[Exam 3-4 marks always]

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Random Signals :-

$\xrightarrow{I/P}$ [P.U.] $\rightarrow$ O/P

System consists of I/P, O/P, P.U.

If I/P (excitation) & P.U. is known then analysis can be done.

Signal can be information/ material / energy

Not a signal $\quad f(t) = \sin t$

Not a signal $\quad f(t) = \sin t \cdot v$ $\quad$ physical parameter changes sinusoidally with time

$v(t) = \sin t \cdot v$ $\quad$ v(t) R

$\rightarrow$ $v(t) = g(t) \sin t\, v$

Random signal / Underterdetermined $\because g(t)$ is unknown

$\longrightarrow$ $e^{\alpha t}$ is signal. Its value is growing. It is not static. It is not random now $\because e^{\alpha t}$ is known.

e.g. Throwing a dice

$\{1, 2, 3, 4, 5, 6\}$

$f(x) = 10 f_i$

$f(f_i) = 10 f_i$

random variable.

Random signal - Signal associated with random variable

$\sin\{10 f_i\}$

$\rightarrow$ If random signal is I/P then O/P will also be random.

e.g. - environmental conditions like noise

ECG | EEG signal low freq. signal / low magnitude signal

$\because$ If ~~info~~ interference (EM waves) - Noise can be random.

Estimation is done to predict the Noise to cancel it from previous evidence.

→ Signals

Probabilistic (for discrete system) | Statistical (For large data)

No. of elements in sample space is ∞ – continuous

– Probability that today's temp lying b/w 25–30°C is easy to find than that at a particular temp like 27.1°C

Basic Concepts of Probability:-

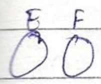
random variable, sample space, event

1) Equally likely event → $\{P(E) = P(F)\} \not\Rightarrow E \neq F$

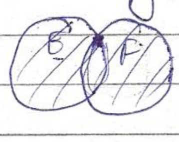
If $E = F \Rightarrow P(E) = P(F)$

2) Mutually Exclusive events – Two events E and F are Mutually exclusive if $E \cap F = \phi$

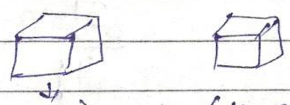
$P(E \cap F) = 0$



3) Collectively exhaustive events – Two events E and F are collectively exhaustive events if $E \cup F = S$



4) Independent events:



(1, 2, - - - 6) (4, 5, 6 - - - 9)

E_1 (2, 4, 6) E_2 (4, 6, 8)

$E_1 \cap E_2 \neq \phi$

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E_1 & E_2 are independent of each other.
Mutually exclusive events & independent events have no relation.

De Morgan's rule :-

$\rightarrow \; (E_1 \cup E_2)^c = E_1^c \cap E_2^c$

$\rightarrow \; (E_1 \cap E_2)^c = E_1^c \cup E_2^c$

$\left[\cancel{P(E_1 \cup E_2)^c} \;\; P(E_1^c \cap E_2^c) = 1 - P(E_1 \cup E_2) \right]$

$A = n_A$

$n \to ?$

$P(A) = \frac{n_A}{n} = \frac{200}{1000} = 1/8$

$n_A' = 250$

$\lim_{n \to \infty} n_A \to n_A'$

$$\boxed{P(A) = \lim_{n \to \infty} \frac{n_A}{n}}$$

Tossing a coin

$P(H) = P(T) = 1/2$

H T Two coin

2 2

Not certain

H H H T T - - - -

$\frac{2}{3} \; \frac{3}{4} \; \frac{3}{5} - - - \; \underline{1/2}$

Pb If out of all possible jumbles of 'BIRD' a random word is picked. What is the probability that this word will start with letter 'B'?

$^4P_3$

$\frac{|4}{|0}$ $4 \times 3 \times 2$ $\underline{24}$

$^3P_3$

$^4P_3$

$\frac{|4}{|1}$

$[\;\;|\;\;|\;\;|\;\;]$

$\underline{4} \;\; 3 \;\; 2 \;\; 1 \; = 24$

$\underline{B}$

$\frac{3!}{4!} = \frac{1}{4}$

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Pb

Grade	A	B	C	D
No. of students	10	20	30	40

What is the probability of students getting grade A

$$\frac{10}{100} = 0.1$$

AXIOMS OF PROBABILITY

(1) $0 \leq P(E) \leq 1$

(2) $P(S) = 1$

(3) For any sequence of mutually exclusive events $E_1, E_2 ------ E_n$ then probability of $[E_i \cap E_j = \phi \quad \text{if } i \neq j]$

$$P(\bigcup_i E_i) = \sum P(E_i)$$

$P(E_1 \cup E_2 \cup E_3 \cup --- E_n)$

$\left[P(A \cup B) = P(A) + P(B) - \underbrace{P(A \cap B)}_{0} \right]$

$\{1, 2, 3, 4, 5, 6\}$

$n(E_1) + n(E_2) + --- n(E_6) = n$

$$\frac{n(E_1)}{n} + \frac{n(E_2)}{n} + -- \frac{n(E_6)}{n} = 1$$

$$P(E_1) + P(E_2) + -- P(E_6) = 1$$

Complete event: No. of trials must be equal to or greater than No. of outcomes. (= when No. of outcomes are equal to sample space)

Pb

R	B	W	G
10	20	40	30

2 Characters
Weight + colour

$P(R) = 0.1 \quad P(B) = 0.2 \quad P(W) = 0.4 \quad P(G) = 0.3$

	R	B	W	G	
1g	5	10	25	0	40
2g	5	5	0	10	20
3g	0	5	15	20	40
	10	20	40	30	

Joint Probability $P(G,3)$ = Prob. that ball is green & has weight 3g $= \frac{20}{100} = 0.2$

$P(G \cap 3)$

Conditional prob. $P(G|3)$ = Probability that ball is green given that weight is 3g $= \frac{20}{40} = 1/2$

Marginal prob.

$P(G) = \frac{30}{100} = 0.3$

$P(3) = \frac{40}{100} = 0.4$

$P(G,3) = P(G/3) \cdot P(3)$

$= 0.5 \times 0.4 = 0.2$

0.20

→ $P(E \cap M) = P(E/M) \cdot P(M)$

→ If E & M are independent events then

$P(E/M) = P(E)$

$P(E \cap M) = P(E) \cdot P(M)$

→ $P(A/M) = \begin{cases} \frac{P(A \cap M)}{P(M)} & \text{If } P(M) \neq 0 \\ \text{Not definite} & \text{If } P(M) = 0 \end{cases}$

i.e. M should not be impossible event

→ $P(A \cap M) = P(A/M) \cdot P(M)$

$= P(M/A) \cdot P(A)$

→ $P(A/M) = P(A)$ if A and M are independent events

Pb $S = \{BB, BG, GB, GG\}$

What is prob. that the first child as well as second child is girl

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What is the prob. that both children are girl given that Ist child is girl?

