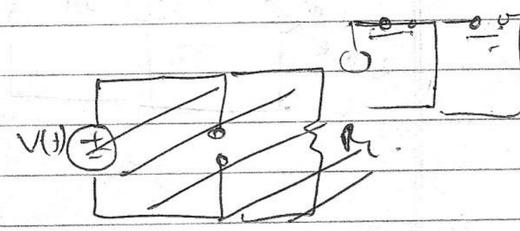
Q $I_{L1}(0^-) \neq 0$

$I_{L2}(0^-) \neq 0$

$v_c(0^-) \neq 0$

find V_{TH}, Z_{TH}.





$I_{L2}(0^+) = I_{L2}(0^-) = \frac{v(t)}{R_L}$

$I_{L1}(0^-) \neq 0$

$v_c(0^-) = v(t)$

~~$v(t) = V(s)$~~

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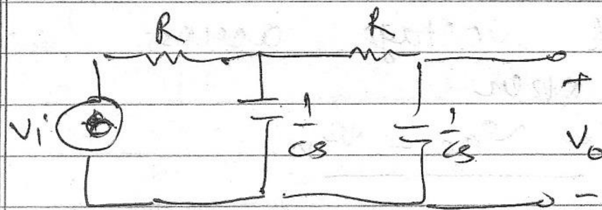
Q3



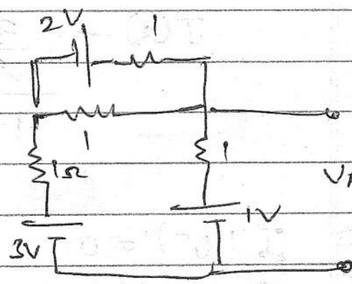
Initially relaxed

1) $\frac{v_o}{v_i} = H(s) = ?$ (Apply Thevenin)

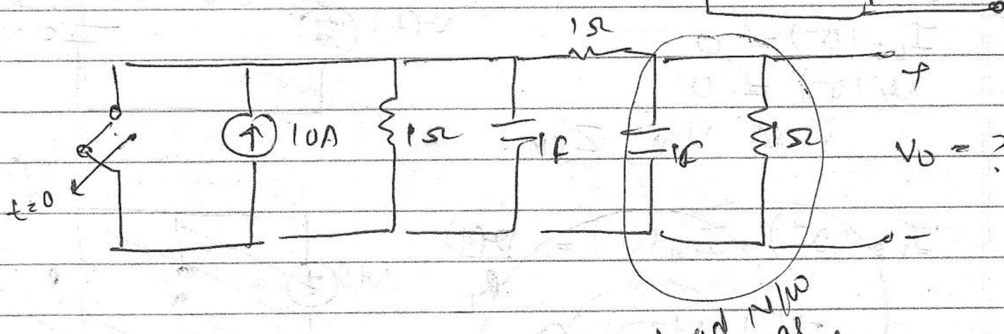
2) $\angle H(s) = \pi$ (find freq.)

3) $|H(s)|$

4. Superposition Thm.

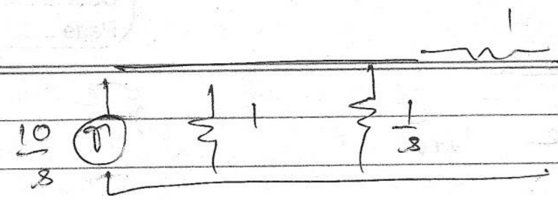


5



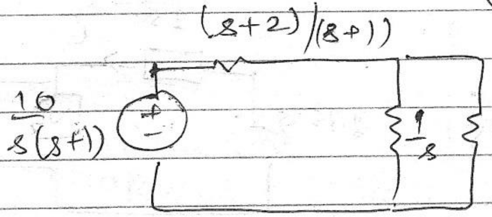
load N/W voltage same

Sol Initial Cond. = 0



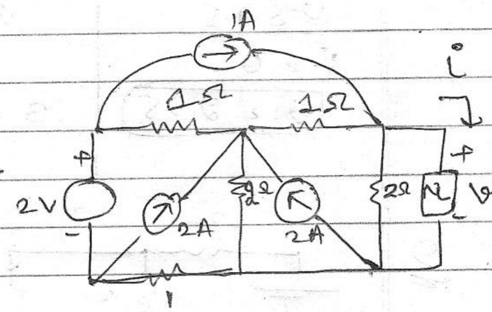
$$Z_{Th} = \frac{\frac{1}{s}}{1+\frac{1}{s}} = \frac{1}{s+1} + 1 = 1 + \frac{s+1}{s+1} = \frac{s+2}{s+1}$$

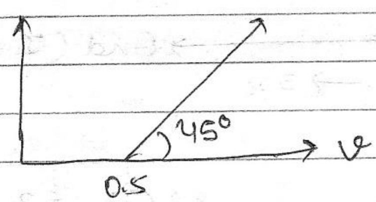
$$V_{Th} = \frac{1}{s}\left(\frac{1}{1+\frac{1}{s}} \times \frac{10}{s}\right) = \frac{1}{s}\left(\frac{\cancel{s}}{s+1} \times \frac{10}{\cancel{s}}\right) = \frac{1}{s} \times \frac{10}{(s+1)}$$



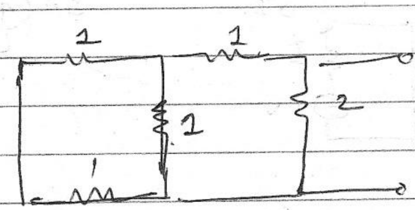
Using Thevenin

Q v-I characteristics of N_L





R_{Th} =



$\hookleftarrow Z_{Th}.$ $\simeq 1$

$$\frac{10}{11}$$

$$\frac{\frac{10}{\cancel{3}}}{\frac{11}{\cancel{3}}} = \frac{10}{11}$$

$$\frac{2 \times 1}{2+1} = \frac{2}{3}$$

$$\cancel{\frac{1}{2}+1 = \frac{3}{2}+2}$$

$$\frac{2}{3}+1 = \frac{5}{3}$$

$$\frac{\frac{5}{3}\times 2}{\frac{5}{3}+2}$$

$$\cancel{\frac{\frac{3}{2}\times 2}{\frac{3}{2}+2} = \frac{3}{\frac{7}{2}}} = \frac{6}{7}$$

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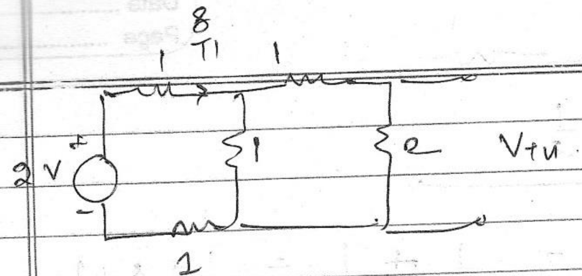
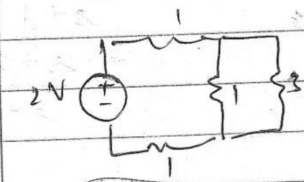


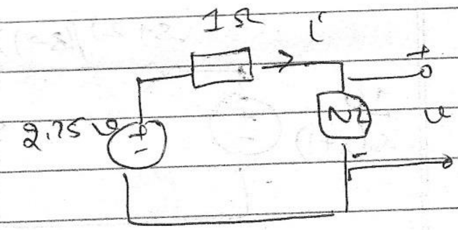


$$\frac{3\times1}{3+1} = \frac{3}{4} + 2 = \frac{11}{4}$$

$$\frac{2\times4}{11} = \frac{8}{11}$$

Current (2) $= \frac{1}{3}\,\frac{8}{11} = \frac{8}{33}$

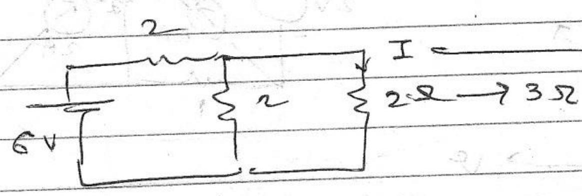
$V_{TH} = 2.75$ V.



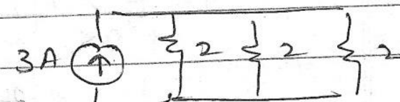
$2.75 - i - v = 0$

$v \le 0.5$	$v > 0.5$
$i = 0$	
$\boxed{v = 2.75}$	$3.25 - 2v = 0$
	$v = \frac{3.25}{2}$

Q

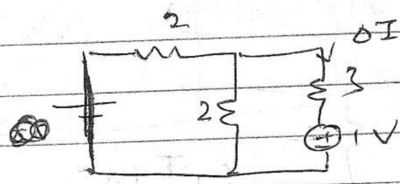


I ⟶ find (I + ΔI)



$\boxed{I = 1A}$

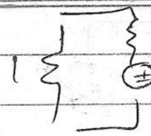
for ΔI



$\cancel{6 = 2I_1 + 2(I_1 - I_2)}$

$\cancel{3I_2 + 1 + 2(I_2 - I_1) = 0}$

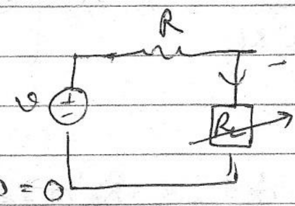
$\cancel{6I_2 - I_1 + 1 = 0}$



$$-\frac{1}{4}\,A = \Delta I$$

$$I + \Delta I = 1 - \frac{1}{4} = \frac{3}{4}$$

Q find value of R_L for which power transfer max

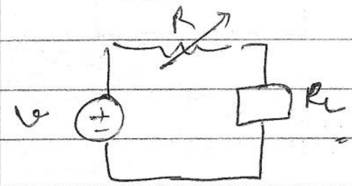


$P = v_L I_L$

differentiate the equate to = 0

$\boxed{R = R_L}$

Q



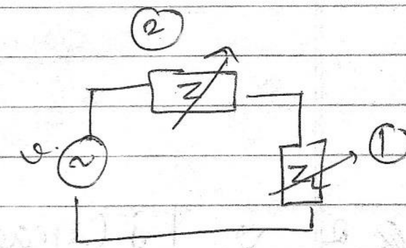
$$\frac{R_L v}{R+R_L} \cdot \frac{v}{R+R_L}$$

$$P = \frac{v^2 R_L}{(R+R_L)^2} \qquad (R+R_L)^{-1}$$

$$\frac{dP}{dR} = -R_L v^2 (R+R_L)^{-3}$$

$$\frac{dP}{dR} = \frac{-R_L v^2}{(R+R_L)^3} = 0 \qquad \boxed{R \to 0}$$

Q find value of Z for which max transfer at Z_L.



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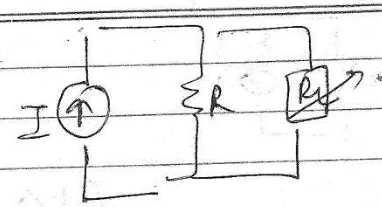
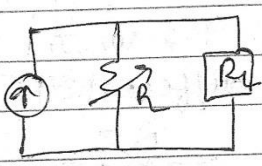
5 $\left[\frac{R}{(R+R_L)} I\right]^2 R_L$



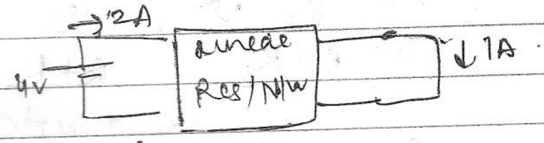
$\frac{R^2 R_L I^2}{(R+R_L)^2}$

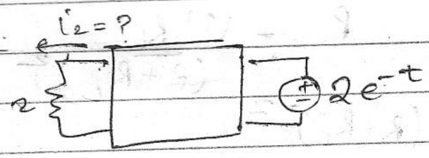
$\frac{dP}{dR_L} = R^2 I_2 \left[\frac{(R+R_L)^2[1] - R_L\, 2(R+R_L)}{(R+R_L)^4}\right]$

6



Ans $R_L \to \infty$.





$4(-i_2) + (0) = 2i_2(2) + 2e^{-t}(1)$.

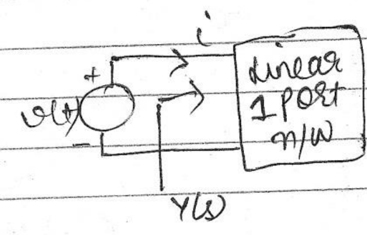
$-4i_2 - 4i_2 = 2e^{-t}$

$-\cancel{8}^{4} i_2 = \cancel{2}e^{-t}$

$\boxed{i_2 = -\frac{e^{-t}}{4}}$

Q

v	i (steady state
6V dc	0
sin t	→ 2
sin 2t	→ ?



$Y(s) \longrightarrow s = -1 \pm j1$ and 2 zeroes.

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$$Y(s) = \frac{I(s)}{V(s)} = \frac{(s-z_1)(s-z_2)}{(s+1+j)(s+1-j)}$$

$$Y(s) = \frac{(s-z_1)(s-z_2)}{(s+1)^2+1}$$

$$Y(s) = \frac{(s-z_1)(s-z_2)}{s^2+1+2s+1} = \frac{(s-z_1)(s-z_2)}{s^2+2s+2}$$

$$\frac{I(s)}{V(s)} = \frac{(s-z_1)(s-z_2)}{s^2+2s+2}$$

6V. $\left.\frac{z_1 z_2}{2}\right|_{s=} = 0$

$$\boxed{z_1 z_2 = 0}$$

$\underline{s = j}$

$$\left|\frac{(j-z_1)(j-z_2)}{1-2j}\right| \sin t = 2$$

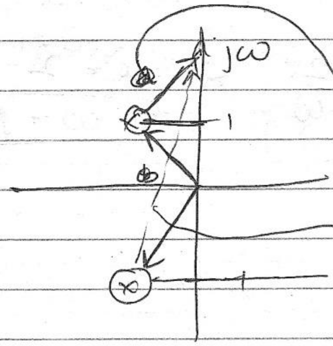
Also $\angle\phi = 0$

Q. $H(s) = \frac{4s}{s^2+2s+2}$ $= \frac{4j\omega}{(j\omega+1-j)(j\omega+1+j)}$

$$\frac{4j\omega}{[j\omega-(-1+j)][j\omega-(-1-j)]}$$

$$\frac{-2 \pm \sqrt{4-4\times1\times2}}{2}$$

$\underline{-1 \pm i}$



$= \frac{4\omega\angle 90^\circ}{\sqrt{(\omega-1)^2+1^2}\ \tan^{-1}(\omega-}$

Mag + phase

$\sqrt{(\omega+1)^2+1^2}$

$\tan^{-1}\angle \omega+$

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At $j2$ $= \dfrac{4(2)\angle 90}{\sqrt{(1)^2+1^2}\angle\tan^{-1}(1)\ \sqrt{(3)^2+1^2}\ \tan^{-1}(3)}$

$= \dfrac{8}{\sqrt{2}\sqrt{10}}\ \angle 90-45\ -\tan^{-1}3$

$= 1.78\ \angle -26.5$

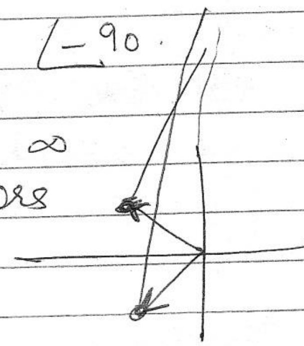
$\sqrt{3+1}$

$j(\infty)$ $\dfrac{\angle 90}{\angle 90\ \ \angle 90} = \angle -90$

Mag $\longrightarrow \infty$
of the vectors

$\dfrac{\infty}{\infty^2}$

$= \longrightarrow 0$



$H(s) = \dfrac{4}{\left(\dfrac{s+2}{s}\right)+2}$

$H(j\omega) = \dfrac{4}{\left(j\omega+\dfrac{2}{j\omega}\right)+2}$

$|H(j\omega)| = \cancel{\dfrac{4}{\sqrt{\omega^2+\left(\frac{2}{\omega}\right)^2+4}}}\quad \dfrac{4}{\sqrt{2^2+\left(\omega-\frac{2}{\omega}\right)^2}}$

$\tan^{-1}\ \angle\left(\dfrac{\omega-\frac{2}{\omega}}{2}\right)$

critical point
$\omega = \dfrac{2}{\omega} = \omega^2 = 2$
$= \boxed{\omega = \sqrt{2}}$

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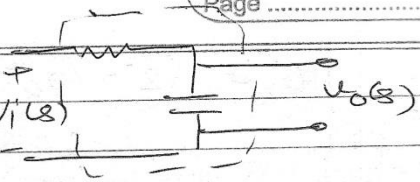


$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{Cs}}{\frac{1}{Cs} + R}$$

$$= \frac{1}{1 + RCs} = \frac{1/RC}{s + 1/RC}$$

$$= \frac{1}{RC}\left[\frac{1}{j\omega + \frac{1}{RC}}\right]$$

$$|H(s)| = \frac{\frac{1}{RC}}{\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}} \qquad \angle -\tan^{-1}(\omega RC)$$

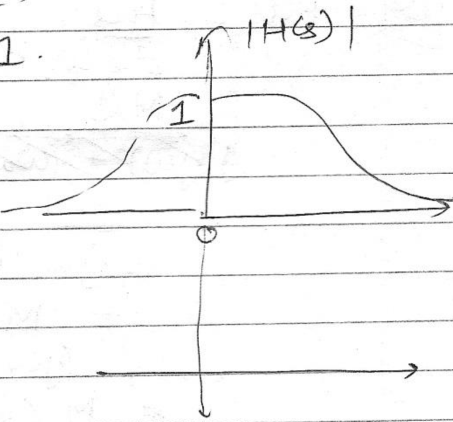
$\omega \to 0$

$$|H(s)| = 1$$

$$\omega \to \infty = 0$$

$$-\tan^{-1}(0) = 0$$

$$-\tan^{-1}(\infty) = -90$$



$$\frac{\frac{1}{RC}}{\sqrt{\omega_c^2 + \left(\frac{1}{RC}\right)^2}} = \frac{1}{\sqrt{2}}$$

$$\frac{1}{RC} \times \sqrt{2} = \sqrt{\omega_c^2 + \left(\frac{1}{RC}\right)^2}$$

$$\frac{2}{(RC)^2} = \omega_c^2 + \frac{1}{(RC)^2}$$

$$\frac{1}{(RC)^2} = \omega_c^2$$

$$\omega_c = \frac{1}{RC}$$

$$\boxed{\cancel{\omega = \frac{1}{\sqrt{RC}}}}$$

$(j\omega)^2 + \omega_0^2$

$-\omega^2 + \omega_0^2$

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Q $H(s) = (s^2 + \omega_0^2)\, H_1(s)$

$\longrightarrow M\angle\phi$

$(s+j\omega_0)(s-j\omega_0)$

$(j\omega + j\omega_0)(j\omega - j\omega_0)$

$[j(\omega+\omega_0)][j(\omega-\omega_0)]$

$\tan^{-1}\frac{\omega+\omega_0}{0}$ Mag $\cancel{\frac{\sqrt{(\omega^2+\omega_0^2)^2}}{\sqrt{(\omega_0^2)^2+(\omega^2)^2}}}$

$= 90 - 90$

$= 0$ $\quad \omega < \omega_0$ $\quad |H| = M(\omega_0^2 - \omega^2)$

$= 90 + 90$ $\quad \omega > \omega_0$

$= 180^\circ$

Q $H(s) = \frac{H_1(s)}{s^2 + \omega_0^2}$

$\cancel{\frac{M\angle\phi}{\sqrt{(j\omega)^2 + (\omega_0)^2}}}$ $\quad \cancel{\angle H(s) = \frac{M}{(\omega^2)^2 -}}$

$\frac{H_1(s)}{(s+j\omega_0)(s-j\omega_0)}$

$= \frac{M\angle\phi}{(j\omega + j\omega_0)(j\omega - j\omega_0)}$

$= \frac{M\angle\phi}{j(\omega+\omega_0)\, j(\omega - \omega_0)}$

$\omega < \omega_0$

$\frac{\angle\phi}{\angle 90\ \angle -90} = \angle\phi$

$\frac{\angle\phi}{\angle 90\ \angle 90} = \underline{\underline{\phi - 180}}$ $\quad \omega > \omega_0$

Q $H(s) = \frac{(j\omega)^2 + \omega_1^2}{(j\omega)^2 + \omega_2^2} H_1(s)$ $\quad \omega_1 < \omega_2$

$\frac{-\omega^2 + \omega_1^2}{-\omega^2 + \omega_2^2} H_1(s) = M\angle\phi$

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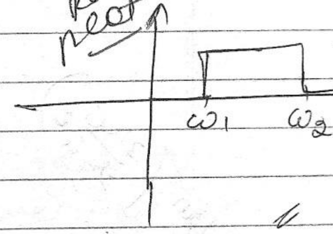
$$\frac{(s+j\omega_1)(s-j\omega_1)}{(s+j\omega_2)(s-j\omega_2)}$$

phase plot



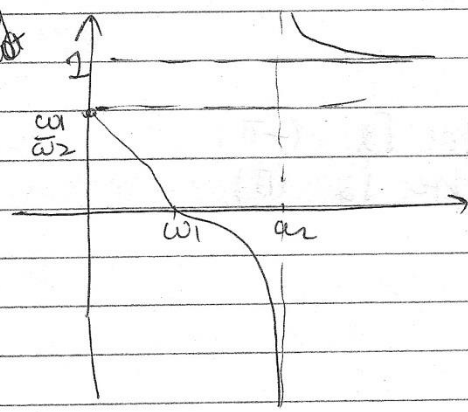
$$\frac{(j\omega+j\omega_1)(j\omega-j\omega_1)}{(j\omega+j\omega_2)(j\omega-j\omega_2)}$$

$$\frac{j(\omega+\omega_1)\,j\,(\omega-\omega_1)}{j(\omega+\omega_2)\,j\,(\omega-\omega_2)}$$

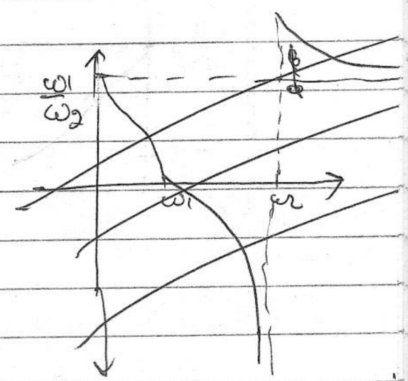
phase 0 $\omega < \omega_1$ 0 $\omega < \omega_2$

phase 180 $\omega > \omega_1$ -180 $\omega > \omega_2$

Mag plot



Mag plot

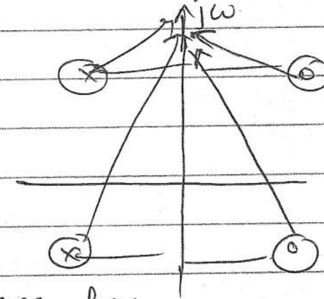


Q $H(s) = \frac{(s-1)^2+1}{(s+1)^2+1}$ $s^2+1-2s+1$

$s^2+2s+1+1$ $\frac{s^2-2s+2}{s^2+2s+2}$

$Z_{12} = 1 \pm j$

$P_{12} = -1 \pm j$



$\frac{\text{Mag (Zeroes)}}{\text{Mag (Poles)}} = 1$

$|H| = k =$ All pass fn.

$$\frac{(j\omega)^2-2j\omega+2}{(j\omega)^2+2j\omega+2} \qquad \frac{-\omega^2-2j\omega+2}{-\omega^2+2j\omega+2}$$

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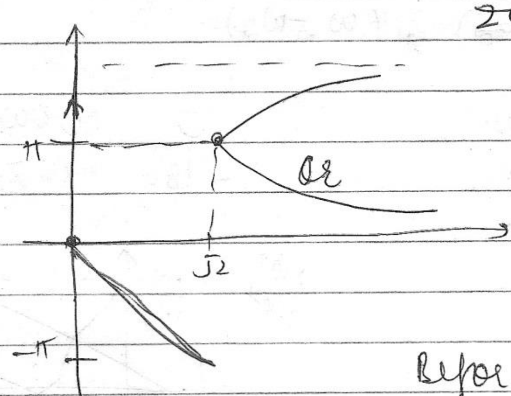
$$\frac{(2-\omega^2) - 2j\omega}{(2-\omega^2) + 2j\omega} = \tan^{-1}\left(\frac{-2\omega}{2-\omega^2}\right) - $$

$$2 - 2\tan^{-1}\left(\frac{2\omega}{2-\omega^2}\right)$$

$$\tan^{-1} \frac{-\cancel{(2-\omega^2)}}{\cancel{2\omega}} \qquad \tan^{-1}\cancel{0}\left(\frac{-\cancel{2\omega}}{\cancel{2-\omega^2}}\right)$$

$$\frac{\cancel{2-\omega^2}}{\cancel{2\omega}} \quad \cancel{\tan}^{-1} \qquad \frac{\cancel{2\omega}}{\cancel{2-\omega^2}}$$



$\omega = \sqrt{2}$

$\tan^{-1} - \left(\frac{\infty}{\infty}\right)$

Befor $\sqrt{2}$ $(-\pi)$
After $\sqrt{2}$ (π)

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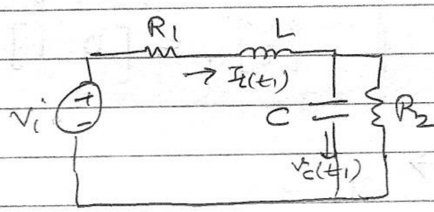
$\tan^{-1}\left(\frac{2\omega}{2-\omega^2}\right)$

State VARIABLE

Info at any point of time that need to be known to find out the response under the influence of excitation at that particular instant of time.

state variable here is $I_L(t_1)$ and $v_C(t_1)$

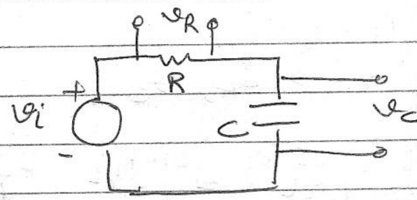


Eg

$S.V = v_c$

$O/P\ V = v_C, v_R$

$I/P\ V = v_i$



$$i_C = C\frac{dv_c}{dt} \qquad \dot{v}_c = \frac{i_c}{C}$$

$$v_i = v_R + v_c$$
$$= i_c R + v_c$$
$$v_i = RC\dot{v}_c + v_c$$

$$\dot{v}_c = \frac{v_i}{RC} - \frac{v_c}{RC}$$

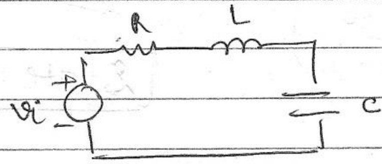
$$v_c = [1]v_c + [0]v_i$$
$$v_R = v_i - v_c$$
$$= [-1]v_c + [1]v_i$$

→ $y \rightarrow v_c$ (O/P)

$u \rightarrow v_i$ (I/P)

x (state variable is)

i_L and v_c



for state variable find dynamic elements

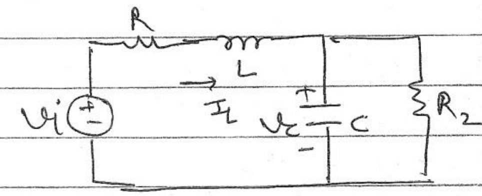
$$\begin{bmatrix} \dot{I_L} \\ \dot{v_c} \end{bmatrix} = \begin{bmatrix} -R/L & -1/L \\ 1/C & 0 \end{bmatrix} \begin{bmatrix} I_L \\ v_c \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} [v_i]$$

$$[v_c] = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} I_L \\ v_c \end{bmatrix} + [0][v_i]$$

→ $y \rightarrow v_c$

$u \rightarrow v_i$

$x \rightarrow I_L, v_c$

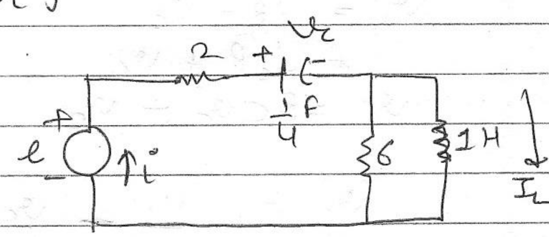


$$\begin{bmatrix} \dot{I_L} \\ \dot{v_c} \end{bmatrix} = \begin{bmatrix} -R_1/L & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{R_2 C} \end{bmatrix} \begin{bmatrix} I_L \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v_i$$

$$v_c = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} I_L \\ v_c \end{bmatrix} + [0] v_i$$

→ I/P e

O/P i

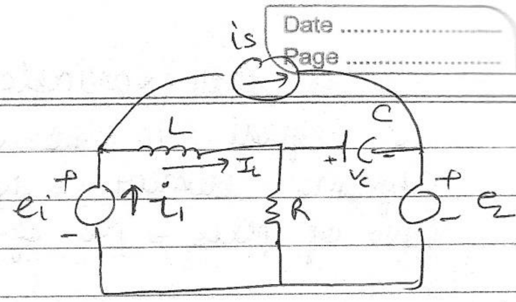


$$A = \begin{bmatrix} -1/2 & 3 \\ -3/4 & -3/2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1/2 \\ 3/4 \end{bmatrix} \qquad C = \begin{bmatrix} -1/8 & 3/4 \end{bmatrix} \qquad D = \begin{bmatrix} \frac{1}{8} \end{bmatrix}$$

→ I/P − e_1, e_2, i_s

O/P − i_1, i_2



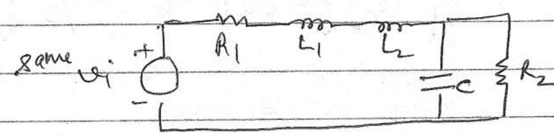
$$U = \begin{bmatrix} e_1 \\ e_2 \\ i_s \end{bmatrix}$$

$$O/P \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \qquad x \rightarrow \begin{bmatrix} v_c \\ i_L \end{bmatrix}$$

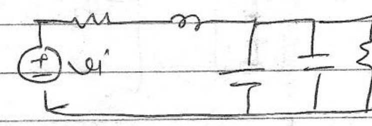
$$A = \begin{bmatrix} -\frac{1}{RC} & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & -\frac{1}{RC} & 0 \\ \frac{1}{L} & -\frac{1}{L} & 0 \end{bmatrix}$$

$$C \rightarrow \begin{bmatrix} 0 & 1 \\ \frac{1}{R} & -1 \end{bmatrix} \qquad D \rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 0 & \frac{1}{R} & -1 \end{bmatrix}$$

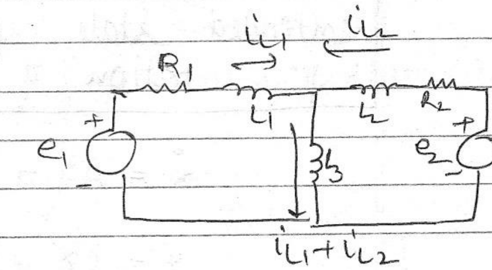
→ 2 state variables
Current through L same



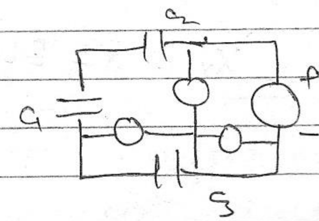
→ similarly 2 state variables



→ 2 state variables



→ 2 state variable
loop including C's.
Voltage of one can be represented in terms of other 2.



