03/09/24

SETS

Set :- A set is a well-defined collection of objects. It is represented as { }

Representation of Sets

Q-1 Write a set that contains all the natural no.s less than 20.

Roster form :- $A = \{1, 2, 3, \ldots, 19\}$
Set Builder :- $A = \{x \mid x \in N \ \& \ x < 20\}$

Q-2 Write a set that contains all the natural no.s less than 30 that are odd

A- $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29\}$
$A = \{x \mid x \in 2n - 1 \ \& \ x < 30 \}, x \in N \ \& \ n \in N \}$

Q-3 Write a set that contains all the natural no.s that are perfect powers & less than 40.

A- $A = \{1, 4, 9, 16, 25, 36, 8, 27, 32\}$
$A = \{x \mid x \in n^m \ \& \ n, m \in N \ \& \ x < 40\} \ \& \ x \in N \}$

Q-4 Write a set that contains all natural no.s less than 60 that are perfect squares & divisible by 4.

A- $A = \{4, 16, 36\}$

$A = \{x \mid x \in N \;\&\; x \in n^2 \;\&\; x = 4n \;\&\; x < 60\}$

Types of Sets

1) Empty Set

⇒ A set that contains no elements is called an empty set.

⇒ It is represented as ϕ or $\{\ \}$.

2) Singleton Set

⇒ A set that contains only one element is called singleton set.

Eg. $A = \{1\}$, $A = \{0\}$, $A = \{\phi\}$

3) Finite Set

⇒ A set that contains a fixed no. of elements is called finite set.

Eg. $A = \{1, 2, 3, 4\}$
$B = \{a, e, i, o, u\}$
$C = \{1, 2, 3, \ldots, 10000\}$

4) Infinite Set

⇒ A set that contains infinite no. of elements is called infinite set

Eg. $A = \{1, 2, 3, ..\}$
$B = \{1, 3, 5, 7, ..\}$

Subset

⇒ Let A be a set. A set B is called a subset of A if all the elements of B are present in A & is represented as $\boxed{B \subseteq A}$

Proper Subset

⇒ Let A be a set. A set B is called a proper subset of A if all the elements of B are present in A but $B \neq A$ & is represented as $\boxed{B \subset A}$.

Q- $A = \{2, 4, 6, 8, 10\}$
$B = \{2, 4, 6\}$
$C = \{x \mid x \in N \ \& \ x \in 2n, x \leq 10\}, n \in N\}$
$D = \{x \mid x \in N \ \& \ x \in 2n, x \geq 10\}, n \in N\}$

Write T or F.

a) $A \subseteq B$
b) $B \subseteq A$
c) $C \subseteq A$
d) $C \subset A$
e) $D \subseteq A$
f) $A \subseteq D$
g) $B \subseteq C$
h) $B \subset C$

A- $C = \{2, 4, 6, 8, 10\}$

$D = \cancel{\{2, 4, 6, 8\}}\ \{10, 12, 14, \ldots\}$

a) F
b) T
c) T
d) F
e) F
f) F
g) T
h) T

Power Set

Let A be a set. A set B is called a power set of A if it contains all the possible subsets of A. It is represented as $B = P(A)$

$A = \{1, 2, 3\}$

$P(A) = \{\{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}, \Phi\}$

NOTE In general, if a set A have n elements then

$$\boxed{P(A) = 2^n \text{ elements}}$$

Q- Find the power set of
$A = \{\Phi, \{\Phi\}, \{\Phi, 1\}\}$

A- $P(A) = 2^3 = 8$

$= \{\{\Phi\}, \{\{\Phi\}\}, \{\{\Phi, 1\}\}, \{\Phi, \{\Phi\}\},$
$\{\Phi, \{\Phi\}, \{\Phi, 1\}\}, \{\Phi, \{\Phi, 1\}\}, \{\{\Phi\}, \{\Phi, 1\}\}, \Phi\}$

04/09/24

Q- Find the no. of elements in $P(P(P(P(P(P(\{\phi\}))))))$

A-
$2^1 = 2$
$2^2 = 4$
$2^4 = 16$
$2^{16} =$
$2 =$
$2 =$

Equal Sets

⇒ 2 sets A & B are said to be equal if they have same elements i.e.

$A = \{1, 2, 3, 4, \ldots, 20\}$
$B = \{x \mid x \in \mathbb{N}, x \leq 20\}$
$C = \{1, 1, 2, 2, 2, 3, 3, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 16, 15, 17, 18, 19, 20\}$

Set Operations

1) Union

⇒ Let A & B be 2 sets. The union of A & B $(A \cup B)$ is the collection of all the elements that are present in A or in B or in both.

Eg. $A = \{x \mid x \in \mathbb{N} \;\&\; x < 10\}$
$B = \{2, 4, 6, 8, 10\}$

$A \cup B = \{x \mid x \in \mathbb{N} \ \& \ x \leq 10\}$

Q- Let $X = \{a_1, a_2, \cdots, a_n\}$ be a set. How many sets A & B can be made from X such that $A \cup B = X$.

A- 3^n elements.

2) Intersection

=> Let A & B be 2 sets. The intersection of A & B ($A \cap B$) is the collection of all the elements that are present in A & in B.

Eg. $A = \{x \mid x \in \mathbb{N}, x \text{ is an odd no less than } 30\}$
$B = \{x \mid x \in \mathbb{N}, x = 3n, n \in \mathbb{N}, x < 30\}$

$A \cap B = \{3, 9, 15, 21, 27\}$

Q- Let $X = \{a_1, a_2, a_3, \cdots a_n\}$ be a set. How many sets A & B can be made from X such that $A \cap B = \Phi$.

A- 3^n elements.

3) Difference

=> Let A & B be 2 sets. The difference of A & B ($A - B$) is the collection of all

The elements that are present in A but not in B.

Eg. $A = \{1, 2, 3, \dots, 20\}$
$B = \{2, 4, 6, 8, 10, 12\}$

$A - B = \{1, 3, 5, 7, 9, 11, 13, 14, 15, 16, 17, 18, 19, 20\}$

4) <u>Complement</u>

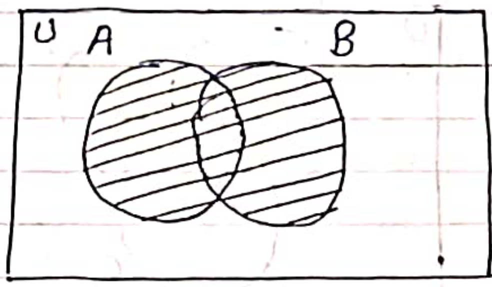
⇒ Let A be a set let U be the universal set. The complement of A (A') is a collection of all the elements that are present in U but not in A

Eg. $A = \{1, 2, 3, 4, \dots, 15\}$
$B = \{3, 5, 7, 11, 17, 19\}$
$U = \{x \mid x \in \mathbb{N}, x \leq 20\}$

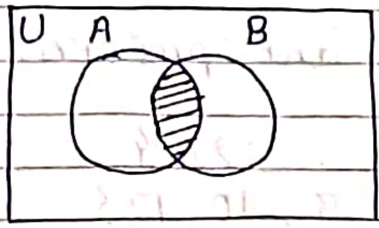
$A' = \{16, 17, 18, 19, 20\}$
$B' = \{1, 2, 4, 6, 8, 9, 10, 12, 13, 14, 15, 16, 18, 20\}$

* Venn Diagrams

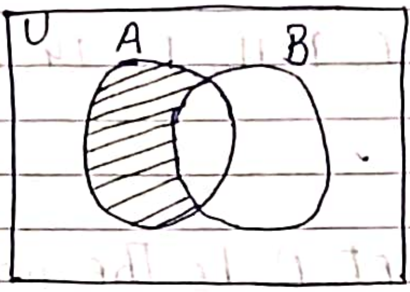
i) <u>$A \cup B$</u>



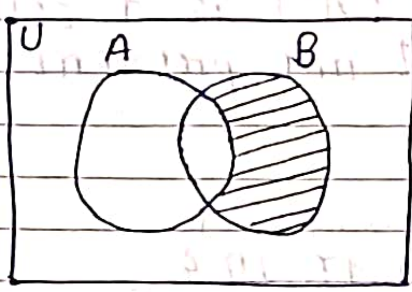
2) $\underline{A \cap B}$



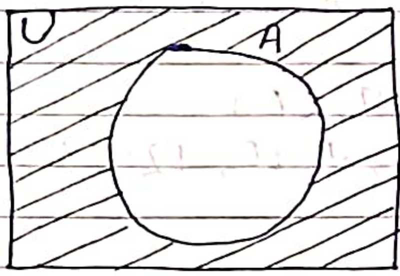
3) $\underline{A - B}$



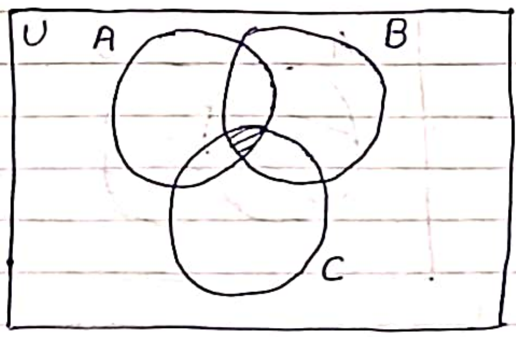
$\underline{B - A}$



4) $\underline{A'}$



5) $\underline{A \cap B \cap C}$



06/09/24

Date
Page

* Multi Sets

Multiplicity of elements

The multiplicity of an element is the no. of times the element is present in set.

Eg. $A = \{1, 1, 2, 2, 3, 3, 3\}$

$m(1) = 2$
$m(2) = 2$
$m(3) = 3$

Union (in case of Multisets) [$\max^m$]

Q- Let $A = \{1, 1, 1, 2, 2, 2, 3, 4, 4, 4, 5, 5, 6\}$
$B = \{1, 1, 1, 2, 3, 3, 3, 4, 4, 4, 5, 6, 6, 6\}$

$A \cup B = \{1, 1, 1, 2, 2, 2, 3, 3, 3, 4, 4, 4, 5, 5, 6, 6, 6$

Intersection (min.)

$A \cap B = \{1, 1, 1, 2, 3, 4, 4, 4, 5, 6\}$

Difference

$A - B = \{2, 2, 5\}$

Q-

$A = \{1, 1, 2, 2, 2, 3, 3, {}^{3, 3,} 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 7\}$

$B = \{1, 1, 1, 1, 2, 3, 3, 4, 4, 4, 5, 5, 6, 6, 7, 7, 8\}$

$C = \{1, 2, 3, 4, 5, 5, 5, 5, 5, 5, 5, 6, 6\}$

Find :-

a) $A - B$

b) $B - A$

c) $A - C$

d) $C - A$

e) $B - C$

f) $\cancel{C}\ A + C$

g) $A \cup C$

h) $B \cup A$

i) $A \cap C$

j) $B \cap C$

k) $C \cup B$

l) $B + C$

A-

a) $A - B = \{2, 2, 3, 3, 5, 5, 5, 6, 6, 6\}$

b) $B - A = \{1, 1, 4, 4, 7, 8\}$

c) $A - C = \{1, 2, 2, 3, 3, 3, 5, 5, 6, 6, 6, 7\}$

d) $C - A = \{5, 5\}$

e) $B - C = \{1, 1, 1, 3, 4, 4, 7, 7, 8\}$

f) $A+C = \{1,1,1,2,2,2,2,3,3,3,3,3,4,4,5,5,5,5,5,5,5,5,5,5,5,5,6,6,6,6,6,6,6,7\}$

g) $A \cup C = \{1,1,2,2,2,3,3,3,3,4,5,5,5,5,5,5,5,6,6,6,6,6,7\}$

h) $B \cup A = \{1,1,1,1,2,2,2,3,3,3,3,4,4,4,5,5,5,5,5,6,6,6,6,6,7,7,7,8\}$

i) $A \cap C = \{1,2,3,4,5,5,5,5,5,6,6\}$

j) $B \cap C = \{1,2,3,4,5,5,6,6\}$

k) $C \cup B = \{1,1,1,1,2,3,3,4,4,4,5,5,5,5,5,5,5,6,6,7,7,8\}$

l) $B+C = \{1,1,1,1,1,2,2,3,3,3,4,4,4,4,5,5,5,5,5,5,5,5,5,6,6,6,6,7,7,8$

* Computer Representation Of Sets

$U = \{1,2,3,\ldots,10\}$

$A = \{1,3,5,7,9\}$

$\boxed{A = 1010101010}$

$P = \{2,4,6,8\}$

$\boxed{P = 0101010101}$

$A \cup B =$

$$
\begin{array}{r}
1010101010 \\
0101010101 \\
\hline
1111111111
\end{array}
$$

$A \cap B =$

$$
\begin{array}{r}
1010101010 \\
0101010101 \\
\hline
0000000000
\end{array}
$$

Q- Let

$U = \{x \mid x \in \mathbb{N} \ \& \ x \leq 20\}$
$A = \{x \mid x \in \mathbb{N} \ \& \ x = 3n, n \in \mathbb{N}, x \leq 20\}$
$B = \{x \mid x \in \mathbb{N} \ \& \ x \% 2 == 0, x \leq 20\}$

Represent A & B using bit string & find $A \cup B$ & $A \cap B$.

A- $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$A = \{3, 6, 9, 12, 15, 18\}$

$B = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

$A = 00100100100100100100$
$B = 01010101010101010101$

$A \cup B =$ ~~101~~

$$
\begin{array}{r}
0\,0\,1\,0\,0\,1\,0\,0\,1\,0\,0\,1\,0\,0\,1\,0\,0\,1\,0\,0 \\
0\,1\,0\,1\,0\,1\,0\,1\,0\,1\,0\,1\,0\,1\,0\,1\,0\,1\,0\,1 \\
\hline
0\,1\,1\,1\,0\,1\,0\,1\,1\,1\,0\,1\,0\,1\,1\,1\,0\,1\,0\,1
\end{array}
$$

$A \cap B = 0000010000010000000100$

10/09/24

LOGIC

Proposition

⇒ It is a declarative statement that have only 2 ans - True or False.

⇒ Represented as p, q, r, s, t etc.

Eg. i) $2 + 3 = 6$? F
ii) Sun rises in the east.

Not a proposition

i) What is your name?
ii) What have you eaten?

Properties of Proposition

1) Negation

⇒ Let p be a proposition. The negation of ($\sim p$) is the statement "It is not the case that p".

p	~p
T	F
F	T

Eg:- p → I am going to class.
~p → I am not going to class.
OR
It is not the case that I am going to class.

2) Conjuction

⇒ Let p & q be 2 propositions. The conjunction of p & q ($p \wedge q$) is the statement "p and q".

Eg. p → I am going to class.
q → I will play cricket.

$p \wedge q$ → I am going to class and I will play cricket.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Date
Page

3) Disjunction

Let p & q be 2 propositions. The disjunction of p & q ($p \vee q$) is the statement "p or q".

