How to use the Correlation and **Model Error** Spreadsheet

This Spreadsheet demonstrates how, starting with a specific input x_i, a linear regression model with standardized variables can be used to make two different types of forecasts or estimates,

A *Point* Forecast: $\widehat{y_i} = \beta x_i$, and
A *Probabilistic* Forecast: $\phi(\beta x_i, 1 - \beta^2)$.

Note that $(1 - \beta^2)$ can also be written as σ_ε^2, notation for "the variance of the model error." So an equivalent way to represent the probabilistic forecast is $\phi(\beta x_i,\ \sigma_\varepsilon^2)$.
Since the linear function for standardized variables can also be expressed $y = Rx$, yet a third equivalent way to represent the probabilistic forecast is $\phi(Rx_i, 1 - R^2)$.

In other words, the probabilistic forecast takes the form of a Gaussian probability distribution function with mean = Rx_i, variance = $1 - R^2$, and standard deviation = $\sqrt{1 - R^2}$. The advantage of the probabilistic forecast is that it allows one to specify the probability that the true result will fall within a certain range around the mean – a "confidence interval."

Example 1. Assume a standardized linear regression model has correlation R = .5 [Cell I38] and the point x_i = .8 [Cell I36]. The point forecast $\widehat{y_i} = Rx_i$ = .4 [Cell I40]. This value is also the mean of the Gaussian probability distribution. The standard deviation of the Gaussian is $\sqrt{1 - R^2}$ = .87 [Cell H46].

Assume one wants 50% confidence [Cell F46] that the true answer would fall within a defined range. Calculated in Excel, the range to specify would over the interval from
Rx_i – (NormSInv(.75) $\sqrt{1 - R^2}$) to
Rx_i + (NormSInv(.75) $\sqrt{1 - R^2}$])

This range, from -.18 [Cell I46] to .98 [Cell K46] is known as the "50% Confidence interval."

Example 2. Assume the correlation is R = .353 [Cell I 38] and the value of x_i = .09 [Cell I36].

Q: What is the mean of the probabilistic forecast?
Answer: .0315 [Cell I40].

Q: What is the 99% Confidence Interval? [Cell F50].
Answer: from -2.38 [Cell I50] to 2.44 [Cell K50].