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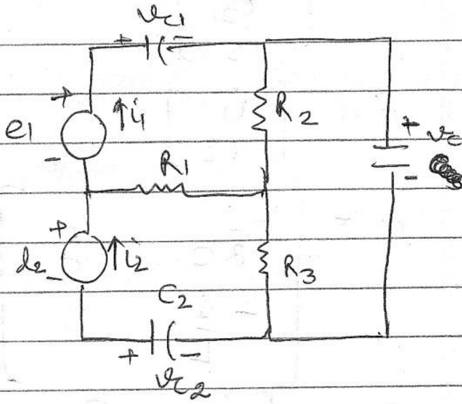
→ No. of state variables for a given system is equal to no. of dynamically independent dynamic element = No. of eigen values = No. of poles = No. of zeroes = No. of natural ν.

Q. $C_1 = C_2 = C_3 = 1F$

$R_1 = R_2 = R_3 = 1\Omega$



$Y = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad U = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$

state variables = 2

$$e_1 + e_2 = v_{c1} - v_{c2} + v_{c3}$$

$$\begin{bmatrix} \dot{v}_{c1} \\ \dot{v}_{c2} \end{bmatrix} = \begin{bmatrix} -1/3 & 0 \\ 0 & -1/3 \end{bmatrix} \begin{bmatrix} v_{c1} \\ v_{c2} \end{bmatrix} + \begin{bmatrix} 1/3 & 0 \\ 0 & -1/3 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ -1/3 & -1/3 \end{bmatrix} \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -1/3 & 0 \\ 0 & 1/3 \end{bmatrix} \begin{bmatrix} v_{c1} \\ v_{c2} \end{bmatrix} + \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} 1/3 & 1/3 \\ 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix}$$

$\boxed{\text{extended state space representation of system}}$

$$\dot{X} = AX + BU + E\dot{U}$$

$$Y = CX + DU + F\dot{U}$$

$$X_n = X - EU \longrightarrow \dot{X}_n = \dot{X} - E\dot{U}$$

$$X = X_n + EU$$

$\dot{x}_n + \cancel{E\dot{U}} = AX_n + AEU + BU + \cancel{E\dot{U}}$

$$\boxed{\dot{x}_n = AX_n + (AE+B)U}$$

$$\boxed{Y = CX_n + (CE+D)U + F\dot{U}}$$

$$sX(s) - X(0^+) = AX(s) + BU(s)$$

↓

0^+ (because above equation valid for $t = 0^+$)

$$X(s) = (SI-A)^{-1}X(0^+) + (SI-A)^{-1}BU(s)$$

$$Y(s) = CX(s) + DU(s)$$

$$= \underbrace{C(SI-A)^{-1}X(0^+)}_{\text{zero I/P sol}^n} + \underbrace{\left[C(SI-A)^{-1}B + D\right]U(s)}_{\text{zero state sol}^n}$$

zero I/P sol^n → Poles λ_i

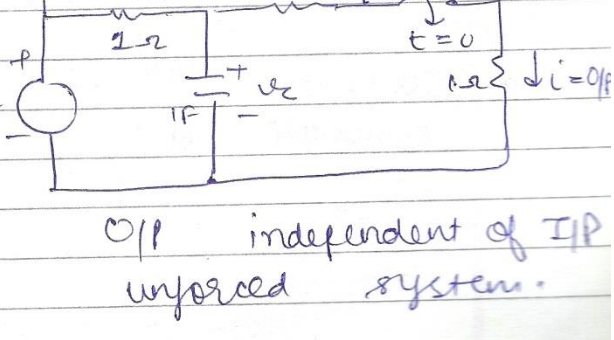
zero state sol^n → Poles of λ_i and poles of $U(s)$

$$= \underbrace{\sum a_i e^{\lambda_i t}\Big|_{\forall i} + \sum b_i e^{\lambda_i t}}_{\text{Transient response}} + \underbrace{\sum c_k e^{P_k t}}_{\text{steady state Respon}}$$

Q Ans

$$\begin{bmatrix} \dot{v}_c \\ \dot{I}_L \end{bmatrix} = \begin{bmatrix} -3/2 & 1/2 \\ -1/2 & -1/3 \end{bmatrix}\begin{bmatrix} \ \\ \ \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}[e]$$

$$i = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} v_c \\ i_L \end{bmatrix}$$



O/P independent of I/P
unforced system.

$$x(0^+) = \begin{bmatrix} e \\ 0 \end{bmatrix}$$

$$\boxed{i(t) = 1 - 0.5e^{-t} + 0.5te^{-t} \qquad t > 0}$$

$$\frac{Y(s)}{U(s)} = C(SI-A)^{-1}B + D$$ [keeping initial cond = 0]

State Transition Matrix

$$\phi(t) = e^{At} = I + \frac{At}{1!} + \frac{A^2t^2}{2!} + \cdots$$

A = Matrix.

All terms of $n \times n$

$$\phi(t) = [\ ]_{n \times n}$$

$$\boxed{\dot{\phi}(t) = A\phi(t)}$$

1) $\phi(t) = [I]_{n \times n}$

2) $\phi(t_2 - t_1) = \phi(t_2 - t_3) \cdot \phi(t_3 - t_2)$

3) $\phi^{-1}(t) = \phi(-t)$

$$x(t) = x(0)e^{At}$$
$$x(t - t_0) = x(t_0)\, e^{A(t - t_0)}$$

→ Laplace Transform.

$$\dot{x} = AX$$
$$x(s) = (SI - A)^{-1}x(0)$$

$$L^{-1}\left((SI-A)^{-1}\right) = e^{At}$$

→ Convert e^{At} into ∞ sum series then summate it

→ Diagonilation of Matrix

$$|sI-A| = 0$$

$$(\lambda_1, \lambda_2 \; ---- \; \lambda_n) \qquad T^{-1}AT = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & 0 \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$

T = Transformation Matrix or similarity transformation Matrix.

$$e^{At} = T \begin{bmatrix} e^{\lambda_1 t} & & \\ & e^{\lambda_2 t} & \\ & & \ddots \; e^{\lambda_n t} \end{bmatrix} T^{-1}$$

→ Cayley Hamilton Thm.

$$|sI-A| = 0$$

↳ Roots $\lambda_1 \lambda_2 --- \lambda_n$

$$e^{At} = C_0(t) + C_1(t)\cdot A + C_2(t) A^2 + --- C_{n-1} A^{n-1}$$

$$\begin{cases} e^{\lambda_1 t} = C_0(t) + C_1(t)\lambda_1 + C_2(t)\lambda_1^2 + \; --- \; C_{n-1}\lambda_1^{n-1} \\ e^{\lambda_2 t} = ------ \\ \vdots \\ e^{\lambda_n t} = ----------- \end{cases}$$

Q $A = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix}$

Soln

$n = 2$

$$e^{At} = C_0 + C_1 A.$$

$$\begin{bmatrix} s+1 & 0 \\ 1 & s+2 \end{bmatrix}$$

$$(s+1)(s+2) - 0 = 0$$

$s = -1$

$s = -2$

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$$e^{-1t} = C_0 + C_1 A$$
$$e^{-2t} = C_0 + C_1 A.$$

$$\phi(t) = \begin{bmatrix} e^{-t} & 2t & 0 \\ e^{-t} - e^{-3t} & & e^{-2t} \end{bmatrix}$$

$$e^{-t} = C_0 - C_1$$
$$e^{-2t} = C_0 - 2C_1$$
$$e^{-t} - e^{-2t} = C_1$$

$$e^{-t} + e^{-t} - e^{-2t} = C_0$$
$$\boxed{2e^{-t} - e^{-2t} = C_0}$$

$$e^{-t} = \theta \cdot (2e^{-t} - e^{-2t}) I + (e^{-t} - e^{-2t}) \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix}$$

Q $A = \begin{bmatrix} 0 & -1 & -3 \\ -6 & 0 & -2 \\ 5 & -2 & -4 \end{bmatrix}$

$\boxed{\lambda_1 = -1 = \lambda_2}$ $\boxed{\lambda_3 = -2}$

$$e^{At} = C_0 + C_1 A + C_2 A^2$$

→ $X = AX + BU$

$$X(t) = \int_0^t e^{t(t-\tau)} B\, U(\tau) d\tau + e^{At} X(0)$$

$$= e^{At} * B\, U(t) + e^{At} X(0)$$

$$\boxed{x(t) = \phi(t) * BU(t) + \phi(t) X(0)}$$

$\rightarrow \quad y(t) = C\phi(t) \cdot x(0) + C\phi(t) * Bu(t) + Du(t)$

Q

$$\dot{x} = f(x, u, t)$$

$$y = h(x, u, t)$$

$$\dot{X} = f(X, u)$$
$$Y = h(X, U)$$

Autonomous system.

$$\rightarrow \begin{bmatrix} 2e^{-t} - e^{-2t} & 0 \\ 0 & 2e^{-t} - e^{-2t} \end{bmatrix} + \begin{bmatrix} -e^{-t} + e^{-2t} & 0 \\ e^{-t} - e^{-2t} & -2e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$= \begin{bmatrix} e^{-t} & 0 \\ e^{-t} - e^{-2t} & e^{-2t} \end{bmatrix}$$

$$\dot{x} = f(x) + G\,x(u)$$

$\uparrow$

Affine System

If $\dot{x} = f(x)$ unforced system.

In case of NL system, harmonic components may be present corresponding to constant freq. Input

NL system can exhibit multiple mode of behaviour

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$\dot{x} = f(x) \qquad \dot{x} = f(x, u)$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

$\cong$ point

$$\left.\begin{aligned} \dot{x}_1 &= f_1(x, u) = 0 \\ \dot{x}_2 &= f_2(x, u) = 0 \\ &\vdots \\ \dot{x}_n &= f_n(x, u) = 0 \end{aligned}\right\} \quad \begin{aligned} &(x_1 e, u_1 e) \\ &(x_2 e, u_2 e) \end{aligned}$$

At $\cong$ point $\quad f(x_e, u_e) = 0$

To find out linear model around $\cong$m point for a NL system will disturb the system from $\cong$ point with very small deviation.

$$(x_e + \Delta x,\ u_e + \Delta u)$$

Now linearize the system around x_e & u_e

$$\left.\frac{dx}{dt}\right|_{x_e, u_e} = f(x_e, u_e) + \left.\frac{\partial f}{\partial x}\right|_{x_e, u_e} \Delta x + \left.\frac{\partial f}{\partial u}\right|_{x_e, u_e} \Delta u + \ldots$$

At $\cong$m $\quad f(x_e, u_e) = 0$

$$\left.\frac{d(x + \Delta x)}{dt}\right|_{x_e + u_e} \qquad \left.\frac{dx}{dt}\right|_{\cong m \text{ point}} = 0$$

$$\boxed{\Delta \dot{x} = \underbrace{\left[\frac{\partial f}{\partial x}\right]_{n \times n}}_{A} \Delta x + \underbrace{\left[\frac{\partial f}{\partial u}\right]_{n \times m}}_{B} \Delta u}$$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & & & \\ \vdots & & & \\ \frac{\partial f_n}{\partial x_1} & - & - & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \qquad B = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial u_1} & & \frac{\partial f_n}{\partial u_m} \end{bmatrix}$$

$$\boxed{\begin{aligned} \Delta \dot{x} &= A \Delta x + B \Delta u \\ \Delta y &= [C] \Delta x + D \Delta u \end{aligned}} \longrightarrow \text{applicable at } \cong m \text{ point}$$

$( )\Delta x^2 + \cdots ( )\Delta u^2$ Neglect

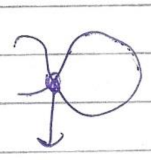
<u>One dimension</u>

$$\dot{x} = ax$$
$$x(t) = x(0)\, e^{at}$$

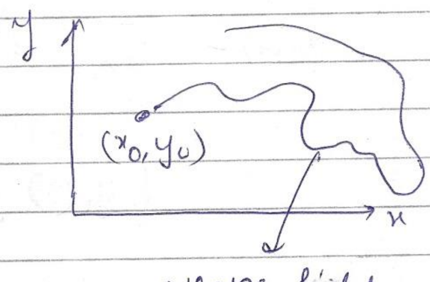
<u>2 dimensional</u>

$$\dot{x} = f_1(x, y) \qquad f_1, f_2 \text{ N.L fn's of } (x, y)$$
$$\dot{y} = f_2(x, y)$$

Then starting from any initial cond. system will follow a particular path in (x, y) plane. that path is called as vector field.



2 $\frac{dx}{dt}$



vector field
(will never intersect)

vector field cannot intersect other than $\cong m$ because that will need 2 direction of vector field at intersection point.

$$\begin{array}{c} x \leftarrow \\ y \leftarrow \end{array} \begin{bmatrix} \Delta\dot{x} \\ \Delta\dot{y} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}_{\text{impnt}}$$

$$X = AX$$

→ If $X_1(t) = \begin{bmatrix} x_1(t) \\ y_1(t) \end{bmatrix}$ and $X_2(t) = \begin{bmatrix} x_2(t) \\ y_2(t) \end{bmatrix}$

are 2 linearly independent soln for given system.

$$X(t) = C_1[X_1(t)] + C_2[X_2(t)]$$

To find out $X_1(t)$ and $X_2(t)$ we have to find out a direction for which system behaviour is one-D. That direction is called eigen direction.

$$|\lambda I - A| = 0$$

Initial cond. Matrix

$$\lambda = \lambda_1, \lambda_2$$

$$\begin{bmatrix} x_1(0) \\ y_1(0) \end{bmatrix}$$

$$|A - \lambda_1 I| X = 0$$

$$X_1(t) = \vec{V}_{\lambda_1} \cdot e^{\lambda_1 t}$$

$$= \begin{bmatrix} x_1(0) \\ y_1(0) \end{bmatrix} e^{\lambda_1 t}$$

$$X_2(t) = v_2(t)\, e^{\lambda_2 t}$$

$$= \begin{bmatrix} x_2(0) \\ y_2(0) \end{bmatrix} e^{\lambda_2 t}$$

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$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = x(t) = c_1 \begin{bmatrix} x_1(0) \\ y_1(0) \end{bmatrix} e^{\lambda_1 t} + c_2 \begin{bmatrix} x_2(0) \\ y_2(0) \end{bmatrix} e^{\lambda_2 t}$$

Q $\dot{x} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -4 & -3 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ find out complete sol^n.

$$\begin{bmatrix} \lambda+4 & 3 \\ -2 & \lambda-3 \end{bmatrix}$$

$$(\lambda+4)(\lambda-3) - (-6) = 0$$
$$\lambda^2 - 3\lambda + 4\lambda - 12 + 6 = 0$$
$$\lambda^2 + \lambda - 6 = 0$$
$$\lambda = 2, -3$$

$\lambda = 2$	$\lambda = -3$
$(A - 2I)x = 0$	$v_2 = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ (can be any)
$\begin{bmatrix} -6 & -3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$	
$-6x - 3y = 0$	
$2x + y = 0$	
If $\boxed{x = 1}$	
$y = \quad -6 - 3y = 0$	
$-\cancel{3}y = \cancel{6}2$	
$\boxed{y = -2}$	
∴ No. of eigen vectors possible.	

$-4 - 2 \times 1$
$-4 + 8$
-6

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = c_1 e^{2t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 e^{-3t} \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$x(t) = c_1 e^{2t} + 3 c_2 e^{-3t}$$
$$y(t) = -2c_1 e^{2t} - c_2 e^{-3t}$$

By using Initial cond.

$$1 = c_1 + 3c_2 \quad \text{—} ①$$
$$1 = -2c_1 - c_2 \quad \text{—} ②$$

$$2 = 2c_1 + 6c_2$$
$$1 = -2c_1 - c_2$$
$$3 = 5c_2$$

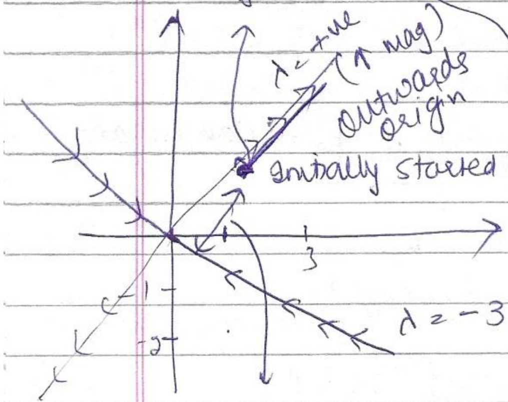
$$\boxed{c_2 = \frac{3}{5}}$$

$$1 = -2c_1 - \frac{3}{5}$$

$$1 + \frac{3}{5} = -2c_1$$

$$\frac{-8}{5 \times 2} = c_1$$

$$\boxed{c_1 = -\frac{4}{5}}$$



this distance w.r.t -ve λ ↑

1) $\lambda_1 = +ve$ $\quad \lambda_2 = -ve$ (3 cases) [saddle pnt]

2) $\lambda_1 = -ve$ $\quad \lambda_2 = -ve$ (attractor) towards origin

3) $\lambda_1 = +ve$ $\quad \lambda_2 = +ve$ (point is called repeller) → unstable

move away from origin

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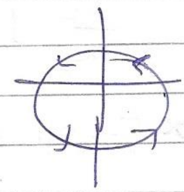
If $\lambda = \sigma \pm j\omega$

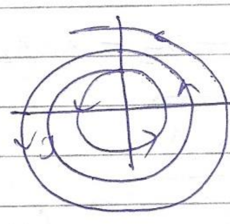
$\sigma = +ve$

$\sigma = -ve$

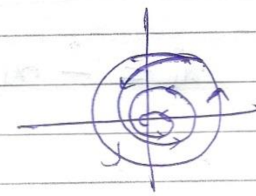
$\sigma = 0$ $\longrightarrow$



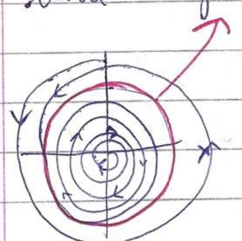
$\sigma = +ve$



$\sigma = -ve$ $\longrightarrow$



limit cycle



Q $\ddot{y} + 3\dot{y} + y^3 = 0$ find $\cong$ pnt. linearize system around $\cong$ pnt & find matrix A. ~~of~~ Then find eigen values of linearized system.

$y = x_1$

$\dot{x}_1 = x_2$ fn (f_1)

stable $\dot{x}_2 = -x_1^3 - 3x_2$ fn (f_2)

To find $\cong$ pnt

$x_2 = 0$

$-x_1^3 - 3x_2 = 0$ $x_1 = 0$

$x_2 = 0$

Transparent electronics
E paper
vini's work.

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

$\dot{X} = AX$

Now $|\lambda I - A| = 0$

Q $\dot{x}_1 = -x_1^2 + x_1 x_2$

$\dot{x}_2 = -2x_2^2 + x_2 - x_1 x_2 + 2$

$\boxed{\text{find } X_e, [A], \lambda}$

Q $\dot{x}_1 = -x_1 + x_2 + x_1(x_1^2 + x_2^2)$

$\dot{x}_2 = -x_1 - x_2 + x_2(x_1^2 + x_2^2)$

find $\equiv$m pnt.

Q $\dot{x} = 3x - 4y$

$\dot{y} = x - y$

$\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

find solution.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} \lambda-3 & +4 \\ -1 & \lambda+1 \end{bmatrix}$$

$(\lambda-3)(\lambda+1) + 4 = 0$

$\lambda^2 + \lambda - 3\lambda - 3 + 4 = 0$

$\lambda^2 - 2\lambda + 1 = 0$

$\lambda = 1, 1$

Soln $= [C_1 + C_2 t] e^{\lambda t}$

Selectivity & Q factor same in notes

Selectivity ckt.

$f(t) = \omega_0$

$y(t) = n\omega_0 \qquad n > 1$

$\oint f(t)g(t) = 0$

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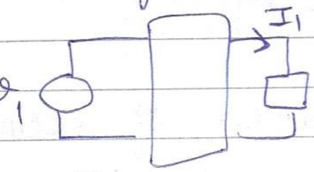
Reciprocity Theorem

A linear n/w containing bilateral 2 terminal elements and reciprocal 2 terminal elements, then the ratio of transform of response & excitation remains unchanged if the location of response and excitation are interchanged.

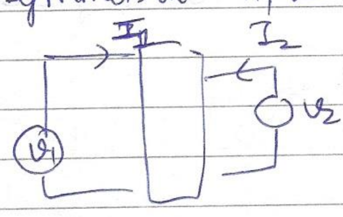


$$\frac{v_1}{I_1}$$

$$\frac{v_2}{I_2}$$

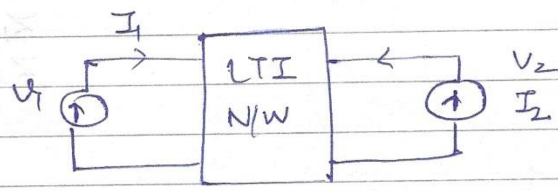
$$\left.\frac{v_1}{I_2} = \frac{v_2}{I_1}\right\} \text{ for reciprocity}$$

Symmeteric n/w



$$\frac{v_1}{I_1} = \frac{v_2}{I_2}$$

$\boxed{\text{Z- parameter}}$



$$v_1 = Z_{11} I_1 + Z_{12} I_2$$
$$v_2 = Z_{21} I_1 + Z_{22} I_2$$

imp → Δ_{11} is not similar to that as in Cramer's
It is cofactor of Z_{11}.

$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$ $\quad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$ $\quad Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$

$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$

for reciprocal n/w

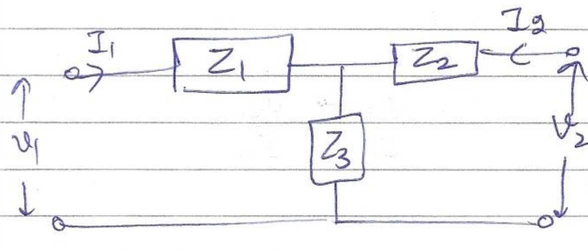
$$\boxed{Z_{12} = Z_{21}}$$

for Y-parameters, condition of reciprocity is

$$\boxed{Y_{12} = Y_{21}}$$

for symmetric N/W

$$\boxed{Z_{11} = Z_{22}} \qquad \boxed{Y_{11} = Y_{22}}$$

Q  Find Z parameters

$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$

$Z_{11} = Z_1 + Z_3$
$Z_{21} = Z_3$
$Z_{12} = Z_3$
$Z_{22} = Z_2 + Z_3$

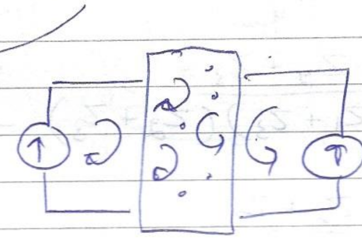
rule.

$y_{ii} \rightarrow$ self admittance of ith node

y_{ij} — relative admittance b/w i and j

$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & --- & y_{1k} \\ y_{21} & y_{22} & --- & \\ \vdots & & & \\ & & & \\ y_{k1} & ------ & & y_{kk} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ \vdots \\ v_k \end{bmatrix}$$

Δ



If there are only two current sources

$$v_1 = \frac{\Delta_{11}}{\Delta} \cdot I_1 + \frac{\Delta_{21}}{\Delta} \cdot I_2 + 0 + 0 + ---$$

We know

$$v_1 = Z_{11} I_1 + Z_{12} I_2$$

$\therefore \quad Z_{11} = \frac{\Delta_{11}}{\Delta}$ ✓ $\qquad Z_{21} = \frac{\Delta_{12}}{\Delta}$

Similarly $\quad Z_{12} = \frac{\Delta_{21}}{\Delta}$ ✓ $\qquad Z_{22} = \frac{\Delta_{22}}{\Delta}$

for Z parameters

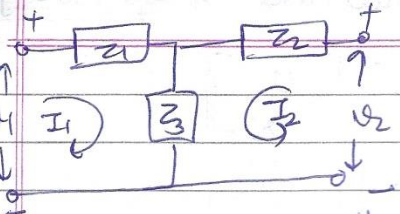
mesh basis sys. determinant

$$\begin{bmatrix} v_1 \\ v_2 \\ 0 \\ 0 \\ \vdots \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & ---- & Z_{1k} \\ \vdots & & & \\ & & & \\ Z_{k1} & ---- & & Z_{kk} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_k \end{bmatrix}$$

Z_{ii} = self impedance of ith mesh.

