FORMATIVE ASSESSMENT 3

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Answer Exercises 7.1 items 2 and 7.

Question Number 2

A binary communication channel carries data as one of two sets of signals denoted by 0 and 1. Owing to noise, a transmitted 0 is sometimes received as a 1, and a transmitted 1 is sometimes received as a 0. For a given channel, it can be assumed that a transmitted 0 is correctly received with probability 0.95, and a transmitted 1 is correctly received with probability 0.75. Also, 70% of all messages are transmitted as a 0. If a signal is sent, determine the probability that:

a. a 1 was received;

Based on the book, the solution will be

Let R0 be the event that a zero is received. Let T0 be the event that a zero is transmitted.

Let R1 be the event that a one is received. Let T1 be the event that a one is transmitted.

the probability that a one was received, we note

$R1 = (T1 \cap R1) \cup (T0 \cap R1)$

so the probability that one is received,

$P(R1) = P(T1)P(R1|T1) + P(T0)P(R1|T0)$

therefore,

$P(R1)=(0.3 \times 0.75)+(0.7 \times 0.05) = 0.225 + 0.035 = 0.26$

in R,

```
Received0_given_T0 <- 0.95
Received1_given_T1 <- 0.75
Received1_given_T0 <- 0.05
Received0_given_T1 <- 0.25

T0 <- 0.7
T1 <- 0.3

R1 <- ((T1*Received1_given_T1) + (T0*Received1_given_T0))

cat("The probability that 1 is received is: ", R1, "\n")
```

```
## The probability that 1 is received is:  0.26
```

Thus, 26% is the probability that 1 is received.

b. a 1 was transmitted given than a 1 was received.

The solution will be,

Let R0 be the event that a zero is received. Let T0 be the event that a zero is transmitted. Let R1 be the event that a one is received. Let T1 be the event that a one is transmitted.

We use Bayes' rule,

$P(T1|R1) = ((P(T1)P(R1|T1))/P(R1))$

Then,

$P(T1)P(R1|T1) = 0.3 \times 0.75 = 0.225$

Given that P(R1) = 0.26, based on the solution in letter A, we will now have

$P(T1|R1) = ((0.3 \times 0.75)/0.26) = (0.225/0.26) = 0.865$

in R,

```
  Received0_given_T0 <- 0.95
  Received1_given_T1 <- 0.75
  Received1_given_T0 <- 0.05
  Received0_given_T1 <- 0.25

  T0 <- 0.7
  T1 <- 0.3

  R1 <- ((T1*Received1_given_T1) + (T0*Received1_given_T0))

  R1_given_T1 <- ((T1*Received1_given_T1)/R1)

  cat("The probability that 1 was transmitted given than a 1 was received: ",        R1_given_T1, "\n")
```

```
## The probability that 1 was transmitted given than a 1 was received:  0.8653846
```

Thus, 86.5% is the probability that 1 is transmitted given that 1 is received.

Question Number 7

There are three employees working at an IT company: Jane, Amy, and Ava, doing 10%, 30%, and 60% of the programming, respectively. 8% of Jane's work, 5% of Amy's work, and just 1% of Ava's work is in error. What is the overall percentage of error? If a program is found with an error, who is the most likely person to have written it?

To solve for the overall percentage error, we use total probability

Assuming E to be the error of the work of the three employees,

$E = (Jane \cap janeErrorWork) \cup (Amy \cap amyErrorWork) \cup (Ava \cap avaErrorWork)$

so,

P(E) = P(Jane)P(E|Jane) + P(Amy)P(E|Amy) + P(Ava)P(E|Ava) = ((0.10.08) + (0.30.05) + (0.6*0.01)) = 0.008 + 0.015 + 0.006 = 0.029

in R,

```
janeProgram <- 0.1
amyProgram <- 0.3
avaProgram <- 0.6

janeErrorWork <- 0.08
amyErrorWork <- 0.05
avaErrorWork <- 0.01

totalError <- ((janeProgram*janeErrorWork) + (amyProgram*amyErrorWork) + (avaProgram*avaErrorWork))

cat("The total probability of error: ", totalError, "\n")
```

```
## The total probability of error:  0.029
```

Thus, 2.9% is the total probability of overall error percentage.

Now, to find the person who most likely have written the error, solve for posterior possibilities,

P(J|E) = ((P(Jane)P(E|Jane))/P(E)) = ((0.10.08)/0.029) = 0.28 P(A|E) = ((P(Amy)P(E|Amy))/P(E)) = ((0.30.05)/0.029) = 0.52 P(AV|E) = ((P(Ava)P(E|Ava))/P(E)) = ((0.6*0.01)/0.029) = 0.21

in R,

```
janeError <- ((janeProgram*janeErrorWork)/totalError)
cat("Jane's written error: ", janeError, "\n")
```

```
## Jane's written error:  0.2758621
```

```
amyError <- ((amyProgram*amyErrorWork)/totalError)
cat("Amy's written error: ", amyError, "\n")
```

```
## Amy's written error:  0.5172414
```

```
avaError <- ((avaProgram*avaErrorWork)/totalError)
cat("Ava's written error: ",  avaError, "\n")
```

```
## Ava's written error:  0.2068966
```

```
errorRate <- c(janeError, amyError, avaError)
mostLikely <- which.max(errorRate)
cat("The most likely person to have written the error is:", c("Jane", "Amy", "Ava")[mostLikely], "\n")
```

```
## The most likely person to have written the error is: Amy
```

Because P(A|E) is the largest, the person who most likely have written an error is Amy.