- Charge (Q) (electrical)

The property of atomic particle of which matter consists, measured in Coulombs.

$$1C = 6.24 \times 10^{18} \text{ electrons}$$

- Current (I):

The drift ~~or~~ or flow of e^- in a conductor in one-direction is known as the electric current.

Electric current = rate of flow of e^-

$$I = \frac{Q}{t}$$

Unit:- C/s

- Potential (V):

The energy required to bring a unit positive charge from infinite distance to the point in the electric field is called as potential i.e., the ratio of work to charge.

Unit: Volts(V) / J/C

- Power (P):

Power is the rate of doing work and is expressed in J/sec or watt (W).

In EE, Power, $P = VI = \frac{V^2}{R} = I^2R$

- Energy (E):

The capacity to do work overall is the total amount of work done.

Unit: Joule

$$E = P \times t$$

- CIRCUIT ELEMENT :

- R, L, C
- VS, CS

} 5 circuit elements.

Circuit element refers to the mathematical model of or physical device. There are 5 basic elements.

R, L, C, VS, CS

Electric Network:

The ~~electric~~ interconnection of 2 or more circuit elements is called electric network.

Electric Circuit:

The interconnection of 2 or more circuit elements with atleast one closed path.

CLASSIFICATION OF ELECTRICAL NETWORKS:

Electric Network
- ① Active / Passive
- ② Lineare / Non-Lineare
- ③ Unilateral / Bilateral
- ④ Lumped / Distributed

① **Active N/W:**

Circuit constituting atleast one energy source is called active N/W. Energy source → VS, CS

Passive N/W:

Circuit having passive elements only, and no E source.

② **Linear N/W:**

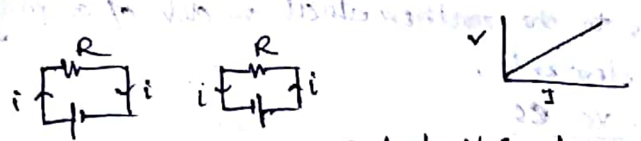
- A Circuit or network whose parameters i.e. elements like R, L and C are always constant irrespective of change in time, voltage, Temperature, etc.
- Ohm's Law applicable.

Non-Linear N/W:

- A circuit whose parameters can change their values with change in time, Voltage, Temp.
- Ohm's Law not applicable

③ **Bilateral N/W:**

Circuit whose characteristics, behaviour is same irrespective of the direction of current through various elements of it is called Bilateral network.



Direction of i changed but V-I characteristics remains same.

Unilateral N/W:

Circuit whose characteristics / behaviour is dependent on the direction of current through different elements.

Eg:- Diode.

④

Lumped N/W:

- A network in which all the ~~network~~ circuit elements are physically seperable.
- Most of the electric network is Lumped.

Distributed N/W:

- A network in which the ~~network~~ circuit elements cannot be physically seperable for analysis purpose.

Eg:- Transmission Line.

Ohm's Law:

The physical conditions remains constant, the Potential difference across the ends of the conductor is directly proportional to the current flowing through it.

$$V \propto I$$
$$V = IR$$

Limitations:

- Cannot be applied to circuits consisting of electronic valves or transistor because these elements are not bilateral.
- Cannot be applied to circuits consisting of non-linear elements such as powdered carbon, Thyrite, Electric arc

Laws of Resistance:

- Resistance of wire depends on its length, area of cross section, type of material of which it is made of and working Temperature.

$$R \propto l/a$$

$$R = \rho \frac{l}{a} \quad \rho \longrightarrow \text{Resistivity}$$

- Resistivity:

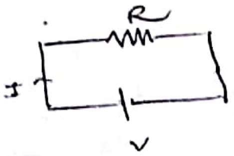
Defined as The Resistance between opposite faces of ~~a~~ unit cube of that material.

OR

Resistance of a material of unit length and unit cross sectional area, is defined as Resistivity.

Resistors :-

If the voltage across the element is directly proportional to the current through it, then that element is a resistor.



$V = IR$

Power absorbed by Resistor :

$$P = I^2 R = \frac{V^2}{R}$$

Inductance (L) :

Inductance is the property of a material by virtue of which it opposes any change of current through it.

Thus the current through inductor cannot change instantaneously.

$$L = \frac{\psi}{I} = \frac{N\phi}{I} \quad \text{(Henry)}.$$

$\psi \rightarrow$ Total flux linkage.

Inductor :

If the terminal voltage (or) voltage across the element is proportional to the time derivative of current through it, then the element is an Inductor.

Characteristics of Inductor :

① No voltage across Inductor if the current through it is not changing with time (DC) i.e., Inductor acts as short circuit to DC supply.

② Inductor opposes the change in current through it.

③ Finite amount of energy can be stored in an Inductor even if the voltage across Inductor is 0.

$$E = \frac{1}{2} L I^2$$

④ The inductor stores in terms of magnetic forms and is given by -

$$E(t) = \frac{1}{2} L\, I(t)^2$$

⑤ The average Power absorbed by the Inductor is 0.

Energy Stored in an Inductor :

*** When a current flows through an inductor, a magnetic field is estd. Work needs to be done in establishing the magnetic field, since the inductor opposes the setting of the field due to its inherent property of electrical inertia i.e., inductance.

Consider an Inductor of Inductance L.

Let a current I is forced through it.

Current increases from 0 to a finite value I.

At any time the emf developed across inductor,

$$e = -L\frac{di}{dt}$$

Instantaneous work done per second,

$$P = e.i \ W$$

$$P = i\left(-L\frac{di}{dt}\right)$$

$$P\,dt = -Li\,di.$$

Integrating both sides,

$$\int P\,dt = -\int Li\,di$$

Total work done on flowing of current from $0 \to I$,

$$W = \int_0^I Li\,di$$

$$W = \frac{LI^2}{2}$$

$$W = \frac{1}{2}LI^2$$

Energy in terms of Magnetic forms.

Capacitance:

Property of a material by virtue of which it opposes the change in voltage through it.

unit - Farads (F).

Capacitor:

If the terminal voltage is proportional to the integral of current through it, then that element is capacitor.

$$v \propto \int i\,dt$$

$$v = \frac{1}{c}\int i\,dt$$

$$\boxed{i = c\frac{dv}{dt}}$$

Characteristics of capacitor:

① Oppose the change in voltage.

② Current through the capacitor is 0, if the voltage across it is not changing with time (DC), i.e., a capacitor acts as open circuit for DC supply.

The energy stored by a capacitor is given by.

$$E = \frac{1}{2} CV^2$$

in terms of Electric field.

ⓑ Average Power absorbed by the capacitor is ZERO,

Energy Stored in Capacitor :-

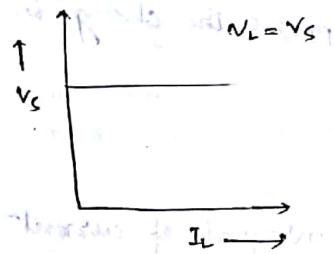
Proof :

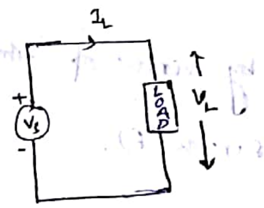
Types of Sources :

1/ Voltage Source → Ideal V.S. / → Practical V.S.

2/ Current Source → Ideal V.S. / → Practical V.S

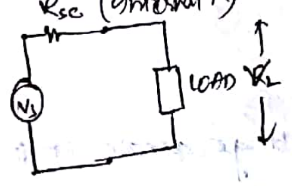
① ⓐ Ideal Voltage Source :

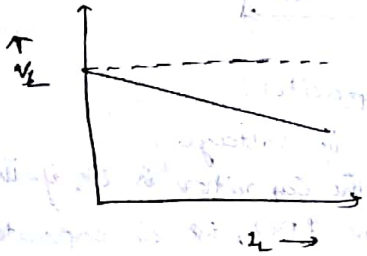




- An ideal voltage source is a device that provides a constant voltage across its terminals ($V_L = V_S$) irrespective of the current flowing through it i.e., the internal resistance is zero.

ⓑ Practical Voltage Source :

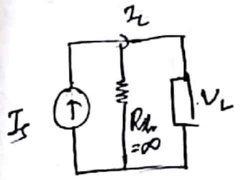


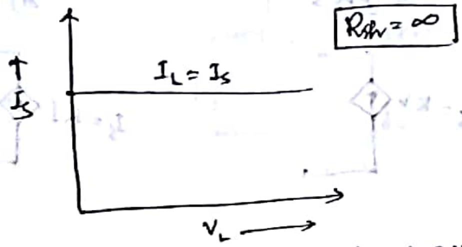


- $\boxed{V_L = V_S - I_L R_{sc}}$

- Practically, small int. R (R_{sc}) is present in every voltage source exists in series with the source. Because of R_{sc}, V_L decreases with $I_L \uparrow$.

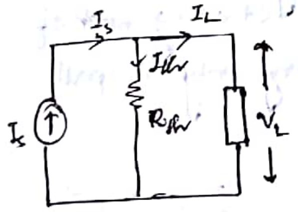
(2) (a) Ideal Current Source:

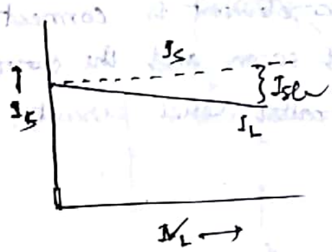




- It is a source which gives constant current at its terminals irrespective of the voltage appearing across its terminals. Symbol for ideal current source and characteristics are shown.

(b) Practical Current Source:





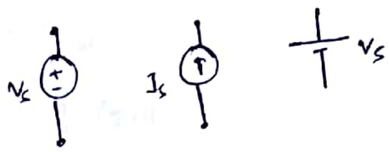
- $I_s = I_L + I_{sh}$

Practically every current source has high internal resistance shown in parallel with current source and it is represented by R_{sh}.

* Independent Sources:

The sources not depending on other voltages (or) current in the circuit for their values are Independent sources.

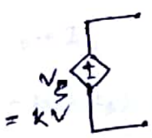
Represented by ◯ with a polarity of voltage (or) direction of current indicated inside.



* Dependent Sources:

Those sources whose value depends on voltage (or) current in the circuit. Such sources are indicated by ◇ and are further classified into:
(a) Voltage dependent voltage source (VDVS)
(b) Current dependent voltage source (CDVS)
(c) Voltage dependent current source (VDCS)
(d) Current dependent current source (CDCS)

(1) VDVS:
It produces a voltage as a function of voltages elsewhere in the circuit.



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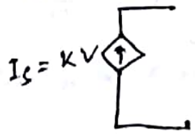
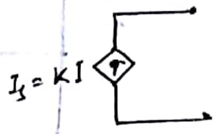
b) CDVS:

c) VDCS:



d) CDCS:



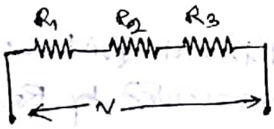
- Dependent sources are also called as Controlled Sources.

* Series and Parallel Circuits:

① Series Circuit:-

When the elements are connected in such a way that the finishing end of one element is connected to the starting end of another element and so on and the current is having only one path, then the circuit is called series circuit.

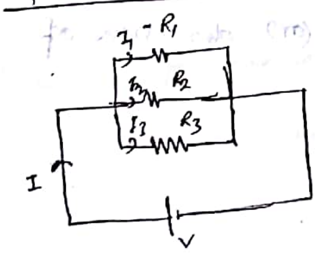
Resistors in Series:



$V_1 = IR_1 \quad V_2 = IR_2 \quad V_3 = IR_3$

$V = I(R_1+R_2+R_3)$ $\quad \boxed{\therefore R_{eq} = R_1+R_2+R_3}$

Resistors in Parallel:



$I = \frac{V}{R_1}+\frac{V}{R_2}+\frac{V}{R_3}$

$\Rightarrow V = \frac{I}{\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)}$

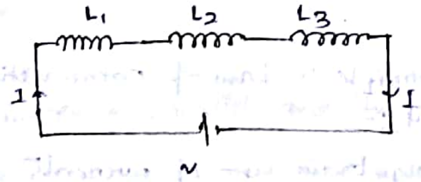
$\therefore \frac{1}{R_{eq}} = \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$

- When the elements are connected in such a way that one end of each of them is joined to a common point and the other ends being joined to another common point then the elements are said to be in parallel.

$R_{eq} < R_1 < R_2 < \dots < R_n$

Inductors in Series :

$V = L\frac{dI}{dt}$



$V_{L_1} = L_1\frac{dI}{dt}$

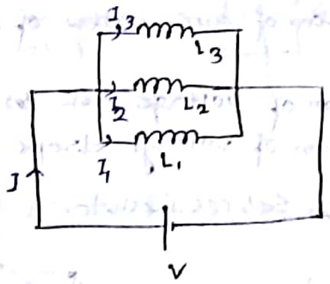
$V_{L_2} = L_2\frac{dI}{dt}$

$V_{L_3} = L_3\frac{dI}{dt}$.

$V = (L_1+L_2+L_3)\left(\frac{dI}{dt}\right)$.

$\boxed{\therefore L_{eq} = L_1+L_2+L_3}$

Inductors in Parallel :



$V_{L_1} = L_1\frac{dI_1}{dt}$

$V_{L_2} = L_2\frac{dI_2}{dt}$

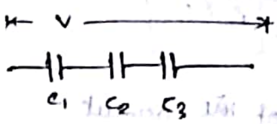
$V_{L_3} = L_3\frac{dI_3}{dt}$

$V = L_{eq}\frac{dI}{dt}$

$I = \frac{1}{L_{eq}}\int V\,dt = \frac{1}{L_1}\int V\,dt + \frac{1}{L_2}\int V\,dt + \frac{1}{L_3}\int V\,dt$.

$\boxed{\therefore \frac{1}{L_{eq}} = \frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}}$

Capacitors in Series :



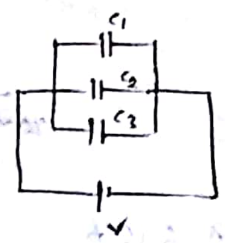
$I = C\frac{dV}{dt}$

$V = \frac{1}{C}\int I\,dt$

$\frac{1}{C_1}\int I\,dt + \frac{1}{C_2}\int I\,dt + \frac{1}{C_3}\int I\,dt = \frac{1}{C_{eq}}\int I\,dt$.

