Using the Algebra with Gaussians Spreadsheet

The new distribution formed by repeatedly combining random draws from two different Gaussian distributions, is itself a Gaussian. This is a very convenient property that is not true of other probability distributions.

We use the Greek letter Phi $\phi$(a, b) notation to stand for a Gaussian distribution with **mean** = a and **variance** (standard deviation squared) = b.

The spreadsheet gives three different types of ways to add two Gaussians.

**First:** if the two Gaussians $\phi_1$ and $\phi_2$ $are$ **Independent,** their correlation R, Covariance ($Cov_{1,2}$) and Mutual Information ($I(\phi_1; \phi_2$) are all 0. In that case, combining a draw from each results in a distribution with mean equal to the sum of the means, and variance equal to the sum of the variances. In Phi notation, we can write,

$$\phi_1(a, b) + \phi_2(c, d) = \phi_{1+2}(a + c, b + d)$$

In the Spreadsheet, enter the mean [Cell C4] and variance [Cell E4] of the first Gaussian and the mean [Cell C6] and variance [Cell E6] of the second Gaussian.
The combined mean is given in [Cell C8] and the combined variance in [Cell E8].

**Second:** it is possible to create **weighted combinations** of the two Independent Gaussians. Most commonly, weights $w_1$ and $w_2$ are used so that $w_1$ + $w_2$ = 1, but any numbers can be used for w. In Phi notation,

$$w_1\phi_1(a, b) + w_2\phi_2(c, d) = \phi_{1+2}(w_1a + w_2c, w_1^2b + w_2^2d)$$

In the Spreadsheet, enter the mean [Cell C13], variance [Cell E13], and weighting [Cell G13] of the first Gaussian and the mean [Cell C15], variance [Cell E15] and weighting [Cell G15] of the second Gaussian. The combined mean is given in [Cell C17] and the combined variance in [Cell E17].

Third: is it possible to create weighted (or unweighted) combinations of **dependent** Gaussian distributions. Dependence is expressed in terms of *Covariance*. In Phi Notation,

$$w_1\phi_1(a, b) + w_2\phi_2(c, d) = \phi_{1+2}\left(w_1a + w_2c, w_1^2b + w_2^2d + 2w_1w_2Cov_{1,2}\right)$$

In the Spreadsheet, enter the mean [Cell C22], variance [Cell E22], and weighting [Cell G22] of the first Gaussian, the mean [Cell C24], variance [Cell E24] and weighting [Cell G24] of the second Gaussian, and the Covariance between them [Cell F22]. The combined mean is given in [Cell C26] and the combined variance in [Cell E26].

Example.

**Question.** I create an investment portfolio that is 65% Exxon stock and 35% Tesla Stock. The stocks' expected annual returns have a Gaussian distribution, with Exxon = $\phi(10\%, .04)$ and Tesla = $\phi(15\%, .09)$ and the two distributions have Covariance = .03. What is the expected return and standard deviation of the investment portfolio?
**Answer.** Use the third spreadsheet section, for weighted combinations of dependent distributions. The expected return = 11.75% [Cell C26] and the expected standard deviation of return = 20.39% [Cell D26