Time Series II

WS 2018/2019

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State of the art / Related work

- Scientifically contributions have to consider the "state of the art" (also called "related work")

- State of the art focus on
  – Analyzation of papers which are related to your work
  – Description of these relations
  – Things to be learned from the listed work
  – Clear differentiation of listed work and your work

State of the art / Related work

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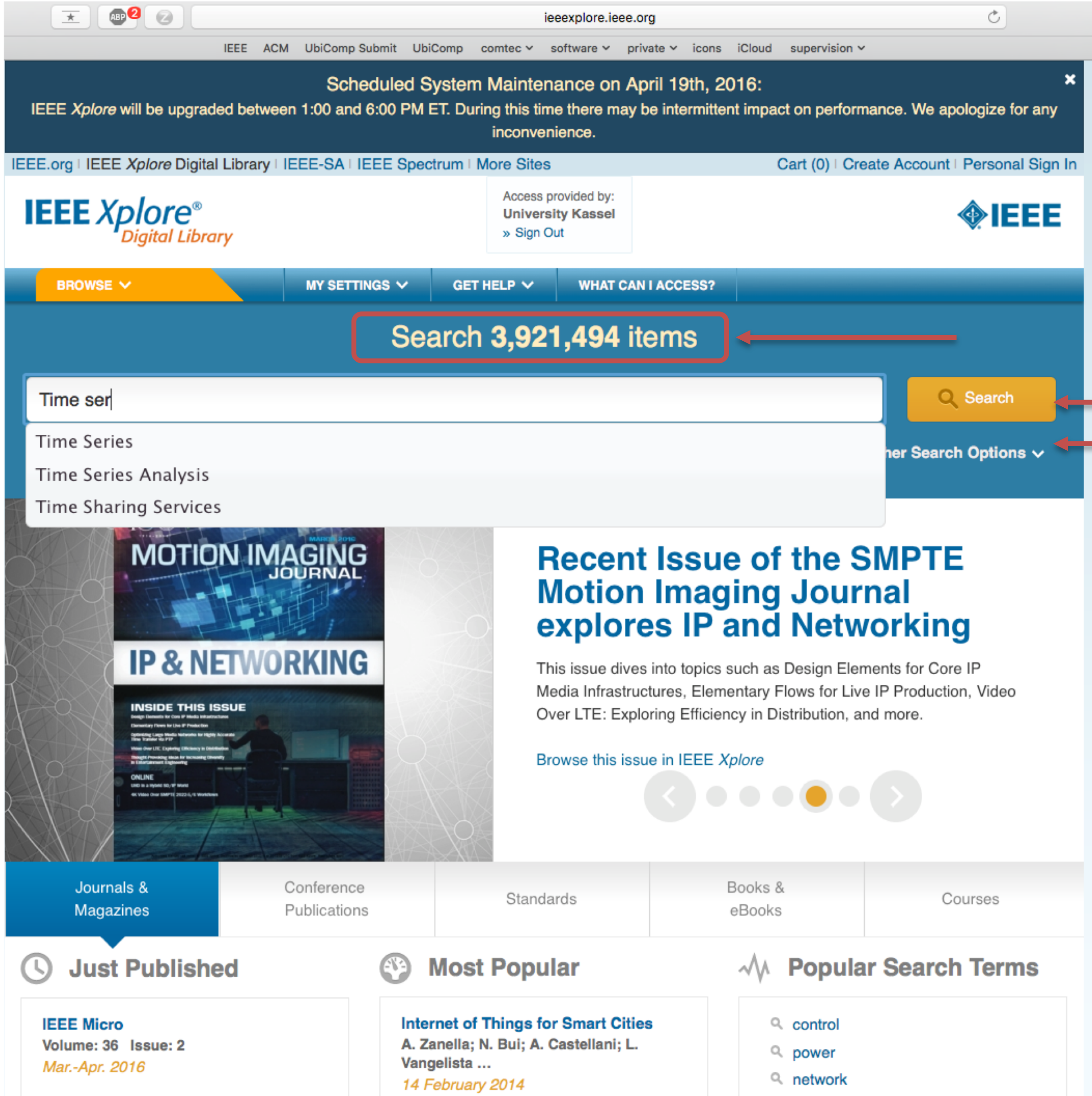

[IEEE]

State of the art / Related work

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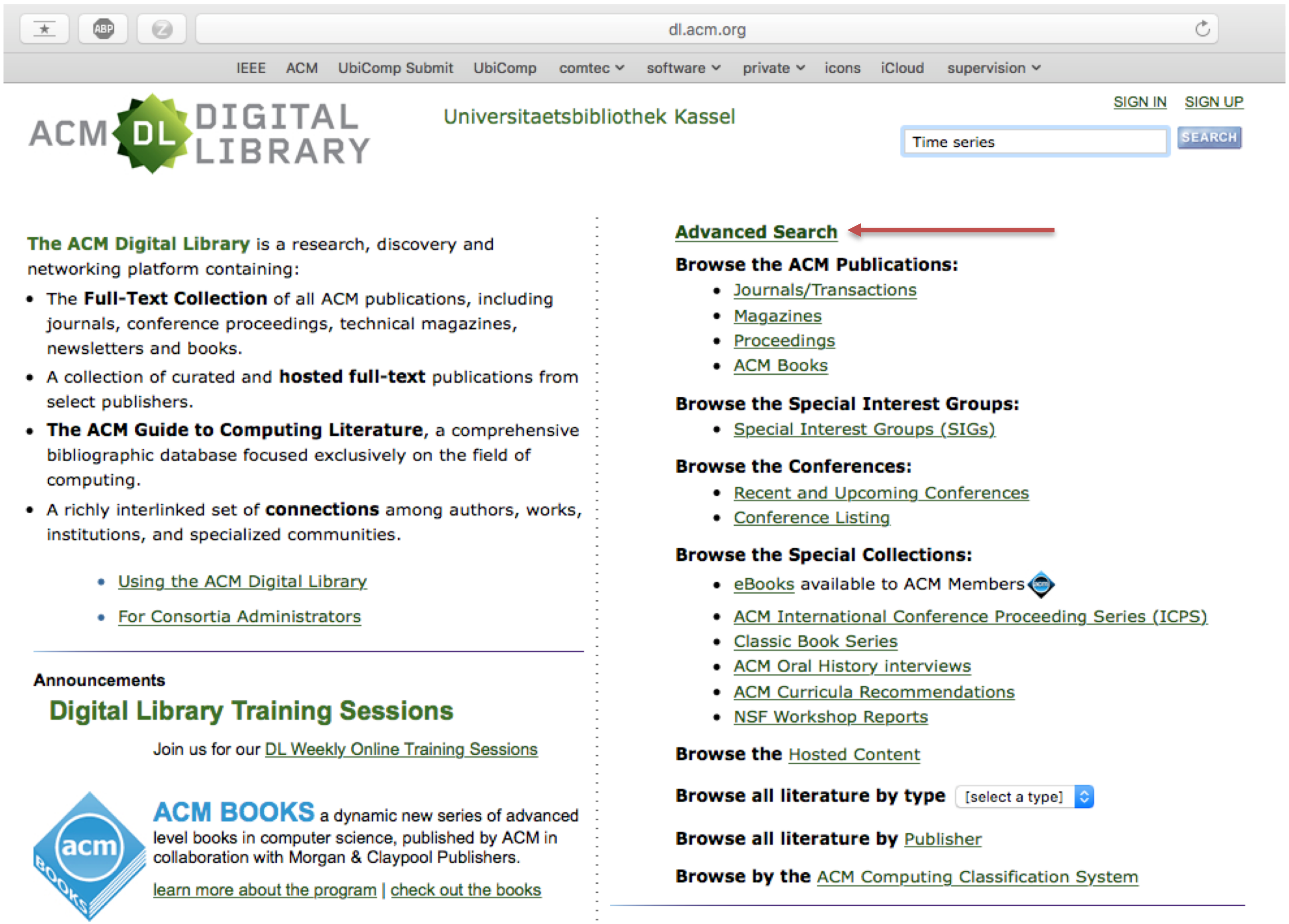