$\Rightarrow \boxed{\frac{1}{C_{eq}} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}$

Parallel



$I = C\frac{dV}{dt}$

$I_1 = C_1\frac{dV}{dt}$; $I_2 = C_2\frac{dV}{dt}$; $I_3 = C_3\frac{dV}{dt}$.

$I = I_1+I_2+I_3$

$C_{eq}\cdot\frac{dV}{dt} = (C_1+C_2+C_3)\frac{dV}{dt}$

$\Rightarrow \boxed{C_{eq} = C_1+C_2+C_3}$

→ **KIRCHHOFF'S LAWS:**

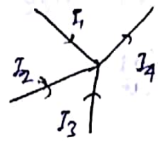
- **KCL:**

Basic Principle :- Law of conservation of charge.

Statement :-

The algebraic sum of currents entering a junction is equal to the algebraic sum of currents leaving the junction in an electric circuit.

$$\boxed{I_1 + I_2 + I_3 - I_4 = 0}$$



KVL:

Basic Principle: Law of conservation of energy.

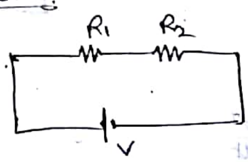
Statement:

The Algebraic sum of voltage rise in any closed circuit is equal to the algebraic sum of voltage drops in the circuit.

Voltage division in Series Resistor:

$$V_{R_1} = V\left(\frac{R_1}{R_1+R_2}\right)$$

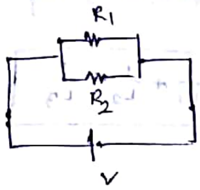
$$V_{R_2} = V\left(\frac{R_2}{R_1+R_2}\right)$$



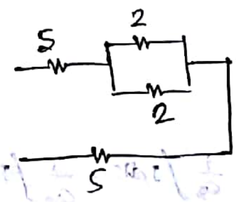
Current division in || Circuit:

$$I_{R_1} = I\left(\frac{R_2}{R_1+R_2}\right)$$

$$I_{R_2} = I\left(\frac{R_1}{R_1+R_2}\right).$$

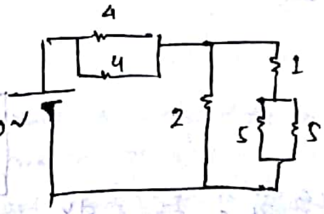


Q/ Find the equivalent R of the circuit.



Soln $R_{eq} = 11\,\Omega$

Q/ Determine ~~Req~~ V. drop across ~~X/M/~~ 1 Ω Resistor and power loss across 2Ω resistor.



Soln

$R_{eq} = \cancel{\frac{33}{8}}\,\Omega \; 3.27\,\Omega$

$I = \cancel{\frac{20}{33/8}} = \cancel{\frac{45}{8}}\,A \quad 6.12\,A$

$I_{2\Omega} = \cancel{\left(\frac{7/2}{2+7/2}\right) \times \frac{45}{8}} \quad 6.12 \times \frac{3.5}{2+3.5}$

$= 3.89.$

Power Loss = $\left(\frac{30}{8}\right)^2 \times 2 = 38'28$ W.

Q/ The R of 3, 4 and 5 Ω are connected in parallel and this combination is put in series with 2 Ω Resistor.

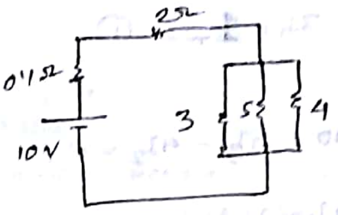
① Find R_{eq}.

② Obtain current in each circuit, when a battery of 10 V with internal resistance of 0'1 Ω is connected across the circuit.

Sol^n

$R_{eq} = \cancel{0'48 + 2 + 2'78\,\Omega}$

$= 1'28 + 2 = 3'28\ \Omega$



~~V/R~~

$I = \frac{10}{2'78 + 0'1} = 3'47\ A.$

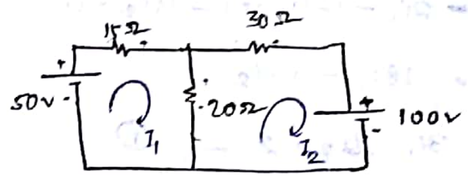
$I_{3\Omega} = \left(\frac{2'22}{5'22}\right) \times 3'47 = 1'48\ A$

$I_{4\Omega} = \left(\frac{1'875}{5'875}\right) \times 3'47 = 1'11\ A$

$I_{5\Omega} = \underline{1'714}$

Q. Apply KVL, find the current flowing through the circuit elements shown in fig.

Sol^n



Applying KVL to loop 1,

$50 - 15I_1 - 20(I_1 - I_2) = 0.$

$50 - 35I_1 + 20I_2 = 0 \Rightarrow 4I_2 - 7I_1 + 10 = 0$ ——— ①

Applying KVL to loop 2,

$-20(I_2 - I_1) - 30(I_2) - 100 = 0$

$-50I_2 + 20I_1 - 100 = 0 \Rightarrow \cancel{+5I_2 + 2I_1 + 10 = 0}$ ——— ⑪

$-5I_2 + 2I_1 - 10 = 0$

①×5 = $20I_2 - 35I_1 + 50 = 0$

②×4 = $-20I_2 + 8I_1 - 40 = 0$

$\cancel{-43I_1 + 10 = 0}$

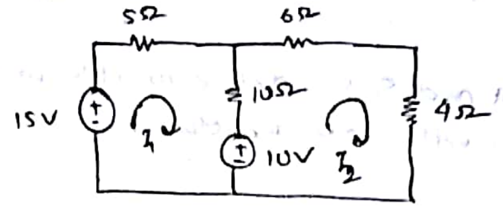
$\cancel{I_1 = \frac{10}{43}}$

$-27I_1 - 10 = 0$

$I_1 =$

Q Find the circuit current using KVL.

Soln



$15 - 5I_1 - 10(I_1 - I_2) - 10 = 0$

$\Rightarrow -10I_2 + 15I_1 = 5$

$\Rightarrow -2I_1 + 3I_1 = 1$ ——— ①

$10 + (I_1 - I_2)10 - 6I_2 - 4I_2 = 0$

$\Rightarrow 10 + 10I_1 - 20I_2 = 0$

$\Rightarrow 2I_2 - I_1 = 1$ ——— ②

Adding ① and ②,

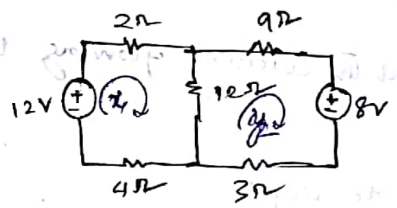
$2I_1 = 2$

$I_1 = 1A$.

H/W

Q

[1.04, 0.211]



$12 - 2i_1 - 12(i_1 - i_2) - 4i_1 = 0$

$\cancel{12 = 18i_1 - 12i_2}$

$\Rightarrow 3i_1 - 2i_2 = 2$ ——— ①

$\cancel{12(i_1 - i_2) - 9i_2 - 8 - 3i_2 = 0}$

$\cancel{12i_1 - 24i_2 = 8}$

$\Rightarrow \cancel{3i_1 - 6i_2 = 2}$ ——— ②

$12 - 2x - 12(x - y) - 4x = 0$

$12 - 2x - 12x + 12y - 4x = 0$

$18x - 12y = 12$

$3x - 2y = 2$ ——— ①

$-12(y - x) - 9y - 8 - 3y = 0$

$-12y + 12x - 9y - 3y = 8$

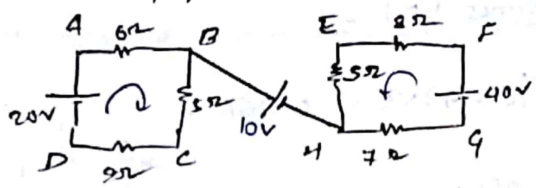
$12x - 24y = 8$

$3x - 6y = 2$

$y = 0$

$x = 2/3 A = 0.67 A$

Q) For the circuit, find V_{CE} and V_{AG}.



Sol:

$$I_1 = \frac{20}{20} = 1A$$

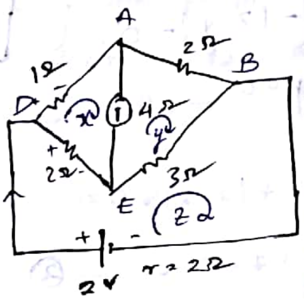
$$I_2 = \frac{40}{20} = 2A.$$

$V_{CE} = 5\times1 - 10 + 5\times2 = 5V$ = -5V direction

$V_{AG} = -6 - 10 - 7\times2 = -30V$ = 30V. because.

$V_A - 6 - 10 - 7\times2 = V_G \Rightarrow \boxed{V_A - V_G = 30V}$

Q) Determine the currents in the unbalanced bridge circuit. Also determine the potential difference across BD, and the resistance from B to D.



Sol:

$2 - 2z - 2(z-x) - 3(z-y) = 0$

$2 - 2z - 2z + 2x - 3z + 3y = 0$

$2x + 3y - 7z + 2 = 0$ —— ①

$2(z-x) - 1x - 4(x-y) = 0$

$7x - 4y - 2z = 0$ —— ② × 3 ⇒ $21x - 12y - 6z = 0$

$4(x-y) - 2y + 3(z-y) = 0$

$4x - 9y + 3z = 0$ —— ③ × 2 ⇒ $8x - 18y + 6z = 0.$

(+)

$29x - 30y = 0$ —— ④

$y = \frac{29}{30}x.$

① × 2 ⇒ $4x + 6y - 14z + 4 = 0$

② × 7 ⇒ $49x - 28y - 14z = 0$

$45x - 34y = 4$ —— ⑤

$45x - 34\left(\frac{29}{30}\right)x = 4$

$\Rightarrow 45x - \frac{493x}{15} = 4 \Rightarrow 12.13x = 4 \Rightarrow x = 0.329A. \approx 0.33A$

$\therefore y = \frac{29}{30} \times 0.33 = 0.319A.$

$z = 0.517A.$

13

The potential diff across BD is –

$$2-(0.517)\times 2 = 0.966\ V.$$

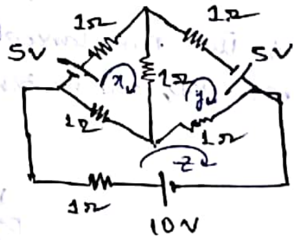
$V_{BD} = 0.966\ V$

$I_{BD} = z = 0.517\ A$

$$R_{BD} = \frac{0.966}{0.517} = 1.8685\ \Omega$$

Q) Determine the branch currents in the network when the value of each branch resistance is 1Ω.

Sol



$10 - z - (z-x) - (z-y) = 0$

$10 - 3z + x + y = 0$ — ①

$5 - x - (x-y) + (z-x) = 0$

$5 - 3x + y + z = 0$ — ②

$(x-y) - y + 5 + (z-y) = 0$

$\Rightarrow x - 3y + z + 5 = 0$ — ③

② - ③ $\Rightarrow$ $-4x + 4y = 0$

$\boxed{x = y}$

$10 - 3z + 2x = 0$ — ①

$5 - 2x + z = 0$ — ②

① + ② $\Rightarrow$ $15 - 2z = 0$

$z = \frac{15}{2} = 7.5\ A.$

$3\times\frac{15}{2} - 2x = 10$

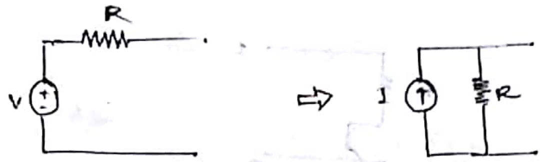
$2x = \frac{75}{2} - 10$

$2x = \frac{55}{2}$

$x = \frac{55}{4} = 13.75\ A$

SOURCE TRANSFORMATION :

A given voltage source with a series resistance can be converted into an equivalent current source with a parallel resistance or vice versa.



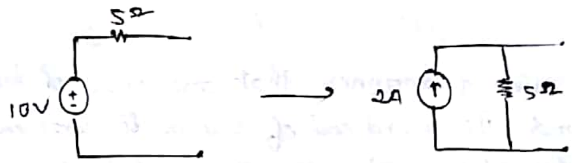
$V = IR$

$I = \frac{V}{R}$

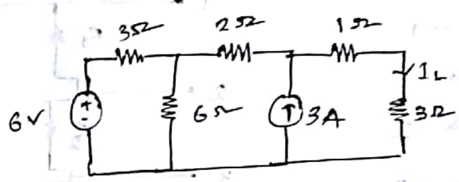
$I = \frac{V}{R}$

Q. Convert the voltage source of fig A into an equivalent current source.

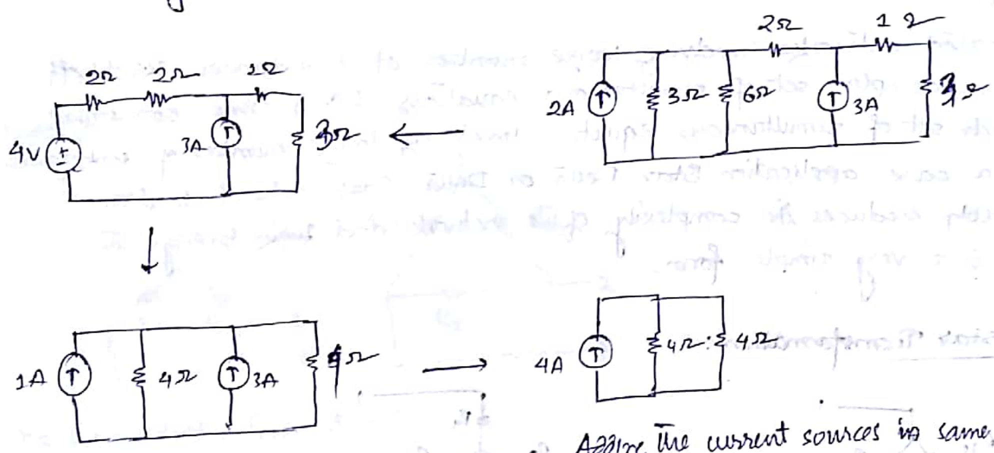
Sol



Q. Use source conversion technique to find the load current I_L in the circuit shown below.



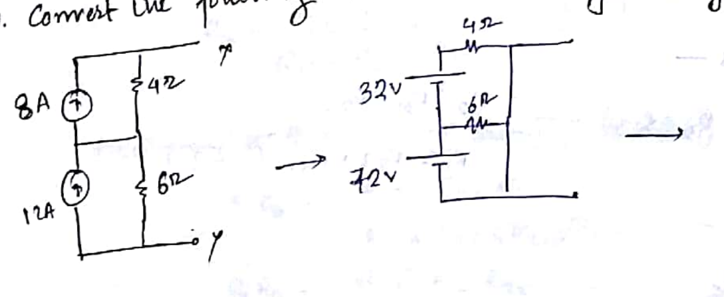
Soln Replacing 6V in series with 3Ω resistor by current source ($I = 2A$).