Eg. p → I am going to class.
q → I will play cricket.

$p \vee q$ → I am going to class or I will play cricket.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

4) Exclusive OR (X-OR)

⇒ Let p & q be 2 propositions. The exclusive OR of p & q ($p \oplus q$) is the statement "p or q" given that it is false when both p & q are true or both p & q are false.

Eg. p → I will have tea.
q → I will have coffee.

$p \oplus q$ → I will have tea or coffee.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

5) <u>Conditional ($\rightarrow$)</u>

$\Rightarrow$ Let p & q be 2 propositions. The conditional of p & q ($p \rightarrow q$) is the statement "if p then q". It is false when p is true & q is false.

Eg. $p \rightarrow$ She tops the class.
$q \rightarrow$ I will give her reward.

$p \rightarrow q$:- If she tops the class then I will give her reward.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Q-1 Find The truth table of

a) $(p \wedge q) \vee (q \wedge p)$

A-

p	q	$p \wedge q$	$q \wedge p$	$(p \wedge q) \vee (q \wedge p)$
T	T	T	T	T
T	F	F	F	F
F	T	F	F	F
F	F	F	F	F

b) $(p \rightarrow q) \rightarrow (q \wedge p)$

p	q	$p \rightarrow q$	$q \wedge p$	$(p \rightarrow q) \rightarrow (q \wedge p)$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	F	T	F	F

11/09/24

6) Biconditional ($\leftrightarrow$)

If p & q are 2 propositions, then the biconditional of p & q ($p \leftrightarrow q$) is the statement "p if and only if q". It is true if both are true or both are false.

Eg. I will take the flight if & only if I will buy the ticket

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Q- Find the Truth table of

i) $(p \rightarrow q) \leftrightarrow (q \leftrightarrow p)$

ii) $(p \vee q) \oplus (q \rightarrow (p \wedge q)$

iii) $(p \vee (q \wedge r)) \leftrightarrow ((q \times p) \longrightarrow (r \wedge p))$

A- i) $(p \rightarrow q) \leftrightarrow (q \leftrightarrow p)$

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \leftrightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

ii) $(p \vee q) \oplus (q \rightarrow (p \wedge q))$

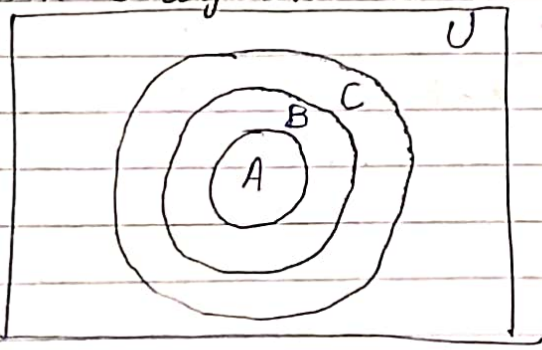
p	q	$p \wedge q$	$p \vee q$	$q \rightarrow (p \wedge q)$	$(p \vee q) \oplus q \rightarrow (p \wedge q)$
T	T	T	T	T	F
T	F	F	T	T	F
F	T	F	T	F	T
F	F	F	F	T	T

iii) $(p \vee (q \wedge r)) \longleftrightarrow ((q \vee p) \longrightarrow (r \wedge p))$

p	q	r	$(q \wedge r)$	$(q \vee p)$	$(r \wedge p)$	$p \vee (q \wedge r)$	$(q \vee p) \rightarrow (r \wedge p)$	$(p \vee (q \wedge r)) \leftrightarrow ((q \vee p) \rightarrow (r \wedge p))$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	F	T	F	F
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	T	F	F
F	T	T	T	T	F	T	F	F
F	T	F	F	T	F	F	F	T
F	F	T	F	F	F	F	T	F
F	F	F	F	F	F	F	T	F

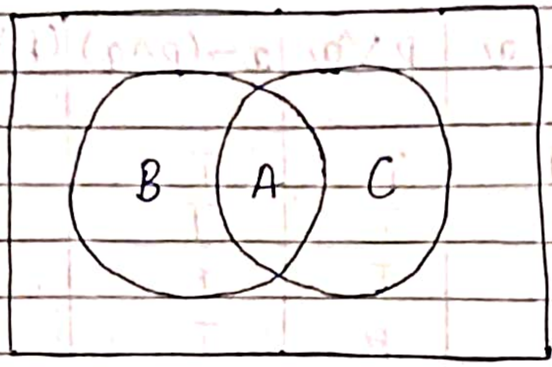
2) Use a venn diagram to show $A \subseteq B$ & $B \subseteq C$

A-



Q3 Use a venn diagram to show $A \subset C$ & $A \subset B$.

A-



Q-4 Determine True or False.

a) $\phi \in \{\phi\}$ T
b) $0 \subseteq \phi$ F
c) $\{0\} \subset \{0\}$ F
d) $\phi \subseteq \{x\}$ T
e) $\phi \in \{x\}$ F
f) $\{\phi\} \in \{\phi\}$ F

Q-5. Find 2 sets A & B such that $A \in B$ & $A \subseteq B$.

A- $A = \phi$
$B = \{\phi, 1\}$

13/07/24

Q-1 Which of These are propositions :-

i) Delhi is capital of ODISHA
ii) $x + 2 = 11$
iii) $2^n \geq 110, \; \forall n \in \mathbb{N}$

Q-2 What is The negation of

i) 121 is a perfect square
ii) There are 13 items in The bank.

Q-3 Let p, q, r be The propositions
p : You have The flu.
q : You miss The final exam.
r : You pass The course

Express The following propositions as an english sentence :

a) $p \to q$
b) $\sim q \leftrightarrow r$
c) $p \to \sim r$
d) $q \to r$

Q-4 Find The truth table of

a) $p \vee (q \wedge r)$
b) $(p \vee q) \wedge (p \vee r)$
c) $(p \oplus q) \vee (p \oplus \sim q)$
d) $\sim p \wedge q \vee r \to p$

ANSWERS

1) i) F (Yes)
ii) F (No)
iii) F (Yes)

2) a) It is not the case that 121 is a perfect square.
b) It is not the case that there are 13 items in the bank.

3) a) $p \rightarrow q$:- If you have flu then you will miss the ~~exc~~ final exam.

b) $\sim q \leftrightarrow r$:- You pass the course if & only if you will not miss the final exam.

c) $p \rightarrow \sim r$:- If you ~~have the flu~~ then you will not pass the course.

d) $q \rightarrow r$:- If you miss the final exam then you pass the course.

4) a) $p \vee (q \wedge r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

b) $(p \vee q) \wedge (p \vee r)$

p	q	r	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	F	T	F
F	F	F	F	F	F

c) $(p \oplus q) \vee (p \oplus \sim q)$

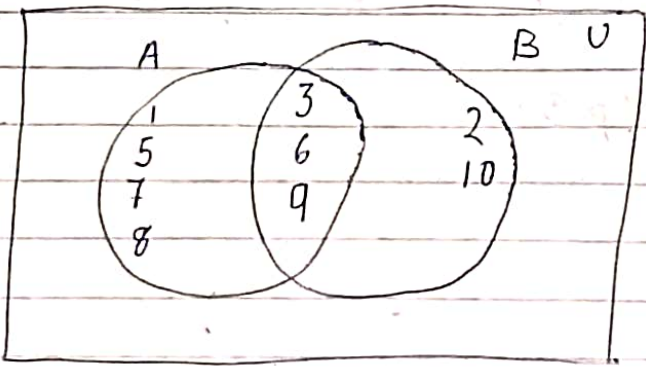
p	q	$\sim p$	$p \oplus q$	$p \oplus \sim q$	$(p \oplus q) \vee (p \oplus \sim q)$
T	T	F	F	T	T
T	F	F	F	F	T
F	T	F	T	F	T
F	F	T	F	T	T

d) $\sim p \wedge q \vee r \rightarrow p$

p	q	r	$\sim p$	$\sim p \wedge q$	$\sim p \wedge q \vee r$	$\sim p \wedge q \vee r \rightarrow p$
T	T	T	F	F	T	T
T	T	F	F	F	F	T
T	F	T	F	F	T	T
T	F	F	F	F	F	T
F	T	T	T	T	T	F
F	T	F	T	T	F	F
F	F	T	T	F	T	F
F	F	F	T	F	F	T

Q.5 If $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$, $A \cap B = \{3, 6, 9\}$. Find A & B.

A-



Note: If the truth values are same for 2 compound propositions then they are called as logically equivalent.

If all the truth values for a compound propositions then it is called tautology. and if all are false then it is called Contradiction.

* Order of Operators

1) $\sim$
2) $\wedge$
3) $\vee$
4) $\rightarrow$
5) $\leftrightarrow$

* Some special Operations

1) Symmetric Difference of Sets

=> Let A & B be 2 sets. The symmetric diff of A & B ($A \Delta B$) is the collection of all the elements that are present in $A \cup B$ but not in $A \cap B$.

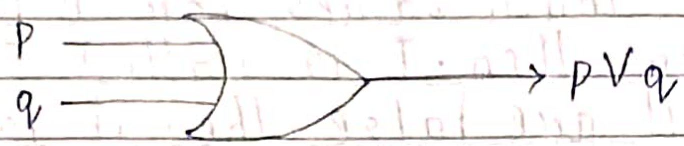
Eg. $A = \{1, 2, 3, 4, 5\}$
$B = \{2, 4, 6, 8\}$
$A \Delta B = A \cup B - A \cap B$
$= \{1, 2, 3, 4, 5, 6, 8\} - \{2, 4\}$
$= \{1, 3, 5, 6, 8\}$

20/09/24

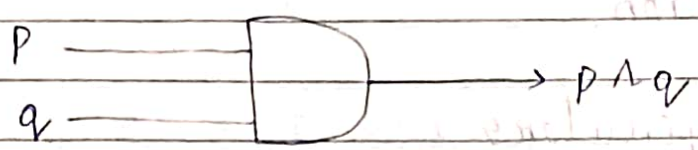
Logic Gates

1) Not

2) Or



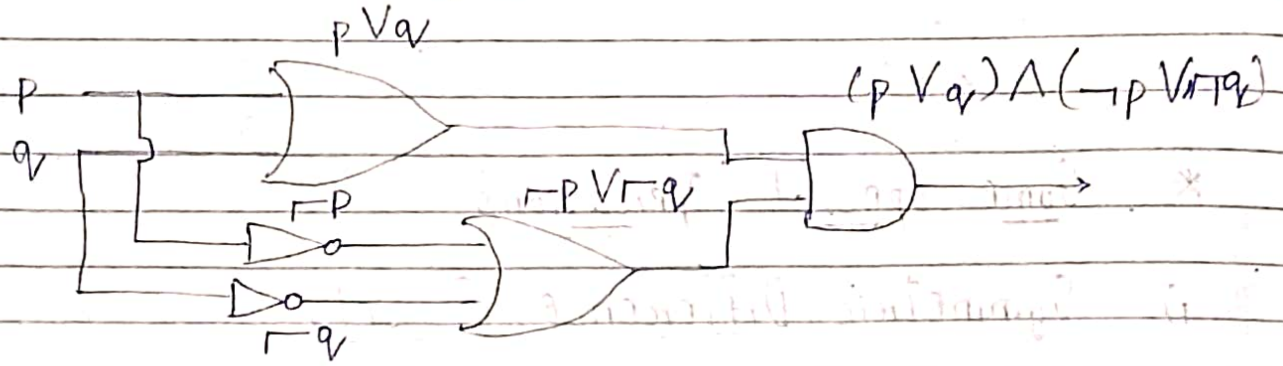
3) And



Q-1 Draw the circuit whose output will be

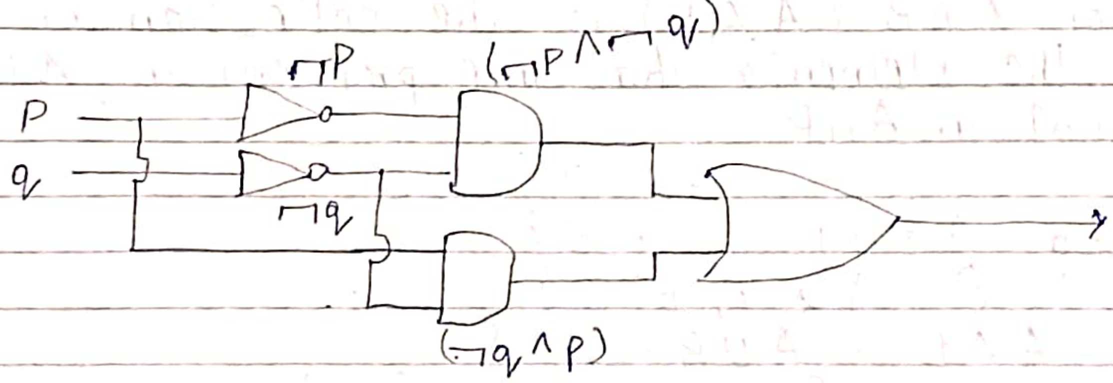
i) $(p \vee q) \wedge (\neg p \vee \neg q)$