[ACM]

Revision of CT1 (SS-2017)

INTRODUCTION TO TIME SERIES

What are time series?

- Oxford dictionary says:

> *"A series of values of a quantity obtained at successive times, often with equal intervals between them [OTS]"*

What are time series?

- Some examples for time series

Apple stock 2013 to 2015 [AS]

What are time series?

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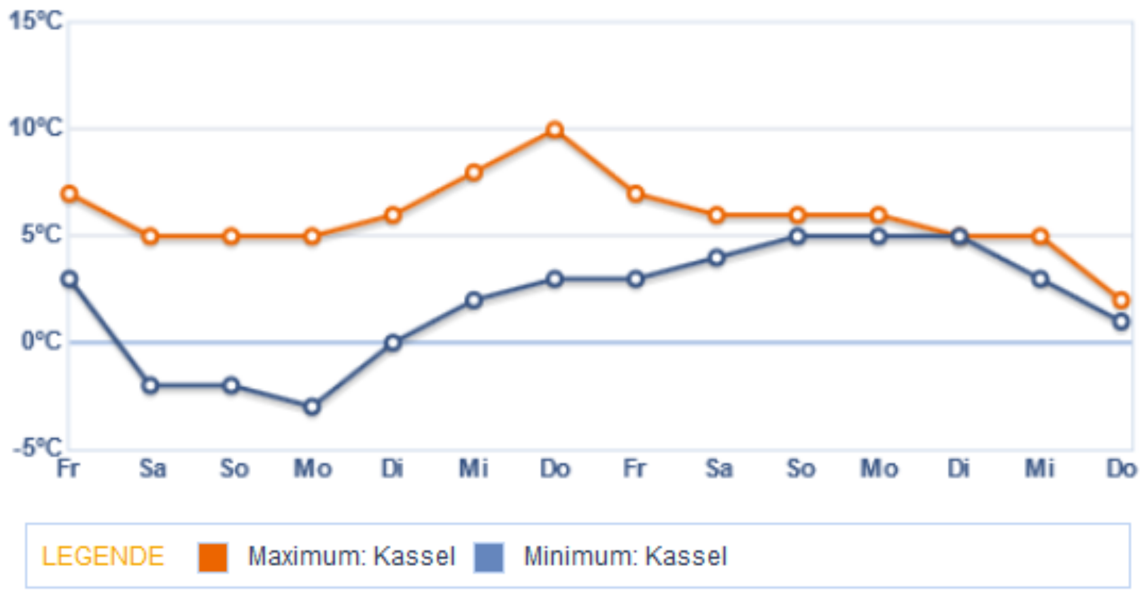
- Some examples for time series



Weather (winter 2015) by wetter.com [WC]

What are time series?

- Some examples for time series

past current future

DONNERSTAG	09	10	11	12	13	14	15	16
Temp. (°C)	6°	8°	9°	10°	11°	12°	12°	12°
RealFeel®	7°	8°	10°	10°	11°	12°	12°	11°
Wind (km/h)	6 SW	6 SW	7 SW	7 WSW	9 WSW	9 WSW	9 W	9 WSW

Weather in Kassel [AccW16]

Time series properties

- Value
  – Numeric, text, binary, dates, ..
  – Missing values

- Time
  – Axis units often seconds, minutes, hours, ..
  – Equally spaced values
    - Specific sampling rate (Hz), seasonal, annual, ..
  – Unevenly spaced values
    - earthquakes, floods, astronomical observations (e.g. supernovas)

Time series properties

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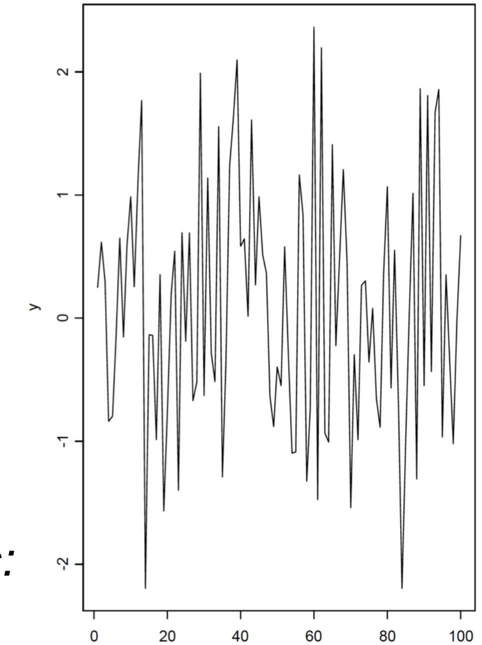
- Noise
  – No noise, white noise, pink noise, ..
- Others
  – Trends, patterns
  – Complete / Continuous (also called “stream data”)
  – Nonstationarity / Stationarity

[Noise]

2 [MASS NOUN] *technical* Irregular fluctuations that accompany a transmitted electrical signal but are not part of it and tend to obscure it:
‘the enhancer can improve the video signal quality, reducing noise and increasing image sharpness’



Example stationary time series: white noise [TSC]

