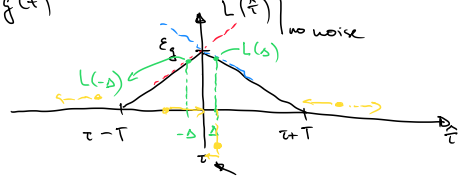
Maximization of $L(\tau)$

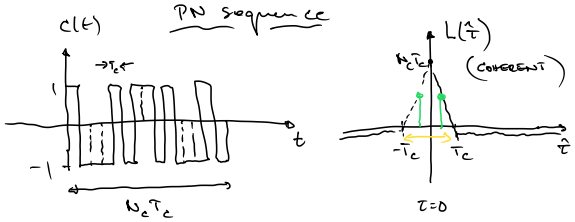
E.g. assume a rectangular-shaped pulse $g(t)$



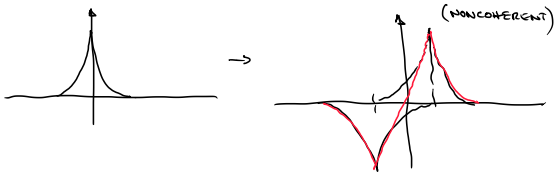
Try to find $\left.\frac{d}{d\tau} L(\tau)\right|_{\tau=\hat{\tau}_{ML}} = 0$

Replace by $L(\tau+\Delta) = L(\tau-\Delta)$

$$\frac{d}{d\tau} L(\tau) = \lim_{\Delta \to 0} \frac{L(\tau+\Delta) - L(\tau-\Delta)}{2\Delta} \stackrel{?}{=} 0$$



In case of non-coherent estimation (BPSK), we squared correlator output:



$$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$$

recap: $\cos x = \frac{1}{2}\left(e^{jx} + e^{-jx}\right)$