Problem 1 p.175

Assume $a_m = +1$ is the transmitted signal.

$\Rightarrow$ first calculate the probability of error conditioned on $a_m = +1$ given by $P_{e|a_m=+1}$:

$$P_{e|a_m=+1} = P_r\{y_m < 0 \mid a_m = +1\}$$

$$= P_r\{a_m + n_m + i_m < 0 \mid a_m = +1\}$$

$$= P_r\{1 + n_m + i_m < 0\}$$

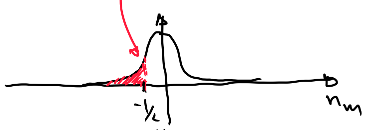
Use "total probability theorem"

$$P_{e|a_m=+1} = P_r\{1 + n_m - \tfrac{1}{2} < 0\}\ \underbrace{P_r\{i_m = -\tfrac{1}{2}\}}_{=\frac{1}{4}}$$

$$+ P_r\{1 + n_m + 0 < 0\}\ \underbrace{P_r\{i_m = 0\}}_{=\frac{1}{2}}$$

$$+ P_r\{1 + n_m + \tfrac{1}{2} < 0\}\ \underbrace{P_r\{i_m = +\tfrac{1}{2}\}}_{=\frac{1}{4}}$$

$$= \frac{1}{4} \underbrace{P_r\{n_m < -\tfrac{1}{2}\}} + \frac{1}{2} P_r\{n_m < -1\} + \frac{1}{4} P_r\{n < -\tfrac{3}{2}\}$$



$$\approx \underbrace{\frac{1}{4}\frac{1}{\sqrt{2\pi}\sigma_n}\int_{-\infty}^{-1/2} \exp\left\{-\frac{z^2}{2\sigma_n^2}\right\} dz}_{Q\left(\frac{1}{2\sigma_n}\right)} + \dots$$

$$P_{e|a_m=+1} = \frac{1}{4} Q\left(\frac{1}{2\sigma_n}\right) + \frac{1}{2} Q\left(\frac{1}{\sigma_n}\right) + \frac{1}{4} Q\left(\frac{3}{2\sigma_n}\right)$$

Due to symmetry, we have

$$P_{e|a_m=-1} = P_r\{y_m > 0 \mid a_m = -1\}$$

$$= P_r\{y_m < 0 \mid a_m = +1\}$$

$$= P_{e|a_m=+1}$$

and thus

$$P_e = \frac{1}{2} P_{e|a_m=+1} + \frac{1}{2} P_{e|a_m=-1} = P_{e|a_m=\pm 1}$$

$$= \frac{1}{4} Q\left(\frac{1}{2\sigma_n}\right) + \frac{1}{2} Q\left(\frac{1}{\sigma_n}\right) + \frac{1}{4} Q\left(\frac{3}{2\sigma_n}\right)$$

Problem 2 p. 175

a) Matched filter demodulator output

$$y(t) = \sum_{k\in\mathbb{Z}} I_k\, x(t - kT_b) + v(t)$$

$$= \sum_{k\in\mathbb{Z}} I_k \int_{\mathbb{R}} g_T(\tau - kT_b)\, g_R(t-\tau)\, d\tau + v(t)$$

where (for roll-off factor $\beta = 1$)

$$x(t)\Big|_{\beta=1} = g_T(t) * g_R(t)\Big|_{\beta=1}$$

$$= \frac{\sin\left(\frac{\pi t}{T}\right)}{\frac{\pi t}{T}} \quad \frac{\cos\left(\frac{\pi t}{T}\right)}{1 - 4\frac{t^2}{T^2}}$$

$m = k: \quad y(mT_b) \quad \Rightarrow \quad x(0) = 1$

$m = k+1: \quad y(mT_b) \quad \Rightarrow \quad x(T_b) = ?$

$m = k-1: \quad y(mT_b) \quad \Rightarrow \quad x(-T_b) = ?$

$$x(T_b) = \frac{\sin\left(\frac{\pi \cancel{T}}{2\cancel{T}}\right)}{\frac{\pi\cancel{T}}{2\cancel{T}}} \lim_{t\to T_b} \frac{\cos\left(\frac{\pi t}{T}\right)}{1 - 4\frac{t^2}{T^2}}$$

$$= \frac{2}{\pi} \lim_{t\to\frac{T}{2}} \frac{\cancel{-}\frac{\pi}{\cancel{T}} \sin\left(\frac{\pi t}{T}\right)}{\cancel{-}\frac{8t}{T^{\cancel{2}}}}$$

$$= \frac{2}{\pi}\,\frac{\pi}{4} = \frac{1}{2} = \lim_{t\to T_b} x(t)$$

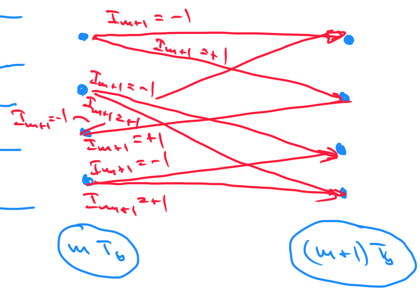
$$y(mT_b) = \sum_{k\in\mathbb{Z}} I_k\, x(mT_b - kT_b) + v(mT_b)$$

$$= I_m + \underbrace{\frac{1}{2} I_{m-1} + \frac{1}{2} I_{m+1}}_{\text{ISI term arising upon doubling the symbol rate from}} + v(mT_b)$$

