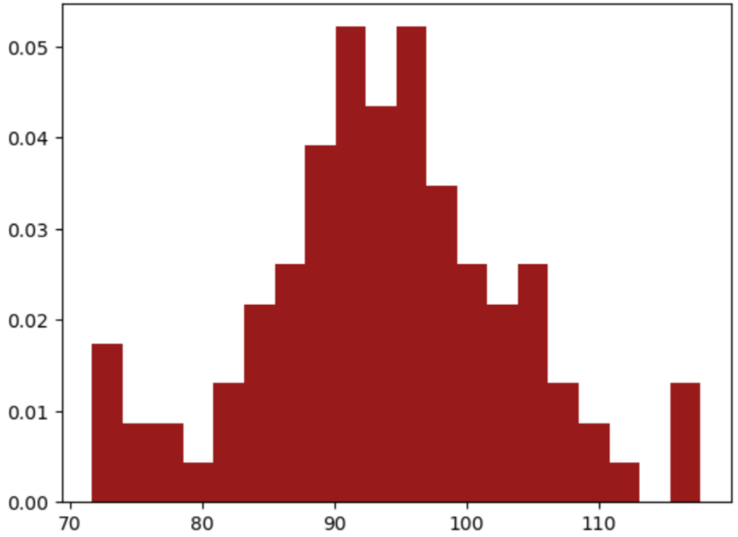
HYPOTHESIS TESTING

Accepting or rejecting a hypothesis

HYPOTHESIS TESTING: WORKING EXAMPLE

A call centre claim that the average waiting time for customers is at most 90 seconds.
You are not so sure of this claim and want to challenge this fact with data.
You collect some data by taking a sample of 100 randomly chosen calls.



The data evidence that you obtain is the following:

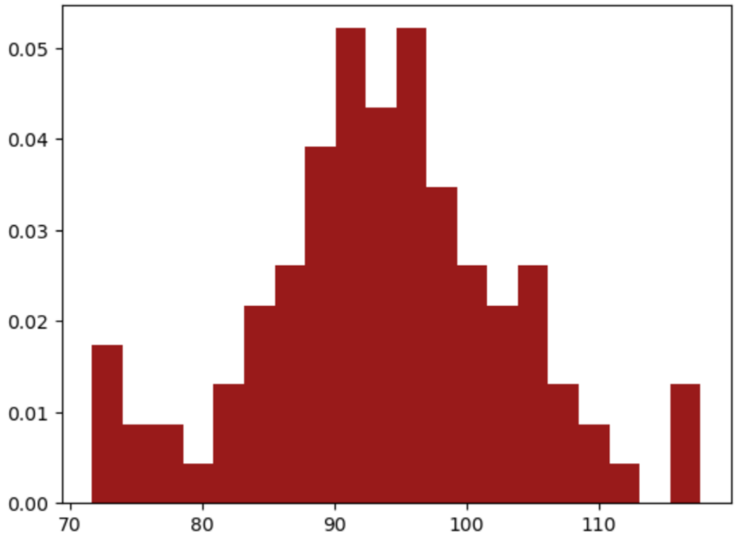
- a data sample that seems normal (by its histogram)
- a sample mean = 93 seconds
- a sample standard deviation = 10 seconds.

With this information, can you reject the call centre claim about customer waiting time?

HYPOTHESIS TESTING: WORKING EXAMPLE

Some facts we can assume so far:

1- Because sample histogram looks normally distributed (at least for a sample of that size), we will assume that the population is normally distributed.