A-

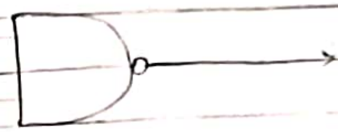


ii) $(\neg p \wedge \neg q) \vee (\neg q \wedge p)$

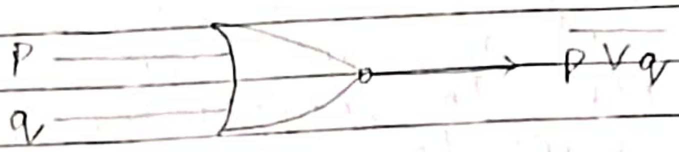
A-



4) NAND



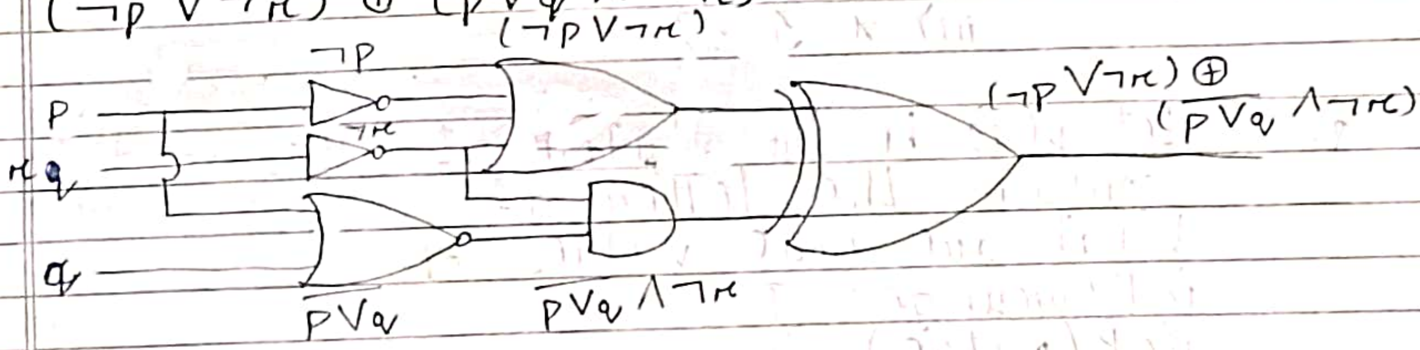
5) NOR



6) XOR

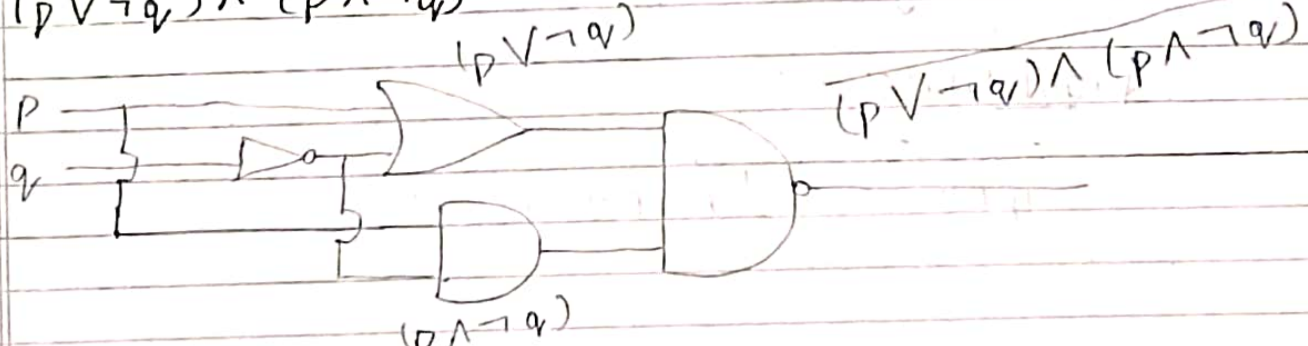
7) XNOR

iii) $(\neg p \vee \neg r) \oplus (\overline{p \vee q} \wedge \neg r)$



iv) $\overline{(p \vee \neg q) \wedge (p \wedge \neg q)}$

A -



Predicates

=> A predicate is a declarative statement that is not a proposition but it can be converted to proposition by assigning certain values to them.

=> Denoted by $p(x)$

q 1) Find the truth value of the predicate
$p(x): x+1=4$
where i) $x=0$ F
ii) $x=2$ F
iii) $x=3$ T

q2) Let $p(x)$ be the statement "The word 'x' contains the letter a"
What are true values of
i) P(orange) T
ii) P(false) T
iii) P(Dog) F

Quantifiers

=> 2 Types :- 1) Universal
2)

1) Universal ($\forall$)

=> The universal quantifier of $P(x)$ is the statement "$P(x)$ for all values of x"

=> It is written as $\forall x\, P(x)$ i.e. "For all x $P(x)$".

Q- Let $P(x): x+1 \geq x$, $x \in \mathbb{R}$ Then $\forall x P(x)$ T or F.

A- $\forall x\,(x+1 \geq x)$, $x \in \mathbb{R}$
True

2) Existensial ($\exists$)

=> The existensial quantifier of $P(x)$ is the statement "There exist a value of x for which $P(x)$". Written as $\exists x\, P(x)$, i.e. "There exist x $P(x)$".

Q- Let $P(x)$ be the statement $x+1 = x$, $x \in \mathbb{R}$ The $\exists P(x)$ T or F.

A- $\exists P(x) \longrightarrow$ False

1) Let $Q(x)$ be the statement "$x+1>2x$"
If $x \in Z$, then what are truth ~~to~~ values

i> $Q(0)$
ii> $Q(-1)$
iii> $\exists x\, Q(x)$
iv> $\forall x\, Q(x)$
v> $\exists x \sim Q(x)$
vi> $\forall x \sim Q(x)$

2) If $x \in R$, then, find the truth value of

i> $\exists x (x^2 = 2)$
ii> $\exists x (x^2 = -1)$
iii> $\forall x (x^2 \neq x)$
iv> $\forall x (x^2 + 2 \geq 1)$

ANSWERS

1) i> True
ii> True
iii> True
iv> False
v> True
vi> False

2 i> True
ii> False
iii> False
iv> True

Nested Quantifiers

$x, y \in \mathbb{R}$

1) $\forall x \exists y (x+y = y+x) \longrightarrow$ True

2) $\forall x \exists y (x+y = 0) \longrightarrow$ True

3) $\exists x \forall y (x+y = y+x) \longrightarrow$ True

4) $\exists x \exists y (x+y = 1) \longrightarrow$ True

5) $\forall x \forall y (x+y = 1) \longrightarrow$ False

25/09/24

Q-1 Let $n, m \in \mathbb{Z}$ Find the truth values?

i) $\forall n \exists m \, (n^2 < m)$ T
ii) $\exists n \forall m \, (nm = m)$ T
iii) $\forall n \exists m \, (n + m = 0)$ T
iv) $\exists n \exists m (n^2 + m^2 = 5)$ T
v) $\exists n \exists m \, (n + m = 4 \wedge n - m = 1)$ F
vi) $\exists n \exists m \, (n + m = 3 \vee n - m = 2)$ T
vii) $\forall n \forall m \exists p \left(p = \frac{m+n}{2}\right)$ F

Q-2 What will be the case if $n, m \in \mathbb{R}$?

i) T
ii) T
iii) T
iv) T
v) T
vi) T
vii) T

* Rules of Inference

Argument

An argument is a sequence of propositions
It is valid, if the conclusion is True.

Eg. Premises { p : I will come to class.
$p \to q$: If I come to class then I will solve the problem

Conclusion q → ∴ I will solve the problem
∴ → Therefore

$((p \to q) \land p) \to q$

p	q	$p \to q$	$(p \to q) \land p$	$((p \to q) \land p) \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	
F	F	T	F	T

	Rules	Tautology	Name
1)	p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus Ponens
2)	$\sim q$ $p \to q$ $\therefore \sim p$	$(\sim q \land (p \to q)) \to \sim p$	Modus Tollens
3)	$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to p \to r$	Hypothetical Syllogism
4)	$p \lor q$ $\sim p$ $\therefore q$	$((p \lor q) \land \sim p) \to q$	Disjunctive Syllogism
5)	p $\therefore p \lor q$	$p \to (p \lor q)$	Addition

6)	$p \wedge q$ $\therefore p$	$(p \wedge q) \rightarrow p$	Simplification
7)	p q $\therefore p \wedge q$	$p \wedge q \rightarrow (p \wedge q)$	Conjuction
8)	$p \vee q$ $\sim p \vee r$ $\therefore q \vee r$	$(p \vee q) \wedge (\sim p \vee r) \rightarrow (q \vee r)$	Resolution

01/10/24

Types of Proof

1) **Direct Proof :-**

Q- Show that if n is odd, then n^2 is also odd.

A- p: n is odd
q: n^2 is odd

Let n is odd
Then $n = 2k+1$, $k = 0, 1, 2, \ldots$

Squaring both sides
$$n^2 = (2k+1)^2 = 4k^2+4k+1$$
$$= 2(2k^2+2k)+1$$
$$= 2t+1 \quad (\because 2k^2+2k = t)$$

So, n^2 is odd
$\therefore$ If n is odd, then n^2 is odd.

Q- Show That if n is even, Then n^2 is also even.

A- p: n is even
q: n^2 is even

Let n is even
Then $n = 2k$ $(k = 0, 1, 2, \ldots)$

Squaring both sides
$n^2 = 4k^2$
$= 2(2k^2)$
$= 2t$ $(\because t = 2k^2)$
So, n^2 is even.
$\therefore$ If n is even, Then n^2 is also even.

Q- Show That if m & n are perfect squares Then mn is also perfect square.

A- p:
q:

Let m & n are perfect squares
$m = k^2$, $n = t^2$

So, $mn = k^2 t^2 = (kt) = s^2$

So, mn is a perfect square

2) Indirect Method

i) Proof of contradiction

Q: Show that $\forall n \in \mathbb{N}$, if $3n+2$ is odd, then n is odd.

Direct

$p: 3n+2$ is odd
$q: n$ is odd

Note: If $\sim q \rightarrow \sim p$ is true, then $p \rightarrow q$ is also true.

p	q	$\sim q$	$\sim p$	$p \rightarrow q$	$\sim q \rightarrow \sim p$
T	T	F	F	T	T
T	F	T	F	F	F
F	T	F	T	T	T
F	F	T	T	T	T

$\sim q$: n is even
$\sim p$: ~~n +~~ $3n+2$ is even

If n is even
Then $n = 2k$
~~Squaring both s~~
$\Rightarrow 3n+2 = 3(2k)+2$
$= 6k+2$
$= 2(3k+1) = 2t \quad (\because t = 3k+1)$

$3n+2$ is even
So, If n is even Then $3n+2$ is even.
$\therefore$ If n is odd Then $3n+2$ is odd.

Q:- Show That if $n = ab$ Then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$.

A:- $p : n = ab$
$q : a \leq \sqrt{n}$ or $b \leq \sqrt{n}$

$\sim q : a > \sqrt{n}$ and $b > \sqrt{n}$
$\sim p : n \neq ab$

$\Rightarrow ab > \sqrt{n}\,\sqrt{n}$
$\Rightarrow ab > n$
$\Rightarrow ab \neq n$

So, if $a > \sqrt{n}$ and $b > \sqrt{n}$ Then $ab \neq n$.

$\therefore$ If $n = ab$ Then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$

ii) Proof by Contradiction

Q:- Show That $\sqrt{2}$ is an irrational no.

A:- $p : \sqrt{2}$ is an irrational no.

$\sim p : \sqrt{2}$ is rational no.

Let $\sqrt{2}$ is a rational no.
$$\sqrt{2} = \frac{a}{b} \quad (\because b \neq 0)$$

$\Rightarrow \sqrt{2}b = a$

Squaring both sides

$\Rightarrow a^2 = 2b^2$ —— (1)

$\Rightarrow a^2$ is even.

$\Rightarrow a$ is even.

Now, let $a = 2c$

Now putting This value in eq^n (1)

$\Rightarrow 4c^2 = 2b^2$

$\Rightarrow b^2 = 2c^2$ —— (2)

$\Rightarrow b^2$ is even

$\Rightarrow b$ is even.

So, a & b have one common factor other Than 1.

So, This is contradiction.

So, $\sqrt{2}$ is an irrational no.

$\therefore \sqrt{2}$ is an irrational no.

04/10/24

Proof By Induction

Show That

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

A- $n=1$

$$1 = \frac{1(1+1)}{2} = 1$$

So, It is true for $n=1$

Let $n = k$ be True

$$1 + 2 + 3 + \dots + k = \frac{k(k+1)}{2}$$

For $n = k+1$

$$1 + 2 + 3 + \dots + k + k+1$$

$$= \frac{k(k+1)}{2} + k+1$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$= \cancel{\frac{k^2 + k + 2k + 1}{2}} \quad \frac{(k+1)(k+2)}{2}$$

$=$

So, it is True for $n = k+1$

$$\therefore 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

Q- Show That

$$1^2 + 2^2 + \dots + n^3 = \frac{n(n+1)(2n+1)}{6}$$

A- For $n = 1$

$$1^2 = \frac{1(1+1)(2+1)}{6}$$

$$\Rightarrow 1 = 1$$

So, it is true for $n = 1$.

Let $n = k$ be true

$$1^2 + 2^2 + \dots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

For $n = k+1$

$$1^2 + 2^2 + \dots + k^2 + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$$

$$= \frac{(k+1)[k(2k+1) + 6(k+1)]}{6}$$

$$= \frac{(k+1)[2k^2 + k + 6k + 6]}{6}$$

$$= \frac{(k+1)[2k^2 + 7k + 6]}{6}$$

$$= \frac{(k+1)[2k^2 + 4k + 3k + 6]}{6}$$

$$= \frac{(k+1)[2k(k+2) + 3(k+2)]}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

So, it is true for $n = k+1$

$$\therefore 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

UNIT – 2

* Cartesian Product of Sets

$A = \{1, 2\}$ $B = \{3, 4\}$

$A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$

Let A & B be 2 sets, The cartesian product of A & B is the set of all ordered pairs of the form (a, b) where $a \in A$ & $b \in B$.

* Relation

A relation from set A to set B is the subset of $A \times B$.

$A = \{1, 2, 3\}$ $B = \{4, 5\}$

$A \times B = \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)\}$

$R = \{(a,b) \mid a \mid b,\ a \in A \ \&\ b \in B).\}$
$= \{(1,4), (1,5), (2,4)\}$

$R = \{(a,b) \mid a > b,\ a \in A \ \&\ b \in B\}$
$= \{\Phi\}$

Relation on a set A

A relation on a set A is the subset of $A \times A$

$A = \{1, 2, 3\}$

$A \times A = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

$R = \{(a,b) \mid a \leq b, \; a, b \in A\}$
$= \{(1,1), (1,2), (1,3), (2,2), (2,3), (3,3)\}$

Properties

1) Reflexive Property

Let A be a set. A relation R on set A is said to be reflexive, if

$\forall a \in A, \; (a,a) \in R.$

$R = \{(a,b) \mid a \mid b, \; a, b \in A\}$
$= \{(1,1), (1,2), (1,3), (2,2), (3,3)\}$

$\therefore$ So, R is reflexive

2) Symmetric Property

Let A be a set. A relation R on set A is said to be symmetric if

$\forall a \in A$, if $(a,b) \in R$ then $(b,a) \in R$

UNIT – 2

* Cartesian Product of Sets

$A = \{1, 2\}$ $\qquad$ $B = \{3, 4\}$

$A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$

Let A & B be 2 sets, The cartesian product of A & B is The set of all ordered pairs of The form (a, b) where $a \in A$ & $b \in B$.

* Relation

A relation from set A to set B is The subset of $A \times B$.

$A = \{1, 2, 3\}$ $\qquad$ $B = \{4, 5\}$

$A \times B = \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)\}$

$R = \{(a, b) \mid a \mid b,\ a \in A\ \&\ b \in B)\}$
$= \{(1,4), (1,5), (2,4)\}$

$R = \{(a, b) \mid a > b,\ a \in A\ \&\ b \in B\}$
$= \{\Phi\}$

Relation on a set A

A relation on a set A is the subset of $A \times A$.