$\frac{1}{T}$ to $\frac{2}{T}$

b) I_{m+1} to be considered the symbol "transfering" the state (I_{m-1}, I_m) to the state (I_m, I_{m+1})

I_{m-1}	I_m
-1	-1
-1	1
1	-1
1	1



Problem 3 p.176

a) Equivalent discrete-time impulse response of the channel given by

$$h(t) = \sum_{n=-1}^{1} h_n \, \delta(t - nT)$$

$$= 0.3\,\delta(t+T) + 0.9\,\delta(t) + 0.3\,\delta(t-T)$$

Let us denote by $\{c_n\}$ the coefficients of the FIR equalizer. Then, the equalized signal reads

$$q_m = h_m * c_m = \sum_{n=-1}^{1} h_{m-n} \, c_n$$

with m ... time index.

From the problem description, we have

$$\begin{aligned}
q_{-1} &= h_{\underbrace{-1-(-1)}_{0}} c_{-1} + h_{-1} c_0 + \cancel{h_{-2}}^{\,0} c_1 \\
q_0 &= h_1 c_{-1} + h_0 c_0 + h_{-1} c_1 \\
\underbrace{q_1}_{\underline{q}} &= \underbrace{\cancel{h_2}^{\,0} c_{-1} + h_1 c_0 + h_0 c_1}_{\underline{\underline{H}}\,\underline{c}}
\end{aligned}$$

$$\Leftrightarrow \begin{bmatrix} 0.9 & 0.3 & 0 \\ 0.3 & 0.9 & 0.3 \\ 0 & 0.3 & 0.9 \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} q_{-1} \\ q_0 \\ q_1 \end{bmatrix} \overset{!}{=} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\Leftrightarrow \quad \underline{c} = \underline{\underline{H}}^{-1} \underline{q} = \begin{bmatrix} -0.4762 \\ 1.4286 \\ -0.4762 \end{bmatrix}$$

b) The values for q_m for $m = \pm 2$ and $m = \pm 3$ are given by

$$q_2 = \sum_{n=-1}^{1} h_{2-n} c_n = c_1 h_1 = -0.1429$$

$$q_{-2} = \sum_{n=-1}^{1} h_{-2-n} c_n = c_{-1} h_{-1} = -0.1429$$

$$q_3 = \sum_{n=-1}^{1} h_{3-n} c_n = 0 = q_{-3} = \sum_{n=-1}^{1} h_{-3-n} c_n$$

Problem 4 p. 176

a) $F(z) = 0.8 - 0.6 z^{-1}$

We know

$$X(z) = F(z) F^*(z^{-1}) = (0.8 - 0.6 z^{-1})(0.8 - 0.6 z)$$

$$= 0.64 + 0.36 - 0.48 z^{-1} - 0.48 z$$

$$= 1 - 0.48 z^{-1} - 0.48 z$$

$$\Rightarrow x_0 = 1, \quad x_{-1} = -0.48 = x_1$$

b) $$\frac{1}{T} \sum_{n \in \mathbb{Z}} \left| H\left(\omega + \frac{2\pi n}{T}\right) \right|^2 = X(e^{j\omega T})$$

$$= 1 - 0.48 e^{-j\omega T} - 0.48 e^{j\omega T}$$

$$= 1 - 2 \cdot 0.48 \underbrace{\frac{1}{2}\left[e^{j\omega T} + e^{-j\omega T}\right]}_{\cos(\omega T)}$$

$$= \underline{1 - 0.96 \cos(\omega T)}$$

$$c)\ J_{min} = \frac{T}{2\pi}\int_{-\pi/T}^{\pi/T} \frac{N_0}{1+N_0-0.96\cos(\omega T)}\,d\omega$$

Subst. $\omega T = \theta;\ d\omega = \frac{d\theta}{T}$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{N_0}{1+N_0-0.96\cos\theta}\,d\theta$$

$$= \frac{N_0}{2\pi(1+N_0)}\int_{-\pi}^{\pi}\frac{d\theta}{1-\underbrace{\frac{0.96}{1+N_0}}_{a}\cos\theta}$$

$$= \frac{N_0}{2\pi(1+N_0)}\int_{-\pi}^{\pi}\frac{d\theta}{1-a\cos\theta}$$

From formulary

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{d\theta}{1-a\cos\theta} = \frac{1}{\sqrt{1-a^2}},\quad a^2<1$$

$$\Rightarrow J_{min} = \frac{N_0}{N_0+1}\,\frac{1}{\sqrt{1-\left(\frac{0.96}{1+N_0}\right)^2}} = \frac{N_0}{\sqrt{(1+N_0)^2-(0.96)^2}}$$