vectors in $\mathbb{R}^3$

$$\underline{x} = x_1 \underline{e}_1 + x_2 \underline{e}_2 + x_3 \underline{e}_3$$

$$\Rightarrow x_i = \langle \underline{x}, \underline{e}_i \rangle \qquad i = 1, 2, 3$$

$$\Rightarrow \underline{x} = \sum_{i=1}^{3} \overset{x_i}{\downarrow}\, x_i \underline{e}_i = \sum_{i=1}^{3} \langle \underline{x}, \underline{e}_i \rangle \underline{e}_i$$

Cholesky decomposition of $\underline{\underline{\Sigma}}_N$

$$\underline{\underline{C}} = \begin{bmatrix} c_{11} & 0 & \cdots & & 0 \\ \vdots & c_{22} & 0 & & \\ \vdots & & c_{33} & 0 & \\ \vdots & & & \ddots & 0 \\ c_{n1} & - & - & - & c_{nn} \end{bmatrix}$$

$$\Rightarrow \underline{\underline{C}}^{-1} = \begin{bmatrix} z_{11} & 0 & 0 & \cdots & 0 \\ z_{21} & z_{22} & 0 & & \\ \vdots & & \ddots & 0 & \\ \vdots & & & \ddots & 0 \\ z_{n1} & z_{n2} & - & - & z_{nn} \end{bmatrix}$$

Consider the product (transform) $\underline{\underline{C}}^{-1} \underline{Y}$

$$= \begin{bmatrix} z_{11} & 0 & 0 & \cdots & 0 \\ z_{21} & z_{22} & 0 & & \\ \vdots & & z_{33} & 0 & \\ \vdots & & & \ddots & 0 \\ z_{n1} & z_{n2} & z_{n3} & \cdots & z_{nn} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \vdots \\ Y_n \end{bmatrix} \begin{matrix} \leftarrow \text{time } 1 \rightarrow \\ \leftarrow \text{time } 2 \rightarrow \\ \\ \\ \\ \end{matrix} = \underline{\bar{Y}} = \begin{bmatrix} \bar{Y}_1 \\ \bar{Y}_2 \\ \vdots \\ \bar{Y}_n \end{bmatrix} \Big\downarrow \text{time}$$

$\Rightarrow$ we can implement the calculation of $\bar{Y}_k$ in a causal way, i.e.

$$\bar{Y}_k = \sum_{x=0}^{n-1} Y_{k-x}\, z_{k+1, x+1}$$

If diagonal values of $\underline{\underline{C}}^{-1}$ would be identical (Toeplitz structure) meaning that $z_{k\lambda} = z_{k+1, \lambda+1}$ the filtering operation would correspond to a time-invariant convolution; otherwise, we have a causal, but time-variant linear transform.