$A = \{1, 2, 3\}$

$A \times A = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

$R = \{(a,b) \mid a \leq b, \ a, b \in A\}$
$= \{(1,1), (1,2), (1,3), (2,2), (2,3), (3,3)\}$

Properties

1) **Reflexive Property**

Let A be a set. A relation R on set A is said to be reflexive, if

$\forall a \in A, \ (a,a) \in R.$

$R = \{(a,b) \mid a \mid b, \ a, b \in A\}$
$= \{(1,1), (1,2), (1,3), (2,2), (3,3)\}$

$\therefore$ So, R is reflexive

2) **Symmetric Property**

Let A be a set. A relation R on set A is said to be symmetric if

$\forall a \in A$, if $(a,b) \in R$ then $(b,a) \in R$

R let $A = \{1, 2, 3, 4\}$

$R = \{(a, b) \mid a < b \text{ or } b < a, \; a, b \in A\}$
$= \{(1,2), (1,3), (1,4), (2,1), (2,3), (3,2), (3,1), (4,1), (4,2), (4,3)\}$

$\therefore$ So, R is symmetric.

29/10/24

Anti-Symmetric

3) Transitive Property

Let A be a set, a relation R on a set A ins said to anti-symmetric if

$(a, b) \in R \;\&\; (b, a) \in R$ Then $a = b$

$A = \{1, 2, 3, 4\}$
$R = \{(1,1), (2,2), (3,3), (4,4), (2,1), (3,1)\}$

$\therefore$ So, R is anti-symmetric

4) Transitive Property

Let A be a set. A relation R on a set A is said to be transitive if

$(a, b) \in R \;\&\; (b, c) \in R$ Then $(a, c) \in R$

Let $A = \{1, 2, 3, 4\}$

$R = \{(1,1), (2,2), (3,3), (2,1), (3,1), (1,2), (3,2), (1,3), (2,3)\}$

∴ So R_e is Transitive

* Equivalence Relation

A relation R on a set A is called an equivalence relation if it is reflexive, symmetric & Transitive

* Poset (Partially Ordered Set)

A relation R on a set A is called Partially ordered relation if it is reflexive, anti-symmetric & transitive & the corresponding set is called poset.

* Reflexive Closure

$A = \{1, 2, 3\}$

$R = \{(1,1), (2,2), (2,3), (3,2)\}$

$R^* = \{(1,1), (2,2), (3,3), (2,3), (3,2)\}$

→ closure of R

R^* is the reflexive closure

* **Symmetric Closure**

$R = \{(1,1), (2,2), (3,3), (3,2)\}$

$R^* = \{(1,1), (2,2), (3,3), (3,2), (2,3)\}$

R^* is symmetric closure

* **Transitive Closure**

$R = \{(1,1), (2,2), (2,3), (3,1)\}$

$R^* = \{(1,1), (2,2), (2,3), (3,1), (2,1)\}$

R^* is Transitive closure.

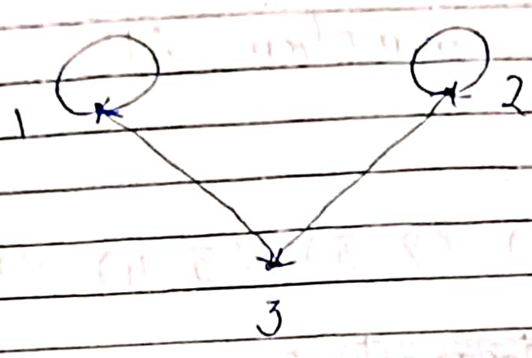
* **Representation of Relation**

1) Matrix Representation

$A = \{1, 2, 3\}$
$R = \{(1,1), (2,3), (2,2), (1,3)\}$

	1	2	3
1	1	0	1
2	0	1	1
3	0	0	0

Graphical Representation



30/10/24

1) For each of the relations on the set $\{1, 2, 3, 4\}$ decide whether it is reflexive, symmetric, antisymmetric or transitive.

a) $\{(2,4), (4,2)\}$
b) $\{(2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\}$
c) $\{(1,1), (2,2), (3,3), (4,4)\}$

2) Determine whether the relation R on set of all integers is reflexive, symmetric, anti symmetric & transitive.

a) $x \neq y$
b) $xy \geq 1$
c) $x = y^2$
d) $x = y+1$ or $x = y-1$
e) $x+y = 0$

3) Let $R = \{(1,1), (2,1), (3,2), (4,3)\}$. Find R^2, R^3, R^4.

1) a) Symmetric
b) Transitive
c) Reflexive, anti-symmetric, All

2) a) $x \neq y$

$R = \{(1,2), (1,3), (2,3), (3,4), (4,5) \ldots\}$

$\therefore$ Symmetric & ~~Transitive~~

b) $xy \geq 1$

$R = \{(1,1), (1,2), (2,3), (2,1), \ldots\}$

$\therefore$ ~~Reflexive~~, symmetric, Reflexive, Transitive

c) $x = y^2$

$R = \{(0,0), (1,1), (4,2), (9,3), \ldots\}$

Anti-symmetric, Transitive

d) $x = y + 1$ or $x = y - 1$

$R = \{(0,1), (0,-1), (1,2), (1,0), \ldots\}$

Nothing

c) $x + y = 0$

$R = \{(0,0), (1,-1), (2,-2), (3,-3), \ldots\}$

Nothing

Composite of Relation

Let R be a relation from A to B

and $R = \{(a,b) \mid a \in A \ \& \ b \in B\}$

Let S be a relation from B to C

$S = \{(b,c) \mid b \in B \ \& \ c \in C\}$

Then

$SoR = \{(a,c) \mid a \in A \ \& \ c \in C\}$

Q- Let R be a relation from $\{1,2,3\}$ to $\{1,2,3,4\}$ given by $\{(1,1), (1,4), (2,3), (3,1), (3,4)\}$ & S be a relation from $\{1,2,3,4\}$ to $\{0,1,2\}$ given by $\{(1,0), (2,0), (3,1), (3,2), (4,1)\}$. Find SoR.

A- ~~SoR = {(1,0), (4,1), (1,2), (2,~~

$SoR = \{(1,0), (1,1), (1,2), (2,1), (2,2), (3,0), (3,1)\}$

3) $R \circ R = \{$

08/11/24

* Poset

1) Comparable Elements & Totally Ordered Set

Let $(S, \preccurlyeq)$ be a poset.

If any 2 elements of this poset are related to each other theyn they are comparable.

If every 2 elements in the poset are comparable then it is called totally ordered set.

Let $A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(a, b) \mid a \geq b,\ a, b \in A\}$

$R = \{(2,1), (3,1), (4,1), (5,1), (6,1),$
$(3,2), (4,2), (5,2), (6,2), (4,3),$
$(5,3), (6,3), (5,4), (6,4), (6,5),$
$(1,1), (2,2), (3,3), (4,4), (5,5),$
$(6,6)\}$

This relation is reflexive (since $(a,a) \in R$)
anti-symmetric (since $(a,b) \in R$ & $(b,a) \in R \Rightarrow b = a$)
Transitive (since $(a,b) \in R$ & $(b,c) \in R \Rightarrow (a,c) \in R$)

$\therefore$ The set A is a totally ordered set.

Q- Let $A = \{1, 2, 3, 5, 6, 8, 10, 12, 14\}$

Let $R_1 = \{(a,b) \mid a \leq b,\ a, b \in A\}$
$R_2 = \{(a,b) \mid a \mid b,\ a, b \in A\}$

Find if (A, R_1) & (A, R_2) are poset.
Also find if (A, R_1) & (A, R_2) are totally ordered set.

A- $R_1 = \{(1,2), (1,3), (1,5), (1,6), (1,8), (1,10), (1,12), (1,14), (2,3), (2,5), (2,6), (2,8), (2,10), (2,12), (2,14), (3,5), (3,6), (3,8), (3,10), (3,12), (3,14), (5,6), (5,8), (5,10), (5,12), (6,8), (6,10), (6,12), (6,14), (8,10), (8,12), (10,12), (10,14), (1,1), (2,2), (3,3), (5,5), (6,6), (8,8), (10,10), (12,12), (14,14)\}$

$R_2 = \{(1,2), (1,3), (1,5), (1,6), (1,8), (1,10), (1,12), (1,14), (2,6), (2,8), (2,10), (2,10), (2,14), (3,6), (3,12), (5,10), (6,12), (1,1), (2,2), (3,3), (5,5), (6,6), (8,8), (10,10), (12,12), (14,14)\}$

R_1 is reflexive, anti-symmetric & transitive.
R_2 is reflexive, anti-symmetric & transitive.

$\therefore$ (A, R_1) & (A, R_2) is poset.

(A, R_1) is totally ordered set.

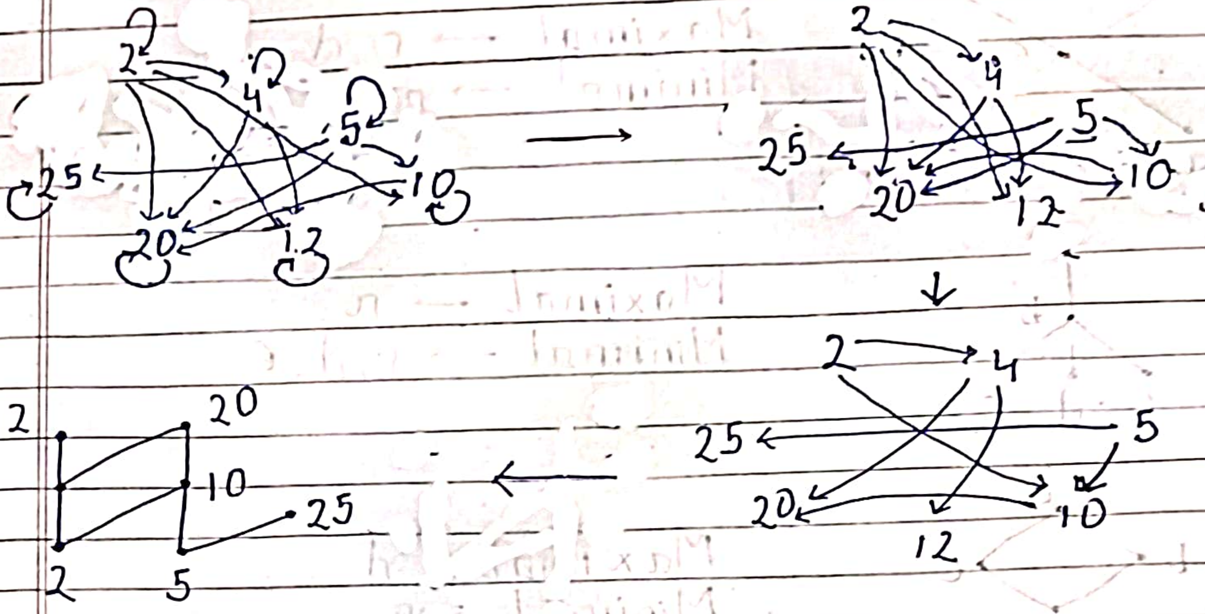
* <u>Hasse Diagram</u>

$R_2 = \{(1,1),(1,2),(1,3),(1,5),(1,6),(1,8),(1,10),$
$(1,12),(2,2),(2,6),(2,8),(2,10),(2,12),$
$(2,14),(1,14),(3,3),(3,6),(5,5),(5,10),$
$(6,6),(6,12),(8,8),(10,10),(12,12),(14,14)\}$

12/11/24

Q: Find the Hasse diagram for (A, R)
where $A = \{2, 4, 5, 10, 12, 20, 25\}$
$R = \{(a, b) \mid a \mid b,\ a, b \in A\}$

A: $R = \{(2,4), (2,10), (2,12), (2,20), (4,12), (4,20), (5,20), (5,25), (10,20), (2,2), (4,4), (5,5), (10,10), (12,12), (20,20), (25,25)\}$

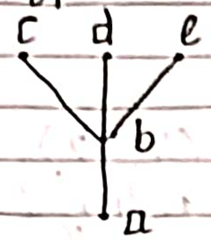


* Maximal & Minimal Element.

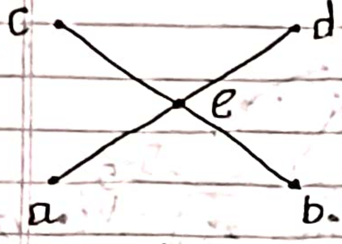
⇒ 'a' is called a maximal in $(S, \leq)$ if there is no $b \in S$ such that $a < b$.

⇒ 'a' is called a minimal in $(S, \leq)$ if there is no $b \in S$ such that ~~$b \leq a$~~ $b < a$.

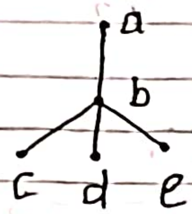
Q- Find the maximal & minimal elements of -



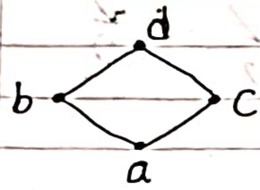
Maximal → c, d, e
Minimal → a



Maximal → c, d
Minimal → a, b

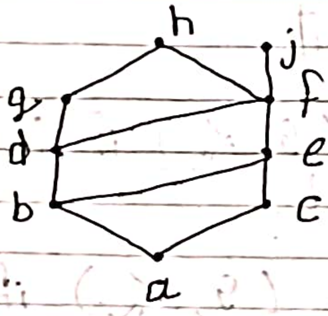


Maximal → a
Minimal → c, d, e



Maximal = d
Minimal = a

Q- Find the upper & lower bound of



{b, c}, {a, d}, {b, c, g}

e, f, j, h   d, f, g, h, j   h
a   a   a

{b, c} => Upper bound = {e, f, h, j}
Lower bound = {a}

{a, d} => Upper bound = {f, g, h, j, d}
Lower bound = {a}

{b, c, g} => Upper bound = {h}
Lower bound = {a}

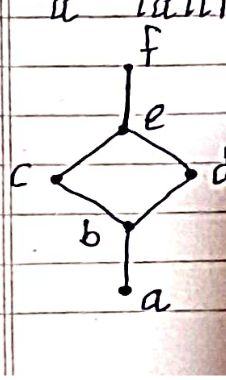
Greatest Lower Bound (GLB) & Lowest Upper Bound (LUB)

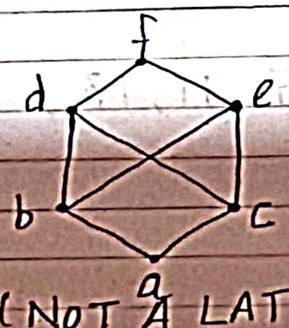
{b, c} => GLB = e
LUB = a
{a, d} => GLB = a
LUB = d
{b, c, g} => GLB = h
LUB = a

13/11/24

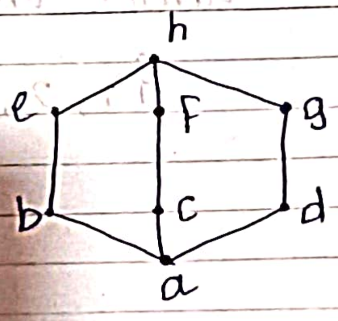
* Lattice

A partially ordered set in which every pair of element has GLB & LUB is called a lattice.