(i) 1/4

(ii) $A = \{GG\}$ $\quad B = \{GG, GB\}$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/2} = 1/2$$

Pb If $P(A/M)$ represents conditional prob. of A. Given M which of following options are true?

a) $P(A/M) \geq 0$ ✓

b) $P(S/M) = 1$ ✓

c) $P\{(A \cup B)/M\} = P(A/M) + P(B/M)$ if A & B are mutually exclusive ✓

d) None

Check arrow

$$\frac{P((A \cup B) \cap M)}{P(M)} = \frac{P(A \cap M)}{P(M)} \cup \frac{P(B \cap M)}{P(M)}$$

distributive property

$$= P(A/M) + P(B/M)$$

Pb An examination consists of two papers Paper1 & Paper2. Prob of failing in Paper1 is 0.3 and that in paper 2 is 0.2. given that student has failed in paper2 the prob. of failing in paper1 is 0.6. Prob. of student failing in both papers is?

$P(1) = 0.3 \qquad P(2) = 0.2$

$P(1/2) = 0.6 \qquad P(1 \cap 2) = ?$

$$P(1 \cap 2) = P(1/2)\,P(2)$$
$$= 0.6 \times 0.2$$
$$= 0.12$$

Certain event $P(A/M) = 1$ A is sample space

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Pb If P & Q are two random events then which of following is true?

× a) Independence of P & Q employs that $P(P \cap Q) = \phi$

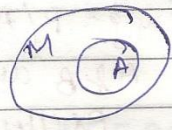
× b) prob. $P(P \cup Q) \geq P(P) + P(Q)$ ≤ for this option to be correct $P \cap Q = \phi$

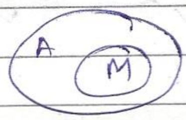
× c) If P and Q are mutually exclusive events then they must be independent

✓ d) Pr $P(P \cap Q) \leq P(P) / P(Q)$ Equality condition when subset

$$P(A \cap M) = P(A/M)\,P(M)$$

$A \subseteq M$ $A \supseteq M$





$P(M) = P(A/M)\,P(M)$

$P(A/M) = 1$

$P(A) = P(A/M)\,P(M)$

$P(A/M) = \dfrac{P(A)}{P(M)} \leq P(M)$

Pb dice value

D.V.	1	2	3	4	5	6
Prob.	1/4	1/8	1/8	1/8	1/8	1/4

Loaded dice i.e. unfair dice [un uniform distribution of weight]

If 3 identical die are thrown the prob. of occurance of event {1,5,6}

$\frac{1}{4}+\frac{1}{8}+\frac{1}{8}=2+1+$

$P((1\cap5)/6)P(6)$

$P(P(1/5))\ 1/4 \cdot 1/4$

$\frac{16}{8}$ $\frac{}{128}$

~~$\frac{1}{4}$~~ $P(1 \cap 5 \cap 6) = P(1) \times P(5) \times P(6)$ ∵ all are independent

$$= \frac{1}{4} \times \frac{1}{8} \times \frac{1}{4} = \frac{1}{128}$$

Pb A fair dice is rolled twice. What is probability that an odd No. will follow an even No.?

OO OE EO EE 1/4

~~$P(O/E) = \frac{P(O \cap E)}{P(E)} P(E)$~~ $= P(E \cap O/E) = \dfrac{P(E \cap O) \cap E}{P(E)}$

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Pb A fair coin is tossed 10 times? What is the prob. that only first two tosses will give head?

{H H T T — — T}

Total elements in sample space = 2^{10}

$= \frac{1}{2^{10}}$ $\left[P(H) \times P(H) \times P(T) - - - P(T) \right]$ $\because$ they are independent

Pb A single dice is thrown twice? What is the probability that sum is neither 8 nor 9?

1 − either 8 or 9

$1 - \frac{\cancel{9}}{\cancel{36}_{4}} = 1 - \frac{1}{4} = \frac{3}{4}$

Pb

A box contains two washers, 3 Nuts & 4 bolts. Items are drawn from box at random one at a time without replacement. What is prob. of drawing two washers first followed by 3 Nuts and then 4 bolts

W N B
2 3 4

W	W	N	N	N	B	B	B	B

$\frac{2}{9} \times \frac{1}{8} \times \frac{3}{7} \times \frac{2}{6} \times \frac{1}{5} \times \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} \times \frac{1}{1} =$

Pb A box contain 4 white & 3 red balls. In succession two balls are randomly selected & removed from box. Given that first removed ball is white what is prob. that 2nd ball is red?

WW
RR $= \frac{2}{4} \cdot \frac{1}{2}$
RW
WR

1/2

W W W W R R R $P(R/W) = \frac{3}{6} = 1/2$

$\frac{P(R) \cdot P(W)}{P(W)} = \frac{\frac{3}{7} \cdot \frac{4}{6}}{4/7}$

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Pb Prob. of an event, $P(A) = 1$, $P(B) = 1/2$ That value of $P(A/B)$ and $P(B/A)$ if A & B are independent

$P(A/B) = P(A) = 1$, $P(B/A) = P(B) = 1/2$

Pb Aishwarya studies either C.S. / Maths on a day. If she studies C.S on a day than prob. that she studies Maths Next day is 0.6. If she studies Maths on a day than prob. that she studies C.S. next day is 0.4. Given that she studies C.S. on Monday. What is prob. she studies C.S. on Wednesday?

$P(M/C.S.) = 0.6$, $P(C.S/M) = 0.4$

$0.6 = P(M/C.S.) = \frac{P(M \cap C.S.)}{P(C.S.)}$ $\quad \frac{P(M \cap C.S.)}{P(M)} = 0.4$

~~0.6 P(C.S.) = P(M∩C.S).~~

0.6

$\frac{0.6}{0.4}$ $\frac{3}{2}$

$\frac{2}{8}$

M	T	W
C	M	C
C	C	C

$P(C \cap M \cap C) + P(C \cap C \cap C)$

$P(\overset{E}{\overline{C \cap M}})$ $\quad P(C \cap C)$

$\frac{P(M/C)\,P(C)}{0.6 \quad 1}$ $\quad \frac{P(C/C)\,P(C)}{1 - P(M/C)}$

$0.4 \times \cancel{0.4} = 0.4$

$P(C \cap E) = P(C/E) \cdot P(E)$

$= P(C/M) \cdot P(E)$

$= 0.4 \times 0.6$

$= 0.24$

$P(C \cap E) = P(C/E) \cdot P(E)$

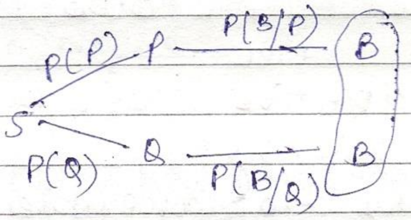
$= P$

Total Probability – If sample space S is collection of events $A_i = \{A_1, A_2 \ldots A_n\}$

$$A_i \cap A_j = \phi \quad i \neq j \qquad \text{exclusive}$$

$$\bigcup_i A_i = S \qquad \text{collectively exhaustive}$$

then $P(B) = P(B/A_1) \cdot P(A_1) + P(B/A_2) \cdot P(A_2) \ldots + P(B/A_n) P(A_n)$



$= P(B \cap P)$

$P\{B \cap (\bigcup A_i)\} \quad P\{B \cap S\} = P\{B\}$

$\Rightarrow \quad P(B) = P(P) \cdot P(B/P) + P(Q) \cdot P(B/Q)$

$$P(P/B) = \frac{P(B \cap P)}{P(B)}$$

$$= \frac{P(B/P) \cdot P(P)}{}$$

Pb We have two bags: Bag 1 contain 2 Red & 5 Green Marbles. Bag 2 contain 2 Red & 6 Green Marbles. A person tosses a coin & if it is head goes to bag 1 and draws a marble & if it is tail goes to bag 2 and draws a marble.

