Prob 3 - $\alpha_p = 3dB$

$\omega_c = \omega_p = 2\pi \times 1000 = 2000\pi$ rad/s

$\alpha_s = 10 dB$

$\omega_s = 2\pi \times 350 = 700\pi$ rad/s

$T = \frac{1}{f_s} = \frac{1}{5000} = 2\times10^{-4}$ sec

Butterworth filter

signal frequencies:-

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p T}{2} = \frac{2}{2\times10^{-4}}\tan\left(\frac{2000\pi\times2\times10^{-4}}{2}\right)$$

$$= 10^4\tan(0.2\pi)$$

$$= 7265 \text{ rad/s}$$

$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s T}{2} = \frac{2}{2\times10^{-4}}\tan\left(\frac{700\pi\times2\times10^{-4}}{2}\right)$$

$$= 10^4\tan(0.07\pi)$$

$$= 2235 \text{ rad/s}$$

So, first need to design LPF with given specification and then using suitable transformation to obtain HPF transfer function.

$$N = \frac{\log\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)} = \frac{\log 3}{\log 3.25}$$

$$= \frac{0.4771}{0.5118}$$

$$= 0.932 \simeq 1$$

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For, 1^{st} order Butterworth filter, $\Omega_c = 1$ rad/s

$$H(s) = \frac{1}{1+s}$$

For, highpass filter, $\Omega_c = \Omega_p = 7265$ rad/s

$$s \rightarrow \frac{\Omega_c}{s} \quad \text{i.e} \quad s \rightarrow \frac{7265}{s}$$

$\therefore$ TF of HPF is -

$$H(s) = \frac{1}{1+s}\Bigg|_{s = \frac{7265}{s}}$$

using BLT :-

$$H(s) = \frac{1}{1+s}\Bigg|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{s}{s+7265}\Bigg|_{s = \frac{2}{2\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{1000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{1000\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 7265}$$

$$= \frac{0.5792\,(1-z^{-1})}{1 - 0.1584z^{-1}}$$

$$\therefore \boxed{H(z) = \frac{0.5792\,(1-z^{-1})}{1 - 0.1584z^{-1}}}$$

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$$\therefore \quad \frac{Y(z)}{X(z)} = \frac{0.5792\,(1-z^{-1})}{1-0.1584z^{-1}}$$

$$Y(z)\left[1-0.1584z^{-1}\right] = X(z)\left[0.5792\,(1-z^{-1})\right]$$

or $$Y(z) - 0.1584z^{-1}\,Y(z) = 0.5792\,X(z) - 0.5792z^{-1}X(z)$$

Applying inverse z-transform:-

i.e. $$y(n) = 0.5792\,x(n) - 0.5792\,x(n-1) + 0.1584\,y(n-1)$$

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