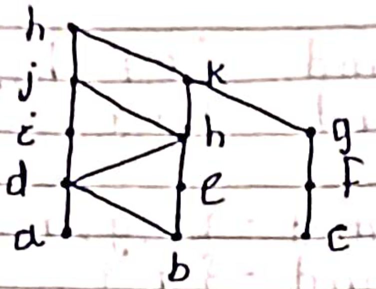




(NOT A LATTICE)



Q-



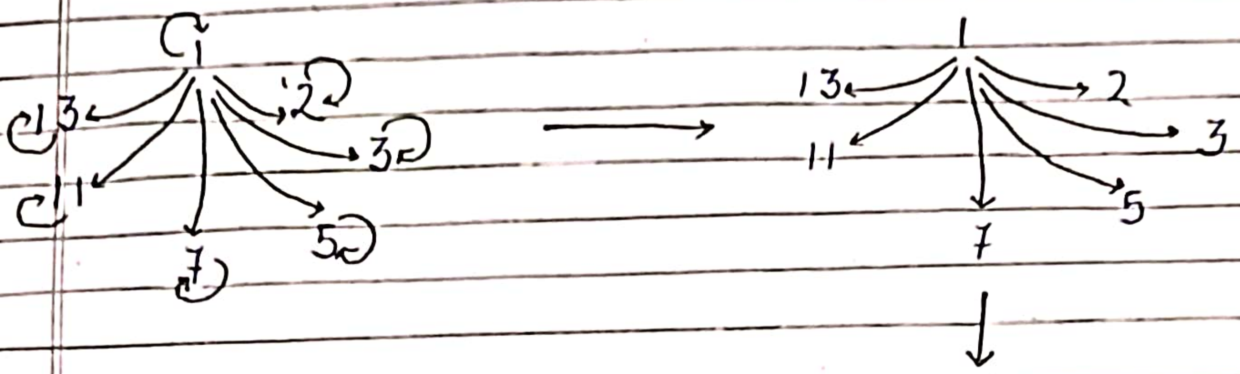
Find - a) Maximal & Minimal
b) Greatest & least
c) Upper bound of {a, b, c}.
d) Lower bound of {f, g, h}
e) Upper bound of {c, f}
f) Lower bound of {j, g}
g) Is it lattice ?

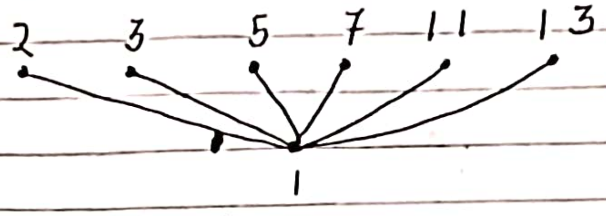
A- a) Maximal = {l, m} Minimal = {a, b, c}
b) No greatest & least
c) {k, l, m}
d) No lower bound
e) l
f) No
g) No

Q- Draw the hasse diagram & verify it's a lattice or not.

{1, 2, 3, 5, 7, 11, 13, - 1}

A- $\{(1,1), (1,2), (1,3), (1,5), (1,7), (1,11), (1,13), (2,2), (3,3), (5,5), (7,7), (11,11), (13,13)\}$





Not a Lattice

* Lexicographic Order

Let $(S_1, \leq)$ & $(S_2, \leq)$ be 2 posets. Then $S_1 \times S_2$ which element is greatest is decided by lexicographic ordering.

$(1, 2, \underline{3}, 4) < (1, 2, \underline{2}, 5)$ ✗

$(\underline{2}, 4, 6, 1) < (\underline{3}, 2, 7, 2)$ ✓

$(1, 2, 9, \underline{10}, 1) < (1, 2, 9, \underline{11}, 1)$ ✓

19/11/24

COUNTING PRINCIPLE

1) **Product Rule**

Suppose a procedure can be break into 2 parts. If there are n_1 ways to do part 1 & for each n_1 ways there are n_2 ways to do part 2 then there are $n_1 \times n_2$ ways to solve the procedure.

Q- There are 2 employees in an office. If there are 12 buildings in the company in how many ways can the building be assigned to the employees?

A- $12 \times 11 = 132$ ways.

Q- How many bit strings can be made of length of 7.

A- $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$

$= 128$ bit strings

Q- How many license plate can be made whose first 3 digits are alphabets & last 3 digits are no.s?

A- 26 26 26 10 10 10

$= 26^3 \times 10^3 =$

$26 \times 25 \times 24 \times 10 \times 9 \times 8$
$=$

2) Sum Rule

If a process can be done in n_1 ways or n_2 ways Then total no. of ways to do it is $n_1 + n_2$.

Q- In how many ways a CEO can be selected out of 350 male employees & 250 female employees ?

A- $n_1 + n_2 = 350 + 250$
$= 600$ ways

Principal of Inclusion - Exclusion

$$|A \cup B| = |A| + |B| - |A \cap B|$$

* Pigeon hole Principle

Statement : Suppose There are k boxes. If k+1 or more objects are to be placed in k boxes Then There is atleast one box That contains 2 or more objects.

Proof

p → k+1 or more objects are to be placed.
q → Atleast one box contain 2 or more objects.

~q → Every box contain exactly one object
~p → k objects are to be placed.

So, ~q → ~p

Suppose There are k boxes, if every box contain exactly one object Then, k objects can be placed in k boxes.

Now, every box contain one object and There are k boxes Then total k objects are placed in k boxes.

So, ~q → ~p is true
⇒ p → q is true.

∴ Therefore, suppose There are k boxes, if k+1 or more objects are to be placed in k boxes Then There is atleast one box That contain 2 or more objects.

Generalised Pigeonhole Principle

If n objects are to be placed in k boxes, then there is atleast one box containing atleast $\left\lceil \frac{N}{k} \right\rceil$ objects.

* Recurrence Relation

A recurrence relation is an eq^n in which the nth term can be obtained by the comb^n of previous terms.

Q- $a_n = a_{n-1} + 2a_{n-2}$

$a_0 = 1$, $a_1 = 3$; Find a_5.

A- $n = 2$ $\quad a_2 = a_1 + 2a_0$
$= 3 + 2$
$= 5$

$n = 3$ $\quad a_3 = a_2 + 2a_1$
$= 5 + 6$
$= 11$

$n = 4$ $\quad a_4 = a_3 + 2a_2$
$= 11 + 10$
$= 21$

$n = 5$ $\quad a_5 = a_4 + 2a_3$
$= 21 + 22$
$= 43$

$n = 6$ $\quad a_6 = a_5 + 2a_4$
$= 43 + 42$
$= 85$

20/11/24

Date
Page

Note A linear homogenous recurrence relation of degree k with const. co-efficients is of the form

$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}$ where

$c_1, c_2, \dots c_k$ are consts.

Q- Solve $a_n = a_{n-1} + 2a_{n-2}$

$a_0 = 2, a_1 = 7$

A- STEP 1 (Find the auxillary eqⁿ)

$r^2 = r + 2$

$\Rightarrow r^2 - r - 2 = 0$ (Auxillary eq^n)

~~$\Rightarrow r(r-1) - 1(r-$~~

$\Rightarrow r^2 - 2r + r - 2 = 0$

$\Rightarrow r(r-2) + 1(r-2) = 0$

$\Rightarrow (r-2)(r+1) = 0$

$\Rightarrow r = 2, -1$ (Auxillary roots)

STEP 2

$\boxed{a_n = \alpha_1 r_1^n + \alpha_2 r_2^n}$ $(r_1 \neq r_2)$

$\boxed{a_n = (\alpha_1 + \alpha_2 n) r^n}$ $(r_1 = r_2)$

$r_1 = 2, \quad r_2 = -1$

$a_n = \alpha_1 2^n + \alpha_2 (-1)^n$

Putting $n = 0$

$a_0 = \alpha_1 2^0 + \alpha_2 (-1)^0$

$\Rightarrow \alpha_1 + \alpha_2 = 2$ ——— 1

Putting $n = 1$

$a_1 = \alpha_1 2^1 + \alpha_2 (-1)^1$

$\Rightarrow 2\alpha_1 - \alpha_2 = 7$

$$\begin{aligned} \alpha_1 + \alpha_2 &= 2 \\ 2\alpha_1 - \alpha_2 &= 7 \\ \hline 3\alpha_1 &= 9 \end{aligned} \Rightarrow \boxed{\alpha_1 = 3}$$

$\alpha_2 = 2 - 3 \Rightarrow \boxed{\alpha_2 = -1}$

So, $\boxed{a_n = 3 \cdot 2^n - (-1)^n}$ (Ans.)

Q- Solve $a_n = 6a_{n-1} - 9a_{n-2}$

$a_0 = 1, \quad a_1 = 6$

A- STEP 1

$r^2 = 6r - 9$

$$r^2 - 6r + 9 = 0$$
$$\Rightarrow r^2 - 3r - 3r + 9 = 0$$
$$\Rightarrow r(r-3) - 3(r-3) = 0$$
$$\Rightarrow (r-3)(r-3) = 0$$
$$\Rightarrow r = 3, 3.$$

STEP 2

$$a_n = (\alpha_1 + \alpha_2 n) 3^n$$

Putting $n = 0$

$$a_0 = \alpha_1 \Rightarrow \boxed{\alpha_1 = 1}$$

Putting $n = 1$

$$a_1 = (\alpha_1 + \alpha_2) 3^1$$
$$\Rightarrow 3(\alpha_1 + \alpha_2) = 6$$
$$\Rightarrow \alpha_1 + \alpha_2 = 2$$
$$\Rightarrow \alpha_2 = 2 - \alpha_1$$
$$\Rightarrow \boxed{\alpha_2 = 1}$$

So, $\boxed{a^n = (1 + n) 3^n}$

Q- Solve $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$
$a_0 = 2, \ a_1 = 5, \ a_2 = 15$

A-
$$r^3 = 6r^2 - 11r + 6$$

$$\Rightarrow r^3 - 6r^2 + 11r - 6 = 0$$

For $r = 1$

$$(1)^3 - 6(1)^2 + 11.1 - 6 \cancel{= 0}$$
$$\Rightarrow 1 - 6 + 11 - 6 \cancel{= 0}$$
$$= 0$$

So, $r = 1$ is a root

$$\frac{r^3 - 6r^2 + 11r - 6}{r - 1}$$
$$= r^2 - 5r + 6$$

$$\cancel{r - 1 \,\big)\, r^3 - 6r^2 + 11r - 6 \,\big(\, r^2 - 5r + 6}$$
$$\cancel{r^3 - r}$$
$$\cancel{-6r^2 - 10r - 6}$$
$$\cancel{5r^2 - 5r}$$
$$\cancel{-r^2 - 15r - 6}$$

$$r - 1 \,\big)\, r^3 - 6r^2 + 11r - 6 \,\big(\, r^2 - 5 + 6$$
$$r^3 \;(\mp)\, r^2$$
$$-5r^2 + 11r - 6$$
$$(\mp) 5r^2 \;(\pm)\, 5r$$
$$6r - 6$$
$$(-) 6r \;(\mp)\, 6$$
$$0$$

$$r^2 - 5r + 6$$
$$= r^2 - 3r - 2r + 6$$
$$= \cancel{r = 1}, r(r-3) - 2(r-3)$$
$$= (r-3)(r-2)$$
$$\Rightarrow r = 1, 2, 3$$

STEP 2

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n + \alpha_3 r_3^n$$

$$a_1 = \alpha_1 1^n + \alpha_2 2^n + \alpha_3 3^n$$

Putting $n = 0$

$$a_0 = \alpha_1 + \alpha_2 + \alpha_3$$
$$\Rightarrow \alpha_1 + \alpha_2 + \alpha_3 = 2 \quad \text{--- } (1)$$

Putting $n = 1$

$$a_1 = \alpha_1 + 2\alpha_2 + 3\alpha_3$$
$$\Rightarrow \alpha_1 + 2\alpha_2 + 3\alpha_3 = 5 \quad \text{--- } (2)$$

Putting $n = 2$

$$a_2 = \alpha_1 + 4\alpha_2 + 9\alpha_3$$
$$\Rightarrow \alpha_1 + 4\alpha_2 + 9\alpha_3 = 15 \quad \text{--- } (3)$$

$$\alpha_1 = 1, \quad \alpha_2 = -1, \quad \alpha_3 = 2$$

So, $\boxed{a_n = 1 \cdot 1^n - 1 \cdot 2^n + 2 \cdot 3^n}$

PERMUTATION AND COMBINATION

$$^nP_r = \frac{n!}{(n-r)!} \qquad ^nC_r = \frac{n!}{(n-r)! \times r!}$$

Q- In how many ways can seven books be arranged on a shelf?

Q- How many 3 digit even no.s can be made using the digits 1, 2, 3, 4, 5, 6, 7 if no digits are repeated?

Q- It is required to sent 5 men & 4 women in a row so that the women occupy the even places. How many such arrangements are possible

ANSWERS

1) $^7P_7 = \frac{7!}{(7-7)!} = 7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

2) $\underline{5} \times \underline{4} \times \underline{3} = 60$

$$^5P_2 \times {^3P_1} = \frac{5!}{\cancel{3!}} \times \frac{\cancel{3!}}{2!}$$

$$= \frac{5 \times 4 \times 3 \times 2!}{2!} = 60$$

3) $\underline{m}\ \underline{w}\ \underline{m}\ \underline{w}\ \underline{m}\ \underline{w}\ \underline{m}\ \underline{w}\ \underline{m}$

$^5P_5 \times {}^4P_4 = 5! \times 4!$

$= 2880$

Q- How many 5 cards set can be selected from a deck of 52 cards?

Q- How many ways are there to select 10 players from 20 member team?

Q- How many cards must be selected from a deck of 52 cards?

Q- How many ways are there to select 10 pages from 20 member team?

Q- How many cards must be selected from a deck of 52 cards to guarantee that atleast 3 cards are of same suit?

Q- How many students must be in class to guarantee that atleast 2 students score same mark, if the range of rank is from 0-100?

26/11/24

UNIT - 3

* Graph Theory

Graph

A graph G(V, E) consists of non-empty set of vertices V & set of edges E.

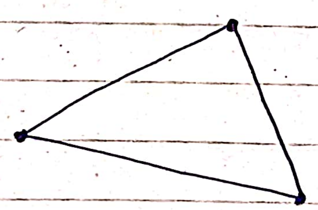
The point where the lines intersect is called vertex.
The lines that are containing vertex is called edges.

Types of Graphs

Graph
- Undirected
  - Simple
  - Multi
  - Pseudo
- Directed
  - Simple
  - Multi
  - Pseudo

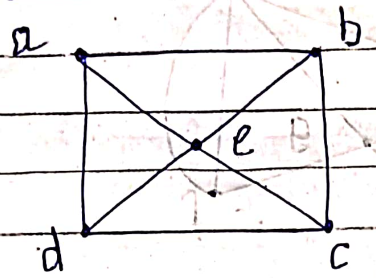
1) Undirected Group

A graph G(V, E) called an undirected group if the edges doesn't contain any dirⁿ.