Z_{ij} = mutual " b/w loop i & j

$$I_1 = \underbrace{\frac{\Delta_{11}}{\Delta}}_{Y_{11}} V_1 + \underbrace{\frac{\Delta_{21}}{\Delta}}_{Y_{12}} V_2$$



Q. find Y parameters by step by step procedure.

$$\begin{array}{ccc} & y_{11} \quad\quad y_{12} & \\ \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = & \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix} & \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \\ & y_{21} \quad\quad y_{22} & \end{array}$$

result

$$y_{11} = \frac{\Delta_{11}}{\Delta} = \frac{Z_2 + Z_3}{(Z_1 + Z_3)(Z_2 + Z_3) - Z_3 Z_3}$$

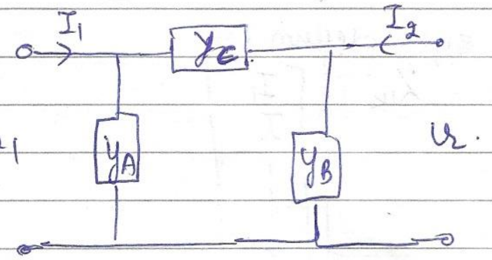
$$= \frac{Z_2 + Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

$$y_{12} = \frac{\Delta_{21}}{\Delta} = \frac{Z_3}{\Delta}$$

$$y_{21} = \frac{\Delta_{12}}{\Delta} = \frac{Z_3}{\Delta}$$

$$y_{22} = \frac{\Delta_{22}}{\Delta} = \frac{Z_1 + Z_3}{\Delta}$$

Q. find y parameters

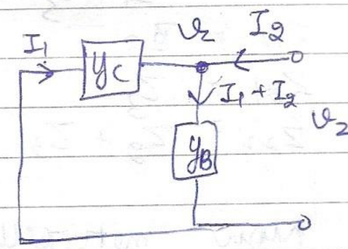


$$Y_{11} = \frac{I_1}{v_1}\Big|_{v_2 = 0}$$

$$I_2 = \frac{y_A}{y_A + y_C} I_1$$

$$\cancel{V_1} = \cancel{I_1}\left[\frac{\cancel{y_A y_C}}{\cancel{y_A + y_C}}\right]$$

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1 = 0}$$



$$I_1 = -V_2\, y_C$$

$$\frac{I_1}{V_2} = -y_C = Y_{12}$$

$$\boxed{\begin{aligned} Y_{21} &= -y_C \\ Y_{22} &\ \ y_B + y_C \end{aligned}}$$

$$\boxed{y_{11} = y_A + y_C}$$

$$y = \begin{array}{cc} 11 & 12 \\ \left[\begin{array}{cc} y_A + y_C & -y_C \\ -y_C & y_B + y_C \end{array}\right] \\ 21 & 22 \end{array}$$

$$Z_{11} = \frac{y_B + y_C}{\Delta m} \qquad Z_{12} = \frac{+y_C}{\Delta m}$$

$$Z_{21} = \frac{+y_C}{\Delta m} \qquad Z_{22} = \frac{y_A + y_C}{\Delta m}$$

$$V_1 = (Z_{11} + Z_{12}) I_1 + Z_{12}(I_2 - I_1)$$

$$V_2 = Z_{12}(I_1 - I_2) + (Z_{22} + Z_{12}) I_2$$

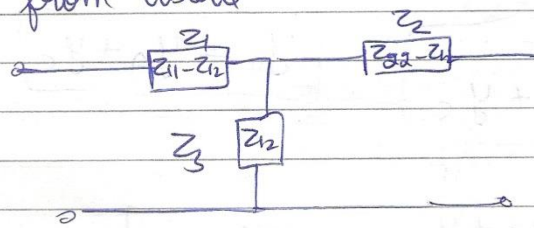
* for T n/w Z parameters easy
for π " Y " "
Use Z or Y for one and then use
conversion for other

⟹ condition taken is n/w is reciprocal

If a black box is given with 2 port n/w s.

$Z_{11} = Z_1 + Z_3$
$Z_{12} = Z_3$
$Z_{21} = Z_3$
$Z_{22} = Z_2 + Z_3$

Now network synthesis, i.e construction of T- n/w from above



Similarly π ckt can be designed using Y-parameters.

$I_1 = y_{11} V_1 + y_{12} V_2$
$I_2 = y_{21} V_1 + y_{22} V_2$

$y_{12} = y_{21}$

similarly for y

$I_1 = y_{11} V_1 + y_{12} V_2$
$I_2 = y_{12} V_1 + y_{22} V_2$

$I_1 = (y_{11} + y_{12}) V_1 + y_{12} (V_2 - V_1)$

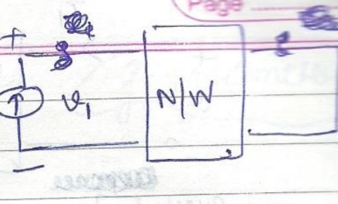
$I_2 =$

$\boxed{\text{h parameters}}$ hybrid parameters

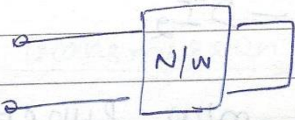
$V_1 = h_{11} I_1 + h_{12} V_2$
$I_2 = h_{21} I_1 + h_{22} V_2$

excitation I_1

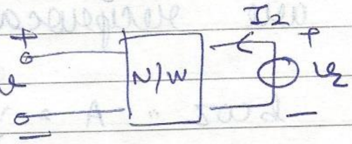
$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2 = 0}$$



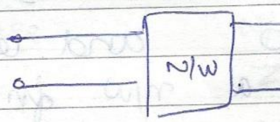
$I_1 \rightarrow 0$ represents poles



$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1 = 0}$$



$V_2 \rightarrow 0$



$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2 = 0}$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1 = 0}$$

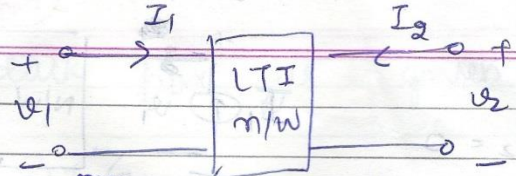
poles of h parameters represents natural // under the conditions first port open and second is short.

$\boxed{t\text{- parameters}}$ transmission line parameters

excitation is V_1, I_1
response is $V_2, -I_2$

because these parameters are used to calculate power

I/P power = $V_1 I_1$
O/P " = $-V_2 I_2$



~~Response~~

In this case excit- ation $\underbrace{v_1}$ $= A \underbrace{v_2} - B \underbrace{I_2}$ Response

$$I_1 = Cv_2 - DI_2$$

★ A, B, C, D are not n/w functions. Instead they are reciprocal of n/w fns.

$$\text{bcoz} \quad A = \left.\frac{v_1}{v_2}\right|_{I_2 = 0}$$

Now $I_2 = 0$ and $v_2 = 0$ not possible

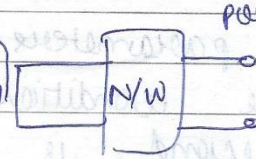
∴ not a n/w fn.

both short $B = -\left.\dfrac{v_1}{I_2}\right|_{v_2 = 0}$ $\qquad$ $C = \left.\dfrac{I_1}{v_2}\right|_{I_2 = 0}$ both open

$$D = -\left.\frac{I_1}{I_2}\right|_{v_2 = 0}$$

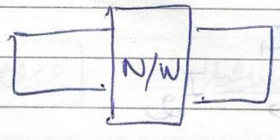
2) under the cond. one port open other short.

zeroes of A represent natural because A is reciprocal $\dfrac{v_2}{v_1}$



$v_1 = 0$ and $I_2 = 0$

zeroes of B →



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$\boxed{\text{Inverse hybrid}}$ parameters. g parameter

Excitation $\rightarrow I_1, V_2$
Response $\rightarrow V_1, I_2$

Inverse Transmission line parameters

Excitation $V_1, -I_1$
Response V_2, I_2

Excitation	Response	N/W parameter
I_1, I_2	V_1, V_2	Z-parameter
V_1, V_2	I_1, I_2	Y parameter
I_1, V_2	I_2, V_1	H parameter
V_1, I_2	I_1, V_2	g-parameter
V_1, I_1	$V_2, -I_2$	ABCD parameter
V_2, I_2	$V_1, -I_1$	A'B'C'D' "

Note
In ABCD & A'B'C'D relationship is written as excitation in terms of response. In all four other, response in terms of excitation

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

$$B = -\left.\frac{V_1}{I_2}\right|_{V_2=0} \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$$

(really hot weather)

Date 9/6/2013

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for reciprocity $\left.\frac{V_1}{I_2}\right|_{I_1=0} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$

$A/B = -\frac{I_2}{V_2}$ $\qquad \frac{C}{D} = -\frac{I_2}{V_2}$

$\Rightarrow \frac{A}{B} = \frac{C}{D}$

$\boxed{AD - BC = 1}$

$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1$$

from def.

$0 = CV_2 - DI_2$

$\frac{V_2}{I_2} = D/C$

$V_1 = AV_2 - BI_2$

$V_1 = AV_2$

$\frac{V_1}{V_2} = A$

$\frac{V_1}{\cancel{V_2}} \times \frac{\cancel{V_2}}{I_2} = \frac{AC}{D}$

$\frac{V_1}{I_2} = \frac{AD}{C}$

for symmetry $\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

$\left.\frac{V_1}{I_2}\right|_{I_1=0} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$

$\cancel{h_{11} = \left.\frac{V_1}{V_2}\right|_{I_1=0}}$ $\qquad \cancel{h_{12} = \left.\frac{V_1}{I_1}\right|_{V_2=0}}$ $\qquad \cancel{h_{21} = \left.\frac{I_2}{V_2}\right|_{I_1=0}}$ $\qquad \cancel{h_{22} = \left.\frac{I_2}{I_1}\right|_{V_2=0}}$

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$$0 = h_{21} v_2 + h_{22} i_1$$
$$-h_{21} v_2 = h_{22} i_1$$
$$\frac{v_2}{I_1} = -\frac{h_{22}}{h_{21}}$$

X.

$$v_1 = h_{11} v_2 \qquad \frac{h_{11}}{h_{21}} = \left.\frac{v_1}{I_2}\right|_{I_1 = 0}$$
$$\frac{v_1}{v_2} = h_{11}$$

$$\frac{h_{11}}{h_{21}} = \frac{-h_{22}}{h_{21}}$$

for reciprocity of h parameters

$$\boxed{h_{12} = -h_{21}}$$

For ~~symmetry~~ $\left.\frac{v_1}{I_1}\right|_{I_2=0} = \left.\frac{v_2}{I_2}\right|_{I_1=0}$

or $\left.\frac{I_2}{v_2}\right|_{v_1=0} = \left.\frac{I_1}{v_1}\right|_{v_2=0}$

$$\frac{-h_{12}h_{21} + h_{11}h_{22}}{\cancel{h_{22}}} = \frac{1}{\cancel{h_{22}}}$$

$$\boxed{h_{11}h_{22} - h_{12}h_{21} = 1}$$

$$\begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 1.$$

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Q. y parameter to h parameter conversion

$$I_1 = y_{11} v_1 + y_{12} v_2$$
$$I_2 = y_{21} v_1 + y_{22} v_2 \quad -①$$

$$v_1 = \frac{I_1 - y_{12} v_2}{y_{11}}$$

h parameters
$$v_1 = h_{12} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2 \quad -②$$

$$I_2 = y_{21}\left(\frac{I_1 - y_{12} v_2}{y_{11}}\right) + y_{22} v_2$$

compare
$$I_2 = \underbrace{\frac{y_{21} I_1}{y_{11}}}_{h_{21}} + \underbrace{\left(y_{22} - \frac{y_{21} \cdot y_{12}}{y_{11}}\right)}_{h_{22}} v_2$$

$$v_1 = \underbrace{I_1}_{h_{11}} y_{11} - \underbrace{\frac{y_{12}}{y_{11}}}_{h_{12}} v_2.$$

Q g parameter to ABCD parameters

$$I_1 = g_{11} v_1 + g_{12} I_2$$
$$V_2 = g_{21} v_1 + g_{22} I_2$$

$$v_1 = \frac{V_2 - g_{22} I_2}{g_{21}}$$

$$v_1 = A v_2 - B I_2$$
$$I_1 = C v_2 - D I_2.$$

$$I_1 = g_{11}\left(\frac{V_2 - g_{22} I_2}{g_{21}}\right) + g_{12} I_2$$

$T_1 = g''$

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$$I_1 = \frac{g_{11}}{g_{21}} V_2 - \frac{g_{11} g_{22}}{g_{21}} I_2 + g_{12} I_2$$

$$I_1 = \frac{g_{11}}{g_{21}} V_2 - \left(\frac{g_{11} g_{22}}{g_{21}} - g_{12} \right) I_2$$

$$\boxed{C = \frac{g_{11}}{g_{21}}} \qquad \boxed{D = \frac{g_{11} g_{22}}{g_{21}} - g_{12}}$$

$$V_1 = \frac{V_2}{g_{21}} - \frac{g_{22}}{g_{21}} I_2$$

$$\boxed{A = \frac{1}{g_{21}}} \qquad \boxed{B = \frac{g_{22}}{g_{21}}}$$

Q Z parameter to ABCD.

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

$$I_1 = \frac{V_2 - Z_{22} I_2}{Z_{21}} \qquad \boxed{C = \frac{1}{Z_{21}}} \quad \boxed{D = \frac{Z_{22}}{Z_{21}}}$$

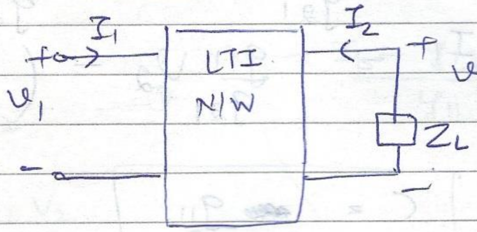
$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \left[\frac{Z_{22}}{Z_{21}} - Z_{12} \right] I_2$$

$$\therefore \boxed{A = \frac{Z_{11}}{Z_{21}}} \qquad \boxed{B = \frac{Z_{22}}{Z_{21}} - Z_{12}}$$

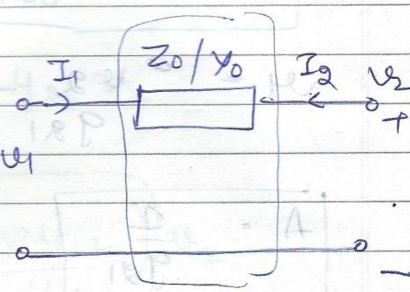
2-port N/W under LOAD.

Extra relationship we get in this case is

$$\boxed{v_2 = -I_2 Z_L}$$



Q. calculate z, y, h, t parameters



$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

$$Z_{11} = \infty$$

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} = -Z_0$$

$$\frac{V_2}{I_1}\Big|_{I_2=0} = Z_{21} = -Z_0 \qquad Z_{22} = \infty$$

$$y_{11} = y_0$$

$$y_{12} = -y_0$$

$$y_{21} = -y_0$$

$$y_{22} = y_0$$

$$h_{11} = y_{11} = y_0$$

$$h_{12} = -\frac{y_{12}}{y_{11}} = -\frac{(-y_0)}{y_0} = 1$$

$$h_{21} = \frac{y_{21}}{y_{11}} = \frac{-y_0}{y_0} = -1$$

$$T = \begin{bmatrix} 1 & Z_0 \\ 0 & 1 \end{bmatrix}$$

$$h_{22} = y_{22} - \frac{y_{21}y_{12}}{y_{11}}$$

$$= y_0 - \frac{(-y_0)(-y_0)}{y_0} = 0$$

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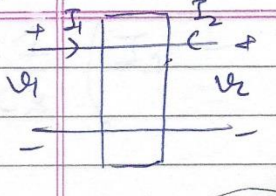
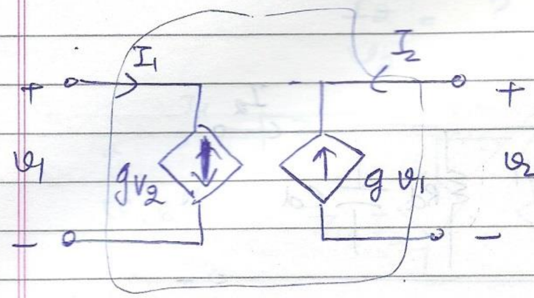
Q



$$h = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Q



Define Y-parameter

$$I_2 = -g v_1$$
$$I_1 = g v_2$$

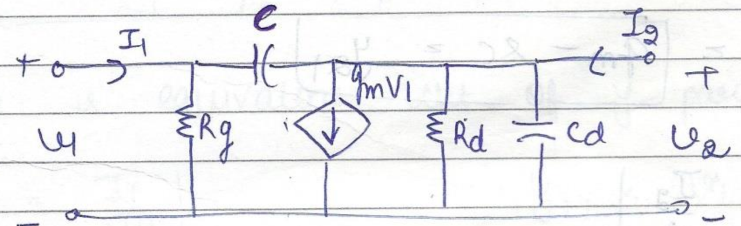
$$y_{11} = \frac{I_1}{v_1}\bigg|_{v_2=0} = \frac{g v_2}{v_1}\bigg|_{v_2=0} = 0$$

$$y_{12} = \frac{I_1}{v_2}\bigg|_{v_1=0} = \frac{g \cancel{v_2}}{\cancel{v_2}}\bigg|_{v_1=0} = g$$

$$y_{21} = \frac{I_2}{v_1}\bigg|_{v_2=0} = -\frac{g \cancel{v_1}}{\cancel{v_1}}\bigg|_{v_2=0} = -g$$

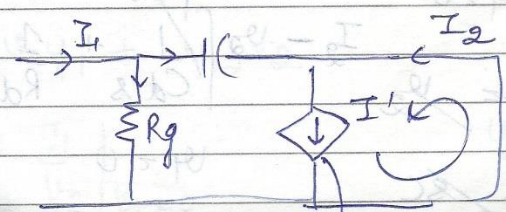
$$y_{22} = \frac{I_2}{V_2}\bigg|_{v_1=0} = -\frac{g v_1}{v_2}\bigg|_{v_1=0} = 0$$

Q



Define Y parameter

Sol



This is also redundant as it circulates in self loop.

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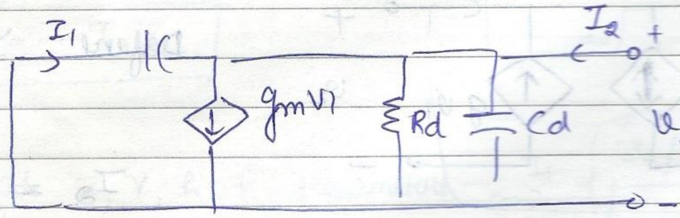
$$I_1 = v_1 \left[\frac{1}{Rg} + \frac{1}{1/sC} \right]$$

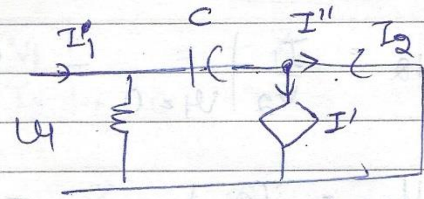
$$y_{12} = \frac{I_1}{V_2}\Bigg|_{v_1 = 0}$$



$$-I_1 \frac{1}{sC} = v_2$$

$$\frac{I_1}{v_2} = -sC$$

$$y_{21} = \frac{I_2}{v_1}\Bigg|_{v_2 = 0}$$



$$-I'' + I' = I_2$$

$$v_1 = \frac{1}{sC} I''$$

$$I'' = sC v_1$$

$$I_2 = g_m v_1 - sC v_1$$

$$I_2 = (g_m - sC) v_1$$

$$\frac{I_2}{v_1} = \boxed{g_m - sC = y_{21}}$$

$$y_{22} = \frac{I_2}{V_2}\Bigg|_{v_1 = 0}$$

$$I_2 - v_2 \left[\frac{1}{\frac{1}{Cds}} + \frac{1}{Rd} \right] + g_m v_1 \} = I_1$$

$$v_1 = 0$$

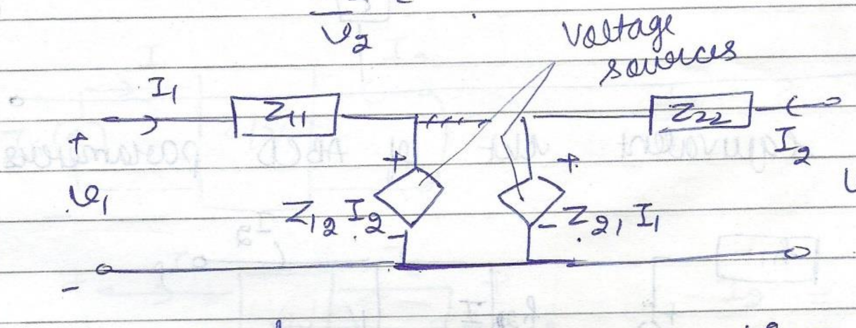
$$\cancel{I_2 \frac{1}{sC} = v_2}$$

$$\cancel{\frac{I_2}{v_2} = sC}$$

$$\frac{1}{Cs}\left(I_2 - v_2 C d s - \frac{v_2}{R d}\right) = v_2$$

$$\frac{I_2}{v_2} = ?$$

Q.



find Z parameters

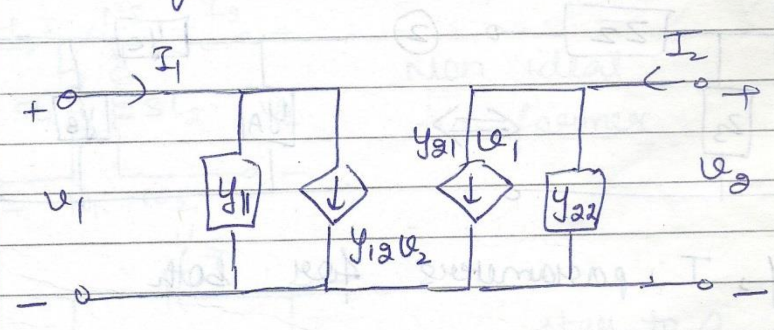
$$Z_{11} = \frac{v_1}{I_1}\Big|_{I_2 = 0}$$

$$v_1 = Z_{11} I_1 + Z_{12} I_2 \qquad Z_{11} = \frac{Z_{11} \cancel{I_1}}{\cancel{I_1}}$$

$$Z_{11} = Z_{11}$$

This is equivalent ckt of Z- parameters.

Q

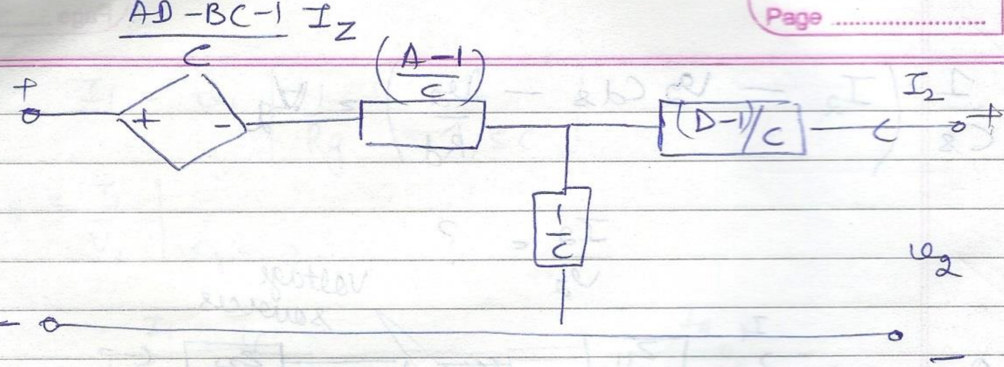


This is equivalent ckt of y- parameters.

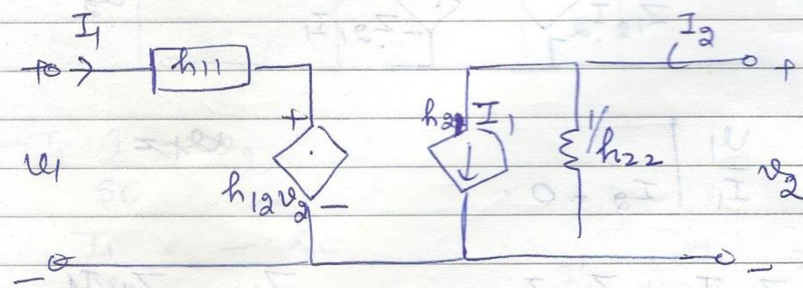
$$y_{11} = \frac{I_1}{v_1}\Big|_{v_2 = 0} \qquad y_{11} = \frac{\cancel{v_1} y_{11}}{\cancel{v_1}}$$

$$I_1 = v_1 y_1 + y_{12} v_2 \qquad y_{11} = y_{11}$$

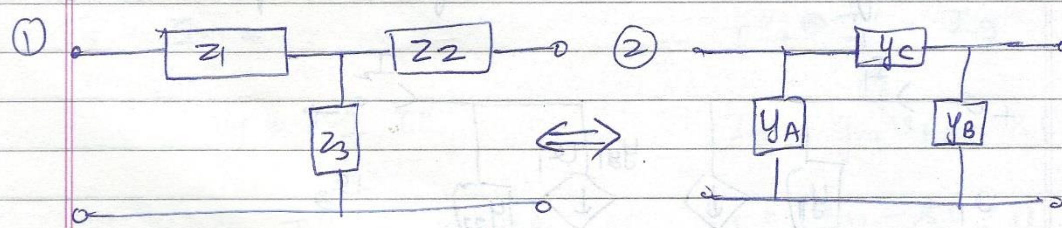
$$y_{12} = \frac{I_1}{v_2}\Big|_{v_1 = 0} \qquad y_{12} = \frac{y_{12} \cancel{v_2}}{\cancel{v_2}} = y_{12}$$



equivalent ckt of ABCD parameters



* Problems related to T and π n/w



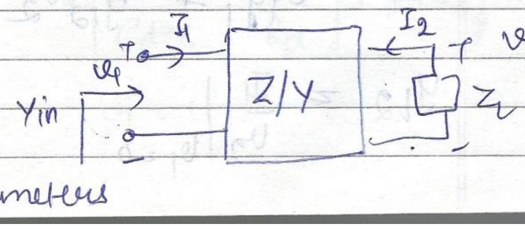
Q1 Find Z, Y, T parameters for both

After calculation, equate the parameters of both T & π.

Q2 Also check cond. of reciprocity for both

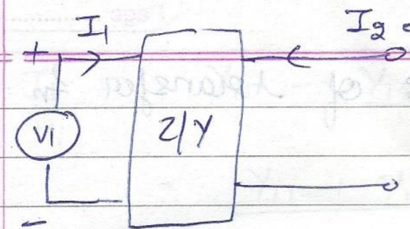
Q3 Calculate Y_{in}

i.e $\left.\dfrac{I_1}{V_1}\right|_{Z_L}$



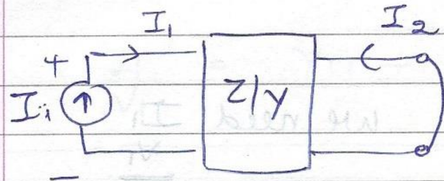
in terms of Y or Z parameters

Q4



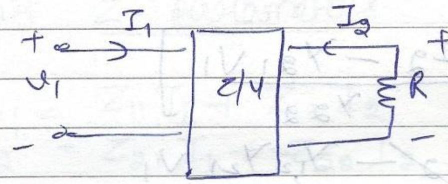
find $\frac{v_2}{v_1}$

Q5



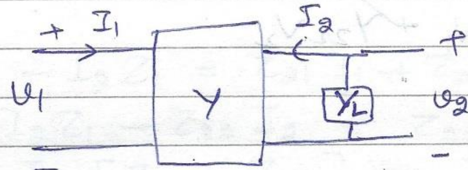
$\frac{I_2}{I_1} = ?$

Q6



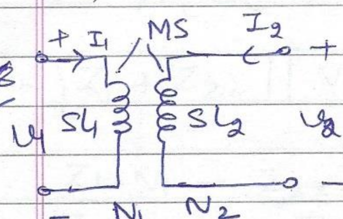
$\frac{I_2}{v_1} = ?$

Q7



$\frac{v_2}{v_1} = ?$, $\frac{I_2}{v_1} = ?$

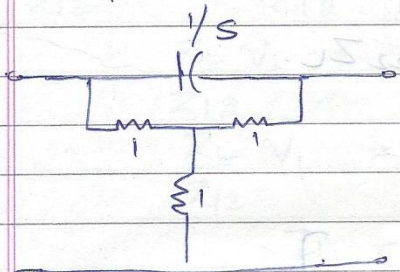
Q8



Non ideal transformer

Define Z parameters

Imp



star to Δ

Δ to star

find z & y parameters

If $Z_{11} = \frac{3s+2}{2s+1}$ If excitation is $5e^{-2/3 t}$

for steady state

Now here ν of excitation = $-2/3$

Zeroes of transfer fn = $-2/3$

excitation v = zeroes of transfer fn.

$\therefore$ response = 0.

Sol3 $v_2 = -I_2 Z_L$.

