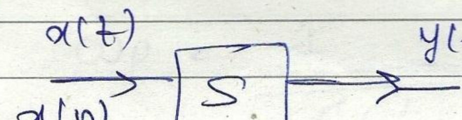
S K Mitra
Schaum Series

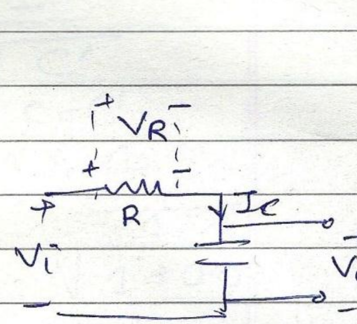
Signals And Systems

Continuous Time Signal
Discrete Time Signal
Cont. Time System
Discrete " "
LTI System
Fourier Analysis of Cont. Time System
" " Discrete " "
Sampling Process
Laplace Transform
Z transform
Probability
Random Signal & System

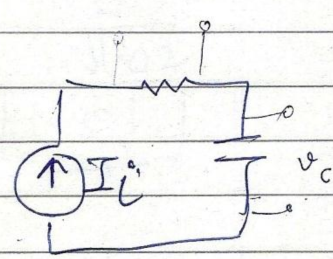
ES → DFT
Filter Design



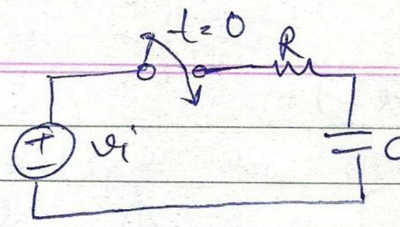
I/P	O/P
v	v_R HPF
v	I_c HPF
v	v_c (Low pass)



I_i v_R } all pass
I_i I_C } all pass
I_i v_C LPF

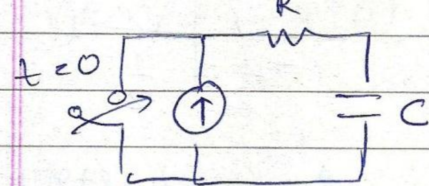


Even at low ν, current across cap. is I_i because otherwise it violates KCL



Mathematical model.

$E\,u(t)$.



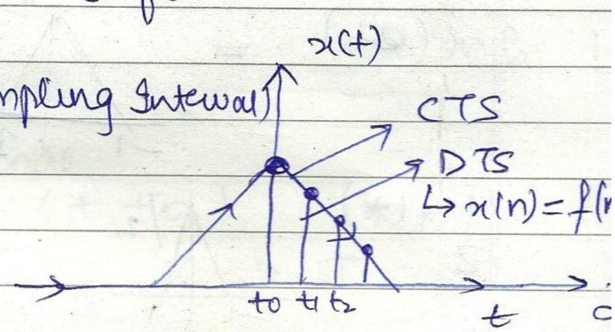
$I\,u(t)$.

Signal

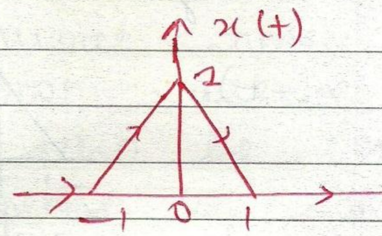
Mathematical fn or functional variable that carries some info.

$t_1 - t_0 \neq t_2 - t_1$ (Sampling Interval)
Irregular Sampling

$t_1 - t_0 = t_2 - t_1$
Regular sampling

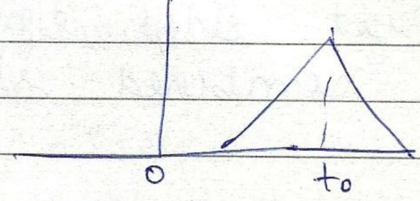


1) Time shift

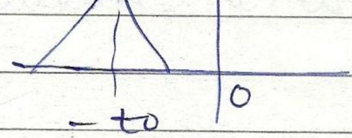


$x(t-t_0)$: (Delay)

↓



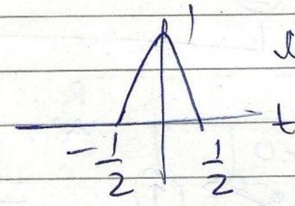
(Advance) $x(t+t_0)$



video fast → compression
slow → stretching

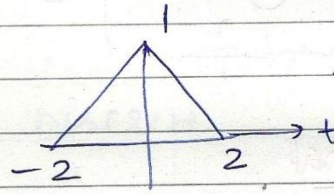
→ Time Scaling $x(\alpha t)$.

$\alpha > 1$ i.e $x(2t)$



compression

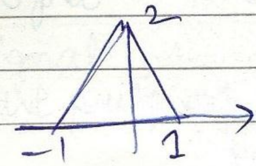
$\alpha < 1$ $x(\frac{1}{2}t)$

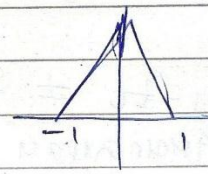


stretching

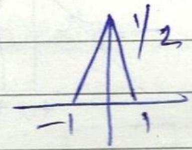
→ Scaling w.r.t magnitude

$2x(t) =$





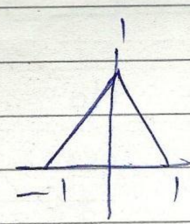
$\frac{1}{2}x(t)$



loudspeaker, mag. ↑

→ folding

$x(-t)$



remains same
reflection with y axis.

Any left sided or right sided signal after folding, the combined signal is even signal.

eg:- run record in backward direction

Odd Signal & Even Signal

$x(t) = x(-t)$ Even

$x(t) = -x(-t)$ ~~Even~~ odd

$x(0) = -x(0)$ only at 0

so odd signal passes through origin

$$\text{Even}[x(t)] = \frac{x(t) + x(-t)}{2}$$

$$\text{Odd}[x(t)] = \frac{x(t) - x(-t)}{2}.$$

Periodic signal

$$x(t) = x(t+T)$$
$$= x(t+2T) \ldots$$

For periodic signal

$$x(t) = x(t+\alpha)$$

Smallest value of $\alpha \Rightarrow T$

$$f = \frac{1}{T}$$

sampling of a continuous time periodic signal does not assure that sampled signal will also be continuous or periodic. Depends on sampling interval.

Signals

$$x(t) = A\,u(t)$$
$$x(t) = \sqrt{2}A \sin(\omega t + \phi)$$

Representation of signal which is discontinuous at particular instants or having discontinuous derivatives by singularity functions.

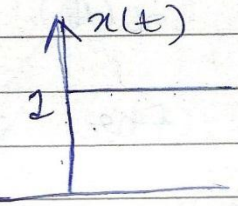
Unit step

1) $u(t) = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases}$

Gibb's phenomenon

At the point of discontinuity, signal is defined as

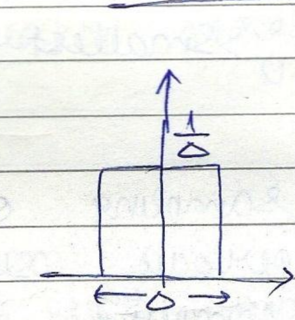
$$x(t_0) = \frac{x(t_0^-) + x(t_0^+)}{2}$$



$$\delta(t) = \lim_{\Delta t \to 0} \frac{\Delta u}{\Delta t} = \frac{1}{0} = \infty$$

or dirac delta fn.

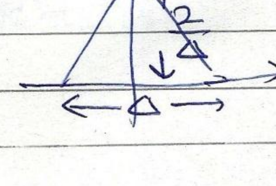
$$\lim_{\Delta \to 0} \Delta \cdot \frac{1}{\Delta} = 1$$



$$\boxed{\int_{-\infty}^{\infty} \delta(\tau)\, d\tau = 1}$$

as $\boxed{\delta(t) \to \infty \quad t \to 0}$

Also defined in terms of Δ pulse



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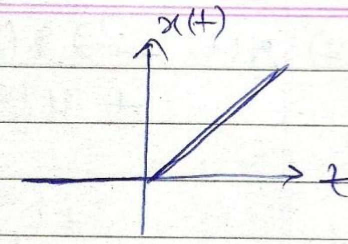
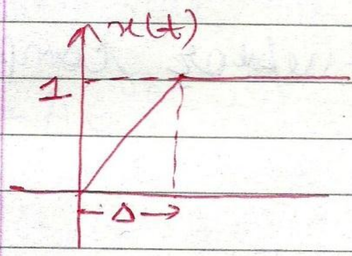
2) Ramp signal.



$$x(t) = \begin{cases} 0 & t<0 \\ t & t\geq 0 \end{cases}$$

Signal is continuous but derivate discontinuous.

Q

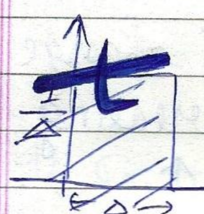


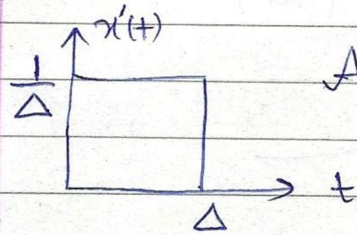
a) $\lim_{\Delta \to 0} x(t) = \delta(t)$

b) $\lim_{\Delta \to 0} x(t) = u(t)$.

c) $\lim_{\Delta \to 0} x'(t) = \delta(t)$.

d) $\lim_{\Delta \to 0} x'(t) = u(t)$.

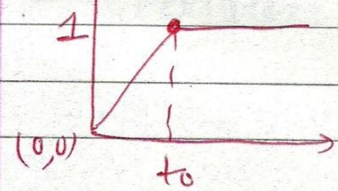




Area under curve = 1

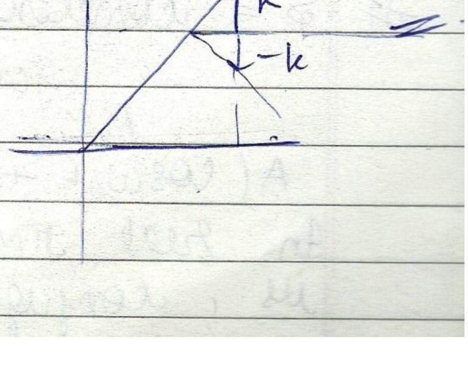
$$x'(t) = \begin{cases} 0 & t<0 \\ \frac{1}{\Delta} & 0<t<\Delta \\ 0 & t>\Delta \end{cases}$$

1)



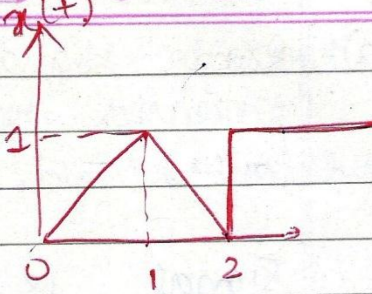
$$\frac{1}{t_0}\left[r(t) - r(t-t_0)\right]$$

$(t_0, 1)$



$1+k-k$
$=1$

2) $r(t) - 2r(t-1) + r(t-2) + u(t-2)$



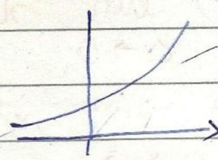
Exponential signals

$$x(t) = Ae^{st}$$

A, s may be real, +ve/-ve or complex quantity

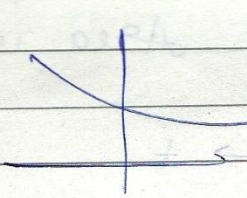
1) Assume $A \rightarrow$ real / +ve
$s \rightarrow$ +ve, real

Ae^{kt}

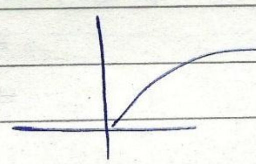


rate of change should be increasing.
$\left|\frac{dx}{dt}\right| \uparrow$

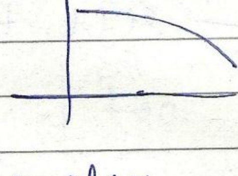
2) A real, +ve
$s \rightarrow$ real, -ve



A +ve



A -ve



s = complex   1) $s = j\omega_0$

$Ae^{j\omega_0 t}$ (hypothetical signal)

$A(\cos\omega_0 t + j\sin\omega_0 t)$

In real time system, if this signal comes, its conjugate must also be present.

$x(t) = A(\cos\theta + i\sin\theta)$
$x^*(t) = A(\cos\theta - i\sin\theta)$

Actual signal $= x(t) + x^*(t)$
$= 2A\cos\theta$

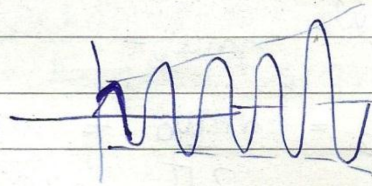
2) A real, +ve $\qquad s = \sigma_0 \pm j\omega_0$

$x(t) = Ae^{\sigma_0 + j\omega_0 t}$
$Ae^{\sigma_0 t} \cdot e^{j\omega_0 t}$

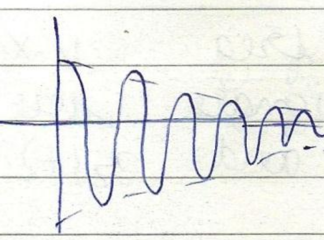
$x^*(t) = Ae^{\sigma_0 t} \cdot e^{-j\omega_0 t}$ $\qquad$ signal value at $t = 0$ $= 1$

$x^*(t) + x(t) = 2Ae^{\sigma_0 t}\cos\omega_0 t$

If σ_0 +ve



If σ_0 -ve



A complex, s complex

$x(t) = (a+jb)\,e^{(\sigma_0 + j\omega_0)t}$

$= |A|\,e^{j\theta} \cdot e^{\sigma_0 t}\, e^{j\omega_0 t}$

$\theta = \tan^{-1}\frac{b}{a}$

$x(t) = |A|\,e^{\sigma_0 t}\, e^{\pm j(\omega_0 t + \theta)}$

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$$x(t) = |A|\, e^{\sigma_0 t}\, e^{j(\omega_0 t + \theta)}$$
$$x^*(t) = |A|\, e^{\sigma_0 t}\, e^{-j(\omega_0 t + \theta)}$$

$$x(t) + x^*(t) = 2|A|\, e^{\sigma_0 t} \cos(\omega_0 t + \theta)$$

Only initial value changed, rest signal remains same.

Here value of $\cos(\omega_0 t + \theta)$ at $t = 0$ at $\cos\theta$.

i.e signal may be shifted to Right or Left

$\Rightarrow$ $x_1(t) = e^{j\omega_0 t}$

$T = \frac{2\pi}{\omega_0}$ $\qquad f_1 = \frac{\omega_0}{2\pi}$

$x_2(t) = e^{jm\omega_0 t}$

$f_2 = \frac{m\omega_0}{2\pi} = m f_1$

ek ka freq. k x(freq. of other) we say signals are harmonically related.
$x_1(t)$ and $x_2(t)$ are " " signals

x_2 is m th harmonic of x_1

Among all harmonically related signals, signal having min. freq. is called fundamental signal & freq = fundamental freq.
All other are its harmonics and its freq. is harmonic 2)

In real life $s = -\sigma_0 \pm j\omega_0$.

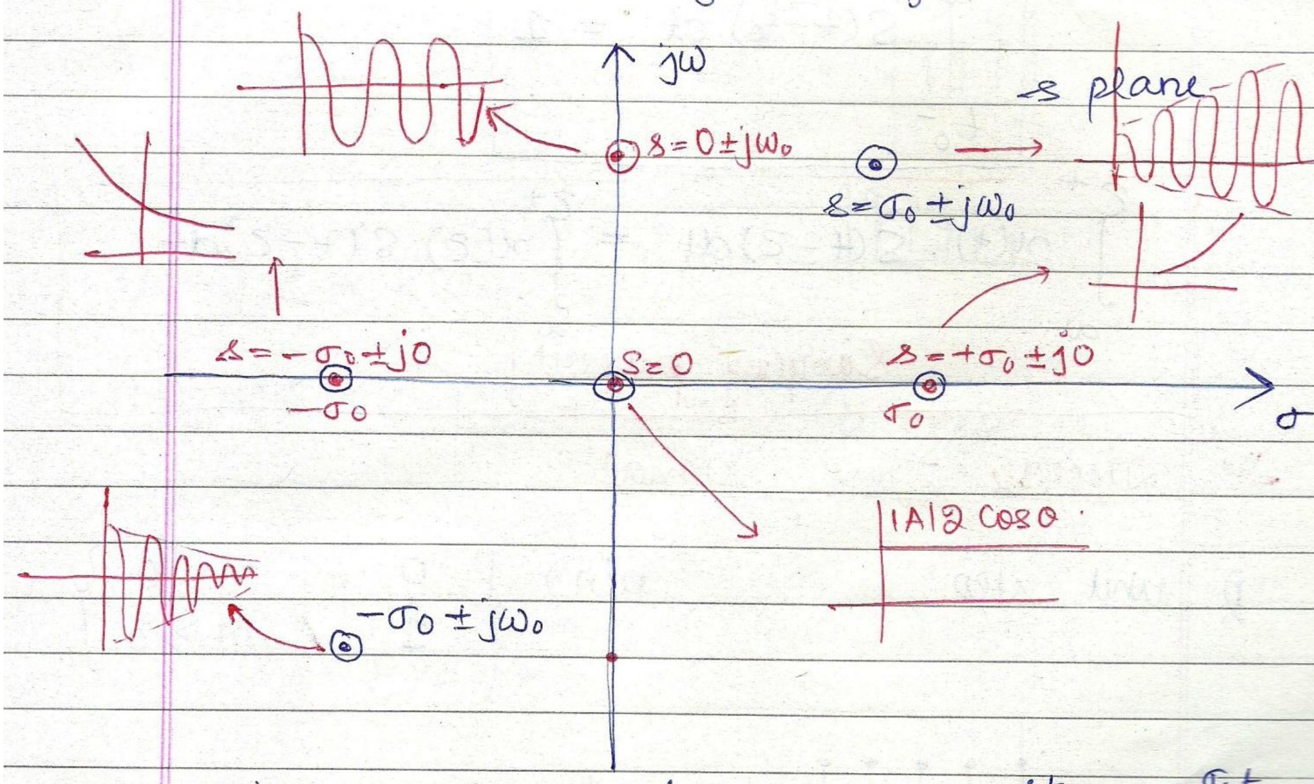
$-\sigma_0$ due to stability of system.
s = complex frequency.
σ_0 = Naplear freq.
↳ Log. signal

$$e^{st} = 1 + \frac{st}{1!} + \frac{(st)^2}{2!} - - - -$$

st dimensionless $\boxed{s = \frac{k}{t}}$

Note Any signal can be represented in the form of exponential signal.