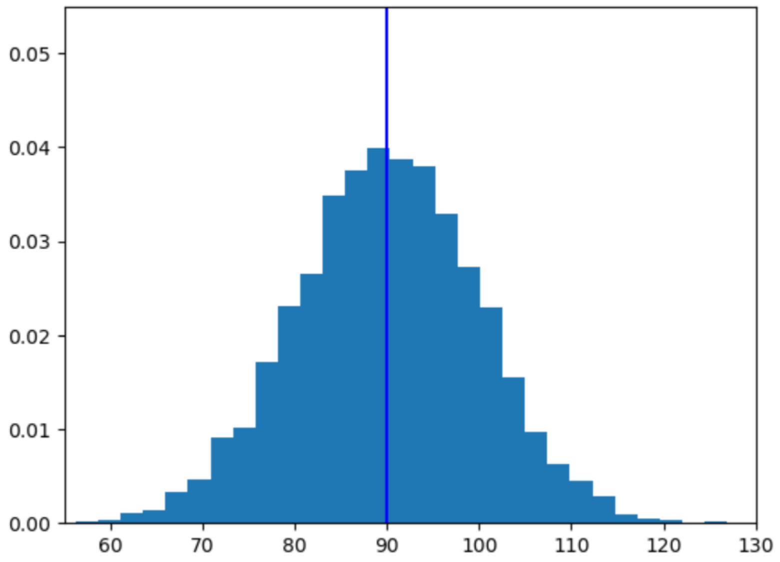
2- Because the population is normally distributed and the sample size is big enough (100 >> 30), then we will assume that the sample standards deviation and the population standard deviation are virtually equal. So, $\sigma = 10\text{secs}$

HYPOTHESIS TESTING: WORKING EXAMPLE

What we have.

The call centre claim is based on the following distribution:

Normal distribution
Mean = 90s
Standard deviation = 10s



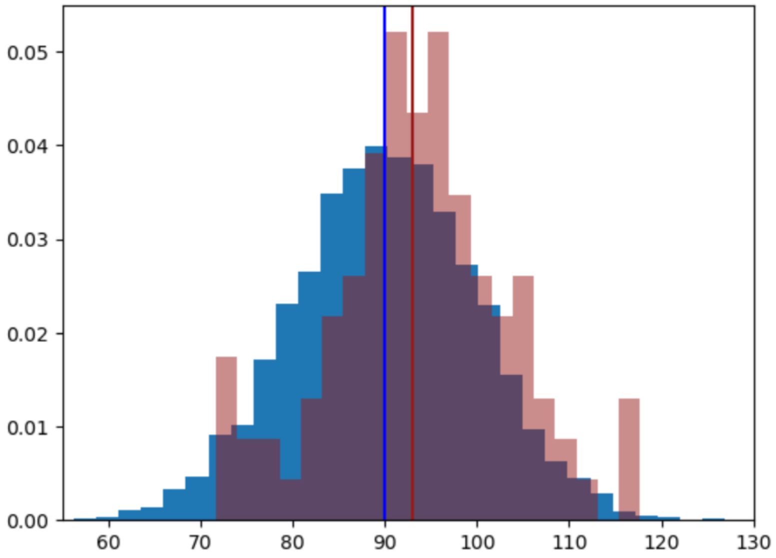
HYPOTHESIS TESTING. WORKING EXAMPLE

What we have.

The call centre claim is based on the following distribution:

Our 100-size sample follows the red distribution, which is a bit more to the right.

Normal distribution
Mean = 90s
Standard deviation = 10s



Normal-ish distribution
Mean = 93s
Standard deviation = 10s

This graph is suspicious but does not prove anything!!
We need to do our hypothesis testing.

HYPOTHESIS TESTING: WORKING EXAMPLE

So, we rephrase the claim as a hypothesis testing problem.

A sample of N=100 is drawn from a population which is normally distributed and has a known standard deviation of 10. The sample mean is 93, and the claim to test is that the population mean is 90 or less.

a) Write the Null and Alternative Hypothesis.
b) Is it a one-sided or a two-sided test?
c) Can we reject the Null Hypothesis?
d) Should we hire this call centre to handle my phone call?

HYPOTHESIS TESTING: WORKING EXAMPLE

So, we rephrase the claim as a hypothesis testing problem.

A sample of N=100 is drawn from a population which is normally distributed and has a known standard deviation of 10. The sample mean is 93, and the claim to test is that the population mean is 90 or less.