Adding the current sources in same direction

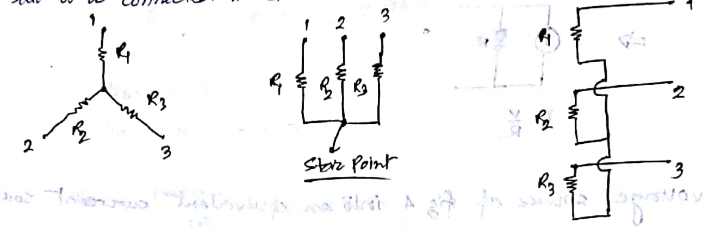
$I_L = \frac{4}{2} = 2A$

Q. Convert the following circuit into a single voltage source.



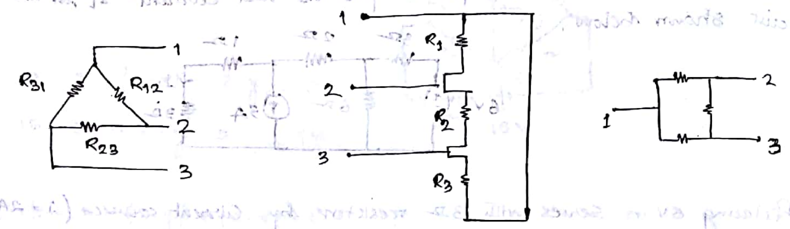
STAR CONNECTION (Y or ⅄)

If 3 Resistances are connected in such a manner that one end of each is connected together to form a junction point called star point, the resistances are said to be connected in Star.



DELTA CONNECTION (Δ)

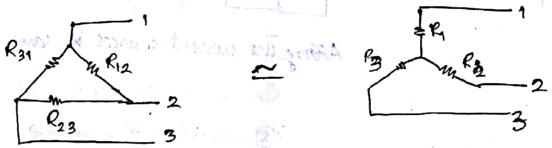
If 3 resistances are connected in such a manner that one end of the first is connected to first end of second, the second end of second to first end of third and so on to complete a loop then the resistances are said to be connected in Delta form.



CONVERSION:

In complicated networks involving large number of resistances, Kirchhoff's Law give us complex set of simultaneous equations. It is time consuming to solve such set of simultaneous equations involving large number of unknowns. In such a case, application Star-Delta or Delta-Star transformation considerably reduces the complexity of the network and ~~links~~ brings the network into a very simple form.

- Delta to Star Transformation:



The 3 resistances, R_{12}, R_{23} and R_{31} connected in Δ are transformed to equivalent star connection as follows -

The R b/w 1 and 2 in Δ is —

$$R_{1-2} = R_{12} \parallel (R_{31}+R_{23}) = \frac{R_{12}(R_{31}+R_{23})}{R_{12}+R_{23}+R_{31}}$$

The R b/w 1 and 2 in Y is —

$$R_{1-2} = R_1 + R_2$$

$$R_{2-3} = \frac{R_{23}(R_{12}+R_{31})}{R_{12}+R_{23}+R_{31}} \quad (\text{in } \Delta)$$

$$R_{2-3} = R_2 + R_3 \quad (\text{in } Y)$$

Also,

$$R_{3-1} = \frac{R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}} \quad (\text{in } \Delta)$$

$$R_{3-1} = R_3 + R_1 \quad (\text{in } Y).$$

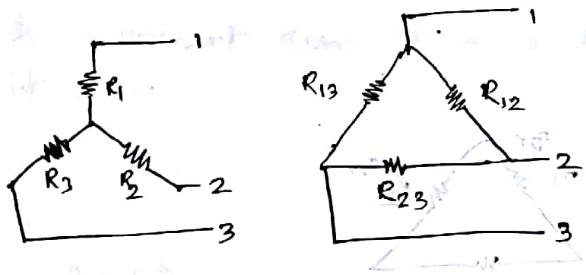
$$R_3 - R_1 = \frac{R_{23}(R_{12}+R_{31})}{R_{12}+R_{23}+R_{31}} + \frac{R_{12}(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}}$$

$$2R_1 = \frac{R_{31}.\cancel{R_{12}} + R_{23}.\cancel{R_{31}} - R_{23}R_{12} - \cancel{R_{23}R_{31}} - R_{12}R_{23} - \cancel{R_{12}}R_{31}}{R_{12}+R_{23}+R_{31}}$$

$$\cancel{R_1 = \frac{R_{23} \times R_{12}}{R_{12}+R_{23}+R_{31}}} \qquad R_1 =$$

$$R_1 = \frac{R_{12} \times R_{31}}{R_{12}+R_{23}+R_{31}} \; ; \quad R_2 = \frac{R_{23} \times R_{12}}{R_{12}+R_{23}+R_{31}} \; ; \quad R_3 = \frac{R_{31} \times R_{23}}{R_{12}+R_{23}+R_{31}}$$

STAR TO DELTA CONVERSION :



We know that, from $\Delta \rightarrow Y$

$$R_1 = \frac{R_{12} R_{31}}{R_{12}+R_{23}+R_{31}} \quad \text{——— (1)}$$

$$R_2 = \frac{R_{12}.R_{23}}{R_{12}+R_{23}+R_{31}} \quad \text{——— (2)}$$

$$R_3 = \frac{R_{23}.R_{31}}{R_{12}+R_{23}+R_{31}} \quad \text{——— (3)}$$

Multiplying eqn (1) and (2), (2) and (3), (3) and (1), we get

$$R_1 R_2 = \frac{R_{12}^2.R_{23}.R_{31}}{(R_{12}+R_{23}+R_{31})^2}$$

$$R_2 R_3 = \frac{R_{23}^2.R_{12}.R_{31}}{(R_{12}+R_{23}+R_{31})^2}$$

$$R_3 R_1 = \frac{R_{31}^2.R_{12}.R_{23}}{(R_{12} \ldots}$$

Adding the three eqns -

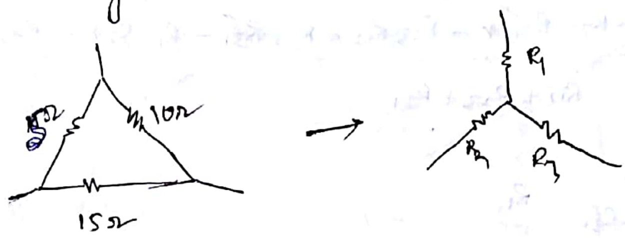
$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}R_{23}R_{31}}{(R_{12}+R_{23}+R_{31})}$$

$$R_{23} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$R_{12} = \frac{\Sigma R_iR_j}{R_3} = R_1 + R_2 + \frac{R_1.R_2}{R_3}$$

$$R_{31} = \frac{\Sigma R_iR_j}{R_2} = R_3 + R_1 + \frac{R_1.R_3}{R_2}$$

Q. Convert the given Delta into star.

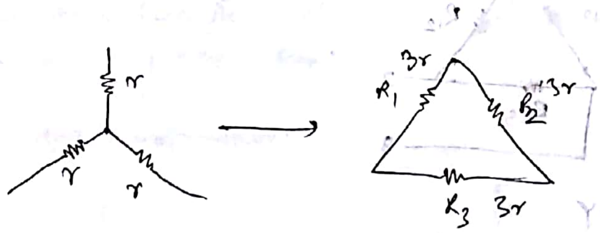


$R_1 = \cancel{\frac{150}{40}}\ \cancel{+\ 3.75\ \Omega}\ \frac{50}{30} = 1.67\,\Omega$

$\cancel{R_2 = \frac{225}{40} = 7.5}$ $\quad R_2 = \frac{75}{30} = 2.5\,\Omega$

$R_3 = \frac{150}{30} = 5\,\Omega$

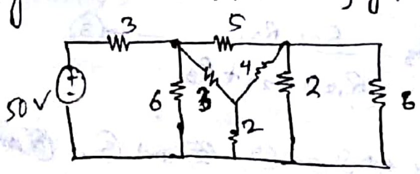
Q/ Convert star into delta.



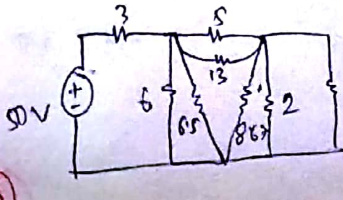
Soln

$$R_1 = r + r + \frac{r^2}{r} = 3r = R_2 = R_3$$

Q/ Determine the current drawn by the circuit shown in fig.



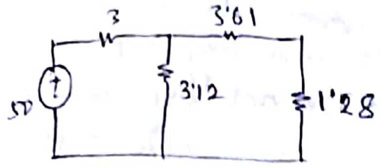
Soln



$R_1 = 7 + \frac{12}{2} = 13\,\Omega$

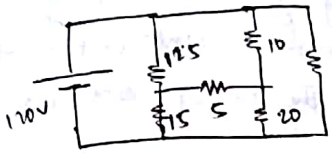
$R_2 = 5 + \frac{6}{4} = 6.5\,\Omega$

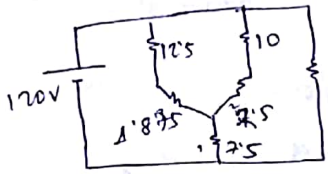
$R_3 = 6 + \frac{8}{3} = 8.67\,\Omega$

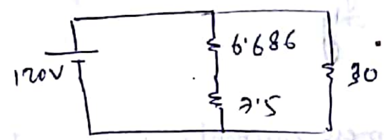


I = 10'2 A.

Q. Obtain the ~~three~~ current shown in the circuit.







$R_1 = \frac{300}{40} = 7.5\,\Omega$

$R_2 = \frac{100}{40} = 2.5\,\Omega$

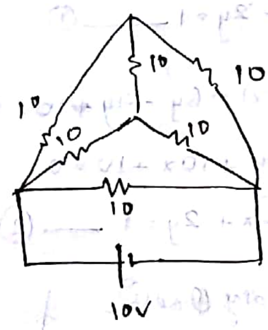
$R_3 = \frac{75}{40} = 1.875\,\Omega$.

$R_{eq} = 9.63\,\Omega$.

$\therefore I = \frac{120}{9.63}$

$= 12.46A$

Q. Find the current drawn from a battery of 10 V connected to the circuit shown.



I = 2A.

→ Branch :

A part of the network which connects various points on the network with one another.

→ Junction :

~~The~~ A point where 3 or more branches meet is called a Junction point.

→ Node :

A point at which 2 or more elements are joined together is called Node.

→ Junction points are also the nodes of the network.

Loop:

Set of branches forming a closed path in a network in such a way that if one branch is removed then remaining branches do not form a closed path.

(OR)

Loop is a closed path with no node passed more than once.

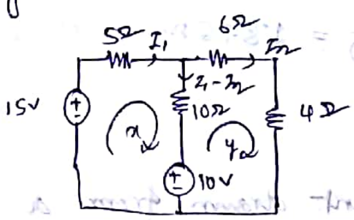
Q

- Mesh Analysis provides general procedure for analysing circuit using mesh currents as the circuit variables. Using mesh current instead of element currents as circuit variables reduces the no. of eqns that must be solved simultaneously.
- Mesh analysis uses KVL to find unknowns and is applicable to planar circuits only.

Procedure:

① Assign mesh currents $I_1, I_2, \dots, I_n$ to the 'n' meshes.
② Apply KVL to each of the 'n' meshes. Use Ohm's Law to express the voltages in terms of the mesh current
③ Solve the resulting 'n' simultaneous equations to get the mesh currents.

Q Find the branch currents using Mesh Analysis.



$$15 - 5x - 10(x-y) - 10 = 0$$

$$5 = 15x - 10y$$

$$3x - 2y = 1 \quad \text{——— ①}$$

$$-10(y-x) - 6y - 4y + 10 = 0$$

$$\Rightarrow -20y + 10x + 10 = 0$$

$$\Rightarrow -x + 2y = 1 \quad \text{——— ②}.$$

Adding ① and ②,

$$2x = 2$$

$$x = 1A$$

$$-1 + 2y = 1.$$

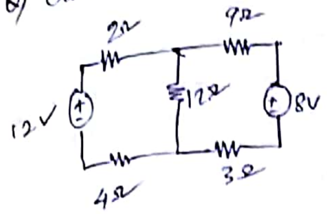
$$2y = 2$$

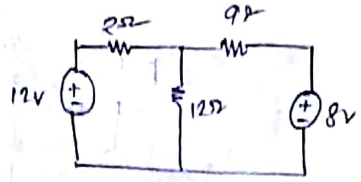
$$y = 1A.$$

$I_1 = 1A \qquad I_2 = 1A \qquad I_1 - I_2 = 0$

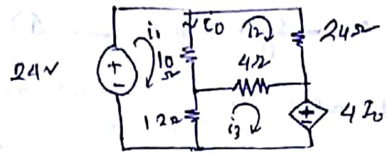
HW

Q/ Calculate Mesh currents i_1 and i_2 in the circuit shown below.





Q/ Use Mesh analysis to find the currents i_0 in the circuit.



Applying KVL to mesh 1,

$$24 - 10(i_1 - i_2) - 12(i_1 - i_3) = 0$$

$$22i_1 - 10i_2 - 12i_3 = 24 \quad \text{—— (1)}$$

Mesh 2,

$$38i_2 - 10i_1 - 4i_3 = 0 \quad \text{—— (2)}$$

Mesh 3,

$$\boxed{i_0 = i_1 - i_2}$$

$$16i_3 - 12i_1 - 4i_2 = 4I_0 \quad \text{—— (3)}$$

$$16i_3 - 16i_1 = 0$$

$$i_1 = i_3 \quad \text{—— (3)}$$

20

(1) ⇒ $10i_1 - 10i_2 = 24$

(2) ⇒ $-14i_1 + 38i_2 = 0.$

$$+14i_1 = 38i_2$$

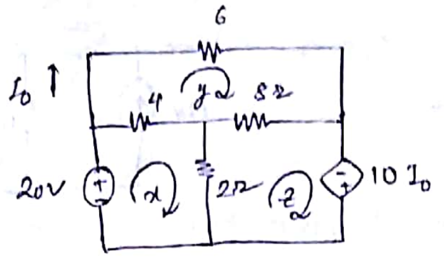
$$i_2 = \frac{14}{38} i_1 = \frac{7}{19} i_1.$$

$$10i_1 - \frac{10 \times 7}{19} i_1 = 24$$

$$\frac{190 - 70}{19} i_1 = 24$$

$$i_1 = \frac{\overset{6}{\cancel{24}} \times 19}{\underset{\cancel{30}\,5}{\cancel{120}}} =$$

Q. Using Mesh analysis, find i_o in the circuit.



Sol.