$x(t) = Ae^{st}$ (characteristic signal or Eigen signal)



consider real signal $x(t) = Ae^{st} = 2|A|e^{\sigma_0 t}\cos(\omega_0 t + \theta)$
here both +ve & neg. freq. are included.

$e^{-2t}, e^{-3t} \longrightarrow$ This will fastly decay

$e^{3t} \rightarrow$ fastly grow

W_0 more $\longrightarrow$ compression

W_0 less $\rightarrow$ expansion

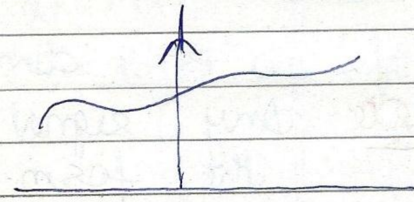
\# $$\int_{-\infty}^{t} \delta(\tau)d\tau = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases}$$

$$= u(t)$$

$$\frac{d\,u(t)}{dt} = \delta(t)$$

$\Rightarrow$ $f(t) = x(t) \cdot \delta(t)$

$= x(0) \cdot \delta(t)$



$\Rightarrow$ $x(t) \cdot \delta(t-t_0)$

$= x(t_0) \cdot \delta(t-t_0)$

$x(0)$ or $x(t_0)$ is now strength of δ fn.

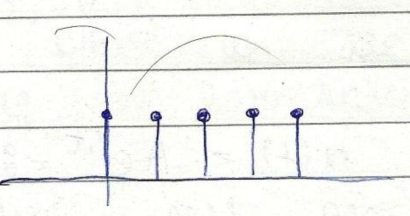
$$\boxed{\int_{t_0^-}^{t_0^+} \delta(t-t_0)\,dt = 1}$$

$$\int_{-\infty}^{\tau^+} x(t) \cdot \delta(t-\tau)\,dt = \int_{-\infty}^{\tau^+} x(\tau)\,\delta(t-\tau)\,dt$$

Sampling property

\# Discrete Time signal

i) unit step $\quad u(n) \begin{Bmatrix} 0 & n<0 \\ 1 & n \geq 0 \end{Bmatrix}$



2) Unit Impulse

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \text{ otherwise} \end{cases}$$

$$\boxed{\begin{array}{l} \delta(n) = u(n) - u(n-1) \quad \text{[Difference operator]} \\ u(n) = \sum_{-\infty}^{n} \delta(n) \end{array}}$$

Ramp signal – sampled version of Cont. Ra

Exponential signal

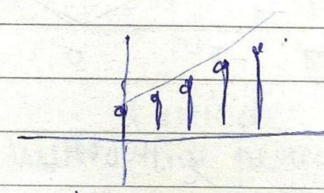
Ae^{st} when discrete Ae^{skT}

T constant

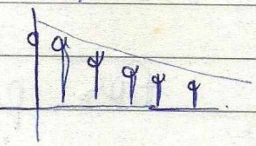
$Ae^{sT(k)} \rightarrow$ only variable.

$x(n) = C\alpha^{n}$

1) $\alpha > 1$

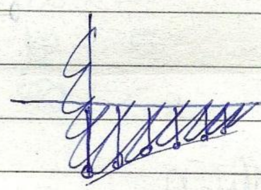


2) $0 < \alpha < 1 \longrightarrow$

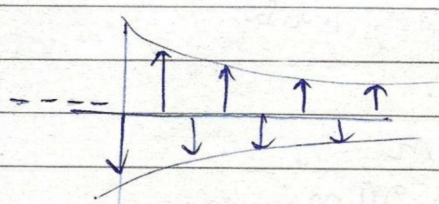


3) $-1 < \alpha < 0$

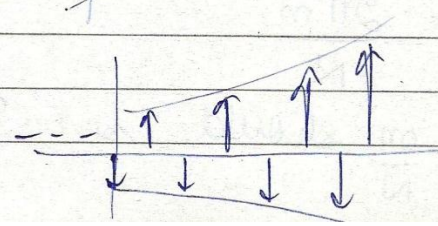
4) $\alpha < -1$



3)



4)



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$C\alpha^n$ $\quad C = 1$ then $\alpha^n = e^{j\omega_0 n}$

$e^{jW_0 n}$

$e^{jW_0 n}$

$$x(n) = e^{jW_0 n}$$

$$x_1(n) = e^{j(W_0 + k2\pi)n}$$

$$= e^{jW_0 n} \cdot e^{jk2\pi n}$$

$$= e^{jW_0 n}$$

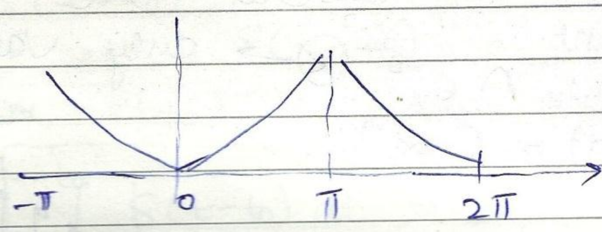
1) $\boxed{0 \le W_0 \le 2\pi}$

2) $e^{j(\pi \pm \Delta)n}$

$$e^{j\pi n} \cdot e^{\pm j\Delta n}$$

$$- e^{\pm j\Delta n}$$

$$-\pi \le W_0 \le \pi$$



$$e^{j\omega_0 t} = e^{j(\omega_0(t+T)}$$

$$= e^{j\omega_0 t}\, e^{j\omega_0 T}$$

For periodicity

$$\omega_0 T = 2\pi k$$

$$\boxed{T = \frac{2\pi k}{\omega_0}}$$

$$e^{jW_0 n} = e^{jW_0(n+N)}$$

$$= e^{jW_0 n}\, e^{jW_0 N}$$

$$W_0 N = 2\pi m$$

$$W_0 = \frac{2\pi m}{N}$$

W_0 cannot 2π so $\frac{m}{N}$ should be rational.

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Q. Find the freq. of $\cos \frac{n}{6}$.

$$\frac{e^{\frac{j \times 1 n}{6}} + e^{\frac{-jn}{6}}}{2}$$ (Non periodi

$$e^{j n/6} = e^{j W_0 n}$$

$$W_0 = \frac{1}{6} = 2\pi \frac{m}{N}$$

Q. $\sin 3n$

$$e^{j3n} \qquad W_0 = 3$$

$$3 = 2\pi \frac{m}{N}$$ (Not periodic

Q. $\cos \frac{2\pi n}{12}$

$$\frac{\cancel{2\pi}}{12} = \cancel{2\pi} \frac{m}{N} \qquad \frac{m}{N} = \frac{1}{12}$$

Periodic.
with time period m = 1
$\boxed{N = 12}$

Q. $\cos \frac{5\pi n}{31}$

$$\frac{5\cancel{\pi}}{31} = 2\cancel{\pi} \frac{m}{N}$$ Periodic

$$\frac{m}{N} = \frac{5}{62}$$

$\boxed{N = 62}$

$x(n) = e^{j n W_0}$ is periodic only if

$$W_0 = 2\pi \frac{m}{N}$$

difference from cont. time

$$N = \frac{2\pi m}{W_0} \qquad f = \frac{2\pi}{N} <$$

#. **Imaginary sequence** – when values at intervals is imaginary.

Bounded sequence Mag finite for all values.

1) $$|x(n)| \leq M < \infty \quad \forall n$$

2) If $\sum_{-\infty}^{\infty} |x(n)| < Q < \infty$ then $x(n)$ is said to be absolutely summable.

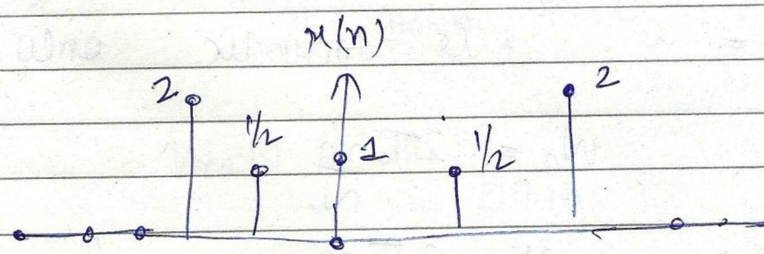
3) If $\sum_{-\infty}^{\infty} |x(n)|^2 < Q < \infty$ then $x(n)$ is called square summable. It is called energy of the signal.

Average power of sequence in the interval $-k$ to k.

$$\frac{1}{2k+1} \sum_{n=-k}^{k} |x(n)|^2$$

Overall power (Total)

$$\lim_{k \to \infty} \frac{1}{2k+1} \sum_{n=-k}^{k} |x(n)|^2$$



$$E = 4 + 1/4 + 1 + 1/4 + 4 = 9.5$$

over finite period

P_{av} over one period $= \frac{9.5}{5} = \frac{E}{2k+1}$

Now average power of overall range

$$\frac{E}{\infty} = 0$$

$\Rightarrow$ Average power of the signal which is of finite length is zero.

Average power of periodic signal with period N

$$\frac{1}{N}\sum_{n=0}^{N} |x(n)|^2$$

$$x_1(n) \rightarrow \otimes \rightarrow x_1(n) \cdot x_2(n)$$
$$\uparrow x_2(n)$$

coeff. multiplicatio

$$x_1(n) \rightarrow \otimes \rightarrow x_1(n) \cdot \alpha$$
$$\uparrow \alpha$$

$$x(n) \rightarrow \boxed{D} \rightarrow x(n-1)$$ Delay.

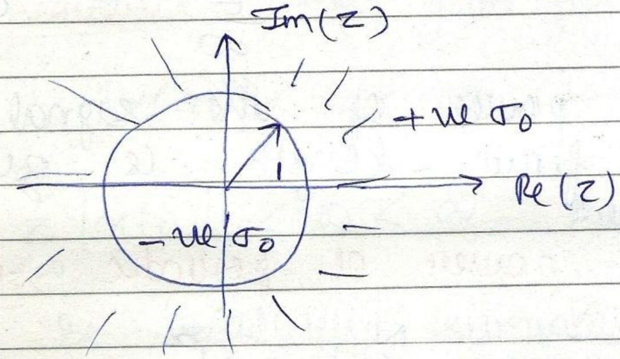
$$x(n) \rightarrow \Sigma \rightarrow x(n) + y(n)$$ summer
$$\uparrow y(n)$$

$$x(t) = Ae^{st}$$
$$= Ae^{skT_s}$$
$$= A|e^{sT_s}|^{k}$$
$$x(n) = A\,z^{k}$$

$$z = e^{sT_s}$$

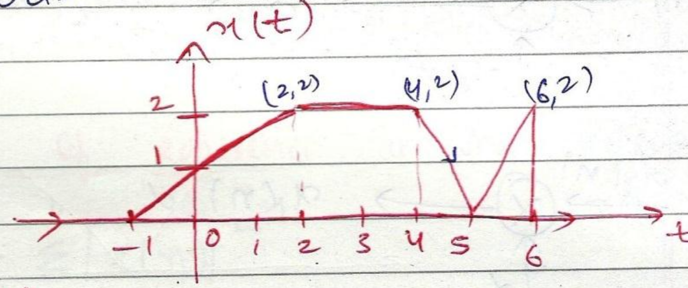
z is complex ν of discrete time signal.

$$Z = Re|z| + j\,Imj|z|$$



→ Re|z| gives rate of growth or decay
→ ν of oscillation depends on phase of z and phase depends on both real & imaginary parts.

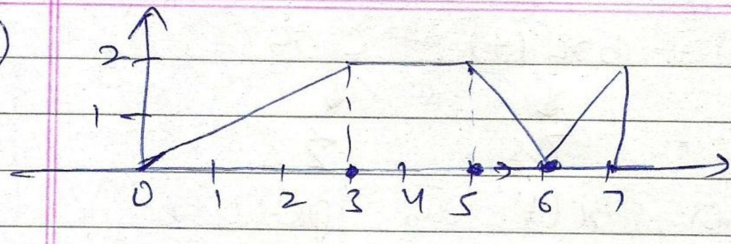
Q Plot



(2,2) (−1,0) (4,2) (5,0) (6,2) (5,0)

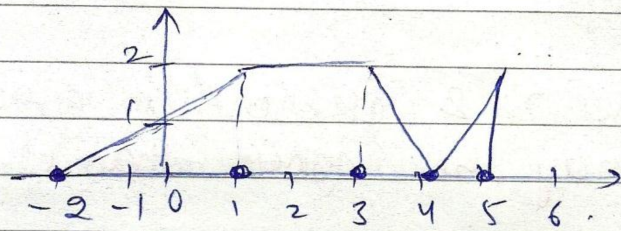
1) $x_1(t) = x(t-1)$
2) $x_2(t) = x(t+1)$
3) $x_3(t) = x(2t)$
4) $x_4(t) = x\left(\frac{1}{2}t\right)$
5) $x_5(t) = 5x(-t)$
6) $x_6(t) = \cancel{5x(2t-4)}$ $5\,x_3(t-4)$
7) $x_7(t) = \cancel{5x(2(t-2))}$ $5\,x_1(2t)$
8) $\cancel{x_8(t)}$ = find even $(x(t))$
9) Odd $(x(t))$
10) Represent $x(t)$ by singularity fns.
11) Plot $x'(t)$ and then represent by singularity fn.

1)



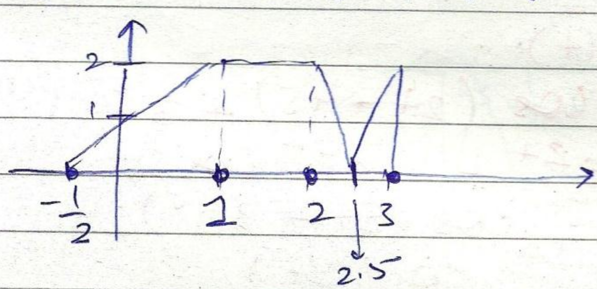
2
1
0
1
2
3
4
5
6
7

2)



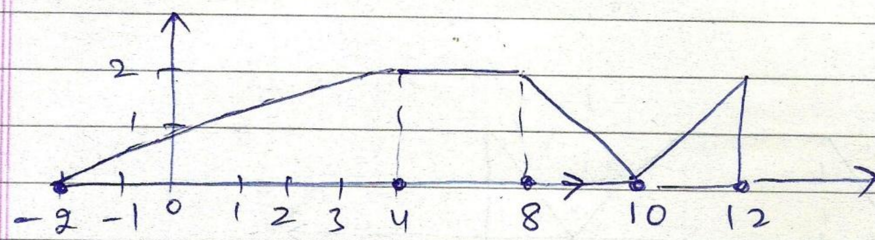
2
1
-2
-1
0
1
2
3
4
5
6

3)



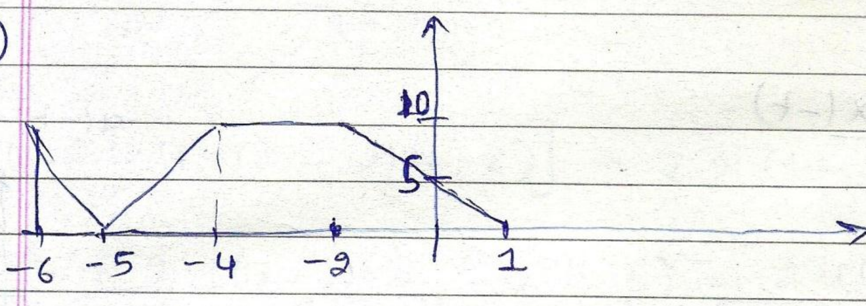
2
1
$-\frac{1}{2}$
1
2
3
2.5

4)



2
1
-2
-1
0
1
2
3
4
8
10
12

5)



10
5
-6
-5
-4
-2
1

6).