Time series representation

- Reasons to change the representation of time series (TSs)
  - Save storage
  - Get time series “human readable”
  - Focus on important information
  - Data representation is key for applying context algorithms
    - Raw time series are rarely applicable for machine learning
    - Therefore, TSs are often divided into segments
    - On the segments, e.g. ”Feature Extraction” is applied
    - Next, features are used to learn and recognize activities (see CT lectures about *activity recognition*)

Time series representation

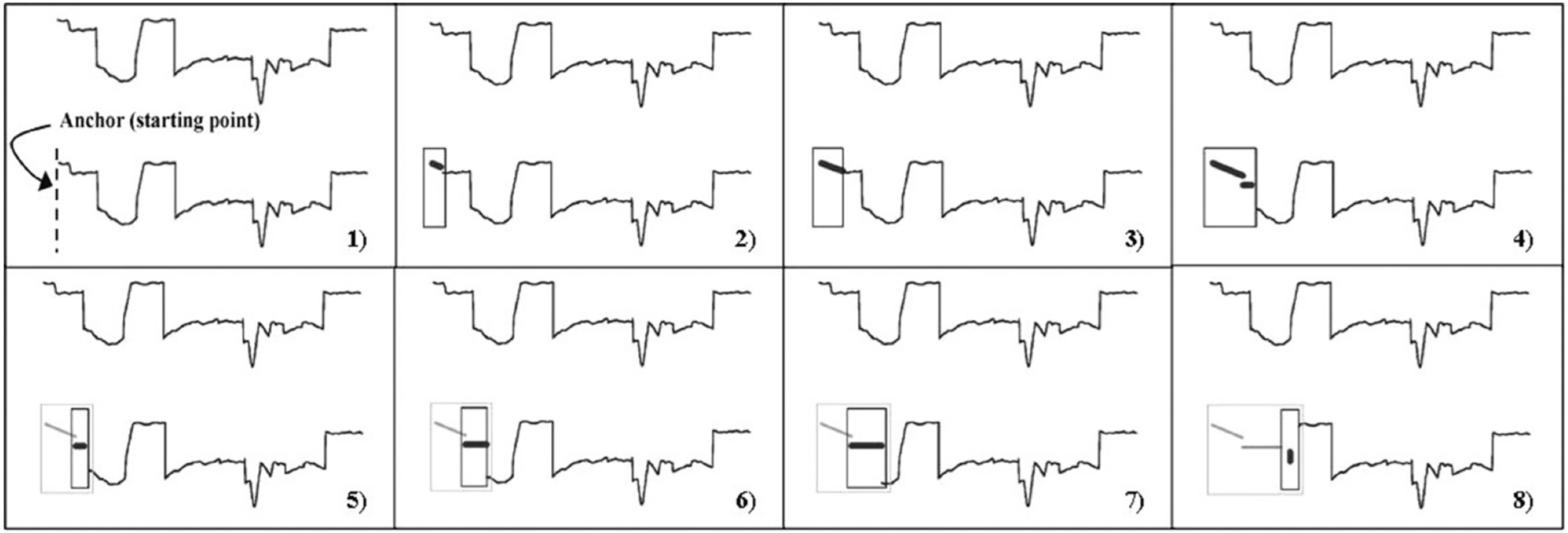
- Example representations
  - Piecewise Linear Approximation (PLA)
    - Segment time series [K01]
    - Approximate segments
  - Autoregressive Models (AR Models)
    - Use time series to parametrize a model
    - Models useful for predicting time series
    - A variety of models exist, in this lecture we focus on AR Models

Time series representation

- Example representations
  - Piecewise Linear Approximation (PLA)
    - **Segment time series [K01]**
    - **Approximate segments**
  - Autoregressive Models (AR Models)
    - Use time series to parametrize a model
    - Models useful for predicting time series
    - A variety of models exist, in this lecture we focus on AR Models

Time series segmentation

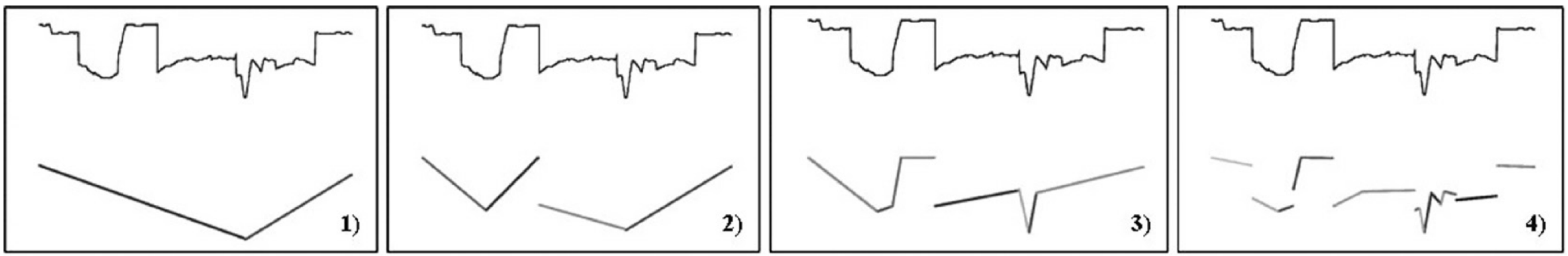
- Sliding Window Segmentation – Short example



[LMS14]

