Problem p. 70

$$\Lambda_L(\underline{\Psi}) = \mathrm{Re}\{e^{i\phi}(A(\tau) + j B(\tau))\}$$

necessary condition:

$$\frac{\partial}{\partial \phi}\Lambda_L(\underline{\Psi}) = 0 \qquad \frac{\partial}{\partial \tau}\Lambda_L(\underline{\Psi}) = 0$$

$$\Lambda_L(\underline{\Psi}) = A(\tau)\cos\phi - \underbrace{B(\tau)\sin(\phi)}_{} \quad \text{... objective function}$$

$$\frac{\partial}{\partial \phi}\Lambda_L(\underline{\Psi}) = -A(\tau)\sin\phi - B(\tau)\cos\phi = 0$$

$$\Rightarrow \boxed{\tan\phi = -\frac{B(\tau)}{A(\tau)}} \leftarrow$$

$$\Rightarrow \boxed{\phi = -\arctan\frac{B(\tau)}{A(\tau)}} \quad \text{for } \tau = \hat{\tau}_{ML}$$

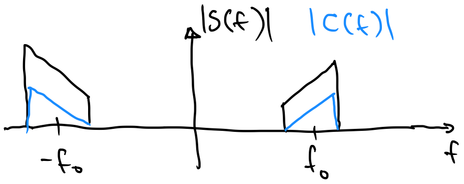
$$\frac{\partial}{\partial \tau}\Lambda_L(\underline{\Psi}) = \frac{\partial A(\tau)}{\partial \tau}\cos\phi - \frac{\partial B(\tau)}{\partial \tau}\sin\phi = 0 \quad \Big| : \cos\phi \neq 0$$

$$\Rightarrow \frac{\partial A(\tau)}{\partial \tau} - \frac{\partial B(\tau)}{\partial \tau}\left[-\frac{B(\tau)}{A(\tau)}\right] = 0$$

$$\Rightarrow \boxed{A(\tau)\frac{\partial A(\tau)}{\partial \tau} + B(\tau)\frac{\partial B(\tau)}{\partial \tau} = 0 \quad \Big|_{\tau = \hat{\tau}_{ML}}}$$

Problem p. 75

$$C(f) = |C(f)|\, e^{j\Theta(f)}$$



① First consider a general signal (not necessarily a bandpass signal like in the figure above) $s(t)$; what is the requirement for $c(\tau)$

with

$$r(t) = s(t) * c(t) = \int_{\mathbb{R}} s(t-\tau)\, c(\tau)\, d\tau$$

to represent a "non-distorting" channel?

$\Rightarrow$ allow for a delay t_0 and a scaling a

$\Rightarrow r(t) = a\, s(t - t_0)$

$$R(f) = \underbrace{a\, e^{-j2\pi f t_0}}_{C(f)} S(f)$$

$\Rightarrow |C(f)| = |a|$ $\longrightarrow$ requ. 1

$\arg C(f) = \Theta(f) = -2\pi f t_0 + \arg a$

Require $c(\tau) \in \mathbb{R} \Leftrightarrow C(f) = C^*(-f)$

$\arg a = \alpha \operatorname{sign} f$ (includes both cases $a \in \mathbb{R}$ and $a \in \mathbb{C}$)

$$\Rightarrow \Theta(f) = \begin{cases} \alpha - 2\pi f t_0 & \text{for } f > 0 \\ -\alpha - 2\pi f t_0 & \text{" } f < 0 \end{cases} \rightarrow \text{req. 2}$$

$$= \begin{cases} \tilde{\alpha} + b(f - f_0) & \text{" } f > 0 \\ -\tilde{\alpha} + b(f + f_0) & \text{" } f < 0 \end{cases}$$

for $f = f_0$: $\Theta(\pm f_0) = \pm\tilde{\alpha}$

2 Bandpass signal with carrier frequ. f_0

$s_1(t) = x(t) \cos(2\pi f_0 t)$

$$S_1(f) = \frac{1}{2}\left[X(f - f_0) + X(f + f_0)\right]$$

With $\Theta(f) = \ldots$ (req. 2) (cf. above)

$$\Rightarrow r(t) = \text{const.}\; x(t - t_0)\cos\left(2\pi f_0 t + \tilde{\alpha}\right)$$

$$= \text{const.}\; x(t - t_0)\cos\left(2\pi f_0\left(t - \frac{-\tilde{\alpha}}{2\pi f_0}\right)\right)$$

GROUP DELAY PHASE DELAY t_{PH}

GROUP DELAY : $t_0 = -\frac{1}{2\pi}\frac{\partial\theta(f)}{\partial f}$

PHASE DELAY : $t_{PH} = -\left.\frac{\theta(f)}{2\pi f}\right|_{f=\pm f_0} = -\frac{\tilde{\alpha}}{2\pi f_0}$