(i) What is prob. that marble drawn is red

(ii) Given that marble drawn is red what is prob. that it came from bag 1.

Pb 3 companies X, Y, Z ~~co~~ supply computers to a university. The percentage of computers supplied by them and the probability of those being defective are given

Comp	%age	Prob. (defective)
X	60	0.01
Y	30	0.02
Z	10	0.03

(i) Given that a comp. is defective what is prob. that it was supplied by company Y.

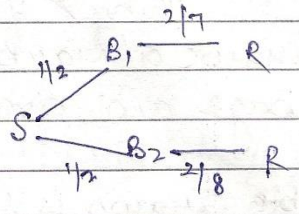
Pb (ii) Consider a company that assembles comp. The prob. of a faulty assembly of any comp. is P. The company so subjects each comp. to a testing process. This testing process provides correct result of any comp. with prob. of Q.

(i) What is the prob. that a comp. is being declared faulty?

(ii) What is prob. that a comp. is being declared Non faulty?

(iii) ~~What~~ If comp. is declared as Non faulty comp. then what is prob. that comp. is actually faulty.

Solu 1.



(i) $P(R) = \frac{1}{2} \times \frac{2}{7} + \frac{1}{2} \times \frac{2}{8}$

$= \frac{1}{7} + \frac{1}{8} = \frac{15}{56}$

(ii) $P(B_1/R) = \frac{P(B_1 \cap R)}{P(R)} = \frac{\cancel{\frac{1}{2}} \times \frac{\cancel{2}}{7}}{\frac{15}{56}} = \frac{8}{15}$

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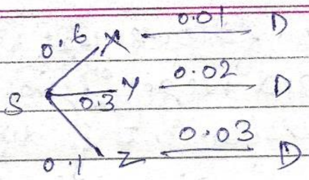
Sol 2



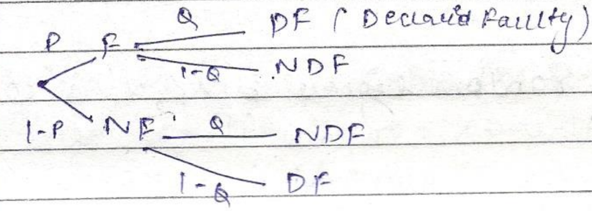
$$P(Y/D) = \frac{P(Y \cap D)}{P(D)}$$

$$= \frac{0.3 \times 0.02}{(0.6 \times 0.01) + (0.3 \times 0.02) + (0.1 \times 0.03)}$$

$$= \frac{0.006}{0.006 + 0.006 + 0.003} = \frac{0.006}{0.015} = \frac{2}{5}$$

$$= 0.4$$

Sol 3



(i) $P(DF) = PQ + (1-P)(1-Q)$

(ii) $P(NDF) = P(1-Q) + (1-P)(Q)$

(iii) $P(F/NDF) = \frac{P(F \cap NDF)}{P(NDF)} = \frac{P(1-Q)}{P(1-Q) + (1-P)Q}$

RANDOM VARIABLE: A random variable is one which is assigned a value corresponding to every outcome of a random exp.

e.g. $\{ \ldots \}$ $\leftarrow X(\xi_i)$

RANDOM SIGNAL - A signal / information carrier which is dependent on random variable.

$\sin \{X(\xi_i)\}$

Random signal is also known as sample func.

RANDOM PROCESS - It is the process which generate random signal.

It is denoted by $\{x(t)\}$

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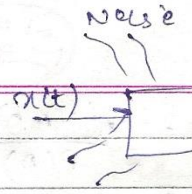
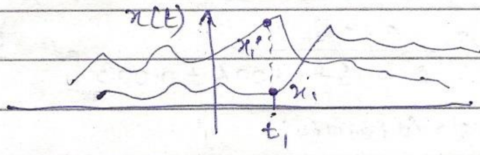
e.g. 

We need to filter so that noise is removed and we get original signal.

Suppose 100 instruments of different values are used. They will give different signals.



(constant time) At t_1 value of random signal is x_1, x_1' is random variable

ENSEMBLE: It is special process in which probablistic value is assigned corresponding to each sample signal.

- It is denoted as $\{x(t), Px_i(t)\ \forall i\}$

→ All ensembles are random process but not vice versa

→ Random Variable

- D.R.V.
- C.R.V.
- M.R.V.

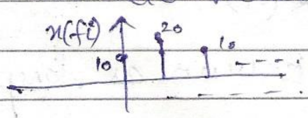
1 Discrete Random Variable (D.R.V.):

e.g. $\{1, 2, 3, 4, 5, 6\}$

$X(f_i) = 10i$ f_i is facevalue, X is random variable

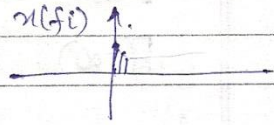
$x = 10, 20, 30, 40, 50, 60$

i.e. discrete value of x. ∴ It is D.R.V.



2 Continuous Random Variable (C.R.V.):- e.g. If dice is thrown at fraction of nanosec. No doubt

it has interval of fraction of nano sec. but it is approx. continuous.

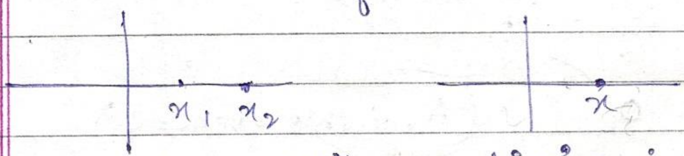


e.g. temp. variation is random & continuous.

$f(T_0)$ is C.R.V.

3 Mixed Random Variable (M.R.V.) : There is piece wise continuity i.e. sometimes continuous & sometimes discrete.

Main concern of our is C.R.V.



probability of finding in a interval is easy or it exists. But at a particular point probability cannot be found in C.R.V.