Time series segmentation

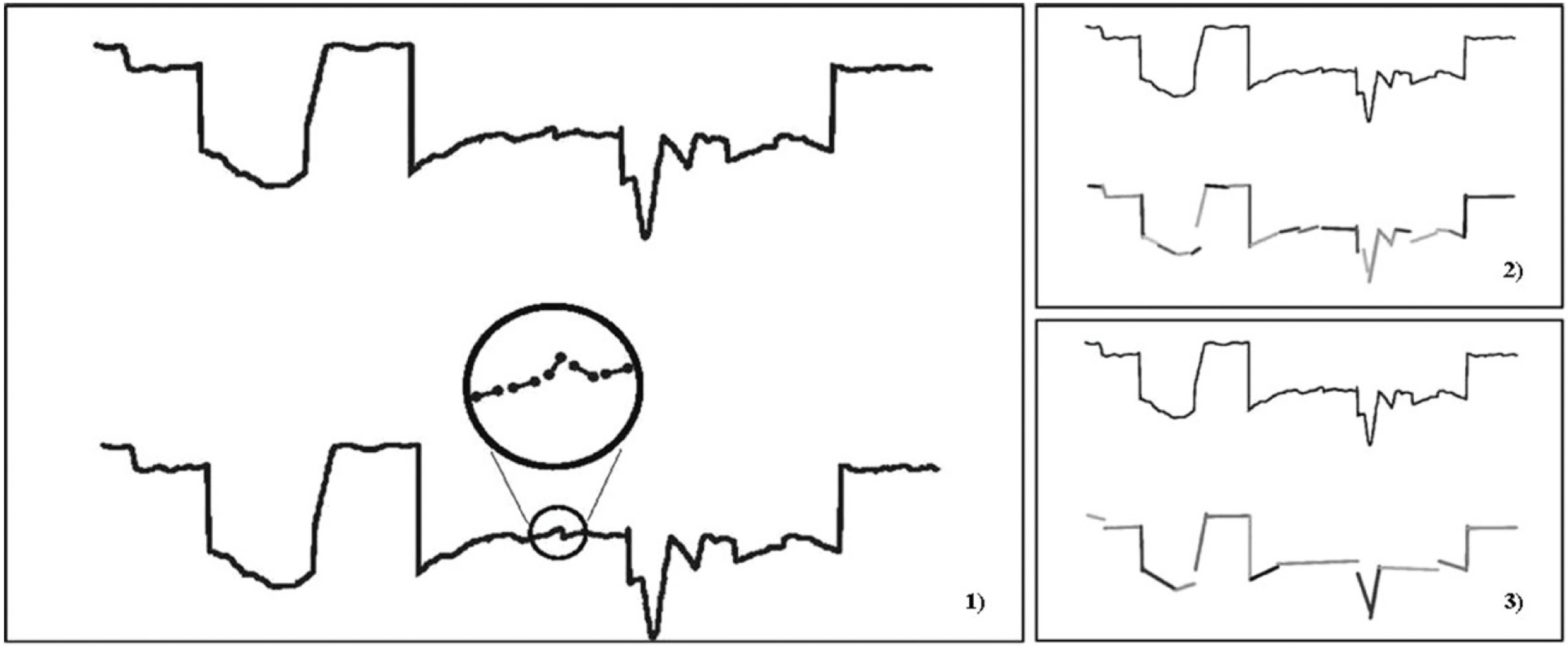
- Top Down Segmentation – Short example



[LMS14]

Time series segmentation

- Bottom Up Segmentation – Short example



[LMS14]

Time series representation and prediction

- Some representations and models also support predicting time series
- In this lecture, two options for time series modelling and prediction are presented
  – A combination of algorithms which can be applied to both, stationary and nonstationary models
  – A model fit for stationary time series

Time series prediction

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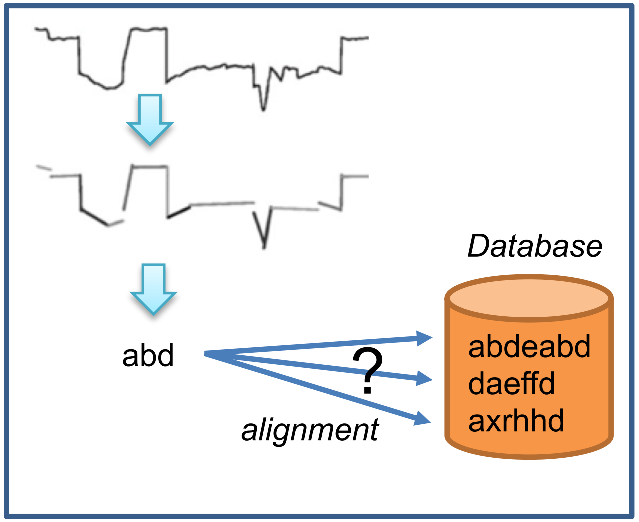
- Option for Nonstationary
  - *Uses: PLA*, label *sequences*, *alignment*
  - Usable, regardless whether TS is stationary



Time series prediction

ComTec

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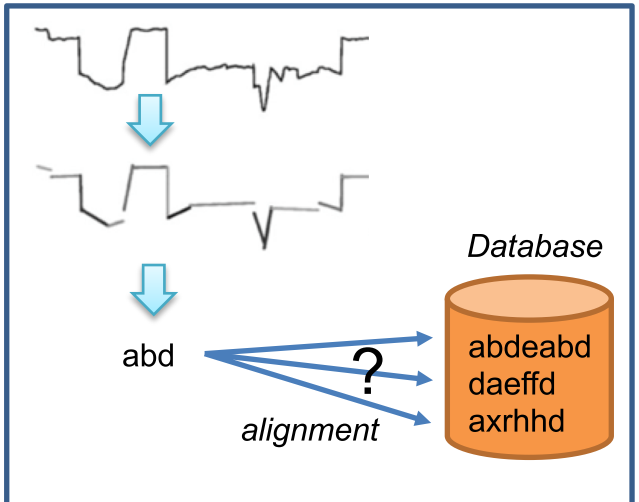
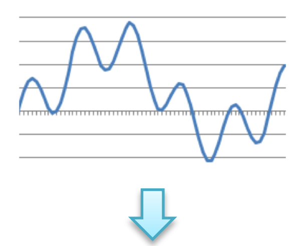


- Option for stationary
  - *Autoregressive models*
  - Parametrize (or train) models with raw TS



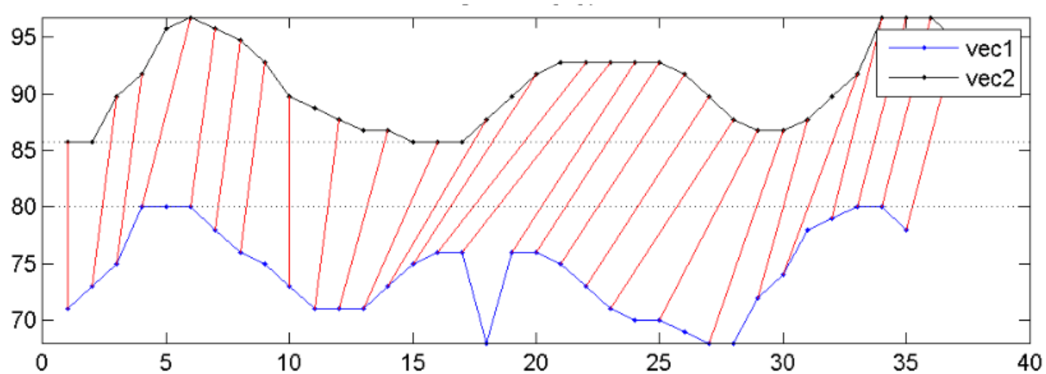
$$x_t = \sum_{i=1}^{p} a_i * x_{t-i} + e_t$$