As Y parameters known

$I_1 = Y_{11}V_1 + Y_{12}V_2$ — we need $\underline{\dfrac{I_1}{V_1}}$

$I_2 = Y_{21}V_1 + Y_{22}V_2$.

$$\cancel{I_1 = Y_{11}V_1 + Y_{12}\left[\frac{I_2 - Y_{21}V_1}{Y_{22}}\right]}$$

$$\cancel{I_1 = Y_{11}V_1 + \frac{Y_{12}}{Y_{22}}I_2 - \frac{Y_{12}Y_{21}V_1}{Y_{22}}}$$

$$\cancel{I_1 = Y_{11}V_1 + \frac{Y_{12}}{Y_{22}}(Y_{21}V_1 + Y_{22}V_2)}$$

$$I_2 = Y_{21}V_1 - Y_{22}I_2 Z_L$$

$$I_2 + Y_{22}I_2 Z_L = Y_{21}V_1$$

$$I_2[1 + Y_{22}Z_L] = Y_{21}V_1$$

$$I_2 = \frac{Y_{21}V_1}{1 + Y_{22}Z_L}.$$

$$I_1 = Y_{11}V_1 + Y_{12}(-I_2 Z_L)$$

$$I_1 = Y_{11}V_1 - Y_{12}Z_L I_2$$

$$I_1 = Y_{11}V_1 - Y_{12}Z_L\left[\frac{Y_{21}V_1}{1 + Y_{22}Z_L}\right]$$

$$I_1 = Y_{11}V_1 - \frac{Y_{12}Y_{21}Z_L}{1 + Y_{22}Z_L}V_1$$

$$I_1 = V_1\left[Y_{11} - \frac{Y_{12}Y_{21}Z_L}{1 + Y_{22}Z_L}\right]$$

$$\frac{I_1}{V_1} = Y_{11} - \frac{Y_{12} Y_{21} Z_L}{1 + Y_{22} Z_L}$$

$$= \frac{Y_{11} + Y_{11} Y_{22} Z_L - Y_{12} Y_{21} Z_L}{1 + Y_{22} Z_L}$$

$$\boxed{\underline{\underline{Y_{in}}} = \frac{I_1}{V_1} = \frac{Y_{11} + Z_L (Y_{11} Y_{22} - Y_{12} Y_{21})}{1 + Y_{22} Z_L}}$$

for $\underline{Z\text{-parameters}}$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$V_2 = -I_2 Z_L$$

$$-I_2 Z_L = Z_{21} I_1 + Z_{22} I_2$$

$$-I_2 Z_L - Z_{22} I_2 = Z_{21} I_1$$

$$-I_2 [Z_L + Z_{22}] = Z_{21} I_1$$

$$+ [Z_L + Z_{22}] \left[\frac{V_1 - Z_{11} I_1}{Z_{12}}\right] = -Z_{21} I_1$$

$$\frac{Z_L}{Z_{12}} V_1 - \frac{Z_{22} Z_{11}}{Z_{12}} I_1 = -Z_{21} I_1$$

$$\frac{Z_L V_1}{Z_{12}} = -Z_{21} I_1 + \frac{Z_{22} Z_{11}}{Z_{12}} I_1$$

$$\frac{Z_L}{Z_{12}} V_1 = I_1 \left[\frac{Z_{22} Z_{11}}{Z_{12}} + Z_{21}\right]$$

$$\frac{I_1}{V_1} = \frac{\frac{Z_L}{Z_{12}}}{\frac{Z_{22} Z_{11}}{Z_{12}} + Z_{21}} = \frac{Z_L}{Z_{22} Z_{11} + Z_{21} Z}$$

4.

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

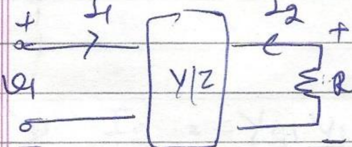
$$I_2 = 0$$

$$Y_{21}V_1 = -Y_{22}V_2$$
$$\boxed{\frac{V_2}{V_1} = -\frac{Y_{21}}{Y_{22}}}$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \qquad I_2 = 0$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$V_1 = Z_{11}I_1 \qquad V_2 = Z_{21}I_1$$
$$\boxed{\frac{V_2}{V_1} = \frac{Z_{21}}{Z_{11}}}$$

6.



$$v_2 = -RI_2$$
$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$\frac{I_2}{V_1} =$

$$I_2 = Y_{21}V_1 + Y_{22}(-RI_2)$$
$$I_2\left[1 + Y_{22}R\right] = Y_{21}V_1$$

$$\frac{I_2}{V_1} = \frac{Y_{21}}{1 + Y_{22}R}$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

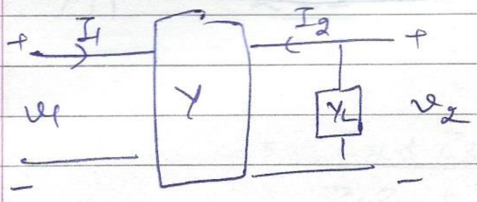
$$V_1 = Z_{11}\left[\frac{V_2 - Z_{22}I_2}{Z_{21}}\right] + Z_{12}I_2$$

$$V_1 = -\frac{Z_{11}}{Z_{21}}\left[R + Z_{22}\right]I_2 + Z_{12}I_2$$

$$V_1 = I_2\left[Z_{12} - \frac{Z_{11}}{Z_{21}}\left(R+Z_{22}\right)\right].$$

$$\boxed{\frac{I_2}{V_1} = \frac{Z_{21}}{Z_{12}Z_{21} - Z_{11}R - Z_{11}Z_{22}}}$$

7.



$$I_2 = -Y_L V_2.$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2.$$

$\frac{v_2}{v_1} \Big| \frac{I_2}{V_1}$

$$\cancel{I_1} = \quad -Y_L V_2 = Y_{21}V_1 + Y_{22}V_2$$
$$-\left[Y_L + Y_{22}\right]V_2 = Y_{21}V_1$$
$$\boxed{\frac{V_2}{v_1} = \frac{-Y_{21}}{(Y_L + Y_{22})}}$$

$$I_2 = Y_{21}V_1 + Y_{22}\,(\cancel{-Y_L V_2})\left[-\frac{I_2}{Y_L}\right]$$

$$I_2 + \frac{I_2}{Y_L}Y_{22} = Y_{21}V_1$$

$$I_2\left[1 + \frac{Y_{22}}{Y_L}\right] = Y_{21}V_1$$

$$\frac{I_2}{V_1} = \frac{Y_{21}}{1 + \frac{Y_{22}}{Y_L}}$$

$$\boxed{\frac{I_2}{V_1} = \frac{Y_{21}Y_L}{1 + Y_{22}}}$$

8.

$$v_1 = sL_1I_1 + sMI_2$$
$$v_2 = sMI_1 + sL_2I_2$$

$$Z_{11} = \frac{v_1}{I_1}\Bigg|_{I_2=0} = sL_1$$

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$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$= Ms$$

$$\boxed{\begin{aligned}\frac{v_1}{v_2} &= \frac{N_1}{N_2} \\ \frac{I_1}{I_2} &= \frac{N_2}{N_1}\end{aligned}}$$

$$Z_{21} = Ms$$

$$Z_{22} = sL_2$$

5.

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$V_2 = 0 \qquad I_1 = Y_{11}V_1$$
$$I_2 = Y_{21}V_1$$

$$\boxed{\frac{I_2}{I_1} = \frac{Y_{21}}{Y_{11}}}$$

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$Z_{21}I_1 = -Z_{22}I_2$$

$$\boxed{\frac{I_2}{I_1} = -\frac{Z_{21}}{Z_{22}}}$$

1.

$$Z_{11} = Z_1 + Z_3 \qquad Oz = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$
$$Z_{22} = Z_2 + Z_3$$
$$Z_{12} = Z_3$$
$$Z_{21} = Z_3 \qquad \begin{bmatrix} Z_1+Z_3 & Z_3 \\ Z_3 & Z_2+Z_3 \end{bmatrix}$$

II
$$\left.\begin{aligned} Y_{12} &= Y_{21} = -Y_3 \\ Y_{11} &= Y_1 + Y_3 \\ Y_{22} &= Y_2 + Y_3 \end{aligned}\right\}$$

I
$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{Z_2 + Z_3}{(Z_1+Z_3)(Z_2+Z_3) - Z_3(Z_3)}$$

$$= \frac{Z_2 + Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_3 Z_2 - \cancel{Z_3^2} + \cancel{Z_3^2}}$$

$$Y_{12} = -\frac{Z_{11}}{\Delta Z} = -\frac{Z_1 + Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_3 Z_2 - Z_3^2}$$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z} = -\frac{Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_3 Z_2 - Z_3^2}$$

$$Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{Z_1 + Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_3 Z_2 - Z_3^2}$$

π n/w
$$Z_{11} = \frac{Y_{22}}{\Delta Y} \begin{bmatrix} Y_1 + Y_3 & -Y_3 \\ -Y_3 & Y_2 + Y_3 \end{bmatrix}$$

$$\Delta Y = (Y_1 + Y_3)(Y_2 + Y_3) - Y_3^2$$
$$= Y_1 Y_2 + Y_1 Y_3 + \cancel{Y_3^2} - \cancel{Y_3^2} + Y_3 Y_2$$

$$Z_{11} = \frac{Y_1 + Y_3}{Y_1 Y_2 + Y_1 Y_3 + Y_3 Y_2}$$

$$Z_{12} = -\frac{Y_{12}}{\Delta Y} = \frac{Y_3}{Y_1 Y_2 + Y_1 Y_3 + Y_3 Y_2}$$

$$Z_{21} = \frac{-Y_{21}}{\Delta Y} = \frac{Y_3}{\Delta Y}$$

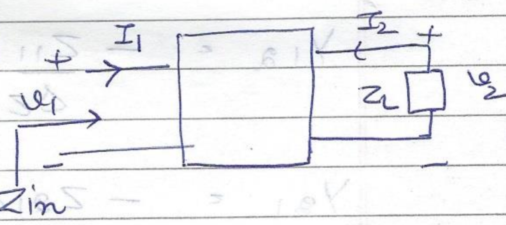
$$Z_{22} = \frac{Y_{11}}{\Delta Y} = \frac{Y_1 + Y_3}{\Delta Y}$$

Now

$$Z_1 + Z_3 = \frac{Y_1 + Y_3}{Y_1Y_2 + Y_1Y_3 + Y_2Y_3}$$

similarly equate other

Q $Z_{in} = \frac{V_1}{I_1} = -Z_L$ (given)



find h parameters.

Sol $V_2 = -I_2 Z_L$.

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

$$\boxed{\frac{V_1}{I_1} = \frac{V_2}{I_2} = k = -Z_L}$$

$V_1 = k I_1$

$V_2 = k I_2$

$\cancel{h_{12}} = \cancel{\frac{V_1}{I_1}}$

$kI_1 = h_{11} I_1 + h_{12} k I_2$

$I_2 = h_{21} I_1 + h_{22} k I_2$

Ans

$$\begin{bmatrix} 0 & k \\ 1/k & 0 \end{bmatrix}$$

$(k - h_{11}) I_1 = h_{12} k I_2$ — (1)

$(1 - h_{22} k) I_2 = h_{21} I_1$.

$\frac{V_1}{V_2} = h_{21}$

$= \frac{I_1}{I_2}$

$(k - \cancel{h_{11}}) I_1 = \cancel{h_{12} k I_2}$

$\cancel{h_{21} I_1 = (\text{k-h}_{11})(1 - h_{22} k) I_2}$

$\cancel{\frac{I_1}{I_2} = \frac{(1-h_{22})k}{h_{21}}}$

from (1) ✓ find I_1 / I_2

then $\frac{I_1}{I_2} = \frac{V_1}{V_2}$.

Date 15/6/2013
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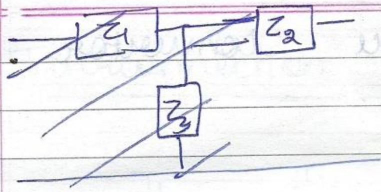
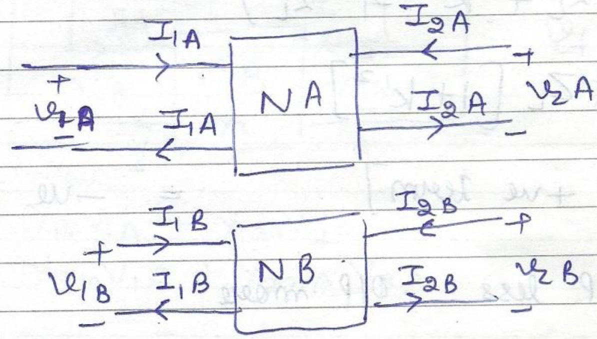


Reciprocity

INTERCONNECTION OF TWO PORT N/W

Parallel Inter Connection.



Voltage at port 1 of n/w A = Voltage at port 1 of n/w B.

$\rightarrow \frac{V_1}{I_1} = \frac{V_2}{I_2} = k = -Z_L$

$\boxed{V_1 = k' V_2 \qquad I_1 = k' I_2}$

$V_2 = k I_2$

Now $\frac{V_1}{I_1} = k = h_{11}$

$h_{22} = 1/k$

$V_1 = h_{11} I_1 + h_{12} V_2$ $\qquad V_1 = k I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$ $\qquad I_2 = h_{21} I_1 + \frac{1}{k} V_2$

$V_1 = V_2 + h_{12} V_2$

$I_2 = h_{21} I_1 + \frac{1}{k} V_2$

Page ....

This is Negative Impedance converter where $Z_{in} = -Z_L$.

ower $= v_1 I_1 + v_2 I_2$

$-\left(I_1^2 Z_L + I_2^2 Z_L\right)$

$-\left(I_1^2 Z_L + k'^2 I_1^2 Z_L\right)$

$-I_1^2 Z_L \left[1 + k'^2\right]$

$-\left[\text{+ve term}\right] = \text{-ve power}$

means I/P less, O/P more

means this is active n/w like transistors, diodes etc are used inside the n/w.

for active n/w

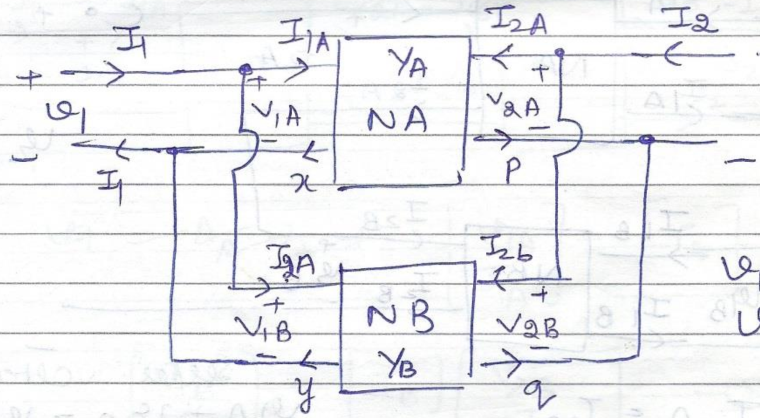
$$P = V_1 I_1 + V_2 I_2 + \text{---} \; V_n I_n < 0$$

or passive

$$P = V_1 I_1 + V_2 I_2 + \text{---} \; V_n I_n \geq 0 \; [\text{RLC n/w}].$$

Interconnection of 2 port N/W

Parallel



$$v_1 = v_{1A} = v_{1B}$$
$$v_2 = v_{2A} = v_{2B}$$

$$I_{1A} = Y_{11A}V_{1A} + Y_{12A}V_{2A}$$
$$I_{2A} = Y_{21A}V_{1A} + Y_{22A}V_{2A}$$

$$I_{1A} = Y_{11A}v_1 + Y_{12A}v_2$$
$$I_{2A} = Y_{21A}v_1 + Y_{22A}v_2$$
$$I_{1B} = Y_{11B}v_1 + Y_{12B}v_2$$
$$I_{2B} = Y_{21B}v_1 + Y_{22B}v_2$$

$$I_1 = y_{11}v_1 + y_{12}v_2$$
$$I_2 = y_{21}v_1 + y_{22}v_2$$

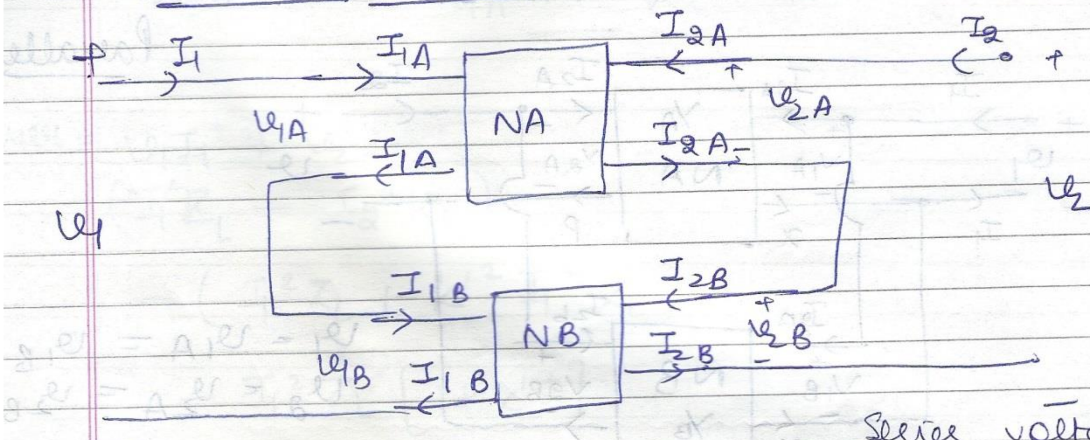
$$I_1 = I_{1A} + I_{1B}$$
$$I_2 = I_{2A} + I_{2B}$$

$$I_1 = (Y_{11A} + Y_{11B})v_1 + (Y_{12A} + Y_{12B})v_2$$
$$I_2 = (Y_{21A} + Y_{21B})v_1 + (Y_{22A} + Y_{22B})v_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Y]\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\boxed{[Y] = [y_A] + [y_B]}$$

Series Connection :



Series voltage add.

$v_{1A} + v_{1B} = v_1$

$v_{2A} + v_{2B} = v_2$

$I_1 = I_{1A} = I_{1B}$

$I_2 = I_{2A} = I_{2B}$

$$\begin{bmatrix} v_{1A} \\ v_{2A} \end{bmatrix} = \begin{bmatrix} Z_A \end{bmatrix} \begin{bmatrix} I_{1A} \\ I_{2A} \end{bmatrix}$$

$\Downarrow$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$[V] = [V_a] + [V_b]$

$[Z]\cancel{[I]} = [Z_a][I_a] + [Z_b][\cancel{I_b}]$

all equal

$\boxed{\text{so } [Z] = [Z_A] + [Z_b]}$

Combined of whole ckt

$$\begin{bmatrix} v_{1B} \\ v_{2B} \end{bmatrix} = \begin{bmatrix} Z_B \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

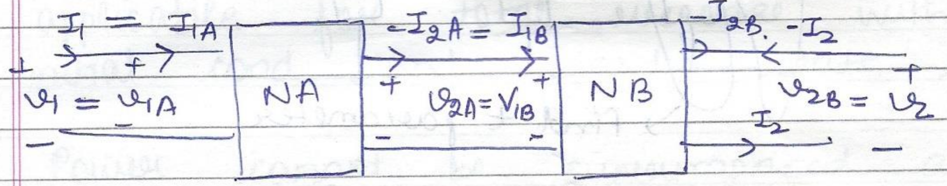
$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \left(\begin{bmatrix} v_{1A} \\ v_{2A} \end{bmatrix} + \begin{bmatrix} v_{1B} \\ v_{2B} \end{bmatrix} \right) \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$= \begin{bmatrix} Z_A + Z_B \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\therefore \boxed{Z = [Z_A] + [Z_B]}$$

Cascade connection

$-I_2 = -I_{2B}$



$\cancel{v_1 = A v_2 +}$ $\begin{bmatrix} v_{1A} \\ I_{1A} \end{bmatrix} = \begin{bmatrix} T_A \end{bmatrix} \begin{bmatrix} V_{2A} \\ -I_{2A} \end{bmatrix}$ — (1)

$$\begin{bmatrix} V_{1B} \\ I_{1B} \end{bmatrix} = \begin{bmatrix} T_B \end{bmatrix} \begin{bmatrix} V_{2B} \\ -I_{2B} \end{bmatrix} \quad — (2)$$

$$\begin{bmatrix} v_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} T \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad — (3)$$

(1) $= \begin{bmatrix} v_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} T_A \end{bmatrix} \begin{bmatrix} V_{2A} \\ -I_{2A} \end{bmatrix}$

$$\Downarrow$$

$$\begin{bmatrix} v_{1B} \\ I_{1B} \end{bmatrix}$$

From (2)

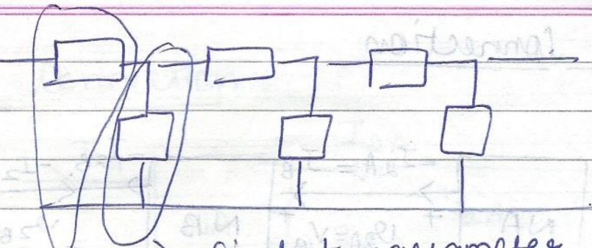
$$\begin{bmatrix} v_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} T_A \end{bmatrix} \begin{bmatrix} T_B \end{bmatrix} \begin{bmatrix} v_{2B} \\ -I_{2B} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} T_A \end{bmatrix} \begin{bmatrix} T_B \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} T \end{bmatrix} = \begin{bmatrix} T_A \end{bmatrix} \begin{bmatrix} T_B \end{bmatrix}}$$

find T parameter individually.

eg



→ find t parameter

Now multiplication of matrices repetitively

NETWORK THEOREMS.

* In AC circuits or sinosoidal replace $s = j\omega$.

Superposition Theorem.

In a linear n/w acted upon by several independent sources, response in any element of the n/w is the sum of responses obtained with one source acting at a time and the other sources being deactivated.

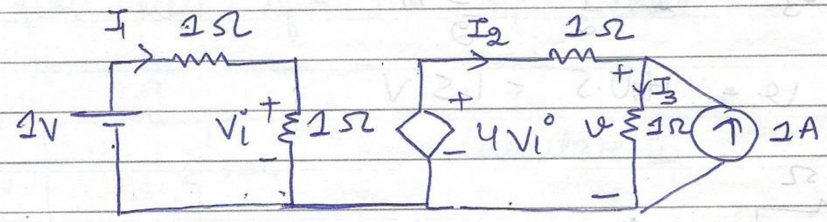
* If dependant sources are present in the N/w then the dependant source will be untouched while calculating the response to one of the independent source.

* Deactivation of source means short circuited cond. for voltage source and open " " " current "

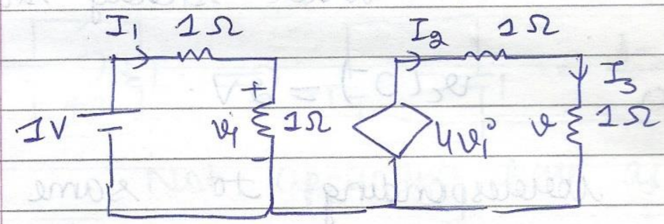
* During transient analysis or while calcula[ting] total response, superposition theorem is applicatble for total response with zero initial cond and steady state response

* Power cannot be superimposed as it is non linearly related to $\underline{\underline{V \& I}}$.

Q.



Calculate v, I_1, I_2, I_3



$1 = I_1 + v_i \; 1 \, I_1$

$2I_1 = 1$

$I_1 = 1/2$;

$v_i = \frac{1}{2} \times 1$

$= \frac{1}{2}$ V

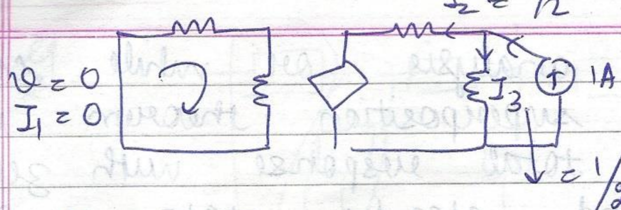
$4v_i = 2V$

$2V = I_2(1+1)$

$\boxed{I_2 = 1A = I_3}$

$v = 1 \times 1$

$\boxed{v = 1V}$.



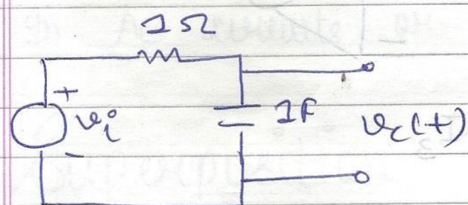
$\therefore \; I_1 = \frac{1}{2} + 0 = \frac{1}{2} A$

$I_2 = 1 - 1/2 = \frac{1}{2} A$

$I_3 = 1 + \frac{1}{2} = \frac{3}{2} A$

$v = 1 + 0.5 = 1.5 \, V$

Q

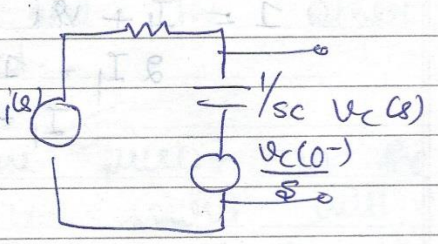


1) Find voltage across capacitor under steady state

$v_i = 4 + \cos t$ $\qquad v_c(0^-) = 2V$.

2) Total response corresponding to same excitation.

soln (i) find $\dfrac{V_C(s)}{V_i(s)}$



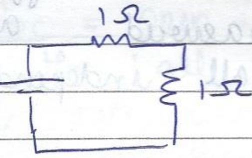
(ii) for total find response corresponding 4, cos t and $V_C(0^-)$ individually then add

$\frac{v_c(0^-)}{s}$

2) Substitution Theorem

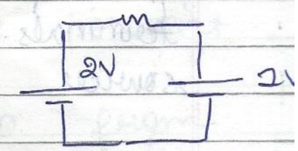
If an element (linear or non linear) in a n/w is replaced by a voltage source whose voltage at any instant of time equal the voltage existing across the element in the original element then the condit for rest of the n/w are unaltered.

Q

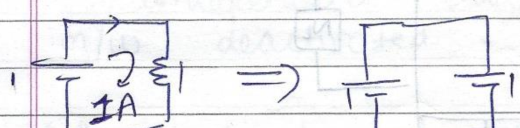


Calculate I

I = 1A



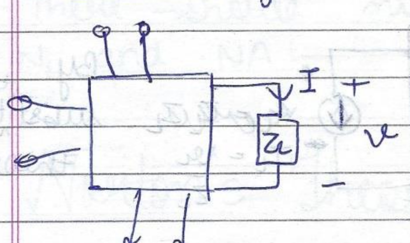
Now I = 1A
same

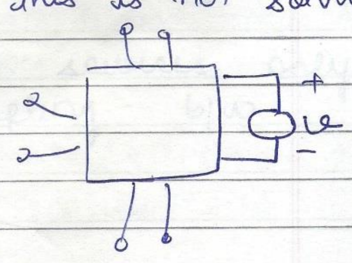


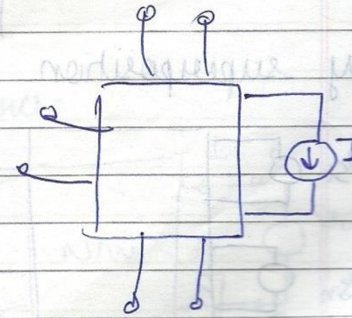
$= \frac{1-1}{0}$ = no defined curren which was actually 1

Not applicable here as this is not solvabl

Alternatively





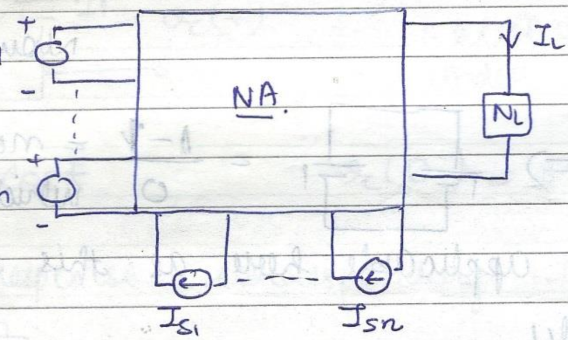


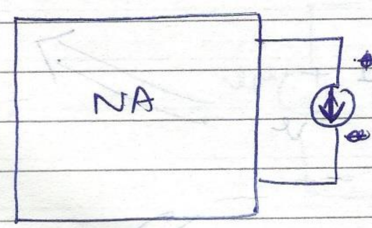
both substitutions are possible

Thevenin's Theorem

A 2 terminal N/W NA containing linear elements & independent source is equivalent to a simpler n/w containing an independent voltage source in series with a 2-terminal n/w having impedance fn $Z_0(s)$. ~~in series~~

The source voltage is open circuit voltage of NA. The impedance $Z_0(s)$ is effective o/p impedance of N/W A measured at its terminals after deactivating all independent sources in NA.