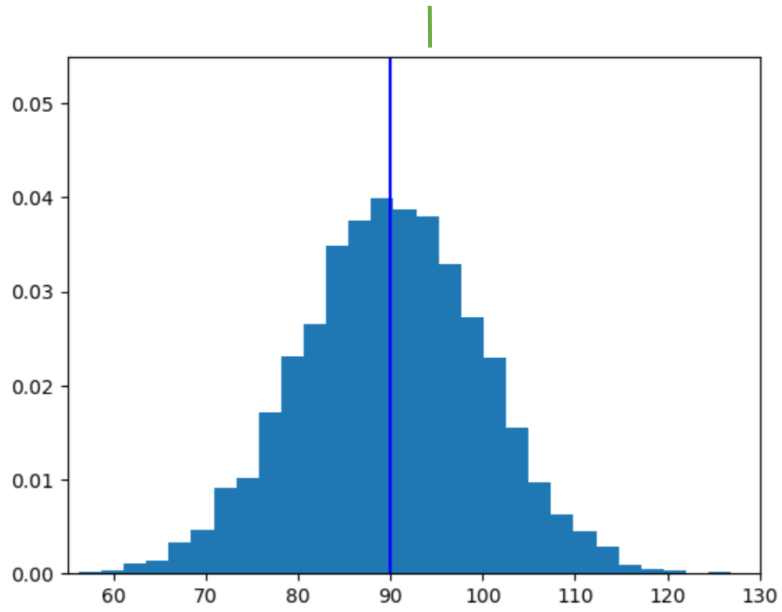
If this is not given, I need to choose myself the significance level BEFORE doing the test.

a) Write the Null and Alternative Hypothesis.
b) Is it a one-sided or a two-sided test?
c) Can we reject the Null Hypothesis with significance level $\alpha = 1\%$?
d) Should we hire this call centre to handle my phone call?

Let's solve this problem!

HYPOTHESIS TESTING: WORKING EXAMPLE

The call centre claim that waiting times are based on the following distribution and the mean is 90 seconds OR LESS.



Population $\mu = 90$
$\sigma = 10$

a) Write the Null and Alternative Hypothesis

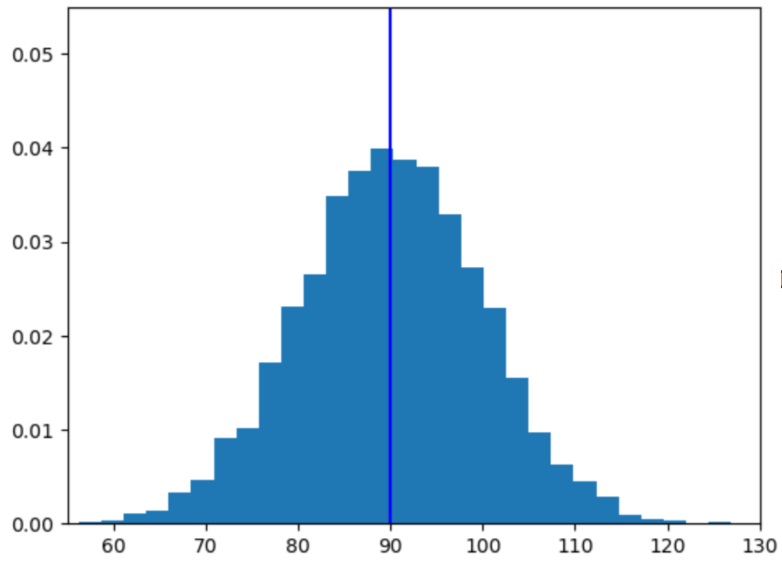
$$\boldsymbol{H_0: \mu \leq 90}$$
$$\boldsymbol{H_1: \mu > 90}$$

b)) Is it a one-sided or a two-sided test?