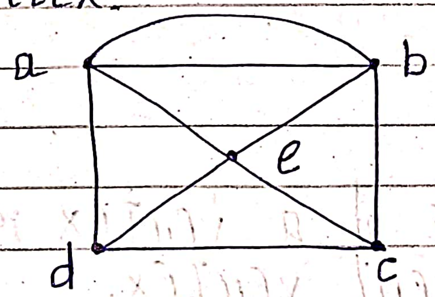
i) Simple Undirected Graph

A graph G(V, E) is called simple graph if There is only one edge betⁿ any 2 vertices.



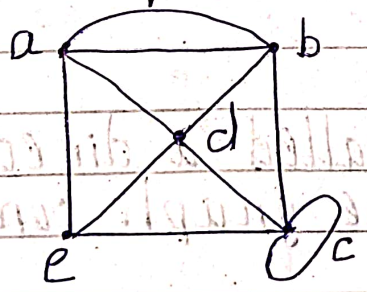
ii) Multi Undirected Graph

A graph G(V, E) is called multi ~~▬~~ graph if There are more Than one edges betⁿ any vertex.



iii) Pseudo Undirected Graph

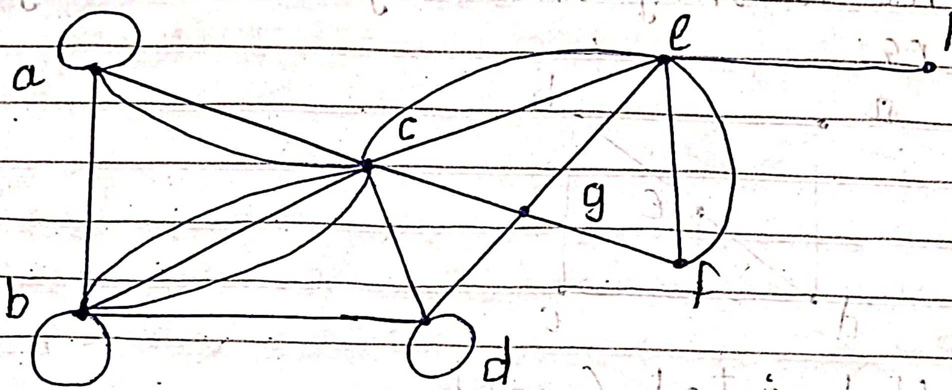
A graph G(V, E) is called pseudo graph if it contain loops.



: contains loop in vertex C,

Degree of Vertex

The degree of vertex is the no. of edges associated with it.



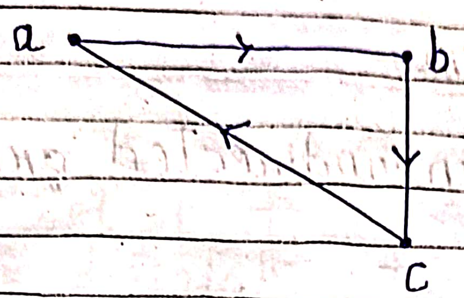
Deg (c) =
Deg (f) =
Deg (h) =
Deg (b) =
Deg (d) =

Whenever the degree of a vertex is 1, it is called as pendent vertex.

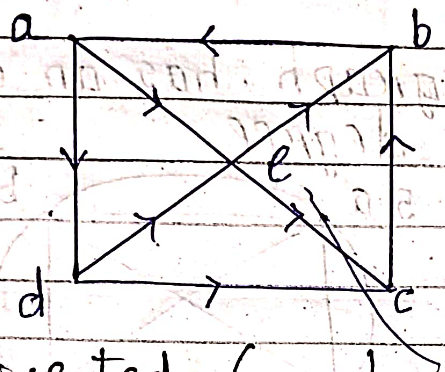
When the degree of vertex is 0, it is called as isolated vertex.

~~Un~~directed Group

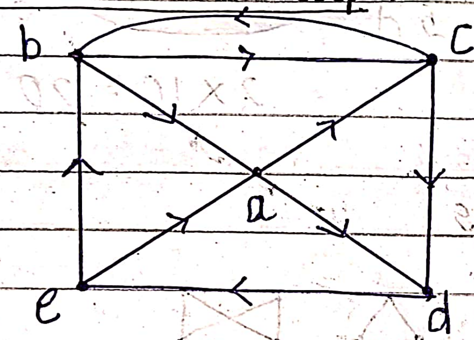
A graph G(v; e) is called a directed graph if the edges of the graph contains dirⁿ.



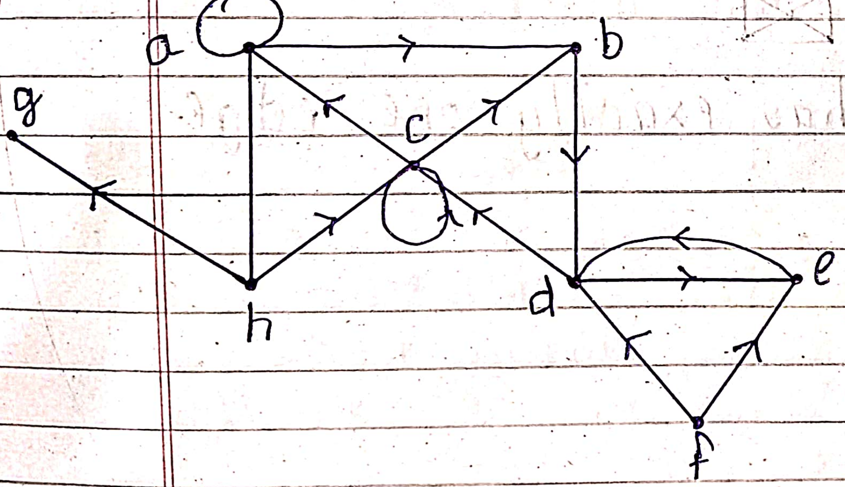
Simple Directed Graph



Multi Directed Graph



Degree of Vertex



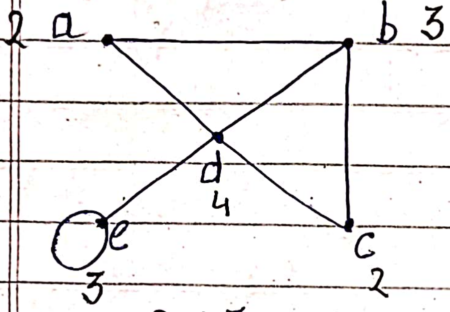
27/11/24

* Hand Shaking Theorem:

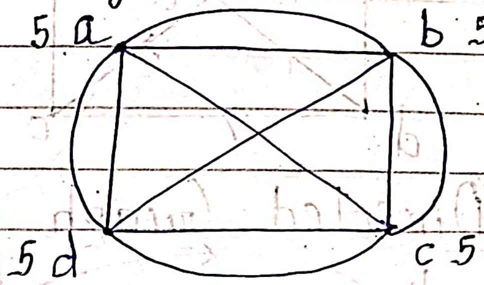
Let $G = (V, E)$ be an undirected graph with m edges Then

$$2m = \sum \deg(v)$$

* An undirected ~~group~~ graph has an even no. of vertices of odd degree.

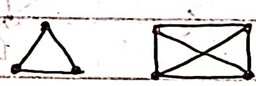
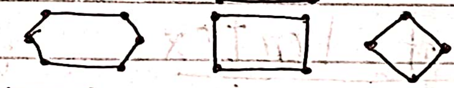
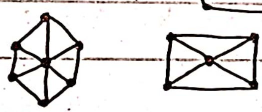


$2 \times 7 = 14$



$2 \times 10 = 20$

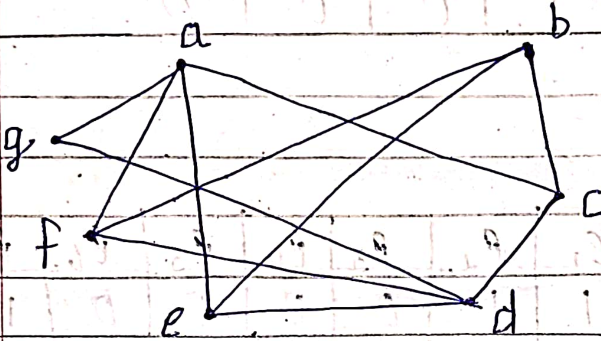
* Some Special Graphs

1) Complete Graph 
2) Cycle 
3) Wheel 

⇒ Every 2 vertex has exactly one edge bet^{n} them.

* Bipartite Graph

A graph G(V, E) is called a bipartite graph if the set of its vertices can be divided into 2 disjoint sets such that there is an edge betⁿ one set to another & there are no edges within itself.



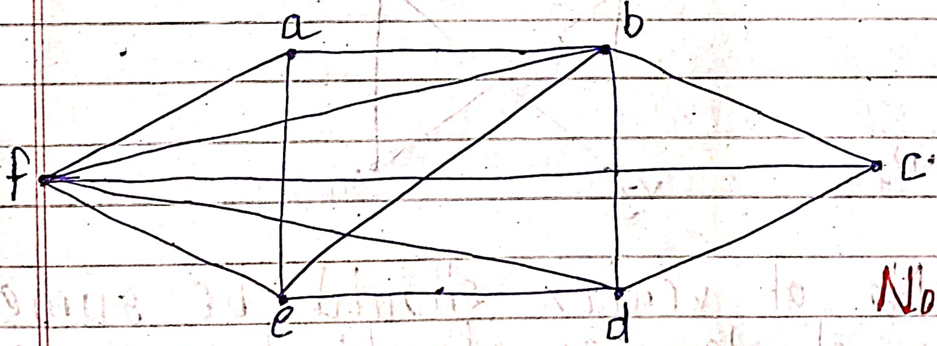
$V_1 \rightarrow$ a b d

$V_2 \rightarrow$ g c f e

2 vertices are said to be adjacent if there is an edge betⁿ them.

Non-adjacent vertex are kept in ~~separate~~ same set.

Adjacent vertices are kept in ~~same~~ separate set.



Not a bipartite Graph

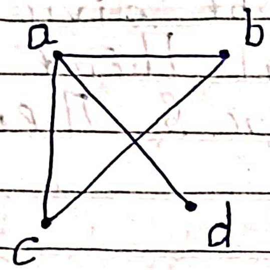
$V_1 \rightarrow$

$V_2 \rightarrow$

28/11/24

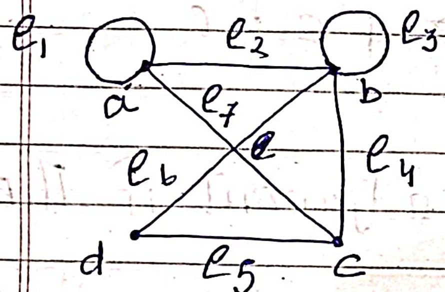
* ## Representation of Graph

1) Adjacency Matrix



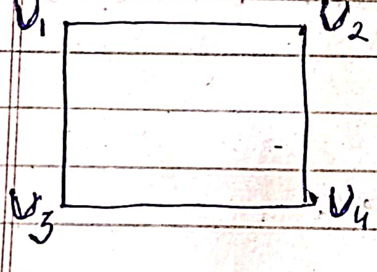
	a	b	c	d
a	0	1	1	1
b	1	0	1	0
c	1	1	0	0
d	1	0	0	0

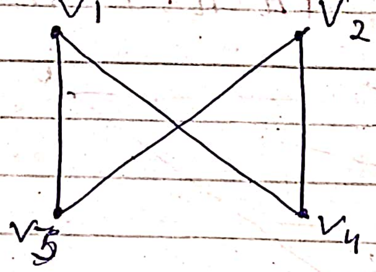
2) Incidence Matrix



	e_1	e_2	e_3	e_4	e_5	e_6	e_7
a	1	1	0	0	0	0	1
b	0	1	1	1	0	1	0
c	0	0	0	1	1	0	1
d	0	0	0	0	1	1	0

* ## Graph Isomorphism





STEP 1 No. of vertex should be same.
No. of degrees should be same

STEP 2 : Find one-one & onto relation bet" 2 graphs

STEP 3 Find the adjacency matrix

deg (U_1) → 2	deg (V_1) → 2
deg (U_2) → 2	deg (V_2) → 2
deg (U_3) → 2	deg (V_3) → 2
deg (U_4) → 2	deg (V_4) → 2

$U_1 = V_1$
$U_2 = V_3$
$U_3 = V_4$
$U_4 = V_2$

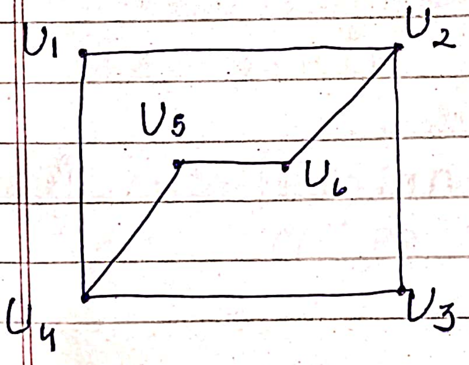
	U_1	U_2	U_3	U_4
U_1	0	1	1	0
U_2	1	0	0	1
U_3	1	0	0	1
U_4	0	1	1	0

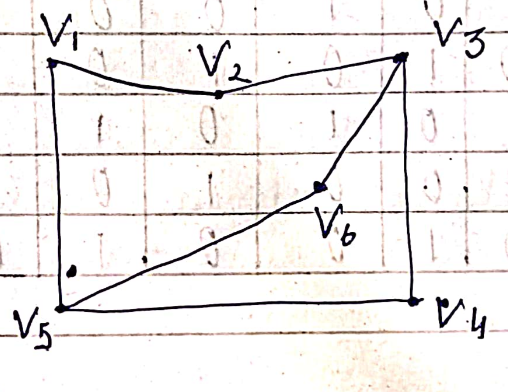
	V_1	V_3	V_4	V_2
V_1	0	1	1	0
V_3	1	0	0	1
V_4	1	0	0	1
V_2	0	1	1	0

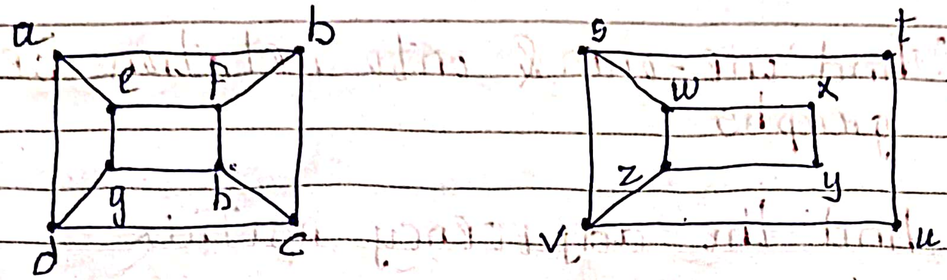
∴ The graph are isomorphic

Q- Check whether the graph are isomorphic.

a)





b) 

a) $U_1 = V_4 / V_6 \longrightarrow V_4$
$U_2 = V_3 / V_5 \longrightarrow V_3$
$U_3 = V_4 / V_6 \longrightarrow V_6$
$U_4 = V_3 / V_5 \longrightarrow V_5$
$U_5 = V_2 / V_1 \longrightarrow V_1$
$U_6 = V_2 / V_1 \longrightarrow V_2$

	U_1	U_2	U_3	U_4	U_5	U_6
U_1	0	1	0	1	0	0
U_2	1	0	1	0	0	1
U_3	0	1	0	1	0	0
U_4	1	0	1	0	1	0
U_5	0	0	0	1	0	1
U_6	0	1	0	0	1	0

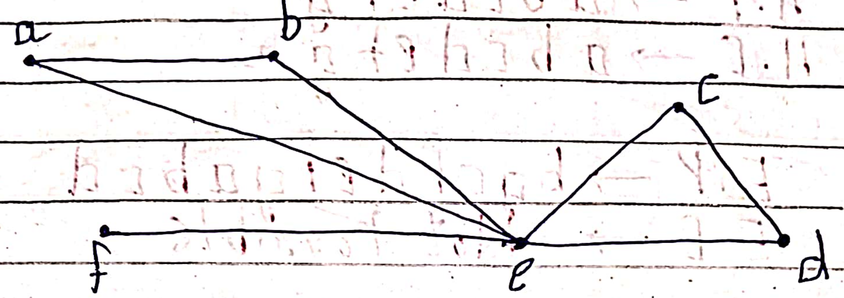
$\therefore$ Both the graphs are isomorphic

	V_4	V_3	V_6	V_5	V_1	V_2
V_4	0	1	0	1	0	0
V_3	1	0	1	0	0	1
V_6	0	1	0	1	0	0
V_5	1	0	1	0	1	0
V_1	0	0	0	1	0	1
V_2	0	1	0	0	1	0

03/12/24

* Path

A path is a sequence of edges that starts at a vertex & travels from vertex to vertex along the edges.