$20 - 6x + 4y + 2z = 0$.

$3x - 2y - z = 10$ —— ①

$18y - 4x - 8z = 0$

$2x - 9y + 4z = 0$ —— ②

$-2(z-x) - 8(z-y) + 10I_o = 0$

$-2z + 2x - 8z + 10I_o + 8y = 0$ ($I_o = y$)

$2x + 18y - 10z = 0$

$x + 9y - 5z = 0$ —— ③

②+③ ⇒ $3x - z = 0$

$x = \frac{z}{3}$

① ⇒ $27x - 18y - 9z = 90$

② ⇒ $4x - 18y + 8z = 0$

(−)

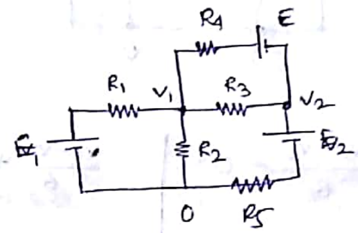
$23x - 17z = 90$

$23x - 51x = 90$

$-28x = 90$

NODAL ANALYSIS:

- Provides a general procedure for analysing circuits using node voltages as the circuit variable.
- Chosing node voltages instead of element voltages as circuit variables reduce the no. of eq^ns that is solved simultaneously.



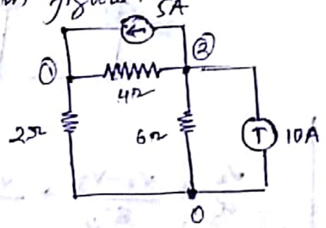
Procedure

1/ Select a node as the reference node.
2/ Assign voltages $V_1, V_2, \ldots V_{n-1}$, n = no. of nodes, to the remaining (n-1) nodes.
3/ The voltages are referenced wrt the reference node.
4/ Apply KCL to the each of (n-1) nodes [non-reference]
5/ Use Ohm's Law to express the branch currents in terms of ~~load~~ node voltages.
6/ Solve the resulting simultaneous equations to obtain the unknown node voltage.

$$\frac{V_1 - E_1}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_2}{R_3} + \frac{V_1 - E - V_2}{R_4} = 0$$

$$\frac{V_2 - V_1}{R_3} + \frac{V_2 - E_2}{R_5} + \frac{V_2 + E - V_1}{R_4} = 0$$

Q/ Calculate the voltages shown in figure.



②

$$\frac{V_2}{6} + \frac{V_2 - V_1}{4} + 5 - 10 = 0$$

$$\frac{V_2}{6} + \frac{V_2 - V_1}{4} = 5$$

$$2V_2 + 3V_2 - 3V_1 = 60$$

$$\Rightarrow 5V_2 - 3V_1 = 60 \quad \text{—— (I)}$$

$$\text{(I)} + \text{(II)} \Rightarrow 4V_2 = 80$$

$$V_2 = 20\,V$$

①

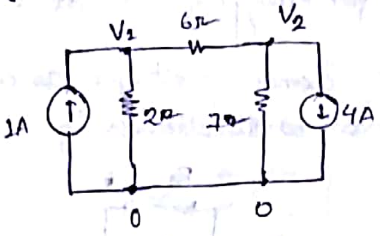
$$\frac{V_1}{2} + \frac{V_1 - V_2}{4} - 5 = 0$$

$$2V_1 + V_1 - V_2 = 20$$

$$3V_1 - V_2 = 20 \quad \text{—— (II)}$$

$$V_1 = \frac{40}{3}\,V$$

Q) Obtain the node voltages in the circuit.



Sol.

$$\frac{V_1}{2} - 1 + \frac{V_1 - V_2}{6} = 0$$

$$\frac{4V_1 - V_2}{6} = 1$$

$4V_1 - V_2 = 6$ —(I)

$52V_1 - 13V_2 = 78.$

$$\frac{V_2 - V_1}{6} + \frac{V_2}{7} + 4 = 0.$$

$7V_2 - 7V_1 + 6V_2 = -168.$

$13V_2 - 7V_1 = -168.$

$7V_1 - 13V_2 = 168$ —(II)

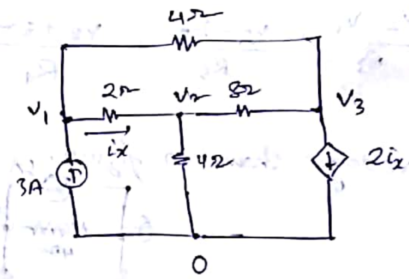
(II) − (I) ⇒ $-45V_1 = 90$

$V_1 = -2V$

$V_2 = -8 - 6$

$V_2 = -14V$

Q)



$$i_x = \frac{V_1 - V_2}{2}$$

$$\frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = 3$$

$2V_1 - 2V_2 + V_1 - V_3 = 12$

$3V_1 - 2V_2 - V_3 = 12$

—(I)

$$\frac{V_2}{\cancel{4}_2} + \frac{V_2 - V_3}{\cancel{8}_4} + \frac{V_2 - V_1}{\cancel{2}_1} = 0$$

$2V_2 + V_2 - V_3 + 4V_2 - 4V_1 = 0$

$7V_2 - 4V_1 - V_3 = 0$

—(II)

$$\frac{V_3 - V_2}{8} + \frac{V_3 - V_1}{4} + V_1 - V_2 = 0$$

⇒ $V_3 - V_2 + 2V_3 - 2V_1 + 8V_1 - 8V_2 = 0$

⇒ $6V_1 - 9V_2 + 3V_3 = 0$

⇒ $2V_1 - 3V_2 + V_3 = 0$ —(III)

(I) + (III) ⇒ $5V_1 - 5V_2 = 12$

$V_1 - V_2 = \frac{12}{5}$

$V_2 = \frac{12}{5}V$

(II) + (III)

⇒ $-2V_1 + 4V_2 = 0$

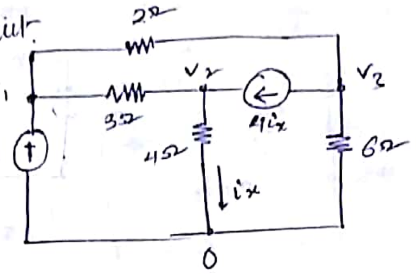
⇒ $V_1 = 2V_2$

$V_1 = \frac{24}{5}V$

$V_3 = \frac{72}{5} - \frac{24}{5} - 12 = \frac{48}{5} - 12$

$= \frac{-12}{5}V$

Q) Obtain the node voltages of given circuit.



Soln

$$\frac{V_1-V_2}{3} + \frac{V_1-V_3}{2} = 10.$$

$$2V_1 - 2V_2 + 3V_1 - 3V_3 = 60$$

$$5V_1 - 2V_2 - 3V_3 = 60 \quad — ①$$

$$\Rightarrow 20V_1 - 8V_2 - 12V_3 = 240$$

$$\frac{V_2-V_1}{3} + \frac{V_2}{4} - V_2 = 0$$

$$\frac{V_2-V_1}{3} - \frac{3V_2}{4} = 0$$

$$-5V_2 - 4V_1 = 0$$

$$4V_1 + 5V_2 = 0$$

$$V_1 = -\frac{5}{4}V_2$$

$$\frac{V_3}{6} + \frac{V_3-V_1}{2} + V_2 = 0$$

$$V_3 + 3V_3 - 3V_1 + 6V_2 = 0$$

$$-3V_1 + 6V_2 + 4V_3 = 0$$

$$\Rightarrow -9V_1 + 18V_2 + 12V_3 = 0$$

$$\Rightarrow 11V_1 + 10V_2 = 240$$

$$\Rightarrow -\frac{55}{4}V_2 + 10V_2 = 240$$

$$\Rightarrow V_2\left(10 - \frac{55}{4}\right) = 240$$

$$\Rightarrow V_2\left(-\frac{\cancel{15}^{3}}{4}\right) = \cancel{240}^{48}$$

$$\Rightarrow V_2 = -\frac{4\times 48}{3}$$

$$V_2 = \frac{-\cancel{192}^{64}}{\cancel{3}} V$$

$$V_2 = -64V$$

$$V_1 = -\frac{5}{\cancel{4}} \times \left(-\frac{\cancel{192}^{48}}{3}\right)$$

$$= \frac{\cancel{240}^{80}}{\cancel{3}} V.$$

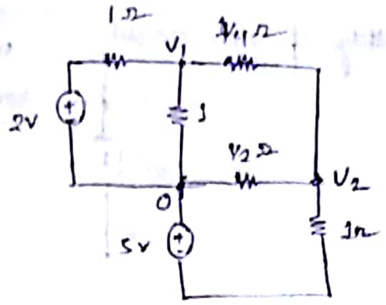
$$400 + 128 - 12V_3 = 240.$$

$$\Rightarrow 12V_3 = 288$$

$$\Rightarrow V_3 = \frac{\cancel{288}^{48}}{\cancel{12}_{2}} = 24V.$$

$V_3 = 15.6$

Q/



$v_1 - 2 + 4(v_1 - v_2) + v_1 = 0$

$2v_1 + 4v_1 - 4v_2 = 2$

$3v_1 - 2v_2 = 1$ — ①

$12v_1 - 8v_2 = 4$

$4(v_2 - v_1) + v_2 + 5 + 2v_2 = 0$

$7v_2 - 4v_1 + 5 = 0$

$21v_2 - 12v_1 + 15 = 0$

$13v_2 + 15 = 0$

$v_2 = -\frac{15}{13}\,V$

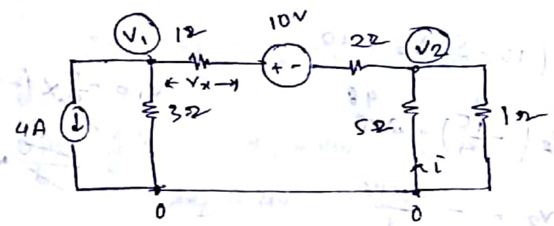
$v_1 = \frac{1 + 2v_2}{3}$

$= \frac{1 - 30/13}{3}$

$= -\frac{17}{39}\,V.$

$i_{2V} = \frac{v_1 - 2}{1} = -\frac{17}{39} - 2 =$

Q/ i and v_x ?



$\frac{v_1}{3} + 4 + \frac{v_1 - 10 - v_2}{3} = 0$

$\frac{2v_1 - v_2}{3} - \frac{10}{3} + 4 = 0$

$\Rightarrow 2v_1 - v_2 = (-2/\cancel{3})\cancel{3}$

$\Rightarrow 2v_1 - v_2 = -2$ — ①

$46v_1 - 23v_2 = -46$

$\frac{v_2}{5} + v_2 + \frac{v_2 + 10 - v_1}{3} = 0$

$3v_2 + 15v_2 + 5v_2 + 50 - 5v_1 = 0$

$23v_2 - 5v_1 = -50.$

$\Rightarrow 41v_1 = -96$

$\Rightarrow v_1 = -\frac{96}{41} = -2.3\,V.$

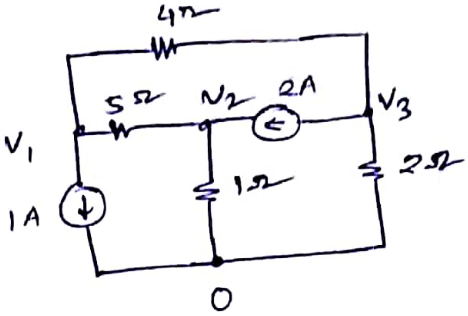
$v_2 = 2v_1 + 2.$

$v_2 = -4.6 + 2$

$v_2 = -2.6\,V$

$i = 0.537\,A$

Q/ Find the current through 5Ω resistor and voltage drop.



$V_1 = -4V$
$V_2 = 1V.$
$i(5Ω) = -1A.$

Sol

$$\frac{V_1 - V_2}{5} + 1 + \frac{V_1 - V_3}{4} = 0$$

$4V_1 - 4V_2 + 20 + 5V_1 -$

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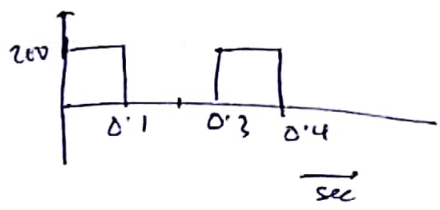
Q/ Compute the effective value of the square voltage wave shown.