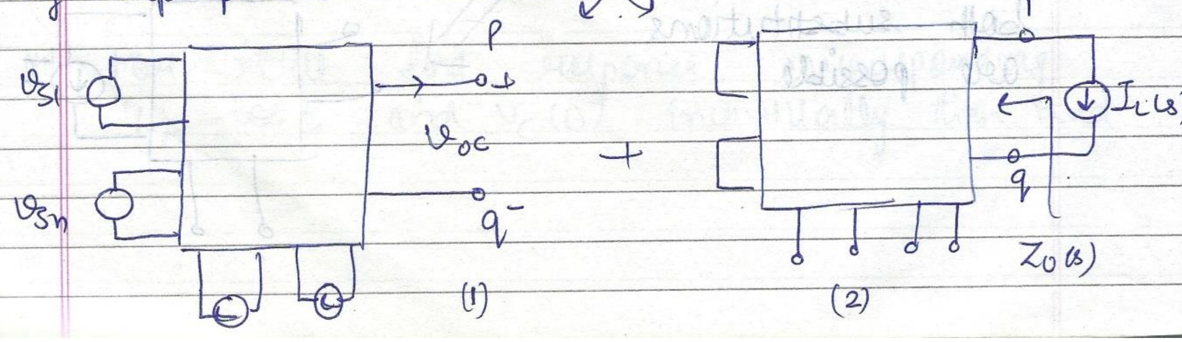




~~v_L~~ $I_L = \frac{v_L}{Z_L}$

By the substitution theorem

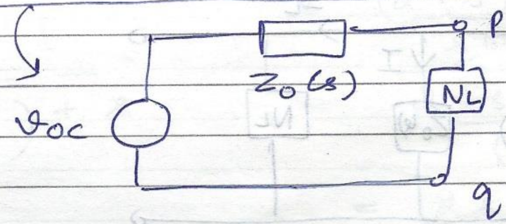
By superposition theorem



In (2)

$$V_{pq} = -I_L(s)\, Z_0(s)$$

$$v_{pq} = \boxed{v_L = v_{oc} - I_L(s)\, Z_0(s)}$$



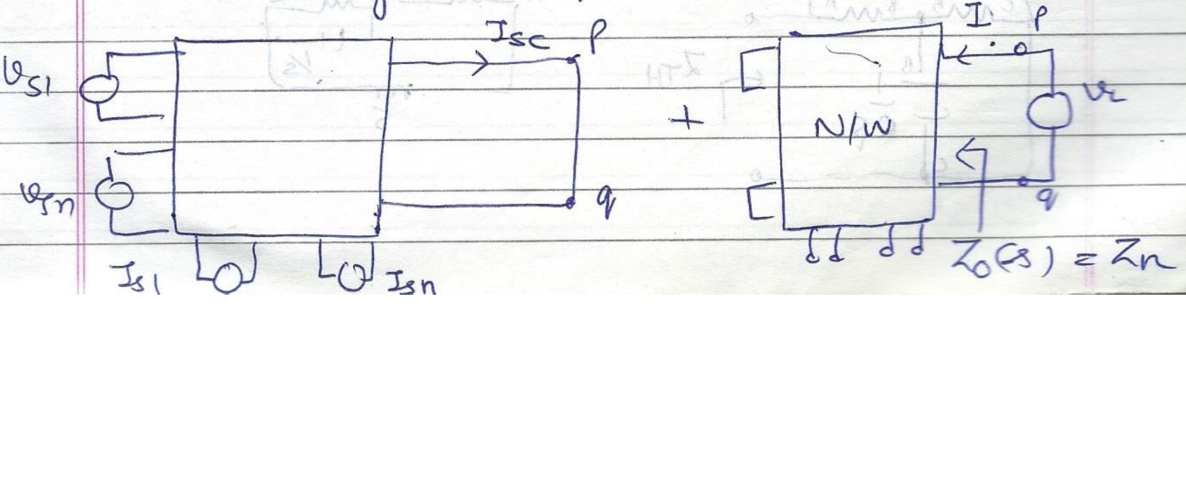
$\underline{v_{TH}} = \underline{v_{oc}}$ = Open ckt voltage b/w terminal p & q with all independent sources present in NA activated

$Z_{TH} = \underline{Z_0(s)}$ = Impedance of NA seen from terminal p & q with all independent sources present in that n/w deactivated.

1) NA must be linear but N_L may be linear or non linear.
2) Deactivation of independent sources only.
3) There should not be coupling b/w N_L and NA.

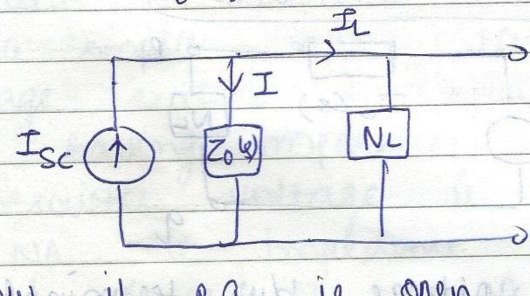
Norton's Theorem

Substitution by voltage source



$$\boxed{I_L = I_{SC} - I}$$

$$I = \frac{v_L}{Z_0(s)}$$



Now if pq is open circuit $I_L = 0$

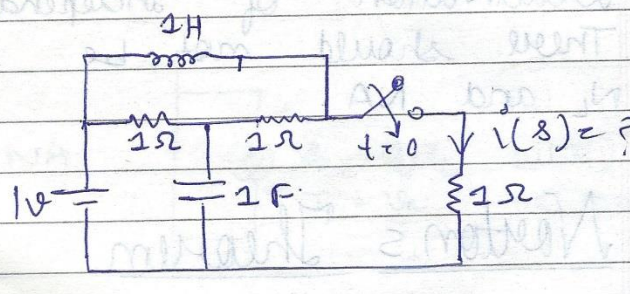
$$I_{SC} = I$$

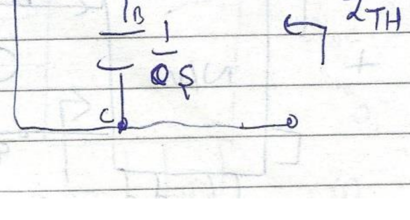
Norton's current $\quad I_{SC} = \frac{v_{OC}}{Z_0}$

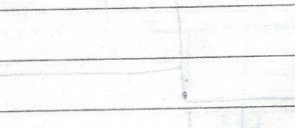
$$v_L = \left[\frac{V_{OC}}{Z_0}\right] \frac{Z_0(s)}{Z_0(s) + Z_L}$$

* All conditions of thevenin also applicable here in NORTON

Q find $i(s)$ by Thevenins





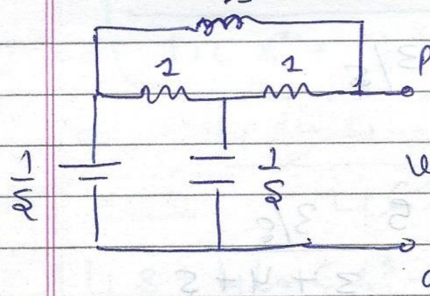


$\dfrac{1 \times \frac{1}{s}}{1+\frac{1}{s}}$ $\dfrac{\frac{1}{\cancel{s}}}{\frac{s+1}{\cancel{s}}}$ $\dfrac{1}{s+1} + 1$ $\dfrac{1+s+1}{s+1} = \dfrac{2+s}{s+1}$

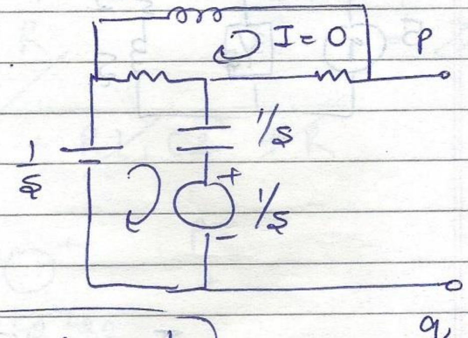
$$\frac{\left(\frac{2+s}{s+1}\right)(s)}{\left(\frac{2+s}{s+1}\right)+s} = \frac{\frac{(2+s)s}{\cancel{s+1}}}{\frac{2+s+s^2+s}{\cancel{(s+1)}}}$$

$$= \frac{2s+s^2}{s^2+2s+2}$$

$$= \frac{s[2+s]}{s^2+2s+2}$$

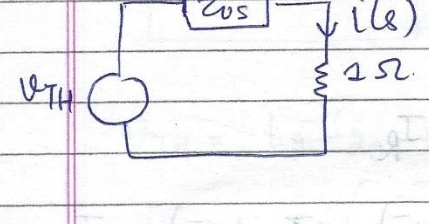


$v_{pq} = ?$



$$I = \frac{\frac{1}{s}}{\frac{s[2+s]}{s^2+2s+2}}$$

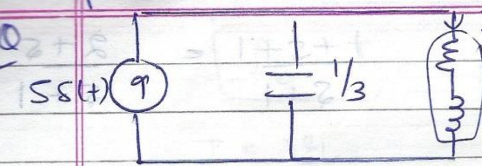
$$v_{TH} = v_{pq} = \frac{1}{s}$$



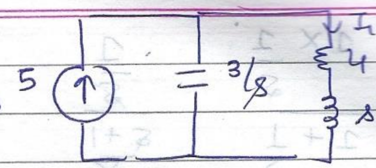
$$i(s) = \frac{v_{TH}}{Z_0(s)+1}$$

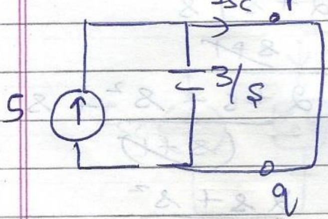
Norton

Q

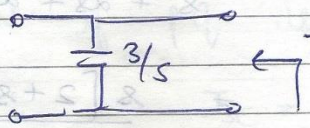


$\Rightarrow$

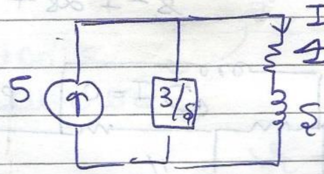




$I_{sc} = 5$



$= 3/s$



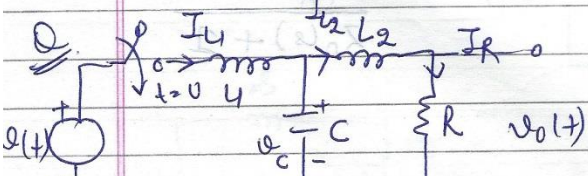
$$I_{L(s)} = \frac{5 \cdot \frac{3}{s}}{\frac{3}{s} + 4 + s}$$

$$= \frac{\frac{15}{\cancel{s}}}{\frac{3 + 4s + s^2}{\cancel{s}}}$$

$$I_{L(s)} = \frac{15}{3 + 4s + s^2}$$

Now calculate $I_L(t)$

Q



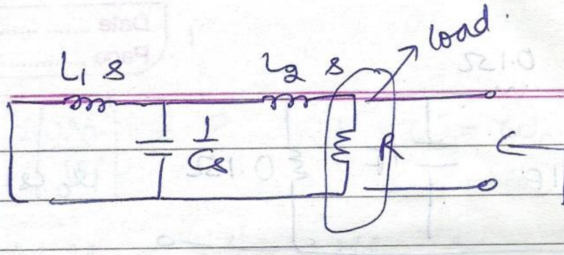
$I_{L2} = I_R$

$I_{L1}(0^-)$ ~~$I_{L2}(0^-)$~~ $= I_{L10}$

$I_{L2}(0^-)$

v_{C0} $\neq 0$

Calculate $v_0(t)$.

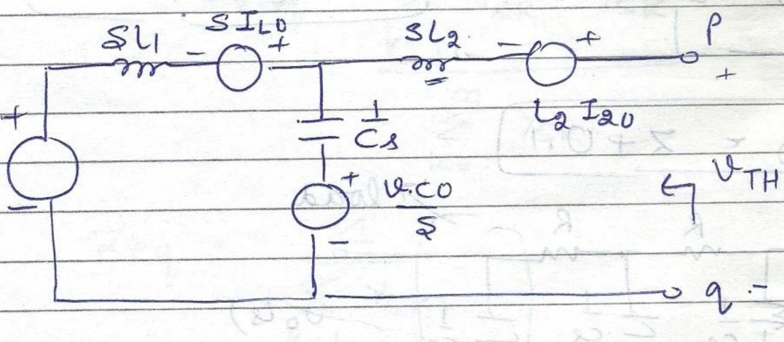


$$\frac{L_1 s \cdot \frac{1}{Cs}}{L_1 s + \frac{1}{Cs}} = \frac{\frac{L_1}{\cancel{C}}}{\frac{L_1 C s^2 + 1}{\cancel{C} s}} = \frac{L_1 s}{L_1 C s^2 + 1} + L_2 s$$

$$\boxed{\frac{L_1 s + L_1 L_2 C s^3 + L_2 s}{L_1 C s^2 + 1}} \left(\cancel{R}\right)$$

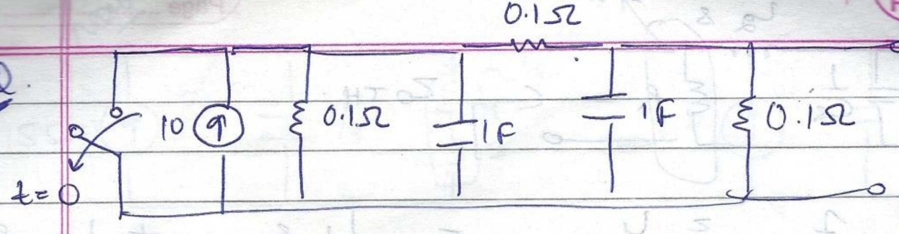
$$\cancel{\frac{L_1 s + L_1 L_2 C s^3 + L_2 s}{L_1 C s^2 + 1}} \cancel{+R}$$

$$\frac{(L_1 s + L_1 L_2 C s^3 + L_2 s)\, R}{L_1 s + L_1 L_2 C s^3 + L_2 s + R L_1 C s^2 + R} = Z_{0TH}$$

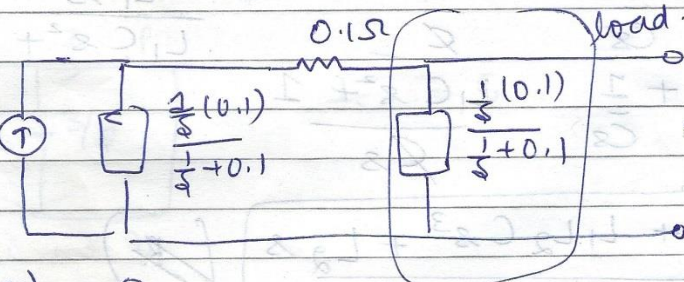


$$V_{TH} = L_2 I_{20} + \frac{V_{C0}}{s} + \underbrace{\left(\frac{V_s + L_1 I_0 - \frac{V_{C0}}{s}}{sL_1 + \frac{1}{Cs}}\right) \cdot \frac{1}{sC}}_{\text{voltage across capacit}}$$

Q.



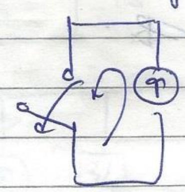
$v_o(s) = ?$

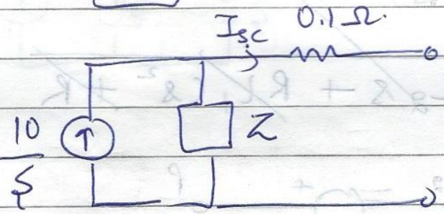


$v_o(s) = ?$

$v_C(0^-) = 0$

as initially current will be flowing in

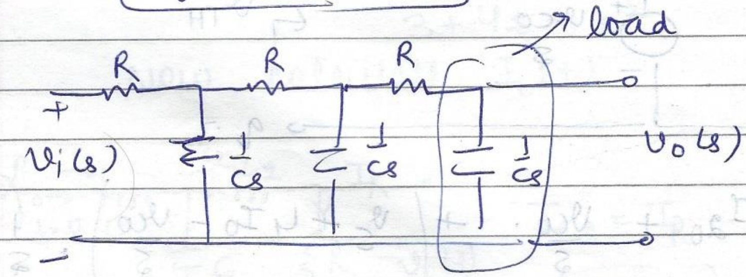




$$\boxed{I_{sc} = \frac{Z}{Z+0.1} \cdot \frac{10}{s}}$$

$$\boxed{Z_0 = Z + 0.1}$$

Q

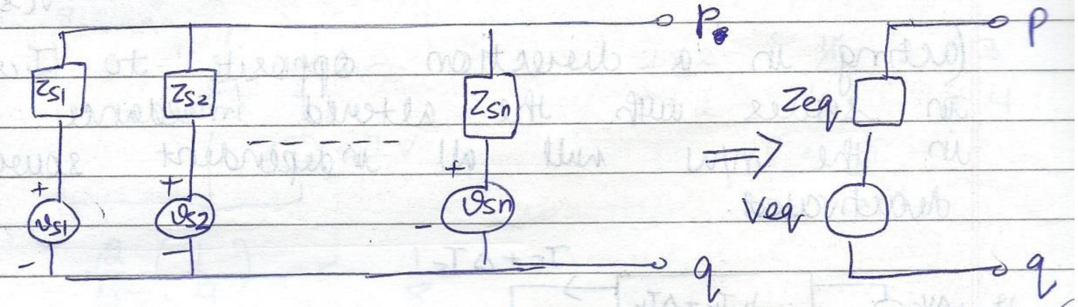


Calculate (i) $\frac{v_o(s)}{v_i(s)}$

(ii) $\angle \frac{v_o(j\omega)}{v_i(j\omega)} = \pi$ find ω_0

(iii) $\left|\frac{v_0}{v_i}\right| = ?$ if $\omega = \omega_0$

Millman's Theorem



Acc. to NORTON theorem

$$Y_{0s} = \sum_{k=1}^{n} Y_{SK}$$

$\frac{I_{sc}}{\text{Admittance}}$ = voltage across

$$V_{eq} = \frac{\sum_{k=1}^{n} V_{SK} \cdot Y_{SK}}{\sum_{k=1}^{n} Y_{SK}}$$

$$Z_{eq} = \frac{1}{\sum_{k=1}^{n} Y_{SK}}$$

Compensation Theorem

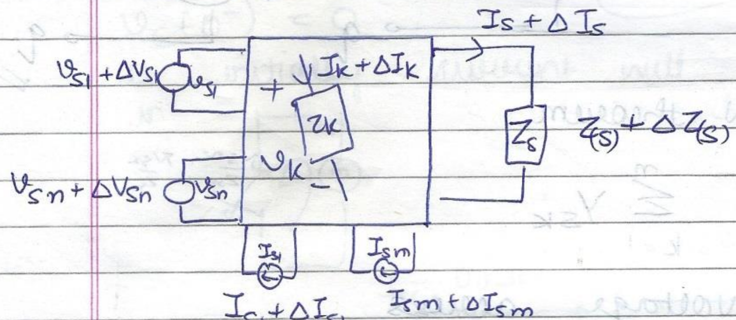
In a linear n/w acted upon by independent sources and impedance $Z_{(s)}$ carrying current is changed to $Z_{(s)} + \Delta Z_{(s)}$ the the

⇒ If Z has initial condition then the equivalent initial condition source remains the same.

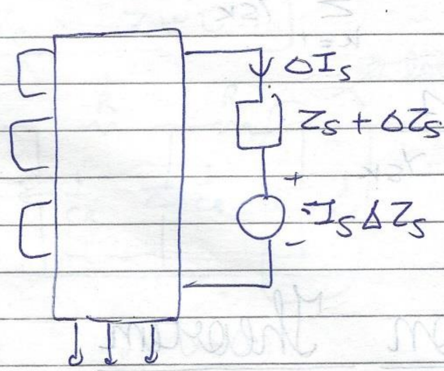
⇒ If ΔZ has some initial condition then the equivalent initial condition sources must be included.

incremental changed produced in the whole n/w will be identical to those produced by a voltage source $\Delta Z_{(s)} \cdot I_{(s)}$ $= V(s)$

(acting in a direction opposite to $I_{(s)}$) in series with the altered impedance in the n/w will all independent sources deactivated.



$$= (I_s + \Delta I_s)(Z_s + \Delta Z_s)$$
$$= I_s Z_s + \Delta I_s (Z_s + \Delta Z_s) + I_s + \Delta Z_s$$



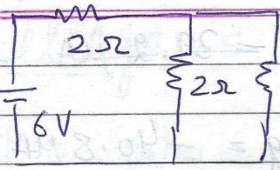
⇒ ΔZ need not to be small

⇒ It can be used to compute the incremental change in entire n/w.

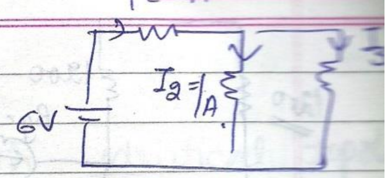
⇒ There should not be any coupling b/w ΔZ and any other elements in the n/w.

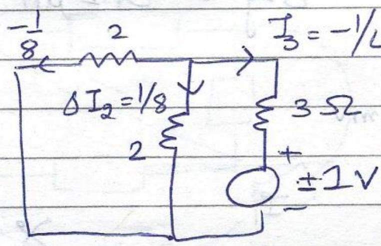
$I_1 = 2A$

Q.



$2\Omega \longrightarrow 3\Omega$





$\frac{2\times 2}{2+2} = \frac{4}{4}$ $\quad 1+3 = 4$

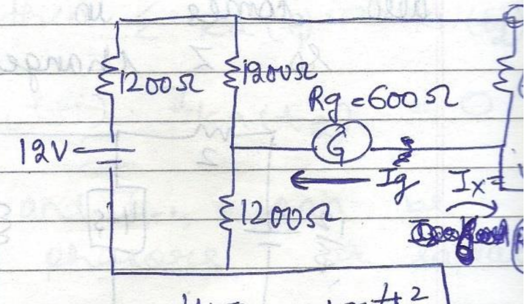
$I = \frac{1}{4}$

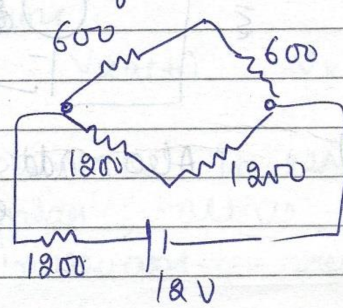
$\frac{2}{4}\left(\frac{1}{4}\right) = \frac{1}{8}$

$I_1' = 2 - 1/8$ $\qquad I_3' = 1 - 1/4$

$I_2' = 1 + 1/8$

Q. Nominal value of $R_x = 600\ \Omega$
with tolerance value $= \pm 5\%$.
Calculate I_g





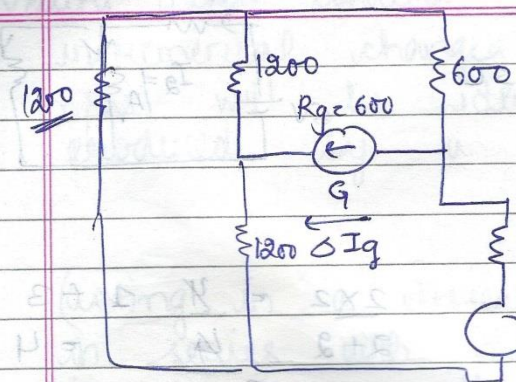
$\frac{1200 \times 2400}{3600} = 800$

$= 800 + 1200$

$= 2000$

$I = \frac{12}{2000} = 6\ mA$

$5\% \leq R_x \leq 630$.



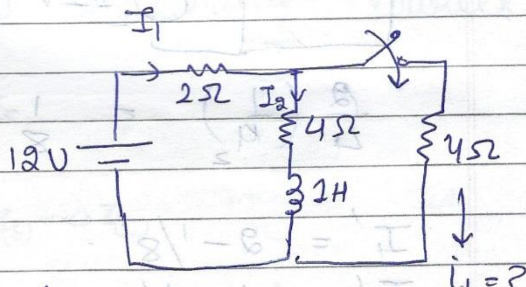
$\cancel{\Delta I_g = 39.2\ \mu A}$

$\Delta I_g = -40.8\ \mu A$

$\Delta I_g = 39.2\ \mu A$

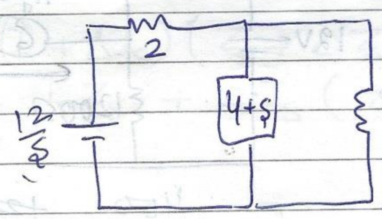
Q. Previously L acts as short circuit

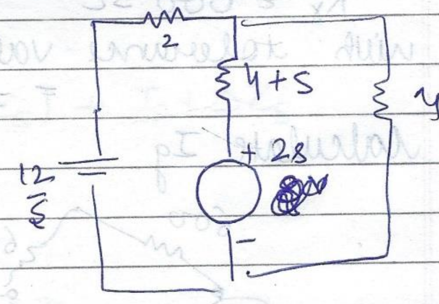
$I_2(0^-) = 2A.$



Now after closing switch impedance of L also comes in existence

so Z changes from 4 to 4+s.

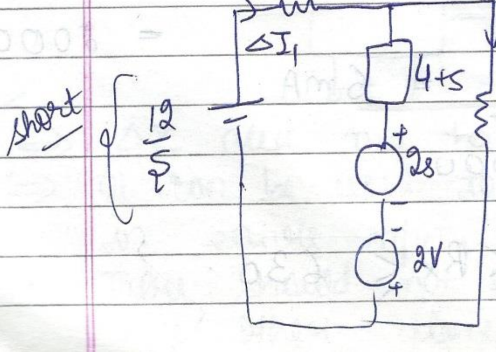




~~2× 4+s = s4~~

$2(\cancel{4}+s-\cancel{4}) = 2s$

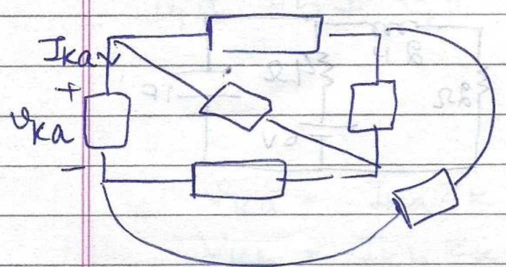
Also add initial effect

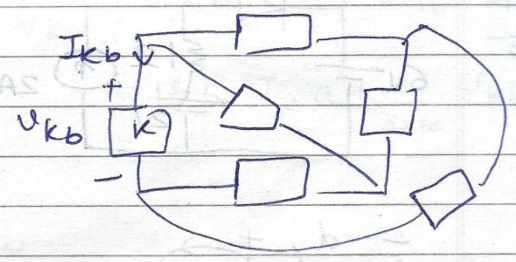


$\Delta I_L(s) = \dfrac{0.5}{3s+16}$

Tellegan's Theorem

Two N/W NA and NB with identical topology





Acc. to Tellegan's theorem

$$\sum V_{ka} I_{kb} = \sum V_{kb} I_{ka} \; (\text{for all } k) = 0$$

* If A & B are different N/W but with identical topology then $\sum V_{ka}(t_1) \cdot I_{kb}(t_2) = \sum v_{kb}(t_1) \cdot I_{ka}(t_2) = 0$

* v_{ka} and I_{ka}, v_{kb} and I_{kb} can be instantaneous values, phasors or laplace transforms.

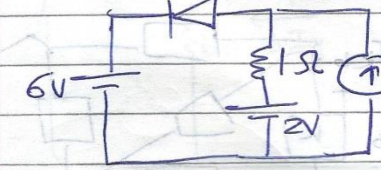
  $V_{ka}(t)$ $\quad v_{ka}(j\omega)$ $\quad V_{ka}(s)$

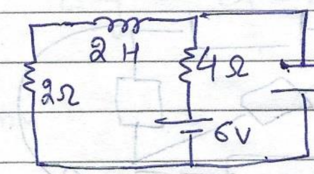
* N/W elements in NA and N_B may be linear, non linear, active, passive, time variant, time invariant, reciprocal, non reciprocal.

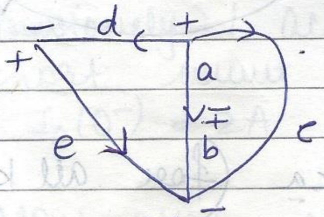
* N_A & N_B both n/w's may be same and V_k, I_k measured at same instant or diff. instants of time, then also Tellegans theorem applicable.

* If KCL / KVL not applicable for eg:- may be in case of non linear elements, then tellegan's theorem not applicable.

Q.