Segmentation, PLA, label sequences, and alignment

TIME SERIES PREDICTION I

Segment comparison

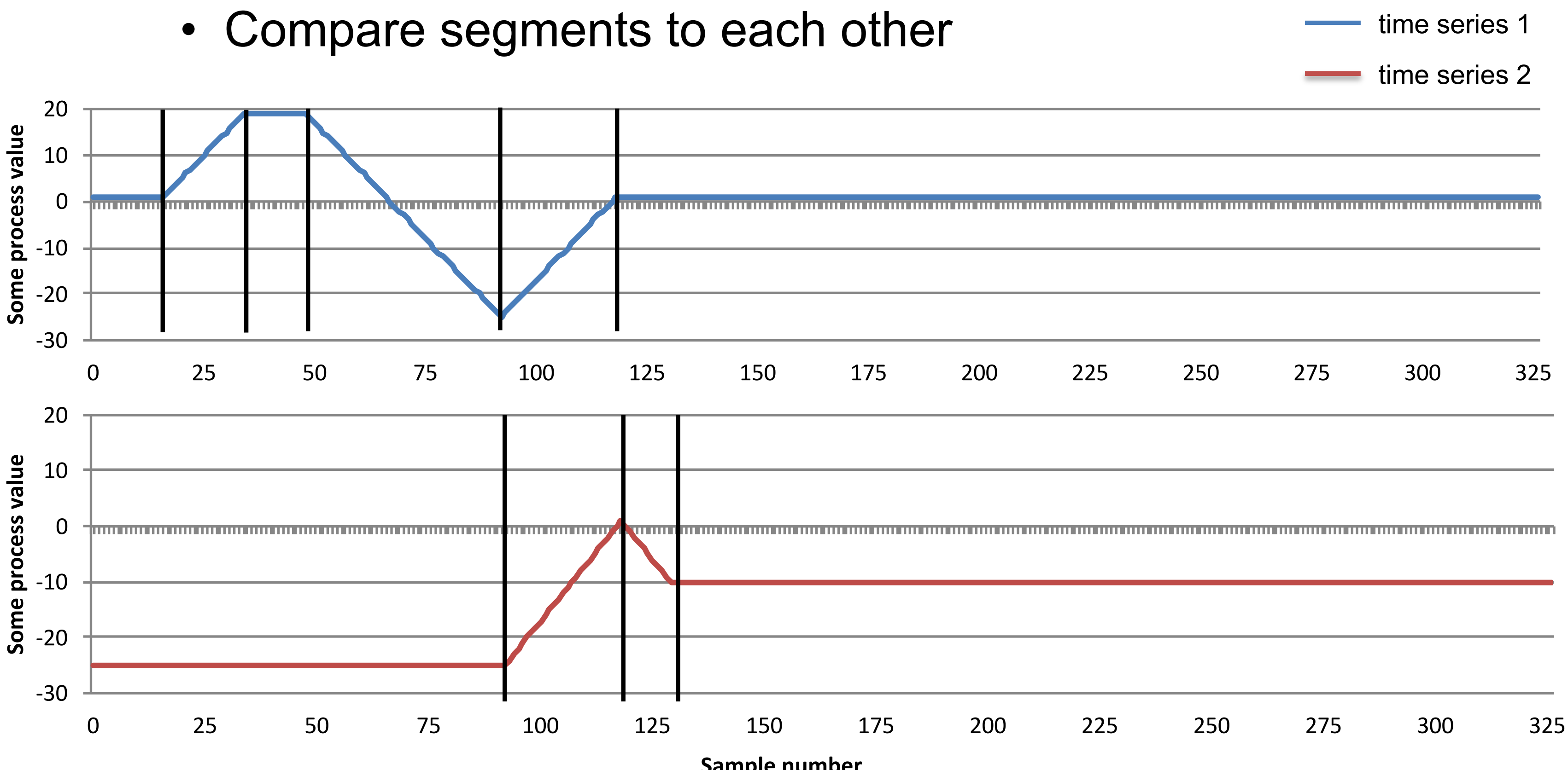
- What we have:
  - Segmentation algorithms and approximations
  - Distance measures to compare segments
    - E.g. euclidean distance
  - Techniques to compare segments with different length
    - E.g. Dynamic-Time-Warping



Example of aligning segments with unequal length
http://mirlab.org/jang/books/dcpr/example/output/dtwBridgePlot02.png

Segment comparison

- What we need next:
  - A way to recognize and label segments
    - Compare segments to each other



Segment comparison and labeling

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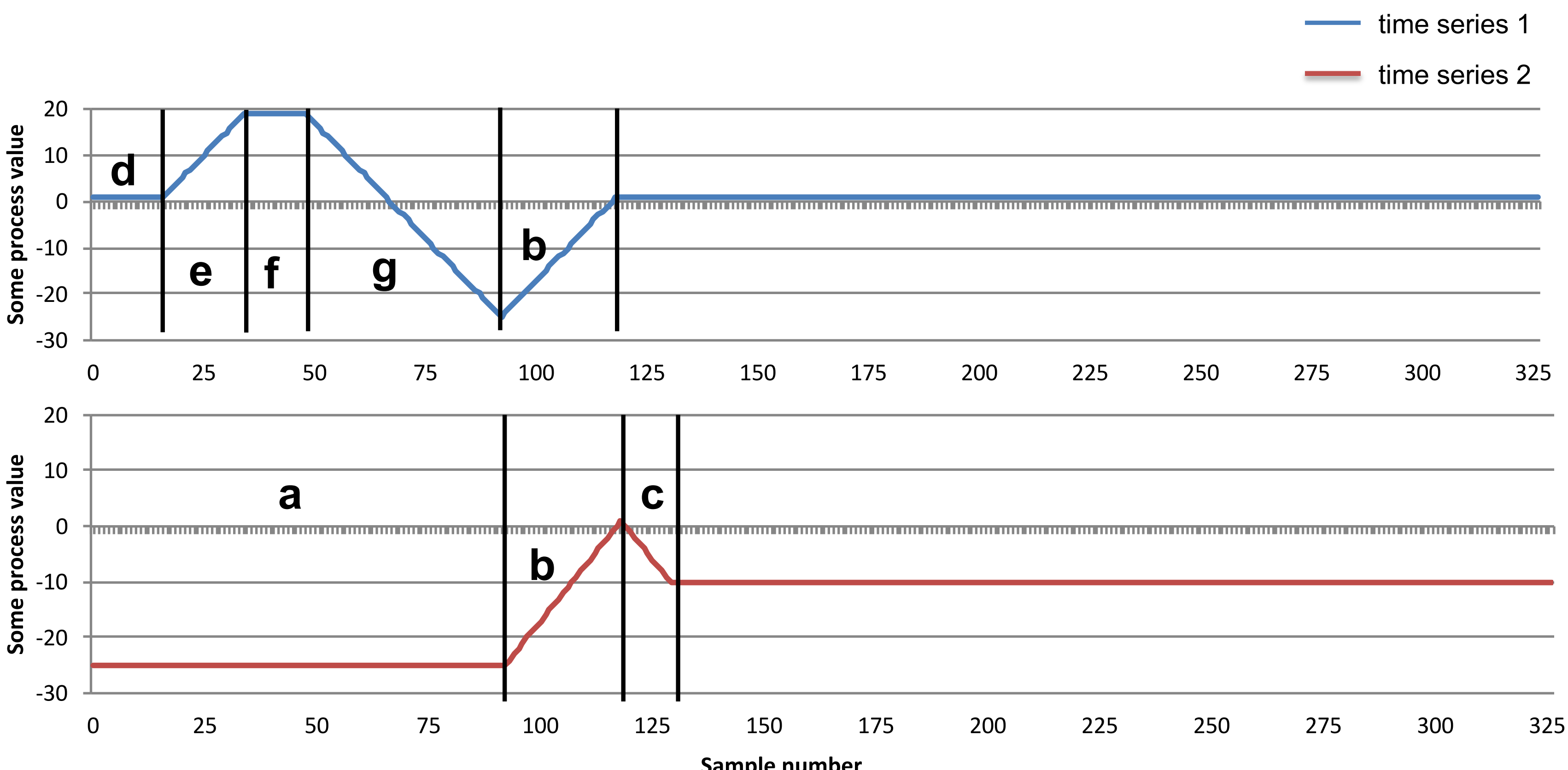
- Similar segments get the same label