Soln

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2 dt$$

$$V_{av} = \frac{1}{T}\int_0^T v\,dt$$

$$V_{rms}^2 = \frac{1}{0.3}\int_0^{0.3} (200)^2 \times dt$$

$$= \frac{1}{\cancel{0.3}_{3}} \times (200)^2 \times (\cancel{0.1})$$

$$V_{rms}^2 = \frac{200^2}{3}$$

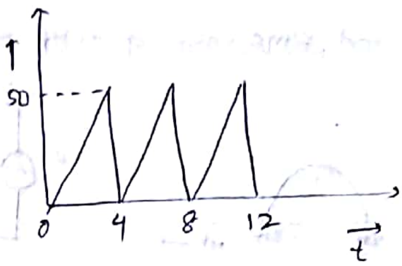
$$V_{rms} = \frac{200}{\sqrt{3}}\ V$$

$$I_{avg} = \frac{1}{4}\int_0^4 \left(\frac{50}{4}\right) t \, dt$$

$$= \frac{50}{16}\left[\frac{t^2}{2}\right]_0^4$$

$$= \frac{50}{\cancel{16}} \times \frac{\cancel{16}}{2}$$

$$= 25 \text{ A.}$$



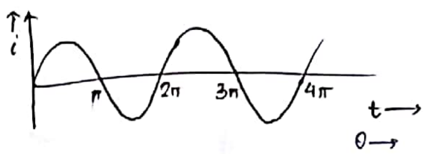
$$I_{rms}^2 = \frac{1}{4}\int_0^4 \left(\frac{50}{4}\right)^2 t^2 \, dt$$

$$= \frac{50^2}{64 \times 3} \cdot \left[t^3\right]_0^4$$

$$I_{rms} = \frac{50}{\sqrt{3}} = 28.86 A.$$

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}} = \frac{28.86}{25} = 1.15$$

Average and RMS value of sinusoidal waveform:



$$i = I_{max} \sin\omega t$$

$$I_{avg} = \frac{1}{T}\int_0^T i \, d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} (I_m \sin\theta) \, d\theta \quad \{\text{considering half cycle}\}$$

$$= \frac{I_m}{\pi} \left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{I_m}{\pi}\left[1 - (-1)\right].$$

$$\boxed{I_{avg} = \frac{2I_m}{\pi}}$$

For sinusoidal or any symmetrical waveform, the averaging is done over one-half cycle instead of a full cycle.

$$i_{rms}^2 = \frac{1}{T}\int_0^T i^2 \, d\theta$$

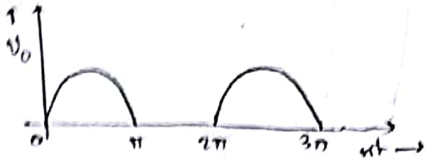
$$= \frac{1}{\pi}\int_0^{\pi} I_{max}^2 \sin^2\theta \, d\theta$$

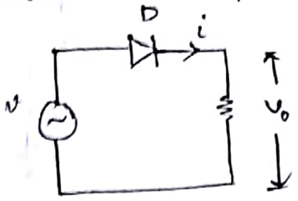
$$= \frac{I_{max}^2}{\pi}\int_0^{\pi}\left[\frac{1 - \cos 2\theta}{2}\right] d\theta = \frac{I_m^2}{2\pi}\left[\pi - 0\right] = \frac{I_m^2}{2}$$

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$

$$\text{Form factor} = \frac{I_{rms}}{I_{avg}} = 1.11$$

#) Average and RMS value of Half wave rectifier:-





$$V_{avg} = \frac{1}{T}\int_0^T v\, d\theta$$

$$= \frac{1}{T}\int_0^T (V_m \sin\theta)\, d\theta$$

$$= \frac{1}{2\pi}\int_0^\pi (V_m \sin\theta)\, d\theta$$

$$= \frac{V_m}{2\pi}\left[-\cos\theta\right]_0^\pi$$

$$= \frac{2V_m}{2\pi} = \frac{V_m}{\pi}$$

$$V_{rms} = \frac{1}{T}\int_0^T v^2 d\theta$$

$$= \frac{1}{2\pi}\int_0^\pi V_m^2 \sin^2\theta\, d\theta$$

$$= \frac{V_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{2}\right]_0^\pi$$

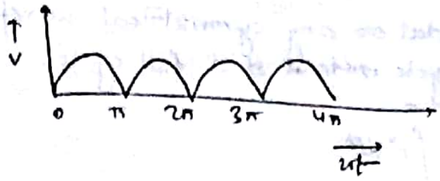
$$= \frac{V_m^2}{4\pi}\left[\cancel{\pi} - 0\right]$$

$$\boxed{V_{rms} = \frac{V_m}{2}}$$

$$\text{Form factor} = \frac{V_{rms}}{V_{avg}} = \frac{V_m/2}{\frac{V_m}{\pi}} = \frac{\pi}{2} = 1.57.$$

$$\text{Peak factor} = \frac{V_{max}}{V_{rms}} = \frac{V_{max} \times 2}{V_m} = 2.$$

#) Average and RMS value of a full wave rectifier:



$$V_{avg} = \frac{1}{T}\int_0^T v\, d\theta$$

$$= \frac{V_m}{2\pi}\int_0^{2\pi} \sin\theta\, d\theta$$

$$= \cancel{\frac{4V_m}{2\pi}}\ \cancel{\frac{2V_m}{\pi}} = \frac{2V_m}{\pi}.$$

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$$V_{rms} = \frac{1}{\pi}\int_0^{2\pi} I_m^2 \sin^2\theta \, d\theta$$

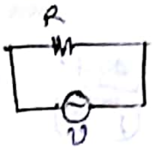
$$= \frac{V_m^2}{\pi} \cdot \frac{\pi}{2}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$$

F.F = 1.11

PF = $\sqrt{2}$.

#) AC through Pure Resistor:

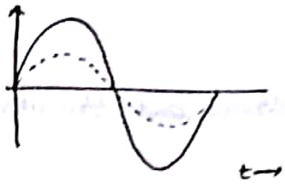


$i_R = v_m \sin\omega t$

$\Rightarrow R = \frac{V_m}{i}\sin\omega t$ — (i)

$v = V_m \sin\omega t$ — (ii)

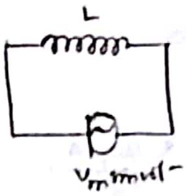
(i) and (ii) are in same phase.



$\phi = 0$

$\boxed{\cos\phi = 1}$

#) AC through pure inductor:



$V = V_m \sin\omega t$

$V = L\frac{di}{dt}$

$V_m \sin\omega t = L\frac{di}{dt}$

$\int V_m \sin\omega t \, dt = \int L \, di$

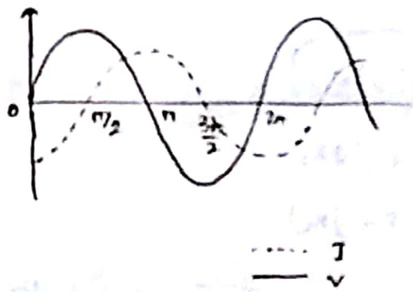
$\Rightarrow -V_m \left[\frac{\cos\omega t}{\omega}\right] = Li$

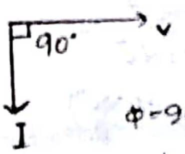
$\Rightarrow i = +\frac{V_m}{\omega L}[-\cos\omega t]$

$\Rightarrow i = \frac{V_m}{\omega L}\sin(\omega t - \pi/2)$

$\boxed{I_{max} = \frac{V_m}{\omega L}}$.

$\boxed{X_L = \omega L}$





$\phi = 90°$

$\cos\phi = 0$

$\boxed{P = 0}$

$P = Vi$

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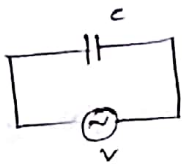
AC through pure Capacitor:



$Q = CV$

$Q = C \cdot V_m \sin\omega t$

$$\frac{dQ}{dt} = C \frac{d}{dt} V_m \sin\omega t$$

$$I = CV_m \omega \cos\omega t$$

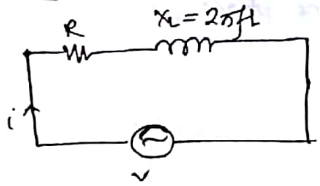
$$I_{max} = \frac{V_m}{X_c}$$

$$\boxed{X_c = \frac{1}{\omega C}}$$

$\boxed{\cos\phi = 0}$

$\boxed{P = 0}$

AC through Resistance and Inductance:

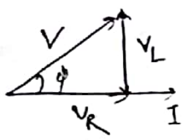


$V = V_R + V_L$

$V_R \rightarrow$ Inphase with I

$V_L \rightarrow$ Leads I by 90°.

Voltage triangle:



Voltage Δ.

$\phi < 90°$

$$V = V_R + jV_L$$

$V = IR + jIX_L$

$= I(R + jX_L)$.

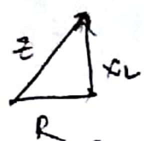
$V = IZ$

$$\boxed{Z = R + jX_L}$$

$$|Z| = \sqrt{R^2 + X_L^2}.$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right).$$

$$\cos\phi = \frac{R}{Z}$$



Impedance Δ

- The current, I lags the applied voltage. V by an angle $\phi < 90°$.

Resistance and capacitance:

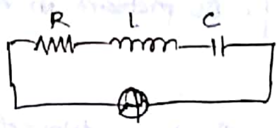
$V = IR - jIX_C$

$= I(R - jX_C)$.

$\therefore Z = R - jX_C$

$|Z| = \sqrt{R^2 + X_C^2}$.

AC through LCR:-



$V = (IR + IX_L + IX_C)$

Power factor (P.F.):

Power factor is the cos of the angle between V and I.

(OR)

It is the ratio of R to Z (or) Active Power to Apparent Power.

$\therefore \cos\phi = \frac{R}{Z}$.

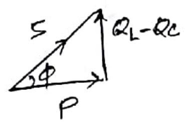
$\cos\phi = \frac{R}{Z} = \frac{P}{S}$.

P → Active power (W)

Q → Reactive power (VAR)

S → Apparent power (VA).

$Q = Q_L - Q_C$



Power factor lagging → Inductive.

Active Power (P):-

Power actually dissipated in the circuit resistance.

$P = I^2R$ (W). $P = VI\cos\phi$ → {Power Δ}

Reactive Power (Q):-

It is the power developed in the inductive reactance of the circuit.

$Q = I^2 X_L$ (VAR) $P = VI\sin\phi$ → {Power Δ}

Apparent Power (S):

It is the net power given by the product of rms value of applied voltage and resultant current.

$\therefore S = VI$ (VA).

$S = \sqrt{P^2 + Q^2}$.

Representation of Alternating quantities :

① Phase representation.
② Rectangular coordinate using j-rotation (complex form).
③ Trigonometric form representation $(a\cos\theta + ib\sin\theta)$.
④ Exponential form. $(e^{\pm i\theta})$
⑤ Polar form.

① Phasor Representation :

- Represent Alternating quantity instead of sine fn to achieve desired result.
- In fact vectors are a short-hand for the representation of Alternating quantity and their use greately signifies the problems in AC work.

② [#] Significance of Operator(j) :

A vector can be specified symbolically in terms of $E_1 = a_1 + jb_1$,

$j \longrightarrow$ an operator indicates that component (b_1) is $\perp$ to component(a_1), and that the two terms are not to be treated like terms in any algebraic expression.

- The vector written in this form is said to be complex form or rectangular coordinate form.
- j is used to indicate the counter-clockwise rotation of a vector through 90°
- $j = \sqrt{-1}$.

$$j = \sqrt{-1} = 90^\circ \text{ ccw}$$
$$j^2 = -1 = 180^\circ \text{ ccw}$$
$$j^3 = -j = 270^\circ \text{ ccw}$$
$$j^4 = 1 = 360^\circ \text{ ccw}.$$

③ Trigonometric form of representation:

- An Alternating qty can be represented in Trigonometric form as

$$E = E\cos\theta \pm jE\sin\theta.$$

④ Exponential form:

- $e^{\pm j\theta} = \cos\theta \pm j\sin\theta$

$$E = E_0 e^{i\theta}$$

⑤ Polar form:

An alt. qty can be represented in polar form as -

$$E = |E| \angle \pm\theta.$$

Q) An AC of freq. 60 Hz has a maximum value of 120 A.
ⓐ write down eqns for the instantaneous value.
ⓑ Instant. value after $\frac{1}{360}$ sec.
ⓒ Time taken to reach 96 A for the first time.

$I_{max} = 120A$.

$f = 60Hz$.

$T = \frac{1}{60} = 0.016$.

$i = i_{max} \sin\omega t = 120 \sin(0.016\, t)$. $\times 2\pi \times$

$\boxed{i = 120 \sin(120\pi t)}$.

At $t = \frac{1}{360}$ sec,

$i = 120 \sin\left(\frac{120\pi}{360\,3}\right)$

$= \frac{120\sqrt{3}}{2} = 60\sqrt{3}$.

$= 103.92 A$.

$\frac{96}{120} = \sin(120\pi t)$.

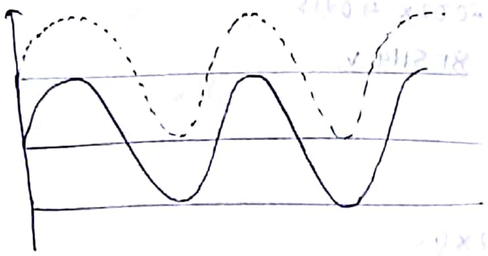
$t = 0.0246$ sec

Q/ A resultant current wave is made up of 2 components, a 5A DC and a 50 Hz AC component, which is of sinusoidal waveform which has a maximum value of 5A.

① Draw a sketch of resultant wave.
② Instantaneous value of combined wave.
③ Average value of Resultant current.
④ RMS value of the resultant current over a cycle.

Soln

①



② $i = 5(1 + \sin\omega t)$,

③ Over one complete cycle, the average value of the AC is ZERO. Hence, the average value of resulting wave = DC value = $I_{av} \times 5A$.

④ $I_{RMS} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} 25(1+\sin\theta)^2 d\theta}$

$= \sqrt{\frac{25}{2\pi} \int_0^{2\pi} 1 + \sin^2\theta + 2\sin\theta \, d\theta}$

$= \sqrt{\frac{25}{2\pi} \left[\theta + \frac{\theta}{2} - \frac{\sin 2\theta}{4} + 2(-\cos\theta)\right]_0^{2\pi}}$

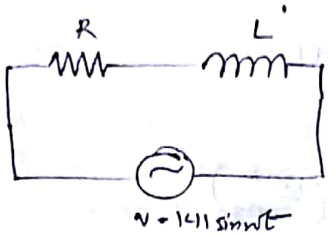
$= \sqrt{\frac{25}{2\pi} \left[\frac{3\cancel{2\pi}}{\cancel{2}}\; 0 + \cancel{2} - 0 + 0 - 0 + \cancel{2}\right]}$

$= \sqrt{\frac{25}{2} \times 3} = \sqrt{\frac{75}{2}} = \sqrt{37.5} = 6.12$.

$I_{rms} = \sqrt{I_{dc}^2 + \left(\frac{I_{max}}{\sqrt{2}}\right)^2}$

Q/ 60 Hz, $v = 141 \sin\omega t$, $R = 3\Omega$, $L = 0.0106$ H, find :
① rms value of current and Power factor angle.
② Exp. for I_{inst}.
③ V_R and V_L.
④ Average Power dissipated.
⑤ Pf of the circuit.

Soln



① $v = 141 \sin\omega t$

$V_{rms} = \frac{V_m}{\sqrt{2}} = 100$ V.

$I_{rms} = \frac{V_{rms}}{Z}$

$|Z| = \sqrt{3^2 + (0.0106)^2 \times (2\pi)^2 \times (60)^2}$

$|Z| = 4.99$

$I_{rms} = 20.02$ A.

Power factor.