10
5
3.5
5
6
6.5
7

12) $x(t) = a\,x_1(t) + b\,x_2(t) + c\,x_3(t)$

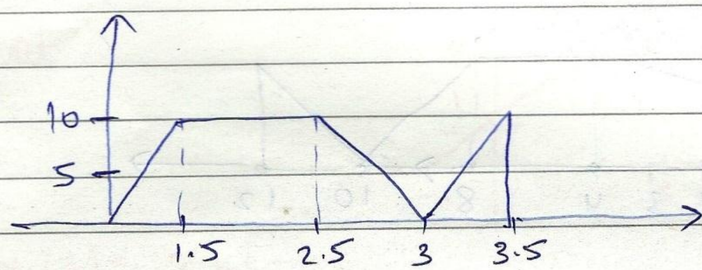
$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ T = 2\,sec & 7\,s & 2\,s. \end{array}$$

check whether $x(t)$ is periodic
find periodicity

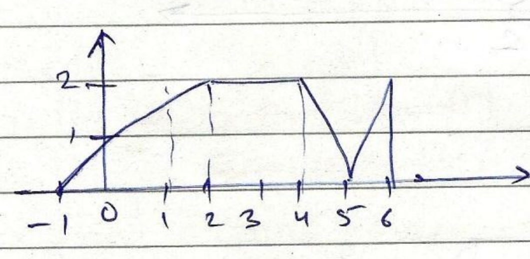
13) Find complex 2 & present in given signal. (complex 2 may be more than 1)

a) $x_1(t) = 5u(t)$
b) $x_2(t) = 2\cos(2t+45) + 3\sin(4t+30)$
c) $x_3(t) = 2e^{2t} + e^{-t}\cos 2t$
d) $x_4(t) = e^{jt}\cos(5t+30°)$
e) $x_5(t) = e^{-2t}\cos(3t)$

7)

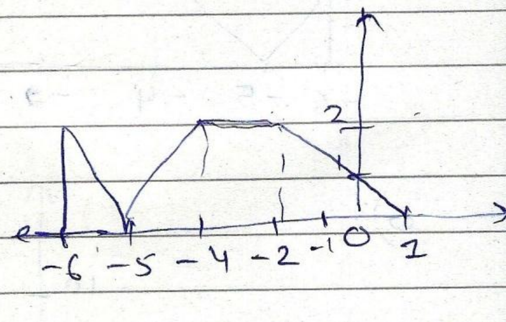


8) $\dfrac{x(t) + x(-t)}{2}$

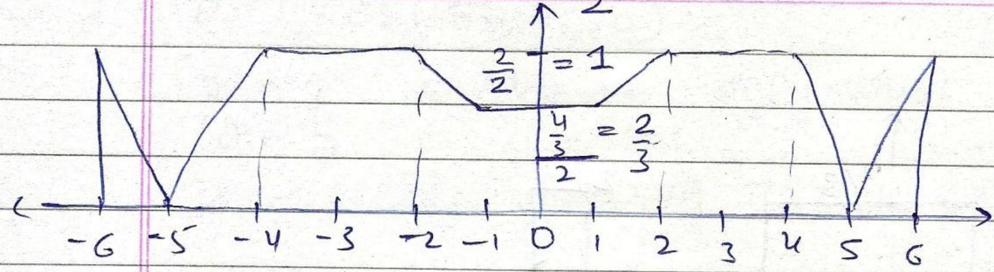


+

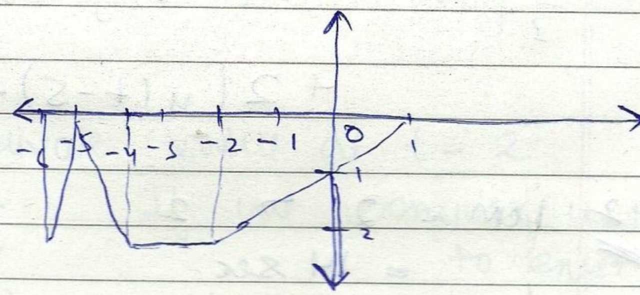
$x(-t)$.

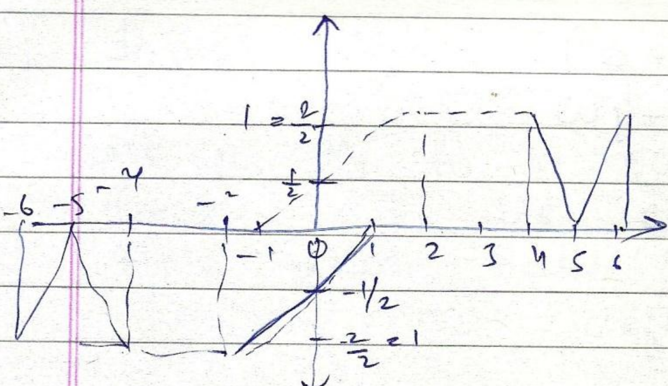


$$\frac{x(t)+x(-t)}{2}$$



9. $-x(-t)$.





10. $\frac{2}{3}\left[r(t+1) - r(t-2)\right] - 2r(t-4) + 2r(t-5)$

$+ 2r(t-5) - 2u(t-6) - 2u(t-6)$.

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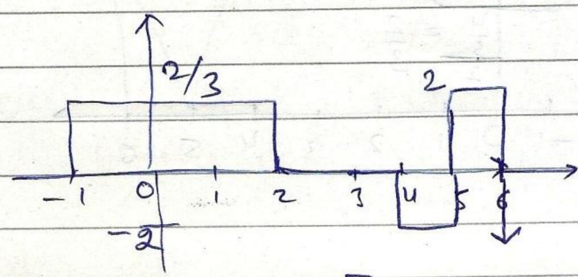
11 $x'(t)$



$$\frac{2}{3}\left[u(t+1)-u(t-2)\right]-2\left[u(t-4)-u(t-5)\right]$$
$$+2\left[u(t-5)-u(t-6)\right]-2\delta(t-6)$$

12 lcm - 2 7 2
= 14 sec.

13 (A) $e^{\sigma t}\cos(\omega t+\phi)$.
complex $\nu = \sigma + j\omega$.

a) $5u(t) = 5e^{0}u(t)$

$\sigma = 0 \quad \omega = 0$

$\nu = 0 + j0$.

b) $2\cos(2t+45) + 3\sin(3t+30)$.

$\sigma = 0$ | $\sigma = 0$
$\omega = 2$ | $\omega = 3$
$\nu = 0 \pm 2j$ | $\nu = 0 \pm 3j$

c) $2e^{2t} + e^{-t}\cos 2t$

$\sigma = 2$ | $\sigma = -1$
$\omega = 0$ | $\omega = 2$
$\nu = 2 \pm j0$ | $\nu = -1 \pm 2j$

$\frac{1}{2} e^{jt} \left(e^{j(5t+30)} + e^{-j(5t+30)} \right)$

$= \frac{1}{2} e^{j(6t)} \cdot e$

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d) $e^{jt} \cos(5t + 30)$.

$\sigma = j$ (σ cant be imaginary)

$\omega = 5$

~~...~~ opening up we get $e^{j6} + e^{-j4}$

e) $e^{-2t} \cos(3t)$

$\sigma = -2$

$\omega = 3$

$\nu = -2 \pm 3j$

→ If we find derivate value at $t = 2$

$\frac{2}{3} - \frac{2}{3}$ if we consider $u(t-2)$ to exist.

$= 0$

If not then $\frac{2}{3}$

but as the fn is not differentiable at $t = 2$, ∴ its derivative value at $t = 2$ does not exist.

∴ not defined.

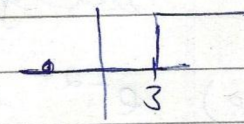
derivative at $t = 6$ is not defined. Rather it is an impulse of ∞ value

⇒ W_0 unit is radian and it is dimensionless.

W_0 is basically representing freq.

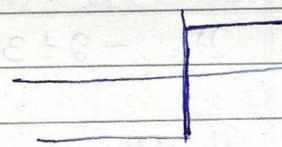
Q. A signal $x(t)$ is known to be 0 for $t<3$. What is range for which $x(t+1)$ is zero.

$$\underline{t<2}$$

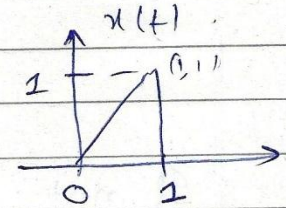


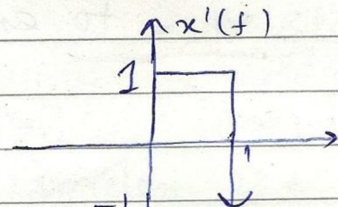
Q. derivative of signum fn at $t=0$

impulse of +2 (not defined)



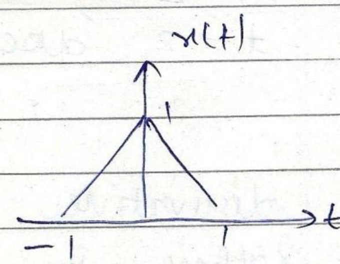
Q. Find $x'(t)$



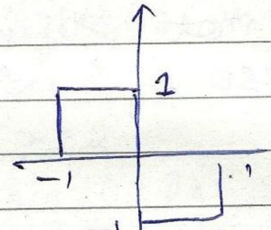


$u(t)-u(t-1)-\delta(t-1)$

Q. Find $x'(t)$, $x''(t)$.

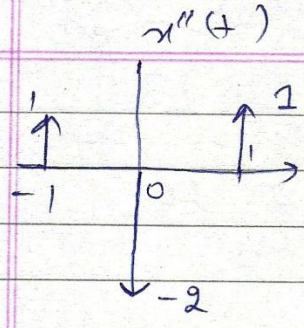


(0,1)
(-1,0)
$\frac{1-0}{0+1}$
1



$[u(t+1)-u(t)]-[u(t)-u(t-1)]$

$u(t+1)-2u(t)+u(t-1)$

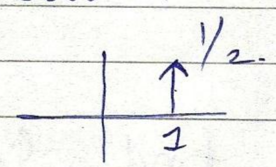


$\delta(t+1) - 2\delta(t) + \delta(t-1)$.

Q Plot area under this curve

$\delta[2(t-1)]$

$\frac{1}{2}\delta(t-1)$



Q $5t^2\,\delta(\cancel{2}(t-2))$.

$5t^2\,\frac{1}{2}\,\delta(t-2)$

$5(2)^2\,\frac{1}{\cancel{2}}\,\delta(t-2)$

$10\,\delta(t-2)$.

Q $4\cos\left(\frac{\pi}{2}t\right)\delta(2t-4))$.

$4\cos\left(\frac{\pi}{2}t\right)\frac{1}{2}\,\delta(t-4)$.

$4\cos\left(\frac{\pi}{\cancel{2}}\times\cancel{4}^{2}\right)\frac{1}{2}\,\delta(t-4)$

${}^{2}\cancel{4}\cos(2\pi)\,\frac{1}{\cancel{2}}$

$2\times 1\,\delta(t-4)$

$2\,\delta(t-4)$

Q $\int_{-4}^{2}\cos(2\pi t)\,\delta(2(t+1))\,dt$

$= \int_{-4}^{2}\cos(2\pi t)\,\frac{1}{2}\,\delta(t+1)$

$$\frac{1}{2}\cos(-2\pi)$$

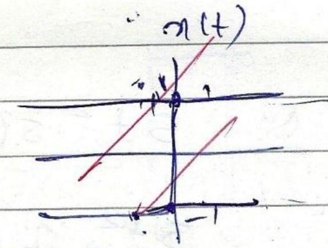
$$= \frac{1}{2}$$

Q $\int_0^{\infty} e^{-\alpha t^2}\, \delta(t+10)\, dt$.

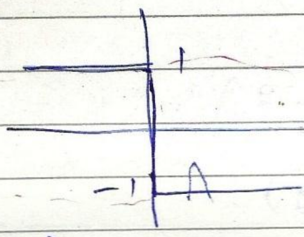
$$= 0.$$

Q find $Ev\{sgn(-t)\} = 0$

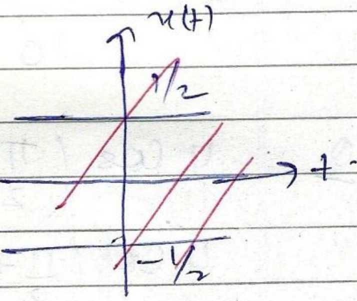
$Odd\{sgn(t)\}$.



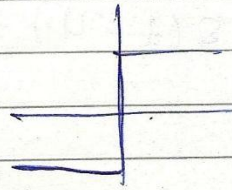
$x(-t)$



$$\frac{x(t)+x(-t)}{2}$$



$-x(-t)$.



$Odd(sgn(t)) = sgn(t)$

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$\int_{-\infty}^{\infty} \delta(2(t-1))\, dt$

$2t - 2 = z$

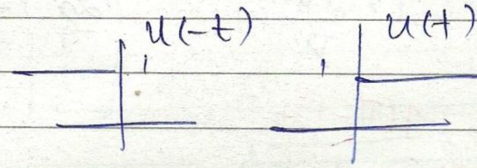
$dt = \frac{dz}{2}$

$\int_{-\infty}^{\infty} \delta(z) \frac{dz}{2}$

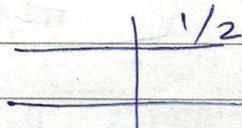
$= \frac{1}{2}(1)$

Q Represent even & odd part of unit step in terms of signum fn.

$\frac{u(t) + u(-t)}{2}$

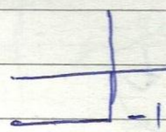


even

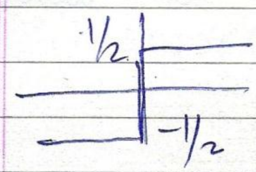


$= \frac{1}{2} x(|t|)$ except at zero.

$-u(-t)$



odd



$\frac{1}{2} \operatorname{sgn}(t)$

value at 0 = 0.

If a signal is combination of 2 or more periodic signals then it will be periodic if and only if ratio of fundamental period is a rational no.

$T = \frac{\text{LCM Numerator } (T_1, T_2 - - -)}{\text{HCF (Denominator } (T_1, T_2 - - -)}$

Q $\sin 2\pi t + \sin t$ (Non periodic)

$T = \frac{2\pi}{2\pi} = 1$ $\quad$ $T = \frac{2\pi}{1} = 2\pi$

Q $2\cos 10t + 1 + \sin 4t$

$\frac{2\pi}{\cancel{10}\,5}$ $\frac{2\pi}{\cancel{4}\,2}$

Period = $\cancel{8}\pi$

Q $e^{j(\pi t - 1)}$

$e^{j\pi t} \cdot e^{-j}$ Initial phase $= -\frac{\pi}{2}$

$\omega = \pi$ $\omega = \Omega$

$T = \frac{2\pi}{\pi}$ $\frac{2\pi}{T} = \pi$ $\boxed{T=2}$

Q $e^{j(\pi-1)t}$ $\omega = (\pi - 1)$ $\boxed{T = \frac{2\pi}{\pi-1}}$

$e^{j\pi t} \cdot e^{-jt}$

Q $e^{(j\pi - 1)t}$ Multiplication of P & NP signal is always non periodic

$e^{j\pi t} \cdot e^{-t}$

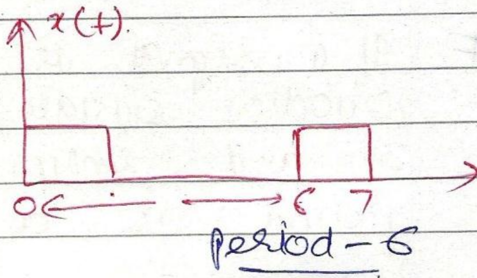
$T = \frac{2\pi}{\pi}$ ↓ $t = \frac{2\pi}{-1}$

P N.P

Q Find periodicity of

$y_1(t) = x(3t)$

$y_2) = x(\frac{t}{3})$



period − 6

Period = $\frac{6}{3} = 2$

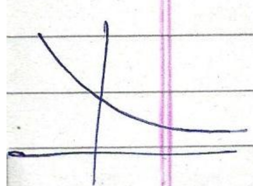
Period = $6 \times 3 = 18$

Q $x(t) = e^{-2t} u(t)$ - find E and power

$$\int_0^\infty |e^{-2t}|^2 \, dt$$

If u(t) not then E =

$\frac{1}{e^{\infty}}$



$$\int_0^\infty e^{-4t} \, dt$$

$$-\frac{1}{4}\int_0^\infty e^{-4t} \, dt = \left[e^{-4(\infty)} - e^{-0}\right]$$

$$-\frac{1}{4}\left[0-1\right]$$

$$E = \boxed{\frac{1}{4}}$$

If $x(t) = e^{-2t} \Rightarrow P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |e^{-2t}|^2 \, dt$ $\boxed{P = 0}$

$$\frac{1}{2T}\int_{-T}^{+T} e^{-4t} \, dt$$

$$\frac{1}{2T} \cdot -\frac{1}{4}\left[e^{-4t}\right]_{-T}^{T}$$

$$\lim_{T\to\infty} \frac{-1}{2T\times 4}\left[e^{-4T} - e^{4T}\right]$$

By L Hospitals

$\boxed{Ans = \infty}$

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

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Q1 $A\cos\omega t$. find power.