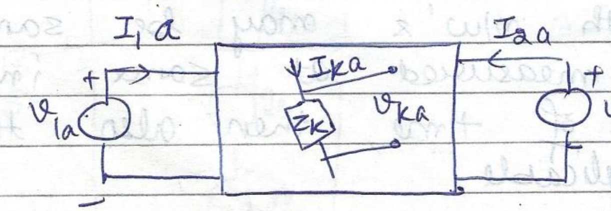




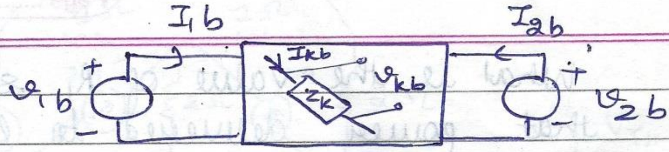
	V_A	I_A	V_B	I_B
a	2V	2A		−1
b	2V	2A		−1
c	4V	−2A		0
d	−2V	0		1
e	6V	0		1

$$\sum_{\forall k} V_A I_A = 0$$

Application of Tellegan's Theorem



N/w is reciprocal (means consists of RLC n/w)



$$v_1 a \cdot I_1 b + v_2 a \cdot I_2 b + \sum_{\forall k} v_{ka} I_{kb} = v_{1b} \cdot I_{1a}$$

$$+ v_2 b \cdot I_2 a + \sum_{\forall k} v_{kb} \cdot I$$

$v_{ka} = I_{ka} Z_k$
$V_{kb} = I_{kb} Z_k$
Put above. Thus both side cancel.

$$\boxed{v_1 a \cdot I_1 b + v_2 a \cdot I_2 b = v_{1b} \cdot I_1 a + v_{2b} \cdot I_{2a}}$$

for Z parameter reciprocity cond.

$I_{2a} = I_{1b} = 0$
Or $I_1 a = I_{2b} = 0$

i.e $\frac{v_{2a}}{I_1 a} = \frac{v_{1b}}{I_{2b}}$

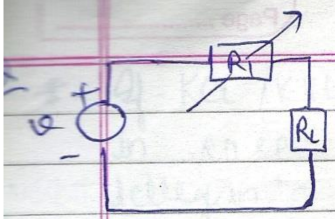
$$\begin{array}{cc} \| & \| \\ Z_{21}\big|_{I_1=0} & Z_{12}\big|_{I_2=0} \end{array}$$

Maximum Power Transfer Theorem

A n/w delivers max power to a load R_L when R_L = Thevenin's eq. resistance of n/w

In AC circuit for power to be max.

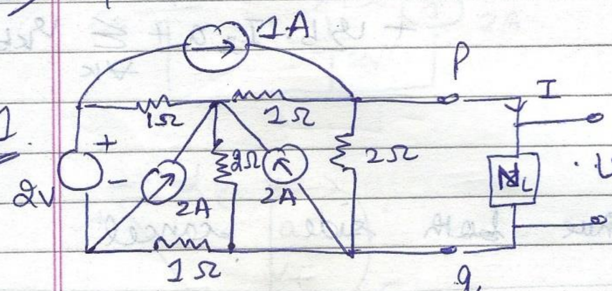
$$Z_L = Z_{TH}^*$$



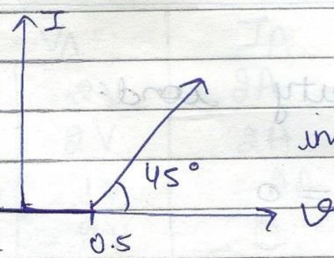
What is the value of R_1 such that power delivered to load is max.

Soln $R_1 = 0$

1.



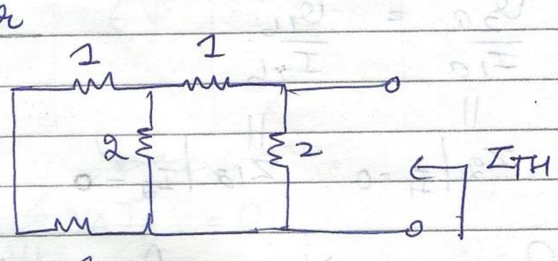
V-I characteristic represented graphically.



Then find v and I

increase linearly after 0.5.

Piece wise linear behaviour
ult overall
ion linear

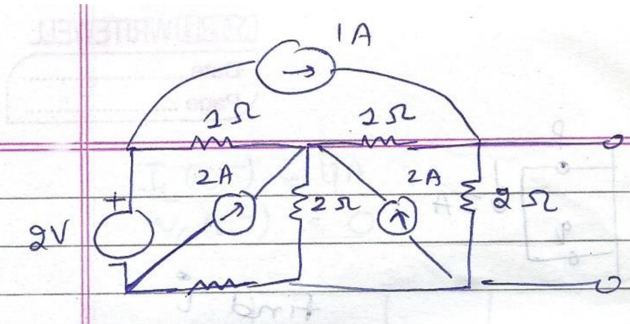


$\frac{2+2}{4} = \cancel{\frac{4}{4}} = \cancel{1+1=2}$

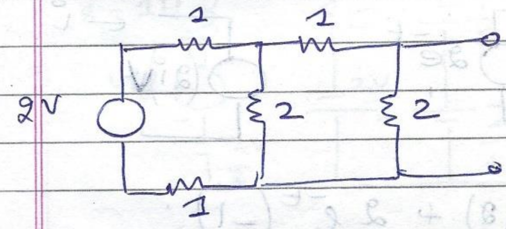
2

$\frac{2 \cdot 2}{2+2} = \frac{4}{4} = 1 \qquad 1+1 = 2$

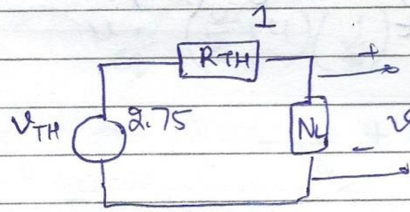
$\frac{2 \cdot 2}{2+2} = \frac{4}{4} = 1\,\Omega.$



Superposition



$V_{TH} = 2.75$ V



$2.75 - i \cdot 1 - v = 0$

here ohm's law not valid as it is non linea

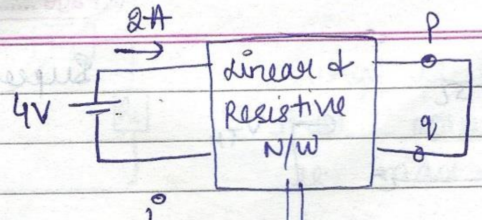
$$i = \begin{cases} 0 & v \leq 0.5 \\ v - 0.5 & v > 0.5 \end{cases}$$

Now two solutions possible as there are 2 regi

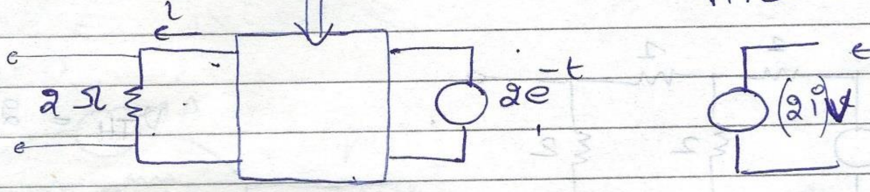
$v \leq 0.5$	$v > 0.5$
$\boxed{i = 0}$	$\boxed{i = v - 0.5}$
$2.75 - i - v = 0$	$2.75 - (v - 0.5) - v = 0$
$v = 2.75$	$2.75 + 0.5 = 2v$
$\Downarrow$	$3.25 = 2v$
This itself is contradictory as $v \leq 0.5$	$v = \frac{3.25}{2}$
	$v = 1.625$

There is no operating pnt for n/w N_L under the application of given n/w N lies in the region of $v \leq 0.5$

2.

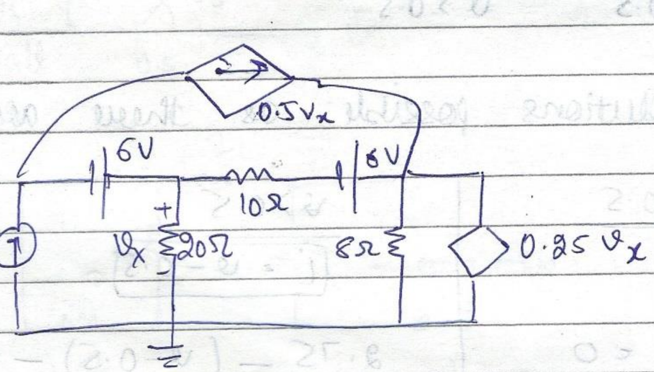


find i



$$4(-i) + 0 = v(2) + 2e^{-t}(-1)$$
$$-4i = 2v - 2e^{-t}$$
$$-2i = v - e^{-t}$$
$$-2i = 2i - e^{-t}$$
$$+4i = +e^{-t}$$
$$\boxed{i = \frac{e^{-t}}{4}}$$

3

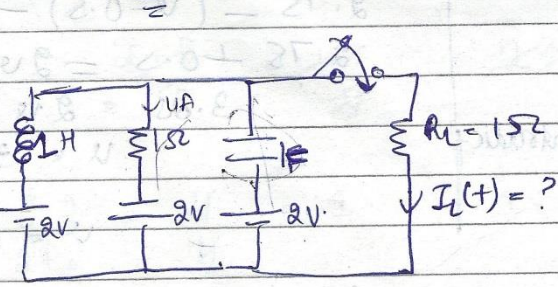


find v_x

Superposition

$4 \times 1 - 2 = 2$

4



Sol To find initial conditions

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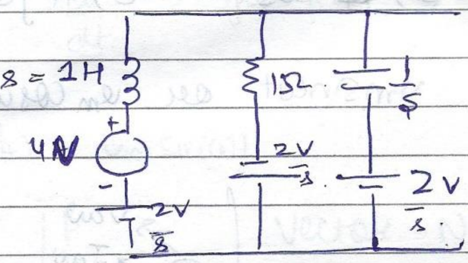
$I_L(0^-) = 4A$
$V_C(0^-) = 0$



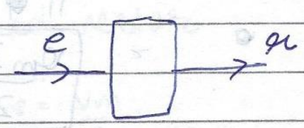
$$Z_{eq} = \frac{1}{\frac{1}{s} + 1 + s}$$

Apply Millmann's Theorem

$$v_{eq} = \frac{\left(\frac{2}{s}+4\right)\left(\frac{1}{s}\right) - \frac{2}{s}\cdot 2 + \frac{2}{s}\cdot s}{\frac{1}{s} + 1 + s}$$

AC circuit Analysis

In this analysis we cal. steady state response.



$R(s) = H(s)\,E(s) \rightarrow$ freq. at $e(t) = \omega$

$R_t = H(j\omega)\,E_{(t)}$

$$R(t) = \frac{a_1 + jb_1}{a_2 + jb_2}\Bigg| v_m \sin\omega t$$

$$= \frac{|A_1|}{|A_2|} \angle(\theta_1 - \theta_2)\; v_m \sin\omega t$$

$\Downarrow$

$$\frac{|A_1|}{|A_2|} e^{j\theta}\, v_m \left(\frac{e^{j\omega t} - e^{-j\omega t}}{2i}\right)$$

$$\frac{|A_1|\, v_m}{|A_2|} \left(\frac{e^{j\theta} e^{j\omega t} - e^{j\theta} e^{-j\omega t}}{2i}\right)$$

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$$\frac{e^{j(\omega t+\theta)} - e^{-j(\omega t-\theta)}}{2i}$$

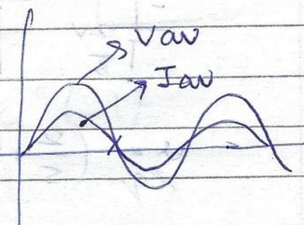
$\cos(\omega t+\theta) + j\sin(\omega t+\theta) - \cos(\omega t-\theta) + j\sin(\omega t-\theta)$

for sinosoidal I/P $v_m \sin\omega t$ or $v_m \cos\omega t$

$v_{RMS} = \frac{v_m}{\sqrt{2}}$

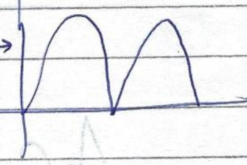
$v_{av} = 0$

$I_{av} = 0$



$P_{av} \neq 0$

Max value of $P_{av} = V_m I_m$



Average value for half cycle $= \frac{2V_m}{\pi}$

Max. value of Pav for half cycle $= \frac{2(V_m I_m)}{\pi}$

~~value~~ RMS " " " $= \boxed{\frac{V_m I_m}{2}}$ RMS value of power

⇒ Form factor $= \frac{\text{Peak value}}{\text{RMS value}}$

$= \frac{V_m}{V_m/\sqrt{2}} = \sqrt{2} = 1.414$

⇒ Crisp factor or peak factor $= \frac{\text{Peak value}}{\text{Average value}}$

$= \frac{V_m}{2V_m/\pi} = \frac{\pi}{2} = 1.57$

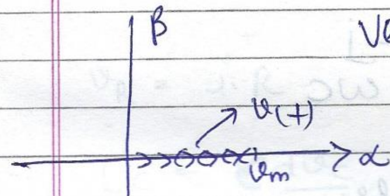
Confirm from book

RMS value is effective value of signal

$\Rightarrow \frac{d}{dt} \longrightarrow s\,X(s) \qquad \int(\ )\,dt \longrightarrow \frac{1}{s}\,X(s)$

$\Rightarrow \frac{d}{dt} \longrightarrow j\omega\, X(j\omega) \qquad \int(\ )\,dt \longrightarrow \frac{1}{j\omega}\,X(j\omega)$

$V_{(t)} = V_m \sin\omega t$



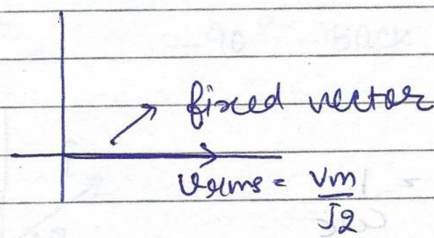
Vector Notation

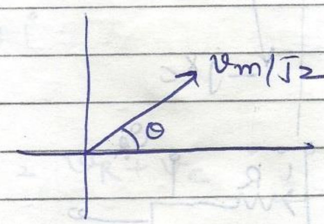
time dependent vector

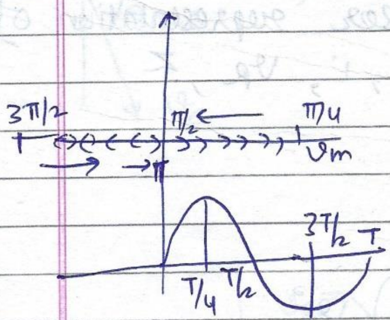
oscillatory type of behaviour
vector notation at instantaneous value

Phasor Notation

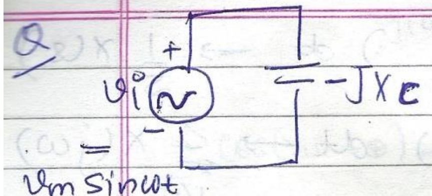
If $v_m \sin(\omega t + \theta)$.







Q



$X_C = \frac{1}{\omega C}$

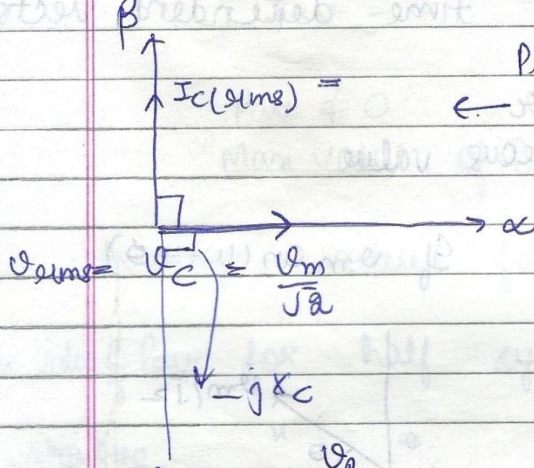
$i_C = j\omega C \cdot v_i$

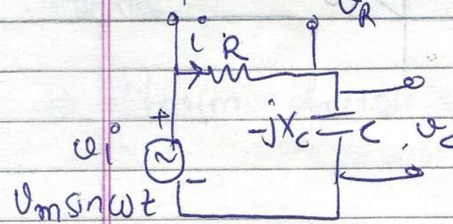
$i_C = C\frac{dV_C}{dt}$

$i_C = C \cdot j\omega \cdot v_C$

$\frac{v_C}{i_C} = \frac{1}{Cj\omega} = \frac{-j}{\omega C}$

Phasor diag.





$X_C = \frac{1}{\omega C}$

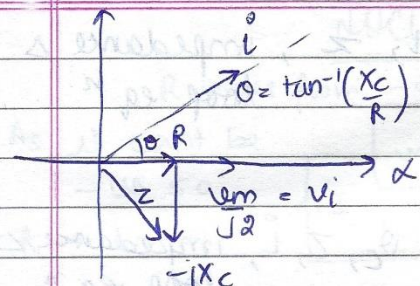
Plot phasor representation of v_i, v_C, i, V_R, Z

$i_C = C\frac{dV_C}{dt}$

$i_C = C \cdot j\omega \cdot v_C$

$i_C = C \cdot j\omega \left(\frac{-jX_C}{R - jX_C}\right) v_i$

$i_C = \frac{C\omega X_C}{R - jX_C} v_i$



$$i = \frac{v_i}{|Z| \angle \tan^{-1}\left(-\frac{X_C}{R}\right)}$$

$$= \frac{v_i}{|Z|} \angle -\tan^{-1}\left(-\frac{X_C}{R}\right)$$

$$= \left|\frac{v_i}{Z}\right| \angle \tan^{-1}\left(\frac{X_C}{R}\right)$$

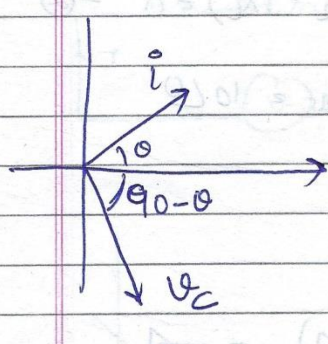
$v_R = i \cdot R$ so same phase as i

$i_C = \cancel{C}\frac{dv_C}{dt}$

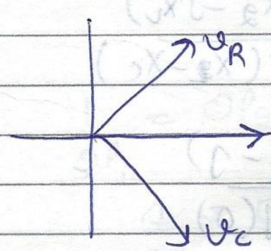
$i_C = Cj\omega \cdot v_C$

$$v_C = \frac{i}{Cj\omega} = \frac{i X_C}{j} = -jX_C i$$

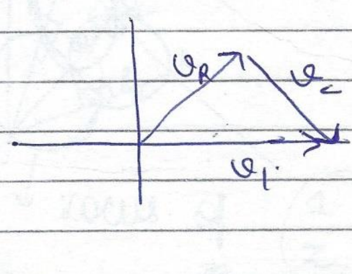
$-90°$ back from i.



$v_i = v_R + v_C$



Parallely shift v_C



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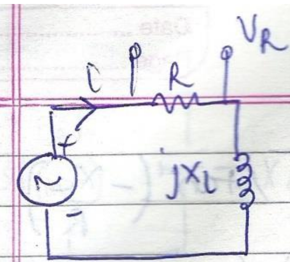
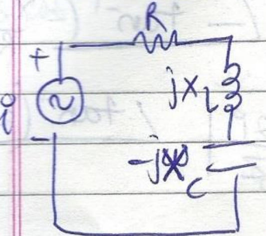


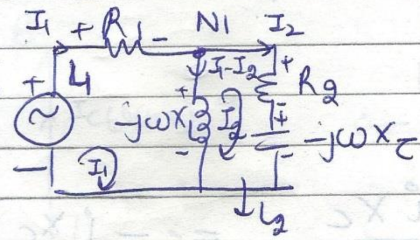
v_i, v_L, v_R, i, z, impedance △ loop eq^n

Q



$v_i, V_L, V_R\ v_C, Z, i$, impedance △ loop eq^n

Q



$v_i, v_R, v_L, v_C, v_{R2}$

$I_1, I_2, Z_{in} = \frac{v_i}{I_1}$

L_1, L_2, N_1

Sol^n

$L_1 = v - I_1R_1 - (I_1 - I_2)JX_L = 0$ —(1)

$JX_L(I_1 - I_2) - I_2R_2 + I_2(JX_L) = 0$ —(2)

$R_1 = R_2 = X_L = X_C = 1$ $\quad V_{RMS} = 10\angle 0$

$$Z_{in} = \frac{JX_L(R_2 - JX_L)}{R_2 + J(X_L - X_C)}$$

$$= \frac{J(1 - J)}{1 + J(0)} = \frac{J(-J)}{1} = -1$$

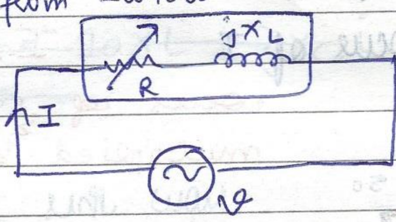
$Z_{in} = 2 + j$

Locus Diagram

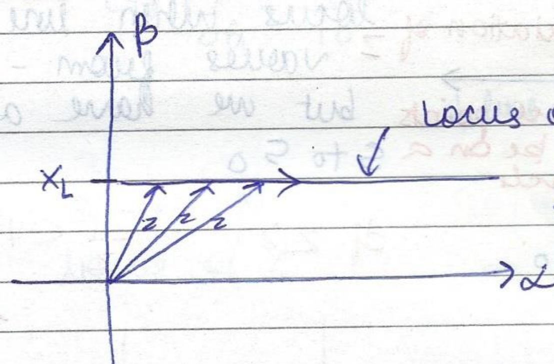
R varies from $-\infty$ to ∞

As it can't be $-ve$ so

$R \to 0 - \infty$



$Z = R + jX_L$

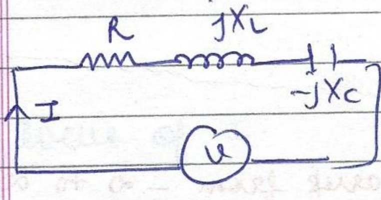


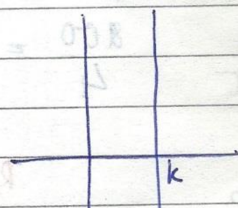
Locus of z as R is varying real part changing Imaginary part is constant.

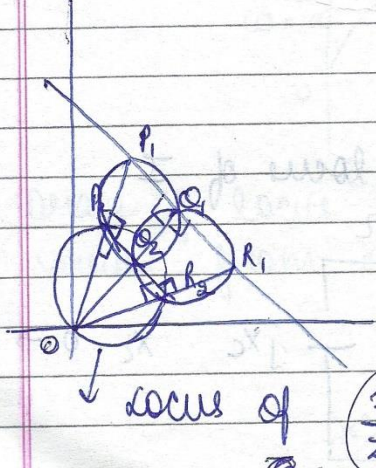
$v = IZ$

$I = vY$

$Y = \dfrac{1}{|Z|\angle\theta} = \dfrac{1}{|Z|}\angle-\theta$







$\dfrac{1}{OP_1} = OP_2$ $\qquad \dfrac{1}{OQ_1} = OQ_2$

$\dfrac{1}{OR_1} = OR_2$

$OP_1\,OP_2 = 1$

$OR_1\,OR_2 = 1$

$OQ_1\,OQ_2 = 1$

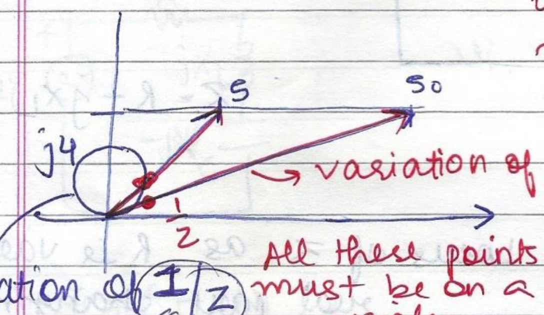
Locus of $\dfrac{1}{Z}$

Q ~~let~~ variation of $R = 5 - 50\ \Omega$. $X_L = 4$ $v = IZ$

Say $v = 200 \angle 0\ V$

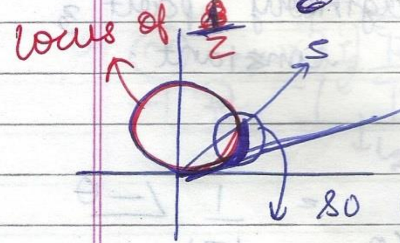
Represent the locus of I $I = \left(\frac{1}{Z}\right) v$.

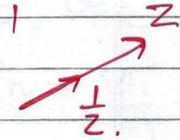
locus of $\frac{1}{Z}$ is locus of I multiplied by v.



Now this is the locus when line varies from $-\infty$ to ∞ but we have a segment 5 to 50

If $Z > 1$ $\frac{1}{Z} < 1$

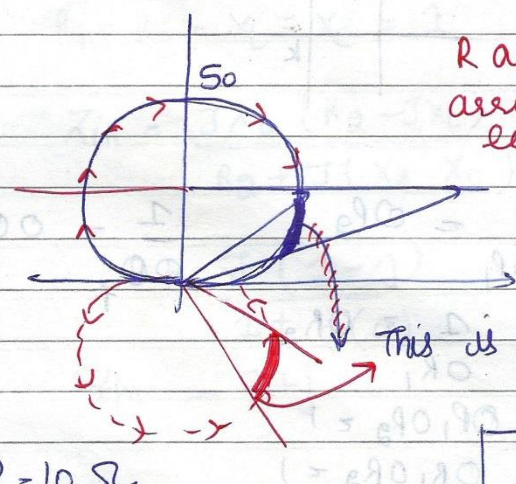




so only this portion is locus of Y.

$I = 200\,Y$

diameter $\frac{200}{4} = 50$



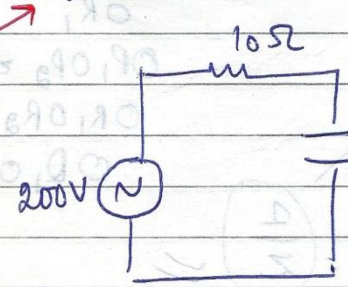
R as varies from $-\infty$ to ∞ arrow represents direction of locus

This is the locus of I

Q $R = 10\ \Omega$

$v = 200\ V$

$X_C = 0$ to ∞



X_C $0 \to \infty$

locus of Inverse of points lying on straight line is a circle passing through origin whose diameter = $\frac{1}{d}$ except one line that passes through origin $y = x$. Then its locus will be a hyperbola

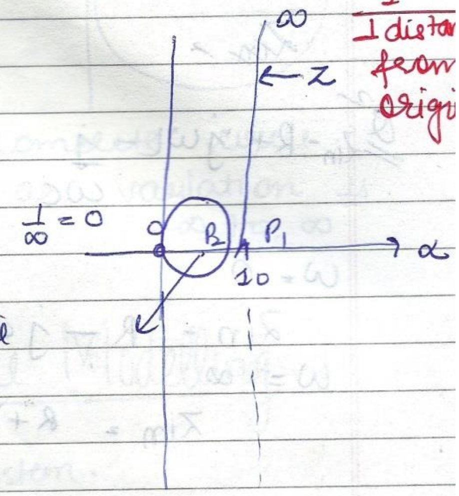
find locus of I. $Z = 10 + jX_C$

This case can occur when both R and L = 0 i.e when both are varied

~~$v = 10 \times i + jX_C\, i$~~

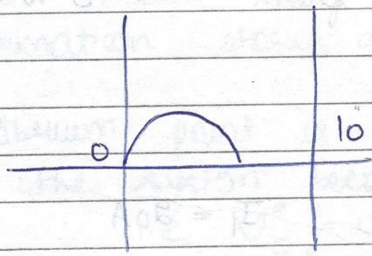
~~$v =$~~

$OP_2 = \frac{1}{OP_1} = \frac{1}{10}$



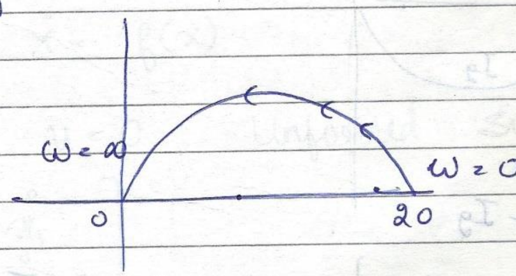
$\frac{1}{\perp \text{ distance from origin}}$

for $0 \rightarrow \infty$.
locus of Y

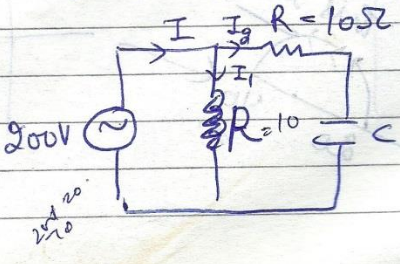


$I = 200Y$
when $X = 0$
$I = \frac{200}{10} = 20$.
$I = 200 \times 0$
$= 0$.

locus of I.