$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

$\phi = \tan^{-1}\left(\frac{4.0715}{3}\right)$

$\phi = 53.61°$

② $i_{inst} = I_{max} \sin(\omega t - \phi)$.

$i = 28.31 . \sin(\omega t - 53.61)$.

③ $V_R = I_{rms} R$
$= 20.02 \times 3$
$= 60.06$ V

$V_L = I_{rms} (X_L)$
$= 20.02 \times 4.0715$
$= 81.5114$ V.

④ Avg. Power dissipated $= V_{rms} I_{rms} \cos\phi$
$= 100 \times 20.02 \times 0.6$
$= 1201.2$ W.

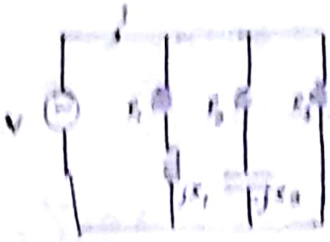
⑤ $\cos\phi = 0.6$ (lagging).

HW

Q/ In a given (RL) circuit, $R = 3.5\Omega$, $L = 0.1$ H, find :
① I_{rms}
② $\cos\phi$ at 50 Hz.
③ $V = 220\angle 30°$ V

Soln

Parallel Circuits :



$Z_1 = R_1 + jX_1$

$Z_2 = R_2 - jX_2$

$Z_3 = R_3$

$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$

$= \frac{1}{R_1 + jX_1} + \frac{1}{R_2 - jX_2} + \frac{1}{R_3}$

$\frac{1}{Z_{eq}} = \frac{(R_2 - jX_2) R_3 + (R_1 + jX_1) R_3 + (R_2 - jX_2)(R_1 + jX_1)}{(R_1 + jX_1)(R_2 - jX_2) R_3}$

G → conductance

B → Susceptance

B_L → Inductive Susceptance

B_C → Capacitive Susceptance

$I = \frac{V}{Z_{eq}}$

$\boxed{I = YV}$

If $Z_{eq} = R + jX$

$Y = G - jB$

$\boxed{G = \frac{R}{\sqrt{R^2 + X^2}}}$

$\boxed{B = \frac{X}{\sqrt{R^2 + X^2}}}$

Y → Admittance

Unit : mho (℧).

$Z = R \pm jX$

$Y = G \mp jB$

Admittance Method :

$I_1 = Y_1 V$

$I_2 = Y_2 V$

$I_3 = Y_3 V$.

$Y_{eq} = Y_1 + Y_2 + Y_3$

$Y_1 = \frac{1}{Z_1}$; $Y_2 = \frac{1}{Z_2}$; $Y_3 = \frac{1}{Z_3}$.

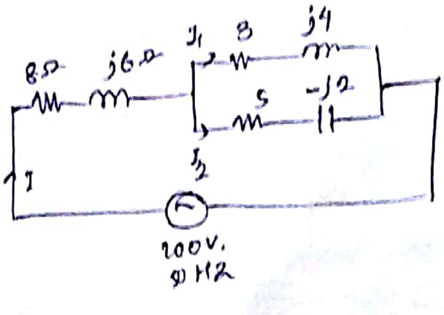
$\boxed{I = Y_{eq} \cdot V}$

Q/ Determine the following in the circuit shown :

(i) The current phasors, I, I_1 and I_2.

(ii) Active Power dissipated in the 3 resistive branches.

(iii) Power factor of the circuit.



Sol (i)

$Z_1 = 3 + 4j$

$Z_2 = 5 - 2j$.

$Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(5-2j)(3+4j)}{8+2j}$

$= \frac{(15+8)+14j}{8+2j} = \frac{(23+14j)(8-2j)}{64+4} = \frac{(184+28)+66j}{68}$

$= \frac{212+66j}{68} = 3.12 + 0.97j$.

$Z_{eq} = (8 + 3.12) + 6.97j$

$= 11.12 + 6.97j$

$|Z_{eq}| = 13.12 \angle 32^\circ$

$I = \cancel{\frac{200}{13.12} = 15.24 A}\ \frac{200 \angle 0^\circ}{13.12 \angle 32^\circ}$

$I = 15.24 \angle -32^\circ$

~~$I_1 = \frac{Y}{Z_1} = 200$~~

$I_1 = \left(\frac{Z_3}{Z_2 + Z_3}\right) . I$

$= (15.24 \angle -32^\circ) \times \left(\frac{5 - 2j}{5 - 2j + 3 + 4j}\right)$.

$= (15.24 \angle -32^\circ) \times \left(\frac{5 - 2j}{8 + 2j}\right)$

$= (15.24 \angle -32^\circ) \times (0.529 - 0.382j)$.

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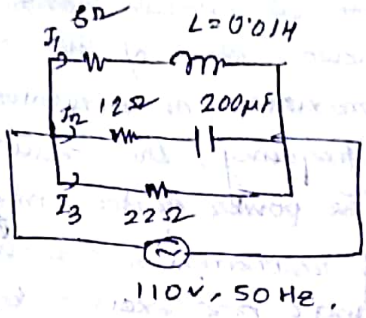
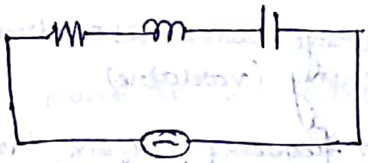
Q/ For the circuit shown, calculate :
i) The current in each branch.
ii) Total current
iii) Total Power consumed by the circuit.
iv) Reactive Power consumed/ generated.
v) Power factor of the circuit.
vi) Phasor diagram of the circuit.



Solo

Reactive Power :- -ve (generated)
+ve (absorbed).

Q/ A coil of resistance 10Ω and inductance of 0.1H is connected in series with the condenser of 150μF across a 200V, 50Hz supply. Determine: ① Impedance
② Current
③ Power factor
④ Voltage across the coil (V_L)
⑤ Voltage across the condenser (V_C).



Electrical Resonance

- Resonance in electrical circuits consisting of passive and active elements represent a particular state of the circuit when the current or voltage in the circuit is maximum or minimum, wrt the magnitude of excitation at a particular frequency, the circuit impedence being either minimum or maximum at the power factor unity.

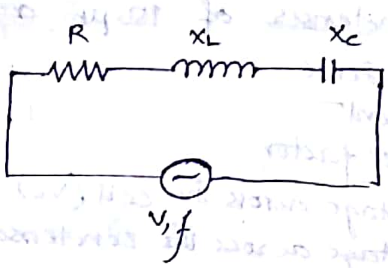
The phenomenon of resonance is observed in both series and parallel AC circuits comprising R, L and C and excited by an AC source.

(OR)

- The circuit is said to be in Resonance when the applied voltage and the resultant current are in phase in a RLC circuit.

#] SERIES RESONANCE

$I = \frac{V}{Z}$



$Z = R + j(X_L - X_c)$.

$I = \frac{V}{R + j(X_L - X_c)}$

$V \rightarrow$ Voltage across RLC circuit (constant)
$f \rightarrow$ frequency (variable)

- At certain frequency, $X_L = X_c$ in magnitude.
  Net Reactance, $\boxed{Z = R}$
  Hence, $\boxed{I = \frac{V}{R}}$ ..... condition for maximum current

- At resonance, V_L and V_c are maximum and they cancel each other.

- Resonant frequency (f_0)

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

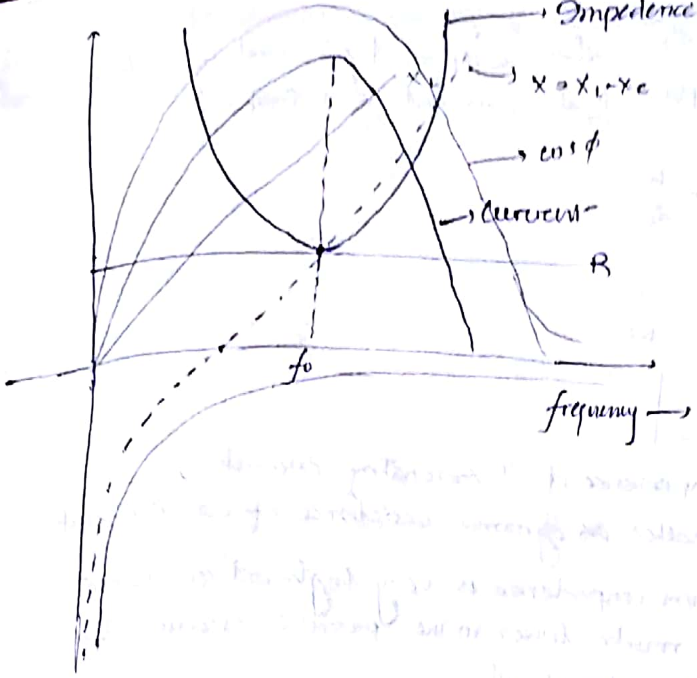
(*) Under series resonance, the circuit is known as acceptor circuit. The resonance is known as voltage resonance or voltage magnification.

Properties of series RLC circuit under Resonance:

- Applied voltage, V and the resultant current are in phase which means the power factor is unity. Net reactance = 0.
- The current is maximum, hence power dissipated is maximum.

$$P = \frac{V^2}{R} = I^2R.$$

- Net reactance is 0, $X_L - X_c = 0$.
- ~~Net~~ Impedence = minimum.

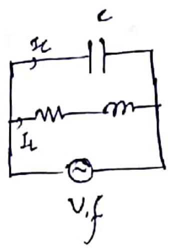


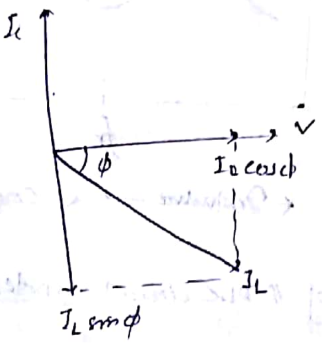
$f < f_0$ → circuit is capacitive

$f > f_0$ → circuit is inductive

→ Parallel Resonance :-

At, resonance capacitive current must be equal to the inductive part of the coil current i.e. the imaginary part of ~~the~~ I_L and I_C must cancel each other. Therefore,





$\cos\phi = \frac{R}{Z_L}$.

$Z_L = R + jX_L$

$I = I_C + I_L$

$I_C = I_L \sin\phi$

$I_C = I_L\left(\frac{X_L}{Z_L}\right)$.

$\frac{V}{X_C} = \frac{V}{Z_L}\cdot\left(\frac{X_L}{Z_L}\right)$.

$\Rightarrow Z_L^2 = X_C \cdot X_L$.

$\Rightarrow Z_L^2 = \frac{L}{C}$

$\Rightarrow Z_L = \sqrt{\frac{L}{C}}$

$\sqrt{R^2 + (\omega L)^2} = \sqrt{\frac{L}{C}}$

$\sqrt{R^2 + (2\pi f_0 L)^2} = \sqrt{\frac{L}{C}}$

$R^2 + (2\pi f_0 L)^2 = \frac{L}{C}$.

$4\pi^2 f_0^2 L^2 = \frac{L}{C} - R^2$.

$\Rightarrow f_0^2 = \frac{L - CR^2}{4\pi^2 L^2 C}$.

$\boxed{\Rightarrow f_0 = \frac{\sqrt{L - CR^2}}{2\pi L\sqrt{C}} = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2}}$

$\boxed{\text{Special case}}$

If $R = 0$,

$\boxed{\omega = \frac{1}{\sqrt{LC}}}$

Again at resonance, since the reactive component of I_L and I_C balances each other, the only remaining part of the current is $I_L \cos\phi = I$

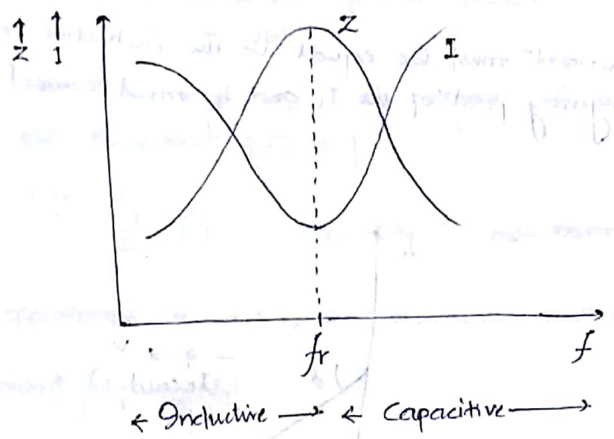
$$\frac{V}{Z} = \frac{V}{Z_L} \times \frac{R}{Z_L}$$

$$Z = \frac{Z_L^2}{R} = \frac{L}{RC}$$

$$\boxed{Z = \frac{L}{RC}}$$

Z is the equivalent impedence of // resonating circuit, This impedence is also called as dynamic resistance of the // circuit. Normally, R being less, this impedence is very high at resonance and thus the current is much lower in the parallel circuit. Then the circuit is also called Rejector circuit.

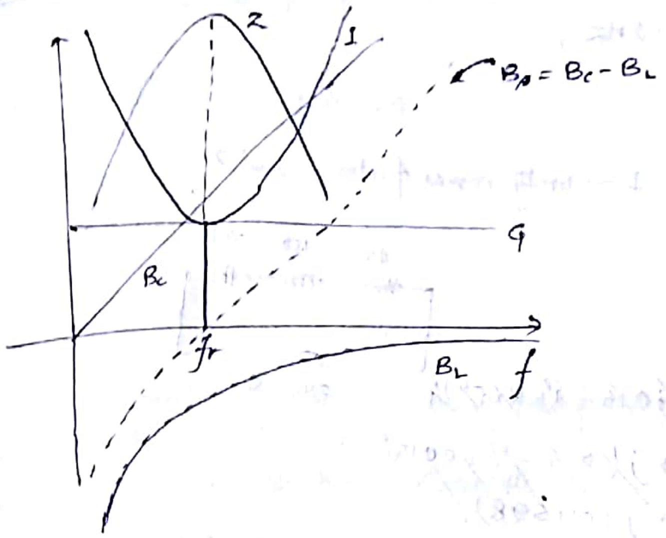
$$I = \frac{V}{Z} = \frac{VRC}{L} = \left(\frac{RC}{L}\right)V.$$

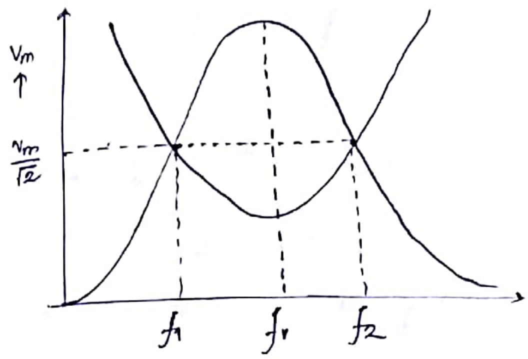


Properties of // RLC circuit under Resonance :

1. Power factor is unity.
2. Current at Resonance is in phase with the applied voltage and the value of current is minimum.
3. Net impedence is maximum, $Z = \frac{L}{RC}$.
4. The Admittance is minimum, $Y = 1/Z$,
5. Net Susceptance is ZERO, $B = B_C - B_L = 0$
6. The resonating frequency of the circuit, $f_r = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2}$
7. If $f < f_r$, the circuit is inductive and the power factor is lagging.
8. If $f > f_r$, the circuit is capacitive and the power factor is leading.