Q2 $x(t) = A$ " "

$$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |A\cos\omega t|^2 dt$$

$$\int_{-T}^{T} A^2 \cos^2\omega t \, dt$$

$E = \infty$
(becoz it is not a converging signal)

$$\frac{A^2}{2}\int_{-T}^{T} \cancel{\cos}\ \frac{1+\cos 2\omega t}{2} = \frac{1}{\cancel{2T}}\frac{A^2}{2}\left[\frac{1}{2}(\cancel{2T}) - 0\right]$$

$$= \boxed{\frac{A^2}{2}}$$

Q3 $x(t) = u(t)$. Calculate power.

$$\underline{2}\quad \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} A^2 dt$$

This is normalized power

$$\frac{1}{\cancel{2T}} A^2(\cancel{2T})$$

$$\boxed{\text{Power} = A^2}$$

Q4 $x(t) \cdot \cancel{2}\, e^{2t}u(t)$.

$$\underline{3}\quad \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} u(t)\, dt$$

Periodic signal power finite
Energy ∞.

$E = \infty$

$$\lim_{T\to\infty} \frac{1}{2T}\int_{0}^{T} dt \qquad \frac{1}{2T}\times T = \boxed{\frac{1}{2}}$$

$$\underline{4}\quad \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |e^{2t}|^2 u(t)$$

$$\left|\frac{e^{4t}}{4}\right|_{-T}^{T} \quad \lim_{T\to\infty} \frac{e^{4T} - e^{0}}{4}\left(\frac{1}{2T}\right)$$

$$\lim_{T\to\infty} \frac{1}{8T}(e^{4T}-1) = \infty$$

$E = \infty$

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for a periodic signal $E = \infty$ and power finite.

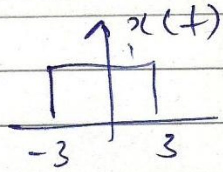
However nothing could be said in case of non periodic signals. They can be E or P.

Q $2 + 3\cos 2\pi t \, e^{-2t} u(t) + 4 \sin 6\pi t$.

Find power of signal.

$$2^2 + \frac{3^2}{2} \times 0 + \frac{4^2}{2} = 4 + \frac{\cancel{16}^{\,8}}{\cancel{2}}$$

$$= \boxed{12}$$

Q $x(t) =$  $E(x)$. Find energy of $x(3t)$

$$\int_{-3}^{3} 1^2 \, dt \qquad 3 + 3 = 6.$$

$$E(3x) = \frac{6}{3} = 2.$$

$$\boxed{\begin{array}{l} x(t) \longrightarrow E \\ x(\alpha t) \longrightarrow \dfrac{E}{\alpha} \end{array}}$$

Power remains same.

$$\lim_{x \to \infty} \frac{e^x - 1}{x} \qquad \frac{\cancel{1} + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} - - - - \cancel{-1}}{x}$$

$$1 + \frac{x}{2!} + \frac{x^2}{3!} = \infty.$$

Q $x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$

$n = 0$

1) $y_1(n) = x(n-2)$
2) $y_2(n) = x(n+3)$
3) $y_3(n) = x(-n)$
4) $y_4(n) = x(2n)$ → Decimation
5) $y_5(n) = x\left(\frac{1}{2}n\right)$ → Interpolation.
6) $y_6(n) = x(2n+1)$
7) $y_7(n) = \left(-\frac{n}{3}+2\right)$

⇒ In case of discrete, scaling of impulse does not change its strength

Imp $\boxed{y_n = \delta(\alpha n) = \delta(n)}$

⇒ $x(n)\,\delta(n) = x(0)\,\delta(n)$

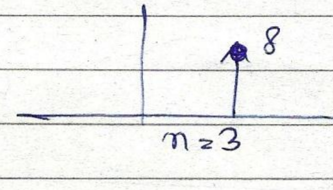
$x_n\,\delta(n-n_0) = x(n_0)\,\delta(n-n_0)$

$$\sum_{-\infty}^{\infty} x(n)\,\delta(n-n_0) = x(n_0)$$

Q $2^n\,\delta(n-3)$

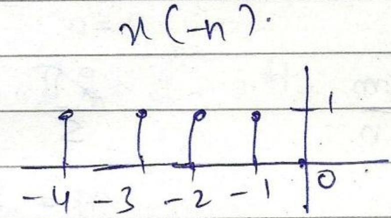
$2^3\,\delta(n-3)$

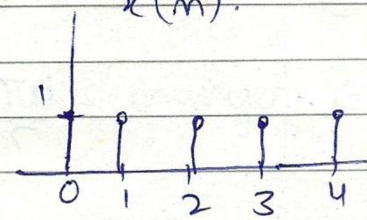
$8\,\delta(n-3)$



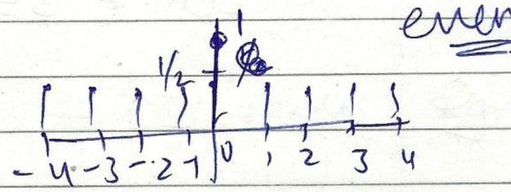
Q $x(n) = u(n) - u(n-5)$

Find odd + even part

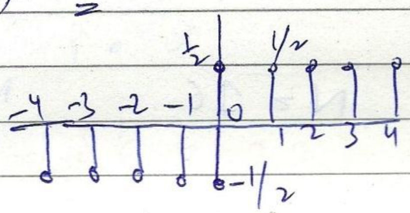


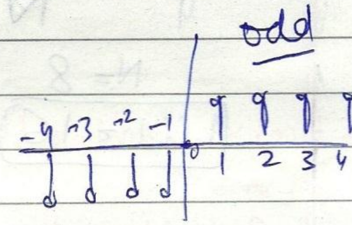


$\dfrac{x(n) + x(-n)}{2}$

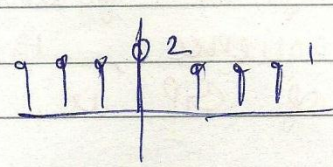


$-x(-n) =$





$x(n) + x(-n)$



\# $\sin\sqrt{t}$ only defined for +ve t.
so signal is N.P.

Q $e^{j7\pi n}$

$$7\pi = \frac{2\pi m}{N}$$

$$\frac{m}{N} = \frac{7}{2}$$

Periodic with period 2.

Q $3e^{j\frac{3}{5}(n+\frac{1}{2})}$

Non periodic

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Q $\cos\left(\frac{n}{8} - \pi\right)$ (Non periodic)

Q $1 + e^{j\frac{4\pi}{7}} - e^{j\frac{2\pi}{5}n}$

$$2\frac{4\pi}{7} = 2\pi\frac{m}{n} \qquad \frac{2\pi}{5} = 2\pi\frac{m}{n}$$

$$n = 7 \qquad n = 5$$

LCM = 35 (Total period)

Q $2\cos\left(\frac{\pi}{4}\right)n + \sin\left(\frac{\pi}{8}n\right) + \cos\left(\frac{\pi}{2}\cdot n\right) + \frac{\pi}{6}$

$$\frac{2\pi}{4} = 2\pi\frac{m}{N}$$

$$N = 8 \qquad N = 16 \qquad N = 4$$

$\boxed{LCM = 16}$

Q If to find power, energy of ∞ sequence, then generally summation of GP is used

Q $x(n) = \frac{1}{2^n} u(n)$

$$\sum_{n=-\infty}^{\infty} \left|\frac{1}{2^n}\right|^2 u(n) = \sum_{n=0}^{\infty} \frac{1}{2^{2n}}$$

$|r| < 1$

$\overset{\infty}{GP}$ sum $= \frac{a}{1-r}$

$$= 1 + \frac{1}{2^2} + \frac{1}{2^4}$$

$$\frac{4}{3}$$

$$= \frac{1}{1 - \frac{1}{2}} = \frac{4}{3}$$

Power = 0.

Q $x(n) = 2^n u(n)$ exp. growing so $E = \infty$.

$$E = \sum_{n=0}^{\infty} |2^n|^2$$

$$= 1 + 2^2 + 2^4 + \dots \text{ (Non convergent GP).}$$

Power $= \infty$

Q $x(n) = 2^{-n}$ (Non Causal signal

$$E = \sum_{n=-\infty}^{\infty} \left|\frac{1}{2^n}\right|^2 = \infty$$

$$P = \infty$$

Q $x(n) = \{\underset{\uparrow}{6}\ 4\ 2\ 2\}$ 1) $x_1(n) = x(n-2)$

cal. energy of 2) $x_2(n) = x(2-n)$

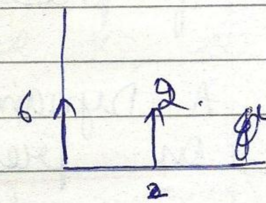
finite length seq. so power 0. 3) $x_3 n = x\left(\frac{n}{2}\right)$

$E = 60$.

1) energy same

2) Energy same.

3) ~~36 + 4 = 40~~



3) Energy $= 60$.

$y(0) = x(0) = 6$ $y(3) = 0$

$y(1) = x\left(\frac{1}{2}\right) = 0$ $y(4) = x(2) = 2$

$y(-1) = x\left(-\frac{1}{2}\right) = 0$ $y(5) = 0$

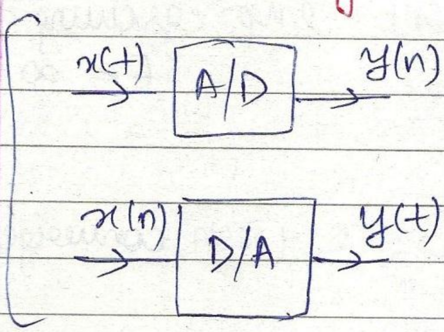
$y(2) = x(1) = 4$ $y(6) = 2$

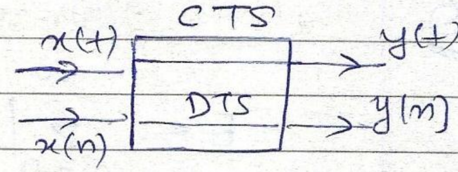
$y(-2) = x(-1) = 0$.

padding has been done.

System

hybrid System





1) Static & Dynamic System.

Static — Memoryless eg:- Resistor
Dynamic — with memory eg:- Cap.

1) $y(n) = x(n) + x(n-1)$ D
2) $y(n) = \sum_{k=1}^{n} x(k)$ D
3) $y(n) = \left[x(n) + x^2(n)\right]^{1/2}$ S
4) $y(t) = x(t^2)$. D

A system is said to be a static system if present O/P depends on present I/P only.

A Dynamic System is one that depends on present I/P, ~~or~~ past I/P or past O/P.

\# Mathematical description of S system is given by simple algebraic eq.

of D by differential eq^n or difference eq^n.

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2) ~~Invertible~~ ~~&~~ + Linear & non linear

additivity & homogenity

$$a_1x_1 + a_2x_2 = a_1y_1 + a_2y_2$$

cond – 0 I/P, then response should be 0.

$$0 = x_1 \rightarrow \boxed{} \rightarrow y_1 = k$$

$$0 = x_2 \rightarrow \boxed{} \rightarrow y_2 = k$$

$$x_1 + x_2 = 0 \rightarrow \boxed{} \rightarrow y = k$$

But by above property

$$a_1x_1 + a_2x_2 \longrightarrow a_1y_1 + a_2y_2$$

$$a_1k + a_2k$$

$$\downarrow 0 \longrightarrow (a_1+a_2)k$$

But this should also be k, so above condition is put.

1) $y(t) = m(x(t)) + c$ $\qquad$ $y(0) = m(x(0)) + c$
$= c$

Initial c
So Non linear

2) $x(t) \rightarrow \boxed{} \rightarrow y(t) = \int_{-\infty}^{3t} x(\tau)\,d\tau$

Time Variant

Whenever initial cond. non zero, it is non linear.

when $x(0)$ O/P $\rightarrow 0$.

Integro differential eqn

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1) $y(t) = x(t)^2 + 2x(t) + 2$.
Non linear even if 2 is removed.
It is Non linear.

2) ~~y(t)~~ $= 2\frac{d^2y}{dt} + \frac{dy}{dt} + 3y = 2\frac{dx}{dt} + 3x + 3$

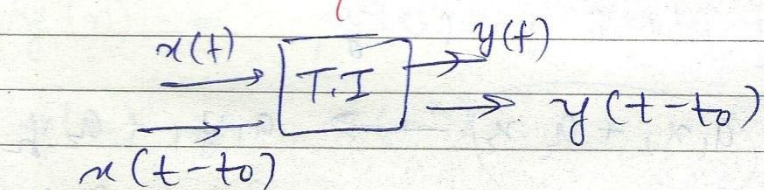
Linear.

If coeff. of all differential operator or dependent variable are independent of dependent variable then the system is linear.

$\frac{dy}{dt} + 2 = 2\sin x \frac{dx}{dt}$ Non linear

$\frac{dy}{dt} + 2 = 2\sin x$ Linear

3) Time Variant / Time Invariant



If coff. of all dif. operator or dependent variable are independent of independent variable then time invariant.

$2n \sin x \,\&\, x_{n-2} + 3x_{(n-1)} + 2 = y_{(n-2)} + 2$
NL / Time variant

Integration & differentiation - Linear system

1) $y(n) = \sum_{l=-\infty}^{\infty} x(l)$ L

2) $y(n) = y(-1) + \sum_{l=0}^{\infty} x(l)$ N.L

3) $y_n = m\,x(n) + c$ N.L

4) $x(n-1) + x(n+1) = y_n$ L

$$\int_{-\infty}^{3t} x(\tau)d\tau = x(\tau - t_0)\,d\tau$$

$$\tau - t_0 = dz$$

$$d\tau = dz \qquad -\infty$$

$$3t - t_0 = dz$$

$$\int_{-\infty}^{3t-t_0} x(\tau - t_0)\,d\tau$$

$$\boxed{\int_{-\infty}^{3t-t_0} x(z)\,dz}$$

Q

1) $y_n = 6x^2(n-2)$ LTI.

2) $n^2 y_n = 6x(n-2)$ L, TV

3) $y_n = 3n\,x(n)$ TV, L

4) $y(n) = 3x(n) - 2x(n-1)$ LTI.

5) $y_n = 3x(n-1) + x(n+1)$ LTI

Q 4) Stable & Non Stable

A system is said to be stable only if bounded I/P results in bounded O/P.

$$|x(t)| \leq M < \infty \Rightarrow |y(t)| < Q < \infty$$

Q $\int_{-\infty}^{3t} x(\tau)\,d\tau$ check stability.

Integral value ∞ so unstable

Q $y(n) = \frac{1}{2M+1} \sum_{k=-M}^{M} x(n-k)$ → Used for data smoothing

Stable

Moving Average Sys.

Q $y(t) = e^{t}$ ~~unbounded~~ ~~as~~ ~~here~~

here t unbounded.

i.e I/P is unbounded.

Unbounded I/P may leads to unbounded O/P

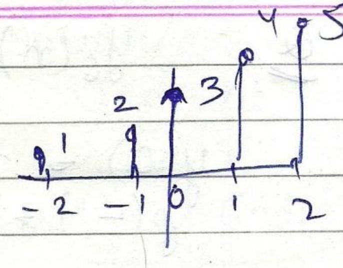
so stable.

$$\xrightarrow{x(t)} \boxed{\text{System}} \xrightarrow{} y(t) = e^{t}$$

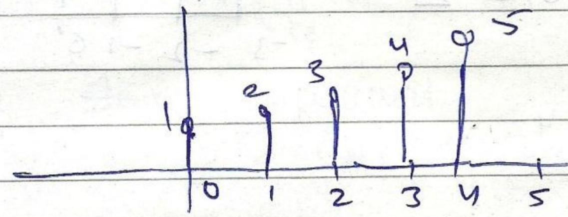
This is bounded I/P

Now for this we are getting unbounded O/P. so this is unstable system.