No. of events ($1/\infty = 0$)

$$P(x) = P(X \leq x)$$

$X \leq x$ represents set of outcomes ξ such that $X(\xi) \leq x$

$x_1 \leq X \leq x_2$ i.e. events within this interval remaining all other sample space

$X \geq x_2$

$$P(X \geq x_2) = 1 - P(X \leq x_2)$$
$$= 1 - P(x)$$

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$P(\infty) = P(X \leq \infty) = 1$ Certain event

$P(-\infty) = P(X \leq -\infty) = 0$ Impossible event

e.g. For dice exp, outcomes are $\{f_1, f_2, f_3 --- f_6\}$
$X(f_i) = 10i$
Find prob. of
(i) $(x \leq 35)$ (ii) $P(x=35)$ (iii) $P(x \leq 5)$
(iv) $P(x=\infty)$ (v) $P(x=-\infty)$ (vi) $P(x=40)$

$X(f_i) = \{10, 20, 30, 40, 50, 60\}$

(i) $P(x \leq 35) = \frac{3}{6} = 1/2$

(ii) $P(x=35) = 0$ (Available event $= \phi$)

(iii) $P(x \leq 5) = 0$

(iv) $P(x=\infty) = 0$ [it is discrete R.V.]

(v) $P(x=-\infty) = 0$

(vi) $P(x=40) = 1/6$

$F_X(x) \equiv P\{X \leq x\}$

$F_X(x) \rightarrow$ Probability Distribution Function
denoted as P.D.F

e.g. Throwing a coin $X(h)=1$, $X(t)=0$

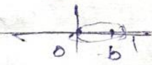
(i) $P\{X>1\}$
$=0$

(ii) $P\{X \leq a \ \forall a \geq 1\}$

$\{h, t\}$

$= 2/2 = 1$

(iii) $P\{X \leq b \ \forall \ 0 \leq b < 1\}$



$\{t\}$

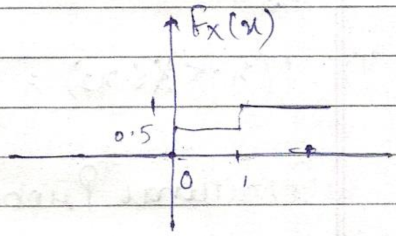
$= 1/2$

(iv) $\{x \leq c \ \forall c < 0\}$

$= \phi$

$\therefore P\{x \leq c \ \forall c < 0\}$

$= 0$



Properties of P.D.F:-

(i) $F(\infty) = 1$ $\quad P(X \leq \infty) = 1$

(ii) $F(-\infty) = 0$ $\quad P(X \leq -\infty) = 0$

(iii) $x_1 \geq x_2 \Rightarrow F(x_1) \geq F(x_2)$ $\quad P(X \leq x_1) \geq P(x \leq x_2)$

It means P.D.F is Monotonically Non decreasing type of func. [↑ing or =]

(iv) $P(x_1 < X \leq x_2) = F(x_2) - F(x_1)$

(v) $F(x^+) = F(x)$ (holds for both continuous & discrete R.V)

(vi) $P(x = x) = F(x) - F(x^-)$ [for D.R.V. only] $\quad$ for C.R.V $F(x) = F(x^-)$

(vii) $P\{x_1 \leq X \leq x_2\} = F(x_2) - F(x_1^-)$

Probability Density Function (pdf)

$f_X(x) = \frac{d}{dx}\{F_X(x)\} \geq 0$ $\quad$ [$\because F_X(x)$ is Monotonically non decreasing]

Properties:-

1) $f_X(x) \geq 0 \quad \forall x$

2) $F_X(x) = \int_{-\infty}^{x} f_X(u)\,du$ $\quad \left[F_X(x) - \underset{0}{F_X(-\infty)} \right]$

3) $\int_{-\infty}^{\infty} f_X(u)\,du = 1$

4) $P(x_1 < X \leq x_2) = F_X(x_2) - F_X(x_1) = \int_{x_1}^{x_2} f_X(u)\,du$

Conditional Probability Distribution function

(1.) $F(x/M) = P\{X \leq x/M\} = \dfrac{P\{(X \leq x) \cap M\}}{P(M)}$

(2.) $F(\infty/M) = \dfrac{P(\infty \cap M)}{P(M)} = 1$

(3.) $F(-\infty/M) = \phi \quad \therefore \dfrac{P(-\infty \cap M)}{P(M)} = 0$

(4.) $F\left(\dfrac{x_1 < X \leq x_2}{M}\right) = F(x_2/M) - F(x_1/M)$ $\quad$ [For discrete $x_1 \leq X \leq x_2$: $F(x_2/M) - F(x_1^-/M)$]

Conditional Probability Density function (pdf)

$f(x/M) = \dfrac{d}{dx} F(x/M)$

eg. A dice is thrown $M = \{f_2, f_4, f_6\}$

$X(f_i) = 10i$

$F(x/M) - ?$

(i) $\forall X \geq 60$

$X \leq x = \{10, 20, 30, 40, 50, 60\}$

$(X \leq x) \cap M = \{f_2, f_4, f_6\}$

$F(x/M) = 1$

(ii) $\forall\ 40 \leq X \leq 60$

$X \leq x = \{f_1, f_2, f_3, f_4, f_5\}$

$(X \leq x) \cap M = \{f_2, f_4\}$

$F(x/M) = \dfrac{2/6}{3/6} \dfrac{P(\cdot)}{P(M)}$

$\Rightarrow \boxed{2/3}$

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(iii) $\forall\ 20 \leq X \leq 40$

$\{f_1, f_2, f_3, f_4\}$

$P(x/M) = \dfrac{2/6}{3/6} = \dfrac{2}{3}$

(iv) $\forall\ X < 20$

$\{f_1\}$

$= 0$

Total probability: $\{a_1, a_2 \ldots a_n\}$

$a_i \cap a_j = 0 \quad i \neq j$ Mutually exclusive

$\bigcup_i a_i = \Sigma a_i$ Collectively exhaustive

$$P(X \leq x) = P(X \leq x/a_1)\, P(a_1) + \ldots \ldots P(X \leq x/a_n) \cdot Pa_n$$

Base Theorem $P(a/X \leq x) = \dfrac{P(X \leq x/a) \cdot P(a)}{P(X \leq x)}$

$$= \frac{P(x/a) \cdot P(A)}{P(x)}$$

$$P(a/x_1 < X \leq x_2) = \frac{[P(x_2/a) - P(x_1/a)]\, P(a)}{P(x_2) - P(x_1)}$$

$$= \frac{[F(x_2/a) - F(x_1/a)]\, P(a)}{F(x_2) - F(x_1)}$$

$$P(a/X = x) = \frac{P(X = x/a) \cdot P(a)}{P(X = x)} = \frac{F(X = x/a) \cdot P(A)}{F(X = x)}$$

In continuous R.V $F(X = x)$ is NOT defined

$\therefore$ Modification is Needed

$x_2 = x + \varepsilon, \quad x_1 = x$

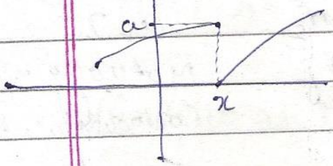
$$P(a/x_1 < x \leq x_2) = \frac{F(x + \varepsilon/a) - F(x/a)}{F(x + \varepsilon) - F(x)}$$

$$\int_{x}^{x+\epsilon} f(x|a)\,\Delta x$$
$$f(x|a)\int_{x}^{x+\epsilon}\Delta x$$
$$f(x|a)\,dx$$

$$= \frac{f(x/a)\,dx}{f(x)\,dx} = \frac{f(x/a)}{f(x)}$$

If $F(x)\big]_{x=a} = 0$ then $F'(x)\big|_{x=a} \neq 0$ always

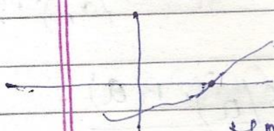
[where slope is not zero $\frac{dy}{dx} \neq 0$]