A circuit is a path that starts ~~from~~ & ends at the same vertex.

1) Euler path & Circuit

⇒ An Euler path is a path that contains every edge exactly once.

⇒ Euler circuit is a circuit that contains every edge exactly once.

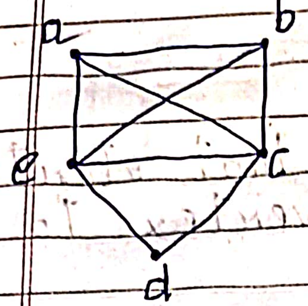
2) Hamiltonian Path & Circuit

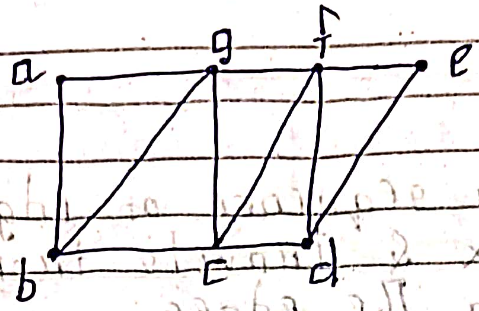
⇒ A hamiltonian path is a path that contains every vertex exactly once.

⇒ A hamiltonian circuit is a circuit that contains every vertex exactly once.

Whenevere

Date
Page



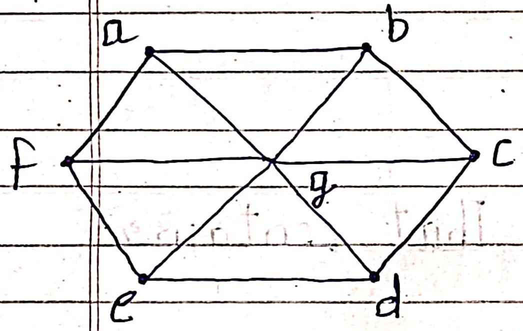


H.P → a b c d e f g
H.C → a b c d e f g a

H.P → a b c d e
e d c a b
H.C → a b c d e a

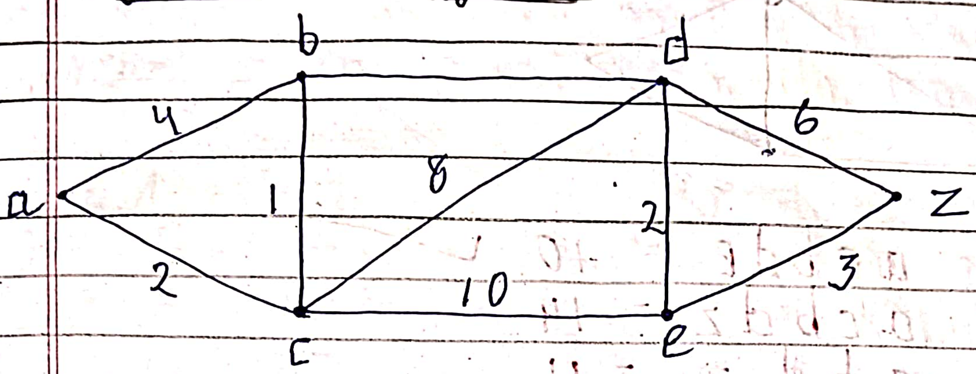
E.P → b g c f d e f g a b c d
E.C → Not Possible

E.P → a b c d e a c e b
E.C →



NOTE In a graph; if there are exactly 2 vertex of odd degree, then it doesn't contain E.C It only contain E.P.

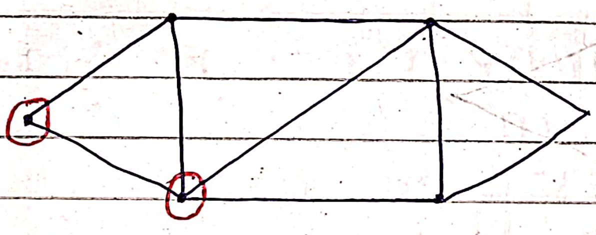
* Dijkstra's Algorithm



Find the shortest path from a to z.

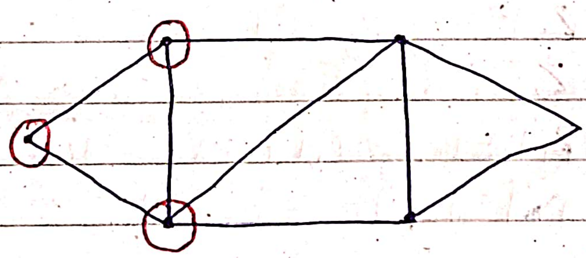
STEP 1

a - c = 2 ✓
a - b = 4



STEP 2

a c b = 3 ✓
a c d = 10
a c e = 12
a b = 4

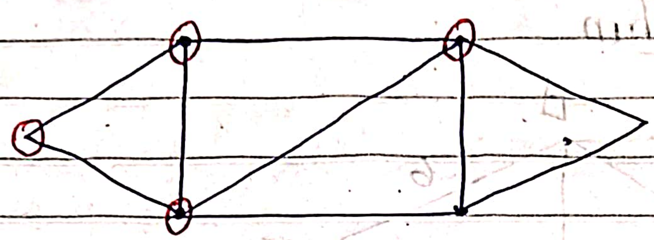


STEP 3 acbd = 8

acd = 10

ace = 12

abd = 9



STEP 4 acbde = 10

acbdz = 14

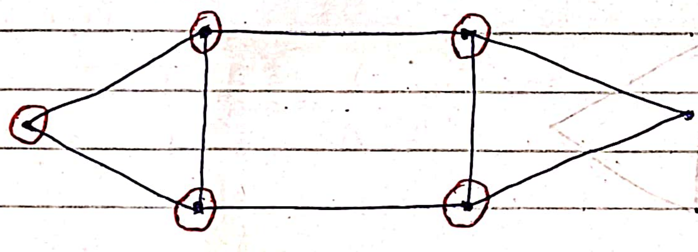
abdz = 11

ace = 12

acde = 12

acdz = 16

abde = 15



STEP 5 acbdez = 13

acbdz = 14

acez = 15

abdz =

abdez =

acdez =

acdz =

04/12/24

Date ______
Page ______

* Connected Graph

An undirected graph is called connected if there exist atleast one path betⁿ every 2 of its vertices.

* Planar Graph

A graph that can be drawn on a plane without edges intersecting each other.

* Euler Formula

Let G be a connected planar graph then

$$\boxed{r = e - v + 2}$$

where r is the region
e is edges
v is vertices

* Four Colour Theorem

The chromatic no. of a planar graph is at most four.

The graph betⁿ the graphs is known as "region".

* Trees

⇒ A tree is a connected undirected graph with no simple circuit.

A rooted tree is a tree with one vertex as root and all other vertex directed away from it.

If v is a vertex, then parent of v is the unique vertex u such that there is a edge bet^n u & v.

v is called the child of u.

Vertices with same parents are called sibling

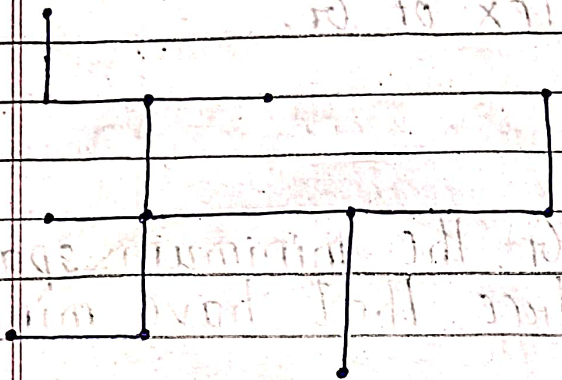
The ancestor of vertex v are all those vertices that contain v.

The descendents of vertex v are all the vertic that are contained by v.

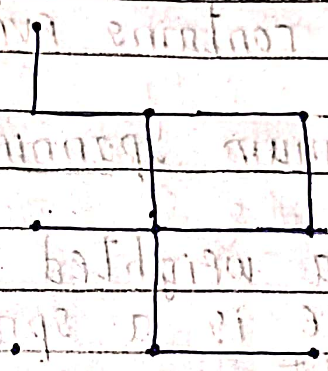
The vertices that have children are called leaf.

A rooted tree is called n-ary tree if every internal vertex have at most n-children.

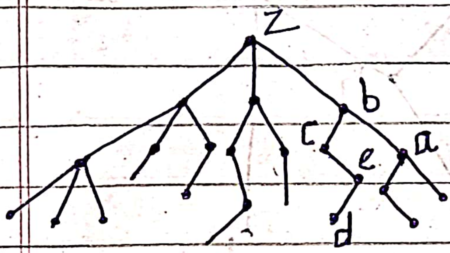
A rooted tree is called full m-ary tree if every internal vertex have exactly m-children.



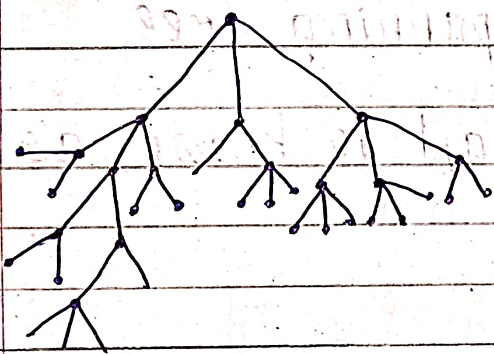
(This is tree)



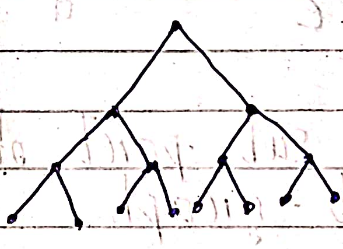
(Not a tree)



b is parent of a.
a is child of b
c is child of b



3-ary tree
Every vertex should have min.m child.



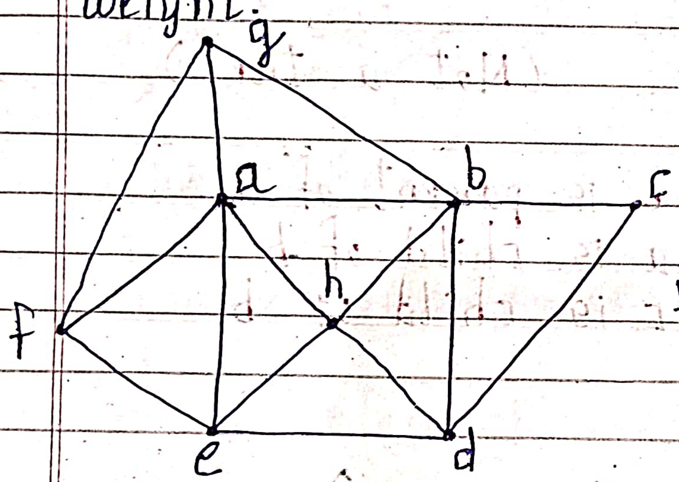
Full m-ary tree
Every vertex should have exactly m child.

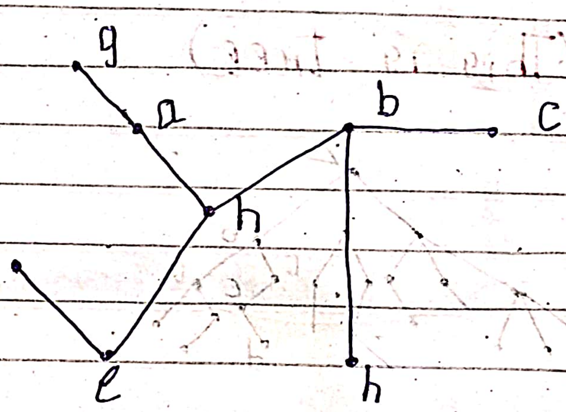
Spanning Tree

Let G be a simple graph. A spanning Tree of G is a subgraph of G That is a Tree and contains every vertex of G.

Minimum Spanning Tree

In a weighted graph G, The minimum spanning tree is a spanning tree That have minimum weight.