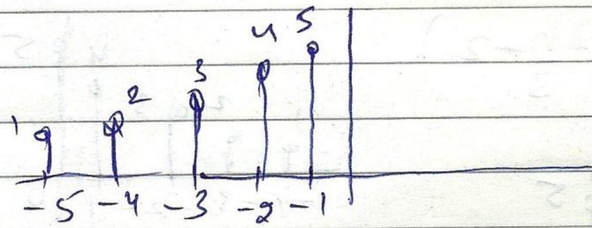
Q $x(n) = \{1, 2, \underset{\uparrow 0}{3}, 4, 5\}$



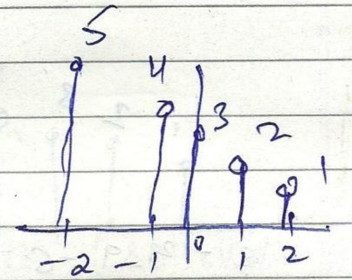
1) $y_1(n) = x(n-2)$



2) $y_2(n) = x(n+3)$



3) $y_3(n) = x(-n)$

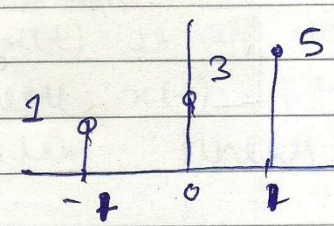


4) $y_4(n) = x(2n)$

$y(0) = x(0) = 3$

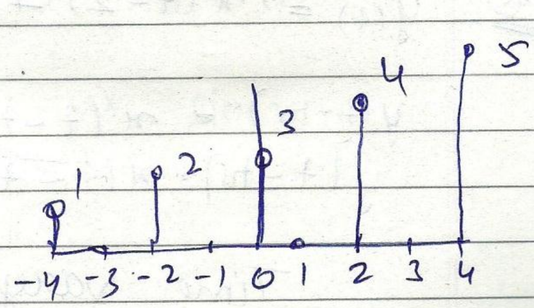
$y(1) = x(2) = 5$

$y(-1) = x(-2) = 1$



5) $y_5(n) = x\left(\frac{n}{2}\right)$

2 - 1 = 1 zero padding



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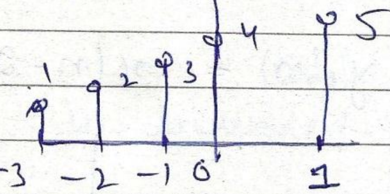
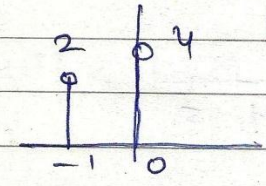
Q. $y_6(n) = x(2n+1)$



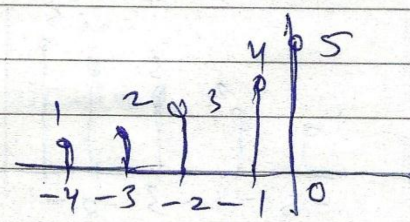
$y(0) = x(0) = 4$

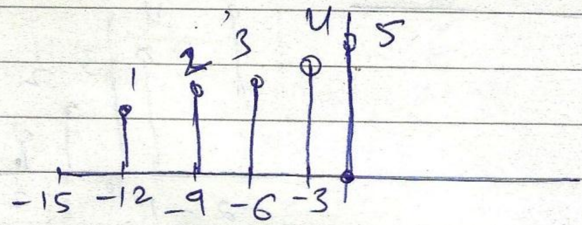
$y(-1) = x(-2) = 2$

$y_6(n) =$

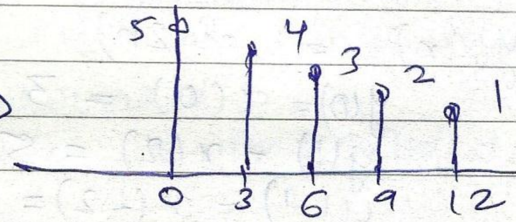


Q. $y_7(n) = x\left(\frac{-n+2}{3}\right)$





ans $\Rightarrow$



Q. $y(t) = x(t-2) + x(2-t)$

$y(t-t_0) = x(t-t_0-2) + x(2-t+t_0)$

$y(t-t_0) = x(t-t_0-2) + x(-t-t_0+2)$

Time varying.

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$\Rightarrow$ $y(t) = x(t-t_0) \rightarrow$ Time Invariant
$\Rightarrow$ $y(t) = x(t_0 - t) \rightarrow$ Time Variant

$y(n) = \cos\left(\frac{\pi}{4} n^2\right)$ ~~Non periodic~~ periodic with period 4

Time ~~Inv~~ Variant.

$$y(n-n_0) = \cos\left(\frac{\pi}{4}(n-n_0)^2\right)$$

$$y(n-n_0) = \cos(n-n_0)$$
$$\Downarrow$$
$$\cos\left(\frac{\pi}{4} n^2 - n_0\right).$$

$$y(n+N) = y_n$$

$$\frac{\pi}{4}(n+N)^2 = \frac{\pi}{4} n^2$$

No. sol^n except $N = 0$ (so non periodic)

$x(t)\frac{dy}{dt} + y = 2$ If given that $x(t)$ is I/P variable where $x(t) = t^2$

then $t^2\frac{dy}{dt} + y = 2$ is Non linear.

If ~~not~~ given that $x(t)$ is I/P then

$$t^2\frac{dy}{dt} + y = 2 \quad \text{is linear.}$$

Causal & Non Causal

Causal system is one whose O/P $y(t)$ is determined only by what is implied to I/P at present and what has been applied to it in the past but not on what shall be applied in future.

All real time systems are causal systems.

Non causal system can be designed only to process recorded data.

C.T.S

If $x_1(t) \longrightarrow y_1(t)$

$x_2(t) \longrightarrow y_2(t)$

and $x_1(t) = x_2(t)$ for $t \leq t_0$

then

$y_1(t) = y_2(t)$ for $t \leq t_0$.

D.T.S

$x_1(n) \longrightarrow y_1(n)$

$x_2(n) \longrightarrow y_2(n)$

$x_1(n) = x_2(n)$ $n \leq n_0$

then

$y_1(n) = y_2(n)$ for $n \leq n_0$

Linear causal system

$x(t) = x_1(t) - x_2(t) = 0$ for $t \leq t_0$

$\therefore \quad y(t) = y_1(t) - y_2(t) = 0 \quad$ for $t \leq t_0$

t_0 may or may not be zero.

Causal signal

$x(t) = 0 \quad$ for $t < 0$

Causal linear system with causal I/P $\longrightarrow$ $\xrightarrow{x(t)}$ $x(n)$ both causal $\boxed{\text{Causal linear system}} \longrightarrow y(t) = 0$ for $t < 0$

Q $\quad y(n) = \frac{1}{L} \sum_{k=-M}^{M} x(k) \qquad L = 2M + 1$

$$y(n) = \frac{1}{2M+1} \sum_{k=-M}^{M} x(k)$$

$M \leq n$ causal $\qquad\qquad M > n$ non causal

summation goes till $M \leq n$, past values so causal

⟹ Active And Passive

~~Active~~ Passive $\quad \sum_{-\infty}^{\infty} |y(n)|^2 \leq \sum_{-\infty}^{\infty} |x(n)|^2$

$\Downarrow$ Total energy O/P from system

$\rightarrow$ lossless system at equality

I/P energy of system.

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Q $y(n) = \alpha\, x(n-N)$ — P a) $|\alpha| < 1$
A 2) $|\alpha| > 1$
Loss less 3) $|\alpha| = 1$

Q $y(t) = \int_{-\infty}^{3t} x(\tau)\, d\tau$

Linear, Dynamic, TV, Unstable
Non causal. Depends on future value i.e $3t$.

Q $y(n) = x(n^2)$ NC

Q $y(n) = \frac{x(n) + x(n-1) + x(n+1)}{3}$ NC

Q $y(n) = 6x^2(n-2)$ C

Q $\overset{\text{constant}}{n^2}\, y(n) = 6x(n-2)$ C

Q $y(n) = 3n\, x(n)$ C

Q $y(n) = 3x(n) - 2x(n-1)$ C

Q $y(n) = 3x(n-1) + x(n+1)$ NC

If $x(n) = n$

and $y_n = [x(n)]^3 \longrightarrow$ Then this is causal system

and if

$y(n) = n^3 \longrightarrow$ Non causal signal.

non zero for $t<0$

$y(t) = \frac{dx}{dt}$ $\quad x = \int_{-\infty}^{t} y\,dt$ defined over a range so Dynam

Time Invariant system also called constant parameter system.

Q $y(n) = x(-n)$ Linear / NC / TV Stable

This is passive but lossless.

Q $y(t) = t \cdot x^3(t)$ Static / Non linear TV

Q $y(t) = Re\{x(t)\}$ L/NL linear

Q $y(t) = |x(t)|$ Non linear

Q $y(t) = x(2t)$ TV/TIV Time Variant

$x(2(t-t_0))$

$x(2t-t_0)$

Linearity / causality / TV, TIV

Q $y(t) = x(\alpha t)$ $\alpha > 0$ L / NC / D / TV

$y(t) = x^2(t)$ NL / Causal / TI / S

$y(t) = x(t^2)$ L / NC / TV / D

$y(t) = \frac{dx(t)}{dt}$ L / C / D / TI

$y(t) = \int_{-\infty}^{t} x\tau\, d\tau$ L/C/D/TI

$y(t) = \cos x(t)$ NL/C/S/TI

$y(t) = \cos(t) + 0 \cdot x(t)$ NL

$y(t) = x(-t)$ L/NC/D/TV

$y(t) = \int_{-\infty}^{2t} x(\tau)\,d\tau$ L/NC/D/TV

Delay

System — shifting operator

$x(n) \rightarrow \boxed{\Delta^{-1}} \rightarrow x(n-1) = \Delta x(n)$

$x(t) \rightarrow \boxed{D} \rightarrow y(t) = \dfrac{dx}{dt}$

↳ Differentiator or delay unit

Differentiation in CT is analogous to Δ (shift) in DT mathematically.

Modulation — multiplication

Difference operator $(x(n) - x(n-1))$

$x(n) \rightarrow \boxed{D_n} \rightarrow y(n) = x(n) - x(n-1)$

$Dx(n) = x(n) - \Delta x(n)$

$D_n = (1 - \Delta)$

Accumulator

$x(n) \rightarrow \boxed{\Sigma} \rightarrow \Sigma x(n)$

If $\delta(n)$ $\quad \sum_{-\infty}^{\infty} \delta(n) = u(n)$

$\delta(n) \rightarrow \boxed{D_n} \xrightarrow{\delta(n) - \delta(n-1)} \boxed{\Sigma} \rightarrow \delta(n)$

D_n: equivalent to $\dfrac{d}{dt}$

Σ: $\int$

$$\boxed{D_n = \frac{1}{\Sigma}}$$

$$\boxed{\xi = \frac{1}{1-\Delta}}$$

Up Sampler

$$y(n) = \begin{cases} x\left(\frac{n}{L}\right) & n = \pm 0, \pm L, \pm 2L \ldots \\ 0 & \text{otherwise} \end{cases}$$

sampling interval ↑ by L amount $\boxed{\uparrow L}$

$$x(n) \rightarrow \boxed{\uparrow L} \rightarrow y(n)$$

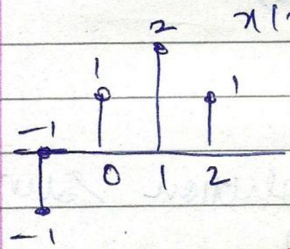
Down Sampler

$$x(n) \rightarrow \boxed{\downarrow M} \rightarrow y(n)$$

$$y(n) = \begin{cases} x(nM) & n = 0, \pm M, \ldots \\ 0 & \text{otherwise} \end{cases}$$

Q

$x(n)$

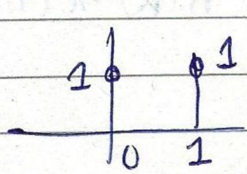


$\rightarrow \boxed{\uparrow 2} \rightarrow y_1(n)$

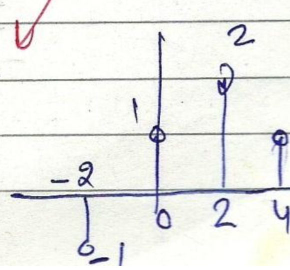
$\rightarrow \boxed{\downarrow 2} \rightarrow y_2(n)$ in DT

$x(2n) = y(0) = x(0) = 1$

$y(1) = x(2) = 1$



$x\left(\frac{n}{2}\right) =$

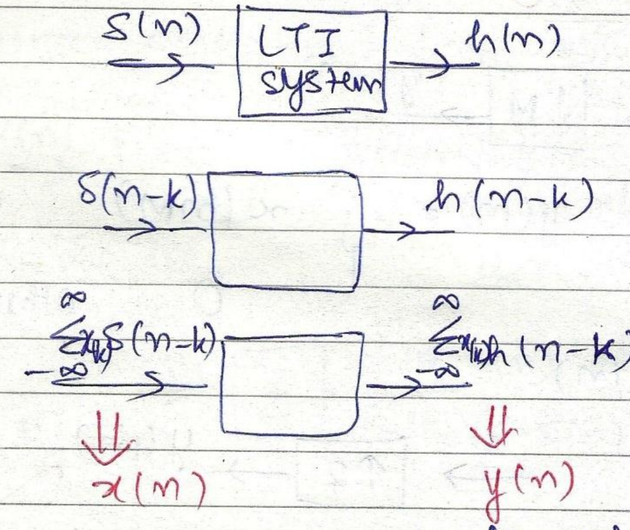


Sampler in CT will also collect additional info.
However case of interpolation is different.

LTI SYSTEM

A system is said to be realizable if it is causal & stable.

DTS



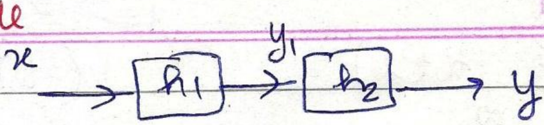
Convolution sum

$$y(n) = x(n) * h(n)$$

Convolution is commutative

$$\sum_{-\infty}^{\infty} x(k)\, h(n-k) = \sum_{-\infty}^{\infty} h(k)\, x(n-k)$$

Cascade

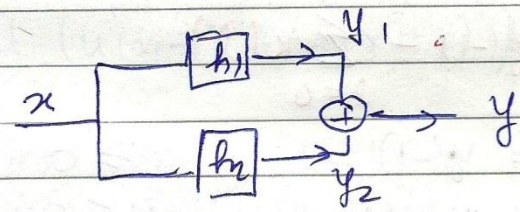


$y_1 = x * h_1 \qquad y = h_2 * y_1$

$y = h_2 * x * h_1$

$$\boxed{y = (h_1 * h_2) * x}$$

Parallel



$y_1 = x * h_1 \qquad y_2 = x * h_2$

$y = y_1 + y_2$

$= x * h_1 + x * h_2$

$$\boxed{y = x * (h_1 + h_2)}$$

If $x = x_a \longrightarrow y_a = h * x_a$

& $x_b \longrightarrow y_b = h * x_b$

$x_a + x_b \longrightarrow y = x * h$

$= (x_a + x_b) * h$

$\Downarrow$

Distributive $\longrightarrow$ $x_a * h + x_b * h$

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Q $y(n) = \sum_{l=-\infty}^{\infty} x(l)$ (L)

Impulse Response $h(n) = u(n)$

Q $y(n) = \sum_{k=0}^{\infty} x(n-k)$ (L)

$h(n) = u(n)$

Q $y(n) = y(-1) + \sum_{l=0}^{n} x(l)$ (N.L)

$y(n) = y(-1) + 1 \quad n \geq 0$
$\quad\;\; = y(-1) \quad n < 0$

Q $y(n) = x\left(\frac{n}{L}\right) \quad n = \pm 0, \pm L, \pm 2L \ldots$

0 otherwise

$h(n) = \delta\left(\frac{n}{L}\right)$ now signal exists at $L, 2L, 3L$

~~$= \sum \delta(n-kL)$~~

Q $y(n) = x(n) + \frac{1}{2}\{x(n-1) + x(n+1)\}$ $n-1$

convert it into + or −ve

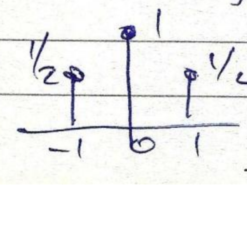
$x(n-1) + \frac{1}{2}(x(n-2) + x(n))$

Order = 2

Above $y(n)$ is interpolator.