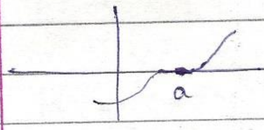
$0 - a = -a$

$$\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = -a\,u_x$$

$$-a \lim_{\Delta x \to 0} \frac{u_x}{\Delta x} = -a\frac{du_x}{dx} = -a\,\delta(x)$$



Here also $F'(x)\big|_{x=a} \neq 0$



Here $F'(x)\big|_{x=a} = 0$

Hence $P(A/X=x) = \frac{f(x/A)\cdot P(A)}{f(x)}$

$$P(A) = \int_{-\infty}^{\infty} P(A/X=x)\,f(x)\,dx \qquad [C.R.V]$$

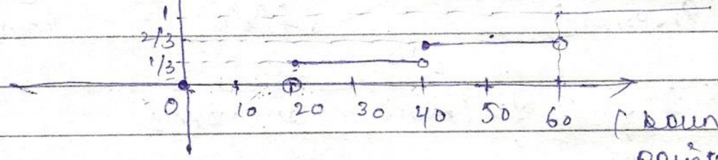
$$P(x/A) = \frac{P(A/X=x)\,f(x)}{P(A)}$$ Base Thm in C.R.V

Changes are possible in discrete but not in continuous [equality]

$F(x|M)$ when value varies from $-\infty$ to ∞

$0 \le F(x|M) \le 1$

PDF plot of previous example $M = \{f_2, f_4, f_6\}$



(boundary points - points at which change is possible)

$X < x_1$
$x_1 \leq X < x_2$
$x_2 \leq X < x_3$
$x_3 \leq X < x_4$
$X \geq x_4$

[$x(\theta)$ may not be equal to $x(\theta)$ discrete]

Since PDF is nondecreasing & start from right

We can't start from left

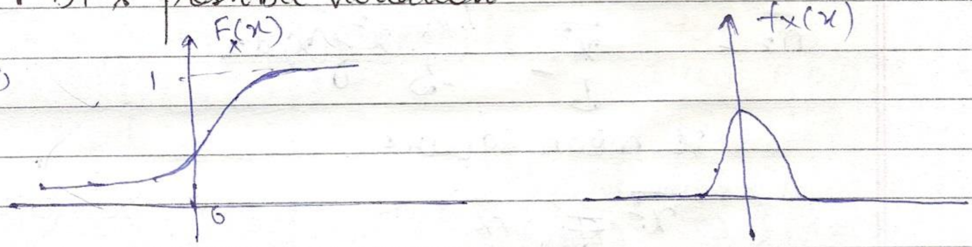
$X \leq x_1$
$x_1 < X \leq x_2$
$x_2 < X \leq x_3$
$x_3 < X \leq x_4$
$x_4 < X$

∵ there occurs problem $20x \leq 40$

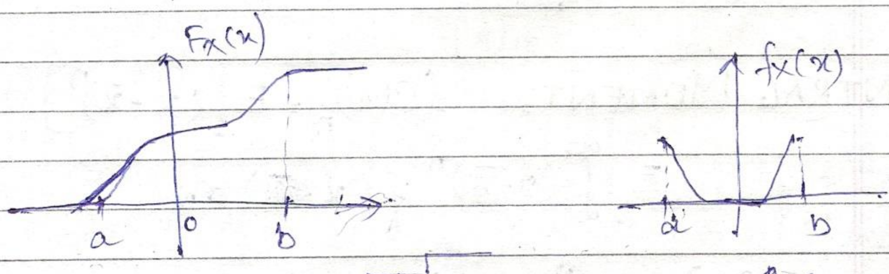
PDF's possible variation

3 forms of PDF

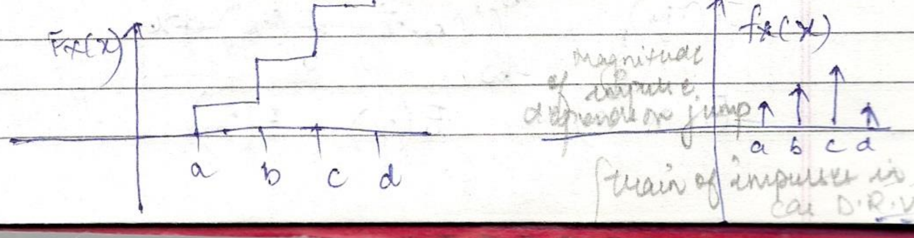
(i) physical



(ii)



(iii)



[train of impulses in case D.R.V.]

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MEAN VALUE :-

$$\underset{(\text{Ensemble})}{R.P} \longrightarrow X \begin{cases} \nearrow F(x) \\ \searrow f(x) \end{cases}$$

$X \rightarrow$ sample func / random variable. Value of x at particular instant

X is func of x / random outcome

$$\bar{X} = \frac{\int_{-\infty}^{\infty} X f(x)dx}{\int_{-\infty}^{\infty} f(x)dx} = \int_{-\infty}^{\infty} X f(x)dx \qquad \left[\because \int_{-\infty}^{\infty} f(x)dx = 1\right]$$

$\bar{X}$ = expected value of 'X' = $E(X)$

(Mean of random variable)

$$E(g(x)) = \overline{g(x)}$$

$g(x)$ – random variable or random function

If $g(x) = x^n$ the $\overline{g(x)} = \int_{-\infty}^{\infty} x^n f(x)dx = \overline{x^n}$

Put $n=1$ $\quad \bar{x} = \int_{-\infty}^{\infty} x f(x) dx$.

$n=2$ $\quad \overline{x^2} = \int_{-\infty}^{\infty} x^2 f(x) dx$.