Use alignment for prediction

- Now TSs are presented as a label sequence
  - Label sequence of time series 1 = “defgb”
  - Label sequence of time series 2 =“abc”

- This representation
  - ➢ enables several methods to go on (Markov, Rules)
  - ➢ Is input for ‘alignment’ [S07]

Use alignment for prediction

- Need to identify the similarity of label sequences
  - Are two sequences similar?
  - Is a sequence similar to a part of another sequence?
  - If so, we can predict!

- Alignment algorithm supports this
  - Input: two label sequences
  - Tasks
    - align sequences to each other
    - calculate a distance (or cost) for the sequences
    - When sequences match, predict rest of the known sequence

Introduction to Autoregressive Models

TIME SERIES PREDICTION II

Forecast – Auto regressive models

- The auto regressive models (AR models)
  - “Regression”
    - A variable depends on others
    - Value of variable is regressed to others
  - “Auto”:
    - Regress values of series *ts1* from its <u>own</u> past (stationarity!)
  - “model”:
    - Mathematical description of a process
    - Approximates the process
    - Has parameters to optimize the approximation

Forecast – Auto regressive models

- Definition

> *“An observation at time t is predictable from a weighted sum of the **p** previous observations (called an AR(p) process) [G10]”*

- The AR model order
  - AR(1) model: use weighted last observation
  - AR(2) model: use weighted sum of last two observations
  - AR(3) model: …

Forecast – Auto regressive models

ComTec

- The first-order AR model (called AR(1))

$$x_t = a_1 * x_{t-1} + e_t$$

weight for last observation
(autocorrelation coefficient)