Q) Draw of locus of I if freq. of supply voltage varies from 0 to ∞

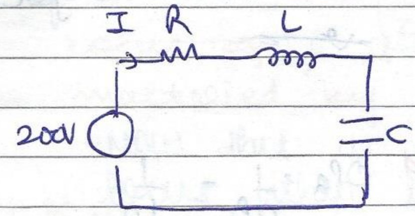


$\frac{20}{10-\frac{1}{\omega C}}$ $\frac{20}{10-j0}$ $\frac{20}{20/10}$

~~When~~ ω varies

$Z_{in} =$

Q2. $Z_{in} = R + j\omega L - j\frac{1}{\omega C}$



$\omega = 0$

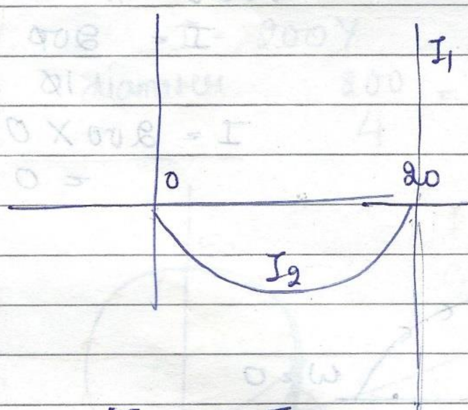
$Z_{in} = R - j\infty$

$\omega = \infty$

$Z_{in} = R + j\infty$

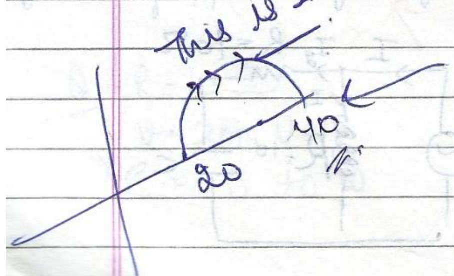
Imag. part varies from $-\infty$ to ∞.

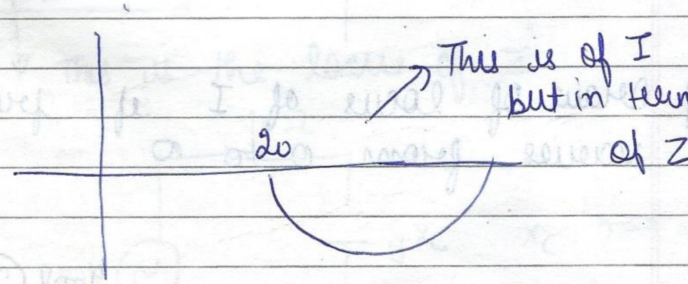
Q.



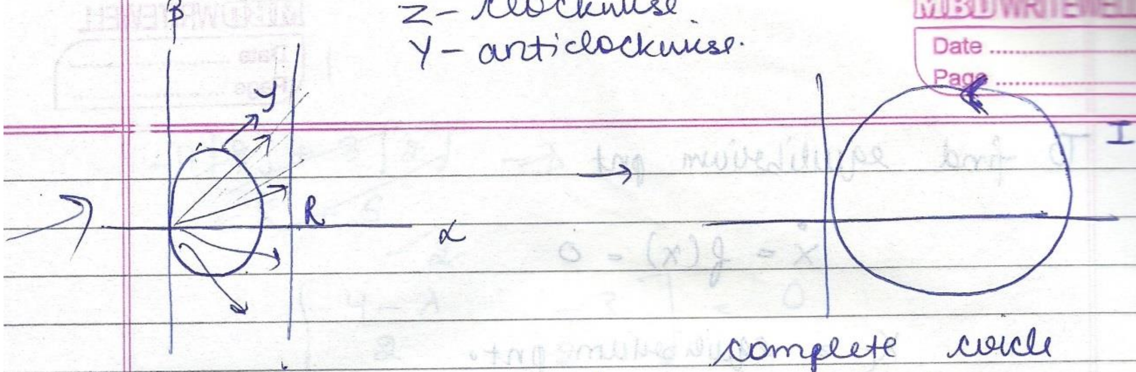
$I_1 = 20A$

$I = I_1 + I_2$





Z - clockwise
Y - anticlockwise.



complete circle locus as variation is $-\infty$ to ∞

State Space Modelling

Applied to non linear system.
State variable defines the state of a system i.e information stored at particular instant

Equilibrium point is the point where dynamic of the system becomes zero.

i.e $\dot{v}_c = 0$, $\dot{i}_L = 0$

$\dot{X} = f(x, u, t)$ — x: state variable, u: I/P variable, t: time

autonomous system. $\dot{x} = f(x)$

$u = 0$ Unforced System.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_n \end{bmatrix} = f(x_1, x_2, x_3 \dots x_n) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ \vdots \\ f_n(x) \end{bmatrix}$$

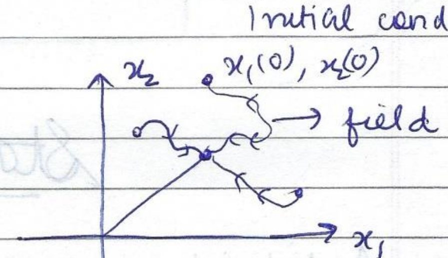
To find equilibrium pnt

$$\dot{x} = f(x) = 0$$

X_e = equilibrium pnt.

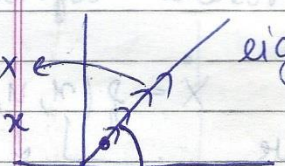
Operating pnt = Eq. pnt

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$



Now as 2 variables are changing.
So if we find a direction
on which if initial condition is present
then both move in that direction forever

$\dot{x} = kx$
$\dot{x} = \lambda x$



eigen value direction. λ = constant matrix

Now it gets converted to single variable
slope represents rate of change of
x_1 & x_2.

Q

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -4 & -3 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\dot{x} = Ax = \lambda x$$

$$[A - \lambda I]\, x = 0$$

characteristic eq^n
$\underline{A - \lambda I = 0}$

$$\begin{bmatrix} -4 & -3 \\ 2 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

Date ....
Page ....

$\cancel{-4[2]+3[2]}$ $\rightarrow \lambda$

$\cancel{-8+6}$

$\cancel{-2}$

$$\begin{bmatrix} -4-\lambda & -3 \\ 2 & 3-\lambda \end{bmatrix} = 0$$

$$-4-\lambda\,[3-\lambda) + 3[2] = 0$$

$$-12+4\lambda-3\lambda+\lambda^2+6 = 0$$

$$\lambda^2+\lambda-6 = 0$$

$$(1)^2-4\times1\times-6$$

$$1+24 = 25$$

$$\frac{-1\pm5}{2} = \frac{-1+5}{2} = \frac{4}{2} = 2$$

$$= \frac{-1-5}{2} = -3$$

means 2 eigen directions possible.

$$[A-2I][X] = 0$$

$$\begin{bmatrix} -4 & -3 \\ 2 & 3 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -6 & -3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$2 - 2 = 0$

$-6 = 3y_1$

$-2 = y_1$

$$-6x_1-3y_1 = 0$$

$$2x_1+y_1 = 0 \;\cancel{\times 3}$$

multiple sol^{n}s possi

$\cancel{6x_1+3y_1=0}$

$\cancel{-6x_1-3y_1=0}$

1, -2 is one of the sol^{n}s

$v_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$(A+3I)[X]$

$$\begin{bmatrix} -4 & -3 \\ 2 & 3 \end{bmatrix} + 3[I]$$

$$\begin{bmatrix} -4+3 & -3 \\ 2 & 3+3 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -3 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-x_2 - 3y_2 = 0$ 1

$2x_2 + 6y_2 = 0$

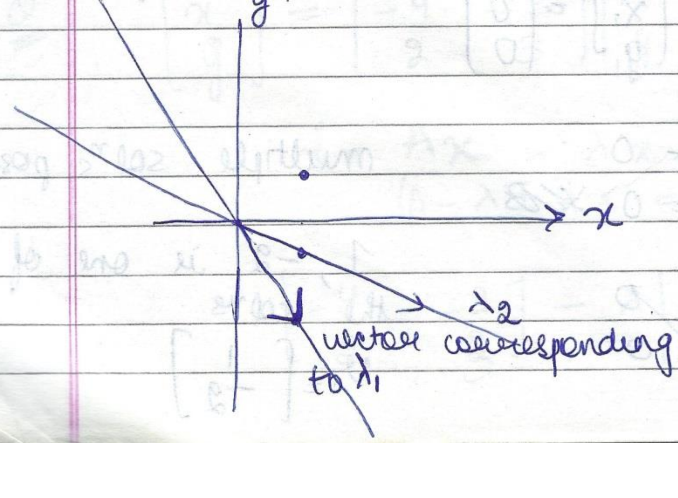
$-1 = 3y_2 \quad y_2 = -\frac{1}{3}$

$\cancel{\frac{2}{3} + 6y_2 = 0} \quad \cancel{6y_2 = \frac{2}{3}}$

$2(1) + 6\left(-\frac{1}{3}\right) = 0$

$2 - 2 = 0$

$= \left(1, -\frac{1}{3}\right) \qquad v_2 = \begin{bmatrix} 1 \\ -1/3 \end{bmatrix}$



$\dot{x} = \lambda_1 x$

$x_1(t) = [1]\, e^{-\lambda_1 t}\, C_1$

$y_1(t) = [-2]\, e^{-\lambda_1 t}\, C_2$ This is not general soln

$$X_1(t) = \begin{bmatrix} x_1(t) \\ y_1(t) \end{bmatrix} = C_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-\lambda_1 t}$$

so C_1 C_2 cannot be found by putting initial conditions as this is not general soln.

$$X_2(t) = C_2 \begin{bmatrix} 1 \\ -1/3 \end{bmatrix} e^{-\lambda_2 t}$$

Both are linear independent as $X_1(t) \neq k X_2(t)$

So general soln $x(t) = C_1 X_1(t) + C_2 X_2(t)$

soln of multi dimensional system

$$x(t) = C_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-\lambda_1 t} + C_2 \begin{bmatrix} 1 \\ -1/3 \end{bmatrix} e^{-\lambda_2 t}$$

Q $A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$ proceed similarly

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = C_1 \begin{bmatrix} e^{-\lambda_1 t} \\ -2e^{-\lambda_1 t} \end{bmatrix} + C_2 \begin{bmatrix} e^{-\lambda_2 t} \\ -\frac{1}{3} e^{-\lambda_2 t} \end{bmatrix}$$

Take initial cond. x_0, y_0

$$(s+4)(s-3) - 3(-2)$$

$$s^2 + \cancel{8}s - 12 + 6$$

$$s^2 + s - 6$$

Inverse $= \begin{bmatrix} \dfrac{s-3}{s^2+s-6} & \dfrac{2}{s^2+s-6} \\ \dfrac{-3}{s^2+s-6} & \dfrac{s+4}{s^2+s-6} \end{bmatrix}$

Network Function:

$$H(s) = \frac{R(s)}{E(s)}$$

$$E(s) \rightarrow \boxed{H(s)} \rightarrow R(s)$$

Assumption: Linear, $\underbrace{\text{lumped}}$, finite, passive, bilateral n/w

actual size negligible as compared to wavelength of excitation

wavelength of excitation in India = 6000 km.

$\boxed{R(s) = E(s)\,H(s)} \rightarrow$ H(s) will be real and rational

rational = ratio of 2 polynomials.
or real & rational fn. of s, z (discrete)
or jω (Sinosoidal)

Put value of $s = -1$, $(-1)^{3/2}$, $= j^3 = -j$

$s^2 + 2s + 1 \rightarrow$ Polynomial, real

$s^2 + s^{3/2} + 1 \rightarrow$ Not poly, not real

$s^3 + s^2 + \sqrt{s} \rightarrow$ Not poly, not real

infinite series — $e^s + 1 \rightarrow$ Not poly,

Polynomial length of series be finite

real – If we put real value of s, quantity should also be real.

$s + s^{-1}$ – Not polynomial, rational fn

$s^2 + js + 1$ – Polynomial, Not real

✓ Polynomial is a finite series with +ve integral power

✓ A poly. is said to be real if we put real value of variable, then value of poly. is real

$$R(s) = H(s) \cdot E(s)$$

$$= \sum_i \frac{A_i}{(s - P_i)} + \sum_J \frac{A_j}{(s - P_J)}$$

$Re[P_i] < 0$ for stable system BIBO system

P_i = Poles corresponding to $H(s)$

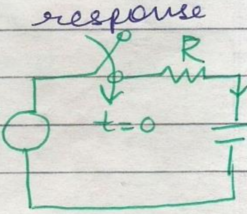
i.e $\sigma < 0$

P_J = Poles corresponding to $E(s)$

i.e bounded signal

$$= \underbrace{\sum A_i e^{P_i t}}_{\text{Natural response}} + \underbrace{\sum A_j e^{P_j t}}_{\text{Forced response}}$$

Q



find $i(t)$

$$H(s) = \frac{I(s)}{V(s)}$$

$$\frac{I(s)}{V(s)} = \frac{1}{R + SC}$$

$$= \frac{1}{R\left[1 + \frac{SC}{R}\right]}$$

$$= \frac{1}{C\left[\frac{R}{C} + s\right]}$$

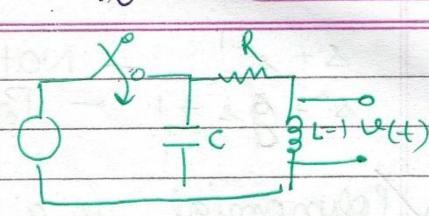
$$\frac{I(s)}{V(s)} = \frac{1/C}{\left(s + \frac{R}{C}\right)} = H(s)$$

$$E(s) = \frac{1}{s+1}$$

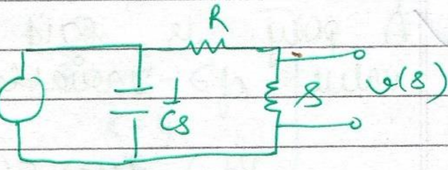
$$I(t) = A e^{-R/C\, t} + B e^{-t}$$

should

Q $v(t)$ = response
find z transfer fn.



i.e find $H(s) = \dfrac{V(s)}{I(s)}$



$$v(s) = \left[\frac{\frac{1}{C\cancel{s}} I(s)}{\left(\frac{1}{Cs} + R + s\right)}\right] \cancel{s}$$

$$\frac{V_s}{I(s)} = \frac{\frac{1}{C}}{\frac{1}{Cs} + R + s} = \frac{\frac{1}{\cancel{C}} Cs}{1 + CsR + Cs^2}$$

$$= \frac{s}{Cs^2 + sCR + 1}$$

$$= \frac{s}{C\left[s^2 + sR + \frac{1}{C}\right]}$$

$$= \frac{s/C}{s^2 + sR + \frac{1}{C}}$$

Q If poles of z transform fn are $-2, -1$ find R and C

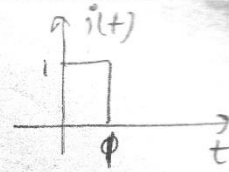
$$\cancel{(R)^2 - 4 \times 1 \times \frac{1}{C}} \qquad (s+2)(s+1) \qquad \cancel{R^2 - \frac{4}{C}}$$

$$s^2 + 3s + 2 = s^2 + sR + \frac{1}{C}$$

$$\frac{1}{C} = 2 \qquad R = 3$$

$$\boxed{Z(s) = \frac{2s}{s^2 + 3s + 2}}$$

Q. If $i(t)$ excitation is given then find $v(t)$ for same n/w



$$H(s) = \frac{2s}{s^2+3s+2}$$

$$i(t) = u(t) - u(t-1)$$

$$I(s) = \frac{1}{s} - \frac{e^{-s}}{s} = \frac{(1-e^{-s})}{s}$$

$$V(s) = \left(\frac{2s}{s^2+3s+2}\right) I(s).$$

$$\cancel{v(t) = Ae^{-2t} + Be^{-t}}$$

$$V(s) = \left(\frac{2s}{s^2+3s+2}\right)\left(\frac{1}{s} - \frac{e^{-s}}{s}\right)$$

$$\frac{2s}{s^2+3s+2}\left(\frac{1}{\cancel{s}}\right) - \frac{2\cancel{s}}{s^2+3s+2}\left(\frac{e^{-s}}{\cancel{s}}\right)$$

$$= \boxed{Ae^{-2t} + Be^{-t} - C\left[e^{-2(t-1)} + e^{-(t-1)}\right]u(t-1)}$$

$$\boxed{2\left[(e^{-t} - e^{-2t})u(t) - (e^{-(t-1)} - e^{-2(t-1)})u(t-1)\right]}$$

Ans

3 cases in N/W function

$$H(s) = \frac{N(s)}{D(s)}$$

both are real polynomial $N(s)$ & $D(s)$. Thus it is rational function.

a) If degree of $N(s) < D(s)$

$$H(s) = \sum_i \frac{A_i}{s-P_i}$$

b) If degree of $N(s) \geq D(s)$

$$H(s) = K + \frac{N_1(s)}{D(s)} \qquad D\{N_1(s)\} < D\{D(s)\}$$

then partial fraction of $\frac{N_1(s)}{D(s)}$ is again is

$$\sum_i \frac{A_i}{s-P_i}$$

c) If deg. of $N(s) > D(s)$

$D\{N(s)\} = 2 + D\{D(s)\}$

$$H(s) = A_2 s^2 + A_1 s + A_0 + \frac{N_2 s}{D(s)}$$

where $D\{N_2(s)\} < D\{D(s)\}$

$$\mathcal{L}(\delta'(t)) = s$$

$$\underline{\mathcal{L}^{-1}(s) = \delta'(t)}$$

$$H(j\omega) = \frac{N(j\omega)}{M(j\omega)}$$

$$\Rightarrow |H(j\omega)| e^{j\phi(j\omega)}$$ Euler representation

$$\mathrm{Re}[H(j\omega)] + j\,\mathrm{Im}[H(j\omega)]$$

$$|H(j\omega)| \cos\phi(j\omega) + |H(j\omega)| \sin\phi(j\omega)$$ Cartesian representation

<u>How to convert $H(s)$ to above representations</u>

$$H(s) = \frac{N(s)}{M(s)}$$

$$N(s) = s^2 + 2s + 1$$

$$= s^2 + 2s + s^0$$

$$\underbrace{[s^2 + s^0]}_{(n_1)\ \text{even part}} + \underbrace{2s}_{\text{Odd part } (n_2)}$$

$$H(s) = \frac{n_1(s) + n_2(s)}{m_1(s) + m_2(s)}$$

$$= \frac{n_1(s) + n_2(s)}{m_1(s) + m_2(s)} \frac{(m_1(s) - m_2(s))}{(m_1(s) - m_2(s))}$$

$$= \frac{n_1 m_1 - n_1 m_2 + n_2 m_1 - n_2 m_2}{m_1^2 - m_2^2}$$

even fn of s means when we put s or −s ans. is f(s).

$$= \frac{E - O + O - E}{\text{even fn of } s}$$

$$= \left[\frac{(n_1 m_1 - n_2 m_2) + (n_2 m_1 - n_1 m_2)}{m_1^2 - m_2^2}\right] \rightarrow \text{real term}$$

= even fn of s + odd fn of s

Put $s = j\omega$

$$H(j\omega) = Re[H(j\omega)] + j\,Im[H(j\omega)]$$

$$|H(j\omega)| = \sqrt{[Real(H)]^2 + [Im(H)]^2} \rightarrow |H(j\omega)|^2 \text{ even fn of } \omega$$

$$\phi(j\omega) = \tan^{-1}\frac{Im(H(j\omega))}{Real(H(j\omega))} \rightarrow \text{odd fn of } \omega$$

$H(s) = \dfrac{a_m s^m + a_{m-1} s^{m-1} + \cdots a_0 s^0}{b_n s^n + b_{n-1} s^{n-1} + \cdots b_0 s^0}$

m = No. of finite zeroes

n = No of finite poles

$$= \left(\frac{a_m}{b_n}\right) K \left[\frac{s^m + a'_{m-1} s^{m-1} \cdots a'_0 s^0}{s^n + a'_{n-1} s^{n-1} \cdots b'_0 s^0}\right]$$

$\dfrac{a_m}{b_n} = k = $ gain of the fn.

1) • In general if $m < n$ then system fn is said to be proper rational fn.

2) • If $m = n$ then $H(s)$ can be written as

$$H(s) = C + \text{Proper rational fn}$$

i.e one impulse present in response

constant

Inverse laplace of constant = Impulse fn

3) • If $n < m$ (Improper fn) (Non realizable)

$$H(s) = \left(a_k s^k + a_{k-1} s^{k-1} + \text{-----} \quad a_0 s^0\right)$$

+ Proper rational fn.

In this case not only impulse present in response but also its derivative.

Above two cases are not physically realisable as impulse practically not available. single Impulse could be tolerated but derivative can never be tolerated.

LLFPB — linear lumped, finite, passive, bilateral

This is physically realisable sys.

$\boxed{\text{FREQUENCY DOMAIN ANALYSIS}}$

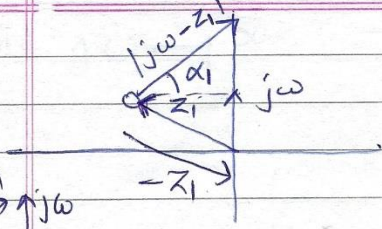
Now if $H(s) = \dfrac{a_m s^m + a_{m-1} s^{m-1} + \text{---} \; a_0 s^0}{b_n s^n + b_{n-1} s^{n-1} + \text{---} \; b_0 s^0}$

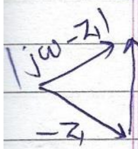
$$= k \left\{ \frac{s^m + a'_{m-1} s^{m-1} + \text{--} \; a'_0 s^0}{s^n + b'_{n-1} s^{n-1} + \text{--} \; b'_0 s^0} \right\}$$

$$= k \frac{\prod_{i=1}^{m} (s - z_i)}{\prod_{j=1}^{n} (s - p_j)} \Bigg|_{s = j\omega}$$

$$= \frac{k \prod (j\omega - z_i)}{\prod (j\omega - p_i)}$$

$$\boxed{|H(j\omega)|\, e^{\phi(j\omega)} = Re(H(j\omega)) + j\, Im(H(j\omega))}$$





$N_1 = |j\omega - z_1|$ $\qquad |j\omega - p_1| = M_1$

$\angle j\omega - z_1 = \alpha_1$ $\qquad \angle j\omega - p_1 = \theta_1$

$$\frac{k\, N_1\angle\alpha_1 \cdot N_2\angle\alpha_2 \cdot N_3\angle\alpha_3 \,----\, N_m\angle\alpha_m}{M_1\angle\theta_1 + M_2\angle\theta_2 \,--------\, M_n\angle\theta_n}$$

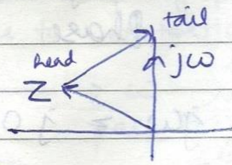
n, m = No. of poles + zeroes

N, M = Mag. of poles + zeroes

$$= k\,\frac{N_1 N_2 N_3 --- N_m}{M_1 M_2 M_3 --- M_m} \frac{\angle \alpha_1 + \alpha_2 --- \alpha_m}{\angle \theta_1 + \theta_2 ---- \theta_n}$$

$$= \boxed{\underbrace{\frac{K\, N_1 \cdot N_2 --- N_m}{M_1 \cdot M_2 --- M_n}}_{\text{Magnitude of } (j\omega)} \underbrace{\angle \sum_i \alpha_i - \sum_j \theta_j}_{\phi(j\omega)}}$$

head of $(j\omega - z)$ is at z and tail at z



<u>In case of proper rational fn</u>

m = Total no. of finite zeroes

$n - m$ = N.o of zeroes lying at ∞.

Q $F(s) = \dfrac{4s}{s^2+2s+2}$ $\quad j\omega = j2$. find the mag. of $F(j\omega)$ and phase of $F(j\omega)$

$s^2 + 2s + 2$

$(2)^2 - 4 \times 1 \times 2$

$4 - 8 = -4 = 2i$

$\dfrac{-2 \pm 2i}{2}$

$-1 \pm 1i$

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$$\frac{4s}{(s+1-i)(s+1+i)}$$

$$F(j2) = \frac{4j2}{(j2+1-j)(j2+1+j)}$$

$$= \frac{8j}{(j+1)(3j+1)}$$

$\frac{1}{0}$ ∞

$\tan^{-1}\infty$

$$Mag = \frac{8}{\sqrt{1+1}\ \sqrt{9+1}} = \frac{8}{\sqrt{2}\sqrt{10}} = \frac{2\sqrt{2}}{\sqrt{2}\sqrt{10}} = \frac{2}{\sqrt{10}}$$

$$Phase = \tan^{-1}\frac{8}{0} - \tan^{-1}\frac{1}{1} - \tan^{-1}\frac{3}{1}$$

$$= 90 - \tan^{-1}1 - \tan^{-1}3$$

$$M^2(j\omega) = Real^2[H(j\omega)] + Im^2[H(j\omega)]$$

Mag. plot is even fn
Phase plot is odd fn.

(ii) $j\omega = j0$

$$\frac{8(j0)}{(j0+1-j)(j0+1+j)}$$

$$= \frac{0j}{(1-j)(1+j)} \longrightarrow \text{null vector having mag. 0. but direction } 90^\circ$$

$$= \frac{0}{\sqrt{2}\ \sqrt{2}} = 0$$

Phase $\underbrace{\tan^{-1}\frac{0}{0}}_{\text{not defined}} + \cancel{\tan^{-1}1} - \cancel{\tan^{-1}1}$

Ans = 90°

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(iii) $j\omega = j\infty$

$$\frac{4j\infty}{(j\infty+1-j)(j\infty+1+j)}$$

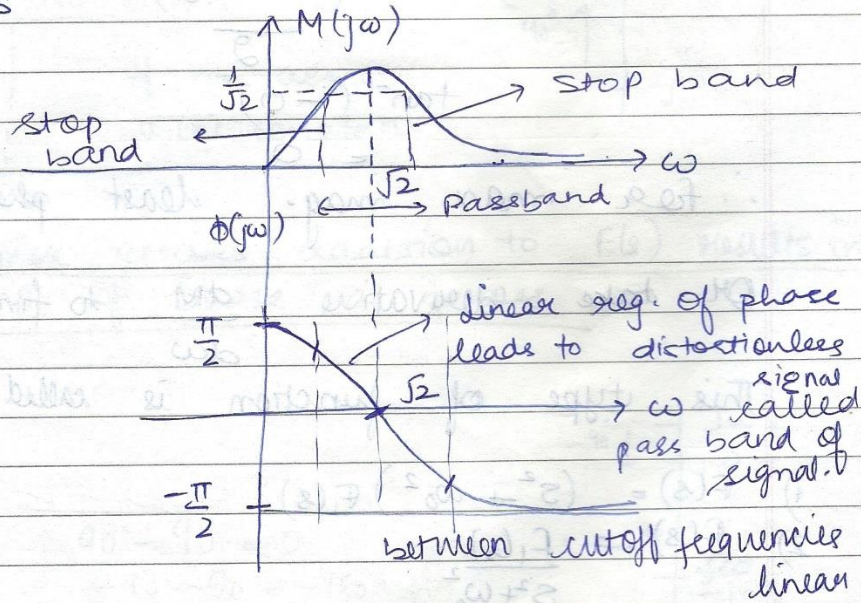
$$= \frac{\infty}{\infty \cdot \infty} = 0.$$

$$\text{Phase} = \tan^{-1}\frac{\infty}{0} - \tan^{-1}\frac{\infty}{1} - \tan^{-1}\frac{\infty}{1}$$

$$= 90 - 180$$

$$= \underline{-90^\circ}$$

Now behaviour of F(s) can be plotted on these basis



Freq. at which phase zero.

$$\frac{4(j\omega)}{(j\omega)^2 + 2j\omega + 2} = \frac{4j\omega}{2j\omega - \omega^2 + 2}$$

$$= \frac{4}{2 + j\left(\omega - \frac{2}{\omega}\right)}$$

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$$= \frac{4}{\sqrt{2^2 + \left(\omega - \frac{2}{\omega}\right)^2}} \angle \tan^{-1}\left(-\frac{\left(\omega - \frac{2}{\omega}\right)}{2}\right)$$

For mag. max den. least

$$\omega - \frac{2}{\omega} = 0$$

$$\omega = \frac{2}{\omega} \qquad \boxed{\omega = \sqrt{2}}$$

Putting $\omega = \sqrt{2}$ in phase

$$\tan^{-1} - \frac{\left(\sqrt{2} - \frac{2}{\sqrt{2}}\right)}{2}$$

$$\tan^{-1}(-0)$$

$$= 0$$

$\therefore$ for max. mag. least phase.

Or take derivative $\frac{dM}{d\omega}$ to find max. value.

This type of function is called BAND PASS.