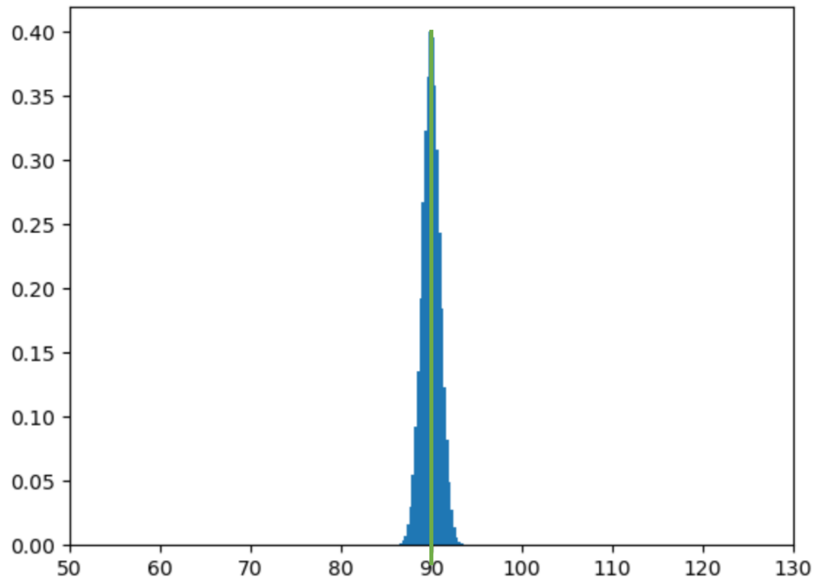
One sided

HYPOTHESIS TESTING: WORKING EXAMPLE

If the call centre claim is true, how likely is that I get an average value of 93s for a sample of size N= 100 => find the sampling distribution and the standard error, (se)



Population $\mu = 90$
$\sigma = 10$



Sampling Distribution

$n = 100$

$mean = 90$

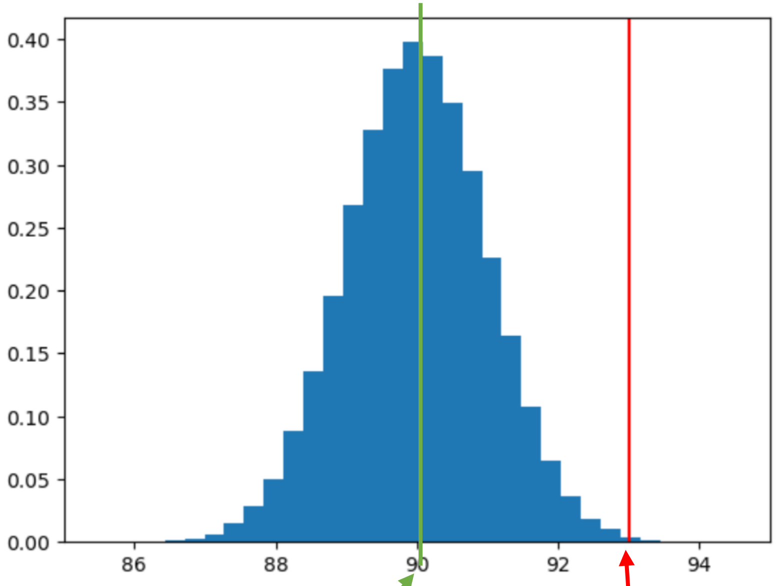
$se = \sigma / \sqrt{100} = 1$

HYPOTHESIS TESTING: WORKING EXAMPLE

Let's zoom in on the SAMPLING DISTRIBUTION.

Sampling Distribution

$n = 100$
$mean = 90$
$se = \sigma / \sqrt{100} = 1$



$\mu = 90s$

$\bar{X} = 93s$

Let's measure the distance from $\bar{X}$ to μ in terms of how many standard errors separate them.

Let's call this 'distance' the Z value

$$Z = \frac{\bar{X} - \mu}{se} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{N}}} = \frac{93 - 90}{1} = 3$$