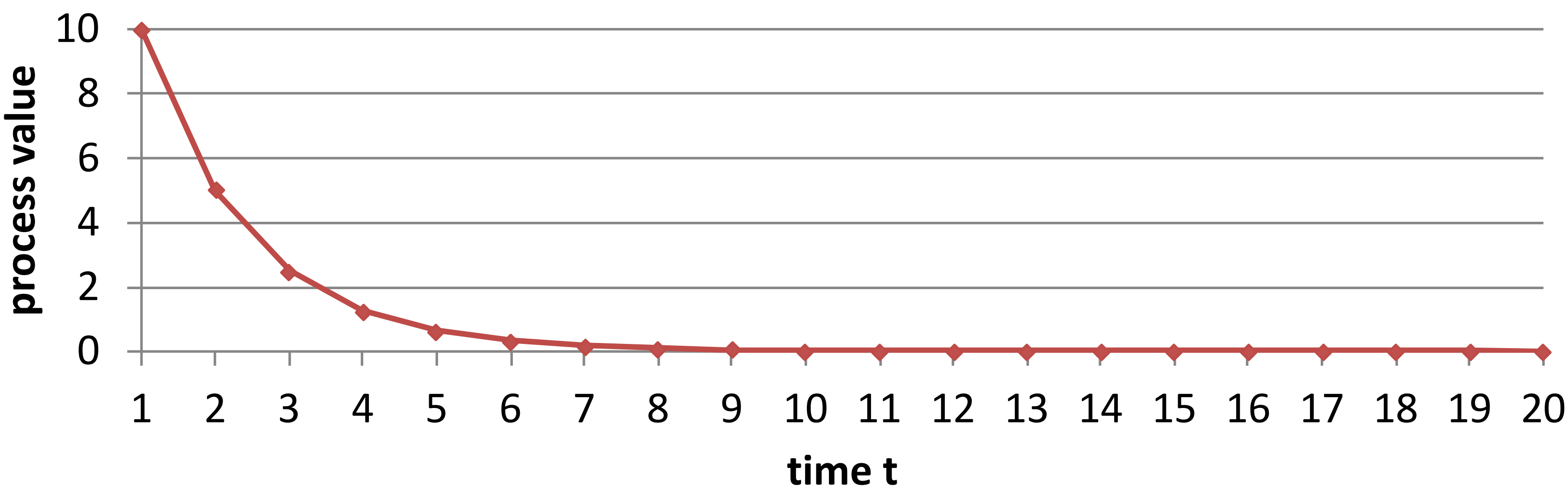
the noise

the last observation

Forecast – Auto regressive models

- AR(1) model example. Observed $x_0 = 10$
  - $a_1 = 0{,}5$
  - $e_t = 0$

$$x_t = 0{,}5 \ * x_{t-1} + e_t$$



Forecast – Auto regressive models

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- First-order AR(1) model

$$x_t = a_1 * x_{t-1} + e_t$$

- Second-order AR(2) model

$$x_t = a_1 * x_{t-1} + a_2 * x_{t-2} + e_t$$

- General AR(p) model

$$x_t = \sum_{i=1}^{p} a_i * x_{t-i} + e_t$$

Forecast – Auto regressive models

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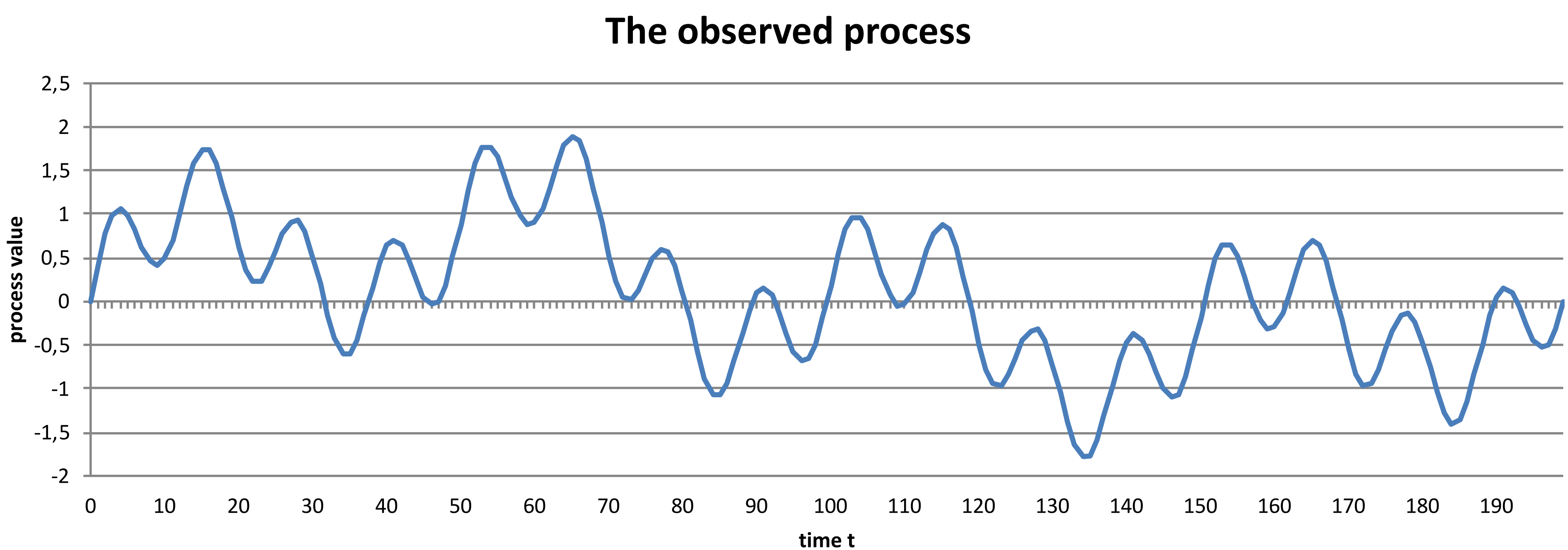
- Example process
  - first 200 Points of data available at [KBG04]



Forecast – Auto regressive models

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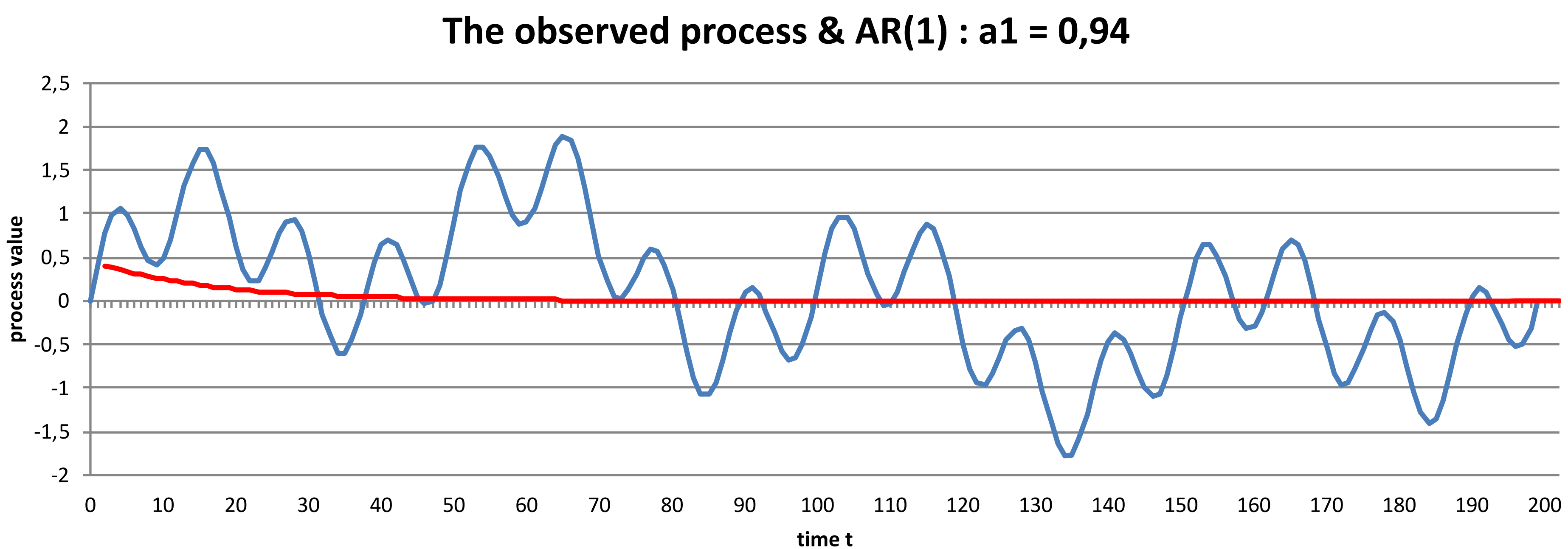

Forecast – Auto regressive models

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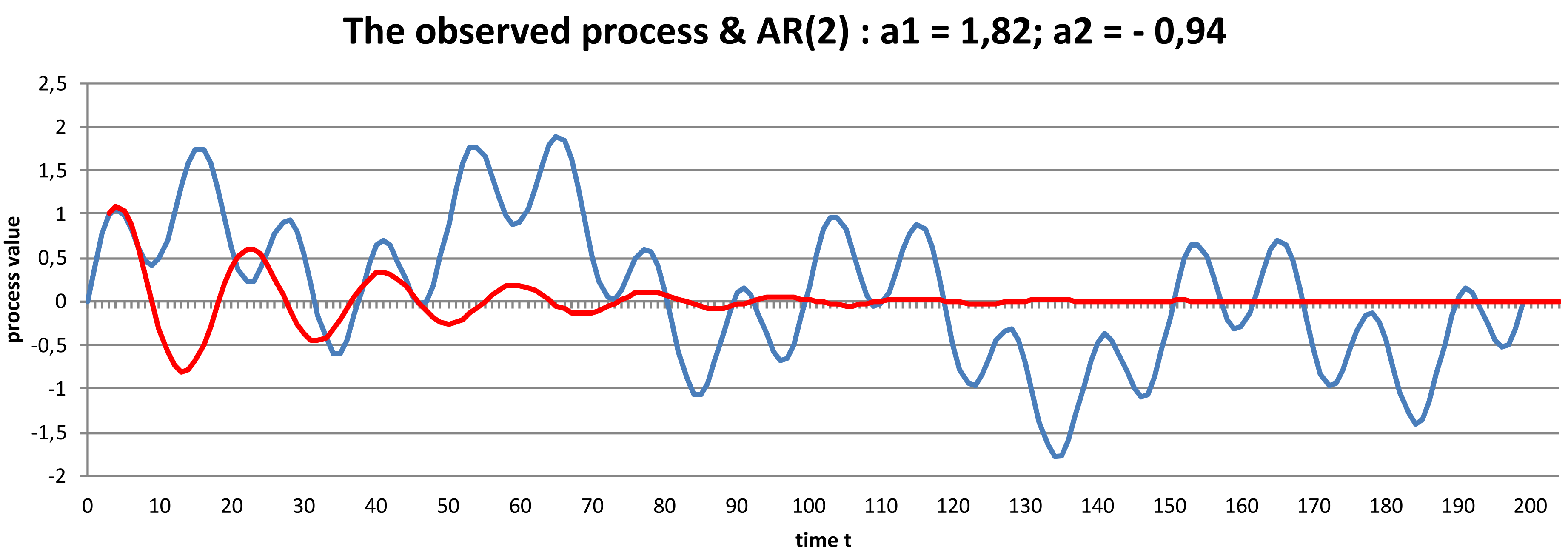