$\downarrow$

mean square

$$\overline{x^2} \neq \bar{x}^2$$

$\bar{x}^2$ (Square of mean)

CENTRAL MOMENT:

$$(CM)_n = E\left[(x-\bar{x})^n\right]$$

$$= \int_{-\infty}^{\infty} (x-\bar{x})^n f(x) dx$$

$$(CM)_1 = \cancel{\int_{-\infty}^{\infty} (x-\bar{x}) f(x) dx}$$

$$E(x-\bar{x}) = E(x) - E(\bar{x})$$

$$\int_{-\infty}^{\infty} x f(x)\,dx - \int_{-\infty}^{\infty} \bar{x} f(x)\,dx \quad \left[\int_{-\infty}^{\infty} f(x)\,dx = 1\right]$$

constant

$$\bar{x} - \bar{x} = 0$$

$$(C.M)_2 = E(x-\bar{x})^2 = E\left(x^2+\bar{x}^2-2x\bar{x}\right)$$

$$= \int_{-\infty}^{\infty} x^2 f(x)\,dx + \int_{-\infty}^{\infty} \bar{x}^2 f(x)\,dx - \int_{-\infty}^{\infty} 2x\bar{x} f(x)\,dx$$

$$= \overline{x^2} + \bar{x}^2 - 2\bar{x}\,\bar{x} = \overline{x^2} + \bar{x}^2 - 2\bar{x}^2$$

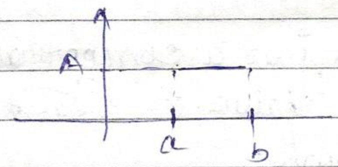
$$= \overline{x^2} - \bar{x}^2 \neq 0$$

$(C.M)_2 = \overline{x^2} - \bar{x}^2$ is known as variance of R.V x represented as σ^2

σ represents standard deviation, $\sigma = \sqrt{\overline{x^2} - \bar{x}^2}$

Pb $f(x) = \begin{cases} A & a < x < b \\ 0 & \text{otherwise} \end{cases}$ find $\bar{x}$, draw P.D.F for x

Uniform density func.



To find value of A

$$A(b-a) = \int_{-\infty}^{\infty} f(x)\,dx$$

[Area]

$$A = \frac{1}{b-a}$$

(i) $$\bar{x} = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{a}^{b} x \cdot \frac{1}{b-a}\,dx = \frac{1}{b-a}\left|\frac{x^2}{2}\right|_a^b$$

$$= \frac{1}{\cancel{b-a}}\left[\frac{b^2-a^2}{2}\right] = \frac{b+a}{2}$$

Slope $\frac{1}{b-a} = A$

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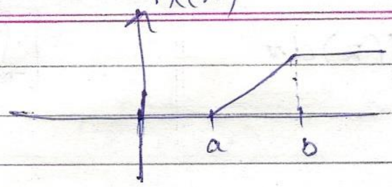
(ii) $F_X(x)$



Standard density func $\int$ limits are given

Pb $\overline{x^2}$ $\cancel{\overline{x^2}}$

$$\int_a^b x^2 f(x)\,dx = \int_a^b x^2 \cdot \frac{1}{b-a}\,dx = \frac{1}{b-a}\left|\frac{x^3}{3}\right|_a^b$$

$$= \frac{1}{3(b-a)}\left[b^3 - a^3\right] = \cancel{\frac{(b-a)^2}{3}} = \cancel{b^2+a^2}.$$

$$= \frac{b^2+a^2+ab}{3}$$

$$\sigma^2 = \overline{x^2} - \overline{x}^2 = \frac{b^2+a^2+ab}{3} - \left(\frac{b+a}{2}\right)^2$$

$$= \frac{4(b^2+a^2+ab) - [3b^2+3a^2+6ab]}{12}$$

$$= \frac{b^2+a^2-2ab}{12} = \frac{(b-a)^2}{12}$$

$$\sigma = \frac{b-a}{\sqrt{12}}$$

Uniform density func is just a conceptual. It is Not part of physical values or real life. $\therefore$ Slope of P.D.F. is known.

Discrete type distribution func

e.g.

X	1	2	3	4	5	6
$P(x)$	1/6	1/6	1/6	1/6	1/6	1/6

(i) $P(X=3)$

$1/6$

(ii) $P(X \geq 3)$

$\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$

$\frac{4}{6} = \frac{2}{3}$

(iii) $P(X \leq 3)$

$3/6 = 1/2$

In discrete $E(x) = \Sigma$
conti. $E(X) = \int$

(iv) $P(X \cancel{\leq} < 4) = 1/2$ Find $E(X)$, variance

$E(X) = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6}$

$\bar{x} = \frac{21}{6} = 3.5$

$\overline{x^2} = x_1^2 f(x_1) + x_2^2 f(x_2) + x_3^2 f(x_3) + \ldots x_6^2 f(x_6)$

$= 1 \times \frac{1}{6} + 4 \times \frac{1}{6} + 9 \times \frac{1}{6} + 16 \times \frac{1}{6} + 25 \times \frac{1}{6} + 36 \times \frac{1}{6}$

$= \frac{1}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6} + \frac{25}{6} + \frac{36}{6} = \frac{91}{6}$

$\bar{x}^2 = \left(\frac{21}{6}\right)^2$

$\sigma^2 = \frac{91}{6} - \left(\frac{21}{6}\right)^2 = 2.917$

$\sigma = 1.707$

Pb An examination paper has 150 MCQs of 1 Mark each with each Ques. having 4 choices. Each incorrect ans. fetches -0.25 marks. Suppose 1000 students selects all their ans. randomly with uniform probability. The sum total of expected Marks obtained by all these students?

discrete

E	Correct ans.	Incorrect ans.
P(E)	1/4	3/4
Marks	1	-0.25

$\bar{x} = 1 \times \frac{1}{4} - \frac{3}{4} \times 0.25 = \frac{1}{16}$

(Mean value of marks obtained by 1 student in 1 Que.)

$\therefore$ Mean value of marks obtained by 1000 students in 150 Que. $= \frac{1}{16} \times 1000 \times 150 = 9375$

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In physical system logically we cannot assign probablistic values :. we use standard distribution func.

1 Binomial Distribution function :- It is applicable under particular conditions :

(a) If only two outcomes are possible. - Success / Failure
(P) / $(1-P)$

(b) Each trial is statistically independent i.e. outcome of 1 trial doesn't influence subsequent trials.

[If balls taken from bag are replaced]

Under these conditions

$$P(X=x) = {}^{n}C_{x} P^{x} (1-P)^{n-x}$$

(Particular event)

n = no. of trials

P = prob. of obtaining x times success in n trials

Q 10 dice are thrown. What is prob. of getting exactly 2 6's ?

$$^{10}C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^8 = 0.2907$$

Q A slot has 10% defective items. 10 items are selected randomly from this slot. Prob. that exactly two of selected items are defective ?

$$^{10}C_2 (0.1)^2 (0.9)^8 = 0.1937$$

Q A coin is tossed 4 times. What is prob. of getting heads exactly 3 times ?

$$^{4}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1 = 0.25$$

Q. 3 coins are tossed simultaneously. Prob. of getting at least 1 head?

$$1-\left[{}^3C_0\left(\tfrac{1}{2}\right)^0\left(\tfrac{1}{2}\right)^3\right] = \frac{7}{8}$$

Q. A fair coin is tossed independently 4 times. The prob. of event the no. of times heads showed up is more than no. of times tails are showed up?

$${}^4C_4\left(\tfrac{1}{2}\right)^4\left(\tfrac{1}{2}\right)^0 + {}^4C_3\left(\tfrac{1}{2}\right)^3\left(\tfrac{1}{2}\right)^1$$

$$= \left(\tfrac{1}{2}\right)^4 + \left(\tfrac{1}{2}\right)^3 \times 2 = \frac{5}{16}$$

