Problem p. 109: case 2 minimizes the noise-to-signal ratio $\Leftrightarrow$ maximizes the signal-to-noise ratio

$$\frac{\sigma_v^2}{d^2} = \frac{1}{P_{av}T} \underbrace{\int_{-W}^{W} \frac{|X_{rc}(f)|^2}{|C(f)|^2 |G_R(f)|^2} df}_{\text{squared norm of } U_2(f)} \underbrace{\int_{-W}^{W} \Phi_{nn}(f) |G_R(f)|^2 df}_{\text{squared norm of } U_1(f)}$$

Use Cauchy-Schwartz's inequality:

$$\boxed{\begin{array}{l}\text{For arbitrary vectors } U_1(f) \text{ and } U_2(f) \\ \|U_2(f)\|^2 \|U_1(f)\|^2 \geq |\langle U_1(f), U_2(f)\rangle|^2\end{array}}$$

with equality iff $U_1(f) \sim U_2(f)$

Define both functions:

$$U_1(f) = \sqrt{\Phi_{nn}(f)}\,|G_R(f)|$$

$$U_2(f) = \frac{|X_{rc}(f)|}{|C(f)|\,|G_R(f)|}$$

$\Rightarrow$ minimum noise-to-signal ratio $\frac{\sigma_v^2}{d^2}$ achieved for $U_1(f) \sim U_2(f)$

proportionality constant

$$\Leftrightarrow \sqrt{\Phi_{nn}(f)}\,|G_R(f)| = K^2 \frac{|X_{rc}(f)|}{|C(f)|\,|G_R(f)|}, \quad K > 0$$

solve for $|G_R(f)|$ $\Leftrightarrow$

$$|G_R(f)| = K \frac{\sqrt{|X_{rc}(f)|}}{[\Phi_{nn}(f)]^{1/4}\sqrt{|C(f)|}} \quad \text{for } |f| \leq W$$

From $|G_R(f)||C(f)||G_T(f)| = |X_{rc}(f)|$

$$\Rightarrow |G_T(f)| = \frac{|X_{rc}(f)|}{|C(f)||G_R(f)|}$$

$$= \frac{1}{K} \frac{\sqrt{|X_{rc}(f)|}}{\sqrt{|C(f)|}} \left[\Phi_{nn}(f)\right]^{1/4} \text{ for } |f| \leq W$$

Determine the maximum achievable SNR

$$\Rightarrow \frac{d^2}{\sigma_v^2} = \frac{P_{av} T}{\left[\int_{-W}^{W} \frac{X_{rc}(f)\sqrt{\Phi_{nn}(f)}}{|C(f)|} df\right]^2}$$

Special case of AWGN with power spectral density $\frac{N_0}{2}$:

$$\Rightarrow G_R(f) = K_1 \sqrt{\left|\frac{X_{rc}(f)}{C(f)}\right|} \quad \text{for } |f| \leq W$$

$$G_T(f) = K_2 \sqrt{\left|\frac{X_{rc}(f)}{C(f)}\right|} \quad \text{"} \quad \text{"}$$

$$\Rightarrow \text{SNR} = \frac{d^2}{\sigma_v^2} = \frac{2 P_{av} T}{N_0} \frac{1}{\left[\int_{-W}^{W} \frac{X_{rc}(f)}{|C(f)|} df\right]^2}$$