Forecast – Auto regressive models

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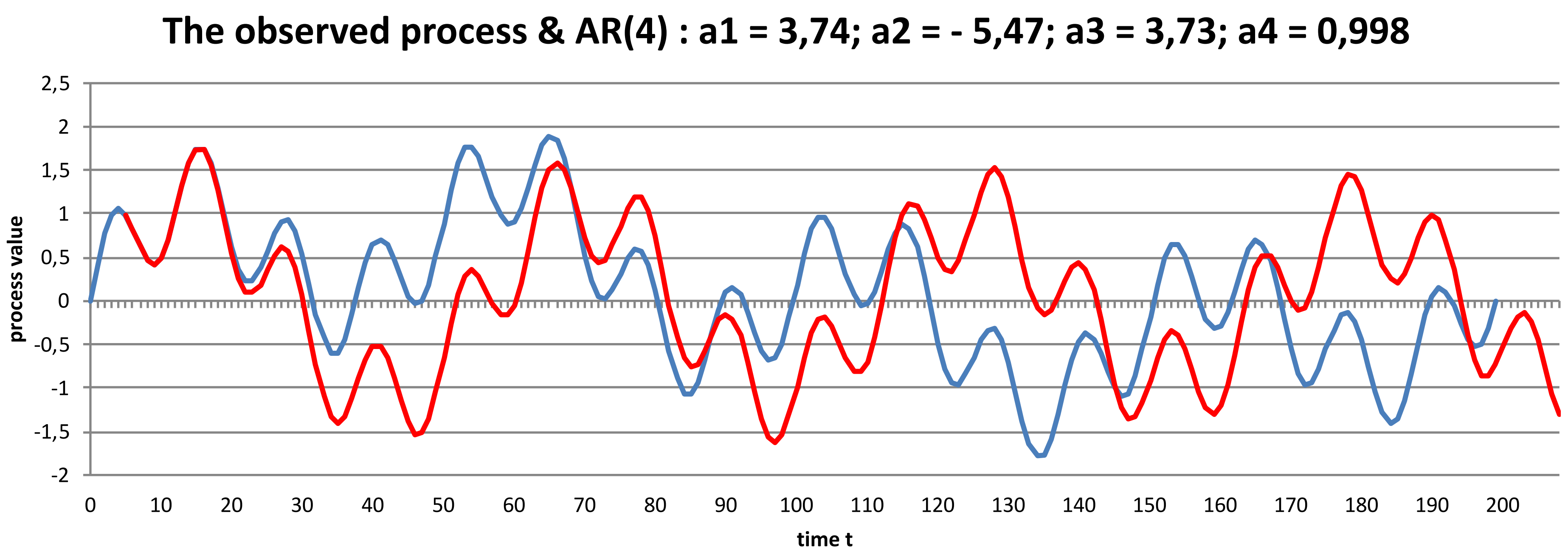

Forecast – Auto regressive models

- To create an AR model
  - Choose order **p** of model
  - Use **N** observed past values to estimate **p** weights

- Example
  - Use data from [KBG04] (**N** = 1000 points)
  - Choose order **p = 1**
  - Calculate parameters of the AR(1) model

Forecast – Auto regressive models

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- Calculate $\boldsymbol{a_1}$ in AR(1) : $\boldsymbol{x_t = a_1 * x_{t-1} + e_t}$
- Therefore, use N observed points:

$$\begin{pmatrix} x_2 \\ x_3 \\ \cdots \\ x_N \end{pmatrix} \approx \begin{pmatrix} x_1 \\ x_2 \\ \cdots. \\ x_{N-1} \end{pmatrix} * a_1$$

Forecast – Auto regressive models

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- Calculate $\boldsymbol{a_1}$ in AR(1) : $\boldsymbol{x_t = a_1 * x_{t-1} + e_t}$
- Use *least squares method* [LSF] to get $\mathbf{a_1}$

$$\underbrace{\begin{pmatrix} x_2 \\ x_3 \\ \cdots \\ x_N \end{pmatrix}}_{b} \approx \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ \cdots. \\ x_{N-1} \end{pmatrix}}_{A} * a_1$$

$$\boxed{a_1 = (A^T A)^{-1}(A^T b)}$$

Forecast – Auto regressive models

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- Simple for AR(1)

$$\underbrace{\begin{pmatrix} x_2 \\ x_3 \\ \cdots \\ x_N \end{pmatrix}}_{b} \approx \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_{N-1} \end{pmatrix}}_{A} * a_1$$

$$(A^T A)^{-1}(A^T b) = \cdots = \frac{\sum_{i=1}^{n-1} x_i x_{i+1}}{\sum_{i=1}^{n-1} {x_i}^2} = a_1$$

Forecast – Auto regressive models

- Simple for AR(1)

Observed data about 1000 points

0
0,423468
0,773358
0,994145
1,06138
0,987356
0,817725
0,619848
0,46583
0,414411
0,496197
1,59398
…..
……

$$\frac{\sum_{i=1}^{1000-1} x_i x_{i+1}}{\sum_{i=1}^{1000-1} {x_i}^2} = a_1 = 0{,}94$$

Forecast – Auto regressive models

- Least squares method (or *fitting*)
  - Expand matrix A for higher AR model order
  - Expand matrix A -> more complex calculations

- Yule-Walker equations [YW]
  - Suitable for general AR(p)
  - Out of scope of this lecture

Thanks for your attention

END

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