#] GRAPHICAL REPRESENTATION OF // RESONANCE :





$f_1, f_2 \rightarrow$ half power frequencies.

Bandwidth $= f_2 - f_1$.

$$f_1 = f_r - \frac{R}{4\pi L}$$

$$f_2 = f_r + \frac{R}{4\pi L}.$$

#] Q-FACTOR :

The Quality factor is defined as the ratio of voltage across the inductor or a capacitor to the applied voltage (RLC Series circuit) (or) Voltage magnification.

(OR)

It is defined as the ratio of current flowing through the capacitor to the total current under resonance condition i.e., current magnification (RLC // circuit).

Series

$$Q = \frac{V_L}{V} = \frac{V_C}{V} = \frac{I_c}{I}.$$

$$Q = \frac{IX_L}{IR} = \frac{X_L}{R} \cdot \left(Z = R \text{ in series} \right). = \frac{\omega L}{R}.$$

Parallel

$$Q = \frac{\omega_r L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

Also, $\boxed{Q = \frac{f_r}{B_W}}$

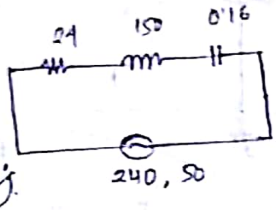
Q. $R = 24\Omega$, $C = 150\,\mu F$, $L = 0.16\,H$ are connected in series with each other. $V = 240\,V$, $f = 50\,Hz$.

① $I = ?$

② $V_L, V_C, V_R = ?$

③ f at which $I \rightarrow$ unity power factor, $I = ?$

Soln



$I = \frac{V}{Z}$.

$Z = 24 + j0.16 - (5 \times 10^{-4})j$.

$\cancel{= 24 + j(0.16 - 0.00015)}$

$\cancel{= 24 + j(0.1598)}$

$\cancel{|Z| = \sqrt{24^2 + (0.1598)^2}}$

$\cancel{= 24.00053}$.

$X_L = \omega L = 2\pi f L = 50.265$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = 21.22$.

$X_L - X_C = 29.04$.

$|Z| = \sqrt{24^2 + (29.04)^2}$

$= 37.674\,\Omega$.

$I = \frac{240}{37.67} = 6.37\,A$.

$V_L = IX_L =$

Q. In a series RLC circuit, $R = 2\Omega$, $L = 2.0\,mH$, $C = 10\,\mu F$.

① Resonant f.

② Q-factor

③ B_W.

④ Half-Power f.

Soln,

$f = \frac{1}{2\pi\sqrt{LC}} = 1125.39\,Hz$.

$Q\text{-factor} = \frac{\omega L}{R} = \cancel{0.179}\ 7.07$

$BW = \frac{f_0}{Q} = \frac{1125.39}{7.07} = 159.179.$

f_1

f_2

Q/ 10 mH coil is connected in series with a loss-free capacitor ~~with~~ to a variable frequency source of 20V. The current in the circuit has maximum value of 0.2A at a frequency of 100 KHz. Calculate:

① Capacitance;
② Q-factor
③ Half-Power f.

Sol ①
$$I_{max} = \frac{V}{R} = \frac{20}{R} = 0.2$$

$$R = 100\Omega,$$

$$f_r = 100KHz = 10^5 Hz.$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

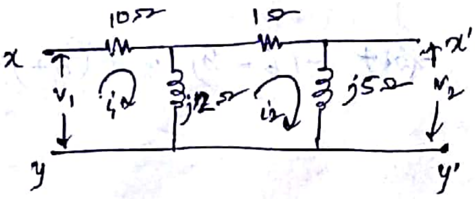
$$C = \frac{1}{4\pi^2 f^2 L} = 253.3\times10^{-12}F = 253.3\ pF.$$

$$Q\text{-factor} = \frac{\omega L}{R} = 62.83.$$

MESH ANALYSIS FOR AC NETWORK:

- Can be done as in DC network
- Only variation lies in the coeff. in the mesh current equation of the AC network have complex numbers (Impedences), instead of real nos. (Resistance) and having phasors instead of real currents or voltages as the unknowns.

Q/ Find the transfer function (V_2/V_1) for the network shown using mesh analysis. (efficiency)



Sol

$$V_1 - 10i_1 - j2(i_1 - i_2) = 0$$

$$V_1 = 10i_1 + j2i_1 - j2i_2$$

$$V_1 = (10+j2)i_1 - j2i_2 \quad \text{——— ①}$$

$-(1+7j)i_2 + j2i_1 = 0$

$j2i_1 - (1+7j)i_2 = 0$ ——— (2)

$i_1 = \left(\frac{1+j7}{j2}\right) i_2$.

In matrix form (1) and (2) can be written as –

$$\begin{pmatrix} 10+j2 & -j2 \\ j2 & -(1+j7) \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} V_1 \\ 0 \end{pmatrix}$$

$$i_2 = \frac{\begin{vmatrix} 10+j2 & V_1 \\ j2 & 0 \end{vmatrix}}{\begin{vmatrix} 10+j2 & -j2 \\ j2 & -(1+j7) \end{vmatrix}} = \frac{\cancel{-}V_1 j2}{\cancel{-}[(10+72j-14) - j^2 4]}$$

$$= \frac{v_1 j2}{72j - \cancel{14} + \cancel{4}}$$

$$= \frac{v_1 j2}{72j + \cancel{26}}$$

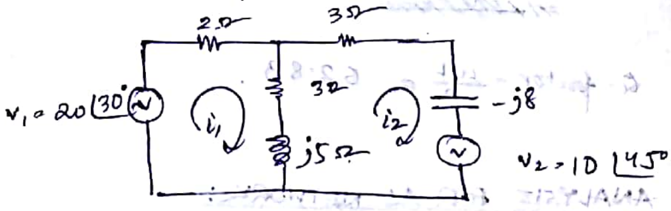
$$i_2 = \frac{v_1 \cancel{j}}{36\cancel{j} + \cancel{13}} \approx \frac{v_1}{36}.$$

$v_2 = i_2 j5$.

$$\therefore \eta = \cancel{\frac{j5 \, i_2 \, 36}{v_1}} \; \frac{V_o}{V_1} = \frac{i_2 j5}{V_1} = \frac{v_1 j5}{36 v_1} = \frac{5j}{36}$$

Q/ Using mesh analysis find the drop in the capacitor :-

Soln



KVL in (I)

$20\angle 30^\circ - 2\times i_1 - 3i_1 + 3i_2 - j5i_1 + j5i_2 = 0$

$20\left[\frac{\sqrt{3}+j}{2}\right] - (5+j5)i_1 + (3+j5)i_2 = 0$ ——— (1)

KVL in (II)

$\cancel{-(6+4)}\; -(6-3j)i_2 + (3+j5)i_1 + 10\angle 45^\circ = 0$

$I_2 = 2.92 \angle 103.29^\circ$

$$i_2 = \frac{\begin{vmatrix} 5+j5 & 20\angle 30^\circ \\ -(3+j5) & 10\angle 45^\circ \end{vmatrix}}{\begin{vmatrix} 5+j5 & 3+j5 \\ -(3+j5) & 3j-6 \end{vmatrix}}$$

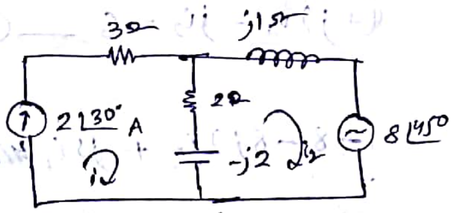
$$= \frac{\frac{10}{\sqrt{2}}(\sqrt{3})\sqrt{2}(5+j5) + 10(\sqrt{3}+j)(3+j5) \cdot \frac{10}{\sqrt{2}}(1+j)}{-30-15-15j+(3+j5)^2}$$

$$= \frac{\frac{50}{\sqrt{2}}(1-1+2j) + 10((3\sqrt{3}-5)+(5\sqrt{3}+3)j)}{-45-15j+9-25+30j}$$

$$= \frac{50\sqrt{2}j + 30\sqrt{3} - 50 + 50\sqrt{3}j + 30j}{-31-15j}$$

$$= -\left[\frac{(30\sqrt{3}-50) + (50\sqrt{3}+50\sqrt{2}+30)j}{(31+15j)}\right]$$

Q) Develop mesh equations and find the voltage drop across 2Ω resistor in the network shown.



$\cancel{2\angle 30^\circ - 3i_1 + 2i_2 = 0}$

$\cancel{5i_1 - 2i_2 = \sqrt{2}(\sqrt{3}+i)}$ —— ①

$i_1 = 2\angle 30^\circ = \sqrt{2}(\sqrt{3}+i)$ —— ①

Applying KVL to mesh ②,

$$(+2-j2)(i_2-i_1) - j i_2 = 8\angle 45^\circ$$

$$\Rightarrow (2-3j)i_2 - (2-2j)i_1 = \frac{8}{\sqrt{2}}(1+j)$$

$$\Rightarrow (2-3j)i_2 = (2-2j)\sqrt{2}(\sqrt{3}+i) + \frac{8}{\sqrt{2}}(1+i)$$

$$\Rightarrow \sqrt{2}(2-3j)i_2 = (8-8j)(\sqrt{3}+j) + 8 + 8j$$

$$= 8\sqrt{3} + 8 - (8\sqrt{3}-8)j + 8j$$

$$\sqrt{2}(2-3j)i_2 = 8\left[(\sqrt{3}+1) + (2-\sqrt{3})j\right].$$

$$\Rightarrow i_2 = \frac{8}{\sqrt{2}}\left[\frac{(\sqrt{3}+1)+(2-\sqrt{3})j}{2-3j}\right].$$

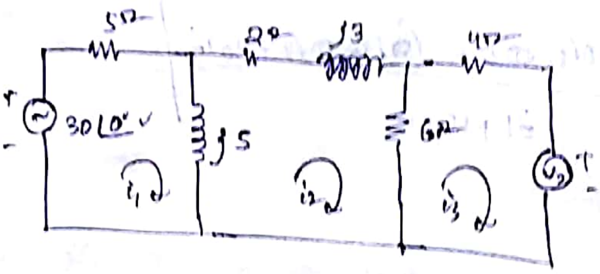
$i_2 = \frac{-8}{5\sqrt{2}}\left[\{(\sqrt{3}+1)+(2-\sqrt{3})j\}\{2+\sqrt{3}j\}\right].$

$i_2 = \frac{-8}{5\sqrt{2}}\left[2\sqrt{3}+2 - 2\sqrt{3}+3 + \{2(2-\sqrt{3})+\sqrt{3}(\sqrt{3}+1)\}j\right]$

$= \frac{-8}{5\sqrt{2}}\left[2+(4-2\sqrt{3}+3+\sqrt{3})j\right]$

$= \frac{-8}{5\sqrt{2}}\left[2+(7-\sqrt{3})j\right].$

Q) What is the value of V_2 such that the current in $(2+j3)\,\Omega$ is zero.



Sol

$(5+j5)i_1 - j5 i_2 = 30.$

$(1+j)i_1 - j i_2 = 6. \quad \text{—— (I)}$

$(-8-8j)i_2 + j5 i_1 + 6i_3 = 0$

$\Rightarrow \cancel{(8+8j)i_2 - (5j)i_1} \quad \text{—— (II)}$

$-10i_3 + 6i_2 - V_2 = 0$

$6i_2 - 10i_3 = V_2 \quad \text{—— (III)}$

$$\begin{bmatrix} 1+j & -j & 0 \\ -5j & 8+8j & -6 \\ 0 & 6 & -10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ V_2 \end{bmatrix}$$

$i_2 = 0$

$i_2 = 0$

Putting $i_2 = 0$,

$(1+j)i_1 = 6$

$i_1 = \frac{6}{1+j}$

$-10 i_3 = V_2$

$i_3 = -\frac{V_2}{10}$

$j5 \cdot \frac{6}{1+j} + \frac{-6V_2}{10} = 0$

$\Rightarrow \frac{30j}{1+j} - \frac{6V_2}{10} = 0$

$\Rightarrow \frac{\cancel{30}^{10} j}{1+j} = \frac{\cancel{6}^{3} V_2}{\cancel{10}_{5}}$

$\Rightarrow \frac{50j}{1+j} = V_2$

$\Rightarrow V_2 = \frac{\cancel{50}^{25} j(1-j)}{\cancel{2}}$

$V_2 = 25(j+1)$

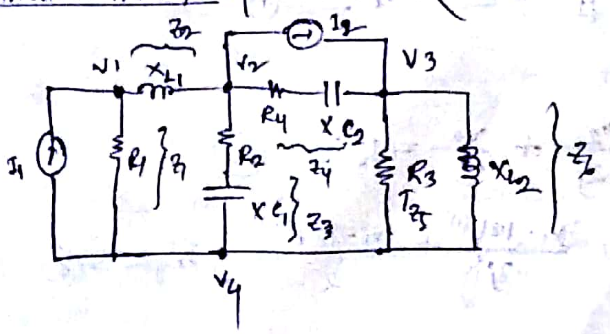
$V_2 = 25(1+j)$

$V_2 = 25\sqrt{2}\angle 45°$

(iv) NODE ANALYSIS FOR AC CIRCUIT:

- identical to that of DC network.
- in the frequency domain, network having 'n' principal nodes, one of them is designated as reference node and we require (n-1) node voltage equations to solve for the desired result.

Q Write the node-transformation matrix of the network shown in figure.