$y(n) = \delta(n) + \frac{1}{2}\{\delta(n-1) + \delta(n+1)\}$

Ans →



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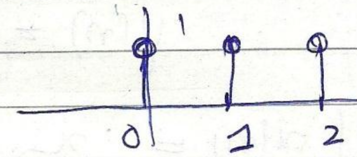
Q $y(n) = x\left(\frac{n}{L}\right)$ find step response

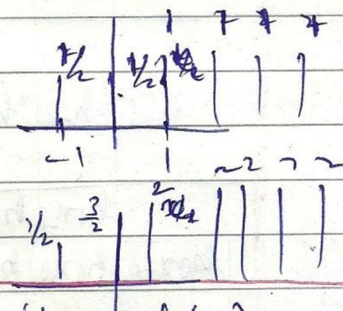
Sol^n $u\left(\frac{n}{L}\right)$, $0, \pm L, \pm 2L - -$

$$\sum_{k=0}^{\infty} \delta(n-kL)$$



$$u(n) + \frac{1}{2}\left(u(n-1) + u(n+1)\right)$$

~~If given that this is LTI system then it is non causal.~~



$\boxed{\text{LTI system is causal if } h(n) = 0 \text{ for } n<0}$

$x(n) \rightarrow$ LTI causal $\rightarrow y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

$h(n) = 0$ for $n<0$ (Here I/P causal not considered

Now $n-k < 0$
$n<k$ for $k>n$ the multiplication is going to be zero

So. $y(n) = \sum_{k=-\infty}^{n} x(k)\, h(n-k)$

or $y(n) = \sum_{k=0}^{\infty} x(n-k)\, h(k)$

If $x(n)$ is causal

then $k > 0$

$$y(n) = \sum_{k=0}^{n} x(k)\, h(n-k)$$

Ques

$$\begin{cases} x(n) = x_{-1} \quad \underset{\uparrow}{x_0} \quad x_1 \quad x_2 \\ h(n) = h_{-2} \quad h_{-1} \quad \underset{\uparrow}{h_0} \quad h_1 \end{cases}$$

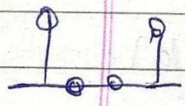
-2	-1	0	1	2
	x_{-1}	x_0	x_1	x_2
h_{-2}	h_{-1}	h_0	h_1	

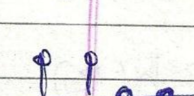
✓ Method to find convolution of Discrete Time

			h_1x_{-1}	h_1x_0	h_1x_1	h_1x_2
		h_0x_{-1}	h_0x_0	h_0x_1	h_0x_2	×
	$h_{-1}x_{-1}$	$h_{-1}x_0$	$h_{-1}x_1$	$h_{-1}x_2$	×	×
$h_{-2}x_{-1}$	$h_{-2}x_0$	$h_{-2}x_1$	$h_{-2}x_2$	×	×	×
$y(-3)$	$y(-2)$	$y(-1)$	$y(0)$	$y(1)$	$y(2)$	$y(3)$

If $x(n)$ is of length N_1
$h(n)$ " " N_2

$$\boxed{\text{Length of } x(n) * h(n) = N_1 + N_2 - 1}$$


Length of seq = 4


Length = 2

Theoretically, no. of non zero values is equal to length of sequence.

Continuous Time.

$$x(t) \rightarrow \boxed{\begin{matrix} LTI \\ h(t) \end{matrix}} \rightarrow y(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$y(t) = x(t) * h(t)$$

$$\int_{-\infty}^{t} x(\tau)\, h(t-\tau)\, d\tau \qquad \text{If causal system}$$

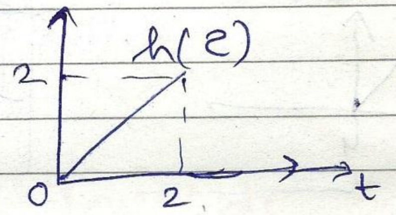
$$\int_{0}^{t} x(\tau)\, h(t-\tau)\, d\tau \qquad \text{Both system \& signal causal}$$

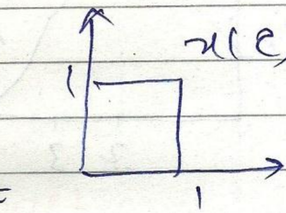
Q $x(t) = u(t) - u(t-1)$

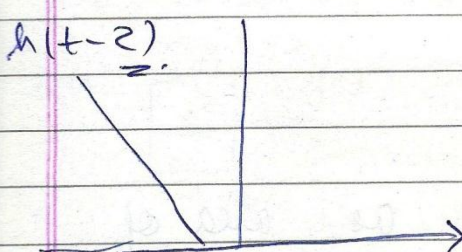
$h(t) = r(t) - r(t-2) - 2u(t-2)$.

Soln

$$\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

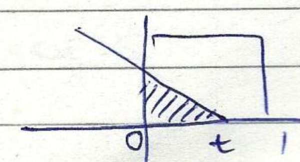






← If t = +ve.

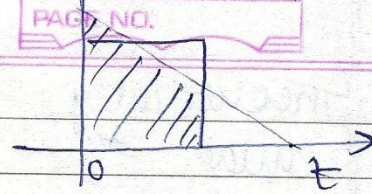
If t = -ve

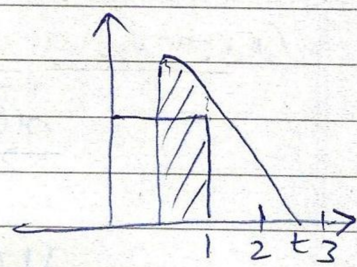


here conv = 0

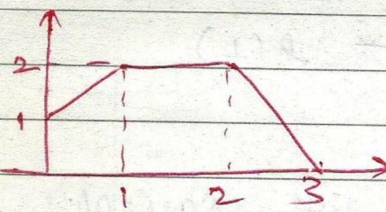
wrong cal.

$$\begin{cases} 0 & t<0 \\ \frac{1}{2}t^2 & 0<t\le 1 \\ 1-\frac{1}{2}(2-t)(1-t) & 1<t\le 2 \\ (3-t) & 2<t\le 3 \\ 0 & t>3 \end{cases}$$

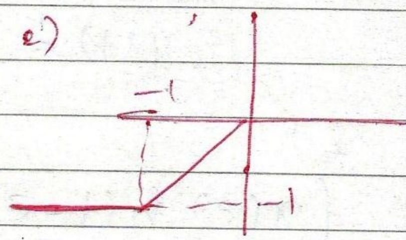




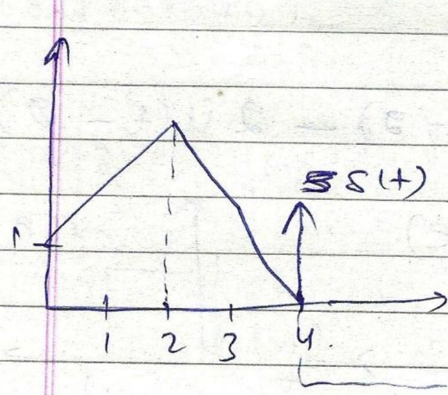
1)

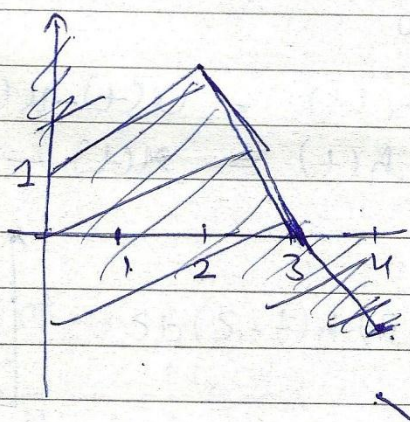


2)



3) $x(t) = u(t) + r(t) - 2r(t-2) - r(t-3) + 2r(t-4) + 5\delta(t-4)$





$$\int_{4^-}^{4^+} x(t)\,dt = 5$$

$$\int_{0^-}^{0^+} u(t)\,dt = 0$$ as area of line is 0.

$(1,2)\quad(0,1)\quad \frac{2-1}{1-0} = 1$

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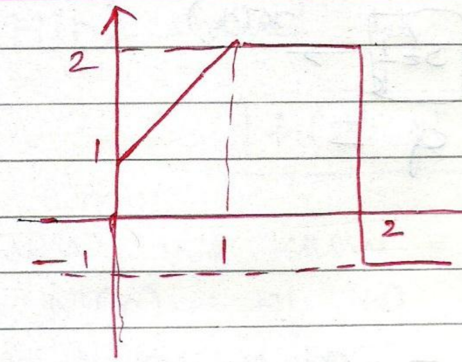
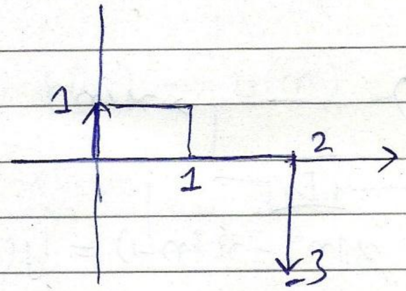
Q



find $x'(t)$



$$\int_{-\infty}^{\infty} x'(t)\,dt$$

$$1+1-3$$
$$= -1.$$

⟹ Static / Dynamic in terms of $h(n)$

$h(n) = k\,\delta(n)$ → Here k is a value

k is strength of Impulse.

$$k = h(0)$$

$h(t) = k\,\delta(t)$

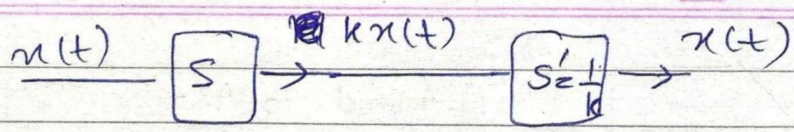
$$k = h(0) = \int_{-\infty}^{\infty} h(t)\,dt$$

Invertible & Non Invertible System.

$x(t) \rightarrow \boxed{S} \rightarrow y(t)$ $\qquad$ $y(t) \rightarrow \boxed{S'} \rightarrow x(t)$

S' is inverse system of S.

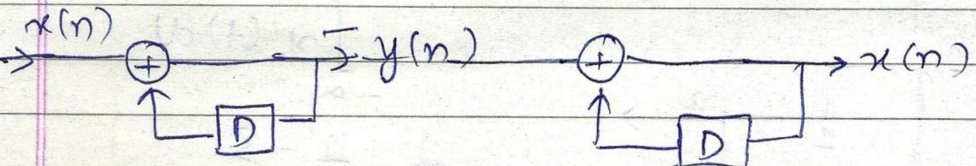
Inverse possible if relation one one & onto.



S' inverse of S.

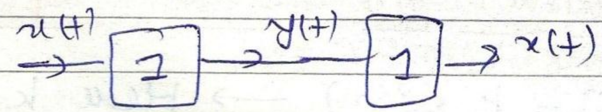
$$y(n) = \sum_{k=-\infty}^{n} x(k)$$

$$y(n) - y(n-1) = x(n)$$



$$x(n) - x(n-1) = y(n)$$

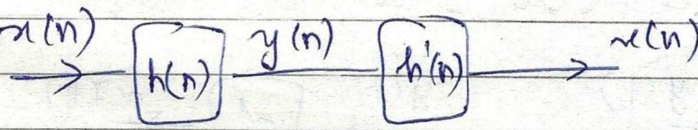
Q $y(t) = x(t)$ Invertible



Q $y(t) = 0$ Non Invertible
does not follow ~~one~~ to one relationship

Q $y(t) = x^2(t)$ No one to one relation
± 2

$\Rightarrow$



$$H(n) = h(n) * h'(n)$$

Now for $x(n) * H(n) = x(n)$

H(n) should be δ(n)

$$\boxed{h(n) * h'(n) = \delta(n)}$$

Now we know h(n) & δ(n). find h'(n). If it exists then system is invertible.

Q $h(n) = u(n)$

$$x(n) \longrightarrow \boxed{u(n)} \longrightarrow y(n)$$

$$y(n) = x(n) * u(n)$$

$$h(n) * h'(n) = \delta(n)$$
$$u(n) * h'(n) = \delta(n)$$

A system whose Impulse response is h(n) is said to be invertible if there exist ~~one~~ h'(n) such that

atleast one

$$h(n) * h'(n) = \delta(n)$$

Q

$$y(n) = \sum_{k=0}^{\infty} x(n-k)$$

$$h(n) = \sum_{k=0}^{\infty} \delta(n-k)$$

$$= u(n)$$

$$\sum_{k=0}^{\infty} u(k)\, h'(n-k) = \delta(n)$$

$$\sum_{k=0}^{\infty} h'(n-k) = \delta(n)$$

Assume inverse system causal

$$h'(n) + h'(n-1) + h'(n-2) \; --- \; = \delta(n)$$

Put $n = 0$

$$h'(0) + h'(-1) \; --- \; = \delta(0)$$

$$h'(0) = 1$$

$$h'(-1) = 0 \quad \text{(causal)}$$

$n = 1$

$$h'(1) + h'(0) + h'(-1) = \delta(1)$$

$$h'(1) + 1 + 0 = 0$$

$$h'(1) = -1$$

Put any other value $h'(n) = 0$

so. $h'(n) = \{ \underset{\uparrow}{1}, -1 \}$

Ans $\Rightarrow$ $\boxed{\delta(n) - \delta(n-1)}$

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Stability in terms of Impulse Response

stable

If Impulse response is absolutely summable.

$$\sum_{-\infty}^{\infty} |h(n)| \leq M < \infty$$

Q $y(n) = x(n-N_0)$ Stable.

$$h(n) = \delta(n-n_0)$$

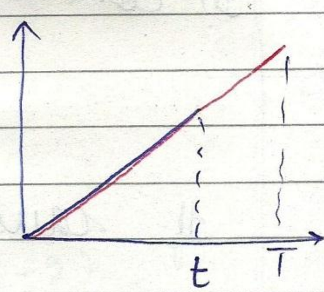
$$\sum_{-\infty}^{\infty} |\delta(n-n_0)| = 1 \quad \text{(only at } n_0\text{)}$$

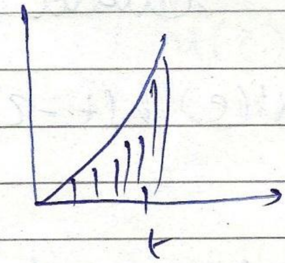
Q $\int_{-\infty}^{t} x(\tau)\, d\tau$

$$h(t) = u(t) \quad t>0$$
$$\quad\quad\;\; 0 \quad t<1$$

$u(t)$ not summable unstable

when multiplied area from 0–t will be considered and fn is t^2





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Analysis in terms of step response

$$\delta(n) \to h(n) \qquad \delta(t) \to h(t)$$
$$u(n) \to a(n) \qquad u(t) \to a(t)$$

corresponding to step response

$\cancel{x(n)} \to x(n) =$ I/P

cal. $y(n) =$

$$a(n) = \sum_{-\infty}^{\infty} u(k)\, h(n-k) \longrightarrow \sum_{0}^{\infty} u(k)$$

$$= \sum_{-\infty}^{\infty} h(k)\, u(n-k) \longrightarrow \sum_{-\infty}^{n} h(k)$$

$$a_n = h(n) + a_{n-1}$$

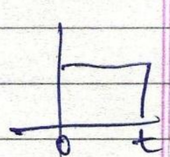
$$h(n) = a_n - a_{n-1}$$

$$\boxed{y_n = x(n) * [a_n - a_{n-1}]}$$

In cont. Time

$$y(t) = x(t) * a'(t)$$
$$= \int_{-\infty}^{\infty} a'(\tau)\, h(t-\tau)\, d\tau$$

If causal I/P, causal system.



$$y(t) = \int_{0}^{t} a'(\tau)\, h(t-\tau)\, d\tau.$$

Now there can be impulse at 0 or at t

$\therefore$

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$$y(t) = \int_{0^-}^{t^+} a'(\tau)\, h(t-\tau)\, d\tau$$

$$a'(t)\Big|_{t\geq 0} = \left(a(0^+) - a(0^-)\right)\delta(t)\Big|_{t=0} + a'(t)\Big|_{t>}$$

Do not miss impulse in terms of derivative fns.

can be proved by taking laplace $\boxed{y(t) = x(t) * a'(t)}$ For causal system. $t \geq 0$ → do not miss equality.

or $\boxed{y(t) = x'(t) * a(t)}$

same

Q $u(t) * u(t)$

$$\int_0^t u(\tau)\, u(t-\tau)\, d\tau$$

$$= t = r(t)$$

Q $u(t) * u(t-2)$

$\int \cancel{u(\tau-2)}\ \cancel{u(t-\tau)}$

$u(\tau) \cdot u((t-\tau)-2)$

$u(\tau) \cdot u(t-\tau-2)$

$$\int_0^{t-2} u(\tau) \cdot u(-\tau+t-2)$$

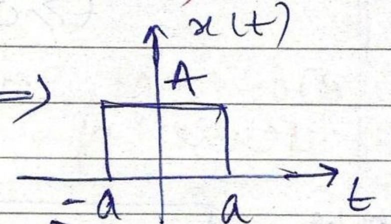
$$= (t-2) = r(t-2)$$

$$\Rightarrow \quad u(t+\alpha) * u(t+\beta) = r(t+\alpha+\beta)$$

$$\Rightarrow \quad x(t) * \delta(t) = x(t)$$

$$\Rightarrow \quad x(t) * \delta(t-t_0) = x(t-t_0)$$

Q. $u(t+a) - u(t-a) \Rightarrow$

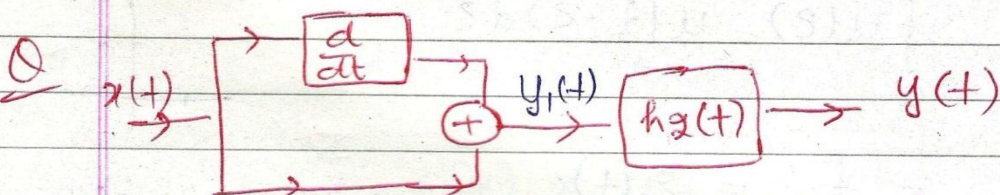


$$A\left[u(t+a) - u(t-a)\right] * A\left[u(t+a) - u(t-a)\right]$$

$$A\left[r(t+2a) - u(t) - u(t) + u(t-2a)\right]$$

$$A\left[r(t+2a) - 2r(t) + r(t-2a)\right]$$

Q.