Q. If a fair coin is tossed 4 times. What is prob. that 2 heads and 2 tails will be the result?

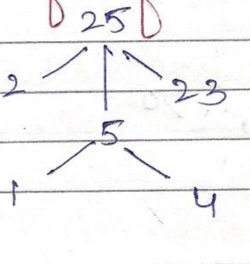
$${}^4C_2\left(\tfrac{1}{2}\right)^2\left(\tfrac{1}{2}\right)^2 = \frac{3}{8} = 0.375$$

2. Hypergeometric distribution – It is used when prob. changes trial to trial
[e.g. taking out balls from bag without replacement]

$E(x) = n\left(\frac{r}{N}\right)$

$$P(X=x) = \left({}^rC_x \cdot {}^{N-r}C_{n-x}\right) / {}^NC_n$$

e.g. There are 25 calculators in box. 2 of them are defective. Suppose 5 cal. are randomly picked for inspection. Each has same chance of being selected. What is prob. that only 1 of defective cal. will be included in inspection



$$P = \frac{{}^2C_1 \cdot {}^{23}C_4}{{}^{25}C_5} = 1/3$$

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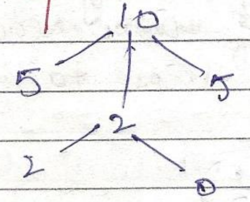
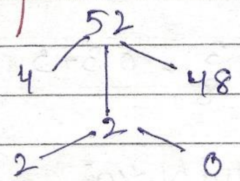
e.g. A box contains 5 black & 5 red balls. Two balls are randomly picked one after another from box without replacement. Find the prob for both balls being red without replacement?



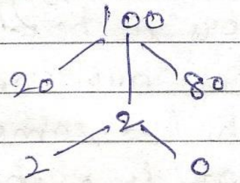
$$\frac{{}^5C_2 \cdot {}^5C_0}{{}^{10}C_2} = 2/3$$

e.g. From a pack of regular plain cards, two cards are drawn at random. What is prob. that both cards are king if first card king is not replaced



$$\frac{{}^4C_2 \cdot {}^{48}C_0}{{}^{52}C_2} = 1/221$$

e.g. A box contain 20 defective and 80 are non defective. If 2 items are selected at random without replacement. What will be prob. that both items are defective?



$$\frac{{}^{20}C_2 \cdot {}^{80}C_0}{{}^{100}C_2} = \frac{19}{495}$$

3 Poisson Distribution :

$$P(X = x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}, \lambda > 0$$

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λ = avg. No. of occurrance of an event in an Observation period of Δt

$\lambda = \alpha \Delta t$

rate of happening of an event

(i) $E(x) = \lambda$ (ii) $\sigma^2 = \lambda$ when P.D.F = $\frac{e^{-\lambda} \cdot \lambda^x}{x!}$, $\lambda > 0$

e.g. A certain air~~craft~~ port receives on an avg. 4 aircrafts per hour. What is prob. that no aircraft lands in a particular instant of 2 hours?

$\alpha = 4, \quad \Delta t = 2$

$\therefore \lambda = 8$

$$P(X=0) = \frac{e^{-8} \cdot 8^0}{0!} = e^{-8}$$

For Continuous Random Variable

e.g. If a pdf of a random variable x is given by $f(x) = \begin{cases} x^2 & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$

$P\left[-\frac{1}{3} \leq x \leq \frac{1}{3}\right] = ?$ [No effect of equality in cont.]

$$F(x) = \int_{-1/3}^{1/3} f(x)\,dx = \int_{-1/3}^{1/3} x^2\,dx = \left|\frac{x^3}{3}\right|_{-1/3}^{1/3} = \frac{1}{3}\left[\frac{1}{27} + \frac{1}{27}\right]$$

$$= \frac{1}{3}\left[\frac{2}{27}\right] = \frac{2}{81} = 0.0247 = 2.47\%$$

e.g. A random variable with p.d.f

$f(t) = \begin{cases} 1+t & -1 \leq t \leq 0 \\ 1-t & 0 \leq 1 \end{cases}$ (Here equality at 0 at both is not contradictory)

Find standard deviation of random variable

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$\therefore \sigma = \sqrt{\overline{x^2} - \bar{x}^2}$ $\sigma^2 = \sqrt{\overline{x^2} - \bar{x}^2}$

$\overline{x^2} = \int_{-\infty}^{\infty} t^2 f(t)\,dt$ $\bar{x} = \int_{-\infty}^{\infty} t f(t)\,dt$

$\overline{x^2} - \bar{x}^2 = \int_{-1}^{1} t^2 f(t)\,dt - \left[\int_{-1}^{1} t f(t)\,dt\right]^2$

$\sigma^2 = \int_{-1}^{0} t^2(1+t)\,dt + \int_{0}^{1}(1-t)\,t^2\,dt - \left[\int_{-1}^{0}(1+t)t\,dt + \int_{0}^{1}(1-t)t\,dt\right]^2$

$= \left|\frac{t^3}{3} + \frac{t^4}{4}\right|_{-1}^{0} + \left|\frac{t^3}{3} - \frac{t^4}{4}\right|_{0}^{1} - \left[\left(\frac{t^2}{2} + \frac{t^3}{3}\right)_{-1}^{0} + \left|\frac{t^2}{2} - \frac{t^3}{3}\right|_{0}^{1}\right]^2$

$= \frac{1}{3}(1) + \frac{1}{4}(-1) + \frac{1}{3}(1) - \frac{1}{4} - \left[\left(\cancel{\frac{-1}{2}} + \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right)\right]^2$

$\frac{8-6}{12}$ $\frac{2}{6}$ $= \frac{2}{3} - \frac{2}{4} - 0 \qquad = \frac{8-6}{12} = \frac{\cancel{2}}{\cancel{12}_{6}} = \frac{1}{6}$

$\therefore \sigma = \frac{1}{\sqrt{6}}$

e.g. pdf is of form $P(x) = K e^{-\alpha|x|}$, $x \in (-\infty, \infty)$ find value of K?

$P(x) = f(x) = K e^{-\alpha|x|}$

$f(x)\,dx$ $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{\infty} P(x)\,dx = 1$

$\int_{-\infty}^{\infty} P(x)\,dx = \int_{-\infty}^{\infty} K e^{-\alpha|x|}$

$1 = \int_{-\infty}^{0} K e^{\alpha x} + \int_{0}^{\infty} K e^{-\alpha x}\,dx$

$1 = \left|K\frac{e^{\alpha x}}{\alpha}\right|_{-\infty}^{0} + K\frac{e^{-\alpha x}}{-\alpha}\Big|_{0}^{\infty}$

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$$1 = \frac{k}{\alpha}\left[1-0\right] + \frac{-k}{\alpha}\left[0-1\right]$$

$$1 = \frac{k}{\alpha} + \frac{k}{\alpha} \Rightarrow 1 = \frac{2k}{\alpha} \Rightarrow k = \frac{\alpha}{2} = 0.5\alpha$$

If x, y are two independent random variables then which of following are true?

✓ $E(xy) = E(x)\,E(y)$ (And)

✓ $Var(x+y) = Var(x) + Var(y)$ Linear

Pb A box contain 4 red and 6 white balls. If 3 balls are drawn randomly with replacement then what is prob. that we will get exactly 2 red balls?

