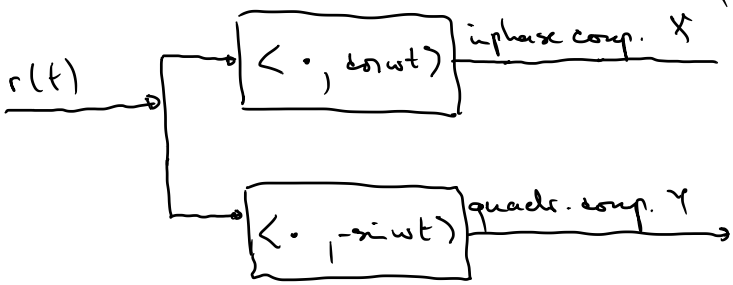
The "arctan-problem"

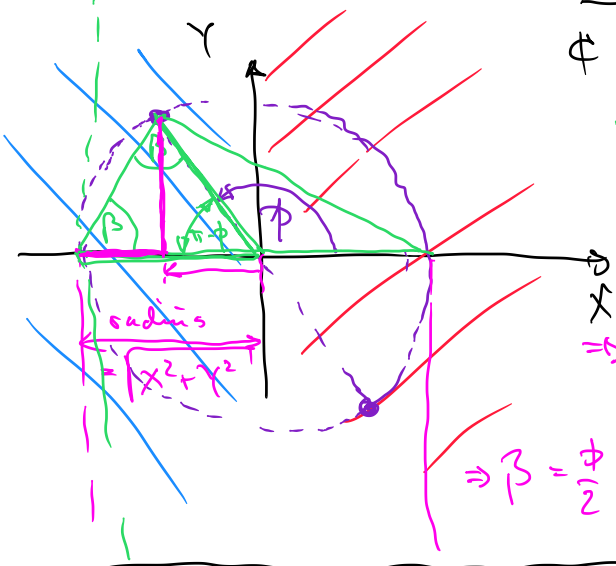
$$r(t) = \cos(\omega t + \phi) = \underbrace{\cos\phi}_{X}\cos\omega t - \underbrace{\sin\phi}_{Y}\sin\omega t$$



only true for $X \geq 0$

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{Y}{X} \quad \overset{?}{\Rightarrow} \quad \boxed{\phi = \arctan\left(\frac{Y}{X}\right)}$$

↳ true for $\phi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



$$2\beta + \pi - \phi = \pi$$

$$\beta = \frac{\phi}{2}$$

$$\Rightarrow \tan\beta = \frac{Y}{\sqrt{X^2+Y^2}+X}$$

$$\Rightarrow \beta = \frac{\phi}{2} = \arctan\frac{Y}{\sqrt{X^2+Y^2}+X}$$

$$\Rightarrow \quad \boxed{\phi = 2\arctan\frac{Y}{\sqrt{X^2+Y^2}+X}} \quad \rightarrow \text{true for } \phi \in (-\pi, \pi]$$