$$h_2(t) = e^{-t}u(t)$$

Cal. $y(t)$ in terms of $x(t)$

$$y_1(t) = x(t) + \frac{d}{dt}x(t)$$

Now to find impulse, put $x(t) = \delta(t)$

$$h_1(t) = \delta(t) + \frac{d}{dt}\delta(t)$$

$$y(t) = x(t) * \left(h_1(t) * h_2(t)\right)$$

$$x(t) * \left[\left(\delta(t) + \frac{d}{dt}\delta(t)\right) * e^{-t}u(t)\right]$$

x(t) consider so $\delta(t) * x(t) = x(t)$

$$x(t) * \left[\delta(t) * e^{-t}u(t) + e^{-t}u(t) * \frac{d}{dt}\delta(t)\right]$$

$$\left[e^{-t}u(t) + \frac{d}{dt}\left(e^{-t}u(t)\right)\right]$$

$$\left[e^{-t}\cancel{u(t)} + e^{-t}\delta(t) + u(t)\,\cancel{e^{-t}}(-1)\right]$$

$$x(t) * e^{-t}\delta(t)$$

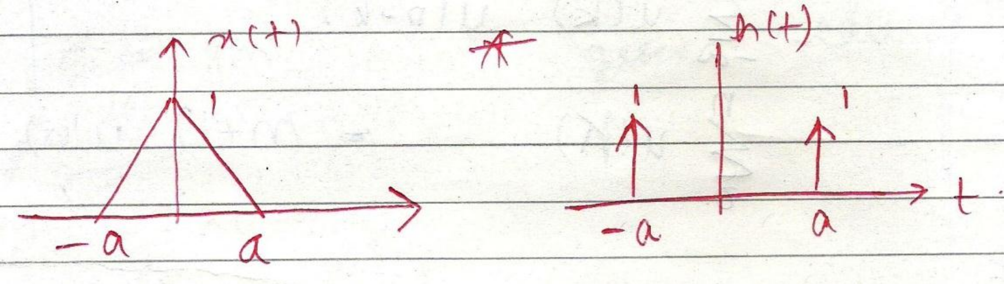
$$x(t) * \delta(t)$$

$$= x(t)$$

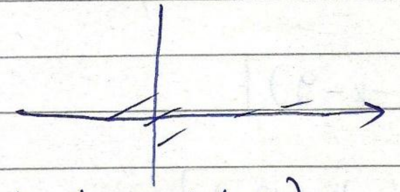
Q ~~u(t)~~ $[u(t+a) - u(t-a)] * 2u(t)$

$$2\left[\cancel{a}\, r(t+a) - r(t-a)\right]$$

Q



(0, 1)



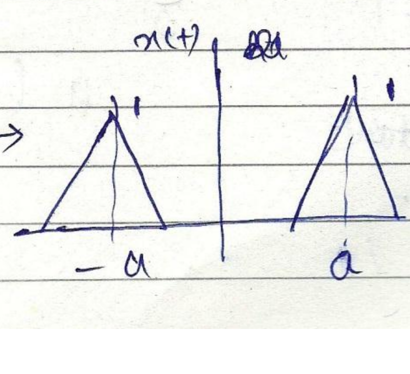
$x(t) * [\delta(t+a) + \delta(t-a)]$

$x(t+a) + x(t-a)$

$$y - 1 = \frac{1}{a}(x)$$

$$y = \frac{x}{a} + 1$$

Ans →



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3. $e^{-2t} u(t) * u(t)$

$$\int e^{-2\tau} u(\tau) \cdot u(t-\tau) \, d\tau$$

$$\int_0^t e^{-2\tau} \, d\tau$$

$$\left| \frac{e^{-2\tau}}{-2} \right|_0^t$$

$$-\frac{1}{2} \left[e^{-2t} - e^{0} \right]$$

$$-\frac{1}{2} \left[e^{-2t} - 1 \right]$$

4. $u(n) * u(n) = u(n) \cdot (n+1)$

$$\sum_{-\infty}^{\infty} u(k) \cdot u(n-k)$$

$-n$

$$\sum_{0}^{n} u(k) \qquad = (n+1) \, u(n)$$

5. $u(n) * u(n-2) = (n-1) \, u(n-2)$

$$\sum_{-\infty}^{\infty} u(k) \left[u(n-k-2) \right]$$

$y(0) = 0$

$y(1) = 0$

$y(2) = 1$

$$\sum_{0}^{n-2} u(k) \quad u(-k+n-2]$$

$n-2$

$$(n-2+1)$$

$$(n-1) \, u(n-2)$$

signal starts from 2.

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6. $\left[\delta(n) - \frac{1}{2}\delta(n-1)\right] * \left(\frac{1}{2}\right)^n u(n)$

7. $x(n) \rightarrow \boxed{h_1} \rightarrow \boxed{h_2} \rightarrow \boxed{h_3} \rightarrow y(n)$ cal. h

$h_1 = 0.5u(n)$, $h_2 = u(n+3)$, $h_3 = \delta(n) - \delta(n-1)$

7. $h = h_1 * h_2 * h_3$

$= \frac{1}{2}u(n) * u(n+3) * (\delta(n) - \delta(n-1))$

8. $u(n) * a^n u(n)$ → on seeing this we can conclude that this is causal ~~I/P~~ system with causal I/P.

9. $x(n) \rightarrow \boxed{h_1(n)} \xrightarrow{y(n)} \boxed{h_2(n) = (0.5)^n u(n)} \rightarrow$

$h_1(n)$: $x(n) - 0.5x(n-1)$

cal. $h(n)$

10. $x(n) = n\,u(n) \rightarrow \boxed{h(n) = 2^n u(n)} \rightarrow y(n) = ?$

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11. $x(n) = n^2 u(n) \longrightarrow \boxed{\begin{matrix} h(n) \\ \left(\frac{1}{2}\right)^n u(n) \end{matrix}} \longrightarrow y(n)$

$$n^2 u(n) * \delta(n)$$

$$= n^2 u(n)$$

This is not unstable system because we are giving unbounded I/P.

$\Longrightarrow$ Changing σ, ω, and ϕ of the signal changes its characteristics.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$x(t) = e^{st}$

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau.$$

Eigen or characteristic signal

$$= \int_{-\infty}^{\infty} h(\tau)\, e^{s(t-\tau)}\, d\tau$$

$$= e^{st} \int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau$$

$\Downarrow$ fn of (s)

$$y(t) = e^{st}\, H(s)$$

$\downarrow$ Eigen function
(defines characteristic of system)

form of signal after passing through system does not change.

$x(t) \rightarrow \boxed{} \rightarrow H \cdot x(t) \longrightarrow$ steady state response

H → constant for a particular signal

CTS (Continuous Time System)

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

Order = higher value of N or M.
Order = 0 system static

$$\frac{d^k y}{dt^k} = D^k y$$

$$y(t) = \underbrace{\left[\frac{g(D)_M}{f(D)_N}\right]}_{H(D)} x(t)$$

$$f(D)_N = 0 \quad \text{Natural } \nu \text{, eigen value, poles}$$

roots of this eq^n - $\{\alpha_1, \alpha_2 \; --- \; \alpha_n\}$

Case I – All roots are distinct

Case II – repeated roots $\underbrace{\alpha_1, \alpha_1 \alpha_1}_{L}$, $\underbrace{\alpha_2, \alpha_2}_{N-L}$

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CFS	PIS
1) $k_1 e^{\alpha_1 t} + k_2 e^{\alpha_2 t} + \dots k_N e^{\alpha_N t}$	$y_{PI} = H(D)\big\vert_{D=s_0} \cdot x(t)$
2) $e^{\alpha_1 t}(K_1 + K_2 t + \dots K_L t^{n-1}) + e^{\alpha_2 t}(K_{L+1} + \dots K_N t^{N-L})$	$= H(s_0) \cdot x(t)$

CFS is called transient response, natural respone / Impulse response
It is not zero I/P response because I/P is Impulse.

Q $\dot{y}(t) + 2y(t) = x(t)$

a) $x(t) = k\cos(\omega_0 t)\,u(t)$

b) $x(t) = u(t+1) \qquad y(0) = 0$

Soln a) $(D+2)\,y(t) = x(t)$

$$y(t) = \frac{x(t)}{D+2}$$

$$D = -2$$

$$y_{CFS} = k_1 e^{-2t}$$

$$y_{PIS} = \frac{1}{(j\omega_0 + 2)} k\cos(\omega_0 t)\,u(t)$$

This is causal

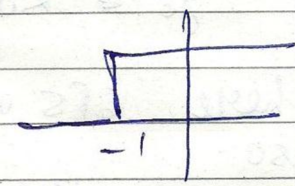
This is also correct

b)

$$\begin{cases} y_{CFS} = k_1 e^{-2t} \\ y_{PIS} = \frac{1}{2} u(t+1) \end{cases}$$

for $t < -1$ $\quad x(t) = 0$

$$\boxed{y(t) = k e^{-2t}}$$



for $t > -1$ $\quad x(t) = 1$

$$y(t) = k e^{-2t} + \frac{1}{2}$$

$$0 = k(1) + \frac{1}{2}$$

$$k = -\frac{1}{2}$$

$$\boxed{y(t) = \frac{1}{2}(1 - e^{-2t})}$$

This is non causal

$$y(t) = -\frac{1}{2} e^{-2t} + \frac{1}{2} u(t+1)$$

For LTI system to be causal

If $x(t) = 0$ $\quad t < t_0$
then $y(t) = 0$ $\quad t < t_0$

} Initial rest condition

A linear causal system is also Time invariant.

All static systems are causal but converse is not true.

All non causal systems are dynamic.

Special case 3) $x(t) = e^{-2t} u(t)$

$$y_0 = K_0 e^{-2t}$$

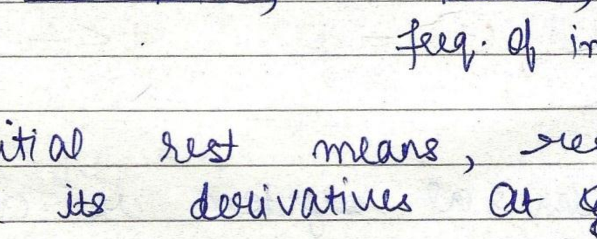
here CFS and PIS have same form
so
$$y(t) = e^{-2t}(K_0 + k_1 t)$$

4) $x(t) = 0 \quad t \geq 0$

$$y(0) = 0$$

$$y(t) = k e^{-2t}$$
$$0 = k e^{-2(0)}$$
$$k = 0$$
$$\boxed{y(t) = 0}$$

⇒ Natural ν is ν of response of a system when excitation becomes zero.



freq. of impulse

Initial rest means, response and all its derivatives at 0 zero should be zero.

If Nth order system
then y and $y'(0), y''(0)$
$$\text{---} \; y^{\text{n}th}(0) = 0$$

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Discrete Time System

$$\sum_{k=0}^{n} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k)$$

$$N > M$$

If $N = 0$

$$a_0 y(n) = \sum_{k=0}^{M} b_k x(n-k)$$

$$y(n) = \sum_{k=0}^{M} \frac{b_k}{a_0} x(n-k)$$

$$\boxed{y(n) = \sum_{k=0}^{M} c_k x(n-k)}$$

Non-Recursive as no feedback is there
i.e no $y(n+1), y(n+2)$

If $N \neq 0$ then it is recursive.

$$y(n) = \sum_{k=0}^{n} x(k) \longrightarrow N.R$$

$$y(n) - y(n-1) = x(n) \longrightarrow R$$

same fn can be represented as both R and N.R.

$$\boxed{y(n) = \sum_{k=0}^{M} c_k \delta(n-k)}$$

generally expressed in NR way

This is finite Impulse Response (FIR)

If length of Impulse response is ∞ then IIR

in Recursive

FIR is not necessarily NR
IIR " " " R

R and NR way of representation are used to describe the process of computation whether we need feedback or not.

Q $y(n) = x(n) + x(n-1)$

This is NR

$y(n-1) = x(n-1) + x(n-2)$

$y(n) - y(n-1) = x(n) - x(n-2)$

Q $y(nT) = \int_0^{t=kT} x(\tau)\, d\tau$ [digital Integrator]

find diff. eq^n and Impulse response.

$$y(n) = \int_0^{(n-1)T} x(\tau)\, d\tau + \int_{(n-1)T}^{nT} x(\tau)\, d\tau$$

Average value

$$y(nT) = y(n-1)T + \left\{ \frac{x(nT) + x((n-1)T)}{2} \right\} T$$

If $T = 1$

$$y(n) = y(n-1) + \frac{1}{2}\{x(n) + x(n-1)\}$$

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$$2y(n) = 2\,y(n-1) + x(n) + x(n-1)$$

Impulse Response

$$2h(n) = 2h(n-1) + \delta(n) + \delta(n-1)$$

Assume system causal

$$2\,h(0) = 1$$

$$h(0) = \frac{1}{2} = \frac{T}{2}$$

$$2\,h(1) = 2\,h(0) + 1$$

$$2\times\frac{1}{2} + 1 = \frac{2}{2} = 1 = T$$

$$\boxed{h(1) = 1}$$

$$h(2) = T$$

$$h(3) = T$$

How to solve difference eq^n

$$\frac{d(y)}{dt} = D \cdot y$$

$$y(n-1) = \Delta y$$

degree of Δ, gives order of.

$$y(n) = \left[\frac{g(\Delta)}{f(\Delta)}\right] x(n)$$

$$x(n) = A z^n \qquad z \rightarrow \text{Complex } v.$$

$$F(\Delta) = 0$$

$\rho \alpha^{nt}$

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<u>C.F.S</u> | <u>P.I.S</u>

1) $k_1(\lambda_1)^n + \text{----} \; k_n(\lambda_N)^n$ $\quad = H(\Delta)\Big|_{\Delta = z_0} \cdot x(n)$

If distinct roots
N = order of system

2) Repeated roots

$\lambda_1^n(k_1 + k_2 n + k_3 n^2 \text{---} k_L n^{L-1})$
$+ \text{------}$

Q. $y(n) - \frac{1}{2} y(n-1) = x(n)$

$y(-1) = a \;,\; x(n) = k\delta(n)$

$\left(1 - \frac{1}{2}\Delta^{-1}\right) y(n) = x(n)$

$y(n) = \dfrac{x(n)}{1 - \frac{1}{2}\Delta^{-1}}$

$1 = \dfrac{1}{2\Delta}$

$\Delta = 1/2$

$y(n)$ CFS $= k_1 (1/2)^n$

$\left(\frac{1}{2}\right)^n \left(k + \frac{1}{2}a\right)$

$y(0) - \frac{1}{2} y(-1) = k$

$y(0) - \frac{1}{2}a) = k \quad = k + \frac{1}{2}a$

$y(1) - \frac{1}{2} y(0) = \cancel{(k + \frac{1}{2}a) + \frac{1}{2}} \; 0$

$y(n) = \left(\frac{1}{2}\right)^n \left(k + \frac{1}{2}a\right)$

$(1)^n$ = unit step.
$(0)^n = 0$ exactly zero.