$^4C_1$ without calculation

T_1	T_2	T_3
R	R	R
R	R	W
R	W	R
R	W	W
W	R	R
W	R	W
W	W	R
W	W	W

$$= \frac{4}{10} \times \frac{4}{10} \times \frac{6}{10} = \frac{96}{1000}$$

$$P = \frac{3 \times 96}{1000} = 0.288$$

or

$${}^3C_2\left(\frac{4}{10}\right)^2\left(\frac{6}{10}\right)^1$$

$\frac{96}{96}$ $\frac{3\times 96}{1000}$ 0.288

Pb A program consists of 2 modules executed sequentially. Let $f_1(t)$ & $f_2(t)$ denotes pdf of time taken to execute 2 Loading ~~exec~~ respectively. pdf for overall time taken to execute the program is?

$M_1 \rightarrow f_1(t_1) = \int P(x \le t_1)$

$M_2 \rightarrow f_2(t_2) = \int P(x_1 \le t)$

$f(M_1, M_2)$ $\int$

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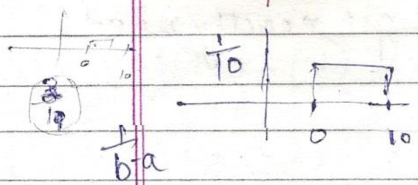
Pb Let $f(x)$ be a continuous prob. density func. of random variable X. The prob. that $a < X \leq b$

$$\int_a^b f(x)\,dx$$

Pb If x is uniformly distributed from 0 to 10. Calculate the prob that (i) $X < 3$ (ii) $X > 6$ (iii) $3 < X < 8$



(i) $\int_{-\infty}^{3} f(x)\,dx = \int_0^3 \frac{1}{10}\,dx$

$= \frac{3}{10}$

(ii) $X > 6$

$1 - P(X \leq 6)$

$1 - \int_0^6 \frac{1}{10}\,dx = 1 - \frac{6}{10} = \frac{4}{10}$

(iii) $3 < X < 8$

$\int_3^8 \frac{1}{10}\,dx = \frac{1}{10}[8-3] = \frac{5}{10} = \frac{1}{2}$

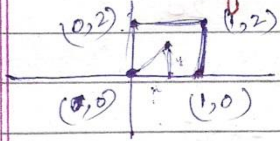
Pb Standard deviation of uniformly distributed random variable between 0 to 1?

$\sigma = \frac{b-a}{\sqrt{12}} = \frac{1}{\sqrt{12}}$

Pb If P and Q are two random events then which of following is true?

a) Independence of P and Q employs that Prob $(P \cap Q) = 0$
b) Prob. of $P \cup Q > P(P) + P(Q)$

Pb A point is randomly selected with uniform prob. in xy plane within rect. with corners (0,0) (1,0) (1,2) (0,2). If P is length of position vector of a point. Expected value of P^2?



$\frac{b-a}{2}$ $\quad P = \sqrt{x^2+y^2}$

$\frac{5}{3}$

$$E(P^2) = E(x^2) + E(y^2)$$

$E(x^2) \Rightarrow$

$$\sigma^2 = \overline{x^2} - \overline{x}^2$$

$$\frac{(b-a)^2}{12} = \overline{x^2} - \left(\frac{b+a}{2}\right)^2$$

$$\frac{1}{12} + \frac{1}{4} = \overline{x^2} \quad = \quad \overline{x^2} = \frac{4}{12} = \frac{1}{3}$$

$$E(y^2) \Rightarrow \quad \frac{4}{12} + 1 = \frac{1}{3} + 1 = \frac{4}{3}$$

$$\therefore E(P^2) = \frac{4}{3} + \frac{1}{3} = \frac{5}{3}$$

Or

$$E(x^2) = \int_0^1 x^2 f(x)\,dx + \int_0^2 y^2 f(y)\,dy$$

$$= \int_0^1 x^2 + \int_0^2 y^2 \cdot \frac{1}{2}\,dy$$

$$= \frac{1}{3} + \frac{1}{2} \cdot \frac{8}{3} = \frac{5}{3}$$

Here $f(x) = \frac{1}{b-a} = \frac{1}{1}$

$f(y) = \frac{1}{2}$

STANDARD EXPONENTIAL FUNCTION

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\boxed{E(X) = \frac{1}{\lambda} \qquad \sigma^2 = \frac{1}{\lambda^2}}$$

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$$P(X \leq a) = \int_{0}^{a} f(x) \cdot dx = 1 - e^{-\lambda a}, \quad a \geq 0$$

Pb Suppose the length of phone call in a minute is exponential random variable with parameter $\lambda = 1/10$. If someone arrives immediately ahead of public telephone booth find the prob. that he will have to wait for

a) > 10 min b) $10 < \quad < 20$

a) $P(X \leq 10)$

0.368

$$\int_0^{10} f(x)\,dx$$

$$\int_0^{10} \lambda e^{-\lambda x} dx = \cancel{1 - e^{-\lambda a}}$$

$$= \frac{\lambda e^{-\lambda x}}{-\lambda}\Big|_0^{10} = -\left[e^{-\lambda(10)} - 1\right]$$

$$= 1 - e^{-10/10}$$

$$= 1 - e^{-1}$$

b) $P(10 < X < 20)$

$F(20) - F(10)$

0.233

$$-\left[e^{-\lambda x}\right]_{10}^{20}$$

$$-\left[e^{-20\lambda} - e^{-10\lambda}\right]$$

$$-\left[e^{-20 \times \frac{1}{10}} - e^{-10 \times \frac{1}{10}}\right]$$

$$-\left[e^{-2} - e^{-1}\right] = 0.233$$

$$\text{Co variance} = \frac{\sigma}{\mu\,(\text{Mean})}$$

NORMAL / GUASSIAN DISTRIBUTION FUNCTION

– suitable for physical random process

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

at $\infty, -\infty$
$x = 0$

$$E(X) = \mu$$

$$Var(X) = \sigma^2$$

Pb A class of Ist Yr B.Tech students is formed of 4 batches A, B, C & D each consisting of 30 students. It is found that sessional

Statistics Ques

Marks of students in S.D. in batch C have a mean of 6.6 and standard deviation 2.3. The Mean and standard deviation of Marks for the entire class is 5.5 & 4.2 resp. It is decided by a course instructor to normalise the marks of students of all batches to have the same mean & standard deviation as that of entire class. Due to this marks of a student in batch C is changed from 8.5 to ___ ?

Batch C

$$\frac{8.5\times 1 + 29\times y}{30} = 6.6$$

$$8.5 + 29y = 198$$

$$29y = 189.5$$

$$y = 6.534$$

Other Batches (A, B, D)

$$\frac{120\times 5.5 - 30\times 6.6}{90} = \cancel{5.133} ?$$

$$= 5.133$$

(of 90 students)

New

$$\frac{x\cdot 1 + 29\times 6.534 + 90\times 5.133}{120} = 5.5$$

$$\therefore x = 8.5$$