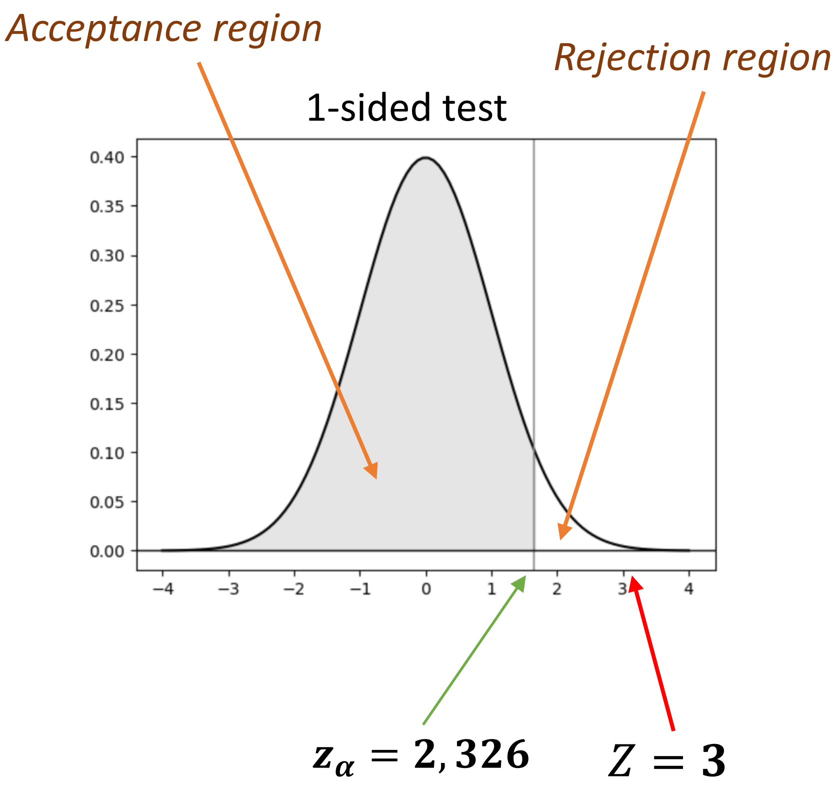
*A sample mean of 93 seconds: **3** standard errors to the right of the 'expected' mean.*

Is that a lot? => **it depends on my significance level!!**

HYPOTHESIS TESTING: WORKING EXAMPLE

Is the sample mean 'far' from the expected mean?

The right question to ask: Is the value $Z = 3$ in the acceptance or rejection region?



- The rejection region is obtained by selecting a confidence level.

```
import scipy.stats
alpha=0.01
z_right= scipy.stats.norm.ppf(1-alpha)
print(z_right)
```

```
2.3263478740408408
```

- For $\alpha = 0.01$, the critical z_α =2.326
- Our value Z is inside the rejection region.
- **We should then reject the Null hypothesis that the waiting call has an average of 90 seconds!**
- Think twice before hiring that call centre!

HYPOTHESIS TESTING: WORKING EXAMPLE

Let's solve the complete example following the full procedure.

Statement of the problem (short version)

Call centre claim $\mu \leq 90$ seconds, and $\sigma = 10$. Normal distribution assumption is ok. I measure sample N=100 and find average $\bar{X} = 93$ second. Should I hire this call centre?

Step 1. Set null hypothesis and type of test.

H_0: $\mu \leq 90$

H_1: $\mu > 90$

One side test. I reject the hypothesis (don't hire centre) if waiting time clearly >90 => Rejection region to the right.

Step 2. Fix a confidence level.

Let's take $\alpha = 0.01$(small number because I want to be confident that I do not commit a type 1 error).

Step 3. Compute critical level

Using Python (or memory, as this is a common alpha value) z_α =2,326

Step 4. Compute the Z-value

$$\mathrm{Z} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{N}}} = \mathbf{3} > \mathbf{2.326} = \mathrm{z}_\alpha \longrightarrow$$

Reject Null Hypothesis
Don't hire that call centre

SOME USEFUL CRITICAL VALUES

Type of test	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.01$
One sided. Rejection region at right H_0: $\mu \le \mu_0$ H_1: $\mu > \mu_0$	1,282	1.654	2,326
One sided. Rejection region at left H_0: $\mu \ge \mu_0$ H_1: $\mu < \mu_0$	-1,282	-1.654	-2,326
Two sided. Rejection regions at both extremes H_0: $\mu = \mu_0$ H_1: $\mu \ne \mu_0$	-1,645 +1,645	-1,96 +1,96	-2,576 +2,576

Calculated using this code.
Extend the table by computing your own significance levels!

```
import scipy.stats
alpha=.05

# One sided test. Right Side
z_critic = scipy.stats.norm.ppf(1-alpha)

#One sided test; Left Side
z_critic = scipy.stats.norm.ppf(alpha)

#Two sided
z_left = scipy.stats.norm.ppf(alpha/2)
z_right = scipy.stats.norm.ppf(1- alpha/2)
```