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2) $y(n) \cancel{+} 0.5\, y(n-1) = 2u(n)$

$\boxed{y(n) = ?\,,\ n \geq 0}\,,\quad y(-1) = 2$

3) $y(n) + 0.5\, y(n-1) = (-0.5)^n u(n)$

$y(n) = ?\,,\quad n \geq 0 \qquad y(-1) = 2$

Impulse occurs only at one point, so σ and ω have no sense becoz there is no length defined.
so complex ν of $\delta(t)$ not defined

Replace $\quad y(n-x) = \Delta^{-1}$

$y(n+x) = \Delta$

3) $(1 + 0.5\Delta^{-1})\, y = (-0.5)^n u(n)$

$1 + \dfrac{0.5}{\Delta} = 0$

$\boxed{\Delta = -0.5} \qquad A z^n = (-0.5)^n$

$z = (-0.5)$

$y(n) = \cancel{8}(-0.5)^n [k_1 + k_2 n]$

$2 = (-0.5)^{-1} [k_1 - k_2]$

$\cancel{2} = -\dfrac{\cancel{2}}{\cancel{0.5}} [k_1 - k_2]$

$\cancel{k_2 - k_1} = \boxed{k_2 - k_1 = 1}$

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$$y(0) + 0.5\, y(-1) = (-0.5)^0\, u(0)$$

$$y(0) + 0.5(2) = 1$$

$$\boxed{y(0) = 0}$$

$$y(0) = (-0.5)^0 (k_1)$$

$$\cancel{y(0)} \quad \boxed{0 = k_1}$$

$$\boxed{k_2 = 1}$$

Solve using Recurssive Method

$$y(n) = x(n) + \alpha\, y(n-1)$$

Q $x(n) = \delta(n)$, $y(-1) = 0$

Solve

$$y(0) = x(0) + \alpha y(-1)$$
$$= 1$$
$$y(1) = x(1) + \alpha y(0)$$
$$= \alpha$$
$$y(2) = \alpha^2$$
$$y(3) = \alpha^3$$
$$\vdots$$
$$y(n) = \alpha^n$$

Q $x(n) = u(n)$

Q $x(n) = \sin(wn)\, u(n)$

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General rule

$u(n+\alpha) * u(n+\beta) = (n+\alpha+\beta+1)\, u(n+\alpha+$

Sol 6

$\left[\delta(n) - \frac{1}{2}\delta(n-1)\right] * \left(\frac{1}{2}\right)^n u(n)$

$\delta(n) * \left(\frac{1}{2}\right)^n u(n) - \frac{1}{2}\delta(n-1) * \left(\frac{1}{2}\right)^n u(n)$

$\left(\frac{1}{2}\right)^n u(n) - \frac{1}{2}\left(\frac{1}{2}\right)^{n-1} u(n-1)$

$\left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{2}\right)^n u(n-1)$

$\left(\frac{1}{2}\right)^n \left[u(n) - u(n-1)\right]$

$\left(\frac{1}{2}\right)^n \cdot \delta(n)$

$\left(\frac{1}{2}\right)^0 \delta(n) = \underline{\underline{\delta(n)}}$

Sol 7

$\frac{1}{2} u(n) * u(n+3) * (\delta(n) - \delta(n-1))$

$\delta(n) * \left[\frac{1}{2} u(n) * u(n+3)\right] - \delta(n-1) * \left[\frac{1}{2} u(n) * u(n+$

$\frac{1}{2} u(n) * u(n+3) - \frac{1}{2} u(n-1) * u(n-1+3)$

$u(n+2)$

$\frac{1}{2}(n+0+3+1)\, u(n+3) - \frac{1}{2}(n-1+2+1)\, u(n+2-1)$

$\frac{1}{2}(n+4)\, u(n+3) - \frac{1}{2}(n+2)\, u(n+1)$

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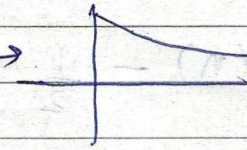
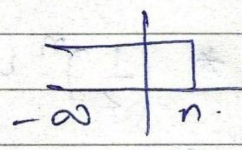
Sol 8 $a^n u(n) * a^n u(n)$

$$\sum_{-\infty}^{\infty} a^k\, u(k) .\, u(n-k)$$

for $0 < a < 1$ $\quad a^k u(k) \rightarrow$



$0 < n-k < \infty \qquad u(n-k)$



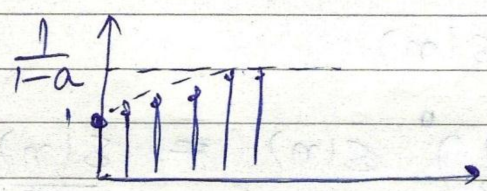
$-n < -k < \infty$

$n > k > -\infty$

for $n > 0$

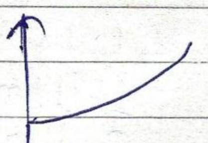
$$\sum_{0}^{n} a^k = \frac{1 - r^{n+1}}{1 - r}$$

$$= \frac{1 - a^{n+1}}{1 - a}$$



for large n $\quad a^{n+1} \rightarrow 0$

for $a > 1$ $\quad a^k u(k)$



$$\sum_{0}^{n} a^k$$

$$= \frac{a^{n+1} - 1}{a - 1}$$

Soln 9 $h = h_1(n) * h_2(n)$

$$[\delta(n) - 0.5\,\delta(n-1)] * [(0.5)^n\, u(n)]$$

$$[(0.5)^n u(n) - 0.5\,(0.5)^{n-1}\, u(n-1)]$$

$(0.5)^n u(n) - (0.5)^n u(n-1)$

$(0.5)^n [u(n) - u(n-1)]$

$(0.5)^n \delta(n)$

$= \delta(n)$

~~y(n)~~ =

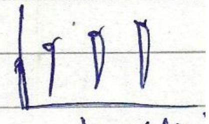
10 $y(n) = h(n) * x(n)$

$= n u(n) * 2^n u(n).$

$\sum_{-\infty}^{\infty} k\, u(k) \cdot 2^{n-k} u(n-k)$

$2^n \sum_{-\infty}^{\infty} k\, u(k)\, 2^{-k} u(n-k)$

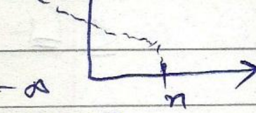
$2^n \sum_{-\infty}^{\infty} k\, u(k) \left(\frac{1}{2}\right)^k u(n-k)$



k u(k)

$2^n \sum_{0}^{n} k \left(\frac{1}{2}\right)^k.$

$S = 2^n \left[0 + 1 \times \left(\frac{1}{2}\right)^1 + 2 \times \left(\frac{1}{2}\right)^2 + \cdots n \left(\frac{1}{2}\right)^n\right]_{-\infty}$



$\frac{1}{2} \times S = 2^n \left[\left(\frac{1}{2}\right)^2 + 2 \times \left(\frac{1}{2}\right)^3 + \cdots n \left(\frac{1}{2}\right)^{n+1}\right]$

$S\left[1 - \frac{1}{2}\right] = 2^n \left[\left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots + n \left(\frac{1}{2}\right)^{n+1}\right]$

$\frac{S}{2} = 2^n \left[\frac{1}{2} + \left(\frac{1}{2}\right)^2 \cdots \left(\frac{1}{2}\right)^n\right] + 2^n n \left(\frac{1}{2}\right)^{n+1}$

$\frac{S}{2} = 2^n \left[\frac{\frac{1}{2}\left[1 - \left(\frac{1}{2}\right)^n\right]}{1 - \frac{1}{2}}\right] + 2^n \cdot n\ 2^{-n-1}$

$= 2^n \left[1 - \left(\frac{1}{2}\right)^n\right] + \underbrace{n \cdot 2^{-1}}_{\frac{n}{2}}$

$$S = 2^{n+1}\left[1 - \left(\tfrac{1}{2}\right)^{n}\right] - n$$

$$= 2^{n+1} - 2^{\cancel{n}+1-\cancel{n}} - n$$

$$\boxed{S = 2^{n+1} - n - 2}$$

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$$y(n) = n^2 u(n) * \left(\tfrac{1}{2}\right)^{n} u(n)$$

$$\sum_{-\infty}^{\infty} k^2 u(k) \cdot \left(\tfrac{1}{2}\right)^{n-k} u(n-k)$$

$$\left(\tfrac{1}{2}\right)^{n} \sum_{-\infty}^{\infty} k^2 u(k) \left(\tfrac{1}{2}\right)^{-k} u(n-k)$$

$$\sum_{-\infty}^{\infty} k^2 u(k)\; 2^{k}\; u(n-k)$$

$$\left(\tfrac{1}{2}\right)^{n} \sum_{0}^{n} k^2 2^{k}$$

$$S = \left(\tfrac{1}{2}\right)^{n}\left[1^2 \times 2^1 + 2^2 2^2 + 3^2 2^3 \;--\; n^2 2^n\right]$$

$$2S = \left(\tfrac{1}{2}\right)^{n}\left[1^2 \times 2^2 + 2^2 2^3 + 3^2 2^4 \;--\; n^2 2^{n+1}\right]$$

$$(1-2)S = \left(\tfrac{1}{2}\right)^{n}\left[2 + 2^2[2^2-1^2] + 2^3[3^2-2^2] + 2^4[4^2-3^2] \;---\; 2^n[n^2-(n-1)^2] - n^2 2^{n+1}\right]$$

$$-S = \left(\tfrac{1}{2}\right)^{n}\left[2 + 2^2[3] + 2^3[5] + 2^4[7] \;---\; 2^n[2n-1] - n^2 2^{n+1}\right]$$

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$$-2S = \left(\frac{1}{2}\right)^n \left[2^2 + 2^3[3] + 2^4[7] --- 2^{n+1}[2n-1] + n^2\, 2^{n+2} \right]$$

$$S[-1+2] = \left(\frac{1}{2}\right)^n \left[2 + 2^2[3-1] + 2^3[5-3] + --- 2^n[2] - 2^{n+1}[2n-1] + n^2 [2^{n+1} - 2^{n+2}] \right]$$

$$= \left(\frac{1}{2}\right)^n \left\{ [2 + 2^3 + 2^4 + 2^5 -- 2^{n+1}] - 2^{n+1}[2n-1] + n^2\, 2^{n+1}[-1] \right\}$$

$$= \left(\frac{1}{2}\right)^n (2) + \left(\frac{1}{2}\right)^n \left[2^3 + 2^4 + 2^5 -- 2^{n+1} \right] - \left(\frac{1}{2}\right)^n 2^{n+1}[2n-1] + \left(\frac{1}{2}\right)^n n^2\, 2^{n+1}(+1)$$

$$= 2^{-n+1} + \left(\frac{1}{2}\right)^n \left[2^3 \left[\frac{(2)^{n+1+3} - 1}{2-1} \right] \right]$$

$$= 2^{-n+1} + \left(\frac{1}{2}\right)^n \left[(2^{n+3} - 1)\, 2^3 \right] - 2^{n+1-n}[2n-1]$$

$$= 2^{-n+1} + 2^{-n+n+3+3} - 2^{-n+3} - 2(2n-1) + n^2(2)$$

$$+ 2^{n+1-n}\, n^2$$

$$= 2^{-n+1} + 2^6 - 2^{-n+3} - 2(2n-1) + n^2(2)$$

$$= -3[2^{-n+1}] + 2^6 - 2[-n^2 + 2n - 1]$$

$$2^6 + 2(n^2 + 1 - 2n)$$

$$= -3[2^{-n+1}] + 2^6 + 2(n-1)^2$$

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Q $y(n) + 0.5\, y(n-1) = 2u(n)$ $\qquad y(-1) = 2$

$y(0) + 0.5\, y(-1) = 2u(0).$

$y(0) + 0.5 \times 2 = 2$

$y(0) = 2(1-0.5)$

$y(0) = \frac{1}{2} \times 2 = 1$

$y(1) + 0.5\, y(0) = 2 \times 1$

$\frac{1}{2} \times (1) = 2$

$y(1) = 2 - \frac{1}{2} = \frac{3}{2}$

$y(2) + \frac{1}{2} \times y(1) = 2$

$\frac{1}{2} \times \frac{3}{2} = 2 = 2 - \frac{3}{4} = \frac{5}{4}$

$y(3) + \frac{1}{2}\, y(2) = 2$

$\frac{1}{2} \times \frac{5}{4} = 2$

$2 - \frac{1}{2}\left(\frac{5}{4}\right) = \frac{11}{8}$

$y(4) + \frac{1}{2} \times \frac{11}{8} = 2 = 2 - \frac{1}{2} \times \frac{11}{8}$

$= \frac{32-11}{16} = \frac{21}{16}$

$y(5) + \frac{21}{16} \times \frac{1}{2} = 2 = 2 - \frac{21}{16 \times 2} = \frac{43}{32}$

Q $x(n) = u(n)$ $\qquad y(n) = x(n) + \alpha\, y(n-1)$

$y(n) - \alpha\, y(n-1) = x(n)$

$\left(1 - \frac{\alpha}{\Delta}\right) = 0 \qquad \boxed{\Delta = \alpha}$

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$$= \frac{x(n)}{1-\frac{\alpha}{\Delta}} \qquad x(n) = u(n)$$

$$Az^n = u(n)$$

$$z = 1$$

$$= \frac{x(n)}{(1-\alpha)} = \frac{u(n)}{(1-\alpha)}$$

$$y(n) = k(\alpha)^n + \frac{u(n)}{1-\alpha}$$

Q $x(n) = \sin(\omega n)\, u(n) \qquad y(-1) = 0.$

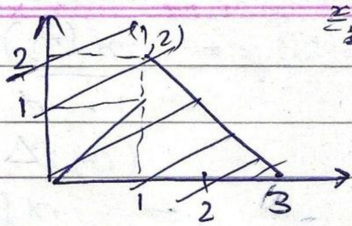
$$\left(\frac{e^{j\omega n} + e^{-j\omega n}}{2j}\right) u(n)$$

$$\frac{e^{j\omega n} u(n)}{2j} + \frac{e^{-j\omega n} u(n)}{2j}$$

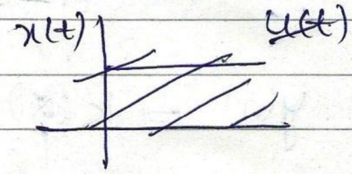
Unit step response $a(t)$

$(1,2)$ $(3,0)$ $\frac{2-0}{1-3} = \frac{2}{-2} = -1$

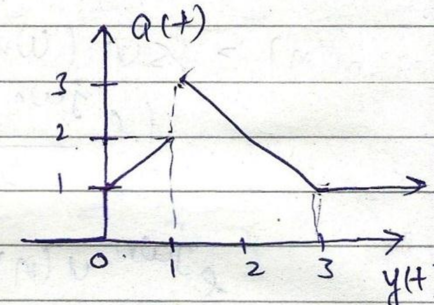
Q $y(t) = ?$



$x(t) \mid e^{-t}u(t)$



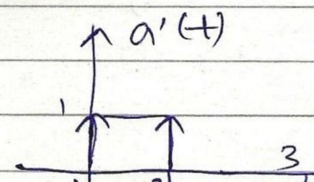
$$y(t) = x(t) * a'(t)$$

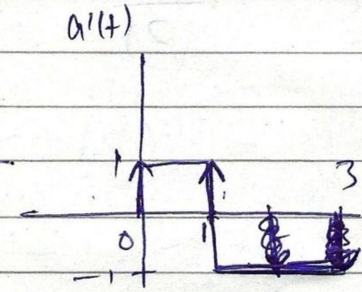


$$a'(t) = \delta(t) + u(t) - 2u(t-1) + u(t-3) + \delta(t-1)$$

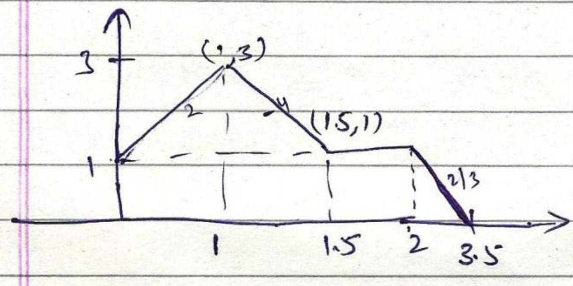
$(0,1)$ $(1,2)$ $\frac{2-1}{1-0} = 1$

$(1,3)$ $(3,1)$ $\frac{3-1}{1-3} = \frac{2}{-2} = -1$





Q



$(0,1)$ $(1,3)$ $\frac{3-1}{1-0} = 2$

$\frac{1-0}{1.5-3} = \frac{2}{3}$

$$u(t) + 2r(t) - 2r(t-1) - 4r(t-1) + 4r(t-1.5) - \frac{2}{3}r(t-2) + \frac{2}{3}r(t-3.5)$$

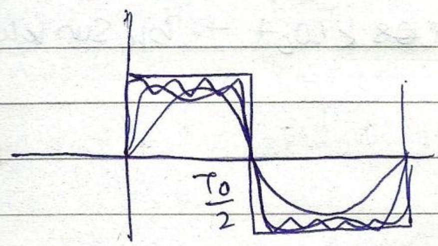
$\frac{1-3}{1.5-1} = \frac{-2}{1/2} = -2 \times 2 = -4$

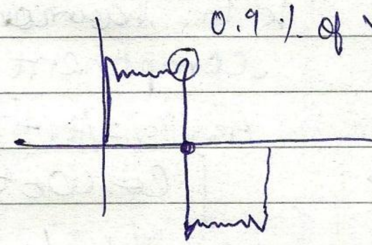
signal Invertibility & system Invertibility are two diff. things

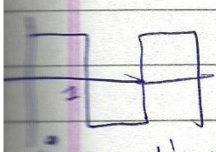
FOURIER

Any periodic signal $\tilde{x}(t) = \tilde{x}(t+T)$ can be represented as

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{+jk\omega_0 t}$$







After gibbs phenomenon value of $\frac{T_0}{2}$ defined $= 0$.

discontinuous signal at 1.

$$x(t) = x(t+T)$$

$T_0 \rightarrow$ smallest T

$$f_0 = \frac{1}{T_0}$$

$$\omega_0 = \frac{2\pi}{T_0} = 2\pi f_0$$

$$x(t) = a_0 + [a_1 \cos\omega_0 t + a_2 \cos 2\omega_0 t + \dots]$$

$$+ [b_1 \sin \omega_0 t + b_2 \sin\omega_0 t + \dots]$$