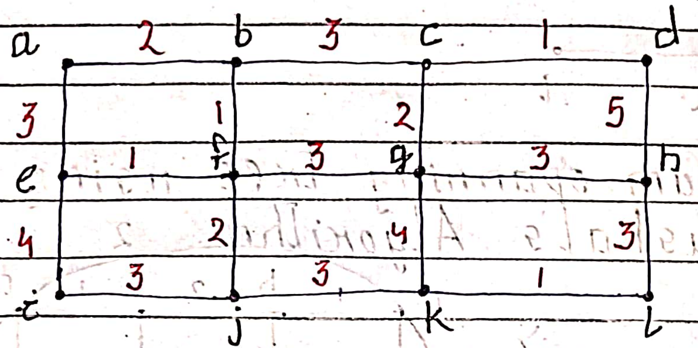
Spanning Tree

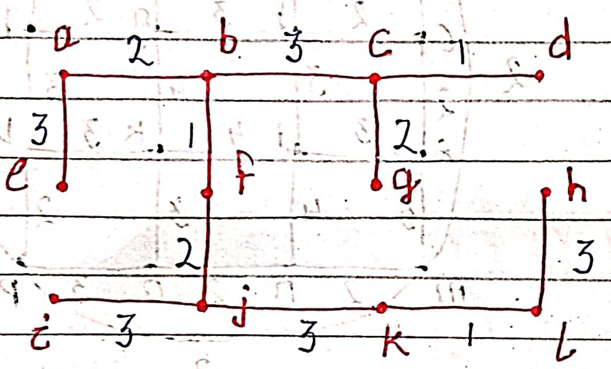
The sub part of The graph is known as sub-graph.

06/12/24

* Algorithm to Find Minimum Spanning Tree

1) Prism's Algorithm

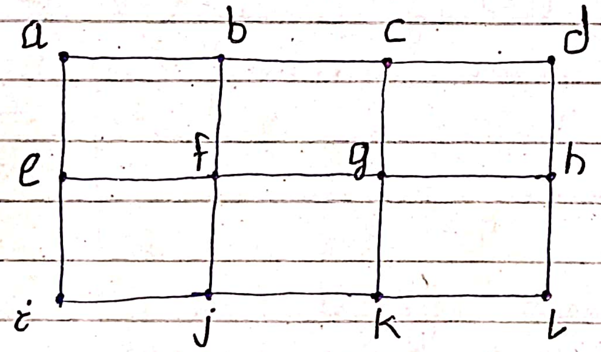




b - f = 1
b - a = 2
f - j = 2
a - e = 3
b - c = 3
c - d = 1
c - g = 2
j - k = 3
k - l = 1
l - h = 3
j - e = 3

$\boxed{\text{Total} = 24}$

2) Kruskal's Algorithm

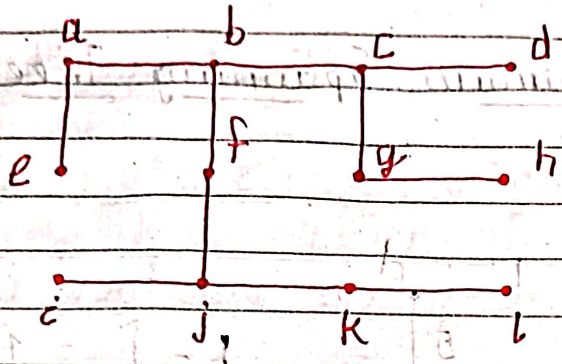


k - l = 1
c - d = 1
b - f = 1
e - f = 1
c - g = 2
b - a = 2
f - j = 2
b - c = 3
g - h = 3
j - k = 3

i - j = 3
a - e = 3

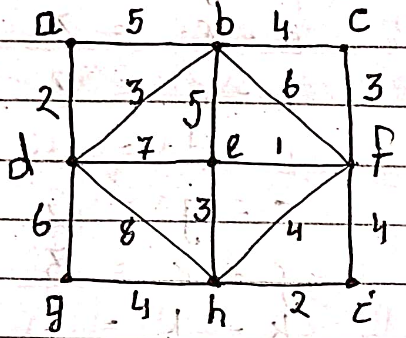
$\boxed{\text{Total} = 24}$

Whichever is smallest, take it first.

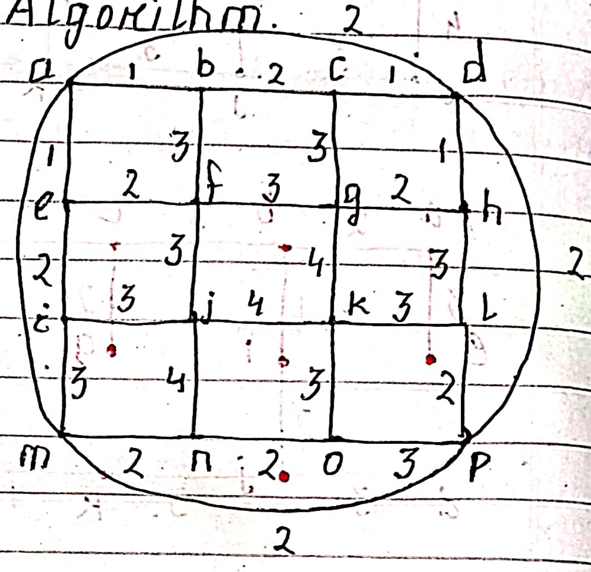


Q- Find the minimum spanning tree using Prism's & Kruskal's Algorithm.

i)



ii)



A- i) Prism Algorithm

11/12/24

BOOLEAN ALGEBRA

$1+1=1$ $\quad$ $1.1=1$
$1+0=1$ $\quad$ $1.0=0$ $\quad$ $\overline{1}=0$
$0+1=1$ $\quad$ $0.1=0$ $\quad$ $\overline{0}=1$
$0+0=0$ $\quad$ $0.0=0$

Q- Find The value of

$(\overline{1.0}) + \overline{(0+1)}\ (1+1) = 1 + (\overline{1.1})$
$= 1+0$
$= 1$

Q- Find The logical complement of

$(\overline{1.1}) + (\overline{0.0}).(\overline{1+0}) + (0+1)$
$= 0 + 1 . 0 + 1$
$= 1$

$\sim(T \wedge T) \vee \sim(F \wedge F) . \wedge \sim(T \vee F) \vee (F \vee T) \equiv T$

Logical Equivalent

Q- Let $f(x,y) = x\overline{y}$. Find all The values of $f(x,y)$.

x	y	$\overline{y}$	$x\overline{y}$
1	1	0	0
1	0	1	1
0	1	0	0
0	0	1	0

Q- $f(x,y,z) = x\bar{y} + \bar{z}$. Find all values.

A-

x	y	z	$\bar{y}$	$\bar{z}$	$x\bar{y}$	$x\bar{y}+\bar{z}$
0	0	0	1	1	0	1
0	0	1	1	0	0	0
0	1	0	0	1	0	1
0	1	1	0	0	0	0
1	0	0	1	1	1	1
1	0	1	1	0	1	1
1	1	0	0	1	0	1
1	1	1	0	0	0	0

Q- Find the duality of

$(\bar{x}.1) + (y+0) + (z+1)$

Duality → Interchange 0 & 1
Interchange · & +

A- $(\bar{x}+0).(y.1).(z.0)$

K-Map

Q- Minimize the SOP

i) $xy\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}$

ii) $xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z$

A- i)

$x \backslash yz$	00	01	11	10
0	1		~~1~~	1
1			~~1~~	1

$\cancel{\overline{xyz} +}$ $y\bar{z} + x\bar{z}$

ii)

$x \backslash yz$	00	01	11	10
0	1	1	1	
1	1	1	1	1

$x + \bar{y} + z$

* Coding Theory

Word → Sequence of 0 & 1.

Code → Collection of words

Weight → No. of 1's in a word

x : 1100110011
$W(x) = 6$

* Hamming Distance

The hamming dist. betn x & y is the weight of $x \oplus y$.

Q- Find the hamming dist. betⁿ
x : 10110001
y : 00110110

$1 \oplus 1 = 0$
$1 \oplus 0 = 1$
$0 \oplus 1 = 1$
$0 \oplus 0 = 0$

A- $x \oplus y = 10000111$

$\therefore W(x \oplus y) = 4$

So, hamming dist. = 4.

* Encoding Funcⁿ

It is defined as

$$e : B^m \rightarrow B^n$$

where $e(m, n)$ is the encoding funcⁿ.

* Error Detection

Let $e : B^m \rightarrow B^n$ be the encoding funcⁿ. If the min. dist. of e is $k+1$, then e can detect k or less errors.

Error Correction

Let $e : B^m \rightarrow B^n$ be the encoding funcⁿ, if the min. dist. of e is $2k+1$, these e can correct k or less errors.

Q- Consider the encoding func^n e(2,4) given by

$e(00) = 0000 = x_1$
$e(01) = 1011 = x_2$
$e(10) = 0110 = x_3$
$e(11) = 1100 = x_4$

How many errors e can detect & correct ?

A- $x_1 \oplus x_2 = 1011 = 3$
$x_1 \oplus x_3 = 0110 = 2$
$x_1 \oplus x_4 = 1100 = 2$
$x_2 \oplus x_3 = 1101 = 3$
$x_2 \oplus x_4 = 0111 = 3$
$x_3 \oplus x_4 = 1010 = 2$

The min. dist. of e = 2
So, for detection
$e = k+1$
$\Rightarrow k+1 = 2$
$\Rightarrow \boxed{k = 1}$

So, e can detect 1 or less errors.

For correction, $e = 2k+1$
$\Rightarrow 2k+1 = 2$
$\Rightarrow \boxed{k = 0.5}$

So, e can correct 0 errors.

12/12/24

Q- Show that the encoding func^n e(2, 5) given by

e(00) = 00000
e(01) = 01110
e(10) = 10101
e(11) 11011
is a group code.

A-

$\oplus$	00000	01110	10101	11011
00000	00000	01110	10101	11011
01110	01110	00000	11011	10101
10101	10101	11011	00000	01110
11011	11011	10101	01110	00000

Closure

Any 2 element will operate & the result will be from the same elements.

From table it is clear that all element satisfy closure. i.e. 10101 $\oplus$ 01110 = 11011

Associativity

(11011 $\oplus$ 01110) $\oplus$ 10101
= 11011 $\oplus$ (01110 $\oplus$ 10101)
0 = 0

Identity

Here the identity is 00000.

Inverse

Inverse exist

$01110 \oplus 01110 = 00000$
$11011 \oplus 11011 = 00000$
$10101 \oplus 10101 = 00000$
$01110 \oplus 01110 = 00000$

Commutative

$01110 \oplus 10101 = 10101 \oplus 01110$

∴ So, its a group code.

Q- Consider the parity matrix given by

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

& determine the encoding func^n.

$m = 2, \; n = 5$

$e(00) = 00$

Date
Page

$$[x_1\ x_2\ x_3] = [0\ 0]\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}$$

$$= [0\times1 \oplus 0\times0 \quad 0\times1 \oplus 0\times1 \quad 0\times0 \oplus 0\times1]$$

$$= [0\ 0\ 0]$$

$\therefore e(00) = 00000$

$e(01) = 01$

$$[x_1\ x_2\ x_3] = [0\ 1]\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}$$

$$= [0\times1 \oplus 1\times0 \quad 0\times1 \oplus 1\times1 \quad 0\times0 \oplus 1\times1]$$

$$= [0\ 1\ 1]$$

$\therefore e(01) = 01011$

$e(10) = 10$

$$[x_1\ x_2\ x_3] = [1\ 0]\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}$$

$$= [1\times1 \oplus 0\times0 \quad 1\times1 \oplus 0\times1 \quad 1\times0 \oplus 0\times1]$$

$$= [1\ 1\ 0]$$

$\therefore e(10) = 10110$

$e(11) = 11$

$$[x_1\ x_2\ x_3] = [1\ 1]\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}$$

$$= [1\times1 \oplus 1\times0 \quad 1\times1 \oplus 1\times1 \quad 1\times0 \oplus 1\times1]$$

$$= [1\ 0\ 1]$$

$\therefore e(11) = 11101$

13/12/24

GROUP THEORY

Date ____
Page ____

Group : A non-empty set G along with the operator '$*$' is called a group if it satisfies the following properties :-

1) Closure

$\forall a, b \in G, \ a * b \in G$

2) Associative

$\forall a, b, c \in G, \ (a*b)*c = a*(b*c)$

3) Identity

$\forall a \in G, \ \exists \ e \in G$, such that
$a*e = a = e*a$

4) Inverse

$\forall a \in G, \ \exists \ b \in G$, such that
$a*b = e$

Q- Verify whether $(\mathbb{Z}, +)$ is a group.

1) Closure

$\forall a, b \in \mathbb{Z}, \ a+b \in \mathbb{Z}$

2) Associative

$\forall a, b, c \in \mathbb{Z}$, $a+(b+c) = (a+b)+c$

3) Identity

$\forall a \in \mathbb{Z}$, $a+e = a$ $\therefore$ So, 0 is the identity
$\Rightarrow e = 0$

4) Inverse

$\forall a \in \mathbb{Z}$, $\exists b \in \mathbb{Z}$ such that
$a+b = e = 0$
$\Rightarrow b = (-a)$
So, $\forall a$, $(-a)$ is the inverse.

$\therefore$ Thus, $(\mathbb{Z}, +)$ is a group.

Q- Verify whether $(\mathbb{Z}, \times)$ is a group.

Q- Verify whether $(\mathbb{R}, \times)$ is a group.

1) Closure

$\forall a, b \in \mathbb{Z}$, $a \times b \in \mathbb{Z}$

2) Associative

$\forall a, b, c \in \mathbb{Z}$, $a \times (b \times c) = (a \times b) \times c$

3) <u>Identity</u>

$\forall a \in \mathbb{Z}$, $a \times e = a$ $\Rightarrow e = 1$ $\therefore$ So, 1 is the identity.

4) <u>Inverse</u>

$\forall a \in \mathbb{Z}$, $\exists b \in \mathbb{Z}$ such that

$a \times b = e = 1$

$\Rightarrow b = \frac{1}{a}$ (which is not possible in $\mathbb{Z}$)

$\therefore$ Thus, $(\mathbb{Z}, \times)$ is not a group.

1) <u>Closure</u>

$\forall a, b \in \mathbb{R}$, $a \times b \in \mathbb{R}$

2) <u>Associative</u>

$\forall a, b, c \in \mathbb{R}$, $a \times (b \times c) = (a \times b) \times c$

3) <u>Identity</u>

$\forall a \in \mathbb{R}$, $a \times e = a$ $\Rightarrow e = 1$ $\therefore$ So, 1 is the identity.

4) <u>Inverse</u>

$\forall a \in \mathbb{R}$, $\exists b \in \mathbb{R}$ such that

$$a \times b = e = 1$$
$$\Rightarrow b = \frac{1}{a}$$

But for 0, inverse will be $\frac{1}{0}$ which is not defined.

$\therefore$ Thus, $(\mathbb{R}, \times)$ is not a group.

Order of Group

The no. of elements present in a group.

Order of an element

Let G be a group. Let $a \in G$. The order of 'a' is the smallest element 'n' such that $a^n = e$

Q- Verify whether $(\underbrace{\{1, -1, i, -i, j, -j, k, -k\}}_{A}, \times)$ is a group.

1) Closure

$-1 \times 1 = -1 \in A$
$i \times -i = -i^2 \in A \quad = (1)$
$j \times -j = -j^2 \in A \quad (1)$
$k \times -k = -k^2 \in A \quad (1)$

$\therefore$ Closure is satisfied