1) $F(s) = (s^2 + \omega_0^2) F_1(s)$ $\qquad s^2 = -\omega_0^2$

2) $F(s) = \frac{F_1(s)}{s^2 + \omega_0^2}$ $\qquad s = \pm j\omega_0$

3) $F(s) = \frac{s^2 + \omega_1^2}{s^2 + \omega_2^2}$ $\quad \omega_1 < \omega_2$

$F_1(s) \Rightarrow |F_1(j\omega)| = M_1$ & Mag. and phase of

$\angle F_1(j\omega)$ ϕ_1 $\quad F_1(j\omega)$ known

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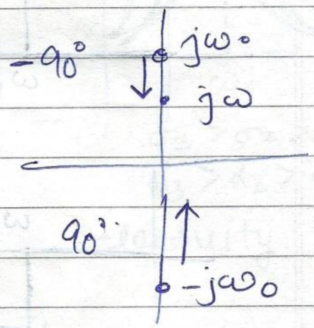
$(j\omega - z_1)$ form

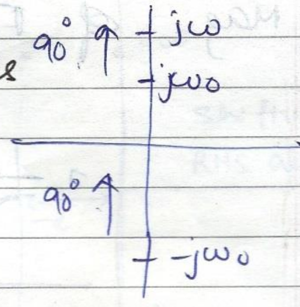
$(j\omega - j\omega_0)\,(j\omega + j\omega_0)$
$(j\omega - (-j\omega_0))$

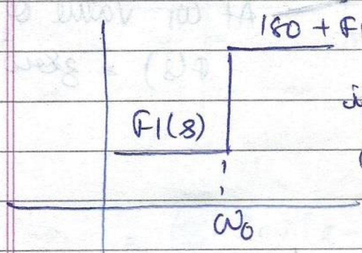


If $\omega < \omega_0$ $\quad -90 + 90 = 0$

$\phi F(j\omega) = 0^\circ + \phi_1$ if $\omega < \omega_0$
$= 180 + \phi_1$ if $\omega > \omega_0$

There is a jump in phase of $F(s)$ by 180° at $\omega = \omega_0$
i.e location of zeroes

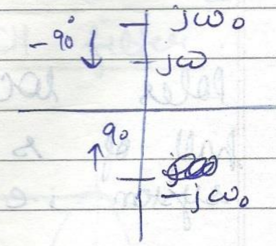




if we assume $F_1(s)$ constant

\# $\therefore$ complex zeroes addition to $F(s)$ results in jump of phase at zeroes

2) $\dfrac{F_1(s)}{(s^2 + \omega_0^2)}$



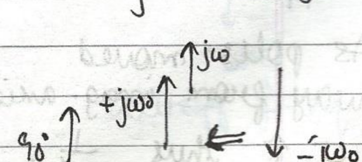
$90 - 90 = 0$
$-90 - 90 = -180$

$0 + \phi_1$ if $\omega < \omega_0$
$-180 + \phi_1$ if $\omega > \omega_0$

Introduction of -180° phase

$j\omega + j\omega_0$
$j\omega - (-j\omega_0)$

$\text{Mag} = \dfrac{k}{(\omega^2 - \omega_0^2)}$



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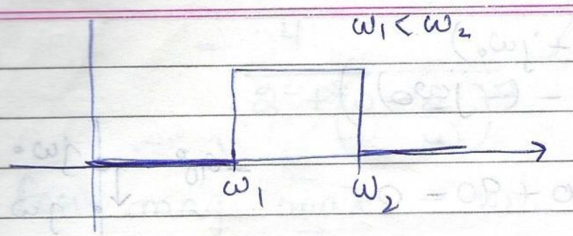
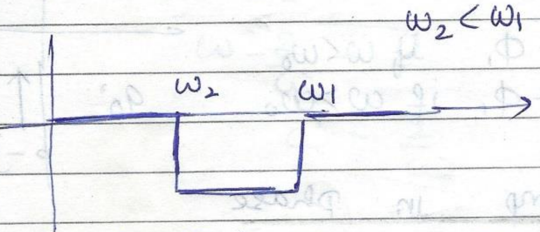
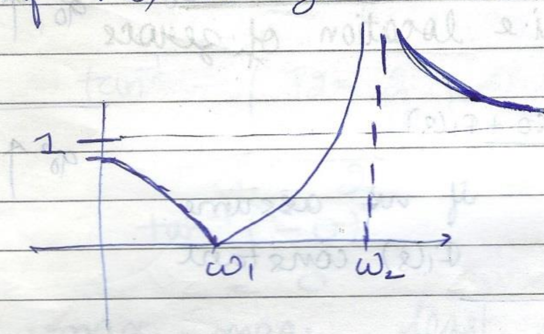


$F(s) = 1$
Phase $= 0$



- Mag. of $F(s)$ at zeroes is zero



At ω_1 value of $F(s) = $ zero

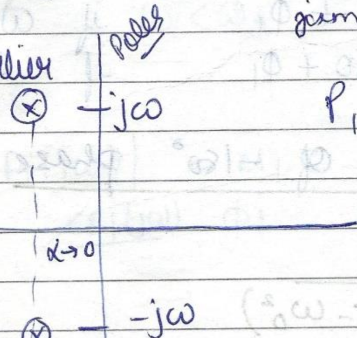
Now at $+ve\ \infty$ i.e. form ∞/∞

$$\lim_{s\to 0} \frac{s^2+\omega_1^2}{s^2+\omega_2^2} = \frac{2\cancel{s}}{2\cancel{s}} = 1$$

means including $j\omega$ axis

Poles location will always be in closed half of s plane for a physically realisable system i.e (LLFPB)

- whatever be the phase earlier to ω_0 it will $\downarrow$ rapidly by approx. $180°$



jump at zeroes in finite duration.

$P_{1,2} = -\alpha \pm j\omega$

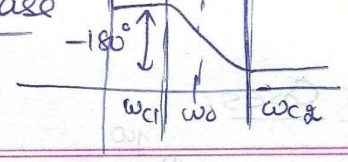
Now value not ∞ but very large

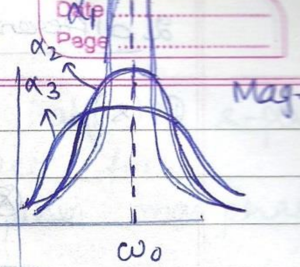
As poles moved away from imag. axis

$\sigma \uparrow$ thus $\frac{1}{\sigma(2j\omega+\sigma)} \downarrow$

σ value ↑ transition smooth.
σ value less, transition abrupt.

Phase





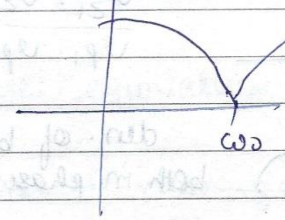
$(s-P_1)(s-P_2)$

Low pass special case of
→ Band pass when $\omega_0 = 0$
→ High pass when $\omega_0 = \infty$

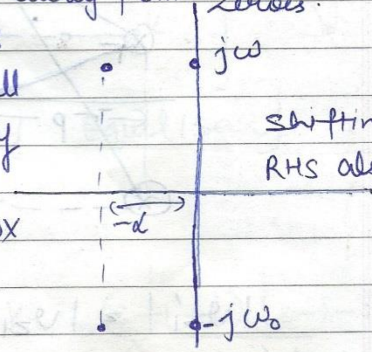
$\sigma_3 > \sigma_2 > \sigma_1$
$\alpha_3 > \alpha_2 > \alpha_1$

- Mag. large – better range of selectivity or Narrow band
- small mag – poor rang of selectivity. or wider band
- when zeroes loc. σ is slightly away from jω axis

zeroes



phase will ↑ rapidly at ω_0 by approx 180°

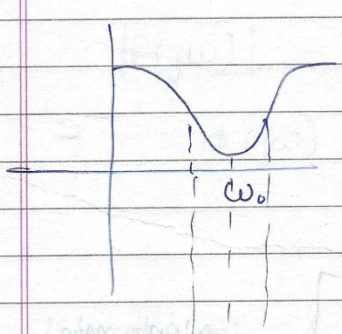


shifting on RHS also possib

$Z_{1,2} = (s-Z_1)(s-Z_2)$

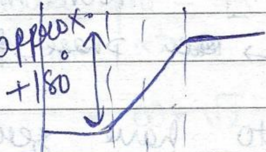
Now mag. not zero but tending to zero.

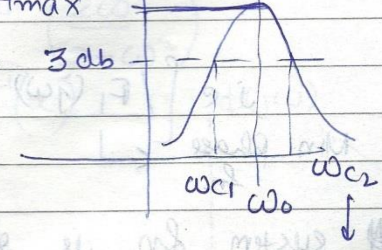
Mag.



called BAND STOP or BAND NOTCH filter.

Phase

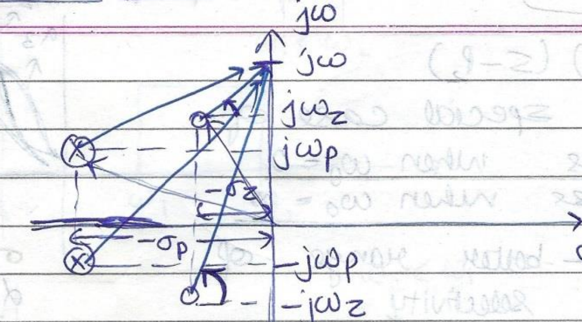




↓
Max. cutoff freq.

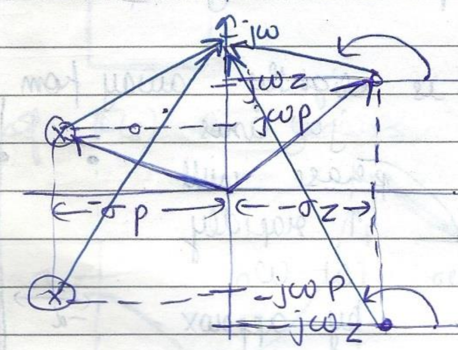
2 different Cases-

$F_1(s)$



$$\frac{v_{z1} \cdot v_{z2}}{v_{p1}\, v_{p2}}$$

$F_2(s)$ -



$$\frac{v_{z1}' \cdot v_{z2}'}{v_{p1} \cdot v_{p2}}$$

den. of both same
both in phase as well as mag.

$|v_{z1}'| = |v_{z1}|$ $\qquad$ $|v_{z2}'| = |v_{z2}|$

i.e overall mag. of $F_1(j\omega) = F_2(j\omega)$

Zeroes $\quad$ Poles
$(\alpha_2 > \alpha_1)$ $\quad$ $(\theta_1 = \theta_2)$

$\alpha_2 - \theta_2 > \alpha_1 - \theta_1$

i.e $\phi_2 > \theta_1$

i.e $\left[\angle F_1(j\omega) < \angle F_2(j\omega)\right]$

Min. phase fn $\qquad$ Non. min. phase fn.

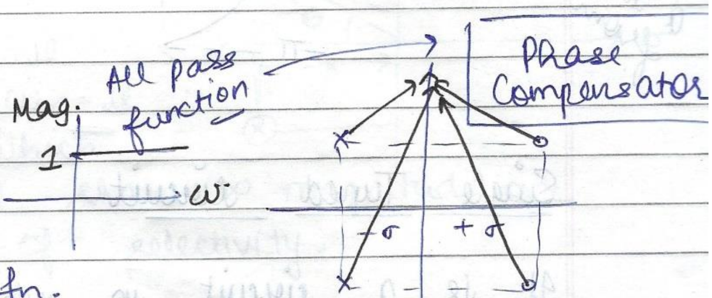
(non-including $j\omega$ axis)

If system fn is restricted to have zeroes only on open left half of s-plane then it is called min phase fn. If system fn.

having zeroes on closed right half of s-plane then it is called non-min. phase fn.

- If ~~no~~ jω axis not included then max phase fn

Q $F(s) \Rightarrow F(j\omega)$



Mag. always $|F(j\omega)| = 1$

Non min. phase fn.

Application $\boxed{\text{Phase equivaliser or Delay Equilisar}}$

Phase = non-zero.

Q. $F(s) = \dfrac{k\left[(s-1)^2+1\right]}{\left[(s+1)^2+1\right]} = k\dfrac{(s-1+i)(s-1-i)}{(s+1+i)(s+1-i)}$ ⇓ All pass

Soln $\dfrac{k(2-\omega^2) - 2j\omega}{2-\omega^2+2j\omega}$

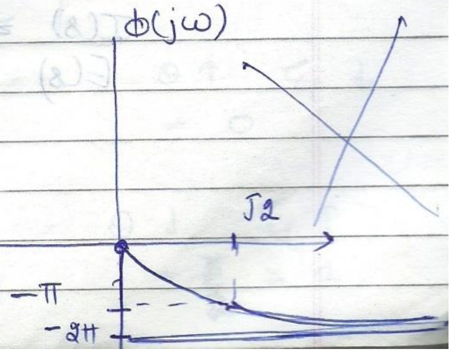
$|F(j\omega)| = k$

$\Rightarrow \phi(j\omega) = \tan^{-1}\left(\dfrac{-2\omega}{2-\omega^2}\right) - \tan^{-1}\left(\dfrac{2\omega}{2-\omega^2}\right)$

$= -2\tan^{-1}\left(\dfrac{2\omega}{2-\omega^2}\right)$

Critical pnt - fn where $\tan^{-1}(x)$ goes to ∞

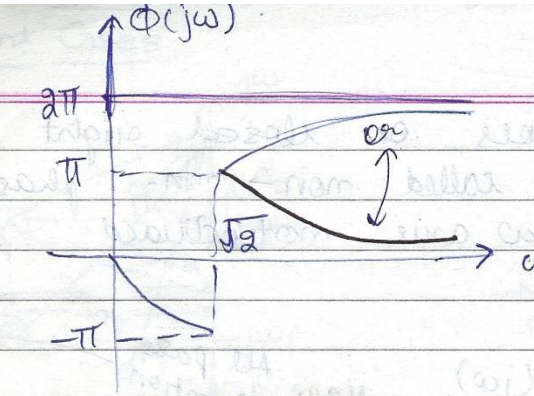
$\omega = \sqrt{2}$



Now this is book soln

After √2 ans -ve. One more -ve sign outside so ϕ = +ve.

check that at critical point there is a transition from $-ve$ to $+ve$



$\lim_{\omega \to \infty} \frac{2\omega}{2-\omega^2}$

$\frac{2}{2-2\omega}$

$\frac{2}{2-\infty} \quad -0$

$-2 \times (-0)$
$= 0$

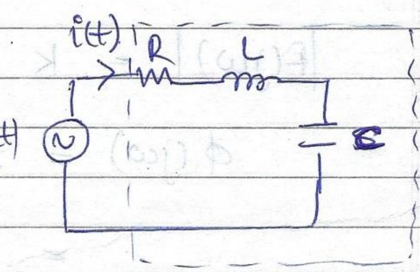
0 can be written as 2π

Single Tuned circuit

It is a circuit in which there is one inductor & one capacitor connected in such a way that they resonate i.e the reactance of the inductor & reactance of capacitor together will show no reactance outside at a particular freq. This is called phenomenon of resonance or tuning.

Single tuned circuit has resonance only at one freq.

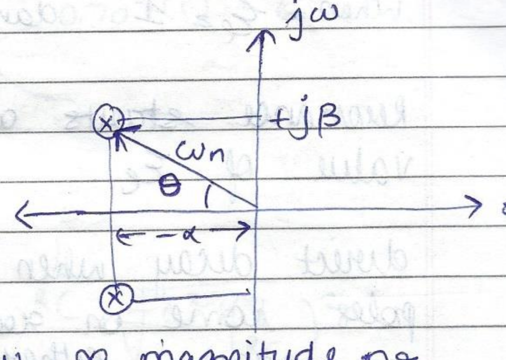
Q $\frac{I(s)}{E(s)} = ?$ Driving pnt fn when excitation & response connected to same port.



$$\frac{I(s)}{E(s)} = \frac{s}{L\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)}$$

$$= \frac{s}{L(s-P_1)(s-P_2)}$$

$P_{1,2} = -\alpha \pm j\beta$



ω_n represents undamped natural frequency.
because if there is no damping then ω_n is the freq. at which the circuit will show ∞ magnitude or i.e ∞ amount of selectivity.

Damping decided by real part. If $\sigma = 0$ damping zero.

$$\alpha = \omega_n \cos\theta$$
$$\beta = \omega_n \sin\theta$$

$$\xi = \cos\theta = \frac{\alpha}{|\omega_n|} = \text{Damping factor (Quality of decay)}$$

$\underline{\text{If } \xi \uparrow, \sigma \uparrow}$

$$= \frac{\alpha}{\sqrt{\alpha^2 + \beta^2}}$$

$$\alpha = \omega_n . \xi$$
$$\beta = \omega_n \sqrt{1 - \xi^2}$$

$$\beta = \omega_n \sin\theta$$
$$= \omega_n \sqrt{1 - \cos^2\theta}$$
$$= \omega_n \sqrt{1 - \xi^2}$$

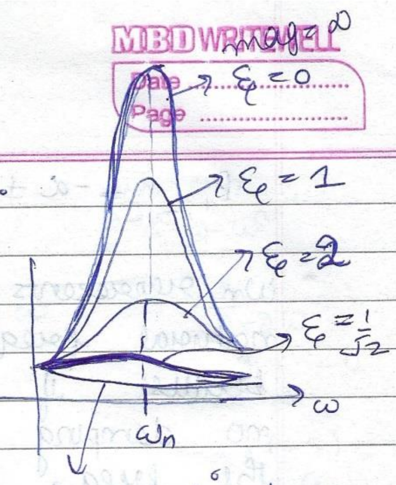
$\therefore$ Any 2nd order transfer fn can be represented as

$$\boxed{\frac{I(s)}{V(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}}$$

Q If $\theta \to \frac{\pi}{2}$ $\quad \xi \to ?$ $\quad \theta \uparrow \quad \xi \downarrow$
$= 0$

$\theta \to 0$ $\quad \xi \to ?$ $\quad \theta \downarrow \quad \xi \uparrow$
$= \frac{\pi}{2} = 1$

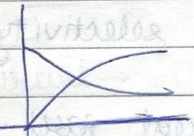
when $\xi_e = 1$ damping max.



Resonance starts at a finite value of ξ_e

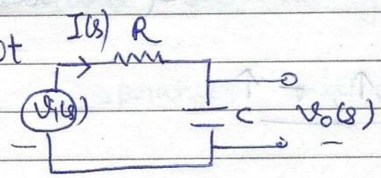
At a pnt direct decay will start

direct decay when $\theta = 0$
poles come on real axis
so graph either of two



Amplitude of peak represents amount of selectivity. Higher amount of mag., good selectivity, lower " " ", bad selectivity.

Q. Draw mag. and phase plot for the transfer fn.

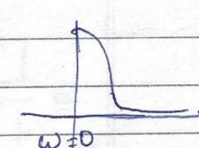


$$\cancel{v_0(s)} = \frac{1}{C} \int I(s)\,dt$$

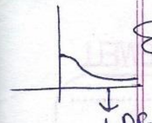
$$v_0(s) = \frac{I(s)}{Cs}$$

$$v_i(s) = R\,I(s) + \frac{I(s)}{Cs}$$

$$\frac{v_0(s)}{v_i(s)} = \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{1}{RCs + 1} \qquad s = \frac{-1}{RC}$$



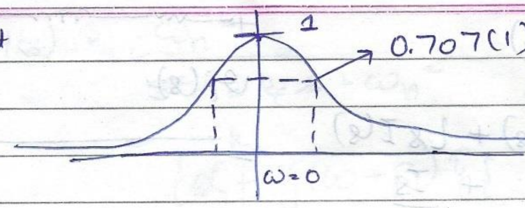
At $\beta = 0$ i.e imaginary part $= 0$ we are getting max. peak. Thus Low pass filter.



$\xi > \frac{1}{\sqrt{2}}$

(case of max flatness)
(poorest LPF)

Mag plot



$$\frac{1}{RC}\left[\frac{1}{j\omega + \frac{1}{RC}}\right] = \frac{1}{RC\sqrt{\omega^2 + \left(\frac{1}{RC}\right)^2}}$$

At $\omega = 0$

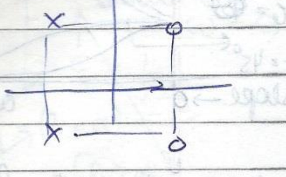
Max value $= \underline{\underline{1}}$

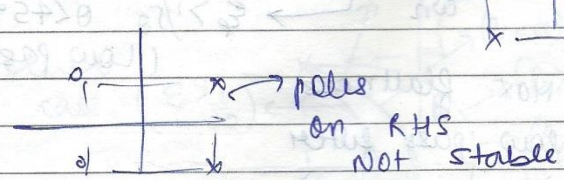
* $\boxed{\text{Cut off freq. also called Half Power Freq.}}$

Q which of the foll. statements is / are true and physically realisable

1) All pass fn cannot be a driving pnt fn.
2) Non min. phase fn cannot be a " " ".
3) Min phase fn cannot be a " " ".
4) All of these are correct.

As plot of Z of all pass is when of Y then both interchange





* Transfer fn may be a ratio fn so in that case it is not necessary that poles & zeroes interchange.

Date ....
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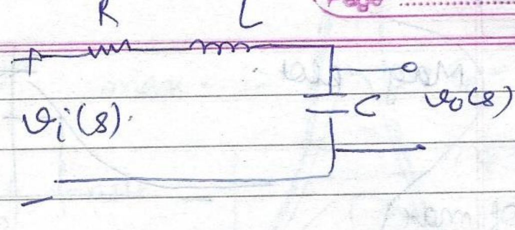


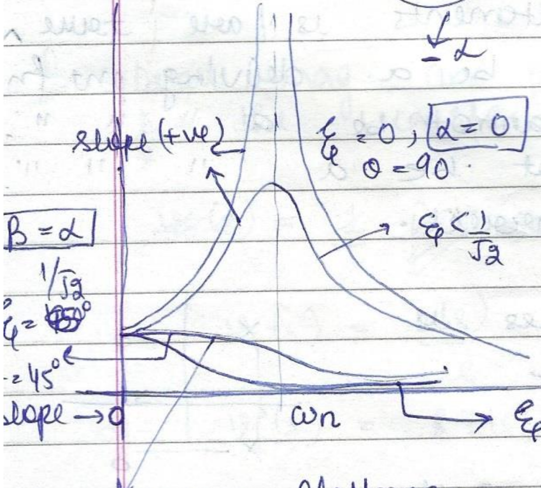
$$v_o(s) = \frac{I(s)}{Cs}$$

$$v_i(s) = RI(s) + Ls\,I(s) + \frac{Is}{Cs}$$

$$\frac{v_o(s)}{v_i(s)} = \frac{\frac{1}{Cs}}{R + Ls + \frac{1}{Cs}} = \frac{1}{RCs + LCs^2 + 1}$$

$$= \frac{1}{LC\left[s^2 + \frac{R}{L}s + \frac{1}{LC}\right]}.$$

$$P_{1,2} = -\underbrace{\frac{R}{2L}}_{\alpha} \pm j\underbrace{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}_{\pm\beta}$$



$\xi < \frac{1}{\sqrt{2}}$ $\theta > 45°$ $\boxed{\beta > \alpha}$

$\xi = 1/\sqrt{2}$
$\boxed{\theta = 45°}$
critical value of ξ

$\xi > 1/\sqrt{2}$ $\theta < 45°$ $\boxed{\alpha > \beta}$
(Low pass filter but good one)

case of Max. flattness
(Bad low pass filter in this case)

$$H(s) = \frac{\omega_n^2}{s^2 + 2\xi_\varphi s + \omega_n^2}$$

$$= \frac{k}{\{\alpha + j(\omega+\beta)\}\{\alpha + j(\omega-\beta)\}}$$

$$|H(j\omega)| = \frac{k}{\sqrt{\alpha^2+(\omega+\beta)^2}\,\sqrt{\alpha^2+(\omega-\beta)^2}}$$

$$M_{(j\omega)}^2 = \frac{k^2}{\left[\alpha^2+(\omega+\beta)^2\right]\left[\alpha^2+(\omega-\beta)^2\right]}$$

for $M(j\omega)$ to be max. $M^2(j\omega)$ must be max.

$$\frac{d[M^2(j\omega)]}{d\omega} = 0$$

$$\boxed{\omega_{max} = \beta^2 - \alpha^2}$$

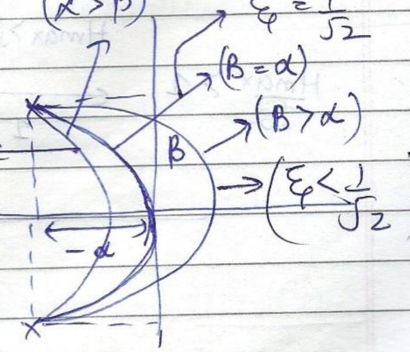
$$\omega_{max} = \omega_n\sqrt{1-\xi_\varphi^2} - \omega_n\xi_\varphi$$
$$= \omega_n \sqrt{1-\xi_\varphi^2} - \omega_n\xi_\varphi$$

$$\boxed{\omega_{max}^2 = \omega_n^2(1-2\xi_\varphi^2)}$$

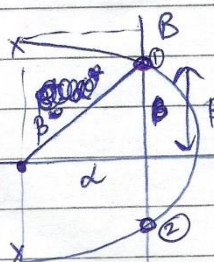
freq. at which peak will occur.

* To easily find ω_{max}

Peaking will occur only in $\beta > \alpha$ case $\left(\xi_\varphi > \frac{1}{\sqrt{2}}\right)$



Point ① and ② represent ω_{max} where they intersect y axis.



radius of circle = β.

Thus this intersection gives ω_{max}
This circle is called Peaking circle.

$|H(j\omega)| \rightarrow$ Peak when $\beta > \alpha \Rightarrow \theta > 45$

$\Rightarrow \xi_\ell < \frac{1}{\sqrt{2}}$

Q factor

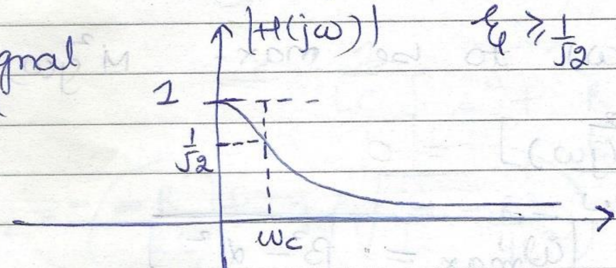
$$Q \text{ factor} = \frac{1}{2\xi_\ell}$$

Half Power frequency / cut off frequency

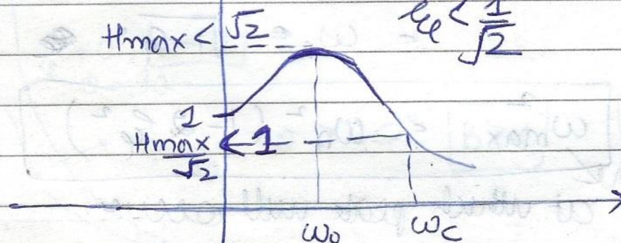
$\xi_\ell \geq \frac{1}{\sqrt{2}}$ $\theta \leq 45$

If voltage signal is considered then at this point $\frac{V_{max}}{\sqrt{2}}$

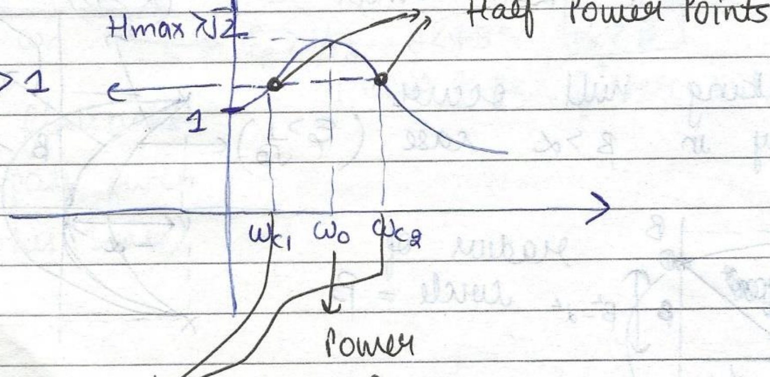
and Power = $\frac{P_{max}}{2}$



$\xi_\ell < \frac{1}{\sqrt{2}}$



$\frac{H_{max}}{\sqrt{2}} > 1$



$$\boxed{\omega_0^2 = \omega_{c1} \cdot \omega_{c2}}$$

GRAPH THEORY

$\xi = \frac{1}{\sqrt{2}}$