Applying KCL to node 1,

$$-I_1 + \frac{V_1 - V_2}{Z_2} + \frac{V_1}{Z_1} = 0$$

$$V_1\left[\frac{1}{Z_1}+\frac{1}{Z_2}\right] - \frac{V_2}{Z_2} = I_1$$

$$\Rightarrow V_1(Y_1+Y_2) - V_2 Y_2 = I_1 \quad \text{——— (i)}$$

Applying KCL to node 2,

$$\frac{V_2 - V_1}{Z_2} + I_2 + \frac{V_2 - V_3}{Z_4} + \frac{V_2}{Z_3} = 0$$

$$V_2 Y_2 - V_1 Y_2 + I_2 + V_2 Y_4 - V_3 Y_4 + V_2 Y_3 = 0$$

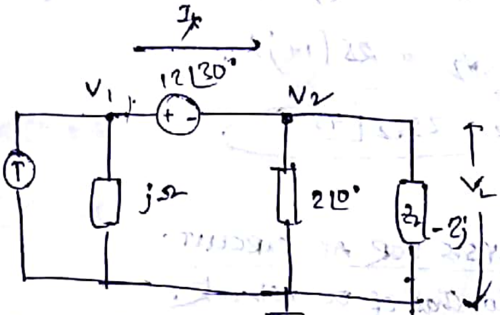
$$V_1(Y_2) - V_2(Y_2 + Y_3 + Y_4) + V_3(Y_4) = I_2 \quad \text{——— (ii)}$$

Applying KCL to node 3,

$$(V_3 - V_2)Y_4 + V_3 \cdot Y_5 + V_3 Y_6 = I_2 \quad \text{——— (iii)}$$

$$\begin{bmatrix} Y_1+Y_2 & -Y_2 & 0 \\ +Y_2 & -(Y_2+Y_3+Y_4) & +Y_4 \\ \cancel{Y_1+Y_2} & Y_3 & Y_6 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{pmatrix} I_1 \\ I_2 \\ I_2 \end{pmatrix}$$

Q If $Z = -j2\,\Omega$, find the V_L :-



$= 2($

KCL

(1)

$$\frac{V_1}{j1} - 2\angle 60° + I_x = 0$$

$$\frac{V_1}{j1} - 2\angle 60° = -I_x \quad \text{——— (1)}$$

(2)

$$\frac{V_2}{-2j} + \frac{V_2}{2} = I_x \quad \text{——— (2)}$$

$$\Rightarrow \frac{V_1 - 12\angle 30°}{-2j} + \frac{V_1 - 12\angle 30°}{2} = I_x \quad \text{——— (3)}$$

(52)

Applying KVL,

$V_1 - 12\angle 30^\circ = V_2$.

$$① + ② \Rightarrow \frac{V_1}{j1} - 2\angle 60^\circ + \frac{V_1 - 12\angle 30^\circ}{-2j} + \frac{V_1 - 12\angle 30^\circ}{2} = 0$$

$$\frac{V_1}{j} - \frac{2}{2}(1+\sqrt{3}i) - \frac{V_1}{2j} + \frac{6\angle 30^\circ}{j} + \frac{V_1}{2} - 6\angle 30^\circ = 0$$

$$\Rightarrow \frac{V_1}{2}\left(\frac{1}{j}+1\right) - 1 - \sqrt{3}i + 6\angle 30^\circ\left[-1+\frac{1}{j}\right] = 0.$$

$$\Rightarrow \frac{V_1}{2}\left(\frac{1+j}{j}\right) - 1 - \sqrt{3}i + 6(\sqrt{3}+i)\left[\frac{1-j}{j}\right] = 0$$

$$\Rightarrow \frac{V_1}{2}\left(\frac{1+j}{j}\right) - 1 - \sqrt{3}i + (6\sqrt{3}+6j)\frac{(1-j)}{(j)} = 0$$

$$\Rightarrow \frac{V_1}{2}\left(\frac{1+j}{j}\right) - 1 - \sqrt{3}j + \frac{6\sqrt{3}}{j} - 6\sqrt{3} + 6 - 6j = 0$$

$$\Rightarrow \frac{V_1}{2}\,-j(1+j) - 1 - \sqrt{3}j - 6\sqrt{3}j - 6\sqrt{3} + 6 - 6j = 0$$

$$\Rightarrow \frac{V}{2}(1-j) - 1 - 7\sqrt{3}j - 6j + 6 = 0$$

$$\Rightarrow \frac{V}{2}(1-j) = -5 + (7\sqrt{3}+6)j.$$

$$\Rightarrow V = 2\left[\frac{-5+(7\sqrt{3}+6)j}{(1-j)}\right].$$

#) Superposition Theorem:

- States that the voltage across (or current) through an element in a linear circuit is the algebraic sum of the voltages across (or current through) that element due to each independent source acting alone.

- Steps to analyze circuit using Superposition theorem.

(i) Turn off all independent sources except one source, find the output (voltage or current) due to that active source using mesh or nodal analysis or KVL and KCL equations.

(ii) Repeat Step (i) for each of the other independent sources.

(iii) Find the total contribution by adding algebraically all the contributions due to independent sources.

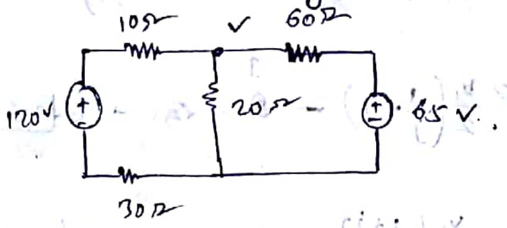
Limitations :-

i/ Cannot be applied to non-linear circuit.

ii/ To find the effect on power due to each source by using Superposition is not applicable.

iii/ Analyzing requires more work.

Q/ Using Superposition theorem, find the current through 20Ω resistor.



① **Removing 65V,**

$$R_{eq} = \frac{\cancel{60}^{15}}{\cancel{4}} + 30 + 10 = 55\Omega$$

$$\therefore i = \frac{120}{55} A = 2.18 A$$

$$i_{20} = \left(\frac{\cancel{60}}{\cancel{80}/4}\right) \times \cancel{2.18}^{0.545} = 1.635 A.$$

② **Removing 120V,**

$$R_{eq} = \frac{40}{3} + 60 = \frac{220}{3}\Omega.$$

$$\therefore i = \frac{65 \times 3}{\cancel{220}_{20}} = 0.886 A$$

$$i_{20} = \left(\frac{\cancel{40}^{2}}{\cancel{60}/3}\right) \times \cancel{0.886}^{0.295} = 0.590$$

~~∴ Total current = 2.18 + 0.886 = 3.067 A~~

$\therefore$ Total current, $I_{20} = 1.635 + 0.590 = 2.225 A$

Cross verification:

$$\frac{V-120}{40} + \frac{V-65}{60} + \frac{V-\cancel{120}}{20} = 0$$

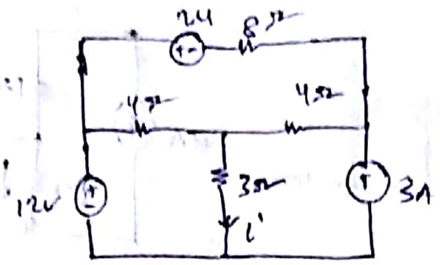
$$\Rightarrow \frac{V-120}{2} + \frac{V-65}{3} + V = 0$$

$$\Rightarrow \frac{5V}{6} - 60 - 22 + V = 0$$

$$\Rightarrow \frac{11V}{6} = 82$$

$$\Rightarrow V = \frac{82 \times 6}{11} =$$

Q) For the circuit shown, use Superposition to find the current (i).

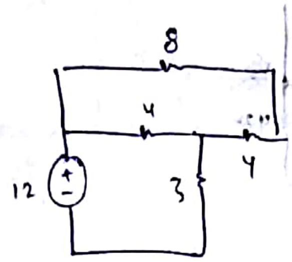


①
Keeping 12V :

~~$R_{eq} = \left(\frac{12}{7}+4\right) \parallel 8$~~

~~$= (4.714) \parallel 8$~~

~~$= 2.967\ \Omega$.~~

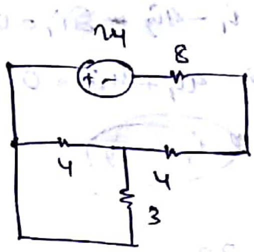


~~$i_{tot} = \frac{12}{2.967} =$~~

$R_{eq} = 3+3 = 6\ \Omega$.

$i = \frac{12}{6} = 2A$.

Keeping 24V ,



$R_{eq} = \left(\frac{12}{7}+4\right)+8$

$= 4.714+8$

$= 12.714\ \Omega$.

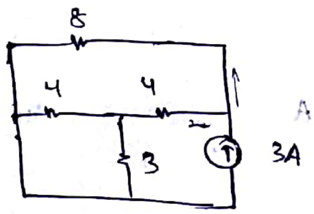
$i = \frac{24}{12.714}$

$= 1.88\ A$.

$i_{3\Omega} = \left(\frac{4}{7}\right) \times 1.88$

$= 1.076\ A$.

Keeping 3A :-



$R_{eq} = \left(\frac{12}{7}+4\right)$

$= 4.714\ \Omega$.

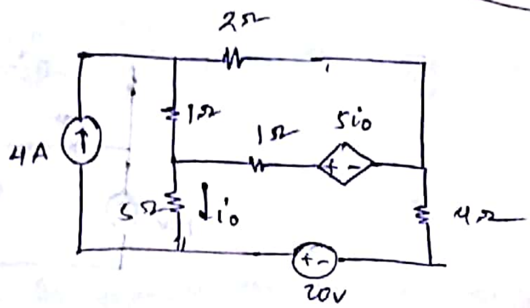
$i_{4\Omega} = \left(\frac{8}{12.714}\right) \times 3 = 1.88A$.

$i_{3\Omega} = \left(\frac{4}{7}\right) \times 1.88 = 1.076A$

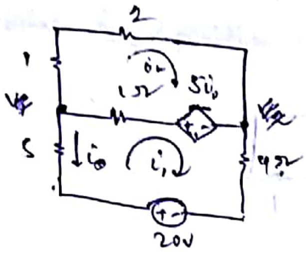
Q/ find I_0 using superposition:-

$[i_0 = -0.4706]$

Sol:



Keeping 20V:



$i_1 = -i_0$

$-10i_1 + 5i_2 + i_2 + 20 = 0.$

$\Rightarrow 5i_1 - i_2 = 20$ —— (1)

$-4i_2 + i_1 + 5i_0 = 0$

$\Rightarrow i_1 - 4i_2 - 5i_1 = 0$

$\Rightarrow 4i_1 + 4i_2 = 0$

$i_1 = -i_2$

$5i_1 + i_1 = 20$

$6i_1 = 20$

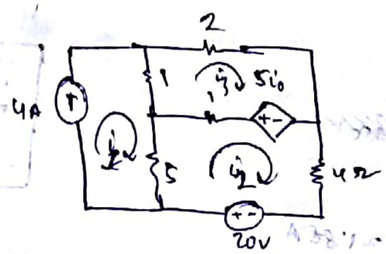
$i_1 = \frac{20}{6} = 3.33A.$

$i_0 = -3.33A.$

Keeping 4A:

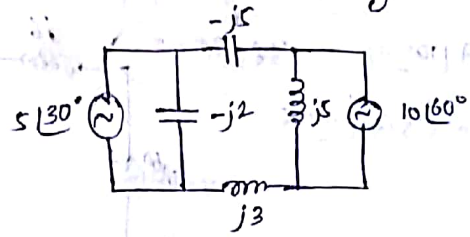
$i_1 = 4A$

$5\times4 -$

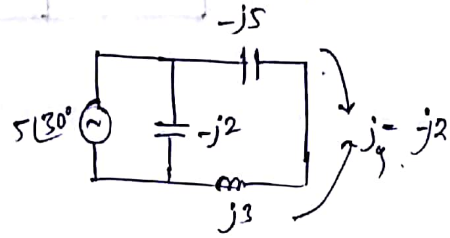


Q/ Find the current through $j3\Omega$ inductive reactance using Superposition.

Sol



Removing $10\angle 60°$:



$\cancel{Z_{eq} = -j}$

$\cancel{V_{eq} = \frac{5}{2}(\sqrt{3}+j)}$

$\cancel{i = \frac{\frac{5}{2}(\sqrt{3}+j)}{-j}}$

$i =$

$$i_x = \frac{5\angle 30°}{-j2}$$

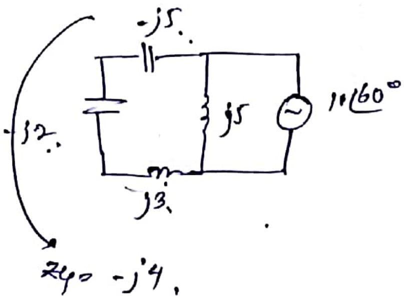
$$= \frac{-5(\sqrt{3}+j)}{2j \times 2}$$

$$= -\frac{5}{4}(-j\sqrt{3}+1)$$

$$= -\frac{5}{4}(1-j\sqrt{3}).$$

$$|i_x| = -\frac{5}{4} \times \frac{2}{2} = -5/2 = 2.5\angle 120° \ A$$

Removing $5\angle 30°$:



$$\therefore i_x' = \frac{\cancel{10}^{5}\angle 60°}{-j\cancel{4}2}$$

$$= -\frac{5}{4}\left(\frac{1+\sqrt{3}j}{j}\right)$$

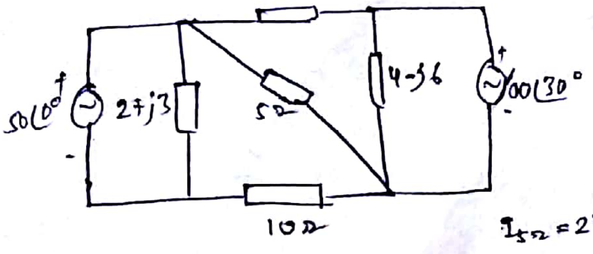
$$= -\frac{5}{4}(-j+\sqrt{3})$$

$$= \frac{-5}{4}(\sqrt{3}-j)$$

$$(i_x') = -\frac{5}{4} \times 2 = -5/2$$

$$= 5\angle 170°.$$

Q/ Find by the principle of superposition, the current through 5Ω



$$I_{5\Omega} = 2.5\angle 0 + 5\angle 30$$

$$= 7.273\angle 20°$$

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Q/ Find current in R_L using superposition:

$I_L = 1.9\angle 108.43 + 1.58\angle 78.43$

$= 3.562\angle 94.78$